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**CAN PRODUCTIVITY PROGRESS IN CHINA HURT THE US? PROFESSOR SAMUELSON'S
EXAMPLE EXTENDED**

ABSTRACT

This paper develops a general equilibrium 3-good Ricardian model that extends Professor Samuelson's example on the impact of productivity progress published in JEP (summer 2004). Our model highlights Professor Samuelson's insight that productivity progress can change the pattern of trade which in turn can have dramatic welfare implications. It also shows that while Professor Samuelson is correct that productivity growth in one country can hurt another, the loss is not as permanent as his example appears to suggest. Continuing productivity growth in one country is likely to benefit all trading countries in the long run.

Key words: 3-good Ricardian model, impact of productivity growth, globalisation

JEL classification: F10, F11

CAN PRODUCTIVITY PROGRESS IN CHINA HURT THE US? PROFESSOR SAMUELSON'S EXAMPLE EXTENDED

1. INTRODUCTION

In his recent article, "Where Ricardo and Mill Rebut and Confirm Arguments of Mainstream Economists Supporting Globalization" published in the *Journal of Economic Perspectives* (Summer 2004), Professor Samuelson raises an important question: can free trade globalisation convert a productivity gain in one country (say China) into a permanent welfare loss in another (say the US)? Professor Samuelson's answer to this question is a definite yes. In a standard setting of a Ricardian model, Professor Samuelson gives an example that demonstrates that an invention that improves China's productivity in a good that she does not traditionally export can permanently reduce per capita real income in the US. In this paper, we extend Professor Samuelson's example by developing a general equilibrium 3-good Ricardian model.* Our model will show that:

1. Professor Samuelson's insight lies in his observation that a productivity gain in one country can alter the pattern of trade, which in turn has welfare implications. In his example, prior to the productivity improvement in China as a result of an invention, good 1 and good 2 are traded whereas good 3 is not. After the invention, all 3 goods are traded. The change in the pattern of trade is more fundamental than a mere improvement or deterioration of the terms of trade, thus Professor Samuelson's example is not a simple restatement of the well-known result that an exogenous change in productivity abroad may harm a country, depending on what happens to the country's terms of trade.
2. Professor Samuelson's example is an incomplete representation of possible welfare implications of a productivity improvement in a 3-good Ricardian model. His example considers only two trade patterns: a pre-invention equilibrium involving two traded goods, and a post-invention equilibrium involving three traded goods. In a complete general equilibrium Ricardian model with 3 goods, there are 25 possible patterns of production, and each production pattern can be associated with multiple trade patterns.
3. Because Professor Samuelson's example is an incomplete representation of possible welfare implications of a productivity improvement in a 3-good Ricardian model, his conclusion is incomplete and can be misleading. Our model confirms that given Professor Samuelson's assumptions, his result is correct that an invention in China that *doubles* China's productivity in good 3 can hurt the US relative to the pre-invention equilibrium.

* Professor Samuelson describes a 2-good Ricardian model in the main text of his article, and presents a 3-good Ricardian model in Appendix 1. Our extension is based on the 3-good model, as it has, in Professor Samuelson words, "greater verisimilitude" than the 2-good version.

However we will show that if further improvement occurs in China in good 3 so that her productivity is, for instance, *five times* the pre-invention level, then both US and China will benefit from the invention (relative to the pre-invention equilibrium). This suggests that the possible loss to the US resulting from China's productivity improvement is not as "permanent" as Professor Samuelson's example appears to suggest. Free trade and the forces of creative destruction may in the longer term provide benefits to all trading countries after all as most economists believe, although episodes of net losses in some countries can occur in the process as Professor Samuelson correctly argues.

The remainder of the paper is organised as follows. Section 2 summarises Professor Samuelson's example. Section 3 extends the example to a full general equilibrium model (with model details presented in the Appendix) and discusses how productivity improvement in one country may hurt or benefit another. Section 4 provides some concluding remarks.

2. A SUMMARY OF PROFESSOR SAMUELSON'S EXAMPLE

Professor Samuelson constructs a Ricardian model with 2 countries (the US and China), and 3 goods. For simplicity the model assumes that the workforce is 100 in the US and 1000 in China. The Ricardian productivity parameters for the three goods are initially (2, $\frac{1}{2}$, 1) in the US, and (1/20, 2/10, 1/10) in China. After an invention, the productivity parameter for good 3 in China doubles from 1/10 to 2/10.

Given these assumptions, Professor Samuelson then tells the tale of two equilibria: the before-invention equilibrium and the post-invention equilibrium. At the before-invention equilibrium, the US produces good 1 and good 3, and exports good 1; China produces good 2 and good 3, and exports good 2. Good 3 is not trade internationally. Both countries benefit from trade and each has a net national product of 52.915. At the post-invention equilibrium, the US produces good 1 only, and exports good 1 in exchange of good 2 and good 3. China produces good 2 and good 3 and exports both in exchange of good 1. The national product in the US falls from the pre-invention level of 52.915 to 41.997. Therefore Professor Samuelson concludes that an invention abroad can cause a permanent loss of income in the US.

3. EXTENDING PROFESSOR SAMUELSON'S EXAMPLE TO A GENERAL EQUILIBRIUM MODEL

Professor Samuelson's example is based on an economist's intuition without resort to a fully developed model. The power of intuition lies in the observation that there are different equilibrium trade patterns in a 3-good Ricardian model, and that a change in productivity in one country can

lead to a discontinuous change in the equilibrium trade structure. In his example as outlined above, Professor Samuelson observes that before the invention, the equilibrium trade structure involves each country exporting its comparative advantage good, and one good (good 3) is not traded. After the invention, the equilibrium trade structure jumps to one in which the non-traded good becomes traded. The jump in equilibrium trade structure leads to not only changes in relative prices of the goods, but also the pattern of production and trade. It is no wonder these fundamental changes can have drastic welfare implications.

It should be emphasised that Professor Samuelson's observation is different from the conventional wisdom that exogenous productivity changes can hurt a country depending on how the country's terms of trade is affected. This is because a jump in equilibrium production and trade pattern is a more fundamental change than mere changes in the terms of trade. In other words, Professor Samuelson's example points to a cause of income changes at the different level, rather than simply restating the findings of Professor Johnson (1954, 1955).

Powerful as Professor Samuelson's intuition may be, without a fully developed model, it is easy to miss some implications that are not immediately intuitive. His example considers only 3 trade structures (including autarky) out of scores of possible trade structures that can occur in equilibrium in a 2-country, 3-good Ricardian model. In addition, the example only demonstrates the effect of a single invention that happens to double productivity in good 3. Therefore one would suspect that there is at least a possibility that his result may be biased or even misleading. To test the robustness of his result, we develop a general equilibrium 2x3 Ricardian model. The model confirms our suspicion that Professor Samuelson's result, while correct in a narrow sense, can be misleading. The logic of the model is outlined below and the detailed calculation is presented in the Appendix.

Consider two countries i ($i=1, 2$), each endowed with a workforce L_i which can be used to produce three consumer goods X, Y, and Z. On the demand side, the representative consumer in country i maximises utility subject to the constraint that his total expenditure equals his wage income, that is

$$\begin{aligned} \max_{x_i, y_i, z_i} U_i &= x_i^\alpha y_i^\beta z_i^{1-\alpha-\beta} \\ \text{s.t. } p_x x_i + p_y y_i + p_z z_i &= w_i \end{aligned}$$

The solution to the utility maximization problem gives us the demand functions for goods X, Y and Z as follows:

$$x_i^d = \frac{\alpha w_i}{p_x}, y_i^d = \frac{\beta w_i}{p_y}, z_i^d = (1 - \alpha - \beta) w_i$$

where we normalise the price of good Z to be 1.

On the supply side, there are 3 types of representative firms in country i each producing good X or Y or Z and maximizing profit subject to the following production technologies:

$$x_i = a_{ix}L_{ix}, \quad y_i = a_{iy}L_{iy}, \quad z_i = a_{iz}L_{iz}$$

where a_{ix}, a_{iy}, a_{iz} are productivity coefficients, and L_{ix}, L_{iy}, L_{iz} are labor devoted to producing good X, Y and Z respectively.

For instance, the decision problem for the representative firm producing good X is:

$$\begin{aligned} \max_{L_{ix}} \pi_{ix} &= p_x a_{ix} L_{ix} - w_i L_{ix} \\ \text{s.t. } L_{ix} &\geq 0 \end{aligned}$$

The non-negativity constraint captures the possibility that X is not produced in country i when $L_{ix} = 0$.

Since L_{ix}, L_{iy}, L_{iz} can be zero or positive, there are 64 ($=2^6$) possible production structures, but only 25 of which are consistent with autarky or trade between the two countries. These 25 structures consists of: 1 structure where both countries produce all three goods; 6 ($=2 C_3^3 C_3^2$) structures where one country produces all three goods and the other produces only 2 goods; 6 ($=2 C_3^3 C_3^1$) structures where one country produces all three goods and the other produces only 1 good; 6 ($=2 C_3^2 C_3^2$) structures where each country produces two goods; and 6 ($=2 C_3^2 C_3^1$) structures where one country produces 2 goods and the other produces the remaining 1 good. The feasible structures are listed in Table 1. In Table 1, the letters in the first bracket indicate goods produced in country 1, and those in the second bracket indicate goods produced in country 2. It should be noted that each production structure may be associated with multiple trade patterns. For instance, structure (XYZ)(XYZ), where both countries produce all 3 goods, can be associated with (i) country 1 exporting good 1 in exchange of good 2 and good 3; (ii) country 1 exporting good 1 in exchange of good 2 with good 3 being non-traded; (iii) country 1 exporting good 1 in exchange with good 3 with good 2 being non-traded; etc.

Table 1: Production Structures

Sequence Number	Characters of Production Structures	Structures
1	$L_{1x} > 0, L_{1y} > 0, L_{1z} > 0, L_{2x} > 0, L_{2y} > 0, L_{2z} > 0$	(XYZ)(XYZ)
2	$L_{1x} > 0, L_{1y} > 0, L_{1z} > 0, L_{2x} > 0, L_{2y} > 0, L_{2z} = 0$	(XYZ)(XY)
3	$L_{1x} > 0, L_{1y} > 0, L_{1z} > 0, L_{2x} > 0, L_{2y} = 0, L_{2z} > 0$	(XYZ)(XZ)
4	$L_{1x} > 0, L_{1y} > 0, L_{1z} > 0, L_{2x} = 0, L_{2y} > 0, L_{2z} > 0$	(XYZ)(YZ)
5	$L_{1x} > 0, L_{1y} > 0, L_{1z} = 0, L_{2x} > 0, L_{2y} > 0, L_{2z} > 0$	(XY)(XYZ)
6	$L_{1x} > 0, L_{1y} = 0, L_{1z} > 0, L_{2x} > 0, L_{2y} > 0, L_{2z} > 0$	(XZ)(XYZ)
7	$L_{1x} = 0, L_{1y} > 0, L_{1z} > 0, L_{2x} > 0, L_{2y} > 0, L_{2z} > 0$	(YZ)(XYZ)
8	$L_{1x} > 0, L_{1y} > 0, L_{1z} = 0, L_{2x} > 0, L_{2y} = 0, L_{2z} > 0$	(XY)(XZ)
9	$L_{1x} > 0, L_{1y} > 0, L_{1z} = 0, L_{2x} = 0, L_{2y} > 0, L_{2z} > 0$	(XY)(YZ)
10	$L_{1x} > 0, L_{1y} = 0, L_{1z} > 0, L_{2x} > 0, L_{2y} > 0, L_{2z} = 0$	(XZ)(XY)
11	$L_{1x} > 0, L_{1y} = 0, L_{1z} > 0, L_{2x} = 0, L_{2y} > 0, L_{2z} > 0$	(XZ)(YZ)
12	$L_{1x} = 0, L_{1y} > 0, L_{1z} > 0, L_{2x} > 0, L_{2y} = 0, L_{2z} > 0$	(YZ)(XZ)
13	$L_{1x} = 0, L_{1y} > 0, L_{1z} > 0, L_{2x} > 0, L_{2y} > 0, L_{2z} = 0$	(YZ)(XY)
14	$L_{1x} > 0, L_{1y} > 0, L_{1z} > 0, L_{2x} > 0, L_{2y} = 0, L_{2z} = 0$	(XYZ)(X)
15	$L_{1x} > 0, L_{1y} > 0, L_{1z} > 0, L_{2x} = 0, L_{2y} > 0, L_{2z} = 0$	(XYZ)(Y)
16	$L_{1x} > 0, L_{1y} > 0, L_{1z} > 0, L_{2x} = 0, L_{2y} = 0, L_{2z} > 0$	(XYZ)(Z)
17	$L_{1x} > 0, L_{1y} > 0, L_{1z} = 0, L_{2x} = 0, L_{2y} = 0, L_{2z} > 0$	(XY)(Z)
18	$L_{1x} > 0, L_{1y} = 0, L_{1z} > 0, L_{2x} = 0, L_{2y} > 0, L_{2z} = 0$	(XZ)(Y)
19	$L_{1x} = 0, L_{1y} > 0, L_{1z} > 0, L_{2x} > 0, L_{2y} = 0, L_{2z} = 0$	(YZ)(X)
20	$L_{1x} > 0, L_{1y} = 0, L_{1z} = 0, L_{2x} > 0, L_{2y} > 0, L_{2z} > 0$	(X)(XYZ)
21	$L_{1x} = 0, L_{1y} > 0, L_{1z} = 0, L_{2x} > 0, L_{2y} > 0, L_{2z} > 0$	(Y)(XYZ)
22	$L_{1x} = 0, L_{1y} = 0, L_{1z} > 0, L_{2x} > 0, L_{2y} > 0, L_{2z} > 0$	(Z)(XYZ)
23	$L_{1x} > 0, L_{1y} = 0, L_{1z} = 0, L_{2x} = 0, L_{2y} > 0, L_{2z} > 0$	(X)(YZ)
24	$L_{1x} = 0, L_{1y} > 0, L_{1z} = 0, L_{2x} > 0, L_{2y} = 0, L_{2z} > 0$	(Y)(XZ)
25	$L_{1x} = 0, L_{1y} = 0, L_{1z} > 0, L_{2x} > 0, L_{2y} > 0, L_{2z} = 0$	(Z)(XY)

Given each production pattern, we can solve the representative firms' decision problems using the Kuhn-Tucker conditions. The solutions to consumers and firms decision problems, together with the clearing conditions in both the goods and labor markets will give us the equilibrium prices and quantities for that structure, and also define the conditions under which the structure will occur in equilibrium.

To illustrate, consider the pre-invention trade structure in Professor Samuelson's example. Prior to the invention, country 1 (the US) produces X and Y (goods 1 and 2), exports X; country 2 (China) produces Y and Z (good 3) and exports Z. The production structure is structure 9 listed in Table 1, which is characterised by:

$$L_{1x} > 0, L_{1y} > 0, L_{1z} = 0, L_{2x} = 0, L_{2y} > 0, L_{2z} > 0.$$

Given the above constraints and setting the price of good Z to be 1, we can solve the firms' decision problems and obtain:

$$p_x = \frac{a_{1y}}{a_{2y}} \frac{a_{2z}}{a_{1x}} < \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{2z}}{a_{2y}}, w_1 = \frac{a_{1y}}{a_{2y}} a_{2z} > a_{1z}, w_2 = a_{2z}$$

These solutions defines one condition for the structure (XY)_x(YZ)_z to occur in equilibrium:

$$\frac{a_{1x}}{a_{2x}} > \frac{a_{1y}}{a_{2y}} > \frac{a_{1z}}{a_{2z}} \quad (1)$$

From the market clearing conditions in the goods and labor markets, we can obtain the equilibrium quantities of productions and trade, as well as another condition for the trade pattern (XY)_x(YZ)_z (the subscripts denote the good exported) to occur in equilibrium

$$\frac{L_2}{L_1} = \frac{(1-\alpha-\beta)a_{1y}}{\alpha a_{2y}} \quad (2)$$

Therefore if the parameter values satisfy both conditions (1) and (2), the equilibrium structure will be (XY)_x(YZ)_z, where country 1 produces goods X and Y and exports good X; and country 2 produces goods Y and Z, and exports Z. The equilibrium per capita real income (or utility) levels are:

$$\text{Country 1 (US): } u_1 = \alpha^\alpha \beta^\beta (1-\alpha-\beta)^{(1-\alpha-\beta)} a_{1y}^{1-\alpha} a_{1x}^\alpha a_{2z}^{1-\alpha-\beta} a_{2y}^{\alpha+\beta-1} \quad (3)$$

$$\text{Country 2 (China): } u_2 = \alpha^\alpha \beta^\beta (1-\alpha-\beta)^{(1-\alpha-\beta)} a_{1y}^{-\alpha} a_{1x}^\alpha a_{2z}^{1-\alpha-\beta} a_{2y}^{\alpha+\beta} \quad (4)$$

Now let us adopt Professor Samuelson's following assumptions:

(i) Workforce in US and China are: $L_1 = 100$, $L_2 = 1000$,

(ii) Preference parameters are: $\alpha = \beta = \frac{1}{3}$

(iii) Pre-invention productivity parameters are:

$$a_{1x} = 2, a_{1y} = 1, a_{1z} = \frac{1}{2}, a_{2x} = \frac{1}{20}, a_{2y} = \frac{1}{10}, a_{2z} = \frac{2}{10}.$$

It is easy to show that these parameter assumptions satisfy conditions (1) and (2), therefore Professor Samuelson is correct about the equilibrium trade pattern given his assumptions. Substituting the parameter values into equation (3) and (4), we have the per capita real income in US to be 0.52915, and that in China to be 0.052915. These imply that both countries have the same net national income of 52.915, which confirms Professor Samuelson's result.

Similarly we can solve the equilibrium for the post-invention structure $(X)_X(YZ)_{YZ}$, where the US produces good X and exports X; China produces goods Y and Z, and exports both. The condition for this structure to occur in equilibrium is:

$$\max\left\{\frac{(1-\alpha)a_{1y}}{\alpha a_{2y}}, \frac{(1-\alpha)a_{1z}}{\alpha a_{2z}}\right\} \leq \frac{L_2}{L_1} < \frac{(1-\alpha)a_{1x}}{\alpha a_{2x}} \quad (5)$$

And the equilibrium per capita real income (or utility) levels are:

$$\text{Country 1 (US): } u_1 = \alpha^\alpha \beta^\beta (1-\alpha-\beta)^{(1-\alpha-\beta)} \left(\frac{\alpha L_2}{(1-\alpha)L_1}\right)^{1-\alpha} a_{1x}^\alpha a_{2z}^{1-\alpha-\beta} a_{2y}^\beta \quad (6)$$

$$\text{Country 2 (China): } u_2 = \alpha^\alpha \beta^\beta (1-\alpha-\beta)^{(1-\alpha-\beta)} \left(\frac{\alpha L_2}{(1-\alpha)L_1}\right)^{-\alpha} a_{1x}^\alpha a_{2z}^{1-\alpha-\beta} a_{2y}^\beta \quad (7)$$

Post invention, China's productivity in good Y doubles from $a_{2y} = \frac{1}{10}$ to $a'_{2y} = \frac{2}{10}$, while other parameters remain unchanged. It is easy to show that now the parameters satisfy condition (5), thus Professor Samuelson is right that following the invention, the equilibrium trade structure will jump from $(XY)_X(YZ)_Z$ to $(X)_X(YZ)_{YZ}$. Substituting the post invention parameters into equations (6) and (7), we again confirm Professor Samuelson's result: following the invention, the national product in the US falls to 41.997, while the net national product in China rises to 83.995.

Now suppose further invention occurs (or the initial invention is more drastic) such that China's productivity in good Y improves five-fold to $a'_{2y} = \frac{1}{2}$ while other parameters remain unchanged, then the post invention parameters will still satisfy condition (5), and the equilibrium trade structure will still be $(X)_X(YZ)_{YZ}$. However, the post invention equilibrium net national income in the US will be 56.999, which is greater than the pre-invention level of 52.915. In comparison, the net national income in China will be 113.998, which is also greater than the pre-invention level of 52.915. Therefore contrary to what Professor Samuelson's example appears to imply, our calculation shows that large and/or continuing productivity progress in China in a good that China does not

traditionally export is likely to benefit both US and China in the long run. In other words, while it is possible that a given productivity gain in China can cause a welfare loss in the US, the welfare loss is not as permanent as Professor Samuelson's example would let us believe. If productivity continues to improve as it usually does in a dynamic globalised economy, the view of most mainstream economists is correct after all – free trade is likely to generate net gains to all trading nations in the long run. Of course this does not preclude that episodes may occur where some trading countries may experience losses from exogenous changes in technology or other factors as Professor Samuelson's example correctly shows.

Following a similar approach, we can also solve the equilibrium prices, quantities and income levels for other structures and the define conditions for each of these structure to occur in equilibrium. The solutions are presented in the Appendix.

4. CONCLUDING REMARKS

In this paper we have outlined a general equilibrium 2-country, 3-good Ricardian model, which is used to test the robustness of Professor Samuelson's example on the impact of exogenous productivity change. Our model highlights Professor Samuelson insight that an exogenous productivity change in one country can lead to a discontinuous jump in equilibrium trade structure, which unsurprisingly can have drastic welfare implications. It also shows that contrary to what Professor Samuelson's example appears to imply, large and/or continuing productivity progress in one country (even in the good that the country does not traditionally exports) is likely to benefit both trading countries in the long run.

Aside from the issue of welfare implication of productivity progress, our model points to the amazing explanatory power of the general Ricardian model which appears to have been overlooked in the literature. Variants of the Ricardian model outlined in this paper can be used to study a wide range of topics including the effect of trade on wage inequality, and welfare implications of outsourcing.[†] The power of the Ricardian model, in our view, lies in that despite its simplicity, it is able to capture complex trade phenomena. As our model shows, in a simple 3-good, 2-country setting, the model captures 25 production patterns and many more possible trade

[†] Professor Panagariya (2004) criticises that Professor Samuelson's 2-good example is a wrong model to represent outsourcing. The criticism does not apply to the 3-good example because the example can be interpreted as modelling the outsourcing service (good Y) as initially non-traded, then productivity change turning the non-traded service into a traded one. This is essentially the same construct as that of model 3 in Bhagwati, Panagariya and Srinivasan (2004) which, in Professor Pangariya's view, correctly models outsourcing.

patterns that can occur in equilibrium. A detailed analysis of the model may produce other interesting results.

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APPENDIX

Consider two country i ($i=1, 2$) each endowed with labor L_i which can be used to produce three consumer goods X, Y, and Z. The goods can be freely traded between the two countries.

The decision problem of a representative consumer in country i is

$$\begin{aligned} \max_{x_i, y_i, z_i} U_i &= x_i^\alpha y_i^\beta z_i^{1-\alpha-\beta} \\ \text{s.t. } p_x x_i + p_y y_i + p_z z_i &= w_i \end{aligned} \quad (\text{A1})$$

where x_i , y_i and z_i are quantities of goods X, Y and Z, respectively; p_x is the price of good X; p_y is the price of good Y; p_z is the price of good Z which is normalised to be 1; and w_i is wage rate in country i .

To solve the consumer's problem, we define a Langrangean function

$$Z_i = x_i^\alpha y_i^\beta z_i^{1-\alpha-\beta} + \lambda_i (w_i - p_x x_i - p_y y_i - z_i)$$

The first order conditions are

$$\frac{\partial Z_i}{\partial x_i} = \alpha x_i^{\alpha-1} y_i^\beta z_i^{1-\alpha-\beta} - \lambda_i p_x = 0 \quad (\text{A2})$$

$$\frac{\partial Z_i}{\partial y_i} = \beta x_i^\alpha y_i^{\beta-1} z_i^{1-\alpha-\beta} - \lambda_i p_y = 0 \quad (\text{A3})$$

$$\frac{\partial Z_i}{\partial z_i} = (1 - \alpha - \beta) x_i^\alpha y_i^\beta z_i^{-\alpha-\beta} - \lambda_i = 0 \quad (\text{A4})$$

$$\frac{\partial Z_i}{\partial \lambda_i} = w_i - p_x x_i - p_y y_i - z_i = 0 \quad (\text{A5})$$

From equations (A2)-(A5), we obtain the following demand functions for goods X, Y and Z:

$$x_i^d = \frac{\alpha w_i}{p_x}, y_i^d = \frac{\beta w_i}{p_y}, z_i^d = (1 - \alpha - \beta) w_i \quad (\text{A6})$$

Next, we consider the supply side. The production functions for X, Y and Z in country i are assumed to be:

$$x_i = a_{ix} L_{ix}, y_i = a_{iy} L_{iy}, z_i = a_{iz} L_{iz} \quad (\text{A7})$$

where L_{ix}, L_{iy}, L_{iz} are labor devoted to the production of good X, Y, and Z, respectively; a_{ij} ($i=1,2; j=x,y,z$) is the total factor productivity coefficient. Since a_{ij} is country specific, it captures the productivity difference between the two countries.

Constrained by the production technology, the representative firm producing X in country i maximizes its profit, i.e.,

$$\begin{aligned} \max_{L_{ix}} \pi_{ix} &= p_x a_{ix} L_{ix} - w_i L_{ix} \\ \text{s.t. } L_{ix} &\geq 0 \end{aligned}$$

Define a Lagrangean function for above optimal problem as

$$Z_{ix} = p_x a_{ix} L_{ix} - w_i L_{ix} + \lambda_{ix} L_{ix}$$

The Kuhn-Tucker conditions for this problem are

$$\frac{\partial Z_{ix}}{\partial L_{ix}} = p_x a_{ix} - w_i + \lambda_{ix} = 0 \quad (\text{A8})$$

$$L_{ix} \geq 0, \lambda_{ix} \geq 0, \lambda_{ix} L_{ix} = 0 \quad (\text{A9})$$

Similarly, the decision problem for the representative firm producing Y in country i is:

$$\begin{aligned} \max_{L_{iy}} \pi_{iy} &= p_y a_{iy} L_{iy} - w_i L_{iy} \\ \text{s.t. } L_{iy} &\geq 0 \end{aligned}$$

Define a Lagrangean function for above optimal problem as

$$Z_{iy} = p_y a_{iy} L_{iy} - w_i L_{iy} + \lambda_{iy} L_{iy}$$

The Kuhn-Tucker conditions for this problem are

$$\frac{\partial Z_{iy}}{\partial L_{iy}} = p_y a_{iy} - w_i + \lambda_{iy} = 0 \quad (\text{A10})$$

$$L_{iy} \geq 0, \lambda_{iy} \geq 0, \lambda_{iy} L_{iy} = 0 \quad (\text{A11})$$

The decision problem for the representative firm producing Z in country i is:

$$\begin{aligned} \max_{L_{iz}} \pi_{iz} &= a_{iz} L_{iz} - w_i L_{iz} \\ \text{s.t. } L_{iz} &\geq 0 \end{aligned}$$

Define a Lagrangean function for above optimal problem as

$$Z_{iz} = a_{iz} L_{iz} - w_i L_{iz} + \lambda_{iz} L_{iz}$$

The Kuhn-Tucker conditions for this problem are

$$\frac{\partial Z_{iz}}{\partial L_{iz}} = a_{iz} - w_i + \lambda_{iz} = 0 \quad (\text{A12})$$

$$L_{iz} \geq 0, \lambda_{iz} \geq 0, \lambda_{iz} L_{iz} = 0 \quad (\text{A13})$$

Since L_{ix}, L_{iy}, L_{iz} can be zero or positive, there are 64 ($=2^6$) possible production structures, but only 25 of which are consistent with autarky or trade between the two countries. These 25 structures are listed in Table 1 in the main text.

Given a structure, we can solve both the consumer's and firms' decision problems described above. The solutions, together with the following market clearing conditions will give us the equilibrium prices, quantities and income levels in both countries as well as the conditions for the structure to occur in equilibrium.

$$x_1^d + x_2^d = x_1^s + x_2^s \quad (\text{A14})$$

$$y_1^d + y_2^d = y_1^s + y_2^s \quad (\text{A15})$$

$$z_1^d + z_2^d = z_1^s + z_2^s \quad (\text{A16})$$

$$L_{1x} + L_{1y} + L_{1z} = L_1 \quad (\text{A17})$$

$$L_{2x} + L_{2y} + L_{2z} = L_2 \quad (\text{A18})$$

Note that according to Walras's law, only 4 of the above 5 equations are independent.

To illustrate the methodology for solving the equilibrium, we provide two examples below.

Example 1: Structure $(XY)_X(YZ)_Z$.

From Table 1, this structure is characterised by $L_{1x} > 0, L_{1y} > 0, L_{1z} = 0, L_{2x} = 0, L_{2y} > 0, L_{2z} > 0$.

From equations (A8)-(A13), we obtain

$$p_x = \frac{w_1}{a_{1x}} < \frac{w_2}{a_{2x}} \quad (\text{A19})$$

$$p_y = \frac{w_1}{a_{1y}} = \frac{w_2}{a_{2y}} \quad (\text{A20})$$

$$w_1 > a_{1z}, w_2 = a_{2z} \quad (\text{A21})$$

which in turn imply:

$$p_x = \frac{a_{1y}}{a_{2y}} \frac{a_{2z}}{a_{1x}} < \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{2z}}{a_{2y}}, w_1 = \frac{a_{1y}}{a_{2y}} a_{2z} > a_{1z}, w_2 = a_{2z} \quad (\text{A22})$$

$$\frac{a_{1x}}{a_{2x}} > \frac{a_{1y}}{a_{2y}} > \frac{a_{1z}}{a_{2z}} \quad (\text{A23})$$

From the market clearing conditions (A14)-(A18), we obtain

$$a_{1x}L_{1x} = \alpha a_{1x}L_1 + \alpha a_{1x} \frac{a_{2y}}{a_{1y}} L_2, \quad a_{1y}L_{1y} = \beta a_{1y}L_1 \quad (\text{A24})$$

$$a_{2z}L_{2z} = (1 - \alpha - \beta)a_{2z} \frac{a_{1y}}{a_{2y}} L_1 + (1 - \alpha - \beta)a_{2z}L_2, \quad a_{2y}L_{2y} = \beta a_{2y}L_2 \quad (\text{A25})$$

Equations (A23) and (A24) define the conditions under which structure $(XY)_X(YZ)_Z$ will occur in equilibrium:

$$\frac{a_{1x}}{a_{2x}} > \frac{a_{1y}}{a_{2y}} > \frac{a_{1z}}{a_{2z}}, \quad \frac{L_2}{L_1} = \frac{(1 - \alpha - \beta)a_{1y}}{\alpha a_{2y}} \quad (\text{A26})$$

From equations (A6) and (A22), we solve the equilibrium demand and then obtain the utility (real per capital income) of individual in each country as follows.

$$u_1 = \alpha^\alpha \beta^\beta (1 - \alpha - \beta)^{(1 - \alpha - \beta)} a_{1y}^{1 - \alpha} a_{1x}^\alpha a_{2z}^{1 - \alpha - \beta} a_{2y}^{\alpha + \beta - 1}$$

$$u_2 = \alpha^\alpha \beta^\beta (1 - \alpha - \beta)^{(1 - \alpha - \beta)} a_{1y}^{-\alpha} a_{1x}^\alpha a_{2z}^{1 - \alpha - \beta} a_{2y}^{\alpha + \beta}$$

Example 2: Structure $(X)_X(YZ)_{YZ}$.

This structure is characterised by $L_{1x} > 0, L_{1y} = 0, L_{1z} = 0, L_{2x} = 0, L_{2y} > 0, L_{2z} > 0$.

From equations (A8)-(A13), we obtain

$$p_x = \frac{w_1}{a_{1x}} < \frac{w_2}{a_{2x}} \quad (\text{A27})$$

$$p_y = \frac{w_2}{a_{2y}} < \frac{w_1}{a_{1y}} \quad (\text{A28})$$

$$w_1 > a_{1z}, \quad w_2 = a_{2z} \quad (\text{A29})$$

From equations (A14)-(A18), we have

$$a_{1x}L_{1x} = \alpha a_{1x}L_1 + \alpha a_{1x} \frac{w_2}{w_1} L_2 \quad (\text{A30})$$

$$a_{2y}L_{2y} = \beta a_{2y} \frac{w_1}{w_2} L_1 + \beta a_{2y}L_2 \quad (\text{A31})$$

$$a_{2z}L_{2z} = (1 - \alpha - \beta)a_{2z} \frac{w_1}{w_2} L_1 + (1 - \alpha - \beta)a_{2z}L_2 \quad (\text{A32})$$

$$L_{1x} = L_1 \quad (\text{A33})$$

which in turn imply

$$p_x = \frac{w_1}{a_{1x}}, p_y = \frac{a_{2z}}{a_{2y}}, w_1 = \frac{\alpha L_2}{(1 - \alpha)L_1} a_{2z}, w_2 = a_{2z} \quad (\text{A34})$$

$$\max\left\{\frac{(1 - \alpha)a_{1y}}{\alpha a_{2y}}, \frac{(1 - \alpha)a_{1z}}{\alpha a_{2z}}\right\} \leq \frac{L_2}{L_1} < \frac{(1 - \alpha)a_{1x}}{\alpha a_{2x}} \quad (\text{A35})$$

From equations (A6) and (A34), we solve the equilibrium demand and then obtain the utility (real per capital income) of individual in each country as follows.

$$u_1 = \alpha^\alpha \beta^\beta (1 - \alpha - \beta)^{(1 - \alpha - \beta)} \left(\frac{\alpha L_2}{(1 - \alpha)L_1}\right)^{1 - \alpha} a_{1x}^\alpha a_{2z}^{1 - \alpha - \beta} a_{2y}^\beta$$

$$u_2 = \alpha^\alpha \beta^\beta (1 - \alpha - \beta)^{(1 - \alpha - \beta)} \left(\frac{\alpha L_2}{(1 - \alpha)L_1}\right)^{-\alpha} a_{1x}^\alpha a_{2z}^{1 - \alpha - \beta} a_{2y}^\beta$$

Following the same approach as in examples 1 and 2 above, we can solve the equilibrium prices for each production structure listed in Table 1, and also define the conditions under which a trade structure occurs in the equilibrium. The results are presented in Table A1 and Table A2 below.

Table A1: Production Structures and General Equilibrium Prices

Structures	Equilibrium prices
(XYZ)(XYZ)	$p_x = \frac{a_{1z}}{a_{1x}} = \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{1z}}{a_{1y}} = \frac{a_{2z}}{a_{2y}}, w_1 = a_{1z}, w_2 = a_{2z}$
(XYZ)(XY)	$p_x = \frac{a_{1z}}{a_{1x}}, p_y = \frac{a_{1z}}{a_{1y}}, w_1 = a_{1z}, w_2 = \frac{a_{2x}}{a_{1x}} a_{1z} = \frac{a_{2y}}{a_{1y}} a_{1z} > a_{2z}$
(XYZ)(XZ)	$p_x = \frac{a_{1z}}{a_{1x}} = \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{1z}}{a_{1y}} < \frac{a_{2z}}{a_{2y}}, w_1 = a_{1z}, w_2 = a_{2z}$
(XYZ)(YZ)	$p_x = \frac{a_{1z}}{a_{1x}} < \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{1z}}{a_{1y}} = \frac{a_{2z}}{a_{2y}}, w_1 = a_{1z}, w_2 = a_{2z}$
(XY)(XYZ)	$p_x = \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{2z}}{a_{2y}}, w_1 = \frac{a_{1x}}{a_{2x}} a_{2z} = \frac{a_{1y}}{a_{2y}} a_{2z} > a_{1z}, w_2 = a_{2z}$
(XZ)(XYZ)	$p_x = \frac{a_{1z}}{a_{1x}} = \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{2z}}{a_{2y}} < \frac{a_{1z}}{a_{1y}}, w_1 = a_{1z}, w_2 = a_{2z}$
(YZ)(XYZ)	$p_x = \frac{a_{2z}}{a_{2x}} < \frac{a_{1z}}{a_{1x}}, p_y = \frac{a_{1z}}{a_{1y}} = \frac{a_{2z}}{a_{2y}}, w_1 = a_{1z}, w_2 = a_{2z}$
(XY)(XZ)	$p_x = \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{1x}}{a_{2x}} \frac{a_{2z}}{a_{1y}} < \frac{a_{2z}}{a_{2y}}, w_1 = \frac{a_{1x}}{a_{2x}} a_{2z} > a_{1z}, w_2 = a_{2z}$
(XY)(YZ)	$p_x = \frac{a_{1y}}{a_{2y}} \frac{a_{2z}}{a_{1x}} < \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{2z}}{a_{2y}}, w_1 = \frac{a_{1y}}{a_{2y}} a_{2z} > a_{1z}, w_2 = a_{2z}$
(XZ)(XY)	$p_x = \frac{a_{1z}}{a_{1x}}, p_y = \frac{a_{2x}}{a_{1x}} \frac{a_{1z}}{a_{2y}} < \frac{a_{1z}}{a_{1y}}, w_1 = a_{1z}, w_2 = \frac{a_{2x}}{a_{1x}} a_{1z} > a_{2z}$
(XZ)(YZ)	$p_x = \frac{a_{1z}}{a_{1x}} < \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{2z}}{a_{2y}} < \frac{a_{1z}}{a_{1y}}, w_1 = a_{1z}, w_2 = a_{2z}$
(YZ)(XZ)	$p_x = \frac{a_{2z}}{a_{2x}} < \frac{a_{1z}}{a_{1x}}, p_y = \frac{a_{1z}}{a_{1y}} < \frac{a_{2z}}{a_{2y}}, w_1 = a_{1z}, w_2 = a_{2z}$
(YZ)(XY)	$p_x = \frac{a_{2y}}{a_{1y}} \frac{a_{1z}}{a_{2x}} < \frac{a_{1z}}{a_{1x}}, p_y = \frac{a_{1z}}{a_{1y}}, w_1 = a_{1z}, w_2 = \frac{a_{2y}}{a_{1y}} a_{1z} > a_{2z}$
(XYZ)(X)	$p_x = \frac{a_{1z}}{a_{1x}}, p_y = \frac{a_{1z}}{a_{1y}} < \frac{a_{2x}}{a_{1x}} \frac{a_{1z}}{a_{2y}}, w_1 = a_{1z}, w_2 = \frac{a_{2x}}{a_{1x}} a_{1z} > a_{2z}$
(XYZ)(Y)	$p_x = \frac{a_{1z}}{a_{1x}} < \frac{a_{2y}}{a_{1y}} \frac{a_{1z}}{a_{2x}}, p_y = \frac{a_{1z}}{a_{1y}}, w_1 = a_{1z}, w_2 = \frac{a_{2y}}{a_{1y}} a_{1z} > a_{2z}$
(XYZ)(Z)	$p_x = \frac{a_{1z}}{a_{1x}} < \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{1z}}{a_{1y}} < \frac{a_{2z}}{a_{2y}}, w_1 = a_{1z}, w_2 = a_{2z}$

(XY)(Z)	$p_x = \frac{w_1}{a_{1x}} < \frac{a_{2z}}{a_{2x}}, p_y = \frac{w_1}{a_{1y}} < \frac{a_{2z}}{a_{2y}}, w_1 = \frac{\alpha + \beta}{(1 - \alpha - \beta)} \frac{L_2}{L_1} a_{2z} > a_{1z}, w_2 = a_{2z}$
(XZ)(Y)	$p_x = \frac{a_{1z}}{a_{1x}} < \frac{w_2}{a_{2x}}, p_y = \frac{w_2}{a_{2y}} < \frac{a_{1z}}{a_{1y}}, w_1 = a_{1z}, w_2 = \frac{\beta L_1}{(1 - \beta) L_2} a_{1z} > a_{2z}$
(YZ)(X)	$p_x = \frac{w_2}{a_{2x}} < \frac{a_{1z}}{a_{1x}}, p_y = \frac{a_{1z}}{a_{1y}} < \frac{w_2}{a_{2y}}, w_1 = a_{1z}, w_2 = \frac{\alpha L_1}{(1 - \alpha) L_2} a_{1z} > a_{2z}$
(X)(XYZ)	$p_x = \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{2z}}{a_{2y}} < \frac{a_{1x}}{a_{2x}} \frac{a_{2z}}{a_{1y}}, w_1 = \frac{a_{1x}}{a_{2x}} a_{2z} > a_{1z}, w_2 = a_{2z}$
(Y)(XYZ)	$p_x = \frac{a_{2z}}{a_{2x}} < \frac{a_{1y}}{a_{2y}} \frac{a_{2z}}{a_{1x}}, p_y = \frac{a_{2z}}{a_{2y}}, w_1 = \frac{a_{1y}}{a_{2y}} a_{2z} > a_{1z}, w_2 = a_{2z}$
(Z)(XYZ)	$p_x = \frac{a_{2z}}{a_{2x}} < \frac{a_{1z}}{a_{1x}}, p_y = \frac{a_{2z}}{a_{2y}} < \frac{a_{1z}}{a_{1y}}, w_1 = a_{1z}, w_2 = a_{2z}$
(X)(YZ)	$p_x = \frac{w_1}{a_{1x}} < \frac{a_{2z}}{a_{2x}}, p_y = \frac{a_{2z}}{a_{2y}} < \frac{w_1}{a_{1y}}, w_1 = \frac{\alpha L_2}{(1 - \alpha) L_1} a_{2z} > a_{1z}, w_2 = a_{2z}$
(Y)(XZ)	$p_x = \frac{a_{2z}}{a_{2x}} < \frac{w_1}{a_{1x}}, p_y = \frac{w_1}{a_{1y}} < \frac{a_{2z}}{a_{2y}}, w_1 = \frac{\beta L_2}{(1 - \beta) L_1} a_{2z} > a_{1z}, w_2 = a_{2z}$
(Z)(XY)	$p_x = \frac{w_2}{a_{2x}} < \frac{a_{1z}}{a_{1x}}, p_y = \frac{w_2}{a_{2y}} < \frac{a_{1z}}{a_{1y}}, w_1 = a_{1z}, w_2 = \frac{\alpha + \beta}{(1 - \alpha - \beta)} \frac{L_1}{L_2} a_{1z} > a_{2z}$

Table A2: Conditions for Existence of General Equilibrium Trade Structures

Parameter subspaces		Structures	
$\frac{a_{1x}}{a_{2x}} = \frac{a_{1y}}{a_{2y}} = \frac{a_{1z}}{a_{2z}}$	$L_{1x} = \alpha L_1, L_{1y} = \beta L_1, L_{1z} = (1 - \alpha - \beta)L_1$	AA	
	$L_{1x} = \alpha L_1, L_{1y} > \beta L_1, L_{1z} < (1 - \alpha - \beta)L_1$	$(XYZ)_Y(XYZ)_Z$	
	$L_{1x} = \alpha L_1, L_{1y} < \beta L_1, L_{1z} > (1 - \alpha - \beta)L_1$	$(XYZ)_Z(XYZ)_Y$	
	$L_{1x} > \alpha L_1, L_{1y} = \beta L_1, L_{1z} < (1 - \alpha - \beta)L_1$	$(XYZ)_X(XYZ)_Z$	
	$L_{1x} < \alpha L_1, L_{1y} = \beta L_1, L_{1z} > (1 - \alpha - \beta)L_1$	$(XYZ)_Z(XYZ)_X$	
	$L_{1x} > \alpha L_1, L_{1y} < \beta L_1, L_{1z} = (1 - \alpha - \beta)L_1$	$(XYZ)_X(XYZ)_Y$	
	$L_{1x} < \alpha L_1, L_{1y} > \beta L_1, L_{1z} = (1 - \alpha - \beta)L_1$	$(XYZ)_Y(XYZ)_Z$	
	$L_{1x} > \alpha L_1, L_{1y} > \beta L_1, L_{1z} < (1 - \alpha - \beta)L_1$	$(XYZ)_{XY}(XYZ)_Z$	
	$L_{1x} > \alpha L_1, L_{1y} < \beta L_1, L_{1z} < (1 - \alpha - \beta)L_1$	$(XYZ)_X(XYZ)_{YZ}$	
	$L_{1x} > \alpha L_1, L_{1y} < \beta L_1, L_{1z} > (1 - \alpha - \beta)L_1$	$(XYZ)_{XZ}(XYZ)_Y$	
	$L_{1x} < \alpha L_1, L_{1y} > \beta L_1, L_{1z} < (1 - \alpha - \beta)L_1$	$(XYZ)_Y(XYZ)_{XZ}$	
	$L_{1x} < \alpha L_1, L_{1y} > \beta L_1, L_{1z} > (1 - \alpha - \beta)L_1$	$(XYZ)_{YZ}(XYZ)_X$	
	$L_{1x} < \alpha L_1, L_{1y} < \beta L_1, L_{1z} > (1 - \alpha - \beta)L_1$	$(XYZ)_Z(XYZ)_{XY}$	
	$L_{1x} > \alpha L_1, L_{1y} < \beta L_1, L_{1z} < (1 - \alpha - \beta)L_1$	$(XYZ)_X(XYZ)_{YZ}$	
	$L_{1x} > \alpha L_1, L_{1y} < \beta L_1, L_{1z} > (1 - \alpha - \beta)L_1$	$(XYZ)_{XZ}(XYZ)_Y$	
	$L_{1x} < \alpha L_1, L_{1y} > \beta L_1, L_{1z} < (1 - \alpha - \beta)L_1$	$(XYZ)_Y(XYZ)_{XZ}$	
	$L_{1x} < \alpha L_1, L_{1y} > \beta L_1, L_{1z} > (1 - \alpha - \beta)L_1$	$(XYZ)_{YZ}(XYZ)_X$	
	$L_{1x} < \alpha L_1, L_{1y} < \beta L_1, L_{1z} > (1 - \alpha - \beta)L_1$	$(XYZ)_Z(XYZ)_{XY}$	
$\frac{a_{1x}}{a_{2x}} = \frac{a_{1y}}{a_{2y}} > \frac{a_{1z}}{a_{2z}}$	$\frac{L_2}{L_1} \leq \frac{(1 - \alpha - \beta)a_{1z}}{(\alpha + \beta)a_{2z}}$	$(XYZ)_{XY}(Z)_Z$	
	$\frac{(1 - \alpha - \beta)a_{1z}}{(\alpha + \beta)a_{2z}} < \frac{L_2}{L_1} \leq \frac{(1 - \alpha - \beta)a_{1x}}{(\alpha + \beta)a_{2x}}$	$(XY)_{XY}(Z)_Z$	
	$\frac{L_2}{L_1} > \frac{(1 - \alpha - \beta)a_{1x}}{(\alpha + \beta)a_{2x}}$	$L_{1x} > \alpha L_1, L_{1y} > \beta L_1$	$(XY)_{XY}(XYZ)_Z$
		$L_{1x} = \alpha L_1, L_{1y} > \beta L_1$	$(XY)_Y(XYZ)_Z$
		$L_{1x} > \alpha L_1, L_{1y} < \beta L_1$	$(XY)_X(XYZ)_{YZ}$
		$L_{1x} > \alpha L_1, L_{1y} = \beta L_1$	$(XY)_X(XYZ)_Z$
		$L_{1x} < \alpha L_1, L_{1y} > \beta L_1$	$(XY)_Y(XYZ)_{XZ}$

$\frac{a_{1x}}{a_{2x}} > \frac{a_{1y}}{a_{2y}} = \frac{a_{1z}}{a_{2z}}$	$\frac{L_2}{L_1} < \frac{(1-\alpha)a_{1z}}{\alpha a_{2z}}$	$L_{2y} < \beta L_2, L_{2z} > (1-\alpha-\beta)L_2$	$(XYZ)_{xy}(YZ)_z$
		$L_{2y} = \beta L_2, L_{2z} > (1-\alpha-\beta)L_2$	$(XYZ)_x(YZ)_z$
		$L_{2y} > \beta L_2, L_{2z} > (1-\alpha-\beta)L_2$	$(XYZ)_x(YZ)_{yz}$
		$L_{2y} > \beta L_2, L_{2z} = (1-\alpha-\beta)L_2$	$(XYZ)_x(YZ)_y$
		$L_{2y} > \beta L_2, L_{2z} < (1-\alpha-\beta)L_2$	$(XYZ)_{xz}(YZ)_y$
	$\frac{(1-\alpha)a_{1z}}{\alpha a_{2z}} \leq \frac{L_2}{L_1} < \frac{(1-\alpha)a_{1x}}{\alpha a_{2x}}$	$(X)_x(YZ)_{yz}$	
$\frac{L_2}{L_1} \geq \frac{(1-\alpha)a_{1x}}{\alpha a_{2x}}$	$(X)_x(XYZ)_{yz}$		
$\frac{a_{1x}}{a_{2x}} > \frac{a_{1y}}{a_{2y}} > \frac{a_{1z}}{a_{2z}}$	$\frac{L_2}{L_1} \leq \frac{(1-\alpha-\beta)a_{1z}}{(\alpha+\beta)a_{2z}}$	$(XYZ)_{xy}(Z)_z$	
	$\frac{(1-\alpha-\beta)a_{1z}}{(\alpha+\beta)a_{2z}} < \frac{L_2}{L_1} \leq \frac{(1-\alpha-\beta)a_{1y}}{(\alpha+\beta)a_{2y}}$	$(XY)_{xy}(Z)_z$	
	$\frac{(1-\alpha-\beta)a_{1y}}{(\alpha+\beta)a_{2y}} < \frac{L_2}{L_1} < \frac{(1-\alpha-\beta)a_{1y}}{\alpha a_{2y}}$	$(XY)_x(YZ)_{yz}$	
	$\frac{L_2}{L_1} = \frac{(1-\alpha-\beta)a_{1y}}{\alpha a_{2y}}$	$(XY)_x(YZ)_z$	
	$\frac{(1-\alpha-\beta)a_{1y}}{\alpha a_{2y}} < \frac{L_2}{L_1} < \frac{(1-\alpha)a_{1y}}{\alpha a_{2y}}$	$(XY)_{xy}(YZ)_z$	
	$\frac{(1-\alpha)a_{1y}}{\alpha a_{2y}} \leq \frac{L_2}{L_1} < \frac{(1-\alpha)a_{1x}}{\alpha a_{2x}}$	$(X)_x(YZ)_{yz}$	
	$\frac{L_2}{L_1} \geq \frac{(1-\alpha)a_{1x}}{\alpha a_{2x}}$	$(X)_x(XYZ)_{yz}$	

It should be noted that Table A2 only presents the equilibrium trade structures given the assumption of $\frac{a_{1x}}{a_{2x}} \geq \frac{a_{1y}}{a_{2y}} \geq \frac{a_{1z}}{a_{2z}}$. There are other structures under different assumptions about the relativity of productivity coefficients in the two countries. However because of symmetry in production technology and consumer preference, the equilibrium solutions and existence conditions are symmetric as well. Therefore Table 2A simplifies the presentation without loss of generality.