Credit, Vacancies and Unemployment Fluctuations*

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Abstract

Propagation in equilibrium models of search unemployment is significantly altered when vacancy costs require some external financing on frictional credit markets. The easing of financing constraints during an expansion reduces the opportunity cost of resources allocated to job creation, raising the elasticity of market tightness through (i) a cost channel, increasing incentive to recruit for a given benefit from a new hire; (ii) a wage channel, whereby an improved bargaining position of firms limits the upward pressure of market tightness on wages. The calibrated model can match the volatility and persistence of market tightness observed in the data.

Keywords: Vacancies and unemployment dynamics, search and matching, credit market frictions

JEL classification: E32, E44, J63, J64

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1 Introduction

The standard Mortensen and Pissarides (1994) search and matching model of equilibrium unemployment has been argued in many places to be inconsistent with key business cycle facts. In particular, it cannot explain the high volatilities of unemployment, vacancies and market tightness (Shimer, 2005), nor the persistence in the adjustment of these variables to exogenous shocks (Fujita and Ramey, 2007). Subsequent research has focused on whether the lack of internal propagation, both in terms of amplification and persistence, stems from the structure of the model itself or whether it is a question of setting an appropriate calibration.

Firms in these models must expend resources to fill job vacancies, a time-consuming process in the presence of search frictions on labor markets. Under Nash bargaining as a wage mechanism, wages absorb much of the change in the expected benefit to a new worker induced by fluctuations in labor productivity. As a result, Shimer (2005) argues, the incentive to post vacancies changes little over the business cycle. Quite naturally, subsequent research has focused on the dynamics of wages as a means of generating amplification of exogenous innovations. Such studies have either altered the particulars of the wage determination mechanism (e.g. Shimer 2004), or as in Hagedorn and Manovskii (2008), followed an alternative calibration strategy that results in a rigid wage.[1] In order to address the second empirical shortcoming, the persistence in labor market adjustments to productivity shocks, a second strand of research has focused on the structure of vacancy costs. Fujita and Ramey (2007), for example, develop a story about sunk costs to vacancy creation such that the strongest change in market tightness occurs several periods after the original shock.

[1] Examples of alternate wage determination include a demand-game auction (Hall, 2005) or staggered wage contracting (Gertler and Trigari, 2009). In essence, the parametrization in Hagedorn and Manovskii (2008) of the value of non-market activities and the relative Nash bargaining weight ensures that the wage is highly inelastic to its time varying components, i.e. labor productivity and the degree of market tightness.

Their approach, however, does not generate any additional amplification.[2]

This paper extends the baseline search and matching model of equilibrium unemployment by assuming that external finance must be called upon to fund part of a firm's vacancy costs, and that agency problems cause credit markets to be frictional. The thrust of this paper is to show that evolving conditions on credit markets over the business cycle change the opportunity cost of resources used by firms to create new jobs in the face of small changes in the expected benefit to a new worker, simultaneously addressing the lack of amplification and persistence to productivity shocks outlined above.[3] Acemoglu (2001) and Wasmer and Weil (2004) have shown that credit market imperfections lead to higher equilibrium unemployment by restricting firm entry.[4] This paper shows that such frictions matter for the cyclical dynamics of the labor market. This paper also raises a broader case for the role credit market imperfections in understanding aggregate dynamics operating through worker as opposed to investment flows, as has been the focus in models of financial intermediation and agency costs such as Kiyotaki and Moore (1997) or Bernanke et al. (1999).

The model developed in this paper works as follows. Due to a problem of costly state verification in lending relationships, firms write standard debt contracts (Gale and Hellwig, 1985, Williamson, 1987) to fund vacancies over accumulated assets. The higher shadow cost of external over internal funds increases the cost of vacancies, leading to a higher rate of equilibrium unemployment. However, the degree of agency costs is alleviated during economic upturns, lowering the shadow cost of resources allocated to job creation. This

[2]Fujita and Ramey (2007) argue that by combining their modeling of job vacancies with the calibration in Hagedorn and Manovskii (2008), their model can address both issues pertaining to the propagation of productivity shocks. Alternate approaches to modeling vacancy costs include Yashiv (2006) and Rotemberg (2006) in which the cost of vacancies is a declining function of the number of vacancies a firm posts.

[3]This result is independent of whether external funding applies to recruiting costs alone or include the wage bill. Section 3 develops and presents the results of a model in which both recruiting costs and the wage bill require external funding. Linking current costs to financial markets is also a features of bank loan models as in Christiano et al. (2005), or commercial debt models as in Carlstrom and Fuerst (2000).

[4]Acemoglu (2001) provides evidence that credit constrained industries have lower employment shares and Rendon (2001) finds that labor demand is both restricted and more elastic at credit constrained firms.

opens two channels through which the elasticity of job vacancies to productivity is increased: (i) a cost channel, driving a time-varying wedge in the job creation condition in which the lowered opportunity cost of resources allocated to job creation during an upturn increases the incentive to post vacancies; (ii) a wage channel - under Nash bargaining as a wage mechanism, the lowered opportunity cost of vacancies limits part of the upward pressure of market tightness on wages by improving the bargaining position of firms. Note that the source of wage rigidity is a consequence of frictional credit markets and not an inherent feature of the wage rule or a particular calibration of the model. In addition, the opportunity cost of resources used for recruiting is distinct from the *fixed* unit cost of a job vacancy and the *average cost* of recruiting a worker, which is a function of the degree of congestion on labor markets. Just as in the canonical model, this average cost, which appears in the job creation condition, will be pro-cyclical. However, it will be more rigid due to the presence of a counter-cyclical premium on external funds. Finally, the progressive easing of financing constraints as firms accumulate assets induces persistence in the adjustments of labor market variables to productivity shocks. Whereas in standard search models of equilibrium unemployment, or models with increased wage rigidity for that matter, the largest response of market tightness is contemporaneous to the exogenous shock, the height of the response in this setting is reached with a lag after the innovation.[5]

Section 3 details the model's quantitative results and sets them against a comparable framework without credit frictions. This sections finds the cost channel to be the most important for the model's ability to replicate the volatility relative to output and persistence of labor market variables observed in the data. For example, the relative volatility of market tightness reaches 12.45 (against 15.41 in the data) while only 3.76 in the standard model with

[5]The staggered nature of wage contracts in Gertler and Trigari (2009) is an exception in this literature in that persistence to productivity shocks does arise.

perfect credit markets, and the relative volatility of unemployment, which is 6.82 in the data, rises to 3.26 in the presence of credit frictions compared to 0.82 in the standard model.[6] U.S. quarterly data on market tightness display a high degree of persistence, measured as positive auto-correlations in the growth rate of 0.67, 0.48 and 0.33 at the first, second and third lags respectively. Allowing for frictional credit markets can generate auto-correlations of 0.62, 0.24 and 0.08 at the first, second and third lags, whereas a standard search model generates virtually no auto-correlation. This criticism is akin to that of Real Business Cycles (RBC) models advanced by Cogley and Nason (1995) in their inability to generated persistence in the growth rate of output. In this last respect, the inclusion of credit frictions allows the model to nearly perfectly match the persistence in the growth rate of output by inducing large variations in employment over the business cycle. Section 3 also examines a series of robustness issues. The results are very robust to an extension to externally funding part of the wage bill over and above recruiting costs.

2 Model

The model is populated by two types of agents: firms that produce using labor and households who decide on optimal consumption and purchases of risk free bonds. The allocation of labor from households to firms involves a costly and time-consuming matching process, following the now common approach of Mortensen and Pissarides (1994), adapted to a representative household framework as in Merz (1995) or Andolfatto (1996). The additional assumption is that firms must seek external funds over accumulated assets in order to pay for current vacancies, and that the lending relationship is subject to a credit market friction

[6]Second moments correspond to Hodrick-Prescott filtered data. Time series cover the period 1977:1 to 2005:4. The standard model refers to the Mortensen-Pissarides model in a discrete time setting, DSGE framework, detailed in the appendix.

of the costly state verification type. This incorporation of imperfect credit markets into a DSGE framework builds on work by Carlstrom and Fuerst (1997) with the canonical real business cycle model. The resulting debt contract is characterized by an optimal monitoring threshold and vacancy postings. Although the assumption of a fraction of vacancy costs needing external financing is sufficient to generate the results in this paper, the model is extended below to allow for the external funding of both recruiting costs and the wage bill, with very little effect on the results.

2.1 Labor markets and households

Firms post job vacancies V_t to attract unemployed workers U_t at a unit cost of γ, the nature of which will be discussed in detail when calibrating the model. Jobs are filled via a constant returns to scale matching function taking vacancies and unemployed workers $M(U_t, V_t)$. Define $\theta_t = \frac{V_t}{U_t}$ as labor market tightness from the point of view of the firm, or the v-u ratio. The matching probabilities are $\frac{M(U_t,V_t)}{V_t} = p(\theta_t)$ and $\frac{M(U_t,V_t)}{U_t} = f(\theta_t)$ for firms and workers respectively, with $\partial p(\theta_t)/\partial\theta_t < 0$ and $\partial f(\theta_t)/\partial\theta_t > 0$. Note that $f(\theta_t) = \theta_t p(\theta_t)$. Once matched, jobs are destroyed at the exogenous rate δ per period. Thus employment N_t and unemployment U_t evolve according to

$$N_{t+1} = (1-\delta)N_t + p(\theta_t)V_t \tag{1}$$

$$U_{t+1} = (1-f(\theta_t))U_t + \delta N_t \tag{2}$$

The representative household, given existing rates of employment and unemployment, chooses optimal consumption and purchases of risk free bonds B_t, which pay a rate r_t the following period, in order to maximize the value function:[7]

$$\mathcal{H}_t = \max_{C_t, B_t} \left[\mathcal{U}(C_t) + \beta E_t \mathcal{H}_{t+1} \right],$$

subject to the budget constraint $W_t N_t + bU_t + (1 + r_{t-1})B_{t-1} + \Pi_t = C_t + B_t + T_t$, and the laws of motion for matched labor (1) and unemployment (2). The government raises T_t in taxes to fund unemployment benefits $U_t b$, while employed workers earn the wage W_t. Π_t are firm dividends rebated lump sum at the end of the period. Denoting the multiplier on the budget constraint by λ, the first order conditions are

$$(C_t) \quad : \quad \mathcal{U}_C(C_t) = \lambda_t \tag{3}$$

$$(B_t) \quad : \quad \lambda_t = \beta E_t \lambda_{t+1}(1 + r_t) \tag{4}$$

2.2 Financial contract and vacancy decisions

The informational assumptions are chosen to generate standard debt contracts, in the tradition of Gale and Hellwig (1985) and Williamson (1987), set in a quantitative macroeconomic framework as in Carlstrom and Fuerst (1997). The contracts are written on a competitive capital market (in the sense that there is a large number of insignificant lenders and firms) and lenders are assumed to hold sufficiently large and diversified portfolios to ensure perfect risk pooling, with the result that investors behave as if they were risk neutral. Repayment of the debt is assumed to occur within the period: the contract is negotiated at the beginning of the time period and resolved by the end of the same period.[8] The competitive pressure

[7] As in Andolfatto (1996), each worker is a member of a household that offers perfect insurance against labor market outcomes and is involved in a passive search process. Labor force participation choices are not considered here, individuals are either employed or unemployed. See Garibaldi and Wasmer (2005) or Haefke and Reiter (2006) for models of labor market participation.

[8] The present contract is written for intra-period loans while Bernanke et al (1999) consider inter-period contracts which take into account aggregate uncertainty.

ensures that each lender-firm pair will write a contract which maximizes the expected value of the firm subject to the constraint that the expected return to the lender cover the amount borrowed.

Define firm period net revenues as $x\left(X_t - W_t\right) N_t$, where X_t is the aggregate level of technology, W_t is the wage rate. x is a random variable, i.i.d. across firms and time, drawn from a positive support with $E(x) = 1$, density $h(x)$ and distribution $H(x)$.[9] The crucial assumption for the contractual problem is that agents have asymmetric information over the realization of the random variable x. This state can only be observed by lenders at some cost proportional to realized net revenues, $0 < \mu_t < 1$. Levine et al (2004), using firm level data over the period 1997Q2 to 2003Q3, estimate the resource cost of monitoring consistent with the spread on corporate bonds and the expected risk of default reached a low of $\underline{\mu} = 0.07$ during the late 1990s expansion, and a high of $\overline{\mu} = 0.46$ during the 2001 recession. This variation may capture the fact that the value of liquidated assets following bankruptcy is subject to strong illiquidity effects that are highly cyclical, implying a much greater cost of default to the lender during an economic downturn (Ramey and Shapiro, 2001, Pulvino, 1998).[10] Consequently, and contrary to previous applications of costly state verification problems, it is assumed here that monitoring costs increase during a downturn according to the relationship $\mu_t = g(X_t)$, with $g'(X_t) < 0$ and $g''(X_t) < 0$. The effect of the proposed modification are circumscribed to the dynamics of the external finance premium, in a manner detailed below, as opposed to an alternative approach taken by Faia and Monacelli

[9] Alternatively, the firm's period net revenue could be expressed as $(xX - W)N$ with x drawn from a positive support with lower bound W. Either formulation guarantees a positive payoff function over the support of the idiosyncratic productivity, ensuring that the problem is well defined. This is similar to the approach in Carlstrom and Fuerst (2000) which consists of assuming that firms sell their product at a time varying mark-up over costs.

[10] Pulvino (1998), for example, finds that financially constrained airlines sell air crafts at a 14% discount to the average market price, but that these discounts exist only in times when the airline industry is depressed and not when it is booming.

(2007) .[11]

The timing of events in each period is as follows. Assume that vacancy costs γV_t must be paid before production occurs. All agents observe the aggregate state X_t and, given initial assets A_t, firms borrow $(\gamma V_t - A_t)$ from financial markets to pay for period vacancy postings. Lenders and borrowers agree on a contract that specifies a cutoff productivity $\overline{x}_t$ such that if $x > \overline{x}_t$, the borrower pays $\overline{x}_t\left(X_t - W_t\right)N_t$ and keeps the equity $\left(\overline{x}_t - x\right)\left(X_t - W_t\right)N_t$. If $x < \overline{x}_t$, the borrower receives nothing and the lender claims the residual net of monitoring costs.

Define the expected gross share of returns going to the lender under the contract as

$$\Gamma(\overline{x}_t) = \int_0^{\overline{x}_t} x dH(x) + \int_{\overline{x}_t}^{\infty} \overline{x}_t dH(x)$$

noting that $\Gamma'(\overline{x}_t) = 1 - H(\overline{x}_t) > 0$ and $\Gamma''(\overline{x}_t) = -h(\overline{x}_t) < 0$, and expected monitoring costs as

$$\mu_t G(\overline{x}_t) = \mu_t \int_0^{\overline{x}_t} x dH(x)$$

with $G'(\overline{x}_t) = \overline{x}_t h(\overline{x}_t)$. It is easy to see that the expected gross share to the lender will always be positive.[12] Given this set of definitions we can conveniently express the lender's participation constraint as $\left[\Gamma(\overline{x}_t) - \mu_t G(\overline{x}_t)\right]\left(X_t - W_t\right)N_t = (\gamma V_t - A_t)$, which states that the return net of monitoring costs must equal the value of the loan.

Given the assumptions on the functional forms, notably constant returns to scale in

[11] The latter assume that the mean of the random variable x is increasing in aggregate productivity: $E(x|X_t) = X_t^{\chi}$, where $\chi > 1$. While an effective strategy to generate a counter-cyclical external finance premium, this approach bears the unappealing effect of increasing the elasticity of effective TFP, now $X^{1+\chi}$, to exogenous productivity shocks. Thus any amplification in their model is a conjunction of the increased variance of effective TFP and counter-cyclical external financing constraints. For a detailed analysis of the conditions under which credit market frictions create a financial accelerator which destabilizes the economy, see House (2006).

[12] To do so, take the limits $\lim_{\overline{x}\to 0}\Gamma(\overline{x}) = \int_0^{\infty} \overline{x} dH(x) = 0$, $\lim_{\overline{x}\to\infty}\Gamma(x) = \int_0^{\infty} x dH(x) = 1 > 0$ and recall that $\Gamma(\overline{x})$ is strictly increasing and concave in $\overline{x}$. Note that the expected share of returns going to the borrower under the contract is $\Upsilon(\overline{x}_t) = \int_{\overline{x}_t}^{\infty}\left(x - \overline{x}_t\right)dH(x)$. Note that $\Gamma(\overline{x}_t) + \Upsilon(\overline{x}_t) = 1$.

production and a linear monitoring technology, only the evolution of aggregate assets is needed to know the cost faced by firms on credit markets. As such, all firms will choose the same ratio of vacancies to assets allowing the model to remain in representative firm setting (see Carlstrom and Fuerst, 1997). The evolution of aggregate assets is given by $A_{t+1} = \varsigma \left[1 - \Gamma(\overline{x}_t)\right] (X_t - W_t) N_t$, where the parameter $0 < \varsigma < 1$ ensures that self-financing does not occur, and defines rebated dividends as $\Pi_t = (1 - \varsigma) \left[1 - \Gamma(\overline{x}_t)\right] (X_t - W_t) N_t$.[13] Rearranging as

$$A_{t+1} = \varsigma \left[(X_t - W_t) N_t - \left(1 + \frac{\mu_t G(\overline{x}_t) (X_t - W_t) N_t}{\gamma V_t - A_t} \right) (\gamma V_t - A_t) \right] \tag{5}$$

focuses on the premium associated with external funds, $\iota_t \equiv \frac{\mu_t G(\overline{x}_t)(X_t - W_t) N_t}{\gamma V_t - A_t}$, which for any $\mu > 0$ is strictly positive.

We can now write the optimal incentive compatible contracting problem with non-stochastic monitoring and repayment within the period. Vacancy postings V_t and the threshold $\overline{x}_t$ are chosen to maximize the expected value of the firm, given aggregate states X_t, N_t, U_t, and A_t, subject to the lender's participation constraint and the law of motion for employment:

$$\begin{aligned} J_t &= \max_{V_t, \overline{x}_t} \left[1 - \Gamma(\overline{x}_t)\right] (X_t - W_t) N_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1} \\ &\quad \text{subject to } \left[\Gamma(\overline{x}_t) - \mu_t G(\overline{x}_t)\right] (X_t - W_t) N_t = (\gamma V_t - A_t) \\ &\quad \text{and} \qquad N_{t+1} = (1 - \delta) N_t + V_t p(\theta_t) \end{aligned}$$

[13] The assumption of some depletion in the stock of assets is needed to rule out eventual self-financing. Carlstrom and Fuerst (1997) assume that consumers and entrepreneurs have different time discount factors, while Bernanke et al. (1999) assume that a fraction of the entrepreneurial population exits every period consuming their assets on the way out. This paper assumes that firms retain a fraction of their earnings toward next period's assets while rebating the remaining to households as dividends.

where firms use the stochastic discount factor $\beta\frac{\lambda_{t+1}}{\lambda_t}$.

2.3 Job creation under credit constraints

Denote the multiplier on the lender's participation constraint by ϕ. The optimality condition for vacancy postings describes a job creation condition

$$\frac{\gamma\phi_t}{p(\theta_t)} = \beta E_t\frac{\lambda_{t+1}}{\lambda_t}J_{N,t+1}$$

equating the average economic cost of a vacancy, $\frac{\gamma\phi_t}{p(\theta_t)}$, to the discounted expected marginal value of an additional employed worker $\beta E_t\frac{\lambda_{t+1}}{\lambda_t}J_{N,t+1}$. Note that the average cost of recruiting a worker is in fact $\frac{\gamma(1+\iota_t)}{p(\theta_t)}$.

In order to derive the marginal value of a worker to the firm, $J_{N,t}$, differentiate the firm's value function with respect to N,

$$J_{N,t} = [1-\Gamma(\overline{x}_t)]\,(X_t - W_t) + \phi_t\,[\Gamma(\overline{x}_t) - \mu_t G(\overline{x}_t)]\,(X_t - W_t) + (1-\delta)\beta E_t\frac{\lambda_{t+1}}{\lambda_t}J_{N,t+1}$$

The first term corresponds to the net return on an employee accruing to the firm under the debt contract. The second term captures the value an additional worker brings to the firm by relaxing the financing constraint in terms of an increased ability to reimburse the loan. The final term captures the value of the continued relationship. For the sake of simplifying the notation, call $\Omega(\overline{x}_t) \equiv 1-\Gamma(\overline{x}_t) + \phi_t\,[\Gamma(\overline{x}_t) - \mu_t G(\overline{x}_t)]$. Combining the marginal value of a worker with the optimality condition for vacancies, and making use of the household bond Euler equation (4), yields the intertemporal condition for vacancy postings

$$\frac{\gamma\phi_t}{p(\theta_t)} = \frac{1}{1+r_t}E_t\left[\Omega(\overline{x}_{t+1})\,(X_{t+1} - W_{t+1}) + (1-\delta)\frac{\gamma\phi_{t+1}}{p(\theta_{t+1})}\right] \tag{6}$$

At this stage it is useful to show how this setting with credit frictions compares to a standard search and matching model of equilibrium unemployment.[14] Consider first the credit constraint multiplier ϕ_t on the cost side of the job creation condition, which is the shadow cost of external over internal funds. This measure indicates how binding are credit constraints and, consequently, the term $\frac{\gamma\phi_t}{p(\theta_t)}$ should be interpreted as the opportunity cost to the firm of resources allocated to recruiting workers. From the first order condition for the cutoff productivity, this multiplier may be expressed as

$$\phi_t = \frac{\Gamma'(\overline{x}_t)}{[\Gamma'(\overline{x}_t) - \mu_t G'(\overline{x}_t)]} \tag{7}$$

In the absence of monitoring costs the threshold $\overline{x}$ tends to the lower bound of its support. It is straightforward to show that $\partial\phi_t/\partial\overline{x}_t > 0$, and that in the limit $\lim_{\overline{x}_t \to 0} \phi_t = 1$. As a result, for any positive monitoring cost the presence of credit frictions drives up the average economic cost of vacancy postings to $\frac{\gamma\phi_t}{p(\theta_t)}$, as opposed to $\frac{\gamma}{p(\theta_t)}$. Significantly, for the purpose of this paper, an improvement in the state of credit markets, measured as a decrease in ϕ, is a decrease in the opportunity cost of resources allocated to job creation, but not in the average cost of recruiting a worker $\frac{(1+\iota_t)\gamma}{p(\theta_t)}$ which, as we will see, will remain pro-cyclical.

Second, one can show that $\lim_{\overline{x}_t \to 0} \Omega(\overline{x}_t) = 1$, such that in the absence of monitoring costs the first order condition (6) collapses to the standard job creation condition in a stochastic discrete time setting:

$$\frac{\gamma}{p(\theta_t)} = \frac{1}{1+r_t} E_t \left[X_{t+1} - W_{t+1} + (1-\delta)\frac{\gamma}{p(\theta_{t+1})} \right] \tag{8}$$

The received argument for the lack of amplification of productivity shocks is easily under-

[14]The appendix develops the model referred to as the standard search and matching model of equilibrium unemployment model in discrete time.

stood by this job creation condition equating the average cost of a vacancy to the expected benefit of a new job (see Shimer, 2005, Hall, 2005). A sudden rise in productivity, increasing the revenues generated by a job, increases the incentive to post vacancies. The same rise in productivity, however, leads to a rise in the wage which reduces the profits accruing to the firm. For most applications of the Nash bargaining solution, the wage is highly elastic to productivity such that the profits from a job for the firm are relatively inelastic to productivity shocks and, as a consequence, so are vacancy postings. Quite naturally, a first response to this issue has been to induce greater wage rigidity by either changing the structure of the model, i.e. settling on different wage determination mechanisms (Hall, 2005, Gertler and Trigari, 2009), or following a calibration strategy resulting in a wage less elastic to productivity (Hagedorn and Manovskii, 2008).

There is, however, a second, overlooked, dampening mechanism built into the job creation condition. The same event leading to a rise in the job finding hazard for workers, and their ability to negotiate higher wages, also corresponds to an increase in the congestion facing firms on the labor market. In other words, each job vacancy faces a decreasing probability $p(\theta_t)$ of being filled in a given unit of time. This increase in the average cost of hiring a worker further restricts firm entry, limiting the propagation of productivity shocks. Here, credit market imperfections have the potential to amplify productivity shocks in a manner that is fundamentally different, operating through the cost side of the job creation condition. Recall that in the presence of credit frictions the average economic cost to filling a vacancy is $\frac{\gamma\phi_t}{p(\theta_t)}$, whereas in the standard model it is $\frac{\gamma}{p(\theta_t)}$. The multiplier on the lender's participation constraint, ϕ_t, in effect drives a time varying wedge on the cost side relative to the frictionless model. If these constraints are counter-cyclical, or ϕ_t decreases during an economic upturn, there is a downward push on the opportunity cost of recruiting workers that increases the incentive for firms to post job vacancies independently of changes in the expected benefit of a new worker. The strong congestion effects on labor markets imply, however, that the average cost $\frac{\gamma(1+\iota_t)}{p(\theta_t)}$ remains pro-cyclical, yet more rigid over the business cycle.

2.4 Workers and wages

The model is fully described once the rule for wages is determined. In order to define the values of a job ($\mathcal{H}_N$) and unemployment ($\mathcal{H}_U$) to a worker, differentiate the household's value function with respect to N and U:

$$\begin{aligned}
\mathcal{H}_{N,t} &= \lambda_t W_t + \beta E_t \left[(1-\delta)\mathcal{H}_{N,t+1} + \delta \mathcal{H}_{U,t+1}\right] \\
\mathcal{H}_{U,t} &= \lambda_t b + \beta E_t \left[(1-f(\theta_t))\mathcal{H}_{U,t+1} + f(\theta_t)\mathcal{H}_{N,t+1}\right]
\end{aligned}$$

The current value of a job corresponds to the wage measured in utils and the discounted expected values of next period's state, which with probability $(1-\delta)$ remains employment. The value of unemployment is derived from the value of non-market activities, $\lambda_t b$, and the discounted expected value of next period's state, which with probability $f(\theta_t)$ is employment.

The surplus of a worker-firm match, defined as $S_t = J_{N,t} + \frac{\mathcal{H}_{N,t} - \mathcal{H}_{U,t}}{\lambda_t}$, is split under a generalization of Nash bargaining by choosing a wage that maximizes ${J_{N,t}}^{(1-\eta)} \left(\frac{\mathcal{H}_{N,t} - \mathcal{H}_{U,t}}{\lambda_t}\right)^{\eta}$, where η is the worker's bargaining weight. Wages are negotiated at the beginning of the period once the aggregate state is observed but before the firm draws an idiosyncratic productivity. The wage is not a function of the idiosyncratic productivity, lest it reveal the firm's productivity draw to creditors, but will reflect the terms faced by the firm on credit markets.[15] The first order condition to this problem,

$$\nu_t J_{N,t} \quad = \quad (1-\nu_t)\left(\frac{\mathcal{H}_{N,t}-\mathcal{H}_{U,t}}{\lambda_t}\right)$$

where $\nu_t = \frac{\eta}{\eta+(1-\eta)\Omega(\overline{x}_t)}$, describes a rule for sharing the joint surplus of the relationship that differs from the usual application of Nash bargaining to wage determination in that the sharing weight ν_t depends on the state of credit markets, and can differ from the constant bargaining weight η. An increase in the term $\Omega(\overline{x}_t)$, which reflects a greater degree of credit market imperfection, improves the firm's effective bargaining power as the firm's surplus becomes more sensitive to changes in the wage relative to the worker's. The resulting wage rule is

$$W_t = \eta\left[X_t + \gamma\frac{\phi_t}{\Omega(\overline{x}_t)}\theta_t\right] + (1-\eta)b \tag{9}$$

As with the job creation condition, when monitoring costs tend to 0 the wage rule (9) collapses to

$$W_t = \eta\left[X_t + \gamma\theta_t\right] + (1-\eta)b \tag{10}$$

This is simply the wage rule in a search model of equilibrium unemployment without credit frictions, leading to the following proposition

Proposition 1 - *The canonical Mortensen-Pissarides search and matching model of equilibrium unemployment is a special case of the present model with frictional credit markets when the cost of monitoring tends to zero.*

The steady state and quantitative implications for the dynamics labor markets are dis-

[15] The study of possible separating equilibria on credit markets due to heterogeneous wages is beyond the scope of this paper. It is also assumed that wages cannot be renegotiated ex-post. Details on the derivation of the wage are presented in the appendix.

cussed in the next section. However, one important aspect of the modified wage rule is worth stressing here. A principal force in the cyclical properties of the wage rule is the term $\gamma\frac{\phi_t}{\Omega(\overline{x}_t)}\theta_t$ which, along with the value of non-market activities, captures the relative bargaining positions of workers and firms. During an upturn, market tightness rises making it more costly for firms to pull out of wage negotiations to search for another worker (recall that a rise in θ implies a drop in the probability of meeting a worker $p(\theta)$). In the presence of credit market frictions, the opportunity cost of resources allocated to a job vacancy $\gamma\frac{\phi_t}{\Omega(\overline{x}_t)}$ actually decreases during good times as conditions on credit markets improve, that is , $\frac{\phi}{\Omega(\overline{x})} > 1$ and $\frac{\phi}{\Omega(\overline{x})}$ tends to 1 as $\overline{x}$ tends to zero. The strengthened bargaining position of firms somewhat limits the upward pressure on wages stemming from the rise in market tightness. The end result is to induce some degree of wage rigidity which will contribute to amplifying productivity shocks in the manner outlined above.

2.5 Closing the model

From the household's budget constraint, it is straightforward to derive an aggregate resource constraint

$$Y_t\left[1 - \mu_t G(\overline{x}_t)\right] = C_t + \gamma V_t$$

where $Y_t = X_t N_t$, $\mu_t G(\overline{x}_t)$ are resources consumed in monitoring and γV_t are vacancy costs.

The equilibrium of the model is then defined by equations (3) and (4) from household optimization, a job creation condition (6), optimality condition for the threshold $\overline{x}_t$ in (7), the definition of market tightness, the lender's participation constraint, a wage rule (12), the aggregate resource constraint and laws of motion for asset accumulation, aggregate employment and unemployment.

3 Propagation properties of financial and labor market frictions

Before discussing some of the steady state labor market implications of credit market frictions in this setting, the assumptions on functional forms and calibration are presented in detail. The model is then solved by computing the unique rational expectations solution for a log-linearization around the deterministic steady state, and the dynamics are evaluated with a series of unconditional second moments and impulse response functions. The performance of the model is assessed by presenting results for a standard labor search model as a basis for comparison and performing a series a sensitivity analysis to key parameters and aspects of the model, including an extension to external financing of the wage bill.

3.1 Functional forms and calibration

Following much of the real business cycle literature, aggregate technology is assumed stationary and to evolve according to

$$\log X_t = \rho_X \log X_{t-1} + \varepsilon_t^X,$$

with $\varepsilon_t^X \sim (0, \sigma_X^2)$ and $0 < \rho_X < 1$. Staying within this literature, the relevant parameters are chosen as $\rho_X = 0.975$ and $\sigma_X = 0.0072$ (e.g., King and Rebelo, 1999).

For household preferences, period utility is defined as $\mathcal{U}(C) = \log C$. The idiosyncratic shock x is assumed to follow a log-normal distribution with mean $E(x) = 1$; i.e. $\log(x) \sim \mathcal{N}(-\frac{\sigma^2_{\log(x)}}{2}, \sigma^2_{\log(x)})$. Finally, following much of the labor search literature, the matching technology is a Cobb-Douglas $M(U, V) = \xi U^{\epsilon} V^{1-\epsilon}$, with $0 < \epsilon < 1$ and $\xi > 0$.

The model is calibrated to quarterly data. The discount factor $\beta = 0.992$ is set so as to match an average annual real yield on a risk-free 3-month treasury bill of 3.3%. For parameters pertaining to financial factors, the standard deviation of idiosyncratic productivity shocks and the parameter ς in the asset accumulation equation are set jointly to match two observations: i) a steady state quarterly default rate $H(\bar{x})$ of 1%, corresponding to the values reported in both Carlstrom and Fuerst (1997) and Bernanke et al. (1999); ii) a steady state proportion of vacancy costs funded externally of two thirds. This is consistent with evidence in Devereux and Schiantarelli (1989) and Buera and Shin (2008) on the proportion of firm current expenditure financed externally. This calibration, which results in values of $\sigma_x = 0.23$ and $\varsigma = 0.66$, also implies a steady state leverage ratio $\frac{\gamma V - A}{A}$ of 2, the target employed in Bernanke et al. (1999). Other investigations, such as Christiano et al. (2005), have assumed that all current costs, in their case the entire wage bill, must be financed through bank loans. It is important to note here that model is extended below to funding a fraction of both the wage bill and current vacancy costs to assess the importance of this assumption for the main results.

There is no direct measure of the model's external finance premium in the data, but several proxies are regarded as good indicators (see Gomes, Yaron and Zhang, 2003, Levin et al., 2004). One such indicator, the corporate bond spread, averaged 108 basis point over the period 1971Q1 to 2007Q4. Consequently, the steady state resource cost of monitoring is set to $\mu = 0.25$, targeting this premium of external over internal funds. This also corresponds to the resource cost of monitoring in Carlstrom and Fuerst (1997). Next, the elasticity of resource cost of monitoring to changes in aggregate productivity is calibrated such that the cost of monitoring doubles during a recession due to high degrees of illiquidity in the assets used as collateral.[16] The sensitivity of the model results to the calibration of the credit

[16] An alternative strategy would have been to calibrate this elasticity to match the volatility relative to output of the chosen indicator of the external finance premium. However, as has been the challenge in the

market will examined in detail.

Several authors have argued that the targeted steady state rate of unemployment should include more than the rate of workers counted as unemployed as the model does not account for non-participation. Krause and Lubik (2007), for example, choose an unemployment rate of 12%, above the average rate observed for the United States. The benchmark calibration, however, will target a 10% unemployment rate, a mid point between the later authors and the value of 7% in Gertler and Trigari (2009). This is achieved by adjusting the level parameter ξ in the matching function. According to the study by Baron et al. (1997), the average cost of time spent hiring one worker is approximately 3% of quarterly hours, and up to 4.5% if it is assumed that hiring is done by supervisors with higher wages (Silva and Toledo, 2009). The unit cost of job vacancies is set to $\gamma = 0.25$ such that the labor cost of vacancies $\frac{(1+\iota)\gamma V}{p(\theta)WN}$ is 3.9%.

The elasticity of the labor matching function, ϵ, is set to 0.72, corresponding to the estimated elasticity in Shimer (2005). In the baseline parametrization, the household's bargaining weight in wage negotiations, η, is set to 0.5, a mid-point chosen to strike a balance between the extremes advocated in the literature. However, this parameter will be the focus of a detailed sensitivity analysis. Note the the effective share, ν, is 0.495 under this calibration. Finally, the quarterly rate of job separation is set to 6%, corresponding to the evidence presented in Davis, Faberman and Haltiwanger (2006), and the value of non-market activities $b = 0.75$. This baseline calibration results in a replacement rate b/W of 0.77. It is well known that the properties of labor search models change dramatically as this ratio tends to unity, and setting a high value as advocated by Hagedorn and Manovskii (2008) has the unappealing implication that workers gain little utility from accepting a job (see Mortensen

asset pricing literature, the volatility is more than an order of magnitude above that of aggregate output and, with log-preferences, it would not be possible to match this target. See Jermann (1998).

and Nagypal, 2007). Hagedorn and Manovskii (2008) reconcile the standard search model with key labor market statistics by employing an elevated value of the replacement rate of 0.96. Rotemberg (2006) chooses a value of 0.9, while Elsby and Michaels (2008) set the rate at a lower 0.86. In addition, the former adopt an extremely low value of the bargaining parameter in order to generate a wage with a low elasticity to productivity. Shimer (2005) sets the bargaining weight equal to the weight on unemployment in the matching function as under the 'Hosios rule' (Hosios, 1990) in order to ensure constrained efficiency of the decentralized solution. While there is no definitive value for the replacement rate, it is shown below that the result are robust to much lower values .

3.2 Steady state implications

Proposition 2 - *There exists a unique steady state equilibrium in which the rate of unemployment is strictly increasing in the resource cost of monitoring, μ.*

Proof. The job creation condition in the presence of credit constraints can express the wage as a decreasing function of market tightness

$$W = 1 - \left(\frac{1}{\beta} - (1-\delta)\right)\frac{\phi\gamma}{\Omega(\overline{x})p(\theta)}$$

where aggregate productivity has been normalized to 1. Relative to the case with perfect credit markets, the additional cost induced by the necessity of external funds implies a steeper curve by the factor $\frac{\phi}{\Omega(\overline{x})} > 1$, with $\frac{\phi}{\Omega(\overline{x})}$ strictly increasing in $\overline{x}$ and $\lim_{\overline{x}\to 0} \frac{\phi}{\Omega(\overline{x})} = 1$. Figure 1 plots in (θ, W) space the job creation curve for the model with (solid line) and without (dashed line) credit frictions. The wage rule in the presence of credit frictions, $W = \eta(1 + \gamma\frac{\phi}{\Omega(\overline{x})}\theta) + (1-\eta)b$, has a slope greater than in the absence of credit market friction by the same factor $\frac{\phi}{\Omega(\overline{x})} > 1$. This captures the greater opportunity cost of a match to the firm that workers can exploit and, conditional on $(\eta + (1-\eta)b) < 1$, the intersection of the wage rule and job creation condition is unique.

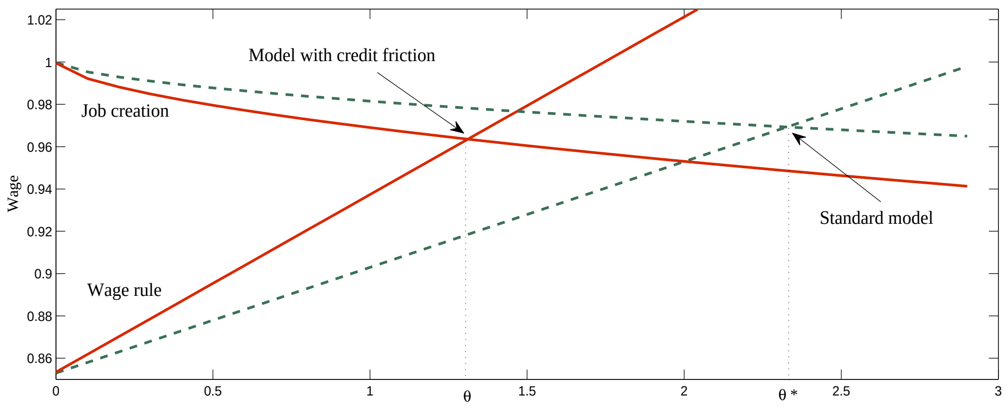

Figure 1: Steady state labor market equilibrium

Combined, the two labor market equilibrium conditions, job creation and the wage rule, pin down equilibrium market tightness $\tilde{\theta}$ as

$$\gamma \left(\frac{(r+\delta)}{\xi} \tilde{\theta}^{\epsilon} + \eta \tilde{\theta} \right) \Phi(\overline{x}) = (1-\eta)\left[1-b\right]$$

where $\Phi(\overline{x}) \equiv \frac{\phi}{\Omega(\overline{x})} \geq 1$. In the absence of credit frictions this is given by

$$\gamma \left(\frac{r+\delta}{\xi} \theta^{*\epsilon} + \eta \theta^{*} \right) = (1-\eta)\left[1-b\right]$$

where θ^* denotes equilibrium market tightness in the frictionless case. $\tilde{\theta} < \theta^*$ follows from the fact that $\Phi(\overline{x}) > 1$ for any strictly positive value of the monitoring cost μ. To see the effect of an increase in μ on market tightness, note that $\frac{\partial \Phi(\overline{x})}{\partial \mu} > 0$. As a result, an increase in monitoring costs leads to a decrease in equilibrium labor market tightness which, through the Beveridge relationship $U = \frac{\delta}{\delta + f(\theta)}$, implies a greater steady state rate of

Table 1: Credit frictions and steady state unemployment rates

Monitoring cost μ	0	0.25	0.5
Premium	0%	0.93%	19%
Unemployment rate	9.93%	10%	10.76%

unemployment.[17] This insight is similar to that in Acemoglu (2001) and Wasmer and Weil (2004) in that credit friction restricts firm entry on labor markets. Table 1 explores the steady state implications quantitatively and finds that reasonable degrees of credit market imperfections have a negligible impact on the rate of unemployment. Removing all frictions reduces the steady state rate of unemployment from 10% to 9.93%. Moreover, increasing the resource cost of monitoring and steady state default rate such that the premium reaches 19% only increase the unemployment rate to 10.76%. Therefore the impact of financial frictions in the long run in this set-up are modest. As the next sections will show this need not be the case in the short run.

3.3 Intuition for propagation on the labor market

Looking at the impact of a permanent change in aggregate productivity on equilibrium market tightness yields some intuition into the sources of propagation induced by imperfect credit markets. But first, consider the elasticity of market tightness to productivity in the absence of credit friction

$$\Sigma_{\theta^*,X} = \frac{(1-\eta)}{\alpha(\theta^*)}$$

[17] The effect on the equilibrium wage is ambiguous as higher recruiting costs both lowers job offers and affects the threat point in wage bargaining to the advantage of workers.

where $\alpha(\theta) = \gamma\left[\eta\theta + \epsilon\left(r+\delta\right)/p(\theta)\right]$ and households have been assumed to be risk neutral. The numerator captures the share of the change in labor productivity retained by the firm after paying the wage. The first term within the brackets in the denominator corresponds to the share of the change in productivity going to the worker. One clearly sees through this expression how a lower bargaining weight η creates a stronger elasticity of the incentive to post job vacancies to changes in productivity. Finally, the term $\epsilon\left(r+\delta\right)/p(\theta)$ corresponds to the increase in the cost of recruiting a worker net of the discounted future value of that worker to the firm as the outcome of a rise in productivity is an increase congestion facing open job vacancies on labor markets.

In the presence of credit market imperfection we have

$$\Sigma_{\tilde{\theta},X} = \frac{(1-\eta)}{\alpha(\tilde{\theta})\Phi(\overline{x})}\left[1-(1-b)\Sigma_{\Phi,X}\right]$$

where $\Sigma_{\Phi,X} = \frac{\partial\Phi(\overline{x})}{\partial X}\frac{X}{\Phi(\overline{x})} \leq 0$, recalling that $\Phi(\overline{x}) \equiv \frac{\phi}{\Omega(\overline{x})} \geq 1$ and that this ratio tends to 1 as conditions on credit markets improve. The first block is similar to the elasticity in the absence of credit market frictions and, given the baseline parametrization, $\alpha(\tilde{\theta})\Phi(\overline{x}) \approx \alpha(\theta^*)$. The difference therefore lies in the bracketed term $\left[1-(1-b)\Sigma_{\Phi,X}\right]$ and in particular, the magnitude of amplification will depend heavily on the elasticity $\Sigma_{\Phi,X}$, which reflects the change in the opportunity cost of resources allocated to recruiting to changes in productivity. This will be discussed, along with the quantitative results, in the following subsection.

3.4 Dynamic results

Several authors, as mentioned earlier, have noted the failure of the Mortensen-Pissarides framework to generate sufficient internal propagation of exogenous shocks to match key labor market statistics. Table 2 reports the Hodrick-Prescott filtered standard deviation relative to aggregate output of variables central to the labor market, along with their contemporaneous correlation with the cyclical component of aggregate output. The first columns set the performance of the standard labor search model against moments from U.S. data and highlight its shortcomings in terms of amplification. The relative volatility of market tightness generated by the standard model is only 24% of that in the data. The dismal performance of the model extends to job vacancies which have a relative volatility of 8.83 in the data and 3.27 in the standard model. The performance in terms of unemployment or employment is hardly any better: the model generates a relative standard deviation for unemployment of 0.82 against a relative standard deviation of 6.83 in the data, or just 12% of the relative volatility observed in the data.

The second significant shortcoming concerns the persistence in the adjustment to exogenous shocks. Evidence uncovered from reduced form VARs show that market tightness (and vacancies) have a sluggish response to productivity shocks, peaking several quarters after the innovation (see Fujita and Ramey, 2007). The last three rows of Table 2 report another measure of this persistence, the auto-correlation in the growth rate of market tightness. The data is characterized by a high degree of positive auto-correlation at the first three lags while the standard search model generates virtually no persistence. With regards to output growth, the standard search model does generate some persistence essentially due to the predetermined nature of employment. However, fluctuations in the later are too weak for the model to be consistent with the data.

3.4.1 Amplification and persistence under imperfect credit markets

We begin by examining, in Figure 2, the responses of vacancies and market tightness to a positive productivity shock in the standard (dashed line) and proposed (solid line) models.

Table 2: Unconditional 2nd moments

1977:1-2005:4	U.S. data		Labor search		Labor search with credit friction	
Variable:	a	b	a	b	a	b
U	6.83	-0.88	0.82	-0.70	2.37	-0.75
V	8.83	0.89	3.27	0.97	8.95	0.97
θ	15.41	0.90	3.76	0.99	10.58	0.99
N	0.48	0.82	0.09	0.70	0.27	0.74
$\sigma(Y)$	1.40		1		1.15	
ν:	θ	Y	θ	Y	θ	Y
$corr(\Delta\nu, \Delta\nu_{-1})$	0.67	0.26	001	0.05	0.48	0.15
$corr(\Delta\nu, \Delta\nu_{-2})$	0.48	0.23	0	0.02	0.03	0.13
$corr(\Delta\nu, \Delta\nu_{-3})$	0.33	0.08	0	0	0	0.05

a: Standard deviation relative to output; b: contemporaneous correlation with output. All moments, but growth rate, are Hodrick-Prescott filtered;
Data sources: BLS, BEA.

The introduction of credit frictions yields two improvements: first, the responses are largely amplified; second, the responses are persistent, or the adjustment to the exogenous innovation is "sluggish." The unconditional second moments for the proposed model, presented in the last columns of Table 2, show a relative volatility of job vacancies remarkably close to its empirical counterpart at 8.95, compared to 8.83 in the data. The increase in the relative volatility of market tightness is equally large, rising to 10.58 compared to 15.41 in the data. In terms of persistence, deviations in market tightness peak several quarters after the shock. More precisely, the model generates elevated positive auto-correlations in the growth rate of market tightness, close to the data at the first lag but decaying too rapidly at the second and third (see the last three rows of Table 2).

Understanding the present results lies in the dynamics of the cost and wage channels of propagation outlined earlier. As the previous section discussed the extent to which both depend on the evolution of the measure of credit market imperfection $\Phi(\overline{x})$, the first panel of Figure 3 plots its response following the same expansionary shock to productivity. While

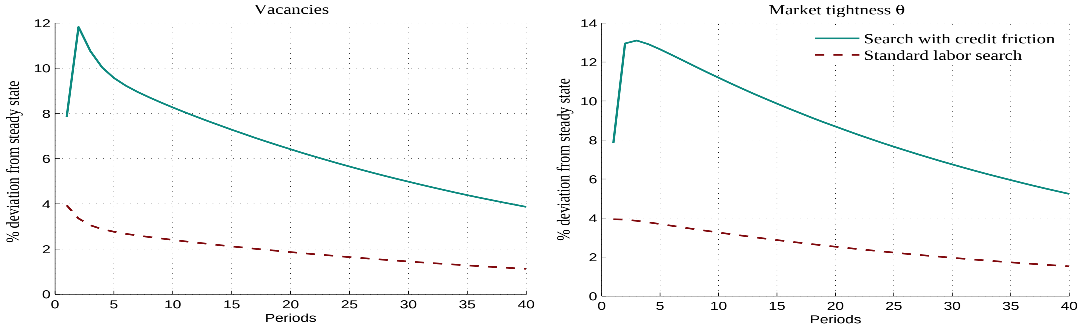

Figure 2: IRF to a positive productivity shock, job vacancies and labor market tightness

the constraint is relaxed on impact, the accumulation of assets pushes the constraint to its lowest level with a lag. The effect on the job creation condition is not strongest, therefore, contemporaneously to the productivity shock, as is in the case in the standard model and illustrated in Figure 2. Over the business cycle, $\Phi(\bar{x})$ has a volatility relative to output of 8 and a contemporaneous correlation of -0.97.

Recall that the average cost of recruiting in the canonical model, $\frac{\gamma}{p(\theta_t)}$, is highly pro-cyclical: its contemporaneous correlation with output is 0.99 and its standard deviation relative to output is 2.7. While the shadow cost of external resources ϕ and the premium on external funds ι are counter-cyclical, the average cost of recruiting $\frac{\gamma(1+\iota_t)}{p(\theta_t)}$ remains pro-cyclical, with corresponding moments of 0.97 and 6.17, respectively.

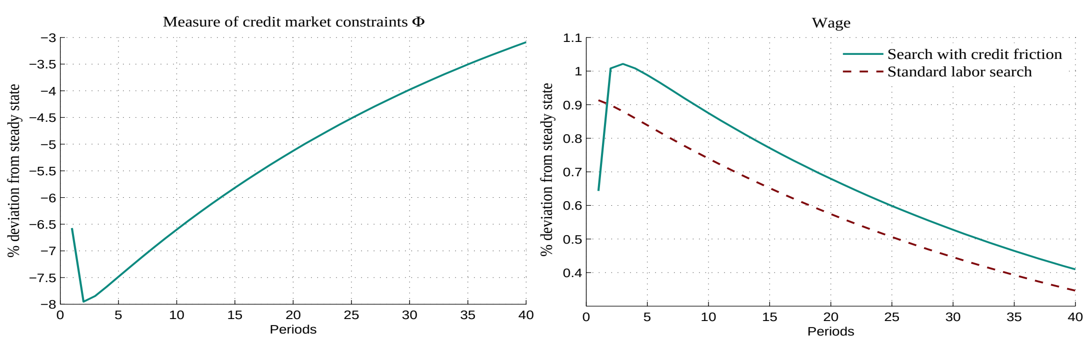

Figure 3: IRFs to a positive productivity shock, credit market constraints and wage

The wage channel is illustrated in the second panel of Figure 3. Following an innovation to productivity, wages do not initially respond as strongly as in the standard model, increasing progressively for several quarters. This rigidity contributes to the elasticity of the initial response of market tightness and vacancies to a productivity shock, which is greater in the model with credit frictions (again, see Figure 2). As market tightness continues to rise more that the reduction in $\Phi(\overline{x})$, the wage peaks after a few quarters such that the wage effect is operative only contemporaneously to the productivity shock. Moreover, the continued rise in market tightness, even as the wage is increasing, leads to the conclusion that the cost channel is largely dominant in generating the propagation of productivity shocks. This reinforces the main argument that endogenously evolving conditions on credit markets contribute to fluctuations on labor markets through the change in the opportunity of resources used by firms to create jobs.

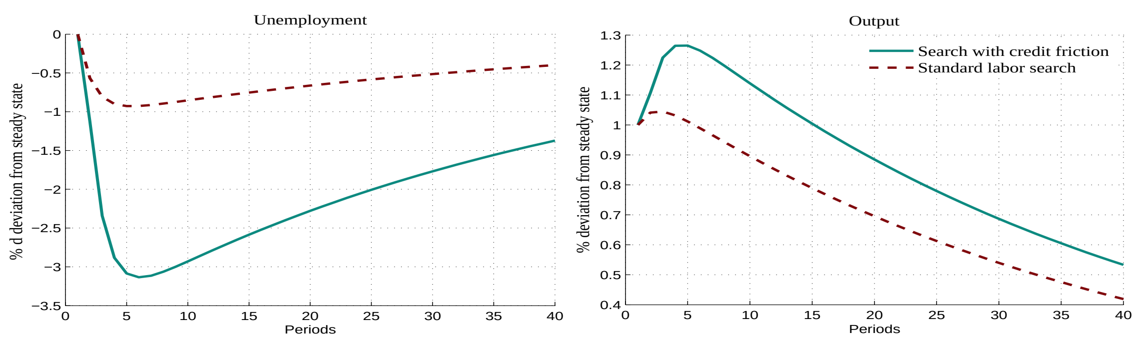

Figure 4: IRFs to a positive productivity shock, unemployment and output

The large propagation potential of financial frictions in this setting results in a standard deviation of aggregate unemployment of 2.37, and standard deviation of aggregate output of 1.15, up from 1 for the model with perfect credit markets. Although a significant improvement upon the standard model, this still falls short of the data. Figure 4, which plots the responses of unemployment and output to a positive productivity shock, illustrates the full impact of this financial accelerator on aggregate activity. The strong rise in hirings leads to a deep and prolonged drop in the unemployment rate. It immediately follows that output continues to expand several quarters after the innovation, a pattern not present in the standard model. As a result, the model with credit frictions generates positive auto-correlations in the growth rate of output at the first three lags of 0.15, 0.13 and 0.05, compared to 0.26, 0.23, 0.08 in the data, going a long way in addressing the lack of endogenous persistence in real business cycle models raised by Cogley and Nason (1995).[18]

3.4.2 Robustness to the calibration of the credit market

This section examines the behavior of the model along the dimension of the calibration of the credit market. The first columns of Table 3 present the effects of calibrating to either 55 or 135 point premia by changing the steady state value of the resource cost of monitoring μ.[19] A lower premium on external funds implies a reduced propagation of productivity shocks, the volatility of the v-u ratio dropping to 7.03 and the relative volatility of unemployment to 1.59. The inverse is observed when the premium on external finance is raised to 135 points, the relative standard deviation of θ rising to 13.42. With respect to the measure of persistence, the change in the premium affects mainly the auto-correlation at the second and third lags, a higher premium generating lower coefficients. This contrasts with the negligible effects of the monitoring cost on the steady state rate of unemployment presented in Table 1.

In order to fully illustrate the range of amplification as a function of the degree of agency costs, Figure 5 plots the effect of varying the steady state resource cost of monitoring μ over the range $[0, 0.5]$ for the main measure of amplification used in this paper. The dashed horizontal line in Figure 5 marks the result for the standard model, to which the model

[18] Addressing the lack persistence in the growth rate of output in the basic RBC model motivates Andolfatto's (1996) work on incorporating search on labor markets to this class of models. He shows that the problem of persistence can be resolved for certain parametrization of the labor market.

[19] A 55 point premium is achieved be setting $\mu = 0.15$ and a 135 point premium for $\mu = 0.35$.

Table 3: Robustness to credit market parametrization

	Baseline		Premium				Leverage ratio				Elasticity of	
			55 points		135 points		0.5		4		μ_t	
	a	b	a	b	a	b	a	b	a	b	a	b
U	2.37	-0.75	1.59	-0.67	3.00	-0.79	1.74	-0.65	2.47	-0.77	1.64	-0.70
V	8.95	0.97	5.90	0.98	11.34	0.95	6.31	0.98	9.46	0.97	6.18	0.98
θ	10.58	0.99	7.03	0.98	13.42	0.99	7.60	0.97	11.10	0.99	7.31	0.99
$\sigma(Y)$	1.15		1.06		1.24		1.06		1.17		1.07	
ν:	θ	Y	θ	Y	θ	Y	θ	Y	θ	Y	θ	Y
$corr(\Delta\nu, \Delta\nu_{-1})$	0.48	0.15	0.48	0.08	0.43	0.22	0.55	0.08	0.30	0.19	0.42	0.09
$corr(\Delta\nu, \Delta\nu_{-2})$	0.03	0.13	0.09	0.07	-0.01	0.20	0.21	0.08	0	0.13	0.02	0.08
$corr(\Delta\nu, \Delta\nu_{-3})$	0	0.05	0.01	0.03	0	0.07	0.07	0.05	0	0.05	0	0.03

a: Standard deviation relative to output; b: contemporaneous correlation with output.
All moments, but growth rate, are Hodrick-Prescott filtered;

converges as the resource cost of monitoring μ tends to 0, as illustrated by the solid line.

Finally, this application of the costly state verification problem to financial markets differs some the standard set-up by assuming a time-varying resource cost of monitoring μ_t on the grounds that, according to the estimates by Levine et al (2004), the cost of default to a lender is greater in a recession than an expansion. The baseline elasticity was chosen to be consistent with their estimates of the magnitude of its variation. As a verification of the sensitivity of the results along this dimension, the last columns of Table 1 reports the effects of reducing this elasticity by 50%. This yields a relative volatility of market tightness of 7.3, still twice the magnitude of the standard search model, while the persistence in the growth rate remains relatively unchanged.

The implications of changing the fraction of vacancy costs requiring external funds is examined in the last columns of Table 3. Such variations alter the elasticity of assets to aggregate shocks, thereby significantly affecting the dynamics of the shadow cost of external funds. For example, calibrating to a leverage ratio of 0.5 generates persistence in the growth rate of market tightness of 0.55, 0.21 and 0.07 at the first, second and third lags. This comes

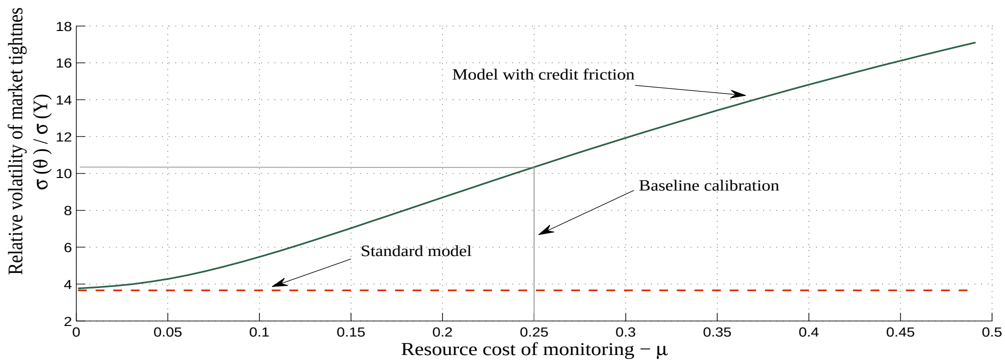

Figure 5: Variations in the resource cost of monitoring μ and the volatility of market tightness

very close to matching the persistence in the growth rate observed in the data. This gain, however, is achieved at the expense of less amplification. The relative standard deviation of market tightness is now 7.60, compared to 10.58 in the baseline calibration. Smaller movements in markets tightness then lead to less movement in unemployment and less persistence in the growth of output. A doubling of the steady state leverage ratio has a modest inverse effect, as can be seen in the last columns of Table 3. Thus, along this dimension, amplification and persistence on labor markets move in opposite direction.

3.4.3 Sensitivity to the calibration of the labor market and the volatility of wages

This section first examines the sensitivity of the main results to changes in the calibration of labor market specific parameters and then examines the dynamics of the wage. With results presented in Table 4, we look at the impact of variations in the unit cost of vacancies, the value of the bargaining weight η and the value of non-market activities b.

Decreasing the unit cost of job vacancies γ from 0.25 to 0.125 implies a lower steady state rate of unemployment such that, though the relative volatility of market tightness is slightly greater than for the baseline calibration, the standard deviation of aggregate output declines from 1.15 to 1.13. Increasing this cost to 0.5 has the exact opposite implication, with a relative standard deviation of market tightness going from 10.58 to 10.46. However, as the stock of unemployed is larger under this scenario, there is slightly more persistence in the growth rate of market tightness.

Table 4: Robustness to labor market parametrization

	Vacancy cost γ				Bargaining weight η				Value of Unemp. b			
	0.125		0.5		0.05		0.9		0.5		0.95	
	a	b	a	b	a	b	a	b	a	b	a	b
U	2.55	-0.76	2.19	-0.73	3.26	-0.73	1.73	-0.72	2.21	-0.74	3.11	-0.86
V	8.88	0.98	9.03	0.96	9.59	0.99	9.00	0.94	7.67	0.98	15.00	0.97
θ	10.72	0.99	10.46	0.98	12.45	0.95	9.96	0.97	9.27	0.98	16.81	0.93
$\sigma(Y)$	1.13		1.17		1.09		1.21		1.09		1.63	
ν:	θ	Y	θ	Y	θ	Y	θ	Y	θ	Y	θ	Y
$corr(\Delta\nu, \Delta\nu_{-1})$	0.40	0.14	0.46	0.14	0.62	0.12	0.43	0.15	0.44	0.10	0.34	0.44
$corr(\Delta\nu, \Delta\nu_{-2})$	-0.01	0.11	0.04	0.15	0.24	0.08	0	0.18	-0.01	0.10	0.05	0.38
$corr(\Delta\nu, \Delta\nu_{-3})$	0	0.02	0	0.08	0.08	0.02	0	0.11	-0.01	0.02	0	0.24

a: Standard deviation relative to output; b: contemporaneous correlation with output.
All moments, but for growth rates, are Hodrick-Prescott filtered;

The current model resulted in a certain degree of wage rigidity contributing to amplification beyond that originating from the cost channel outlined above. In order to gain a sense of the magnitude of the rigidity induced, consider that the elasticity of wages with respect to productivity in U.S. time series is 0.53, as documented in Gertler and Trigari (2009). This elasticity is the cross-product of a contemporaneous correlation between wages and productivity of 0.62 and a relative volatility of wages to productivity of 0.85.[20] The calibration strategy in Hagedorn and Manovskii (2008), which is anchored on this low elasticity of wages in the data, is achieved by a reduction in the relative volatility of wages and not the correlation with productivity (again, see Gertler and Trigari 2009). Both Pissarides (2009) and Haefke et al (2008), however, argue that the empirically relevant wage, that of

[20] For a surveys of wage time series and their properties, see Brandolini (1995) and Abraham and Haltiwanger (1995).

new matches or hires, is characterized by a high degree of volatility and near proportionality with productivity.

The introduction of credit frictions results in a standard deviation of wages relative to productivity of 0.98 , and a cross-correlation with productivity of 0.95. Thus the elasticity of wage in the baseline calibration is 0.93. By comparison, the relative standard deviation of wages in the standard model is 0.92, and the contemporaneous correlation is near unity. Since the time series for the aggregate wage present a low elasticity to productivity, the bargaining weight of workers is reduced to to $\eta = 0.05$, keeping all remaining parameters constant. It is important to stress that this calibration yield interesting results without relying on a small value of the surplus to the firm-worker pair (the replacement ratio is 0.86). First, the elasticity of wages to productivity is now 0.70, the product of a relative volatility of 0.87 and correlation of 0.80, bringing the dynamics of wages much closer to the data. Second, this additional wage rigidity yields a little more amplification and, importantly, much more persistence in the growth rate of market tightness. The latter is now 0.62, 0.24 and 0.08 and the first, second and third lags. The standard deviation of market tightness and job vacancies are now, respectively, 12.45 and 9.59. Thus with a lower value to the worker's bargaining weight, the model does a very good job at matching both the amplification and persistence of labor market variables to productivity shocks. Significantly, the same exercise yields virtually no change in volatility and the persistence of the growth rate of market tightness in the standard model without reducing the size of the labor surplus by increasing the value of non-market activities.

Finally, the significance of a small labor surplus in generating amplification in search models of equilibrium unemployment is illustrated by the last columns of Table 4. If the value of non-market activities is set to $b = 0.95$, the relative standard deviations of market tightness and job vacancies increase to 16.81 and 15, respectively, while the standard deviation of aggregate output increases to 1.63. The changes are less pronounced when the value of non-market activities is reduced to 0.5.

3.4.4 The Beveridge curve and cross-correlations

One concern for extensions to the standard framework is the violation of a robust empirical observation of a strong negative correlation between unemployment and vacancies, or the Beveridge curve. This occurs, for instance, when allowing for jobs to end endogenously as in Mortensen and Pissarides (1994). Table 5 presents the contemporaneous cross-correlations of key labor market variables in the data and generated by the models. In this respect the proposed model offers a moderate improvement on the standard search model of equilibrium unemployment, generating a correlation between unemployment and vacancies of -0.62. This gain is due to the appearance of a positive short run auto-correlation in the growth rate of job vacancies.

The data are also characterized by a very strong negative correlation between the unemployment rate and the measure of labor market tightness, with a contemporaneous correlation of -0.97. The standard model generates a somewhat weak correlation of -0.67. The presence of credit frictions, by inducing persistence in the adjustment of market tightness that mirrors that of unemployment, brings the correlation closer to the data at -0.75. By extension, the proposed model also improves on the correlation between the unemployment and job finding rates.

Finally, the proposed model is able to reduce the correlation between unemployment and labor productivity to -0.57, closer to a correlation of -0.42 in the data. This correlation is too strong in the standard labor search model, which generates a correlation of -0.65. This can be understood from the fact that credit market imperfections, which amplify movements in unemployment that peak several quarters after labor productivity, increase the disconnect between the two variables. Both models fall short, however, of being consistent with the correlations between labor productivity and vacancies or market tightness. These have a mild positive correlation in the data, around 0.4, whereas both models generate very high positive correlations.

Table 5: Labor market cross-correlations

	U.S. data									
	U	V	θ	$f(\theta)$	Y/N					
U	1.00	-0.89	-0.97	-0.95	-0.41					
V	-	1	0.98	0.90	0.36					
θ	-	-	1	0.95	0.40					
$f(\theta)$	-	-	-	1	0.40					
Y/N	-	-	-	-	1					
	Labor search					Labor search - Credit friction				
	U	V	θ	$f(\theta)$	Y/N	U	V	θ	$f(\theta)$	Y/N
U	1	-0.52	-0.67	-0.67	-0.65	1	-0.62	-0.75	-0.75	-0.57
V	-	1	0.98	0.98	0.99	-	1	0.98	0.98	0.98
θ	-	-	1	1	0.99	-	-	1	1	0.96
$f(\theta)$	-	-	-	1	0.99	-	-	-	1	0.96
Y/N	-	-	-	-	1	-	-	-	-	1

All moments are Hodrick-Prescott filtered; Data sources: BLS, BEA and Fujita and Ramey (2008).

3.5 Extension to financing the wage bill and vacancy costs

Vacancy costs represent a small fraction of operating expenses and a natural issue is whether the results are robust to firms financing a fraction of both the wage bill and vacancy costs externally. This first extension reinforces the argument that the key effect of credit market imperfections for job creation is to alter the evolution of the opportunity cost resources used to recruit workers independently of the assumption on the fraction for current costs funded externally.

Define firm period revenue as xX_tN_t and assume now that vacancy costs γV_t and the wage bill W_tN_t must be paid before production occurs. All agents observe the aggregate state X_t and, given initial assets A_t, firms borrow $(\gamma V_t + W_tN_t - A_t)$ from financial markets to pay for period operating costs. Again, lenders and borrowers agree on a contract that specifies job vacancies and a cutoff productivity $\overline{x}$ such that if $x > \overline{x}$, the borrower pays $\overline{x}_tX_tN_t$ and keeps the equity $(x - \overline{x}_t)\, X_tN_t$. If $x < \overline{x}$, the borrower receives nothing and the lender claims the residual net of monitoring costs. The expected gross share of returns going to the lender and expected monitoring costs retain the same form, such that the lender's participation constraint is now $[\Gamma(\overline{x}_t) - \mu_tG(\overline{x}_t)]\, X_tN_t = (\gamma V_t + W_tN_t - A_t)$, which again states that the returns net of monitoring costs must equal the value of the loan. Aggregate assets now evolve according to $A_{t+1} = \varsigma\, [1 - \Gamma(\overline{x}_t)]\, X_tN_t$ and the premium associated with external funds is expressed as $\frac{\mu_tG(\overline{x}_t)X_tN_t}{\gamma V_t + W_tN_t - A_t}$, which for any $\mu_t > 0$ is strictly positive. The modified optimal incentive compatible contracting problem with non-stochastic monitoring and repayment within the period is now:

$$
\begin{aligned}
J_t &= \max_{V_t, \overline{x}_t} [1 - \Gamma(\overline{x}_t)]\, X_tN_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1} \\
& \quad \text{subject to } [\Gamma(\overline{x}_t) - \mu_tG(\overline{x}_t)]\, X_tN_t = (\gamma V_t + W_tN_t - A_t) \\
& \quad \text{and} \qquad N_{t+1} = (1-\delta)N_t + V_tp(\theta_t)
\end{aligned}
$$

3.5.1 Job creation and wages

Retaining the notation for the multiplier on the lender's participation constraint, ϕ, the optimality condition for vacancy postings describes a job creation condition $\frac{\gamma\phi_t}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{N,t+1}$ in which the marginal value of a worker to the firm, $J_{N,t}$ is now $J_{N,t} = [1 - \Gamma(\overline{x}_t)]\, X_t + \phi_t\left([\Gamma(\overline{x}_t) - \mu_tG(\overline{x}_t)]\, X_t - W_t\right) + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{N,t+1}$, with a similar interpretation as earlier. Making use of $\Omega(\overline{x}_t) \equiv 1 - \Gamma(\overline{x}_t) + \phi_t \left[\Gamma(\overline{x}_t) - \mu_t G(\overline{x}_t)\right]$ to simplify the notation, the intertemporal condition for vacancy postings in this extension is

$$\frac{\gamma\phi_t}{p(\theta_t)} = \frac{1}{1+r_t} E_t \left[\Omega(\overline{x}_{t+1}) X_{t+1} - \phi_{t+1} W_{t+1} + (1-\delta) \frac{\gamma\phi_{t+1}}{p(\theta_{t+1})}\right] \tag{11}$$

Relative to earlier, the term $\phi_{t+1}W_{t+1}$ on the right hand side of the expression captures the fact that opportunity cost of wages paid to a new hire for the firm depends on the degree of credit constraint. Note also that the expression for the shadow cost of external funds remains $\phi_t = \frac{\Gamma'(\overline{x}_t)}{[\Gamma'(\overline{x}_t) - \mu_t G'(\overline{x}_t)]}$.

The wage is again determined by splitting the surplus of a worker-firm match under a generalization of Nash bargaining, yielding

$$W_t = \eta \left[\frac{\Omega(\overline{x}_t)}{\phi_t} X_t + \gamma\theta_t\right] + (1-\eta)b \tag{12}$$

As earlier, both the job creation condition (11) and the wage rule (12) collapse to (8) and (10) when monitoring costs μ tend to 0.

3.5.2 Results

This extension follows the calibration strategy for the baseline model, adopting the specification of a low value of the worker's bargaining weight as this brings the model closer generating the degree persistence in the growth rate market tightness seen in the data. With regards to amplification, as reported in Table 6, the results are broadly similar to the previous model, although the relative volatility of market tightness is slightly lower. The main difference appears in the persistence in the growth rate of market tightness, the 'hump' in the response being less pronounced. This is seen in lower measures of order auto-correlation at the first lag. Table 6 also presents the contemporaneous cross-correlations of labor market variables. Once again, the results are robust to this extension to financing a fraction of the wage bill on imperfect credit markets.

Table 6: Unconditional 2nd moments - extension to current costs

	Baseline model with $\eta = 0.05$		Financing wage and vacancy costs	
Variable:	a	b	a	b
U	3.26	-0.73	2.82	-0.73
V	9.59	0.99	9.74	0.99
θ	12.45	0.95	11.86	0.99
$\sigma(Y)$	1.09		1.15	
ν:	θ	Y	θ	Y
$corr(\Delta\nu, \Delta\nu_{-1})$	0.62	0.12	0.24	0.18
$corr(\Delta\nu, \Delta\nu_{-2})$	0.24	0.08	0.16	0.09
$corr(\Delta\nu, \Delta\nu_{-3})$	0.08	0.02	0.11	0.05

Contemporaneous cross-correlations:

	U	V	θ	$f(\theta)$	Y/N
U	1	-0.68	-0.80	-0.80	-0.55
V	-	1	0.98	0.98	0.98
θ	-	-	1	1	0.94
$f(\theta)$	-	-	-	1	0.94
Y/N	-	-	-	-	1

a: Standard deviation relative to output; b: contemporaneous correlation with output. All moments, but growth rates, are Hodrick-Prescott filtered; Data sources: BLS, BEA.

4 Conclusion

It has been argued that the standard search and matching model of equilibrium unemployment cannot generate sufficient propagation as productivity shocks, by inducing a rise in wages, have little effect on firm profits from a new employee and, hence, on the incentive to post job vacancies. This paper has shown that when vacancies must be funded in part on frictional credit markets, agency problems can lead to higher, time-varying, opportunity costs of the resources involved that greatly increase the elasticity of vacancies to productivity.

The quantitative exercise has shown that this financial accelerator contributes significantly to bringing the model closer to the cyclical fluctuations of labor market variables found in the data, both in terms of volatility and persistence. The paper thus concludes that the dynamics of the opportunity of resources allocated to recruiting workers are an essential element in understanding the cyclical behavior of job creation and the dynamics of the labor market, echoing the conclusions in Fujita and Ramey (2007) and Pissarides (2009). The originality here is that these costs evolve endogenously as a function of credit market conditions and can simultaneously address the lack of amplification and persistence to productivity shocks. While the macroeconomic consequences of credit market imperfections have generally focused on their consequences for capital investment, e.g. models of financial intermediation and agency costs by Bernanke et al. (1999) or Kiyotaki and Moore (1997), this paper finds that their implications for labor markets should not be overlooked.

Two questions remain that warrant further investigation in subsequent research. First, how general these results are to the type of friction present on credit markets is an open question. This can, however, be partially addressed by considering that any friction which will generate a counter-cyclical premium on external resources will have the same qualitative implications. Second, if hiring is conditional on the state of credit markets, it may be that worker flows, as opposed to investment in new capital goods, are an alternative channel for the transmission of monetary policy shocks that affect the cost of credit. This avenue seems particularly promising as the propagation mechanism in the paper can be interpreted as increasing the rigidity of the firm's marginal cost to changes in production. Often referred in the New Keynesian literature as a greater degree of real rigidity, this property is known to be essential for understanding the dynamics of inflation and for allowing any significant scope to monetary policy.

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Credit, Vacancies and Unemployment Fluctuations

Detailed appendix - not intended for publication

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A Data sources

Job vacancies are measured using the Conference Board's Help-Wanted Index. The unemployment rate corresponds to the B.L.S series LNS14000000. The job finding rate was provided by Shigeru Fujita and Garey Ramey and is based on C.P.S. data. The raw monthly series were first adjusted by a 12 month backward-looking moving average. Quarterly series were then computed by averaging over monthly observations. Output was obtained from the B.E.A. as Expenditure based and measured in 2000 chained dollars. Interest rate spreads are calculated using data on Moody's Seasoned Aaa and Baa Corporate Bond yield.

B Equilibrium search unemployment with credit market imperfections

This section details the derivation of the wage rule under Nash bargaining, the equilibrium system of equations and the method for computing the steady state.

B.1 Wage determination

Define the surplus to the worker-firm relationship as $S_t = J_{N,t} + \frac{\mathcal{H}_{N,t} - \mathcal{H}_{U,t}}{\lambda_t}$, where the firm's surplus is $J_{N,t}$ and the worker's surplus is $\frac{\mathcal{H}_{N,t} - \mathcal{H}_{U,t}}{\lambda_t}$. Using the definition for each marginal value, the joint surplus is expressed as:

$$
\begin{aligned}
S_t &= \Omega(\overline{x}_t)\left(X_t - W_t\right) + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{N,t+1} \\
&\quad + W_t + \frac{\beta}{\lambda_t} E_t \left[(1-\delta)\mathcal{H}_{N,t+1} + \delta \mathcal{H}_{U,t+1}\right] \\
&\quad - b - \frac{\beta}{\lambda_t} E_t \left[(1-f(\theta_t))\mathcal{H}_{U,t+1} + f(\theta_t)\mathcal{H}_{N,t+1}\right] \\
S_t &= \Omega(\overline{x}_t) X_t + \left(1 - \Omega(\overline{x}_t)\right) W_t - b \\
&\quad + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[J_{N,t+1} + \frac{\mathcal{H}_{N,t+1} - \mathcal{H}_{U,t+1}}{\lambda_{t+1}}\right] \\
&\quad - f(\theta_t)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{\mathcal{H}_{N,t+1} - \mathcal{H}_{U,t+1}}{\lambda_{t+1}}\right]
\end{aligned}
$$

Nash bargaining consists in choosing a wage that satisfies $argmax \left(\frac{\mathcal{H}_{N,t}-\mathcal{H}_{U,t}}{\lambda_t}\right)^{\eta}\left(J_{N,t}\right)^{1-\eta}$. The optimality condition describes a sharing rule $\nu_t J_{N,t} = (1-\nu_t)\left(\frac{\mathcal{H}_{N,t}-\mathcal{H}_{U,t}}{\lambda_t}\right)$, where $\nu_t = \frac{\eta}{\eta+(1-\eta)\Omega(\overline{x}_t)}$, from which we have $J_{N,t} = (1-\nu_t)S_t$ and $\frac{\mathcal{H}_{N,t}-\mathcal{H}_{U,t}}{\lambda_t} = \nu_t S_t$. Using this result, the above expression for the joint surplus can be rewritten as

$$
S_t = \Omega(\overline{x}_t) X_t + \left(1 - \Omega(\overline{x}_t)\right) W_t - b + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1} - f(\theta_t)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\nu_t S_{t+1}
$$

Noting that the optimality condition for vacancy postings can be expressed as $\frac{\gamma\phi_t}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\nu_t)S_{t+1}$, we now have

$$
\begin{aligned}
S_t &= \Omega(\overline{x}_t) X_t + \left(1 - \Omega(\overline{x}_t)\right) W_t - b + (1-\delta)\frac{\gamma\phi_t}{p(\theta_t)(1-\nu_t)} - \nu_t f(\theta_t)\frac{\gamma\phi_t}{p(\theta_t)(1-\nu_t)} \\
(1-\nu_t)S_t &= (1-\nu_t)\left[\Omega(\overline{x}_t) X_t + \left(1 - \Omega(\overline{x}_t)\right) W_t - b\right] + (1-\delta)\frac{\gamma\phi_t}{p(\theta_t)} - \nu_t \gamma\phi_t\theta_t
\end{aligned}
$$

Equating this expression with the marginal value of an addition worker $J_{N,t} = (1-\nu_t)S_t = \Omega(\overline{x}_t)(X_t - W_t) + (1-\delta)\frac{\gamma\phi_t}{p(\theta_t)}$ yields

$$[1 + \nu_t(\Omega(\overline{x}_t) - 1)] W_t = \nu_t [\Omega(\overline{x}_t)X_t + \gamma\phi_t\theta_t] + (1-\nu_t)b$$

Finally, after a little algebra, we obtain the wage rule described in Section 2

$$W_t = \eta\left[X_t + \gamma\frac{\phi_t}{\Omega(\overline{x}_t)}\theta_t\right] + (1-\eta)b$$

B.2 Equilibrium system of equations

The following 20 equations define the endogenous variables Y_t, C_t, N_t, U_t, V_t, θ_t, $p(\theta_t)$, $f(\theta_t)$, W_t, r_t, λ_t, A_t, ϕ_t, $\overline{x}_t$, $\Gamma(\overline{x}_t)$, $\Gamma'(\overline{x}_t)$, $G(\overline{x}_t)$, $G'(\overline{x}_t)$, $\Omega(\overline{x}_t)$, μ_t,

$$\begin{aligned}
\frac{\gamma\phi_t}{p(\theta_t)} &= \frac{1}{1+r_t}E_t\left[\Omega(\overline{x}_{t+1})(X_{t+1} - W_{t+1}) + (1-\delta)\frac{\gamma\phi_{t+1}}{p(\theta_{t+1})}\right] \\
W_t &= \eta\left[X_t + \gamma\frac{\phi_t}{\Omega(\overline{x}_t)}\theta_t\right] + (1-\eta)b \\
1/C_t &= \lambda_t \\
\lambda_t &= \beta E_t\lambda_{t+1}(1+r_t) \\
Y_t[1 - \mu_t G(\overline{x}_t)] &= C_t + \gamma V_t \\
Y_t &= X_t N_t \\
N_{t+1} &= (1-\delta)N_t + p(\theta_t)V_t \\
U_{t+1} &= (1 - f(\theta_t))U_t + \delta N_t \\
A_{t+1} &= \varsigma[1 - \Gamma(\overline{x}_t)](X_t - W_t)N_t \\
\theta_t &= \frac{V_t}{U_t}
\end{aligned}$$

$$
\begin{aligned}
f(\theta_t) &= \xi \theta_t^{1-\epsilon} \\
p(\theta_t) &= \xi \theta_t^{-\epsilon} \\
\phi_t &= \frac{\Gamma'(\overline{x}_t)}{[\Gamma'(\overline{x}_t) - \mu_t G'(\overline{x}_t)]} \\
(\gamma V_t - A_t) &= [\Gamma(\overline{x}_t) - \mu_t G(\overline{x}_t)] (X_t - W_t) N_t \\
\Gamma(\overline{x}_t) &= \int_0^{\overline{x}_t} x dH(x) + \int_{\overline{x}_t}^{\infty} \overline{x}_t dH(x) \\
G(\overline{x}_t) &= \int_0^{\overline{x}_t} x dH(x) \\
\Gamma'(\overline{x}_t) &= 1 - H(\overline{x}_t) \\
G'(\overline{x}_t) &= \overline{x}_t h(\overline{x}_t) \\
\mu_t &= g(X_t) \\
\Omega(\overline{x}_t) &= [1 - \Gamma(\overline{x}_t)] + \phi_t [\Gamma(\overline{x}_t) - \mu_t G(\overline{x}_t)]
\end{aligned}
$$

B.3 Solving the steady state

Given a target steady state quarterly rate of default and the assumption of log-normality of the distribution $H(x)$, ϕ and $\Omega(\overline{x})$ are pinned down for a value of the standard deviation of idiosyncratic productivity shocks σ_x. The steady state market tightness, given choices on the parameters β, γ, ξ, ϵ, η and b, is found by solving $\gamma \left(\frac{(r+\delta)}{\xi} \theta^{\epsilon} + \eta \theta \right) \Phi(\overline{x}) = (1-\eta)(1-b)$. The remaining labor market variables are straightforward to compute, noting that ξ is adjusted to achieve a desired level of unemployment. ς is found such that the asset accumulation and lender participation equations hold, and σ_x is chosen such that we obtain a steady state leverage ratio of 2.

C Deriving the standard search and matching model of equilibrium unemployment in discrete time

This section details what this paper terms the standard Mortensen-Pissarides search model of equilibrium unemployment in discrete time. It is essentially drawn from the work of Monica Merz (1996) and David Andolfatto (1997) in which workers are members of a representative household and search passively on the labor market. Firms post job vacancies V_t to attract unemployed workers U_t at a unit cost of γ. Jobs are filled via a constant returns to scale matching function taking vacancies and unemployed workers $M(U_t, V_t)$. Define $\theta_t = \frac{V_t}{U_t}$ as labor market tightness from the point of view of the firm, or the v-u ratio. The matching probabilities are $\frac{M(U_t,V_t)}{V_t} = p(\theta_t)$ and $\frac{M(U_t,V_t)}{U_t} = f(\theta_t)$ for firms and workers respectively, with $\partial p(\theta_t)/\partial\theta_t < 0$ and $\partial f(\theta_t)/\partial\theta_t > 0$. Note that $f(\theta_t) = \theta_t p(\theta_t)$. Once matched, jobs are destroyed at the exogenous rate δ per period. Thus employment N_t and unemployment U_t evolve according to

$$N_{t+1} = (1-\delta)N_t + p(\theta_t)V_t \tag{13}$$

$$U_{t+1} = (1 - f(\theta_t))U_t + \delta N_t \tag{14}$$

The representative household, given existing employment and unemployment, chooses optimal consumption and purchases of risk free bonds, which pay a rate r_t the following period, in order to maximize the value function:[21]

$$\mathcal{H}_t = \max_{C_t, B_t} \left[\mathcal{U}(C_t) + \beta E_t \mathcal{H}_{t+1}\right],$$

[21] As in Andolfatto (1996), each worker is a member of a household that offers perfect insurance against labor market outcomes and is involved in a passive search process.

subject to the budget constraint $W_t N_t + bU_t + (1+r_{t-1})B_{t-1} + \Pi_t = C_t + B_t + T_t$, and the laws of motion for matched labor (13) and unemployment (14). The government raises T_t in taxes to fund unemployment benefits $U_t b$, while employed workers earn the wage W_t. Π_t are firm dividends rebated lump sum at the end of the period. Denoting the multiplier on the budget constraint by λ, the first order conditions are

$$(C_t) \quad : \quad \mathcal{U}_C(C_t) = \lambda_t \tag{15}$$

$$(B_t) \quad : \quad \lambda_t = \beta E_t \lambda_{t+1}(1+r_t) \tag{16}$$

Firms post job vacancies to maximize their expected value

$$J_t \quad = \quad \max_{V_t} \left(X_t - W_t\right) N_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}$$

subject to the law of motion for employment (13) using the stochastic discount factor $\beta\frac{\lambda_{t+1}}{\lambda_t}$. The optimality condition for vacancy postings describes a job creation condition

$$\frac{\gamma}{p(\theta_t)} \quad = \quad \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{N,t+1}$$

equating the average cost of a vacancy, $\frac{\gamma}{p(\theta_t)}$, to the expected marginal value of an additional employed worker $\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{N,t+1}$. In order to derive the marginal value of a worker to the firm, $J_{N,t}$, differentiate the firm's value function with respect to N: $J_{N,t} = (X_t - W_t) + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{N,t+1}$. Combining the marginal value of a worker with the optimality condition for vacancies, and making use of the household bond Euler equation (16), yields the intertemporal condition for vacancy postings

$$\frac{\gamma}{p(\theta_t)} = \frac{1}{1+r_t} E_t \left[X_{t+1} - W_{t+1} + (1-\delta) \frac{\gamma}{p(\theta_{t+1})} \right] \tag{17}$$

The model is fully described once the rule for wages is determined. In order to define the values of a job ($\mathcal{H}_N$) and unemployment ($\mathcal{H}_U$) to a worker, differentiate the household's value function with respect to N and U:

$$\begin{aligned}
\mathcal{H}_{N,t} &= \lambda_t W_t + \beta E_t \left[(1-\delta)\mathcal{H}_{N,t+1} + \delta \mathcal{H}_{U,t+1}\right] \\
\mathcal{H}_{U,t} &= \lambda_t b + \beta E_t \left[(1-f(\theta_t))\mathcal{H}_{U,t+1} + f(\theta_t)\mathcal{H}_{N,t+1}\right]
\end{aligned}$$

which as the same interpretation as in the text. Splitting the surplus of a worker-firm match, defined as $S_t = J_{N,t} + \frac{\mathcal{H}_{N,t} - \mathcal{H}_{U,t}}{\lambda_t}$, under a generalization of Nash bargaining yields the wage rule

$$W_t = \eta \left[X_t + \gamma \theta_t\right] + (1-\eta) b \tag{18}$$

C.1 Equilibrium system of equations

The following 11 equations define the endogenous variables Y_t, C_t, N_t, U_t, V_t, θ_t, $p(\theta_t)$, $f(\theta_t)$, W_t, r_t, λ_t.

$$\begin{aligned}
\frac{\gamma}{p(\theta_t)} &= \frac{1}{1+r_t} E_t \left[X_{t+1} - W_{t+1} + (1-\delta) \frac{\gamma}{p(\theta_{t+1})} \right] \\
W_t &= \eta \left[X_t + \gamma \theta_t\right] + (1-\eta) b \\
1/C_t &= \lambda_t \\
\lambda_t &= \beta E_t \lambda_{t+1} (1 + r_t)
\end{aligned}$$

$$
\begin{aligned}
Y_t &= C_t + \gamma V_t \\
Y_t &= X_t N_t \\
N_{t+1} &= (1-\delta)N_t + p(\theta_t)V_t \\
U_{t+1} &= (1 - f(\theta_t))U_t + \delta N_t \\
\theta_t &= \frac{V_t}{U_t} \\
f(\theta_t) &= \xi \theta_t^{1-\epsilon} \\
p(\theta_t) &= \xi \theta_t^{-\epsilon}
\end{aligned}
$$

C.2 Computing the steady state

Steady state market tightness, given choices on the parameters β, γ, ξ, ϵ, η and b, is found by solving $\gamma\left(\frac{(r+\delta)}{\xi}\theta^{\epsilon} + \eta\theta\right) = (1-\eta)\left[1-b\right]$. The remaining labor market variables are straightforward to compute, noting that ξ is adjusted to achieve a desired level of unemployment.

D Extension to financing wage bill and vacancy costs

D.1 Deriving the wage rule

This subsection details the steps to obtaining the wage rule of section 4.1. The joint firm-worker surplus in this scenario is given by:

$$
\begin{aligned}
S_t &= \Omega(\overline{x}_t)X_t - \phi_t W_t + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{N,t+1} \\
&\quad + W_t + \frac{\beta}{\lambda_t} E_t \left[(1-\delta)\mathcal{H}_{N,t+1} + \delta \mathcal{H}_{U,t+1}\right] \\
&\quad - b - \frac{\beta}{\lambda_t} E_t \left[(1-f(\theta_t))\mathcal{H}_{U,t+1} + f(\theta_t)\mathcal{H}_{N,t+1}\right] \\
S_t &= \Omega(\overline{x}_t)X_t + (1-\phi_t)W_t - b \\
&\quad + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[J_{N,t} + \frac{\mathcal{H}_{N,t+1} - \mathcal{H}_{U,t+1}}{\lambda_{t+1}}\right] \\
&\quad - f(\theta_t)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{\mathcal{H}_{N,t+1} - \mathcal{H}_{U,t+1}}{\lambda_{t+1}}\right]
\end{aligned}
$$

The solution to the Nash bargaining process results in the surplus being split according to $J_{N,t} = (1-\nu_t)S_t$ and $\frac{\mathcal{H}_{N,t} - \mathcal{H}_{U,t}}{\lambda_t} = \nu_t S_t$, where in this case $\nu_t = \frac{\eta}{\eta + (1-\eta)\phi_t}$. Using this result, the above expression for the joint surplus can be rewritten as

$$
S_t = \Omega(\overline{x}_t)X_t + (1-\phi_t)W_t - b + (1-\delta)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1} - f(\theta_t)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\nu_t S_{t+1}
$$

Noting that the optimality condition for vacancy postings can be expressed as $\frac{\gamma\phi_t}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\nu_t)S_{t+1}$, we now have

$$
\begin{aligned}
S_t &= \Omega(\overline{x}_t)X_t + (1-\phi_t)W_t - b + (1-\delta)\frac{\gamma\phi_t}{p(\theta_t)(1-\nu_t)} - \nu_t f(\theta_t)\frac{\gamma\phi_t}{p(\theta_t)(1-\nu_t)} \\
(1-\nu_t)S_t &= (1-\nu_t)\left[\Omega(\overline{x}_t)X_t + (1-\phi_t)W_t - b\right] + (1-\delta)\frac{\gamma\phi_t}{p(\theta_t)} - \nu_t\gamma\phi_t\theta_t
\end{aligned}
$$

Equating this expression with the marginal value of an additional worker $J_{N,t} = (1-\nu_t)S_t = \Omega(\overline{x}_t)X_t - \phi_t W_t + (1-\delta)\frac{\gamma\phi_t}{p(\theta_t)}$ yields

$$[1+\nu_t(\phi_t - 1)]\,W_t \;=\; \nu_t\left[\Omega(\overline{x}_t)X_t + \gamma\phi_t\theta_t\right] + (1-\nu_t)b$$

Finally, we obtain the wage rule

$$W_t \;=\; \eta\left[\frac{\Omega(\overline{x}_t)}{\phi_t}X_t + \gamma\theta_t\right] + (1-\eta)b$$

D.2 Equilibrium system of equations

The following 20 equations define the endogenous variables Y_t, C_t, N_t, U_t, V_t, θ_t, $p(\theta_t)$, $f(\theta_t)$, W_t, r_t, λ_t, A_t, ϕ_t, $\overline{x}_t$, $\Gamma(\overline{x}_t)$, $\Gamma'(\overline{x}_t)$, $G(\overline{x}_t)$, $G'(\overline{x}_t)$, $\Omega(\overline{x}_t)$, μ_t,

$$\begin{aligned}
\frac{\gamma\phi_t}{p(\theta_t)} &= \frac{1}{1+r_t}E_t\left[\Omega(\overline{x}_{t+1})X_{t+1} - \phi_{t+1}W_{t+1} + (1-\delta)\frac{\gamma\phi_{t+1}}{p(\theta_{t+1})}\right] \\
W_t &= \eta\left[\frac{\Omega(\overline{x}_t)}{\phi}X_t + \gamma\theta_t\right] + (1-\eta)b \\
1/C_t &= \lambda_t \\
\lambda_t &= \beta E_t\lambda_{t+1}(1+r_t) \\
Y_t\left[1-\mu_t G(\overline{x}_t)\right] &= C_t + \gamma V_t \\
Y_t &= X_t N_t \\
N_{t+1} &= (1-\delta)N_t + p(\theta_t)V_t \\
U_{t+1} &= (1-f(\theta_t))U_t + \delta N_t \\
A_{t+1} &= \varsigma\left[1-\Gamma(\overline{x}_t)\right]X_t N_t \\
\theta_t &= \frac{V_t}{U_t}
\end{aligned}$$

$$
\begin{aligned}
f(\theta_t) &= \xi\theta_t^{1-\epsilon} \\
p(\theta_t) &= \xi\theta_t^{-\epsilon} \\
\phi_t &= \frac{\Gamma'(\overline{x}_t)}{[\Gamma'(\overline{x}_t) - \mu_t G'(\overline{x}_t)]} \\
(W_t N_t + \gamma V_t - A_t) &= [\Gamma(\overline{x}_t) - \mu_t G(\overline{x}_t)]\, X_t N_t \\
\Gamma(\overline{x}_t) &= \int_0^{\overline{x}_t} x dH(x) + \int_{\overline{x}_t}^{\infty} \overline{x}_t dH(x) \\
G(\overline{x}_t) &= \int_0^{\overline{x}_t} x dH(x) \\
\Gamma'(\overline{x}_t) &= 1 - H(\overline{x}_t) \\
G'(\overline{x}_t) &= \overline{x}_t h(\overline{x}_t) \\
\mu_t &= g(X_t) \\
\Omega(\overline{x}_t) &= [1 - \Gamma(\overline{x}_t)] + \phi_t\, [\Gamma(\overline{x}_t) - \mu_t G(\overline{x}_t)]
\end{aligned}
$$

D.3 Solving the steady state

Given a target steady state quarterly rate of default and the assumption of log-normality of the distribution $H(x)$, ϕ and $\Omega(\overline{x})$ are pinned down for a value of the standard deviation of idiosyncratic productivity shocks σ_x. The steady state market tightness, given choices on the parameters β, γ, ξ, ϵ, η and b, is found by solving $\gamma\left(\frac{(r+\delta)}{\xi}\theta^{\epsilon} + \eta\theta\right) = (1-\eta)\left(\frac{1}{\Phi(\overline{x})} - b\right)$. The remaining labor market variables are straightforward to compute, noting that ξ is adjusted to achieve a desired level of unemployment. ς is found such that the asset accumulation and lender participation equations hold, and σ_x is chosen such that we obtain a steady state leverage ratio of 2.