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CAN WE TAX THE DESIRE FOR TAX EVASION?*

Ratbek Dzhumashev† and Emin Gahramanov‡

ABSTRACT
A static income tax evasion model à la Yitzhaki (1974) predicts that an increase in the tax rate causes taxpayers to increase their income declaration. In an important contribution, Lin and Yang (2001) obtained exactly the opposite result by extending the Yitzhaki (1974) model to a dynamic one with Ak(t) production technology. In this paper we show that once the Lin and Yang (2001) model becomes fully compatible with the Yitzhaki's (1974) setting, the negative relationship between taxes and evasion still prevails. We then enrich the dynamic model with a productive public sector, and obtain an ambiguous relationship between taxes and evasion incentives as in Allingham and Sandmo (1972). We also prove that the growth-maximizing share of public expenditures in total output satisfies the natural efficiency condition even in the presence of tax evasion. However, the latter result is not robust to the introduction of the costs associated with income declaration and concealment activities.

Key words: Tax Evasion, Optimal Taxation, Economic Growth

JEL code: H26, H21, D91

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† Department of Economics, Monash University, Ratbek.Djumashev@BusEco.monash.edu.au.
‡ of Accounting, Economics and Finance, Deakin University, eming@deakin.edu.au
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1. Introduction

Income tax evasion is one of the most important issues any country faces. This phenomenon is not pertinent to developing countries only. *Internal Revenue Service*, for instance, estimates that the tax gap, the difference between what taxpayers should have paid by law and what they actually ended up paying, was as large as $345 billion in the Unites States for the year 2001. Out of this, roughly 57 percent constituted the individual income tax gap. Andreoni et al. (1998), based on the 1992 IRS study, report that "...91.7 percent of all [U.S.] income that should have been reported, was in fact reported". According to Engel and Hines (1998), individuals in the U.S. underreport about 10.6 percent of their incomes annually. The picture is not more optimistic in the rest of the world. In Greece and U.K., for example, the amount of tax evaded is estimated to comprise 22.5 and 11.5 percent of total tax collections, respectively (Gupta 2004).

It is not surprising, therefore, that the theoretical analysis of income tax evasion has retained its importance since the seminal contribution of Allingham and Sandmo (1972), which has been further developed notably to incorporate some of the most important features of economic environment.[1] One particular aspect of Allingham-Sandmo (1972) study was concerned with the relationship between the tax rate and evasion incentives. The study showed that when the government raises the tax rate, a taxpayer in response can either increase or decrease income declaration because of competing income and substitution effects. However, Shlomo Yitzhaki (1974) pointed out in a two-page note to Allighman and Sandmo, that under some realistic assumptions a typical risk averse individual will declare more income when the tax rate increases. This is because when the penalty is imposed on the amount of evaded tax, as it is under most current tax laws, the substitution effect vanishes.

The latter finding spurred a lot of harsh criticisms[2] since it goes against intuition and much of the existing empirical evidence. Thus, there were no lack of attempts to reverse the Yitzhaki (1974) result. It is worth mentioning, however, that those attempts often deviated fairly significantly from the Allingham-Sandmo-Yitzhaki setting, with many extensions taking place in a static framework.[3] Dynamic models, although relatively few, were no exception in that respect. For example, Niepelt (2005) showed that with time-dependent penalty rates and costs arising from the taxpayer's switching from "no evasion" to "evasion" state, an increase in the tax rate raises the average duration of an evasion spell. In an important contribution, Chen (2003), who to the best of our knowledge was first to develop a general equilibrium continuous time deterministic

[1] See Kolm (1973), Pencavel (1978), Cowell (1985) and Yaniv (1999),—to name just a few contributions.

[2] See, e.g., Yaniv (1999) and Al-Nowaihi and Pyle (2000).

[3] See, e.g., Pencavel (1979), Cowell and Gordon (1988), Gordon (1989) and Yaniv (1999).

model of tax evasion with public capital, shows that higher taxes increase evasion. Although Chen (2003) never claims that the goal of his paper is to reverse the Yitzhaki (1974) result, he establishes a positive relationship between the tax rate and evasion incentives. Such a relationship is achieved via introducing transaction costs, which are strictly increasing in evasion (see expression (7) on p. 386 in Chen 2003).

The first important theoretical study of tax evasion in the context of the dynamic portfolio choice model *à la* Merton (1969), comes from Lin and Yang (2001), who use $Ak(t)$ production technology. Their model does not seem to deviate from the Yitzhaki (1974) setting in terms of modelling tax evasion: it simply extends the environment from a static to a dynamic one, but produces at the same time a strikingly different result that tax compliance is *negatively* associated with the tax rate. We argue, however, that the Lin and Yang (2001) model follows the Yitzhaki (1974) model in the sense that the penalty for tax evasion is set proportional to the amount of evaded tax liability. In deriving the moments of the stochastic process of capital accumulation, however, Lin and Yang (2001) assume that the decision-making taxpayer considers random return on concealed *income*. However, considering return on a unit of income concealed is appropriate for the Allingham-Sandmo (1972) environment, where fines are levied on the amount of undeclared income. In the Yitzhaki (1974) model the taxpayer always keeps in mind that what is under risk is the amount of *tax evaded*: either the taxpayer pockets it all if not caught, or loses it all (plus pays some penalties on the top of that) if caught. Considering the return on a unit of income concealed makes the variance of the uncertain income independent of the tax rate. As it turns out, such a seemingly unimportant detail is exactly what causes the Yitzhaki (1974) result to be reversed. On the contrary, we make the model fully compatible with the Yitzhaki (1974) setting, and obtain that higher taxes *discourage* evasion.[4]

After revisiting the Lin and Yang (2001) model, we extend it to account for the productive public sector as in Barro (1990), and then we analyze the implications for the optimal government size for a range of extensions.[5] Incorporating the productive

[4]Caballé and Panadés (2001) pointed out that the Yitzhaki (1974) result holds in a discrete-time counterpart of the continuous-time Lin and Yang (2001) setting. However, Caballé and Panadés (2001) do not show this result in the continuous-time framework in order to avoid otherwise arising technical difficulties, as they claim. The authors also hypothesize that the discrepancy between their result and that of Lin and Yang (2001) is likely to arise because the variance of the Brownian motion is not proportional to the tax rate in the latter. Further, in the discrete-time presentation Caballé and Panadés (2001) obtain the consumption path which differs from that in Lin and Yang (2001). As was said above, in this paper we show where the result of the tax-independent variance comes from, and namely in a continuous-time original setting of Lin and Yang (2001).

[5]It is worth noting that growth literature has been enriched in a variety of directions, and here our aim is to keep the analytical setting concise. An interested reader might consult, among others, Glomm and Ravikumar (1994) (modeling public input with varying degree of non-rivalry and non-exclusiveness, with public investment in infrastructure financed via uniform capital and labor income

government enables to capture the interrelations between the tax rate, evasion, tax collections and output. We show that as a result the relationship between the tax rate and evasion will become *ambiguous*.[6]

We also introduce tax compliance/evasion costs, and show that accounting for compliance/evasion costs leads to a larger optimal size of the public sector than in Barro (1990).[7]

Our contribution in this paper can be summarized as follows:

- We make the Lin and Yang (2001) framework fully compatible with the Yitzhaki (1974) specification and show that higher taxes discourage evasion when $Ak(t)$ technology is assumed;
- We introduce public goods and to our knowledge, this is the first attempt to enrich the Lin and Yang (2001) model in this way. This alone enables to establish an ambiguous relationship between taxes and evasion. We also account for the costs associated with income declaration/concealment;
- Finally, we investigate the implications of our model in the context of determining the optimal government size. We show that the traditional natural efficiency condition of the government size holds in our model, leading to identical economic growth rates for economies with different tax evasion rates. This differs from the Lin and Yang (2001) and Chen (2003) conclusions.

One of the main implications of our study is that the Yitzhaki (1974) finding is "stubborn" enough to be reversed simply by extending the model to a dynamic setting.

taxation), Glomm and Ravikumar (1999) (introducing general preferences and technologies), and Baier and Glomm (2001) (separately modeling human, public and private capital accumulation process with elastic labor supply and non-unitary elasticity of substitution between public and private input). Turnovsky and Fisher (1995) in their authoritative paper carefully distinguish between the welfare implications of the government consumptive (directly utility augmenting) and productive expenditures, and allow for the former to be a substitute/complement for private consumption and work effort, with the latter exhibiting variable marginal physical product property. Other notable studies in this line of research include Arrow and Kurz (1970), Aschauer (1989), Fisher and Turnovsky (1998),—to name just a few.

[6]To be fair, in reality higher taxes not always unequivocally depress honesty incentives as can be seen in Geeroms and Wilmots (1985) and Feinstein (1991). Previous studies revisiting the Yitzhaki (1974) framework predominantly establish a clear-cut positive relationship between taxes and evasion. Although the absence of ambiguity might theoretically be advantageous, it naturally leaves no room for a potential explanation of differing real-world evidence. Thus, our finding that higher taxes can lower or increase evasion might help to explain a wider range of empirical evidences.

[7]See Cremer and Gahvari (1994), Sandford (1995), Tran-nam et al. (2000), Slemrod and Yitzhaki (2002), and Buyer and Sutter (2004) for the discussions of the costs associated with tax evasion, compliance and administration.

Perhaps future research should shed more light on this. With that said, we now turn to the model.

2. Modified Lin and Yang Economy

Assume the production function has the following form:

$$y(t) = Ak(t)\left(\frac{g(t)}{k(t)}\right)^{1-\alpha} \tag{1}$$

where $A > 0, 0 < \alpha < 1$, $y(t)$ is output per capita, $k(t)$ is per worker capital, $g(t)$ is per worker public input. It is assumed that the production function is stationary within the planning horizon.

The government imposes an income tax at a flat rate $0 < \tau < 1$. To increase their disposable income the agents evade taxes by under-reporting their true earnings. We assume that the agent reports only $(1-e(t))y(t)$ of his total income $y(t)$ in per capita terms, where $0 < e(t) < 1$. To combat tax evasion the government audits taxpayers randomly and detects the evasion with the joint probability, π.

A detected evader pays back the due tax liability and some additional fine. This penalty is determined by a rate $\theta = 1+s$, which includes the tax evaded and a surcharge, s. The tax paid by the agent is then either $T(t) = (1-e(t))\tau y(t) + \theta e(t)\tau y(t)$ with probability π, or $T(t) = (1-e(t))\tau y(t)$ with probability $1-\pi$. Consequently, the expected tax payment for the agent is expressed as

$$\bar{T}(t) = (1-e(t))\tau y(t) + \pi\theta\tau e(t)y(t) \tag{2}$$

The Lin and Yang (2001) model followed the Yitzhaki (1974) tax evasion model in the sense that the penalty for tax evasion is proportional to the evaded tax. However, in derivation of the variance of the stochastic process of tax evasion they considered not the random return on evaded tax but the random return on concealed income. Hence, the variance of the uncertain income in their model is treated as independent of the tax rate. In any case, we cannot ignore the importance of the tax rate in the risk structure. Therefore, our model focuses on capturing the risk associated with the source of uncertainty, which is the amount of tax liability, not the entire concealed income. Hence, the Lin and Yang (2001) model can be considered as a hybrid of the Yitzhaki (1974) and the Alingham-Sandmo (1972) frameworks. In this paper we will allow the random part of the income depend on the evaded tax as the gain or loss due to evasion is related to this amount only.

The random part of the agent's income can be described by the return on Δh dollars of tax evaded: with probability π the return equals to $r = -s\Delta h = (1-\theta)\Delta h$,

and with probability $1-\pi$, equals $r = \Delta h$. Then the expected return on a unit of tax evaded is found as

$$\bar{r} = \pi \cdot (-s) + (1-\pi) \cdot 1 = 1 - \pi(1+s) = 1 - \pi\theta \tag{3}$$

In general, if $\pi\theta > 1$ then the taxpayers should not evade taxes. However, in the real world, there is always some tax evasion, thus we can assume that $\pi\theta < 1$ holds.

The cumulative change of the agent's income, $y(t) - y(0)$, follows a binomial distribution as we have only two outcomes. It is known that the binomial random process converges to a Brownian motion as the number of steps goes to infinity. The agent's disposable income, $y_d(t)$, is a stochastic process and given by[8]

$$y_d(t) = (1-\tau)y(t) + \bar{r}\tau e(t)y(t) + [\sigma\tau e(t)y(t)]W(t) \tag{4}$$

where the first two terms stand for the deterministic part of the income, the last term is the stochastic part, σ is a constant (to be discussed later), $W(t)$ is a standard Brownian motion.

Since in our model all agents are identically risk-averse *ex-ante*, they evade taxes as soon as the return on tax evasion is positive, or $\bar{r} > 0$. Thus, the value of after-tax income is random and depends on being caught and penalized for tax evasion or being successful in the act of tax evasion.

The households in their pursuit of utility maximization face a resource constraint in deciding what part of their income to consume and what part to save. As the households are facing stochastic disposable incomes depending on the success of tax evasion, the amount of capital accumulation also follows a stochastic process. Based on our assumption that the random part of the disposable income of the agents follows the Brownian motion, and following Merton's (1969) model on the portfolio allocation, the amount of capital accumulation can be expressed as

$$dk(t) = [(1-\tau+\bar{r}\tau e(t))y(t) - c(t)]dt + [\sigma\tau e(t)y(t)]dW \tag{5}$$

where σ is the standard deviation of the normalized process of random return on tax evasion.[9]

The argument of the production function in (1), $\frac{g(t)}{k(t)}$, can be expressed in terms of the tax rate. Note that

$$g(t) = (1-\bar{r}e(t))\tau y(t) = (1-\bar{r}e(t))\tau Ak(t)\left(\frac{g(t)}{k(t)}\right)^{1-\alpha} \tag{6}$$

[8]See Chang (2004) and Dixit and Pindyck (1994) for details.

[9]See Appendix for derivation.

Clearly, $(1-\bar{r}e(t))$ is an effective compliance rate, as the effective tax rate is equal to the product of the statutory tax rate and this parameter.

Therefore we can write

$$\left(\frac{g(t)}{k(t)}\right)^{\alpha} = A(1-\bar{r}e(t))\tau \tag{7}$$

Using (7), expression (1) can be written as

$$y(t) = Ak(t)[A(1-\bar{r}e(t))\tau]^{\frac{1-\alpha}{\alpha}} = A^{\frac{1}{\alpha}}k(t)[(1-\bar{r}e(t))\tau]^{\frac{1-\alpha}{\alpha}} \tag{8}$$

2.1. The household's optimization

An individual household maximizes its expected overall utility by choosing consumption level $c(t)$ and tax evasion rate $e(t)$ subject to the resource constraint:

$$\max_{c(t),e(t)} U = \mathbb{E}_0\left[\int_0^{\infty} \ln(c(t))\exp(-\rho t)dt\right] \tag{9}$$

$$s.t.\, dk = [(1-\tau+\bar{r}\tau e(t))y(t) - c(t)]dt + [\sigma\tau e(t)y(t)]dW \tag{10}$$

$$0 \leq c(t) \leq (1-\tau+\bar{r}\tau e(t))y(t), 0 \leq k(t), k(0) = k_0 \tag{11}$$

$$0 \leq e(t) \leq 1 \tag{12}$$

and (8)

where $\mathbb{E}_0$ is the conditional expectation operator for the given initial value of capital, $k(0) = k_0$. This problem is transformed to the Bellman equation:

$$\rho I(k) = \max_{c(t),e(t)} \left\{\ln(c(t)) + I'(k)([1-\tau+\bar{r}\tau e(t)]y(t) - c(t)) + \tfrac{1}{2}I''(k)[\sigma\tau e(t)y(t)]^2\right\} \tag{13}$$

where $I(k)$ is the value function.

Note, in this setup the output is a function of tax evasion as indicated earlier in (8). Then the FOCs of the Bellman equation (13) lead to

$$c(t) = \frac{1}{I'(k)} \tag{14}$$

$$e(t) = -\frac{I'(k)\bar{r}}{I''(k)\sigma^2\tau y(t)} \tag{15}$$

Inserting (14) and (15) back into (13) yields:

$$\rho I(k) = \ln\left(\frac{1}{I'(k)}\right) - 1 + I'(k)(1-\tau)y(t) - \frac{[I'(k)]^2\,\bar{r}^2}{2I''(k)\sigma^2} \tag{16}$$

A general solution to this differential equation can be expressed as $I(k) = \frac{ln(k)+D}{\rho}$, where D is a constant. Using this solution we obtain from (14) and (15) the equilibrium consumption[10]

$$c(t) = \rho k(t) \tag{17}$$

and evasion rate

$$e(t) = \frac{\bar{r}k(t)}{\sigma^2 \tau y(t)} \tag{18}$$

The results in (17) and (18) are obtained via the small agent assumption. Clearly, if we take $y(t) = Ak(t)$ as Lin and Yang (2001) do, higher taxes will *discourage* evasion, contrary to their finding, and consonant with Yitzhaki's (1974) result.

By using (8) we re-write (18) as

$$e(t) = \frac{\bar{r}}{A^{\frac{1}{\alpha}} \sigma^2 \tau^{\frac{1}{\alpha}} (1 - \bar{r}e(t))^{\frac{1-\alpha}{\alpha}}} \tag{19}$$

Hence, we have obtained an implicit expression for the tax evasion rate. We use this expression to carry out comparative statics analysis and state the following proposition.

Proposition 2.1. The sign of $\frac{\partial e(t)}{\partial \tau}$ depends on whether the effective compliance rate, $1 - \bar{r}e(t)$, is greater or less than the output elasticity of public expenditure, $1 - \alpha$.

Proof Let us first re-write (19) as

$$e(t)(1 - \bar{r}e(t))^{\frac{1-\alpha}{\alpha}} = \frac{\bar{r}}{A^{\frac{1}{\alpha}} \sigma^2 \tau^{\frac{1}{\alpha}}} \tag{20}$$

Apply the Chain Rule to (20) with regards to τ, which is after some simplification yields

$$\frac{\partial e(t)}{\partial \tau} = \frac{-\frac{\bar{r}}{\alpha A^{\frac{1}{\alpha}} \sigma^2 \tau^{\frac{1+\alpha}{\alpha}}}}{(1 - \bar{r}e(t))^{\frac{1-\alpha}{\alpha}} (1 - \frac{e(t)\bar{r}(1-\alpha)}{\alpha(1-\bar{r}e(t))})} \tag{21}$$

It is clear that the sign of $\frac{\partial e(t)}{\partial \tau}$ depends on the sign of $1 - \frac{e(t)\bar{r}(1-\alpha)}{\alpha(1-\bar{r}e(t))}$, which can be written as $\frac{\alpha - \bar{r}e(t)}{\alpha(1-\bar{r}e(t))}$. Since the evasion rate and expected return to tax evasion are less than one, the numerator determines the sign of this fraction. The numerator is given by $\alpha - \bar{r}e(t)$. When $\bar{r}e(t) > \alpha$ holds we conclude that $\frac{\partial e(t)}{\partial \tau} > 0$, while in case $\bar{r}e(t) < \alpha$ is true then $\frac{\partial e(t)}{\partial \tau} < 0$. Alternatively,

[10] We make use of the fact that $\frac{g(t)}{k(t)}$ (and thus, $\frac{y(t)}{k(t)}$) is fixed as soon as $\frac{g(t)}{y(t)}$ is fixed for a given τ (as in Barro 1990).

$$\textit{if } 1-\bar{r}e(t) < 1-\alpha, \quad \textit{then } \frac{\partial e(t)}{\partial \tau} > 0,$$
$$\textit{if } 1-\bar{r}e(t) > 1-\alpha, \quad \textit{then } \frac{\partial e(t)}{\partial \tau} < 0.$$

Recall from (6) that $1-\bar{r}e(t)$ stands for the effective compliance rate. Hence, the comparative statics of the increase in the tax rate on the evasion rate depends on how high the effective compliance rate, $1-\bar{r}e(t)$, is and how large the output elasticity of the public expenditure, $1-\alpha$, is. ■

Now, if $\bar{r}$ is sufficiently *high* (evading is attractive), and α is sufficiently *low* (private sector shares a small fraction of $y(t)$ *or* the public sector already creates a *lot* of positive externalities), then a rise in the tax rate leads to more evasion. A rise in τ increases marginal benefit of cheating, but on the other hand greater evasion tends to reduce the public good provision which would have increased output. However, when the condition $1-\bar{r}e(t) < 1-\alpha$ holds the increased private gains from cheating more than offsets the effect of the reduced public good provision.

2.2. The Lin and Yang environment with costly compliance

Cremer and Gahvari (1994) rigorously analyze how costly expenditures on income concealment activities lower the likelihood of getting caught and influence the government design of the optimal linear tax schedule. Their realistic model delivers an important theoretical finding that tax evasion, coupled with concealment costs can considerably alter the redistributive effect of the optimal income tax policy. Unfortunately, theory has largely remained silent since the contribution of Cremer and Gahvari (1994), although there is a relatively larger body of empirical literature emphasizing the importance of the real costs associated with income tax compliance at the individual level.

In this paper we aim to keep our setting simple in order to isolate the real costs of tax evasion and income declaration which simultaneously can influence tax evasion incentives. Therefore, we do not model the case with an endogenous probability of detection, nor do we investigate the redistributive consequences of tax evasion and costly compliance. We rather wonder how tax evasion incentives are affected both by compliance and non-compliance costs, and how the latter two together might alter the optimal taxation rule of the government in the context of our model. As will be seen shortly, our setting is in line with Cremer and Gahvari (1994) in the sense that it accounts for the fact that not only does the burden of taxation fall on taxpayers in the form of higher taxes and penalties, but also in the form of non-compliance expenditures. At the same time our analysis includes compliance expenditures as well.[11]

[11] For the purpose of our paper we ignore the costs associated with the government's audit practices, which can be substantial in size. One simple way to capture the administrative costs is to assume that they drain a constant fraction of tax revenues collected. Lin and Yang (2001) show that such a setting should not alter the main results of their model.

To strike a balance between realism and analytical tractability, we introduce two additional parameters, ξ_1 and ξ_2, governing the structure of the costly compliance and evasion. One can think of ξ_1 and ξ_2 as the marginal inefficiency losses of income declaration and income concealment, respectively. In so doing, we take to the heart Lin and Yang's observation that increasing tax evasion raises evasion costs on the one hand, but lowers compliance costs on the other hand.

The compliance cost is borne by the taxpayer when she pays taxes voluntarily or when caught for evasion and forced to pay the unpaid taxes. For simplicity we assume that the total compliance cost is proportional to the income declared or found to be taxable. Hence, the total cost of compliance is given by $\xi_1(1-\underline{r}e(t))\tau y(t)$, and $\underline{r}=\bar{r}-\xi_2$.

The evasion cost includes inefficiency losses related to hiding income and bribing tax inspectors. The evasion cost is borne when the taxpayer evades tax by concealing her income, and even when she is caught the cost cannot be recovered. So in case of detection the taxpayer incurs both costs. The effect of this cost is a decrease in the expected return to tax evasion without changing the variance, as this cost must be borne at any instance of tax evasion, and hence the variation of the outcomes for tax evasion act does not change.

Hence, (10) becomes

$$dk(t) = [(1-\tau+\underline{r}e(t)(\tau+\xi_1))y(t) - c(t)]dt + [\sigma\tau e(t)y(t)]\,dW \tag{22}$$

$$k(0) = \textit{given} \tag{23}$$

Now, we state the stochastic Bellman equation as

$$\rho J(k,t) = \tag{24}$$
$$\max_{c(t),e(t)} \left\{\ln c(t) + J'(k)[(1-\tau+\underline{r}e(t)(\tau+\xi_1))y(t) - c(t)] + \tfrac{1}{2}J''(k)[\sigma\tau e(t)y(t)]^2\right\}$$

Clearly, the taxpayer's optimal consumption and evasion profiles are given similarly to (17) and (19) by

$$c(t) = \rho k(t) \tag{25}$$

and

$$e(t) = \frac{\underline{r}(\tau+\xi_1)}{A^{\frac{1}{\alpha}}[(1-\underline{r}e(t))\tau]^{\frac{1-\alpha}{\alpha}}(\tau\sigma)^2} \tag{26}$$

Considerring the effect of an increase in the compliance and non-compliance costs on tax evasion, we formulate the following proposition:

Proposition 2.2. *An increase in the compliance and noncompliance costs has an ambigous effect on tax evasion:*

- *if* $1-(1+\beta)\underline{r}e(t)\tau > 0$, $\frac{\partial e(t)}{\partial \xi_1} > 0$, $\frac{\partial e(t)}{\partial \xi_2} < 0$
- *if* $1-(1+\beta)\underline{r}e(t)\tau < 0$, $\frac{\partial e(t)}{\partial \xi_1} < 0$, $\frac{\partial e(t)}{\partial \xi_2} > 0$

Proof Consider the effect of an increase in compliance cost, ξ_1. To determine this we consider the comparative statics, $\frac{\partial e(t)}{\partial \xi_1}$. For that we apply the Chain Rule to (26) after adjusting it to

$$e(t)\left[(1-\underline{r}e(t))\tau\right]^{\beta} = \frac{1}{A^{\frac{1}{\alpha}}}\left(\frac{\underline{r}}{\tau\sigma^2} + \frac{\underline{r}\xi_1}{(\tau\sigma)^2}\right)$$

where $\beta = \frac{1-\alpha}{\alpha}$. This operation yields

$$\frac{\partial e(t)}{\partial \xi_1}\left[(1-\underline{r}e(t))\tau\right]^{\beta} - \beta e(t)\underline{r}\tau\left[(1-\underline{r}e(t))\tau\right]^{\beta-1}\frac{\partial e(t)}{\partial \xi_1} = \frac{\underline{r}}{(\tau\sigma)^2 A^{\frac{1}{\alpha}}}$$

After some manipulations we arrive at

$$\frac{\partial e(t)}{\partial \xi_1} = \frac{\frac{\underline{r}}{A^{\frac{1}{\alpha}}\tau\sigma^2}}{\left[(1-\underline{r}e(t))\tau\right]^{\beta}\left(1-\frac{\beta e(t)\underline{r}}{(1-\underline{r}e(t))}\right)} \tag{27}$$

The sign of the term on the right hand side depends on the sign of

$$\left(1-\frac{\beta e(t)\underline{r}}{(1-\underline{r}e(t))}\right)$$

as the nominator and the first term in the denominator are positive. This further boils down to the following expression that determines the sign of the expression

$$1-(1+\beta)\underline{r}e(t)$$

Analogously, we can determine the effect of an increase in the cost of noncompliance. Applying the Chain Rule to the modified version of (26) with regards to the cost of noncompliance, ξ_2 yields

$$\frac{\partial e(t)}{\partial \xi_2} = -\frac{\frac{(\tau+\xi_1)}{A^{\frac{1}{\alpha}}(\tau\sigma)^2} + \beta e^2(t)\tau[(1-\underline{r}e(t))\tau]^{\beta-1}}{\left[(1-\underline{r}e(t))\tau\right]^{\beta}\left(1-\frac{\beta e(t)\underline{r}}{(1-\underline{r}e(t))}\right)} \tag{28}$$

Again from (28) we infer that the sign of the comparative statics depends on the sign of the expression, $1-(1+\beta)\underline{r}e(t)$.

■

Although, analytically the effect of increase in the compliance cost is not clear, a calibration of the comparative statics by assuming a range of feasible values for β, τ, $\underline{r}$, and $e(t)$, it can be shown that an increase in the compliance cost leads to an increase in tax evasion, whereas an increase in the noncompliance cost-to a decrease. The important message we carry out from this exercise is that decreasing the compliance cost does not always lead to an increase in compliance, as well as increasing the compliance cost may not result in more tax evasion.

2.3. Optimal Tax Policies

The optimal tax rate depends on the optimal size of the government, as taxation is just the way of raising funds to finance the public expenditure. Thus, the question of optimal taxation becomes: what is the growth-maximizing public expenditure share in the total output? Barro (1990) has shown that to be optimal public expenditure should satisfy the natural condition of productive efficiency, that is the size of the public sector expenditure share, $\frac{g(t)}{y(t)}$, is set so that the marginal product of the public input satisfies $\frac{dy(t)}{dg(t)} = 1$. The intuition here is that the marginal benefit of the productive public input should be equalized with the cost it incurs as marginal tax burden. It is clear that for the system with proportional income tax, the optimal tax rate, τ^*, is found based on the optimal public expenditure share, $\tau^* = \frac{g^*(t)}{y^*(t)}$, satisfying the optimal condition we discussed. In other words, the optimal marginal tax burden is equal to the statutory tax rate.

In the presence of tax evasion, the statutory tax rate is not equal to the tax burden, as some part of income is concealed from taxation. This implies that the growth-maximizing government is interested in satisfying the condition of productive efficiency and sets the statutory tax rate such that the tax burden equals to the *optimal* public expenditure share. Assuming that tax evasion does not cause any losses except the tax revenue collected, an introduction of tax evasion does not change the optimality condition for the public sector size.

With no tax evasion (denoted by index "ne" for short), the growth rate per capita is inverted U in the tax rate, as in Barro (1990). Using (5) with $e(t) = 0$, (8) and (17), we obtain the growth rate for this economy as (see Lin and Yang 2001, p. 1833 for details)

$$\gamma_{ne} = (1-\tau)A^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}} - \rho \tag{29}$$

Partially differentiate (29) with respect to τ to obtain

$$\frac{\partial \gamma_{ne}}{\partial \tau} = A^{\frac{1}{\alpha}}\tau^{\frac{1-\alpha}{\alpha}}\left[\frac{1-\alpha}{\alpha}\frac{1-\tau}{\tau} - 1\right] \tag{30}$$

Set (30) equal to zero to find

$$\tau^* = 1 - \alpha \tag{31}$$

Clearly, $\frac{\partial \gamma_{ne}}{\partial \tau}$ is negative when $\tau > 1-\alpha$ and positive when $\tau < 1-\alpha$. For the other extreme, ($\tau = 0$), it is easy to see that $\gamma_{ne}\,|_{\tau=0} < \gamma_{ne}\,|_{\tau=1-\alpha}$.

Now consider the case when there is tax evasion (denoted by index "e" for short) in the economy. We can state here the following proposition:

Proposition 2.3. The growth rate per capita in the presence of tax evasion is maximized when the government sets the tax rate, τ, such that the tax burden satisfies the following condition:

$$\tau_e^* = 1-\alpha$$

Proof Using (5), (17) and (8) we can state the growth rate of the economy with tax evasion as

$$\gamma_e = [1-\tau(1-\overline{r}e(t))]\, A^{\frac{1}{\alpha}} \left[(1-\overline{r}e(t))\tau\right]^{\frac{1-\alpha}{\alpha}} - \rho \tag{32}$$

From (6) it is easy to see that the effective tax burden, $g(t)/y(t)$ ($\equiv \tau_e$), is $(1-\overline{r}e(t))\tau$. We then can rewrite (32) as

$$\gamma_e = (1-\tau_e)\, A^{\frac{1}{\alpha}} \tau_e^{\frac{1-\alpha}{\alpha}} - \rho \tag{33}$$

Then the problem of maximizing is formally identical to the case without tax evasion given by (29), with the only difference that now the tax burden is not equal to the statutory tax rate. It follows that

$$\tau_e^* = 1-\alpha \tag{34}$$

The latter uniquely determines the optimal statutory tax rate for the environment with tax evasion

$$\tau^{**} = \frac{1-\alpha}{1-\overline{r}e(t)} \tag{35}$$

Therefore, by adjusting the statutory tax rate for the losses in the government revenue due to evasion, the optimal public input supply is restored and growth rate is maximized.■

Proposition 2.3 implies that as long as the government ensures that for any given level of evasion, $1-\alpha$ fraction of the output turns into the public good, the growth rate is maximized. Alternatively, the rule for the statutory tax rate is given by (35), which is directly related to the evasion rate.

The important implication of the above analysis can be summarized in the following corollary.

Corollary 2.4. *A*s long as the government ensures that the effective tax burden equals to the degree of the externalities it generates, the per capita growth rate of the economies with tax evasion and costless compliance will exactly equal to that of the economy with full compliance.

Proof Note with (31), the no evasion economy's rate of growth (29) will be simplified to

$$\gamma_{ne} = A^{\frac{1}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} - \rho \tag{36}$$

For the economy with tax evasion but costless compliance, we can substitute (34) into (33) to obtain that the resulting γ_e exactly equals to (36). ■

In other words, if the government credibly commits itself to spending $1-\alpha$ fraction of the national income on public goods, the resulting per capita economic growth will be equalized across the otherwise identical economies with or without tax evasion.

Now consider the case with costly compliance and income declaration ($\xi_1 > 0$ and $\xi_2 > 0$). We can use (22), (25) and (8) to state the growth rate as

$$\underline{\gamma}_e = \left[1 - \underline{\tau}_e + \underline{r}\xi_1 e(t)\right] A^{\frac{1}{\alpha}} \underline{\tau}_e^{\frac{1-\alpha}{\alpha}} - \rho \tag{37}$$

where "underscore" is used to distinguish the economy with costly compliance/evasion, $\underline{\tau}_e = (1 - \underline{r}e(t))\tau$. Clearly, with $\xi_1 = \xi_2 = 0$, $\underline{r}$ becomes equal to $\overline{r}$, and expression (37) becomes identical to expression (33). Since the government is concerned with choosing only $g(t)/y(t)$ ($\equiv \underline{\tau}_e$) optimally in this economy, it can treat evasion as given. Thus we can state here the following corollary.

Corollary 2.5. The government in the economy with costly compliance/evasion devotes a larger fraction of output for the provision of pubic goods.

Proof Partially differentiating (37) with respect to $\underline{\tau}_e$ and letting it equal to zero, we find

$$\underline{\tau}_e^* = (1-\alpha)\left(1 + \underline{r}\xi_1 e(t)\right) \tag{38}$$

which uniquely determines the optimal statutory tax rate

$$\underline{\tau}^{**} = (1-\alpha)\,\frac{1 + \underline{r}\xi_1 e(t)}{1 - \underline{r}e(t)} \tag{39}$$

Note when $\xi_1 = \xi_2 = 0$, (38) and (39) are identical to (34) and (35), respectively. Since ξ_1 and $\underline{r}$ exceed zero, $\underline{\tau}_e^*$ exceeds τ_e^*.

The implication we can derive from this section is that when the government optimally lets the government size grow at the same rate as output like in Barro (1990) when $g(t)/y(t) = 1-\alpha$, the economy without tax evasion and with it (but costless compliance/concealment) will have the same growth rates per capita. But when in the economy tax evasion is coupled with costly concealment/compliance, the growth-maximizing government share in output exceeds the benchmark public expenditure-to output ratio satisfying the natural efficiency condition.

3. Conclusion

A baseline static model of income tax evasion predicts that taxpayers, in response to a higher tax rate, will decrease tax evasion (Yitzhaki 1974). Such a puzzling theoretical relationship has been one of the major focuses of study in the income tax evasion literature for the past thirty years. However, in analyzing the puzzling relationship, the literature often deviated significantly from the initial set of model assumptions. On the contrary, simply by extending the Yitzhaki (1974) model from a static to a dynamic framework, Lin and Yang (2001) find that higher taxes encourage tax evasion in an economy with $Ak(t)$ technology. In this paper we revisit the Lin and Yang (2001) model by making it fully compatible with the Yitzhaki (1974) setting. As a result, we show that higher taxes discourage tax evasion in an economy with $Ak(t)$ technology.

Further, the literature on tax evasion generally ignored the possible implications of the productive input provided by the public sector, as well as the real costs associated with income declaration and concealment activities. We extend the Lin and Yang (2001) model along these lines and establish an ambiguous theoretical relationship between taxes and honesty incentives. The latter result is compatible with a wide array of empirical literature. We also prove that the growth-maximizing size of the public sector is consonant with traditional natural efficiency condition of the government size in an economy with tax evaders but costless compliance/evasion. Once tax evasion and income declaration become costly, however, the growth-maximizing tax rate is above the rate satisfying the natural efficiency condition for the government size.

Appendix

Here we show how the stochastic differential equation (5) can be derived as the continuous limit of a discrete-time problem. Assume that r is a Brownian motion with drift $\bar{r}$ and variance σ^2 defined for all t and is stated as:

$$r(t) = r(0) + \bar{r}t + \sigma W(t) \tag{40}$$

where W is a Wiener process. By definition $E[W(t)] = 0$, and $Var[W(t)] = t$. Hence,[12]

$$E[r(t) - r(0)] = \bar{r}t \tag{41}$$

$$Var[r(t) - r(0)] = \sigma^2 t \tag{42}$$

Let $x = r\Delta h$ be random return on Δh dollars of evaded tax in a time period of Δt, where r is defined as above.The random variable x has a Bernoulli distribution

$$x = \Delta h \text{ with probability } 1 - \pi,$$

[12]See Breiman (1968, prop. 12.4) for a proof.

$$x = -s\Delta h = (1-\theta)\Delta h \text{ with probability } \pi.$$

Then over each time increment Δt, our stochastic process of tax evasion is characterized by the mean

$$E[x] = \pi(1-\theta)\Delta h + (1-\pi)\Delta h = (1-\pi\theta)\Delta h \tag{43}$$

and the variance

$$Var(x) = E[x^2] - (E[x])^2$$

By substituting for

$$E[x^2] = \pi(-s)^2(\Delta h)^2 + (1-\pi)(\Delta h)^2 = (\Delta h)^2[1-\pi(1-s^2)]$$

$$(E[x])^2 = [(1-\pi\theta)\Delta h]^2$$

the expression for variance is simplified to

$$Var(x) = \pi(1-\pi)(\theta\Delta h)^2 \tag{44}$$

For a time interval of length $t = 1$ we have $\frac{1}{\Delta t}$ discrete steps. The cumulated change $x(t) - x(0)$ is a binomial random variable with mean

$$\frac{(1-\pi\theta)\Delta h}{\Delta t} \tag{45}$$

and variance,

$$\frac{\pi(1-\pi)(\Delta h\theta)^2}{\Delta t} \tag{46}$$

To make our binomial variate to converge to a Brownian motion we choose π and Δh. For that we set

$$\pi = \frac{1}{\theta}\left(1 - \frac{\bar{r}\sqrt{\theta-1}}{\sigma}\sqrt{\Delta t}\right) \tag{47}$$

$$\Delta h = \sigma\frac{\sqrt{\Delta t}}{\sqrt{\theta-1}} \tag{48}$$

Substitute these expressions for Δh and π in (45) and (46) and let Δt go to zero. As the time increment Δt shrinks to zero the binomial variate converges to a normal distribution, with mean

$$\left(1 - \left(1 - \frac{\bar{r}\sqrt{\theta-1}}{\sigma}\sqrt{\Delta t}\right)\right)\frac{\sigma\sqrt{\Delta t}}{\Delta t\sqrt{\theta-1}} = \bar{r} \tag{49}$$

and variance

$$\lim_{\Delta t \to 0} \frac{1}{\theta}\left(1 - \frac{\bar{r}\sqrt{\theta - 1}}{\sigma}\sqrt{\Delta t}\right)\left[1 - \frac{1}{\theta}\left(1 - \frac{\bar{r}\sqrt{\theta - 1}}{\sigma}\sqrt{\Delta t}\right)\right]\frac{\theta^2\sigma^2}{\theta - 1}\frac{\Delta t}{\Delta t} \to \sigma^2 \quad (50)$$

Then, following Merton (1969) and Lin and Yang (2001), it can be verified that capital accumulation is given by

$$dk(t) = [(1 - \tau + \bar{r}\tau e(t))y(t) - c(t)]dt + [\sigma\tau e(t)y(t)]dW \quad (51)$$

That is the equation of capital accumulation given in (5).

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