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School of Economics
Australian School of Business
UNSW Sydney NSW 2052 Australia

http://www.economics.unsw.edu.au

Variable Capacity Utilization, Ambient Temperature Shocks and Generation Asset Valuation

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Variable Capacity Utilization, Ambient Temperature Shocks and Generation Asset Valuation

Chung-Li Tseng,* Wei Zhu,† and Alexandre Dmitriev‡

Abstract

This paper discusses generation asset valuation in a framework where capital utilization decisions are endogenous. We use real options approach for valuation of natural gas fueled turbines. Capital utilization choices that we explore include turning on/off the unit, operating the unit at increased firing temperatures (overfiring), and conducting preventive maintenance. Overfiring provides capacity enhancement which comes at the expense of reduced maintenance interval and increased costs of part replacement. We consider the costs and benefits of overfiring in attempt to maximize the asset value by optimally exercising the overfire option. In addition to stochastic processes governing prices, we incorporate an exogenous productivity shock: ambient temperature. We consider how variation in ambient temperature affects the asset value through its effect on gas turbine's productivity.

Keywords: Electricity generation asset valuation, overfire option, price uncertainty.

JEL Codes: D81, Q40

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*Australian School of Business, University of New South Wales, Sydney NSW 2052, Australia. Tel: +61 2 9385-9704. Fax: +61 2 9662-4061, Email: c.tseng@unsw.edu.au

†Corresponding author, First Choice Power Inc, 4100 International Plaza, Fort Worth, TX 76109, USA. Tel: +1 (817) 762-7576, Email:wei.zhu@firstchoicepower.com

‡Australian School of Business, University of New South Wales, Sydney NSW 2052, Australia. Tel: +61 2 9385-3351. Fax: +61 2 9313-6337, Email: a.dmitriev@unsw.edu.au

1 Introduction

The power industry has been undergoing major restructuring. Traditionally, electric utilities were responsible for long-term capacity expansion planning to ensure the adequacy of the generation capacity, anticipating expected growth in electricity demand. After the restructuring, capacity expansion is no longer the responsibility of the utilities. Instead, capacity choice becomes a pure investment decision based on the profit maximizing behavior of suppliers in the market. Therefore, valuing investments in generation assets is an important subject that may ultimately influence the sustainability of the capacity investments and adequacy. Such valuations, however, must account for various uncertainties, such as demand, price, and even environmental factors. Operational constraints, such as operating limits and/or flexibility, are also important factors that may affect the asset values. An important feature of this paper is that capital utilization decisions are endogenous. Typical to the macro-economic literature, these decisions are made by rational forward-looking optimizing agents.

The concept of variable capital utilization has a long tradition in economics. This notion is related to Keynes' concept of 'user cost'. According to Keynes (1936:69-70) "User cost constitutes the link between the present and the future. For in deciding his scale of production an entrepreneur has to exercise a choice between using his equipment now or preserving it to be used later on..." Variable capital utilization has enjoyed a degree of success in explaining a wide range of macroeconomic phenomena. Examples include studies of the Great Depression (Ohanian 2001), real business cycles (Greenwood et al. 1988), and international transmission of productivity shocks (Baxter and Farr 2005). Other examples include dynamic responses of macroeconomic aggregates to monetary policy shocks (Christiano et al. 2005) and asset pricing puzzles (In and Yoon 2007). Baxter and Farr (2005:336) attribute to this concept a greater role by claiming that "variable utilization of capital is believed to be of first-order importance to understanding business cycles".

Conventional modeling involves a reduced form relation known as depreciation-in-use technology. In this specification, utilization rate is a continuous decision variable. It controls the amount of capital services used in production and determines capital depreciation rate. The key parameter that governs the dynamics of such models is the

elasticity of marginal depreciation with respect to utilization rate. This parameter is notoriously difficult to quantify (see Baxter and Farr 2005, and references therein). For instance, Basu and Kimball's (1997) estimation from 21 manufacturing industries in the U.S. concludes that "data are not very informative". Unfortunately, quantitative implications of the models are often sensitive to the choice of this parameter. For instance, Baxter and Farr (2005) show that international factor co-movement puzzle can be resolved for some plausible electricity parameters but not for others.

Our approach to modeling variable capital utilization is different. Instead of relying on a reduced form relationship, we explicitly model the technological restrictions associated with variable capacity utilization. To do so, we narrow the focus to a specific sector of the economy: electric power generation. In particular, this paper focuses on the valuation of natural gas fueled turbines (GTs).

The reason for focusing on natural gas fueled turbines is twofold. First, among thermal power plants, GTs have gained increasing popularity to new market entrants because of its lower installment cost and shorter construction time. Second, GTs are more environmentally friendly than most steam turbines, which in the times of increased environmental concerns and government regulation makes then more attractive investment options.

Traditional economic valuations were done using the discounted cash flow (DCF) method (e.g., Sullivan et al. 2008). The DCF method assumes that the investment opportunity is now-or-never and irreversible. New information and future opportunities are overlooked in the DCF approach. Therefore, DCF often underestimates the value of investments (e.g., Trigeorgis 1996). What is overlooked by the DCF method is the real option value of investment strategies, referring to the profit that may be increased or risks that may be mitigated by flexibly exercising the right strategies at the right moments as new information emerges. Real options valuation methods take into account the value of flexibility embedded in real operational processes or activities[1]. In this paper, we use the real options approach to value a GT. Capital uti-

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[1]Real option valuation methods have been extensively applied in the electric industry. To name a few, Deng et al. (1999) used real options to value spark spread options; Tseng and Barz (2002) used the same concept to value operational flexibility of power plants; Siddiqui and Marnay (2006) valued distributed generation investment; and Davis and Owen (2003) determined an optimal R&D expenditure level for renewable electric projects.

lization choices that we are exploring, in addition to turning on/off the unit, includes operating GT at increased firing temperatures (overfiring) and the flexible timing of conducting preventive maintenance to the unit. Overfiring provides capacity enhancement which comes at the expense of reduced maintenance interval and increased costs of part replacement[2]. In this paper, the costs and benefits of overfiring are put into consideration in hope to increase the asset value by optimally exercising the overfire option.

For valuing generation assets, the new challenge offered by the restructured market is how individual market participants respond to the uncertain prices optimally by choosing the level of capital utilization. Thus far, most papers have focused on the operating constraints[3]. To the best of our knowledge, none of the existing studies considered the effect of maintenance on asset pricing. A new operating option (and constraint) discussed in this paper is the overfire process, which augments the production capacity of the unit. When market conditions are favorable for selling electricity, the overfire option could increase the profit. However, it has an adverse effect on profit through shortening the time to the next preventive maintenance. A maintenance requires a unit to be off-line for sometime during which no revenue is incurred. Therefore, an "optimal" strategy exists for exercising the overfire options, e.g., overfire during the higher price period and shutdown for maintenance during the lower price period. Our formulation includes both the overfire option and maintenance constraints as the new dimensions of variable capital utilization.

In addition to stochastic processes governing electricity and gas prices we incorporate an exogenous productivity shock: ambient temperature. We document how variation in ambient temperature affect productivity of GT and study the role variable capacity utilization as a transmission mechanism of exogenous shocks. From a computational perspective, such real option valuation problems are difficult to tackle.

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[2]There have been studies on various approaches to enhance the capacity of GTs, such as water injection, emulsion firing, increased firing temperatures (overfiring), etc. For example, a series of projects sponsored by the Electric Power Research Institute (EPRI) in the 90's have provided data to quantify the costs and benefits of the approaches for capacity enhancement (e.g., see EPRI Report 1993; also available from www.epri.com). Among them, overfiring provides attractive capacity enhancement and moderate improvement in heat rate.

[3]Operating constraints include capacity constraints, ramp constraints, and minimum up/down time constraints.

This paper has at least two unique contributions. First, we explicitly model variable capital utilization and study its implications for generation asset valuation. To the best of our knowledge, the overfire option and maintenance constraints included in this paper have not yet been considered in the asset pricing literature. Second, we introduce a novel productivity shock: ambient temperature. We formulate operational characteristics of a GT to reflect its dependency on ambient temperature. As a result, this model is capable of identifying how temperature variations affect the asset value. Furthermore, it can shed some light on how global and long-term climate change may impact asset values and investments.

This paper is organized as follows. In Section 2, we provide an overview of the capital utilization choices and technical constraints associated with them. Section 3 presents a dynamic asset pricing model with endogenous capital utilization for valuing GTs. Uncertainty models for the electricity price, natural gas price, and the environmental temperature are presented in Section 4. Numerical results are presented in Section 5. This paper concludes in Section 6.

2 Capacity Utilization Choices: Technological Constraints

In our framework, capital utilization decision involves discrete choice among the four alternatives: keeping GT off-line, normal operating mode, overfiring, and maintenance. Since the last two options are novel, they deserve particular attention. This section describes technological constraints and opportunities associated with overfiring and maintenance processes.

2.1 Overfire process

Consider the operation of a GT with a constant efficiency (or heat rate) H (MMBtu/MWh) in a single period at time t. Assume that p_t^E ($/MWh) and p_t^F ($/MMBtu) are the electricity and fuel prices, respectively, at time t. The GT generates q_t MW such that $q_t \leq q^{\max}$, where $q^{\max}$ is the maximum rated capacity of the

power plant. The profit due to generating at time t is as follows:

$$\text{Profit at time } t = (p_t^E - Hp_t^F)q_t \tag{1}$$

The term $(p_t^E - Hp_t^F)$ in (1) is called the *spark spread* of the power plant at time t. When the spark spread is positive, converting fuel to electricity is profitable. It is clear that to maximize the profit, q_t is set to be its maximum value $q^{\max}$.

There are two types of overfire process in thermal power generation industry. Both are employed to further increase the generating profit in (1). The first is to generate as much power as possible such that $q_t > q^{\max}$. This can be done by simply burning more fuel and is normally performed when the spark spread is high. While it may be used to generate more profit, it comes with a price because overfiring a unit can cause the temperature inside the combustor to be significantly higher than its normal value, based on which $q^{\max}$ was originally derived. Under this condition, the unit's hot section components are subjected to overstressing and high-temperature corrosion. If a unit is overfired for a prolonged period, further damages, such as degradation, deformation, and/or even cracks, may be incurred. Therefore, the duration for overfiring a unit must be restricted based on metal characteristics of the GT.

The second type of overfire is similar to fuel switching process. Consider an alternative fuel (with heat rate $H_1 < H$) that can release more energy than the primary fuel when it is ignited in the combustor but is considerably more expensive (with price $p_t^{F_1} > p_t^F$). When $Hp_t^F > H_1 p_t^{F_1}$, it is more profitable to burn the alternative fuel. If the operator can switch fuel optimally by tracking $p_t^{F_1}$ and p_t^F in real time, the profit function in (1) is modified by replacing the term Hp_t^F with $\min(Hp_t^F, H_1 p_t^{F_1})$. In this type of overfire, q_t may or may not exceed $q^{\max}$ depending on the amount of fuel burned and the temperature constraint of the combustor. To value the option of the second type of overfire one follows the approach used to value fuel-switching units (e.g., Kulatilaka 1993; Zhu and Tseng 2007), as long as the first type of overfire using a single fuel can be valued. Therefore, in this paper we consider the first type of overfire, which may be viewed as the generic type of overfire using one single fuel.

2.2 Maintenance process

When a GT has been overfired, maintenance is an efficient way to correct potential

damages caused by overfire. During a maintenance process, the following steps are taken: (1) let the unit cool down; (2) clean all working parts; (3) test hot section components; (4) replace ineffective ones with new parts; and (5) reassemble the unit and lubricate it.

These steps may take weeks to complete. After the maintenance, we assume that the GT can work efficiently and safely again. Note in reality the practice of overfire is normally subject to constraints defined by maintenance contracts (more details are discussed in Section 3.5). Such contracts keep track the number of overfire hours and the number of startups and shutdowns to determine when the next preventive maintenance should take place. In general, overfire shortens the time to the next preventive maintenance. In this paper we view overfiring a unit as a safe practice that, however, has restrictions and is closely monitored. As long as the restrictions are followed with opportune preventive maintenances, we assume that the reliability of the GT is warranted by the maintenance contracts. Therefore, we assume there is no long term effect for properly overfiring a unit, such as change of life and salvage value. The fact that overfiring a unit alters its maintenance schedule calls for optimization for exercising the overfire option with the optimal timing subject to the maintenance constraints.

3   Model

In this paper, a real options approach is used for valuing a GT. It is assumed that the operator is a price taker and has no market power to influence market prices. Therefore, the electricity and fuel prices are exogenous to the unit commitment decision models. Furthermore, the operator is risk neutral.

We consider an optimization problem with the objective to value the GT unit with overfire option and maintenance constraints subject to environment temperature and market uncertainties. Assume there are hourly spot markets for both electricity and fuel. The power plant purchases fuel from the fuel market, converts it to electricity, then sells it to the electricity market. At time t, in addition to turning on/off the GT (u_t), we also consider the overfire option (v_t) and the maintenance option (w_t). All three decision variables (u_t, v_t, w_t) are binary, with value equal to 1 representing

exercising the corresponding option and 0 otherwise. Three underlying uncertainties are considered: electricity price P_t^E, fuel price P_t^F, and the environmental temperature T_t^e at time t. This valuation involves a typical multi-stage stochastic optimization. Assume that at any time t and state $\mathbf{X}_t = (x_t, y_t, z_t)$, the uncertainty vector $\mathbf{Q}_t = (P_t^E, P_t^F, T_t^e)$ is observed. The state variables will be discussed in Section 3.2. The operator can realize the asset value in the current time period, then maximize the expected asset value for the remainder of the planning horizon by taking suitable actions $\mathbf{V}_t = (u_t, v_t, w_t)$. Assume a certain time horizon $[0, T]$ for the valuation. Let $J_t(\mathbf{X}_t; \mathbf{Q}_t)$ be the so-called value-to-go function at time t indicating the total asset value of the GT for the remaining period $[t, T]$ at state $\mathbf{X}_t$. The asset valuation problem can be formulated using the following recursive relation:

Maximize the expected total profit:

$$(P) \quad J_t(\mathbf{X}_t; \mathbf{Q}_t) = \max_{u_t, v_t, w_t} \{E_t[J_{t+1}(\mathbf{X}_{t+1}; \mathbf{Q}_{t+1})] + f_t(\mathbf{X}_t, \mathbf{V}_t; \mathbf{Q}_t)\}, \qquad (2)$$

where

$$f_t(\mathbf{X}_t, \mathbf{V}_t; \mathbf{Q}_t) \equiv (P_t^E q_t - C_t(q_t, P_t^F))u_t - S_t(u_t, x_t) - M^{\mathrm{mt}}w_t \qquad (3)$$

In (3), f_t represents the profit of the unit at t by selling power to a spot market less the generating cost C_t and other costs, if incurred, including startup cost S_t and maintenance cost M^{mt}. Note dispatching q_t may also be included in the decision vector $\mathbf{V}_t$. This issue will be discussed later.

The objective is to determine $J_0(\tilde{\mathbf{X}}_0; \tilde{\mathbf{Q}}_0)$ at $t = 0$, where $(\tilde{\mathbf{X}}_0, \tilde{\mathbf{Q}}_0)$ is the initial condition of $(\mathbf{X}_0, \mathbf{Q}_0)$. Problem (P) is also subject to a boundary condition at T.

$$J_T(\mathbf{X}_T; \mathbf{Q}_T) = 0, \ \forall \mathbf{X}_T, \mathbf{Q}_T \qquad (4)$$

Equation (4) indicates that there is no system value at the end of the planning horizon.

For a stochastic problem like (P), it is important to identify the sequence of the events. In this paper, decisions and observations are made only at the beginning of each hour. So there is no change of the system status between any two consecutive decision points. At time t, the state vector $\mathbf{X}_t$ is known and the uncertainty vector $\mathbf{Q}_t$ is observed, the operator will make a instant decision $\mathbf{V}_t$. This decision involves operating, say turning on/off, the GT. Since GT is a responsive unit, no decision lead

time is assumed for the decision to take effect. Namely, at time t after observing $\mathbf{X}_t$ and $\mathbf{Q}_t$, $\mathbf{V}_t$ is made and $\mathbf{X}_{t+1}$ is updated, all happening within the instant of time t. Therefore, more precisely $\mathbf{X}_t$ describes the GT's on/off status over $(t-1, t]$.

The details of problem modeling and constraints of the stochastic optimization problem (P) are described next.

3.1 Decision variables

Decision vector $\mathbf{V}_t$ contains three 0/1 decision variables, u_t, v_t, and w_t. They represent actions to be taken by the operator at time t, including to turn on/off the GT ($u_t = 1$ or 0); to overfire the GT or not ($v_t = 1$ or 0); and to perform maintenance or not ($w_t = 1$ or 0). Since a unit cannot overfire unless it is online, and maintenance can only be conducted when the unit is off-line, these three decision variables are not completely independent. Their interrelations can be described by the following two constraints.

$$u_t \geq v_t, \ \forall t \tag{5}$$

and

$$1 - u_t \geq w_t, \ \forall t \tag{6}$$

Equation (5) implies that if $v_t = 1$, then u_t must be 1. And similarly, from (6) if $w_t = 1$, then u_t must be 0.

These decision variables may be viewed as real options available to the operator at time t subject to exercise constraints.

3.2 State variables

State vector $\mathbf{X}_t$ at time t contains three elements, x_t, y_t, and z_t. State variable x_t is used to tracked how long the GT has been online or off-line at time t; y_t and z_t are used for maintenance purpose and will be discussed in a later section. The state transition of x_t is depicted in Figure 1; equivalently it is described in the following

equation.

$$
x_{t+1} = \begin{cases}
\min(t^{\mathrm{on}} + t^{\mathrm{over}}, \max(t^{\mathrm{on}}, x_t) + 1), & \text{if } u_t = 1 \text{ and } v_t = 1, \\
\min(t^{\mathrm{on}}, \max(x_t, 0) + 1), & \text{if } u_t = 1 \text{ and } v_t = 0, \\
\max(-t^{\mathrm{cold}}, \min(x_t, 0) - 1), & \text{if } u_t = 0 \text{ and } v_t = 0, \\
\max(-t^{\mathrm{cold}} - 1, \min(0, x_t) - 1), & \text{if } w_t = 1.
\end{cases}
\tag{7}
$$

Equation (7) shows that the state transitions are driven by the decision variables. Another perspective is to show feasible decisions (or options available) to the operator at different state x_t. For example, on the first column of nodes in Figure 1 corresponding to time t, no options are available at the states corresponding to $x_t = t^{\mathrm{on}} + t^{\mathrm{over}}$, 2, 1, -1, and -2, since the unit must remain on or off at those states. The other nodes have more than one arc incident from them, indicating the existence of real options. The mathematical descriptions of the options available at different states are as follows.

$$
u_t = \begin{cases}
1, & \text{if } 1 \le x_t < t^{\mathrm{on}} \text{ and } t^{\mathrm{on}} < x_t \le t^{\mathrm{on}} + t^{\mathrm{over}}, \\
0, & \text{if } -t^{\mathrm{off}} < x_t \le -1, \\
0 \text{ or } 1, & \text{otherwise.}
\end{cases}
\tag{8}
$$

$$
v_t = \begin{cases}
0, & \text{if } x_t = t^{\mathrm{on}} + t^{\mathrm{over}} \text{ or } x_t < t^{\mathrm{on}}, \\
0 \text{ or } 1, & \text{otherwise.}
\end{cases}
\tag{9}
$$

Equation (8) represents the so-called minimum uptime/downtime constraints, which means the unit must be online (off-line) for t^{on} (t^{off}) consecutive hours before it can be turned off (on). Equation (9) represents the overfire constraints, which states that the unit can be overfired only after the minimal uptime constraint has been satisfied, i.e., after at least t^{on} hours of normal operation. This restriction can prevent significant temperature variation within a short time period due to overfire. In addition, the unit cannot be overfired for more than t^{over} hours continuously. When it is turned back to normal operation status from overfire, it must remain normal operation for at least an hour before it can be overfired again.

Once a maintenance decision $w_t = 1$ is made at time t, the unit goes into a maintenance interval with a duration t^{mt} time periods. That is, the unit must stay off for at least t^{mt} time periods (approximately two weeks in reality). That is, if $w_t = 1$ and $w_{t-1} = 0$, then $x_{t+t^{\mathrm{mt}}} = -t^{\mathrm{cold}}$.
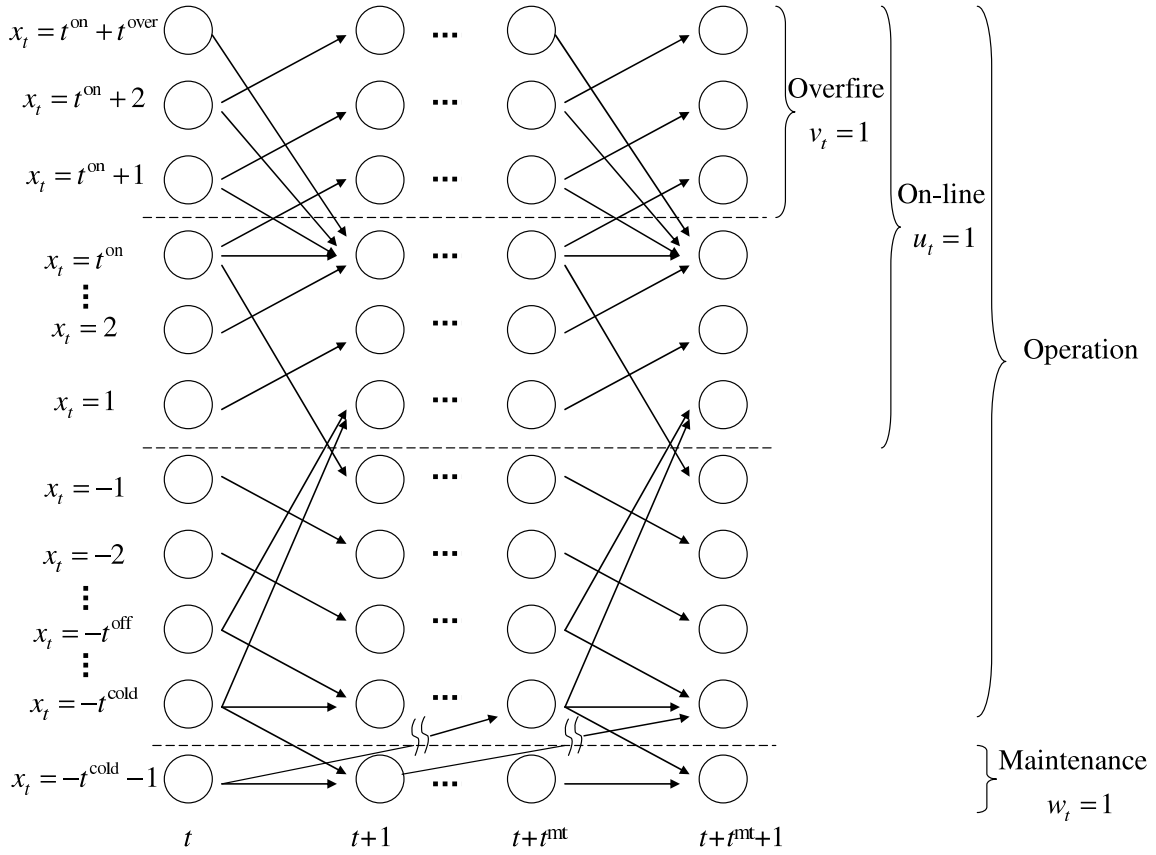
Figure 1: State transition diagram of x_t including operation and maintenance processes.

3.3 Cost model

The costs considered in this paper include the fuel cost C_t, startup/shutdown costs S_t, and maintenance cost M^{mt}. The maintenance cost will be discussed in a later section for maintenance contracts. Fuel cost is modeled as a convex quadratic function as follows (e.g., Wood and Wollenberg 1996), which is more realistic than the constant heat rate used in (1).

$$C_t(q_t, P_t^F) = \beta_c^{\mathrm{e}}(T_t^{\mathrm{e}})(c_0 + c_1 q_t + c_2 q_t^2) P_t^F \tag{10}$$

$$\beta_c^{\mathrm{e}}(T_t^{\mathrm{e}}) = 1 + \left(\frac{T_t^{\mathrm{e}} - T^{\mathrm{d}}}{\Delta T^{\mathrm{d}}}\right) R_c \tag{11}$$

In (10), q_t is the generation level and c_0, c_1, and c_2 are all positive parameters. Furthermore, we consider the impact on the fuel cost $C(q_t, P_t^F)$ of the environment temperature T_t^{e} at time t. In reality, GT's performance is dependent on environment temperature. The generation capacity (see next section) and fuel consumption vary as the environment temperature changes. In (11), the cost variation due to temperature is captured by an adjusting factor $\beta_c^{\mathrm{e}}(T_t^{\mathrm{e}})$, which is a function of the temperature T_t^{e} at time t. If the GT operates at its designed operating temperature T^{d}, i.e., $T_t^{\mathrm{e}} = T^{\mathrm{d}}$, then the adjusting factor $\beta_c^{\mathrm{e}}(T_t^{\mathrm{e}}) = 1$. When the environment temperature deviates from T^{d}, the adjusting factor β_c^{e} will deviate from 1 linearly. When the deviation reaches ΔT^{d}, the adjusting factor deviates from 1 by R_c, where the parameter R_c is a constant between 0 and 1. In general, a GT performs better in a cooler environment.

The startup/shutdown costs are the costs associated with turning on or off the GT (e.g., the labor cost) and are assumed constant in this paper.

$$S_t(u_t, x_t) = \begin{cases} S^{\mathrm{up}}, & \text{if } u_t = 1 \text{ and } x_t < 0 \\ S^{\mathrm{down}}, & \text{if } u_t = 0 \text{ and } x_t > 0 \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

3.4 Generating capacity and overfire

A GT has rated minimum and maximum generating capacities q^{min} and q^{max} for normal operations. Namely, during normal generating operations, the power output should always be within these two levels. However, q^{max} *per se* is a *soft* limit and may

be exceeded if necessary. The scenario that q^{max} is exceeded is called overfire. During the overfire process, q^{max} is stretched to q^{over} ($> q^{\mathrm{max}}$) so that the GT can generate an additional 10% to 20% power above q^{max}. Overfire is particularly useful when the system capacity runs short or the market spark spread is high, indicating converting fuel into electricity is highly profitable. As already mentioned, the benefit of overfire comes with a price: more frequent maintenance may be needed. Detailed formulae for determining the need for maintenance are given in the next section.

Like the fuel cost function, q^{max} is not a constant and may vary from time to time depending on environment temperature. The capacity constraint is modeled below in a similar way to (10).

$$q^{\mathrm{min}}u_t \le q_t \le (q^{\mathrm{max}}(u_t - v_t) + q^{\mathrm{over}}v_t)\beta_q^{\mathrm{e}}(T_t^{\mathrm{e}}), \tag{13}$$

where

$$\beta_q^{\mathrm{e}}(T_t^{\mathrm{e}}) = 1 - \left(\frac{T_t^{\mathrm{e}} - T^{\mathrm{d}}}{\Delta T^{\mathrm{d}}}\right)R_q \tag{14}$$

Equations (13) and (14) state the maximum rated capacity can be increased from q^{max} to q^{over}. Regardless of overfire, both q^{max} and q^{over} are dependent on the environment temperature T_t^{e}. For instance, when T_t^{e} increases, both q^{max} and q^{over} decrease. Again, R_q in (14) is a constant between 0 and 1.

The dispatch problem of the GT is to determine the generation level q_t at time t when $u_t = 1$. At time t, after observing the uncertainties, P_t^E, P_t^F and T_t^{e}, q_t is determined so as to maximize the profit incurred at time t.

$$q_t^* \equiv \arg\max\{P_t^E q_t - C_t(q_t, P_t^F) \mid (13)\} \tag{15}$$

Therefore, a necessary condition for overfiring a unit is that $q_t^* > q^{\mathrm{max}}$.

The optimization problem described in (15) is a simple convex quadratic optimization. Because of the responsiveness of GTs, we also assume that once a GT is turned on it can be instantly dispatched to the optimal level q_t^* in (15), which is an implicit function of the uncertainties observed at time t, $\mathbf{Q}_t$. This explains why q_t was not included in the decision vector $\mathbf{V}_t$.

3.5   Maintenance contract

Proper preventive maintenance ensures a GT to function efficiently and reliably.

A GT owner normally outsources maintenance to avoid staffing full-time crews for maintaining the units. This is more economical especially for the owners of small GTs. This outsourcing involves signing maintenance contracts with some maintenance companies, who are responsible for on-site unit maintenance.

A complete maintenance contract is normally complicated. The contract considered in this paper is a simplified version of a real maintenance contract. Since the purpose of maintenance is to prevent early fatigue or outage of a unit, normally caused by frequent ups/downs and/or quick temperature changes of the unit, the contract keeps track of the aggregated unit operation hours (y_t) and unit startup numbers (z_t), to be defined next. A unit must be shut down for maintenance if either y_t or z_t exceeds some prespecified level N^{op} or N^{start}, respectively.

$$w_t = \begin{cases} 1, & \text{if } y_t \geq N^{\mathrm{op}} \text{ or } z_t \geq N^{\mathrm{start}}, \\ 0 \text{ or } 1, & \text{otherwise.} \end{cases} \tag{16}$$

If the operator operates the GT in compliance with (16), then reliable operation is warranted by the contracts. Therefore, in this paper no outage possibility is considered for the GT, as a result of enforcing the maintenance contracts. According to the contracts, the operator agrees to pay a fixed sum of maintenance cost M^{mt} at each maintenance service. Therefore, the "more" one uses the GT (measured by y_t and z_t), the more frequent maintenance is needed. How to track y_t and z_t to measure the usage of the GT is further explained below.

- The aggregated operation hours is tracked by y_t, whose initial value is 0. While a *normal operation hour* is counted one hour, an *overfire hour* referring to one in which the unit overfires is counted as $W > 1$ hours. This is because that overfiring a unit causes more fatigue than normal operation. Therefore,

$$y_{t+1} = y_t + (Wv_t + (1 - v_t))u_t. \tag{17}$$

- The aggregated startup numbers is tracked by z_t, initially set to 0. Similar to y_t, different levels of startups are categorized depending on the temperature of turbines. They are: warm start, normal start, cold start, and very cold start. Generally, the longer a unit has been shutdown, the lower the temperature of the turbine is. As already mentioned, the higher the temperature variation is,

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the more fatigue is caused. Therefore, a warm startup causes less temperature variation and fatigue than a normal startup, than a cold startup, and than a very cold startup. The following is merely an example how the startup number is considered, depending on how long the unit has been down ($x_t < 0$).

$$n_t(x_t) = \begin{cases} 0.5, & \text{if } x_t > -4 \text{ (warm start)}, \\ 1, & \text{if } -4 \geq x_t > -20 \text{ (normal start)}, \\ 1.5, & \text{if } -20 \geq x_t > -40 \text{ (cold start)}, \\ 2, & \text{if } x_t \leq -40 \text{ (very cold start)}, \\ 0, & \text{otherwise.} \end{cases} \tag{18}$$

In (18), a warm start is considered a half normal start if the unit has been down for less than 4 hours, and a cold start (down for 20 to 40 hours) is considered one and a half normal start. Overall, the startup number is aggregated as follows.

$$z_{t+1} = z_t + n_t(x_t)(1 - u_{t-1})u_t, \tag{19}$$

Both y_t and z_t will be reset to 0 after a maintenance is performed.

4   Uncertainty Generating Processes

Three underlying uncertainties are considered in this paper. They are two price uncertainties, P_t^E and P_t^F, for the market prices of electricity and fuel, and the environmental temperature T_t^{e}. They are further discussed in the following sections.

4.1   Temperature model

This section presents the model of temperature variation over time, since GT's operational characteristics, including fuel consumption and generating capacity, are sensitive to the environment temperature T_t^{e}.

Recent studies have suggested that daily average temperature in US cities can be modeled using time series approach (Campbell and Diebold 2005). Let T_t^{e} be the actual temperature for day t; $\bar{T}_t^{\mathrm{e}}$ be the average temperature for day t; and $\Delta_t^T \equiv T_t^{\mathrm{e}} - \bar{T}_t^{\mathrm{e}}$. Cao and Wei (2000, 2004) and Baldick et al. (2006) have all suggested

to model daily temperature deviation Δ_t^T as a k-lag autoregressive process in terms of $\Delta_t^T, \Delta_{t-1}^T, \cdots, \Delta_{t-k+1}^T$. Particularly, Cao and Wei (2000) suggested that on day t the future temperature deviation on day $t+1$ (from the average temperature of that day) can be forecasted based on temperature deviations over the three previous days, t, $t-1$ and $t-2$ (i.e., $k=3$).

In our study, we collected historical weather data from Maryland to focus on the market of PJM (Pennsylvania-New Jersey-Maryland). Following the general k-lag autoregressive process described above, our analysis found that the temperature deviations on $t-2$ were statistically insignificant to forecast the future temperature deviations on $t+1$ (i.e., $k=2$). Our finding coincided with that reported by Baldick et al. (2006), who used historical weather data from central Texas. The temperature model is as follows.

$$\Delta_{t+1}^T = \alpha_1 \Delta_t^T + \alpha_2 \Delta_{t-1}^T + \gamma_t \epsilon_t \tag{20}$$

$$\gamma_t = \gamma_0 - \gamma_1 |\sin(\pi(t + \phi)/365)|, \tag{21}$$

where ϵ_t is a standard normal random variable; α_1 and α_2 represent the autocorrelation coefficients for deviations from average temperature on day t and $t-1$, respectively. Equation (21) measures the magnitude of the random fluctuations, which is seasonal with a fixed term γ_0 and a seasonal term of magnitude γ_1. The phase ϕ of the sinusoid in (21) is a constant.

4.2 Stochastic price process

In this paper, it is assumed that the prices for electricity P_t^E and fuel P_t^F follow some geometric mean reverting (MR) processes governed by the following stochastic differential equations.

$$d \ln P_t^E = -\eta^E (\ln P_t^E - m_t^E) dt + \sigma^E dB_t^E \tag{22}$$

and

$$d \ln P_t^F = -\eta^F (\ln P_t^F - m_t^F) dt + \sigma^F dB_t^F \tag{23}$$

In (22) and (23), η^E and η^F are reverting coefficients; m_t^E and m_t^F are the mean levels of electricity and fuel prices at time t, respectively; σ^E and σ^F are constant volatilities; and B_t^E and B_t^F are two Wiener processes with correlation ρ. Such mean

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reverting models have been commonly used in representing energy price movements (e.g., Barz 1999; Tseng and Barz 2002; Tseng and Lin 2007).

5   Numerical Results

The problem formulation (P) given in Section 3 is a difficult mixed-integer multi-stage stochastic program. We used the least squares Monte Carlo (LSMC) method initially proposed by Longstaff and Schwartz (2001) to solve the problem (P). Using Monte Carlo simulations to generate scenarios of the evolution of the underlying uncertainties, the conditional expectation at the right-hand side of (2) can be estimated by least squares regressions. This estimation is repeated and is integrated with dynamic programming iterations moving backward in time to obtain the asset value at time 0. For details of the algorithm using LSMC for valuing the overfire option of a GT, the interested reader is directed to Zhu (2004).

5.1   Test system parameters

Consider a small-sized GT with the input-output characteristics following (10) and (11). Based on real data, the following parameters are devised: $c_0 = 200$, $c_1 = 8.149$, and $c_2 = 0.00452$. The generating capacities of the GT are: $q^{\min} = 75\text{MW}$, $q^{\max} = 200\text{MW}$, $q^{\text{over}} = 230\text{MW}$. We also assume that $t^{\text{on}} = 2$, $t^{\text{off}} = t^{\text{cold}} = 1$, and $t^{\text{over}} = 1$ to fully capture the influence of the physical constraints. Let the startup cost be \$1000 and shutdown cost be \$500. Assume the designed operating environment temperature T^{d} to be 66°F, and the allowable operating temperature range ΔT^{d} to be 60°F. The adjusting factors for both fuel cost R_c and generating capacity R_q due to temperature variation are both set to 2%.

Assume the current electricity price P_0^E is \$20/MW and the natural gas price P_0^F is \$2.2/MMBtu. Hourly electricity prices and gas prices are generated by two mean-reverting processes using (22) and (23). The reverting coefficients are $\eta^E = 0.072$ and $\eta^F = 6.95 \times 10^{-4}$; the volatilities are $\sigma^E = 0.27$ and $\sigma^F = 0.019$ for the logarithms of the electricity and natural gas prices, respectively. The mean-reverting process for the electricity price considers a daily price pattern. The 24 hourly mean levels m_t^E are summarized in Table 1, which captures the cyclical nature of the expected

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Table 1: Values of hourly m_t^E for $\ln P_t^E$

t	1	2	3	4	5	6	7	8
m_t^E	1.887	2.656	1.935	2.340	3.503	3.857	3.758	4.660
t	9	10	11	12	13	14	15	16
m_t^E	4.861	4.710	5.811	4.736	5.044	5.738	5.917	4.713
t	17	18	19	20	21	22	23	24
m_t^E	3.723	1.457	1.322	2.511	3.617	0.645	1.603	1.833

electricity prices. For the natural gas prices, $m_t^F = 1.0195$ is set to a constant for all t because there are no hourly markets for natural gas. Furthermore, we assume that the correlation coefficient ρ between electricity and gas prices is 0.4, as observed in the markets. The parameters of the price processes used here are consistent with those in Tseng and Barz (2002).

For the maintenance contracts, the maximum number of operation hours $N^{\text{op}} = 1600$ hours and the maximum startup numbers is $N^{\text{start}} = 120$ between any two maintenance intervals. Assume $W = 4$, i.e., an overfire hour is equivalent to 4 normal operation hours. The startup number is measured using (18). Assume the duration of each maintenance interval T^{mt} is two weeks, i.e., 336 hours. Each time the maintenance is carried out costs $M^{\text{mt}} = \$5,000$.

Following the model described in (20) and (21), we have collected weather data (from January 2000 through December 2002) from the National Climatic Data Center website (www.ncdc.noaa.gov) in the area of Maryland, and use them to calibrate the model. The parameter values for α_1, α_2, γ_1, γ_2, and ϕ are obtained via standard statistical analysis. We further categorize γ_1 into seasonal values. The estimated values of the parameters are summarized in Table 2.

5.2 Valuing a GT with overfire capacity and maintenance contract

Use the LSMC approach and the uncertainties models calibrated in the previous section, we value the GT over a one year (8760 hours).

Table 2: Values of the parameters of the temperature model

Parameters	Value
α_1	0.831
α_2	-0.194
γ_0	8.322
γ_1 (Spring)	5.697
γ_1 (Summer)	5.774
γ_1 (Fall)	5.741
γ_1 (Winter)	5.753
ϕ	-14.2

Asset value vs. time T

Next we consider the capacity value of the GT vs. the length of the time horizon T, ranging from one month to one year. The result is depicted in Figure 2. It can be seen that the capacity value of the GT with overfire capacity and maintenance contract is around \$150/KW for one year. In general, the capacity value increases with the length of time horizon T monotonically and approximately linearly. From Figure 2, it can be seen that the relation is bumpy when T is between 3 and 7 months. This is because a two-week maintenance is incurred within this time period. Since the unit has to be off-line during the maintenance interval, it impacts the overall asset value and creates the bumpiness of the seemingly linear relation when T is not big. However, as T exceeds 8 months, which is considerably longer than the duration of the maintenance interval, the asset value returns to an approximately linear function of T.

Asset value vs. price process parameters

Next we conduct sensitivity analysis to observe how the capacity value of the GT changes with the price parameters, including the reverting coefficients (η^E and η^F) and volatilities (σ^E and σ^F). The asset value vs. the change of reverting coefficients is illustrated in Figure 3. Although both relations (value vs. η^E and value vs. η^F)
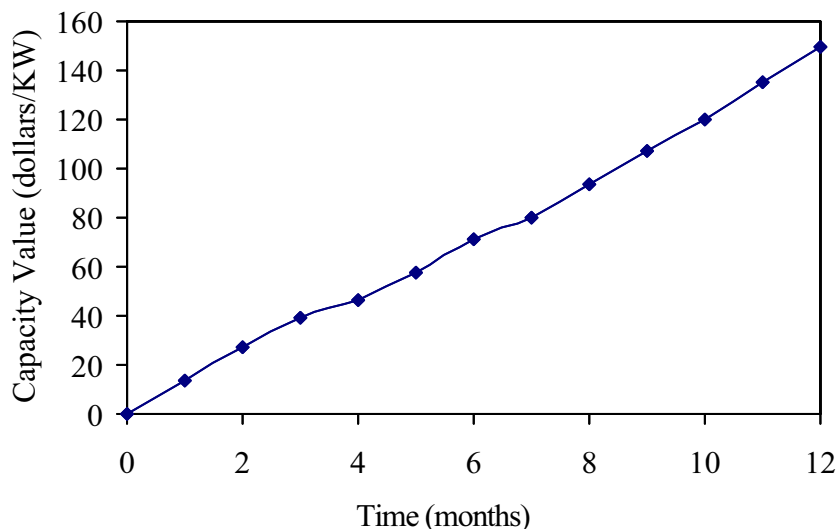
Figure 2: Capacity value of the GT over time.

are depicted in the same figure, only one parameter is changed at a time with all others fixed at their baseline values given Section 5.1. Both of the changes of the asset value and parameter value are measured based on percentage deviated from $150/KW (with $T = 1$ year) and the baseline value, respectively. It can be seen from Figure 3, the asset value decreases as the value of the reverting coefficient increases. The result can be interpreted as follows. For a MR process, a price deviated from the mean is deemed temporary and is subjected to a reverting force to move the price back towards the mean. The bigger the reverting coefficient is, the stronger the reverting force is. When a reverting coefficient is bigger, it also means any price deviation from the mean lasts shorter and, therefore, there are fewer profitable opportunities due to the price deviation. Therefore, the asset value decreases as the reverting coefficient increases. The same arguments applies to both the reverting coefficients of the electricity price and the fuel price. On the other hand, when the value of η^E continues to increase, the decrease of the asset value eventually stops. This may be interpreted as that the electricity price becomes easier to forecast.

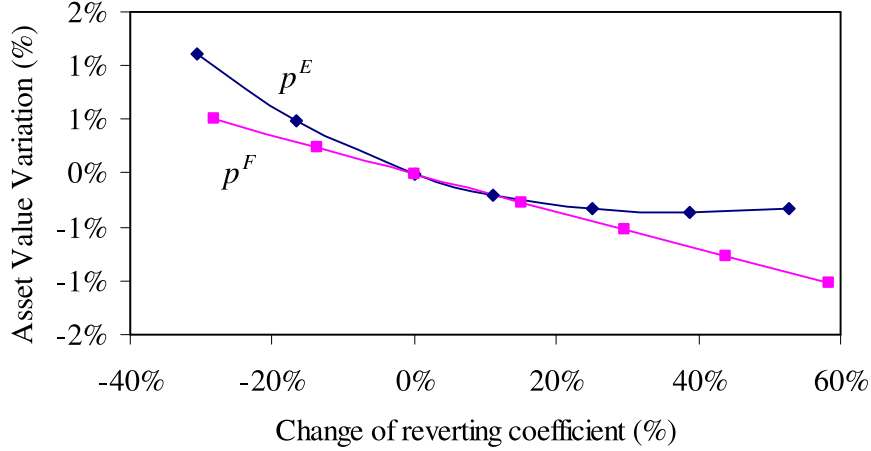In Figure 4, we show the sensitivity analysis of the asset value vs. the price

Figure 3: Capacity value of the GT vs. change of reverting coefficients.

volatilities. When σ^E increases, the asset value increases. Since the GT makes profits by exploring opportunities of big price spread between the electricity and the fuel, a bigger σ^E makes such profitable opportunities more likely to occur. According to Figure 4, the asset value is insensitive to the volatility of the fuel price. This is due to the fact that the value of σ^F remains to be small even after perturbation.

Asset value affected by environment temperature

As mentioned, a GT's operational characteristics (including fuel cost function and generating capacities) are sensitive to the environment temperature T_t^e. To test how the asset value is sensitive to the environment temperature, we design a counterfactual case, in which the environmental temperature is stationary maintained at the recommended operating temperature T_t^d for all times. Therefore, $\beta_c^e = \beta_q^e = 1$ for the counterfactual case, which is then compared with two cases with uncertain temperatures following the proposed temperature model. For simplicity, we choose T to be 8 weeks to exclude the effect of maintenance. One test case uses the temperature model for the summer and another for the winter. Generally, $T_t^e > T_t^d$ in the summer. Therefore, both q^{max} and q^{over} are smaller and the asset value is also lower than the values in the counterfactual case. The situation is reversed in the winter case. The result of the summer case is depicted in Figure 5. It can be seen that as $T = 8$, temperature uncertainty accounts for approximately 2.2% decrease of the asset value (compared with the baseline.) The result of the winter case is given in Figure 6,
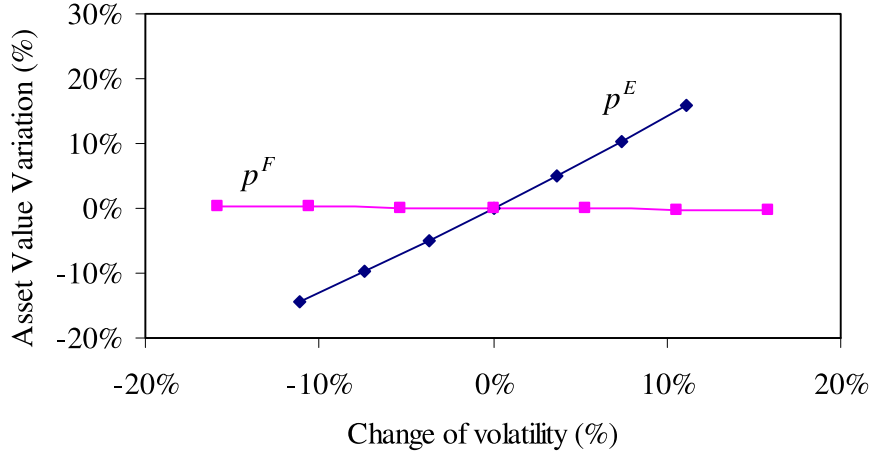
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Figure 4: Capacity value of the GT vs. change of volatilities.

where the asset value is higher than that in the counterfactual case.

5.3 Overfire option value

To extract the value of the overfire option, we introduce an additional constraint that limits the overall number of overfire hours. First, let o_t be a new state variable that tracks the aggregated overfire hours, which is reset to 0 after a maintenance is performed.

$$o_{t+1} = o_t + v_t \tag{24}$$

An upper bound N^{over} is then imposed to o_t such that the unit cannot overfire at time t if o_t exceeds N^{over}.

We then run the model for determining the asset value ($T = 1$ year) repeatedly with the value of N^{over} increased gradually. The capacity value of the GT vs. N^{over} is depicted in Figure 7. Initially, the asset value increases as N^{over} increases from 0, since overfire a real option that has value. When the value of N^{over} equals 100 hours, the asset value reaches a maximum. After that, the value stops increasing even N^{over} increases. Therefore, $N^{\text{over}} = 100$ hours (or roughly one hour every three days) can be viewed as the optimal number of overfire per year. If one continues to
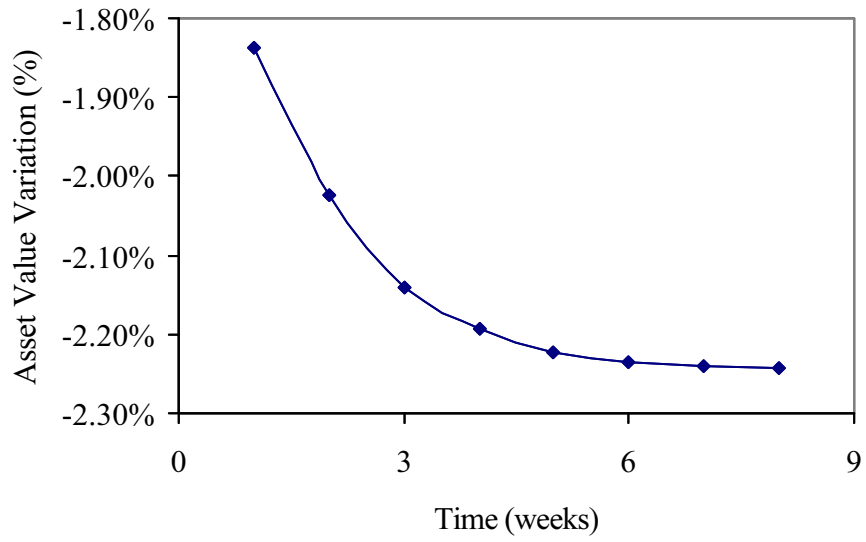
22

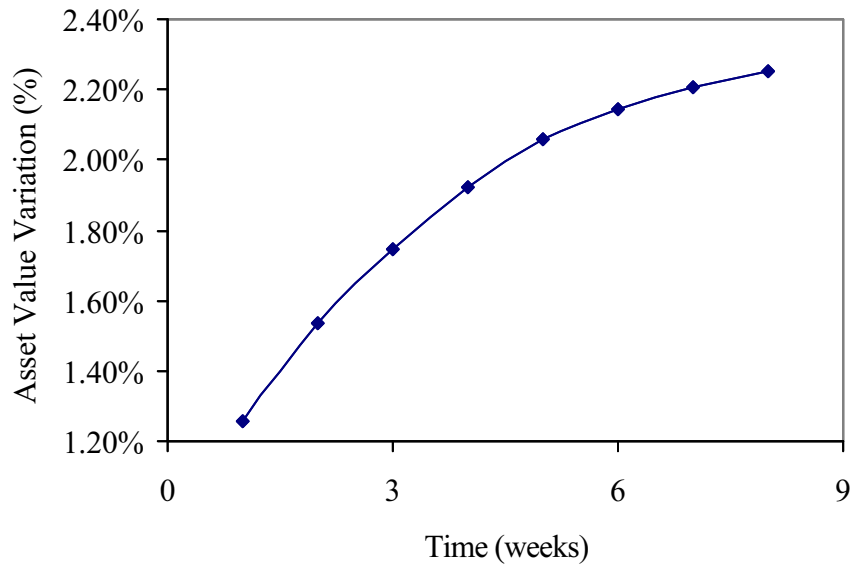Figure 5: Asset value decreased in summer due to temperature.

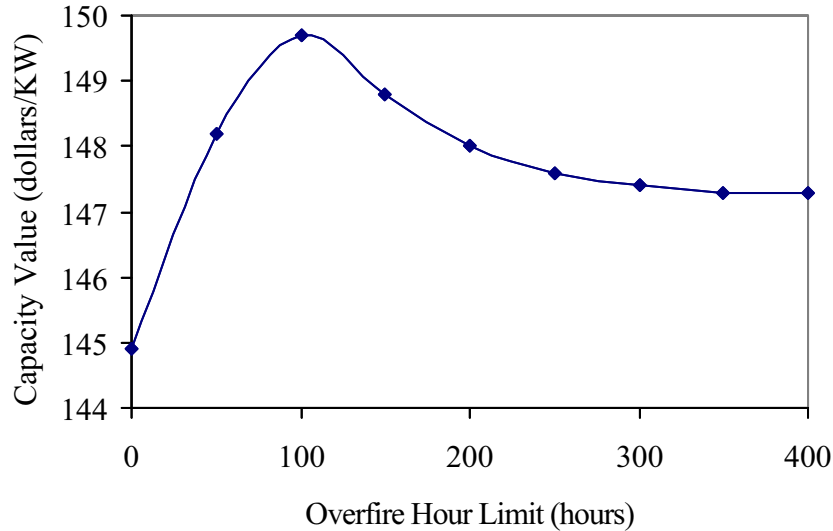Figure 6: Asset value increased in winter due to temperature.

Figure 7: Capacity value affected by overfire limit.

increase N^{over}, the asset value converges because the imposed constraint with N^{over} is no longer binding. Overfiring the GT less than 100 hours is not optimal. On the other hand, overfiring the unit more than 100 hours is not economical, because the maintenance costs outweigh the benefit.

5.4 Maintenance option value

Since maintenance is necessary and required, it may not be perceived to have options. Note that (16) states that maintenance is required by the contracts if at least one of the two maintenance conditions is met, $y_t \geq N^{\text{op}}$ and $z_t \geq N^{\text{start}}$. The same equation, however, does not necessarily imply that maintenance should not take place even when none of these two conditions is met. Clearly, there is an option of flexibly determining the maintenance timing. To measure the value of such an option, we manage to take away this option by modifying (16) such that maintenance is performed only when it is absolutely necessary, i.e., at least one of y_t and z_t has reached its upper limit. We then run the model to determine the asset value (with

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$T = 1$ year) and compare it with the original asset value with (16). The capacity value of the GT is 146.7 \$/KW without the maintenance option, and was 149.8 \$/KW with the option. That is, approximately 2 % of the capacity value can be attributed to the flexible timing of performing necessary maintenance. This option value can also be viewed as the value of flexibility in timing of maintenances.

6   Discussion and Conclusion

In this paper, we develop a generation asset valuation framework where capital utilization decision is endogenous. In particular, we apply real option methods for valuation of natural gas turbines considering overfire option, maintenance constraints, and environment temperature. Numerical results show that the overfire option can be optimally exercised to increase the asset value. This proposed model is also capable of identifying how temperature variations affect the asset value, which has not been tackled in the literature.

Recently, there has been a growing concern about global warming, especially more frequent weather events such as heat wave. The proposed model shows that temperature increase and changing patterns of weather uncertainty will impact the asset value. Our study, however, focuses on an individual's problem by considering the asset value of a price-taking GT. One interesting extension is to consider systemic effects due to global warming, e.g., its impact on both supply and demand. For example, as shown in this paper the change in ambient temperature would potentially affect performance of all suppliers generating power using thermal systems. This would potentially shift the supply-curve of the producers. On the demand side, one possible scenario is that the temperature under extreme weather condition could cause sharper increase in peak demand. In this scenario, both supply and demand effects would seem to make the price higher. Certainly, there are many other factors to consider, such as other weather scenarios and patterns, fuel price, competition, and market mechanism. Nevertheless, the proposed model has the potential to be expanded to include more global and long-term climate change uncertainties, which may have significant impact to the asset values. Overall, the proposed model and the numerical results provide some new insights in the valuation and operation of GTs.

Acknowledgment

Appendix A. Nomencla

A1. List of abbreviations used in the text

DCF   discounted cash flow

GT   Gas turbine

LSMC   Least squares Monte Carlo

MR   Mean reverting

A2. List of symbols used in the model formulation

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t time, $t = 1, \cdots, T$.

Parameters

T the number of hours of the planning horizon.

H, H_1 heat rate of a GT

t^{on} the minimum number of hours the unit must remain on after it has been turned on before it can be turned off.

t^{off} the minimum number of hours the unit must remain off after it has been turned off before it can be turned back on.

t^{cold} the number of hours required to cool the unit from shutdown.

t^{over} maximum number of hours the unit can be overfired continuously.

q^{min} minimum rated capacity of the unit (without overfire).

q^{max} maximum rated capacity of the unit (without overfire).

q^{over} maximum rated capacity of the unit when it is overfired ($q^{\mathrm{over}} > q^{\mathrm{max}}$).

P_t^E spot price at time t for electricity.

P_t^F, P_t^{F1} spot price at time t for fuel.

T_t^{e} environment temperature at time t.

T^{d} designed operating temperature of the gas turbine.

ΔT^{d} designed operating temperature range of the gas turbine.

$\beta_c^{\mathrm{e}}(T_t^{\mathrm{e}})$ adjusting factor for fuel cost when the environment temperature is T_t^{e} at time t.

$\beta_q^{\mathrm{e}}(T_t^{\mathrm{e}})$ adjusting factor for q^{max} when the environment temperature is T_t^{e} at time t.

R_c adjusting factor for fuel cost due to environment temperature variation.

R_q adjusting factor for generating capacity due to environment temperature variation.

$n_t(x_t)$ equivalent startup number at state x_t used by the maintenance contract.

t^{mt} the minimum number of periods the unit must be shutdown for maintenance.

M^{mt} direct maintenance cost which are determined by the maintenance contracts.

N^{op} the maximal aggregate operating hour between any two maintenance intervals, defined by the maintenance contracts.

N^{start} the maximal aggregate number of startups between any two maintenance intervals, defined by the maintenance contracts.

$C_t(q_t, P_t^F)$: fuel cost for operating the unit at output level q_t with fuel price at P_t^F at time t.

$S_t(u_t, x_t)$: startup/shutdown cost associated with turning on/off the unit at time t.

$M_t(y_t, z_t)$: maintenance cost associated with the unit in time period t.

$\mathbf{Q}_t$ an uncertainty vector including P_t^E, P_t^F, and T_t^e indicating all the uncertainties at time t.

$\tilde{\mathbf{Q}}_0$ initial condition of $\mathbf{Q}_t$ at $t = 0$.

Variables

u_t zero-one decision variable indicating whether the unit is to be turned on or off at time t.

v_t zero-one decision variable indicating whether the unit is to be overfired or not at time t.

w_t zero-one decision variable indicating whether the unit is to be shutdown for maintenance or not at time t.

x_t state variable indicating how many hours the unit has been turned on $(x_t > 0)$ or off $(x_t < 0)$ by time t.

y_t state variable indicating the total number of hours at time t that the unit has been overfiring since the most recent maintenance.

z_t state variable indicating the total startup numbers at time t since the most recent maintenance.

q_t variable indicating the amount of power that the unit generates at time t.

$\mathbf{X}_t$ a state vector including x_t, y_t and z_t indicating the status of the unit at time t.

$\mathbf{V}_t$ a decision vector including u_t, v_t and w_t indicating the decisions of the unit at time t.

$\tilde{\mathbf{X}}_0$ initial condition of $\mathbf{X}_t$ at $t = 0$.

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