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Should we transfer resources from college to basic education?*

Marisa Hidalgo-Hidalgo and Íñigo Iturbe-Ormaetxe**

Abstract

This paper analyzes public intervention in education, taking into account the existence of two educational levels: basic education and college education. The government decides per capita expenditure at each level and the subsidy for college education. We explore the effects of transferring resources from one level to the other on equity and efficiency, where efficiency refers to average productivity of college graduates. Except in the special case in which the economy is at the Equity-Efficiency Frontier (EEF), there is always a policy reform that increases the productivity of college graduates without excluding the talented poor from college. For developed countries, this policy consists of transferring resources from college to basic education.

Keywords: Basic education, college education, public expenditure in education

JEL Classifcation: H52, I28, J24

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1 Introduction

In most countries, public expenditure on education accounts for a large proportion of total expenditure on education. For the OECD countries, an average of 87% of expenditure on all levels of education came from public sources in 2004.[1] Public intervention is present at all educational levels, from pre-primary to tertiary education. However, countries differ dramatically according to how they allocate resources across the different educational levels. Columns 1 and 2 of Table 1 show data on yearly per student expenditure at basic and tertiary education, respectively.[2] In Column 3 we compute the ratio between expenditure per student in tertiary education and in basic education. We observe a large heterogeneity. The ratio ranges from 1.00 in Italy to 3.23 in Mexico, with an average for the OECD of 1.68. Columns 4 and 5 show the change in annual expenditure per student from 1995 to 2004 and the ratio between both indexes is reported in Column 6. Fifteen countries out of twenty one have a ratio lower than one, meaning that in this period they have diverted resources from tertiary to basic education, at least in relative terms.

Since Table 1 refers only to annual expenditure, we present some additional evidence in Table 2. Here we show data compiled by the OECD on *cumulative* expenditure at basic and tertiary education, taking into account the duration of each educational level. Again we observe large differences across countries. In Column 3 we compute the ratio between cumulative expenditure at both levels and we see that it ranges from 0.36 in Korea and New Zealand to 0.98 in The Netherlands.

[1] See Table B3.1, Education at a Glance 2007, OECD.

[2] Basic education corresponds to primary, secondary, and post-secondary non-tertiary education.

Table 1. Expenditure on education by level, 2004[a]

	Expenditure per student[b]			Change 1995-2004[c]		
Country	Basic	Tertiary	T/B	Basic	Tertiary	T/B
Australia	6,911	14,036	2.03	138	101	0.73
Austria	8,938	13,959	1.56		122	
Belgium	7,596	11,842	1.62			
Czech Rep.	5,490	6,752	1.68	124	69	0.56
Denmark	8,492	15,225	1.79	121	123	1.02
Finland	6,660	12,505	1.88	122	110	0.90
France	7,262	10,668	1.47			
Germany	6,983	12,255	1.75	105	107	1.02
Greece	4,931	5,593	1.13	192	151	0.79
Hungary	3,833	7,095	1.85	157	73	0.47
Iceland	8,138	8,881	1.09			
Ireland	6,034	10,211	1.69	181	126	0.70
Italy	7,741	7,723	1.00	105	130	1.23
Japan	7,105	12,193	1.72	127	101	0.79
Korea	5,550	7,068	1.27			
Mexico	1,789	5,778	3.23	130	110	0.85
Netherlands	6,914	13,846	2.00	136	101	0.75
New Zealand	5,815	8,866	1.52			
Norway	9,772	14,997	1.53	109	103	0.95
Poland	2,998	4,412	1.47	183	90	0.49
Portugal	5,400	7,741	1.43	154	98	0.64
Slovak Rep.	2,562	6,535	2.55	155	111	0.71
Spain	5,892	9,378	1.59	136	167	1.24
Sweden	7,744	16,218	2.09	117	99	0.84
Switzerland	10,378	21,966	2.12	105	134	1.28
UK	6,656	11,484	1.73	120	93	0.78
USA	9,368	22,476	2.40	130	132	1.01
OECD average	**6,608**	**11,100**	**1.68**	**138**	**109**	**0.79**

[a]Source: Education at a Glance 2007, Tables B1.1a and B1.5 and authors' calculations.

[b]Annual expenditure per student in US dollars, using PPPs.

[c]Index of change in annual expenditure per student, setting expenditure in 1995 at 100.

Table 2: Cumulative public expenditure on education, 2004[a]

	Educational Levels[b]					
Country	Basic	Tertiary	T/B	Subs.[c]	C. Att.[d]	Expenditure[e]
Austria	106,396	73,983	0.70	93.7	37	120.063
Belgium	86,320	35,406	0.41	90.4	34	99.317
Denmark	109,777	56,332	0.51	96.7	55	131.468
Finland	79,900	60,659	0.76	96.3	73	101.830
France[1]	86,406	42,884	0.50	83.9	39	100.551
Germany	87,659	65,732	0.75	86.4	37	100.437
Greece	58,850	29,361	0.50	97.9	33	66.157
Hungary	47,469	36,353	0.77	79.0	68	53.090
Iceland	113,213	32,770	0.29	90.9	79	123.877
Ireland	82,479	33,083	0.40	82.6	44	93.587
Italy	103,871	55,751	0.54	69.4	55	110.102
Japan	84,930	49,624	0.58	41.2	43	95.804
Korea	67,567	24,242	0.36	21.0	48	70.162
Mexico	22,662	19,761	0.87	68.9	29	25.128
Netherlands	74,339	72,555	0.98	77.6	56	94.256
N. Zealand	74,745	27,042	0.36	60.8	89	79.803
Slovak Rep.	32,856	25,484	0.78	81.3	47	36.230
Spain	69,993	43,699	0.62	75.9	44	83.171
Sweden	92,979	75,901	0.82	88.4	79	117.997
Turkey	15,396	12,474	0.81	90.0	26	16.724
UK	81,732	49,872	0.61	69.6	52	93.896
OECD average	**75,216**	**43,951**	**0.61**	**78.2**	**32**	**86.364**

[a]Source: Education at a Glance 2007, Table B1.3b and authors' calculations.

[b]Cumulative expenditure per student in 2004. In equivalent US dollars converted using PPPS for GDP.

[c]Proportion of public expenditure in tertiary education in 2004. Source: Education at a Glance 2007, Table B3.2b.

[d]Entry rates into tertiary-type A programmes for 2004. Source: Education at a Glance 2006, Table C2.1.

[e]Total expenditure on education. Authors' calculations.

[1]Year of reference for C. Att.: 2003.

Tables 1 and 2 document the existence of large differences in educational policies across countries. One possible explanation is that each country has different objectives.[3] In fact, the role of education is at the heart of current and quite diverse debates, such as poverty reduction or the challenge of new technologies. A crucial question when analyzing public intervention is to establish the objectives of educational policies. Most governments care for efficiency and equity issues in a wide sense. However, sometimes the problem is to give a precise meaning to these general principles. To circumvent this problem we propose that equity concerns imply that the objective of the government should be to facilitate everybody the access to education, irrespective of family background. Regarding efficiency we propose two alternative objectives. The first objective consists of maximizing the productivity of college students, while the second one is to maximize the average productivity of the whole population.[4] We study how these two objectives relate to each other and then we analyze which policies should implement the government to achieve efficiency and equity at the same time. In particular, we want to study whether both objectives are compatible or not and, if they are, which policies makes them compatible. Second, we explore whether all countries, rich and poor, should apply the same policy to satisfy these two objectives or if the policy reform is country-specific.

To study these issues, and in line with the data in Tables 1 and 2, we build a model with two educational stages: basic and college education. Basic education comprises all mandatory levels of education and we assume it is fully financed by the government. In contrast, college education is voluntary and students may have to pay a part of the cost. Another difference is that expenditure on basic education affects the quality of education, but not enrollment, since attendance is mandatory. On the other hand, expenditure on college education affects not only quality, but also enrollment. Individuals who go to college get a skilled job, while the rest remain unskilled. Due to capital markets imperfections, some individuals suffer from borrowing constraints.

In our model, any public policy is characterized by two variables corresponding to expenditure on basic and college education, respectively. We define the Equity-Efficiency frontier (EEF) as the set of public policies for which it is not possible to improve the two objectives of the government at the same time. The idea is similar to that of the Pareto set in an Edgeworth Box. In general, except by chance, we should not expect the economy to be at the EEF. Then, it is interesting to study if there is always a policy reform that simultaneously satisfies the objectives of equity and efficiency, where efficiency means to maximize the average productivity of college graduates. We prove that this is always the case. We also find that for rich countries, this policy consists precisely of transferring resources from college education to basic education. The intuition is that this policy reduces the threshold level of income needed to attend college, but at the price of raising the threshold level of ability. Since higher education is heavily subsidized in most of the rich countries, the first effect is smaller in size and attendance reduces. However, due to the increase in the

[3] Alternatively, if two countries have exactly the same fixed and marginal costs, but different proportions of college students, they will not have the same expenditure per capita.

[4] See for example Lloyd-Ellis (2000), Su (2004) and Blankeneau et al (2007) who consider similar criteria.

threshold level of ability the productivity of skilled workers rises. In addition, this policy has a positive effect on the productivity and the number of unskilled workers. We also find that by transferring resources from college to basic education the average productivity across the population as a whole rises. For low income countries, n the contrary, the policy reform that has a positive effect on equity and at the same time improves the productivity of skilled workers consists of transferring resources from basic to college education. However, in general we find that this policy will have a negative effect on the average productivity across the population.

Note that we focus on educational reforms, instead of focusing on the design of an optimal educational policy. We start from a given division of the budget between the two levels of education and we study the effect of diverting resources from one educational level to the other. We believe that this is a sensible approach since most governments, instead of introducing large reforms, introduce small reforms in several steps.[5]

We discuss briefly some previous works related to ours. Lloyd-Ellis (2000) studies the impact of alternative allocations of public resources between basic and higher education on enrollment, income distribution and growth, while Blankenau, Cassou and Ingram (2007) investigate its output and welfare implications. However, none of them consider individual heterogeneity with respect to family background, which is one of our main focuses. Driskill and Horowitz (2002) study optimal investment in human capital in a standard growth model, and they find that developing countries should concentrate on advanced human capital, a result similar to ours. Restuccia and Urrutia (2004) focus on intergenerational mobility and find that an increase in expenditure on early education has more impact than an increase in college subsidies. Su (2004) studies the dynamic effects of allocating public funds between basic and college education. However, she abstracts from private education expenditure which is a crucial factor affecting education outcomes. Finally, Romero (2008) considers that voters decide how to split the budget between basic and college education and he studies how the possibility of opting out from public education affects that decision.

The paper is organized as follows. In Section 2 we describe the economy. In Section 3 we consider the effect of public policies on the different objectives of the government and we illustrate our main result with numerical examples. Finally, Section 4 concludes.

[5] This approach is similar to that in much of the literature on optimal commodity taxation which focuses on local effects of tax changes. See Feldstein (1975), Guesnerie (1977), and King (1983) among others.

2 Model

2.1 Individuals and Educational Sector

We build a model with two periods. In the first period there is a continuum of individuals characterized by income $y \in [0, Y]$ and innate ability $a \in [0, A]$, where $Y, A > 0$. The respective cumulative distribution functions are $F(y)$ and $G(a)$, although to get closed-form solutions we will assume that a is uniformly distributed on its support. We also assume that y and a are independently distributed.[6]

In the first part of the first period all children attend basic education, which is compulsory. In the second and last part of the first period, individuals can either get a job as unskilled workers or, alternatively, they can enrol in higher education to become skilled workers. We call δ the fraction of time of the first period spent at college and, thus, out of the labor force, where $0 < \delta < 1$.

In the second period, those individuals who attended college in the first period get a skilled job, while those who did not remain in an unskilled job. Individuals care for their consumption in the second period (C) which is equal to the value of their lifetime income.

We assume a simple structure for the educational sector. The per capita cost of providing basic education is denoted as $c_L > 0$. Since basic education is compulsory, we assume that its cost is paid in full by the government.[7]

Regarding higher education we want to separate public provision from public financing. The level of public provision is captured by $c_H > 0$, which is the per capita cost of providing higher education. This includes wages paid to teachers, the cost of college equipment, laboratories, etc. The level of public financing of higher education is captured by s, which represents the proportion of the total cost that the government subsidizes. That is, the government pays $c_H s$, while college students pay $c_H(1 - s)$, with $0 \leq s \leq 1$. To simplify things, we assume that the subsidy is the same for all individuals. We define as $\beta = c_H/c_L$ the ratio between both costs.[8]

We want to distinguish between public provision and public financing as each one of them can be used by the government to achieve different objectives. The parameter c_H, as well as c_L in the case of basic education, captures the quality of education. Increasing c_H could be seen as a way of improving the quality of college education which, in turn, may have a positive effect on the human capital of college graduates.

[6] As we will see below, college attendance is the proportion of individuals with ability and income above some given thresholds. The assumption that a and y are independently distributed allows us to study separately the effect of policy changes on college attendance through the effect on the two thresholds.

[7] In 2003, only 7.4% of total expenditure in basic education in the OECD (primary, secondary and post-secondary non-tertiary education) was privately financed.

[8] As in Blankeneau et al (2007) and Lloyd-Ellis (2000) we do not consider the existence of fixed costs. One reason is of tractability. Another reason is that we are interested only in marginal changes in per capita costs.

However, for a fixed level of c_H, an increase in s can be seen as a way of easing access to college for individuals from low-income families.

2.2 College Attendance

Since individuals care only for their consumption in the second period C, their only concern will be to maximize lifetime income. Next we define lifetime income for the two types of workers, unskilled and skilled.

An individual who only attends basic education will be an unskilled worker for all her remaining lifetime. We assume that her productivity and, thus, her wage w_U will be determined exclusively by per capita expenditure at basic education c_L. We write $w_U = w_U(c_L)$, and we assume this function to be increasing and weakly concave. Since they work a fraction δ of the first period, they earn $\delta w_U(c_L)$ and $w_U(c_L)$ in the first and second period, respectively. Their lifetime income is, therefore, $(1+\delta)w_U(c_L)$.[9]

If an individual goes to college she becomes a skilled worker. Her productivity and thus, her wage, rises to $w_S(c_L, c_H, a)$, where we assume this function is increasing and weakly concave with respect to the three arguments. Lifetime income of a skilled worker will be $w_S(c_L, c_H, a) - c_H(1-s)$. An individual will choose higher education if:

$$w_S(c_L, c_H, a) \geq c_H(1-s) + (1+\delta)w_U(c_L). \tag{1}$$

We assume that this condition always holds for those individuals with the highest ability A since, otherwise, nobody will choose a college education. In particular we require:

$$w_S(c_L, c_H, A) > c_H + (1+\delta)w_U(c_L), \tag{2}$$

which means that, even when $s = 0$, the highest ability individual finds always profitable to attend college. Condition (1) will hold for those individuals with ability a above a threshold $\widehat{a}$, with $0 \leq \widehat{a} < A$, which is implicitly defined as:

$$w_S(c_L, c_H, \widehat{a}) = c_H(1-s) + (1+\delta)w_U(c_L). \tag{3}$$

The effect of the different parameters on the threshold value $\widehat{a}$ is as follows. Clearly, δ has a positive effect on $\widehat{a}$ while the effect of s is negative. This means that the higher is s, the lower is the threshold $\widehat{a}$. Some individuals that had decided previously not to attend college because of their low level of ability, now find attending college profitable. The effect of both c_L and c_H depend on the properties of the functions $w_S(c_L, c_H, a)$ and $w_U(c_L)$. For instance, the effect of c_L on $\widehat{a}$ will be positive as long as:

$$\varepsilon_{c_L}^{w_U} > \frac{1}{1+\delta}\frac{w_S}{w_U}\varepsilon_{c_L}^{w_S}, \tag{4}$$

where $\varepsilon_{c_L}^{w_U} = \frac{\partial w_U}{\partial c_L}\frac{c_L}{w_U}$ and $\varepsilon_{c_L}^{w_S} = \frac{\partial w_S}{\partial c_L}\frac{c_L}{w_S}$ are the elasticities of w_U and w_S, respectively with respect to c_L. This would be the case if, for example, money spent on basic education affects mainly the productivity of those who do not go to college. Finally,

[9] To simplify the analysis we assume that individuals do not discount future payoffs.

to see the effect of c_H on $\widehat{a}$ note that an increase in c_H has two effects on $\widehat{a}$. There is a positive effect on w_S and a negative effect on the cost of college $c_H(1-s)$. Which one of them will prevail will depend on the value of the subsidy s. In particular, we see that the effect of c_H on $\widehat{a}$ will be positive as long as:

$$\varepsilon_{c_H}^{w_S} < (1-s)\frac{c_H}{w_S}, \tag{5}$$

where $\varepsilon_{c_H}^{w_S} = \frac{\partial w_S}{\partial c_H}\frac{c_H}{w_S}$ is the elasticity of w_S with respect to c_H. This will not be the case if s is close to 1. In such a case, the effect on the cost will be negligible and the effect on w_S will prevail, reducing $\widehat{a}$. When s is small, the effect on the cost will dominate, and the final effect on $\widehat{a}$ will be to rise it.

However, individuals must also be able to afford the tuition cost $c_H(1-s)$ to attend college. They can use their income y and they have also access to a loan from a bank. However, and due to capital market imperfections we assume that they can borrow only up to an amount $\gamma c_H(1-s)$, where $0 \leq \gamma \leq 1$.[10] The parameter γ captures the "quality" of capital markets. This borrowing constraint, an exogenous feature of the model, is assumed to be the same across individuals.[11] The two polar cases are $\gamma = 0$, which means complete impossibility of borrowing, and $\gamma = 1$ which means that capital markets are perfect. So, the higher is γ the better is the quality of capital markets.[12]

To attend college, therefore, individuals must have pre-tax income satisfying:

$$y \geq \widehat{y}(c_H, s) = (1-\gamma)c_H(1-s). \tag{6}$$

Those with income above $c_H(1-s)$ do not need to ask for a loan. Those with income below $(1-\gamma)c_H(1-s)$ cannot afford college. Finally, those with income between $(1-\gamma)c_H(1-s)$ and $c_H(1-s)$ need a loan to attend college. The proportion of individuals who can afford college is the proportion of individuals with pre-tax income above $\widehat{y}$, namely, $1 - F(\widehat{y})$. When $\gamma = 1$ or $s = 1$, we get $\widehat{y} = 0$ and the constraint is not binding for any individual. To simplify notation we call $p = 1 - F(\widehat{y})$, and in the sequel we assume that $p > 0$. The proportion of individuals attending college π is the proportion of individuals who satisfy at the same time Conditions (1) and (6). Since y and a are independently distributed, we can write π as:

$$\pi(c_L, c_H, s) = \frac{p}{A} \times (A - \widehat{a}). \tag{7}$$

[10] Evidence by Cameron and Taber (2004) and others suggests that credit constraint are not important in determining college attendance. In Section 4 we discuss how the main results of the paper change by removing the assumption that credit constraints bind.

[11] Lochner and Monge-Naranjo (2002) build a model with endogenous borrowing constraints. Individuals of heterogeneous abilities or those making different schooling choices face different borrowing constraints. We implicitly assume that banks cannot condition loans on ability, as they cannot observe it.

[12] The parameter γ could be alternatively interpreted as a policy variable. Many countries are offering students' loans to overcome this constraint. Then $\gamma = 1$ means that there is such a policy in place, while $\gamma = 0$ means a complete absence of it.

It is immediate to check that π is increasing with γ and s. The derivative of π with respect to c_L will be negative if Condition (4) holds. Finally, we want to analyze the effect of c_H on π. An increase in c_H rises the tuition cost reducing p. At the same time, the effect on the term $(A - \widehat{a})$ will depend on whether Condition (5) holds or not. If it holds, these two effects go in opposite directions and the final effect will depend on which of the two effects prevail. If Condition (5) fails, the effect of c_H on π will be unambiguously negative.

However, we want to stress that these results are just partial derivatives, since we are not taking into account that the budget constraint of the government must hold (see Section 2.3 below).

2.3 The Government Budget Constraint

Here we study how the three instruments of the government (c_L, c_H, s) are related through the budget constraint. We define total expenditure in education E as:

$$E \equiv c_L + s c_H \pi. \tag{8}$$

We call T the total fixed budget that government has to spend in education.[13] We assume that the government cannot run a deficit. Then the budget constraint is:

$$E \equiv c_L + s c_H \pi \leq T. \tag{9}$$

For fixed values of c_L and c_H, we call $\widehat{s}$ the value of the subsidy for which the constraint is satisfied with equality. If $E < T$ for all values of the subsidy, then we set $\widehat{s} = 1$. In the next proposition we provide a simple condition that guarantees that $\widehat{s}$ exists and that it is unique.

Proposition 1 *Consider any combination (c_L, c_H) and assume that the fixed budget T satisfies that $T > c_L$. Then, there is a unique value $\widehat{s}(c_L, c_H) \leq 1$ that satisfies the budget constraint.*

Proof. When $s = 0$, we see that $E = c_L$. Since π is increasing with s, the function E is strictly increasing in s. We have two possibilities. Either E is always below T, in which case $\widehat{s}(c_L, c_H) = 1$, or they cross at a value of the subsidy $\widehat{s}(c_L, c_H)$ strictly below 1. ■

The condition that $T > c_L$ is required since, otherwise, even when college education is not subsidized at all, the government would be running a deficit. For given values of the parameters, the fact that the government has to satisfy the budget constraint implies that it has only two free policy instruments. We choose c_L and c_H as

[13] We assume T to be fixed. This could be either because it comes from taxes raised on the previous generations, or because it is financed through lump-sum taxes that do not distort the education decisions of the young. Our focus, however, is not on a comparison of financing schemes, so we abstract from distortionary taxation.

the two free parameters and we assume that the subsidy always adjusts to satisfy the constraint. Since we are interested in policy changes, we want to know what is the effect of changes in c_L and c_H on $\widehat{s}(c_L, c_H)$. To do this we assume that the equilibrium is interior. In particular we define $\widehat{s}(c_L, c_H)$ as the value that satisfies:

$$E(c_L, c_H, \widehat{s}) - T = 0. \tag{10}$$

Computing the corresponding derivatives:

$$\begin{aligned} \frac{\partial \widehat{s}}{\partial c_L} &= -\frac{\frac{\partial E}{\partial c_L}}{\frac{\partial E}{\partial s}} = -\frac{1 + \widehat{s} c_H \frac{\partial \pi}{\partial c_L}}{\frac{\partial E}{\partial s}}, \\ \frac{\partial \widehat{s}}{\partial c_H} &= -\frac{\frac{\partial E}{\partial c_H}}{\frac{\partial E}{\partial s}} = -\frac{\widehat{s}\left(\pi + c_H \frac{\partial \pi}{\partial c_H}\right)}{\frac{\partial E}{\partial s}}. \end{aligned} \tag{11}$$

Since $\frac{\partial E}{\partial s} \geq 0$, the signs of $\frac{\partial \widehat{s}}{\partial c_L}$ and $\frac{\partial \widehat{s}}{\partial c_H}$ will be negative if the terms in the numerator are positive. Consider first that college attendance π is not affected by either c_L or c_H. Then, it is clear that both derivatives are negative. That is, rising either c_L or c_H reduces the resources that can be used to subsidize higher education. However, college attendance can also be affected negatively by the increase in c_L or c_H, reducing the absolute value in the numerator. Intuitively, the negative effect on the subsidy is attenuated, since now fewer individuals receive it. What we do is to assume that the indirect effect through π is not that large so as to offset the initial negative effect.[14]

Assumption 1 (A.1): The following conditions hold: (i) $\frac{\partial \widehat{s}}{\partial c_H} < 0$ and (ii) $\frac{\partial \widehat{s}}{\partial c_L} < 0$.

We define an *"iso-subsidy"* curve as the set of all combinations (c_L, c_H) giving rise to the same value of the subsidy $\widehat{s}$. From Equations (10) and (11), the slope of an iso-subsidy is:

$$\frac{\partial c_H}{\partial c_L}\Big|_{s=\widehat{\widehat{s}}} = -\frac{\frac{\partial \widehat{s}}{\partial c_L}}{\frac{\partial \widehat{s}}{\partial c_H}} = -\frac{1 + \widehat{s} c_H \frac{\partial \pi}{\partial c_L}}{\widehat{s}\left[\pi + c_H \frac{\partial \pi}{\partial c_H}\right]}. \tag{12}$$

By Assumption 1, this slope is negative, implying that there is always a trade-off between expenditure on basic education and expenditure on college education. Holding the subsidy fixed, if we increase one of them we have to reduce the other in order to keep the budget balanced.

From Proposition 1 we also see that a fixed combination (c_L, c_H) corresponds to different values of the subsidy in two countries with different educational budget T. Using Equation (10) above, for a fixed (c_L, c_H), the equilibrium value of the subsidy in a rich country (one with a high T) will be higher than in a less rich country. From Equations (7) and (9) we also obtain that the equilibrium value of college attendance will be also higher in the rich country than in the less rich country.

[14] In the Appendix we discuss which are the conditions that need for this to be true. Basically we need to assume that the elasticities of π with respect to both c_L and c_H are small in size.

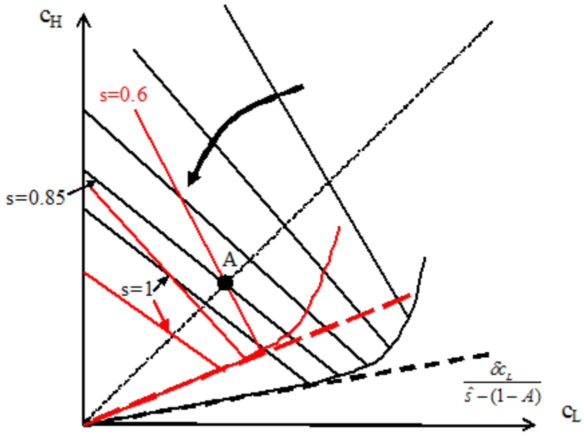

Figure 1: The policy space

In Figure 1 above we illustrate our policy space. We represent in black (respectively, red) a rich (respectively, poor) country. The closer to the origin, the higher is the value of the subsidy. The curved lines in the bottom part of the figure represent the combinations (c_L, c_H) for which Condition (2) holds. That is, below that line nobody attends college and expenditure E is constant at $E = c_L$. Point A in the figure corresponds to a higher subsidy in a rich country than in a poor country. In the example of the figure the values of the subsidy are 0.85 and 0.6, respectively.

Finally we also see that for a fixed combination (c_L, c_H), the iso-subsidy curve $\widehat{s}(c_L, c_H)$ is flatter in a rich country rather than in a less rich country, provided that both $\varepsilon^{\pi}_{c_H}$ and $\varepsilon^{\pi}_{c_L}$ are of small size.[15] The intuition is simple. Consider a rich country where both $\widehat{s}$ and π take high values. If the government rises c_H this policy will have a large impact on expenditure because a lot of people are getting a substantial subsidy. If nothing else is done, the subsidy should be reduced in a large amount. To hold the subsidy fixed, a significant reduction of c_L is needed. In the less rich country, on the contrary, rising c_H has a smaller effect on expenditure, since few people attend college and the subsidy is low. The reduction needed in c_L to keep the subsidy constant is smaller.

3 Policy Reforms

We want to analyze policy reforms from an initial situation described by a particular combination (c_L, c_H) through their effects on different government objectives. As we discussed in the introduction, we assume that the government wants to fulfill several objectives at the same time. In particular, the government has both efficiency and equity concerns. Although we will be more precise below, by efficiency we re-

[15] See the Appendix.

fer to policies that improve the productivity of workers through investment in the educational process. By equity we mean policies that foster equality of opportunities.

3.1 Equity

We assume that one of the government objectives is equity, as measured by equality of opportunities. By this we mean the following. For a fixed value of innate ability a, the probability of attending college is determined exclusively by income y. If the government wants to maximize the probability of attending college for a given ability level a, its objective will be simply to reduce as much as possible the threshold level of income $\widehat{y}$ corresponding to that ability level. Since we are assuming that a and y are independently distributed, that threshold is constant across ability levels. Then, for any value of γ in Equation (6), to reduce the threshold $\widehat{y}$ amounts simply to reduce $c_H(1-\widehat{s})$. Since both $\frac{\partial \widehat{s}}{\partial c_L}$ and $\frac{\partial \widehat{s}}{\partial c_H}$ are negative, this can be done by either reducing c_L or c_H or both. Moreover, we see that the slope of $\widehat{y}$ in the policy space (c_L, c_H) is:

$$\left.\frac{\partial c_H}{\partial c_L}\right|_{\widehat{y}=\overline{\widehat{y}}} = \frac{\frac{\partial \widehat{s}}{\partial c_L}}{\frac{(1-\widehat{s})}{c_H} - \frac{\partial \widehat{s}}{\partial c_H}}, \tag{13}$$

which is negative. This means that, if we increase c_H (respectively, c_L), to hold $\widehat{y}$ constant we have to reduce c_L (respectively, c_H).

3.2 Efficiency

First, we consider that the efficiency objective of the government consists of increasing the average productivity or the average human capital of college graduates. This would be the case if the government is particularly concerned with improving the productivity of skilled workers. Since an individual with ability a who attends college has productivity $w_S(c_L, c_H, a)$, the average productivity of graduates, denoted by Q_S is:

$$Q_S(c_L, c_H) = E[w_S(c_L, c_H, a) \mid a > \widehat{a}(c_L, c_H)]. \tag{14}$$

An increase in either c_L or c_H has a positive effect on Q_S, which is assumed to be concave with respect to both c_L and c_H. There is a positive direct effect through w_S and also an indirect positive effect because the threshold $\widehat{a}$ rises as well. In the space (c_L, c_H) we can define an *"iso-productivity"* curve as the set of combinations (c_L, c_H) giving rise to the same level of Q_S. From (14) the slope of an iso-productivity is:

$$\left.\frac{\partial c_H}{\partial c_L}\right|_{Q_S=\overline{Q}_S} = -\frac{\frac{\partial Q_S(c_L,c_H)}{\partial c_L}}{\frac{\partial Q_S(c_L,c_H)}{\partial c_H}}. \tag{15}$$

Since both c_L and c_H have a positive impact on w_S, this slope is negative. If we reduce c_H (respectively, c_L), to hold Q_S constant we have to increase c_L (respectively, c_H).

In Figures 2a and 2b below we represent the level curves of Q_S (in blue), together with those of $\widehat{y}$ (in red).

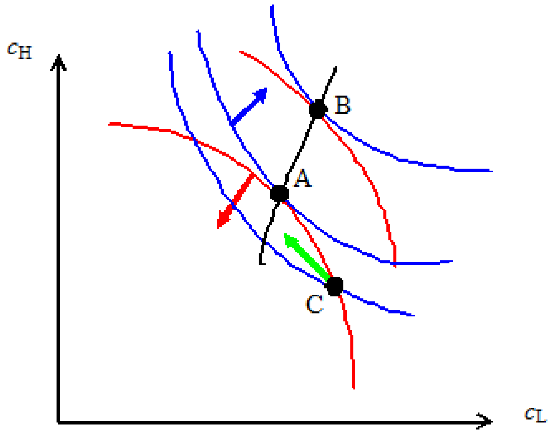

Figure 2a: c_L and c_H are complements

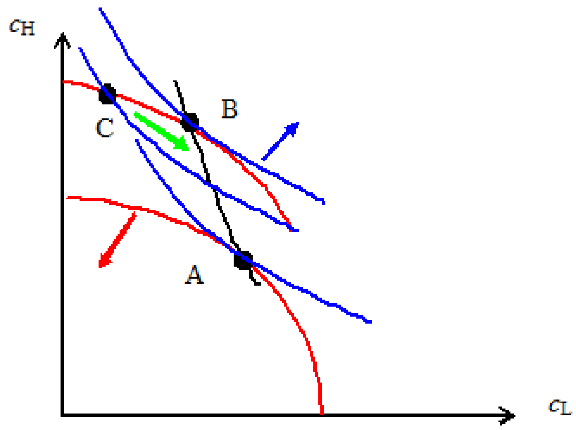

Figure 2b: c_L is relatively more important

A second efficiency objective consists of rising the average level of human capital of the entire cohort of individuals, and not only that of college graduates. Recall that a proportion $1-\pi$ of the cohort has productivity $w_U(c_L)$, while those attending college have productivity $w_S(c_L, c_H, a)$. If we call $Q_T(c_L, c_H)$ the average productivity, we have:

$$Q_T(c_L, c_H) = \int_0^{\widehat{a}} w_U(c_L) f(a) da + p \int_{\widehat{a}}^{A} w_S(c_L, c_H, a) f(a) da, \tag{16}$$

since only a proportion p of those with ability above the threshold $\widehat{a}$ can afford a college education. Using the definition of Q_S above, we can also write Q_T as:

$$Q_T(c_L, c_H) = (1-\pi)w_U(c_L) + \pi Q_S(c_L, c_H), \tag{17}$$

where the first term captures the aggregate level of human capital of unskilled workers and the second term takes into account both the quantity and the quality of college graduates. The differences with Q_S above are that now we also care for the productivity of unskilled workers and for college attendance. In particular, consider a policy change that rises Q_S by transferring resources from college education to basic education. If college attendance π does not change, then Q_T will rise as well. However, in general we should expect a change in college attendance. If college attendance gets lower, the term on the left $((1-\pi)w_U(c_L))$ gets higher, while the term on the right (πQ_S) can either rise or diminish. In fact, this second effect is always of a smaller size, meaning that any policy that transfers resources from college to basic education that has a positive effect on Q_S, will also have a positive effect on Q_T. To see this, note that Q_T is always a convex combination of $w_U(c_L)$ and $Q_S(c_L, c_H)$. In the policy space (c_L, c_H) the level curves of Q_S have negative slope, while those of $w_U(c_L)$ are vertical lines. The level curves of Q_T must have, therefore, a slope between the corresponding slopes of $w_U(c_L)$ and $Q_S(c_L, c_H)$.

In the next section we focus on our narrower definition of efficiency, namely Q_S. We focus on Q_S for two reasons. First, the analysis is much simpler than with Q_T. Second, given the current trend in most Western countries towards cutting expenditure in higher education, we are interested in studying when it is the case that the policy that makes equity and efficiency compatible consists of rising c_L and reducing c_H. Thus, given the relationship between the indifference curves of Q_S and the other two concepts, if that policy has a positive effect on equity and Q_S, it will also have a positive effect on Q_T.

3.3 Equity and College Productivity

We assume that the government is concerned about equity and about the quality of college graduates. Our next objective is to find those combinations (c_L, c_H) that take the economy to the equity-efficiency frontier. We define those combinations as follows:

Definition 2 *The Equity-Efficiency Frontier (EEF) is the set of all combinations* (c_L, c_H) *such that, for a given threshold level of income* $\widehat{y}$*, college productivity* Q_S *is maximized.*

When both Q_S and $\widehat{y}$ are differentiable and quasi-concave functions (i.e, the upper contour sets of both functions are always convex sets), the EEF can be easily characterized by the usual tangency condition. That is, the EEF is defined as the set of all combinations (c_L, c_H) such that:

$$\left.\frac{\partial c_H}{\partial c_L}\right|_{\widehat{y}=\overline{\widehat{y}}} = \left.\frac{\partial c_H}{\partial c_L}\right|_{Q_S=\overline{Q}_S}. \tag{18}$$

For an illustration see Figures 2a and 2b, where points A and B belong to the EEF. The EEF represents all policy combinations such that the government cannot improve equity without hurting efficiency, or the other way round. For example, a movement from A to B in Figure 2a or 2b increases efficiency at the price of reducing equity.

Observe that, as long as Q_S is quasi-concave, we do not need $\widehat{y}$ to be quasi-concave. It is enough to assume that for any combination (c_L, c_H), the upper contour set of Q_S is a proper subset of the lower contour set of $\widehat{y}$. In the Appendix we discuss the issue of quasi-concavity of Q_S. We see that Q_S will be quasi-concave under very mild assumptions.

Next we discuss the shape of the EEF in the policy space (c_L, c_H). This shape will depend on the corresponding shapes of both Q_S and $\widehat{y}$. One example is the case in which Q_S is such that c_L and c_H are strong complements. Just for the purpose of illustration, consider the case $Q_S = \min\{c_L, c_H\}$. Then, rising c_H has no effect on college productivity if at the same time we do not increase c_L. Then, the shape of EEF will be as represented in Figure 2a. That is, its slope will be positive in the space (c_L, c_H). However, for other technologies the contract curve may have a different shape in the policy space (c_L, c_H). For example, if money spent at early stages has a deeper impact on college productivity than expenditure at later stages, the slope of the EEF will be negative as represented in Figure 2b. The reason is that in this case putting more resources into college education has hardly any effect on productivity. The higher is Q_S, the more vertical become its level curves. Now, since the slope of EEF is negative, a policy change that increases c_L and reduces c_H along the EEF, will improve efficiency at the price of reducing equity.

Finally, observe that there is no reason why a given economy should be actually choosing an educational policy on the curve EEF. It is easy to see that, if an economy is not at a policy combination on the EEF, there is no longer a trade-off between equity and efficiency. The following corollary states this result.

Corollary 3 *For all combinations that are not in the EEF:*
(i) There is always a policy that improves the two objectives of the government.
(ii) This policy always consists of transferring resources from college to basic education when the initial combination is to the right of the EEF or from basic to college education if it is to the left of the EEF.

Proof. It is immediate from (18) and Figures 2a and 2b. ■

Consider, for example, the point C in both Figure 2a and Figure 2b. If the government implements a policy reform in the direction of the green arrow, this policy has a positive effect both on equity and efficiency.

This corollary shows that, regardless of the shape of the EEF, the objectives of equity and productivity are always compatible if the initial combination (c_L, c_H) is not in the EEF, and it also provides a characterization of the direction of the policy reform. What is crucial, therefore, is to identify for a given initial situation which is the optimal policy change.

3.4 An illustration: Ability matters only at college

The above model is too general to derive policy recommendations. In the sequel, therefore, we focus our attention to a particular example. In particular, we propose the following functional forms for the productivity levels of unskilled and skilled workers:

$$\begin{aligned} w_U(c_L) &= (c_L)^{\alpha} \\ w_S(c_L, c_H, a) &= w_U(c_L) + (c_H)^{\alpha} a, \end{aligned} \tag{19}$$

where $0 \leq \alpha \leq 1$. The unskilled wage is determined exclusively by per capita expenditure at basic education. We also assume that the marginal productivity of c_L does not depend on ability.[16] Attending college yields a positive premium that depends both on ability a and on expenditure at college c_H.[17] Note also that the marginal productivity of expenditure at college is higher for individuals of high ability than for low-ability individuals. Finally we assume decreasing returns to expenditure at both levels.

Using Equations (3) and (19) above, the threshold $\widehat{a}$ becomes:

$$\widehat{a}(c_L, c_H) = (c_H)^{1-\alpha} (1 - \widehat{s}) + \delta \left(\frac{c_L}{c_H} \right)^{\alpha}, \tag{20}$$

while productivity of college graduates, Q_S, can be written as:

$$Q_S(c_H, c_L) = (c_L)^{\alpha} + (c_H)^{\alpha} \left(\frac{\widehat{a} + A}{2} \right). \tag{21}$$

In the previous section we saw that having the two objectives in mind, equity and efficiency, imposes always a trade-off between basic education and college education as long as the economy is not at a policy combination on the EEF. Here we obtain a condition that characterizes whether the country is to the right or to the left of the EEF. By identifying this condition we are able to provide a policy recommendation for any given initial combination c_L and c_H.

In the next proposition we show that the particular policy to pursue depends crucially on the values of the elasticities of $\widehat{s}(c_L, c_H)$ with respect to c_L and c_H.

Proposition 4 *Suppose that $w_U(c_L)$ and $w_S(c_L, c_H, a)$ are as in Equation (19). Moreover, suppose that the initial situation (c_L, c_H) corresponds to a situation that is not in the EEF. Then, the particular policy reform that has a positive effect both on the productivity of college graduates Q_S and on equity depends on the size of the elasticity of $\widehat{s}(c_L, c_H)$ with respect to c_L. If this elasticity is small in absolute terms, the government should rise c_L and reduce c_H. If the elasticity is large in absolute terms, the government should rise c_H and reduce c_L.*

[16] This specification is similar to that used in Blankenau et al (2007) who also assumed that ability matters only at college (or in the acquisition of specific human capital in their model).

[17] See Su (2004) for a similar specification.

Proof. The proof is very simple. We just have to compare the slopes of Q_S and $\widehat{y}$ in the space (c_L, c_H), when $w_U(c_L)$ and $w_S(c_L, c_H, a)$ are as in Equation (19). The slope of $\widehat{y}$ comes from Equation (13). Regarding the slope of Q_S, using (21) and (20) we get:

$$\left.\frac{\partial c_H}{\partial c_L}\right|_{Q_S=\overline{Q}_S} = \frac{\frac{\partial \widehat{s}}{\partial c_L} - \frac{\alpha(2+\delta)(c_L)^{\alpha-1}}{c_H}}{\frac{\alpha(c_H)^{\alpha-1}A+(1-\widehat{s})}{c_H} - \frac{\partial \widehat{s}}{\partial c_H}} \tag{22}$$

There are two possibilities: either the slope in Equation (22) is smaller than the slope in Equation (13) or it is the other way round. In the first case, the only possibility of achieving both objectives is by increasing c_L while reducing c_H. This is the situation of point C in Figure 3 below, where we represent in green all policies satisfying both objectives. In the second case, the way to achieve both objectives is by increasing c_H while reducing c_L. This is point D in the figure. We then see that which one of these two cases prevails depends on the value of the elasticity of $\widehat{s}(c_L, c_H)$ with respect to c_L. From Equations (13) and (22), we check that the first case will arise as long as:

$$\frac{\frac{\partial \widehat{s}}{\partial c_L} - \frac{\alpha(2+\delta)(c_L)^{\alpha-1}}{c_H}}{\frac{\alpha(c_H)^{\alpha-1}A+(1-\widehat{s})}{c_H} - \frac{\partial \widehat{s}}{\partial c_H}} < \frac{\frac{\partial \widehat{s}}{\partial c_L}}{\frac{(1-\widehat{s})}{c_H} - \frac{\partial \widehat{s}}{\partial c_H}}. \tag{23}$$

This can be written as:

$$(2+\delta)(c_L)^{\alpha-1} c_H \frac{\partial \widehat{s}}{\partial c_H} < (c_H)^{\alpha} A \frac{\partial \widehat{s}}{\partial c_L} + (2+\delta)(c_L)^{\alpha-1}(1-\widehat{s}). \tag{24}$$

Defining the elasticities of $\widehat{s}(c_L, c_H)$ with respect to c_L and c_H as $\varepsilon^s_{c_L} = \frac{\partial \widehat{s}}{\partial c_L}\frac{c_L}{\widehat{s}}$ and $\varepsilon^s_{c_H} = \frac{\partial \widehat{s}}{\partial c_H}\frac{c_H}{\widehat{s}}$, respectively, the expression above can be finally simplified as:

$$\varepsilon^s_{c_L} > \left(\frac{2+\delta}{A}\right)\left(\frac{c_H}{c_L}\right)^{-\alpha}\left(\varepsilon^s_{c_H} - \frac{(1-\widehat{s})}{\widehat{s}}\right). \tag{25}$$

■

Condition (25) is more complicated than it seems, as it depends on $\widehat{s}$ and the elasticities $\varepsilon^s_{c_H}$ and $\varepsilon^s_{c_L}$ which, in turn, depend on the specific values of c_L and c_H. If $A \leq 2+\delta$ and $\frac{c_H}{c_L} \leq 1$, the term $\left(\frac{2+\delta}{A}\right)\left(\frac{c_H}{c_L}\right)^{-\alpha}$ is bigger than 1. In this case, the condition will be true as long as the elasticity of the subsidy with respect to c_L in absolute value is not much higher than the corresponding elasticity in absolute value with respect to c_H. In particular, whenever both elasticities are of comparable size, the condition will be always fulfilled.

If we consider two countries, one rich and one poor, it seems that the condition is more likely to be fulfilled in the rich country. The reason is that in rich countries, college attendance is higher than in poor countries. Then, changes in c_H will have a deeper impact on the subsidy relative to changes in c_L in rich countries, rather than in poor ones. That is, the absolute size of $\varepsilon^s_{c_H}$ in rich countries relative to that of $\varepsilon^s_{c_L}$

will be higher than in poor countries. Below we run some numerical simulations that seem to confirm this intuition.

In Figure 3 point C represents a situation in which Condition (25) holds. This could be seen as the situation in many developed countries. As we move into higher values of both c_L and c_H, the equilibrium level of the subsidy gets lower. At the same time, we find that the iso-equity lines become steeper relatively to the iso-productivity lines. This means that, as we move farther away from the origin, eventually Condition (25) will fail. This is represented as point D in Figure 3.

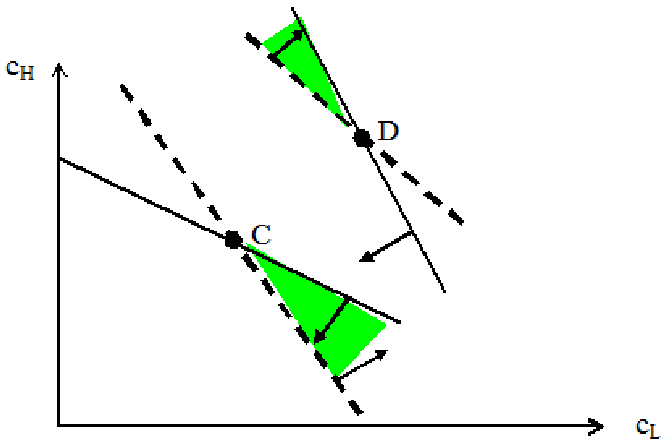

Figure 3: The optimal policy depends on the value of the subsidy

In addition, Proposition 4 shows that which is the policy reform for a given starting point (c_L, c_H) depends on whether the country is rich or poor. Figure 4 represents the policy reforms for two countries: one rich and one poor. In both cases, the dotted lines represent the iso-productivity curves. The rich country (in black) has an educational budget T. The poor country (in red) has educational budget $T' < T$. Thus, for a fixed combination (c_L, c_H), the subsidy in the rich country will be higher than in the poor country and the iso-productivity lines will be steeper than the iso-equity lines. The policy reform will consist of increasing c_L. In the poor country, since the same combination (c_L, c_H) corresponds to a much lower subsidy, the optimal policy reform is just the opposite one.

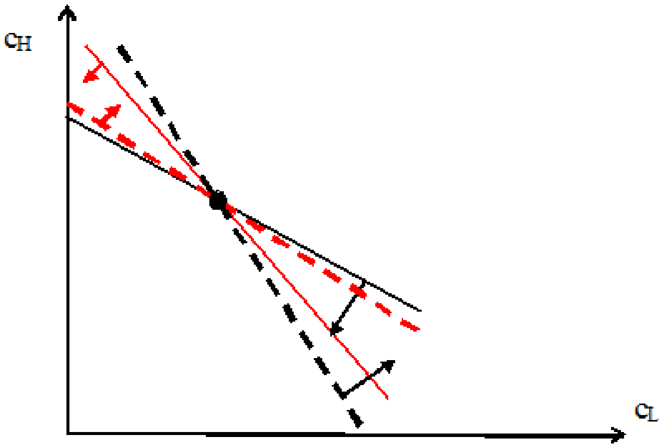

Figure 4: Optimal policy in a rich and a poor country

Finally, we know that any policy that transfers resources from college education to basic education and that respects both equity and college productivity will have a positive effect on Q_T as well. What about when Condition (25) does not hold? We saw that a policy that transfers resources towards higher education improves equity and college productivity. However, this is done at the price of reducing the human capital of those who do not attend college, and this may have at the end a negative effect on Q_T. In fact, the only policy that both respects equity and has a positive effect on Q_T is the one that transfers resources towards basic education.

3.5 A Numerical Example: High Income vs Low Income Countries

Here we present a numerical example to illustrate Proposition 4. To do it we have to find reasonable values of our parameters. This exercise should not be taken as a full-fledged calibration exercise, since the model is too abstract to be calibrated properly.

We need values for δ, γ, α, A and T. Once we have this, for every combination (c_L, c_H) we can compute the equilibrium levels of the subsidy $\widehat{s}(c_L, c_H)$, college attendance and productivity. In Table 3 we present our choice of parameter values. Below we describe briefly our choices.

Table 3: Parameter values

δ	0.1
γ	0.75
α	1
A	1
T (rich)	$\$100,000$
T (poor)	$\$40,000$

The value that we choose for δ reflects the fact that the working life of an unskilled worker is a 10% longer than that of a skilled worker. Since borrowing constraints are not very important for most OECD countries we think that a value of γ close to 1 should be appropriate. In particular, we choose $\gamma = 0.75$. To simplify our computations, we choose $\alpha = 1$, although we comment below how results change when we allow for strictly decreasing returns to public expenditure in education ($\alpha < 1$).

Next we focus on A. The value of A determines the value of the college premium. In particular, the college premium is an increasing function of A. We define the college premium for an individual with ability a as the ratio between net lifetime income attending and not attending college. Since a follows a Uniform distribution on $[0, A]$, the average college premium can be written:

$$\frac{2 + \delta + \beta(A - (1 - s))}{2(1 + \delta)}. \tag{26}$$

To obtain a value for β and s we use OECD data from Table 2. Columns 1 and 2 correspond roughly to what we call in the paper c_L and c_H, respectively. In Column 3 (T/B) we compute the ratio β. Taking the mean values of β and s for the OECD ($\beta = 0.61, s = 0.782$), if we choose $A = 1$ we obtain an average college premium of 1.17, which seems reasonable. Moreover, choosing $A = 1$ simplifies our calculations to a great extent.

Countries are classified as rich or poor according to the value of the educational budget T. For each country in Table 2 we calculate T as follows. Using country data for c_L, c_H, subsidies for higher education (Column Subs.) and college attendance levels (C.Att.), we plug these numbers into Equation (9) to get a value for T. Next, we divide the countries into two groups, according to their values of T. In particular, we consider as poor countries those five in the first quartile, namely: Greece, Hungary, Mexico, Slovak Republic and Turkey. We consider as rich countries the rest of the OECD countries. The average values of T, c_L, and c_H are (approximately) $T = \$40,000, c_L = \$35,000$ and $c_H = \$25,000$ for poor countries and $T = \$100,000$, $c_L = \$70,000$ and $c_H = \$40,000$ for the rich countries.

Once we have values for all our parameters, we can compute the equilibrium values of $\widehat{s}$, p and π corresponding to any given combination (c_L, c_H) for the two groups of countries.[18] Next, we can derive the shape of the EEF. To do this we start by fixing a particular value of c_L. Then, we compute the value of c_H that takes the economy to the EEF. In other words, we find the value of c_H such that the slopes of Q_S and $\widehat{y}$ coincide. Next, we repeat the process with a different value of c_L. By moving c_L through all its support, we can obtain the shape of the whole EEF.

What we obtain in our numerical example is that the slope of the EEF is negative. This means that for those combinations (c_L, c_H) to the left of the EEF curve, the policy reform to implement consists of rising c_L and reducing c_H. For those combinations to the right of the EEF curve, the optimal policy reform is just the opposite. This result is in line with the interpretation we gave to Condition (25) immediately after Proposition 4. In fact, we could think of the EEF curve as a way of separating those combinations (c_L, c_H) where the subsidy is too large and the condition holds (those to the left of the curve), from those where the subsidy is too low and the condition does not hold (those to the right of the curve).

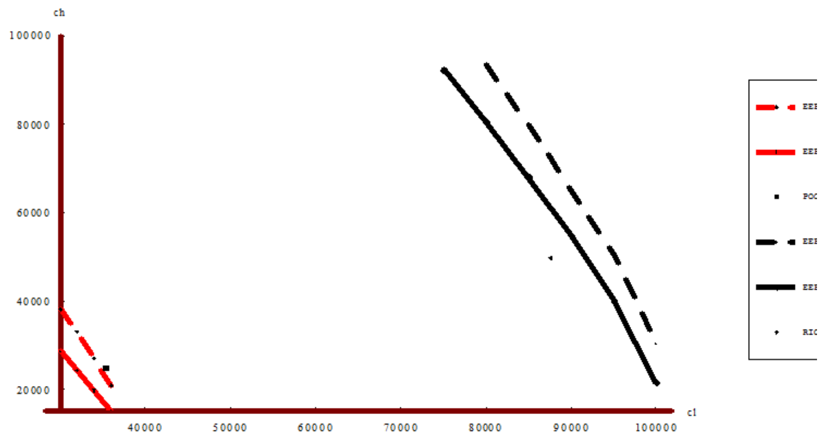

Figure 5: Illustration of Proposition 4

We also find that the position of the EEF depends on the value of the educational budget T. In particular, as we show in Figure 5 above, the EEF curve for rich countries (in black), is above the EEF curve for poor countries (in red).[19] Moreover, if we focus on the group of rich countries, we find that the region where Condition

[18]For simplicity we assume that y follows a uniform distribution on $[0, Y]$. Using the data in Table 2 for c_L, c_H and subsidies for higher education (Column Subs.) we compute a value of $\widehat{a}$ for each country. Using these values of $\widehat{a}$ and given the college attendance levels (C.Att.) we can obtain a value of p. Since $\widehat{y} = (1-\gamma)c_H(1-s)$ and $p = 1 - F(\widehat{y}) = 1 - \frac{\widehat{y}}{Y}$, once we have a value for $\widehat{y}$ and p we can also compute Y.

[19]For both groups, we also obtain that the larger is γ, the larger is the region in the space (c_L, c_H) where Condition (25) holds.

(25) fails (those combinations above the EEF curve) corresponds to extremely low values of college attendance. This allows us to conclude that the empirically relevant region for high income countries corresponds to the situation where Condition (25) holds. However, this is not the case for poor countries, which confirms the result illustrated in Figure 4. That is, as we move farther away from the origin, Condition (25) will eventually fail. Once we reach low enough values of the subsidy, the iso-equity lines become steeper relatively to the iso-productivity lines. Thus, the policy reform for poor countries will eventually consist of increasing c_H. This policy has the effect of increasing the college subsidy so that a higher proportion of poor individuals can afford college, which in turn implies an increase in college attendance. Although this reform reduces the quality of basic education and the ability threshold for college students, due to the increase in c_H, the final quality of college students increases.

Finally we allow for decreasing returns to public expenditure in education ($\alpha < 1$) and in particular we repeat our calculations using a value $\alpha = 0.9$. We find that, the lower is α, the larger is the region in the space (c_L, c_H) where Condition (25) holds. In particular, as we show in Figure 5, the EEF curve for $\alpha = 0.9$ (in dotted line), is above the EEF curve for $\alpha = 1$ (in solid line). In other words, the lower the marginal return to public expenditure in education, the larger the increase in c_L in order to reach the EEF curve.

To illustrate further our results in Corollary 3 and Proposition 4, we present in Table 4 an example of the effects of two different policy changes on the different objectives of the government. We focus on the case of rich countries and we use the numbers from Table 2 to fix an initial situation with $c_L = \$88,000$ and $c_H = \$50,000$, respectively. The corresponding values of $Q_S(c_L, c_H)$, $\widehat{y}(c_L, c_H)$, $\pi(c_L, c_H)$ and Q_T are also computed, together with the values of $\widehat{s}$ and p. This initial situation is in the first column of Table 4.

Table 4: Budget division and public intervention[1]

β	$.56 = \frac{50}{88}$	$.48 = \frac{44}{91}$	$.66 = \frac{57}{86}$
$\widehat{s}$	.739	.711	.748
p	.625	.634	.588
π	.352	.320	.352
Q_S	123,917	123,902	125,957
Q_T	100,650	101,534	100,060

[1] c_L and c_H are in thousand dollars.

We consider two alternative policies. In the first one we transfer resources from higher education to basic education while in the second one we do just the opposite.

In particular, in the second column we consider a 12% reduction in c_H and a 3.4% increase in c_L. New values are $c_L = \$91,000$ and $c_H = \$44,000$. Once all variables reach a new equilibrium, we find that the subsidy becomes lower. However, we observe that the proportion of individuals who can afford college increases from 62.5% to 63.4% meaning that this policy has a positive effect on equity. As we see in the table, it has also a positive effect on both measures of productivity, but at the cost of a negative effect on college attendance.

In the third column we consider a 2.27% reduction in c_L and a 14% increase in c_H. New values are $c_L = \$86,000$ and $c_H = \$57,000$. This policy has a positive impact on the subsidy. Regarding the different objectives of the government, we find a negative effect on equity since the value of p reduces from 62.5% to 58.8%. There is also a positive effect on the productivity of college students. However, the effect on the average level of productivity across the population Q_T is negative. As predicted by Proposition 3, moving resources towards higher education will have a negative effect either on equity or on productivity. On top of this, as seen in the reduction on Q_T, the increase in Q_S (the average productivity of college graduates) is not enough to compensate the reduction in the productivity of unskilled workers (c_L).

4 Conclusion and Discussion

The main result of this paper is that, except in the special case in which the economy is at the EEF curve, there is always a policy reform that increases the productivity of college graduates without excluding the talented poor from college. For most rich countries, this policy consists of transferring resources from college to basic education. In addition we find that this policy has always a positive effect on the average level of human capital across the population.

Throughout the paper we have assumed that capital markets are imperfect ($\gamma < 1$). Here we want to comment briefly on the effect of removing this assumption. If $\gamma = 1$, then equity is no longer a concern for the government since in that case all individuals can attend college, irrespective of family income. The trade-off between equity and efficiency disappears. One interesting implication is that, in this case, the government could fix a large value of both c_L and c_H, such that $s = 0$ in order to achieve efficiency. That is, if capital markets work perfectly college education should be privately financed.

There are many possible extensions of this work. One possibility would be to study other objectives to represent efficiency. For example, we could assume that the government tries to increase average lifetime income within a cohort which implies rising average consumption of the cohort in period 2.[20] Thus, the main difference with average human capital above is that now we are subtracting the monetary cost of higher education paid by students. We have some partial results regarding the effects of policy reforms on this objective. In particular, we find that as long as the indirect effect of changes in both c_L and c_H through s and π is not very large, a policy consisting of transferring resources from college to basic education that has a positive

[20] This objective has been analyzed by Lloyd-Ellis (2000) and Su (2004). Blankeneau et al (2002) analyze the lifetime income (or welfare levels) of each group of workers in a separate way.

effect on Q_S will also have a positive effect on the average lifetime income within a cohort.

Another extension would be to relax the assumption that the two characteristics that define individuals are independent. It is generally assumed that there is correlation between parents' ability and the ability of the child when, for example, IQ is taken as a measure of ability. As parents' income and parents' ability are also correlated, the two characteristics in our model will be positively correlated. However, if the two variables are correlated, the model becomes more complicated as the two terms that define college attendance now cannot be separated. The outcome is that the ability threshold will be lower for rich individuals than for poor individuals. One possibility could be to rely on numerical simulations to see whether the results of the paper change.

We believe that our results are relevant for several recent debates in the literature on the economics of education. There is increasing evidence that shows the early emergence and persistence of gaps in cognitive and non-cognitive skills (see among others, Carneiro and Heckman (2003)). This issue is of special concern as, according to recent evidence, family environments have deteriorated (Heckman and Masterov (2004)).[21] Studies that highlight the importance of increasing expenditure in early childhood care in achieving both equity and efficiency provide an interesting illustration since, obviously, at the current level of resources, the rise of expenditure at that level should be done at the expense of reducing expenditure in later educational levels (see Heckman (2006)).

[21] In the US, the percentage of children born into, or living in, "nontraditional" families has increased greatly in the last 30 years (about 25% of children are now born into single parent homes now). "Nontraditional" families include not only single-parent families but also families where the parents are not married. The evidence found by Heckman and Masterov (2004) suggests that children raised in these types of families fare worse in many aspects of social and economic life.

5 Appendix

Discussion on Assumption 1:
Sign of $\frac{\partial \widehat{s}}{\partial c_L}$:
We have to study the sign of the term in the numerator of $\frac{\partial \widehat{s}}{\partial c_L}$. We see that:

$$\text{Sign}\left[\frac{\partial \widehat{s}}{\partial c_L}\right] = \text{Sign}\left[-1 - \frac{\partial \pi}{\partial c_L}\right]. \tag{27}$$

This is negative if:

$$\varepsilon^{\pi}_{c_L} > -\frac{c_L}{c_H}\frac{1}{\widehat{s}\pi}, \tag{28}$$

where $\varepsilon^{\pi}_{c_L} = \frac{\partial \pi}{\partial c_L}\frac{c_L}{\pi}$ is the elasticity of college attendance with respect to c_L. So, the derivative $\frac{\partial \widehat{s}}{\partial c_L}$ will be negative, except in those situations where the negative effect of c_L on college attendance π is very large. To illustrate the condition above, we take the average values from Table 2, where $\pi = 0.32, \widehat{s} = 0.782$, and the ratio $c_H/c_L = 0,61$. The condition becomes $\varepsilon^{\pi}_{c_L} > -6.55$. That is, the condition could only fail if the elasticity $\varepsilon^{\pi}_{c_L}$ is extremely large in absolute value.

Sign of $\frac{\partial \widehat{s}}{\partial c_H}$:
Now we have that:

$$\text{Sign}\left[\frac{\partial \widehat{s}}{\partial c_H}\right] = \text{Sign}\left[-\pi - c_H\frac{\partial \pi}{\partial c_H}\right]. \tag{29}$$

The sign of $\frac{\partial \widehat{s}}{\partial c_H}$ is clearly negative if $\frac{\partial \pi}{\partial c_H} > 0$. We focus, therefore, on the case in which $\frac{\partial \pi}{\partial c_H}$ is negative. If we define the elasticity of π with respect to c_H as $\varepsilon^{\pi}_{c_H} = \frac{\partial \pi}{\partial c_H}\frac{c_H}{\pi}$, the sign of $\frac{\partial \widehat{s}}{\partial c_H}$ will be negative if $\varepsilon^{\pi}_{c_H} > -1$. That is, we require either that c_H has a positive effect on college attendance π or, if the effect is negative, the size of this effect cannot be very strong.

Slope of the iso-subsidies:
Using the elasticities of π with respect to c_L and c_H as $\varepsilon^{\pi}_{c_L} = \frac{\partial \pi}{\partial c_L}\frac{c_L}{\pi}$ and $\varepsilon^{\pi}_{c_H} = \frac{\partial \pi}{\partial c_H}\frac{c_H}{\pi}$, the slope of the iso-subsidy can be rewritten as:

$$\frac{\partial c_H}{\partial c_L}\mid_{s=\overline{\widehat{s}}} = -\frac{1 + \widehat{s}\pi\beta\varepsilon^{\pi}_{c_L}}{\widehat{s}\pi(1 + \varepsilon^{\pi}_{c_H})}, \tag{30}$$

where $\beta = c_H/c_L$.

First, suppose that both $\varepsilon^{\pi}_{c_H}$ and $\varepsilon^{\pi}_{c_L}$ are zero. Then the slope is:

$$\frac{\partial c_H}{\partial c_L}\mid_{s=\overline{\widehat{s}}} = -\frac{1}{\widehat{s}\pi}. \tag{31}$$

The lower is $\widehat{s}$ the higher is the absolute value of this expression. That is, the lower is $\widehat{s}$, the steeper are the iso-subsidies. Once we take into account the effect of both $\varepsilon^{\pi}_{c_L}$ and $\varepsilon^{\pi}_{c_H}$ the result will hold as long as they are of small size, as we have discussed above.

Quasi-concavity of Q_S

Consider a differentiable function $f(x) : R^2 \to R$. A sufficient condition for quasi-concavity is:

$$2f_1'(x)f_2'(x)f_{12}''(x) - (f_2'(x))^2 f_{11}''(x) - (f_1'(x))^2 f_{22}''(x) > 0. \tag{32}$$

For the case of Q_S we only need to assume the standard regularity conditions, namely:

$$\frac{\partial^2 Q_S(c_L, c_H)}{\partial c_L^2} \leq 0, \frac{\partial^2 Q_S(c_L, c_H)}{\partial c_H^2} \leq 0, \frac{\partial^2 Q_S(c_L, c_H)}{\partial c_H \partial c_L} \geq 0. \tag{33}$$

That is, we require diminishing returns to both factors and a positive cross effect.

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