Nonlinear Pricing with Resale*

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October 7, 2008

Abstract

We consider the problem of a monopolist—choosing an optimal nonlinear pricing scheme—facing two consumers who can resell some or all of the goods to each other in a secondary market. We suppose that the valuations of the consumers are drawn independently from a continuous distribution. We find conditions for the optimum direct mechanism and show that the monopolist can be better off or worse off as compared to the without resale case, depending on the specifics of the cost function of the monopolist and the utility functions of the consumers.

Keywords: Nonlinear pricing, Resale, Mechanism Design, Secondary Markets

JEL Classification Numbers: D42, D82

1 Introduction

Consider the adverse selection problem of a transaction between a seller (the monopolist,) and buyers (consumers,) where the seller does not perfectly know how much the buyers are willing to pay for the goods. Suppose also that the seller sets the terms of the contract—i.e. a menu of quantities and prices. The problem of one principal facing one agent who has private information about his type was first analyzed by Mirrlees (1971). One monopolist and a consumer problem was then analyzed by Mussa and Rosen (1978), and Maskin and Riley (1984), among others. The reader is referred to Bolton and Dewatripont (2005), Laffont and Martimort (2002) and Wilson (1993) for a detailed discussion of economics of adverse selection.

All previous work on monopolist's problem of optimal nonlinear pricing focused either on a single consumer or on multiple consumers who cannot resell the goods to one another, except for Calzolari and Pavan (2006). Calzolari and Pavan (2006)

*PRELIMINARY AND INCOMPLETE, Comments are very welcome.

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recently consider a monopolist who designs an allocation rule and a disclosure policy that optimally fashion the beliefs of the participants in the secondary market. They consider two consumers with two types of valuations and show that it may be impossible to maximize the revenue with a deterministic selling procedure and disclosing only the decision to trade. In their model, the monopolist has one unit of the good at his hand. There is no question of how much to produce.

In this paper, we also consider two consumers who can resell the good to each other in the secondary market. However, we consider consumers drawing valuations from a *continuous* distribution, rather than focusing on a finite set of valuations. In order to simplify optimal nonlinear pricing problem, we do not consider the disclosure policy of the monopolist. Specifically, we suppose that the monopolist proposes a *deterministic* menu of offers and no information is revealed after the trade between the monopolist and the consumers.

We consider two different cases of the nonlinear pricing problem. In the first model, the monopolist is assumed to have a production technology of constant marginal cost, and the consumers are assumed to have concave utility functions. This model was analyzed by Maskin and Riley (1984) for a model without resale, and it results in quantity discounts, which is widely seen in practice. We find conditions for the optimal nonlinear pricing schemes of the model with resale. Moreover, we show that the maximal revenue achievable in an environment with resale is less than the maximal revenue achievable in an environment without resale. This follows from the observation that in an environment without resale, since the utilities of the consumers are concave, the monopolist can offer a certainty equivalent of the transactions of the model with resale at a lower price. This result is in line with understanding that resale is detrimental to monopoly power.

In the second model, the monopolist is assumed to have a convex cost function, and the consumers are assumed to have linear utility functions. For this model, the secondary market optimal behavior is easier to characterize. This is because consumers have linear utility functions and hence announcing a unit price is their optimal selling procedure in the secondary market.[1] This model results in premia rather than quantity discounts. We characterize the optimal menu for this model and show that the maximal revenue achievable in an environment with resale is *more* than the maximal revenue achievable in an environment without resale. This follows from the observation that in an environment with resale, by offering the same quantities as in without resale optimal menu, the monopolist can demand higher amounts of monetary transfers from the consumers. This result runs counter to understanding that resale is detrimental to monopoly power.

We therefore conclude that the profitability of banning the resale (if the monopolist has a power to do so) depends on the specifics of the environment. If the utility functions of the consumers are concave, then the monopolist can gain by offering

[1] Whereas in the first model, the consumers offer a nonlinear menu of quantities and prices to each other in the secondary market.

a certainty equivalent of the transactions in the secondary market. Therefore, the model without resale would give more revenue. If the cost function of the monopolist is convex, then the monopolist can benefit from selling to low value consumers and letting them to sell to high value consumers. Therefore, the model with resale would give more revenue.

2 Economic Environment

Consider a monopolist and two consumers, where the monopolist does not know how much the consumers are willing to pay for the good. Suppose that consumers' preferences depend on the preference characteristics θ. The characteristics θ are private information to the consumers. The monopolist and the other consumer only know the continuous cumulative distribution of θ, $F(\theta)$ on an interval $\left[\underline{\theta}, \overline{\theta}\right]$—we suppose that F satisfies Myerson's (1981) regularity condition.[2] The consumers' preferences are represented by the utility function

$$u(\theta) = \theta v(q) - T,$$

where q is the number of units consumed, and T is the total amount paid. The monopolist's production cost is given by the function $c(q)$. The monopoly profit from selling q units against a sum of money T is then given by

$$\pi = T - c(q).$$

This paper considers the question of finding the profit maximizing pair (T, q) that the monopolist will be able to induce the consumers to choose, in an environment in which the consumers can resell some or all of the good that they have to each other. We suppose that resale takes place in an imperfect information setting. That is, no information about the values or actions taken in the first stage (buying from the monopolist stage) are revealed before the second stage (reselling to each other stage.) We consider no discounting of payoffs between the two stages.

In the resale stage, one of the bidders is chosen randomly–with .5 probability–as a proposer and the other is the responder. The proposer offers an optimal menu to the responder. This menu can have both buy and sell offers. We restrict analysis to direct revelation mechanisms which are truthful in both stages.

In the first stage, the monopolist maximizes[3]

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left[T(\theta_1) + T(\theta_2) - c(q(\theta_1) + q(\theta_2))\right] dF(\theta_1)\, dF(\theta_2)$$

subject to incentive compatibility and individual rationality constraints of the consumers. These constraints are determined with considering the resale stage as well.

[2] That is, the virtual value function $\theta - \frac{1-F(\theta)}{f(\theta)}$ is increasing. This assumption is fairly common in nonlinear pricing literature.

[3] We suppose that the monopolist treats the consumers symmetrically.

3 Linear Cost, Concave Utility Case

In this section, we suppose that the monopolist's production cost is linear and given by $c(q) = cq$ with the unit cost $c > 0$. Hence, monopolist's profit from selling q units against a sum of money T is given by

$$\pi = T - cq.$$

Moreover, we suppose that consumers' preferences are represented by the utility function which is concave in units consumed. Specifically,

$$u(\theta) = \theta v(q) - T,$$

where v satisfies $v(0) = 0$, $v'(0) = \infty$, and $v'(q) > 0$, $v''(q) < 0$ for all q.

We work backwards as usual. Given the menu offered by the monopolist, the consumers will behave optimally in the resale stage.

3.1 Behavior in the resale stage

Since the consumers' choices $\{T(\theta_i), q(\theta_i)\}$ are their private information in the second stage, the consumers will offer each other menus with imperfect information. Let us consider bidder 1 (bidder 2's problem is exactly the same.) Consider the consumer 1 with preference parameter θ_1, who announces his type as θ_1' in the first stage and gets $q(\theta_1')$ units of the product at the price of $T(\theta_1')$.

In the second stage, when bidder 1 is chosen to be the proposer, he will offer a menu $\{S(\cdot \mid \theta_1, \theta_1'), r(\cdot \mid \theta_1, \theta_1')\}$ to consumer 2 where r denotes the amount of the good transferred from 2 and S denotes the amount of the money transferred to consumer 2. Note that r and S can be negative. Consumer 1's problem in the second stage is then

$$\max_{q(\theta_2) \geq r(\theta_2) \geq -q(\theta_1'),\ S(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta_1 v\left(q(\theta_1') + r(\theta_2)\right) - S(\theta_2)\right] dF(\theta_2)$$

subject to individual rationality and incentive compatibility constraints; for all $\theta, \theta' \in [\underline{\theta}, \overline{\theta}]$

$$\theta_2 v\left(q(\theta_2) - r(\theta_2)\right) + S(\theta_2) \geq \theta_2 v\left(q(\theta_2)\right)$$

and

$$\theta_2 v\left(q(\theta_2) - r(\theta_2)\right) + S(\theta_2) \geq \theta_2 v\left(q(\theta_2') - r(\theta_2')\right) + S(\theta_2')$$

We denote the optimal $(S(\cdot), r(\cdot))$ by $(S(\cdot \mid \theta_1, \theta_1'), r(\cdot \mid \theta_1, \theta_1'))$ and the maximand by $C(\theta_1, \theta_1')$.

When bidder 1 is chosen to be the responder, his expected payoff is given by

$$\int_{\underline{\theta}}^{\overline{\theta}} \max\{0, \max_{\theta_1''} \left(\theta_1 v\left(q(\theta_1') - r(\theta_1'' \mid \theta_2, \theta_2)\right) + S(\theta_1'' \mid \theta_2, \theta_2)\right)\} dF(\theta_2)$$

Let us denote the maximand of the above expression by $D(\theta_1, \theta_1')$. Note that when $\theta_1' = \theta_1$, then θ_1'' above should be also equal to θ_1 and when $\theta_1'' = \theta_1$, it gives a value not less than 0.

We can write consumer 1's payoff as

$$U(\theta_1, \theta_1') = \frac{1}{2} C(\theta_1, \theta_1') + \frac{1}{2} D(\theta_1, \theta_1') - T(\theta_1') \tag{1}$$

3.2 Optimal Menu

Therefore, the monopolist's problem in the first stage is given by (since the cost function is linear)

$$2 \max_{q(\cdot), T(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} [T(\theta) - cq(\theta)] \, dF(\theta)$$

subject to

$$\text{(IR)} \quad U(\theta, \theta) \geq 0 \text{ for all } \theta \in [\underline{\theta}, \overline{\theta}],$$

and

$$\text{(IC)} \quad U(\theta, \theta) \geq U(\theta, \theta') \text{ for all } (\theta, \theta') \in [\underline{\theta}, \overline{\theta}]^2.$$

IC constraint implies that $\frac{1}{2} C(\theta, \theta') + \frac{1}{2} D(\theta, \theta') - T(\theta')$ is maximized at $\theta' = \theta$. Using envelope theorem ($v'(0) = \infty$ makes sure that solution for the maximization problem has an interior solution and we can treat r and S as constant while differentiating with respect to θ'), we obtain the following FOC (let us denote $r(\theta' \mid \theta, \theta)$ by $r(\theta' \mid \theta)$ and $S(\theta' \mid \theta, \theta)$ by $S(\theta' \mid \theta)$.)

$$\theta q'(\theta) \frac{1}{2} \left(\int_{\underline{\theta}}^{\overline{\theta}} \left(v'(q(\theta) + r(\theta' \mid \theta)) + v'(q(\theta) - r(\theta \mid \theta')) \right) dF(\theta') \right) = T'(\theta)$$

Moreover, by using $U(\theta) = \frac{1}{2} C(\theta, \theta') + \frac{1}{2} D(\theta, \theta') - T(\theta')$ and envelope theorem, we obtain

$$\begin{aligned} U'(\theta) &= \frac{1}{2} \int_{\underline{\theta}}^{\overline{\theta}} \left(v(q(\theta) + r(\theta' \mid \theta)) + v(q(\theta) - r(\theta \mid \theta')) \right) dF(\theta') \\ &\equiv m(\theta) \end{aligned}$$

Hence we obtain

$$U(\theta) = \int_{\underline{\theta}}^{\theta} m(\theta') \, d\theta' + U(\underline{\theta})$$

and

$$T(\theta) = \frac{1}{2} C(\theta) + \frac{1}{2} D(\theta) - \int_{\underline{\theta}}^{\theta} m(\theta') \, d\theta' - U(\underline{\theta})$$

Moreover,

$$\begin{aligned}\frac{1}{2}C(\theta)+\frac{1}{2}D(\theta) &= \theta m(\theta)-\frac{1}{2}\int_{\underline{\theta}}^{\overline{\theta}}\left[S(\theta'\mid\theta)-S(\theta\mid\theta')\right]dF(\theta') \\ &\equiv \theta m(\theta)+l(\theta)\end{aligned}$$

Since the transfers between the bidders add up to zero, we have

$$\int_{\underline{\theta}}^{\overline{\theta}} l(\theta)\,dF(\theta)=0$$

Thus, we obtain

$$\begin{aligned}\int_{\underline{\theta}}^{\overline{\theta}} T(\theta)\,dF(\theta) &= \int_{\underline{\theta}}^{\overline{\theta}}\left[\theta m(\theta)-\int_{\underline{\theta}}^{\theta} m(\theta')\,d\theta'+l(\theta)\right]dF(\theta)-U(\underline{\theta}) \\ &= \int_{\underline{\theta}}^{\overline{\theta}}\left[\theta m(\theta)-\int_{\underline{\theta}}^{\theta} m(\theta')\,d\theta'\right]dF(\theta)-U(\underline{\theta}) \\ &= \int_{\underline{\theta}}^{\overline{\theta}}\theta m(\theta)\,dF(\theta)-\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\theta} m(\theta')\,d\theta' dF(\theta)-U(\underline{\theta})\end{aligned}$$

and by integration by parts, we obtain

$$\begin{aligned}\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\theta} m(\theta')\,d\theta' dF(\theta) &= (1-F(\theta))\int_{\underline{\theta}}^{\theta} m(\theta')\,d\theta'\Big|_{\theta=\underline{\theta}}^{\theta=\overline{\theta}}+\int_{\underline{\theta}}^{\overline{\theta}} m(\theta)(1-F(\theta))\,d\theta \\ &= \int_{\underline{\theta}}^{\overline{\theta}} m(\theta)(1-F(\theta))\,d\theta\end{aligned}$$

Therefore, we obtain

$$\int_{\underline{\theta}}^{\overline{\theta}} T(\theta)\,dF(\theta)=\int_{\underline{\theta}}^{\overline{\theta}}\left(\theta-\frac{1-F(\theta)}{f(\theta)}\right)m(\theta)\,dF(\theta)-U(\underline{\theta})$$

Thus, monopolist's problem can be rewritten as

$$\max_{q(\cdot)}\int_{\underline{\theta}}^{\overline{\theta}}\left[\left(\theta-\frac{1-F(\theta)}{f(\theta)}\right)m(\theta)-cq(\theta)\right]dF(\theta)-U(\underline{\theta}) \tag{2}$$

3.3 Characterization of S, r and m

Let us consider a proposer with type $\widetilde{\theta}$ who announced his type as $\widetilde{\theta}$ in the first stage. The proposer's problem in the second stage is then

$$\max_{q(\theta)\geq r(\theta)\geq -q(\widetilde{\theta}),\ S(\cdot)}\int_{\underline{\theta}}^{\overline{\theta}}\left[\widetilde{\theta}v(q(\widetilde{\theta})+r(\theta))-S(\theta)\right]dF(\theta)$$

subject to

$$\text{(IR2) } Y(\theta) \geq \theta v(q(\theta)) \text{ for all } \theta \in \left[\underline{\theta}, \overline{\theta}\right],$$

and

$$\text{(IC2) } Y(\theta) \geq Y(\theta, \theta') \text{ for all } (\theta, \theta') \in \left[\underline{\theta}, \overline{\theta}\right]^2.$$

for

$$Y(\theta, \theta') = \theta v(q(\theta') - r(\theta')) + S(\theta')$$

and $Y(\theta) = Y(\theta, \theta)$.

IC2 constraint implies the following first-order condition:

$$S'(\theta) + \theta q'(\theta) v'(q(\theta) - r(\theta)) - \theta r'(\theta) v'(q(\theta) - r(\theta)) = 0$$

Hence, we obtain

$$\begin{aligned} Y'(\theta) &= v(q(\theta) - r(\theta)) \\ &\equiv k(\theta) \end{aligned}$$

therefore we obtain

$$Y(\theta) = \int_{\underline{\theta}}^{\theta} k(\theta') \, d\theta' + Y(\underline{\theta})$$

and

$$S(\theta) = \int_{\underline{\theta}}^{\theta} k(\theta') \, d\theta' + Y(\underline{\theta}) - \theta v(q(\theta) - r(\theta))$$

Thus, IC2 implies

$$\int_{\underline{\theta}}^{\overline{\theta}} S(\theta) \, dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\underline{\theta}}^{\theta} k(\theta') \, d\theta' - \theta v(q(\theta) - r(\theta))\right] dF(\theta) + Y(\underline{\theta})$$

and by integration by parts, we obtain

$$\int_{\underline{\theta}}^{\overline{\theta}} S(\theta) \, dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{1 - F(\theta)}{f(\theta)} k(\theta) - \theta v(q(\theta) - r(\theta))\right] dF(\theta) + Y(\underline{\theta})$$

Thus, the proposer's problem can be rewritten as

$$\max_{q(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\widetilde{\theta} v(q(\widetilde{\theta}) + r(\theta)) + \left(\theta - \frac{1 - F(\theta)}{f(\theta)}\right) v(q(\theta) - r(\theta))\right] dF(\theta) - Y(\underline{\theta})$$

The first order condition is

$$\widetilde{\theta} v'(q(\widetilde{\theta}) + r(\theta)) - \left(\theta - \frac{1 - F(\theta)}{f(\theta)}\right) v'(q(\theta) - r(\theta)) = 0$$

3.4 Revenue Superiority of Without Resale Model

In both with resale and without resale models, monopolist's problem in the first stage is given by

$$2\max_{q(\cdot),T(\cdot)}\int_{\underline{\theta}}^{\overline{\theta}}\left[T(\theta)-cq(\theta)\right]dF(\theta)$$

subject to IR and IC constraints—of course, different constraints in two models.

Given a menu in with resale model, (T,q) the consumers will behave optimally and choose optimal (S,r)

Let us introduce the following handy notation

$$\begin{aligned}
\int_{\underline{\theta}}^{\overline{\theta}}\left(T(\theta)-\frac{1}{2}S(\theta'\mid\theta)+\frac{1}{2}S(\theta\mid\theta')\right)dF(\theta') &= \widetilde{T}(\theta)\\
\int_{\underline{\theta}}^{\overline{\theta}}\left(q(\theta)+\frac{1}{2}r(\theta'\mid\theta)-\frac{1}{2}r(\theta\mid\theta')\right)dF(\theta') &= \widetilde{q}(\theta)\\
\frac{1}{2}\int_{\underline{\theta}}^{\overline{\theta}}\left(v\left(q(\theta)+r(\theta'\mid\theta)\right)+v\left(q(\theta)-r(\theta\mid\theta')\right)\right)dF(\theta') &= \widetilde{v}(\theta)
\end{aligned}$$

Consider a consumer with type θ. Since behaving as if his value is θ' in both stages would give less payoff than behaving as if his value is θ' in the first stage and behaving optimally in the second stage, the IC constraint of the model with resale is not less restricted than the following constraint:

$$\theta\widetilde{v}(\theta)-\widetilde{T}(\theta)\geq\theta\widetilde{v}(\theta')-\widetilde{T}(\theta')\text{ for all }(\theta,\theta')\in\left[\underline{\theta},\overline{\theta}\right]^2$$

and IR constraint is given by

$$\theta\widetilde{v}(\theta)-\widetilde{T}(\theta)\geq 0\text{ for all }\theta\in\left[\underline{\theta},\overline{\theta}\right].$$

Consider the optimal menu of the monopolist in the model with resale $\{T^R,q^R\}$ and corresponding $\{\widetilde{T}^R,\widetilde{q}^R,\widetilde{v}^R\}$. We will argue below that without resale, the monopolist can make a bigger profit.

Since v is concave, from Jensen's inequality, we obtain

$$v\left(\widetilde{q}^R(\theta)\right)\geq\widetilde{v}^R(\theta)$$

which implies that there is some $\widehat{q}(\theta)\leq\widetilde{q}(\theta)$ with

$$v\left(\widehat{q}(\theta)\right)=\widetilde{v}^R(\theta).$$

Now, we claim that monopoly without resale would give a higher revenue than monopoly with resale. The monopolist can obtain a higher revenue by offering the menu $\{\widehat{q}, \widetilde{T}^R\}$. This is because, the transfers among the two consumers add up to zero, and therefore

$$\int_{\underline{\theta}}^{\overline{\theta}} \widehat{q}(\theta)\, dF(\theta) \leq \int_{\underline{\theta}}^{\overline{\theta}} \widetilde{q}^R(\theta)\, dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} q^R(\theta)\, dF(\theta)$$

and

$$\int_{\underline{\theta}}^{\overline{\theta}} \widetilde{T}^R(\theta)\, dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} T^R(\theta)\, dF(\theta).$$

Hence, we have

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(\widetilde{T}^R(\theta) - c\widehat{q}(\theta)\right) dF(\theta) \geq \int_{\underline{\theta}}^{\overline{\theta}} \left(T^R(\theta) - cq^R(\theta)\right) dF(\theta)$$

and

$$\begin{aligned} \theta v(\widehat{q}(\theta)) - \widetilde{T}^R(\theta) &\geq \theta v(\widehat{q}(\theta')) - \widetilde{T}^R(\theta') \\ \theta v(\widehat{q}(\theta)) - \widetilde{T}^R(\theta) &\geq 0 \end{aligned}$$

Therefore, the menu $\{\widehat{q}, \widetilde{T}^R\}$ satisfies IC and IR constraints of the without resale problem, and it gives a revenue bigger than the optimal revenue of the with resale model. The inequality is strict unless for any given θ, $q(\theta) + \frac{1}{2}r(\theta' \mid \theta) - \frac{1}{2}r(\theta \mid \theta')$ is the same for all θ' (otherwise Jensen's inequality would give a strict inequality.) Since this would not be true generically, we can conclude that without resale model is revenue superior to with resale model. We hence obtain the following proposition.

Proposition 1 *If the monopolist has a constant marginal cost, and the consumers have concave utility functions, then the maximal revenue achievable in an environment with resale is less than the maximal revenue achievable in an environment without resale.*

4 Convex Cost, Linear Utility Case

In this section, we suppose that consumers' preferences are linear in units consumed, and represented by the utility function

$$u(\theta) = \theta q - T$$

Assuming that the monopolist's production cost is given by the convex function $c(q)$ with $c(0) = 0$, $c'(q) > 0$ and $c''(q) > 0$ for all q, his profit from selling q units against a sum of money T is then given by

$$\pi = T - c(q)$$

We consider the question of finding the profit maximizing pair (T, q) that the monopolist will be able to induce the consumers to choose. We find the optimal menu for both without resale case and with resale case[4].

4.1 Optimal menu without resale

The monopolist's problem is given by

$$\max_{q,T} \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left[T(\theta_1) + T(\theta_2) - c(q(\theta_1) + q(\theta_2))\right] dF(\theta_1)\, dF(\theta_2)$$

subject to

$$\textit{(IR)}\ u(\theta) \equiv \theta q(\theta) - T(\theta) \geq 0$$

and

$$\textit{(IC)}\ \theta q(\theta) - T(\theta) \geq \theta q(\theta') - T(\theta').$$

IC constraint implies the following first-order condition:

$$T'(\theta) - \theta q'(\theta) = 0 \tag{3}$$

We then obtain

$$u'(\theta) = q(\theta)$$

and it can be shown that for the optimal menu, we have $u(\underline{\theta}) = 0$, hence we obtain

$$u(\theta) = \int_{\underline{\theta}}^{\theta} q(\tau)\, d\tau$$

and

$$T(\theta) = \theta q(\theta) - \int_{\underline{\theta}}^{\theta} q(\tau)\, d\tau$$

By using standard techniques we obtain

$$\int_{\underline{\theta}}^{\overline{\theta}} T(\theta) f(\theta)\, d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta - \frac{1 - F(\theta)}{f(\theta)}\right) q(\theta)\, dF(\theta)$$

Thus, monopolist's problem can be rewritten as

$$\max_{q(\theta)} \left[2 \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta - \frac{1 - F(\theta)}{f(\theta)}\right) q(\theta)\, dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} c(q(\theta_1) + q(\theta_2))\, dF(\theta_1)\, dF(\theta_2)\right]$$

[4] The solution for without resale case for this model was not given in the literature for two consumers case.

4.2 Optimal menu with resale

We work backwards as usual. Given the menu offered by the monopolist, the consumers will behave optimally in the resale stage.

4.2.1 Behavior in the resale stage

Consider the consumer 1 with preference parameter θ_1, who announces his type as θ_1' in the first stage and gets $q(\theta_1')$ units of the product at the price of $T(\theta_1')$. In the second stage, when bidder 1 is chosen to be the proposer, he will offer a menu to consumer 2. It can be shown that the optimal menu of is setting two prices $p_s(\theta_1)$ and $p_b(\theta_1)$ such that $p_s(\theta_1)$ maximizes

$$q(\theta_1')(1 - F(p_s))(p_s - \theta_1)$$

and $p_b(\theta_1)$ maximizes

$$\int_0^{p_b} q(\theta_2)(\theta_1 - p_b)\, dF(\theta_2).$$

When bidder 1 is chosen to be the responder, his expected extra payoff is given by

$$\int_0^{\theta_2^s} q(\theta_2)(\theta_1 - p_s(\theta_2))\, dF(\theta_2) + q(\theta_1') \int_{\theta_2^b}^{\overline{\theta}} (p_b(\theta_2) - \theta_1)\, dF(\theta_2)$$

where $p_s(\theta_2^s) = p_b\left(\theta_2^b\right) = \theta_1$

Thus, bidder 1's overall payoff is

$$\begin{aligned}
u(\theta_1, \theta_1') &= q(\theta_1')\theta_1 + \frac{1}{2} q(\theta_1')(1 - F(p_s(\theta_1)))(p_s(\theta_1) - \theta_1) \\
&\quad + \frac{1}{2} \int_0^{p_b(\theta_1)} q(\theta_2)(\theta_1 - p_b(\theta_1))\, dF(\theta_2) \\
&\quad + \frac{1}{2} \int_0^{\theta_2^s} q(\theta_2)(\theta_1 - p_s(\theta_2))\, dF(\theta_2) + \frac{1}{2} q(\theta_1') \int_{\theta_2^b}^{\overline{\theta}} (p_b(\theta_2) - \theta_1)\, dF(\theta_2)
\end{aligned}$$

Let

$$\theta_1 + \frac{1}{2}(1 - F(p_s(\theta_1)))(p_s(\theta_1) - \theta_1) + \frac{1}{2} \int_{\theta_2^b}^{\overline{\theta}} (p_b(\theta_2) - \theta_1)\, dF(\theta_2) = h(\theta_1)$$

and

$$\frac{1}{2} \int_0^{p_b(\theta_1)} q(\theta_2)(\theta_1 - p_b(\theta_1))\, dF(\theta_2) + \frac{1}{2} \int_0^{\theta_2^s} q(\theta_2)(\theta_1 - p_s(\theta_2))\, dF(\theta_2) = g(\theta_1)$$

With this new notation, a bidder with value θ who announces his type as θ' in the first stage achieves a payoff

$$u(\theta, \theta') = q(\theta') h(\theta) + g(\theta) - T(\theta')$$

Moreover, since

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left[T(\theta_1) + T(\theta_2)\right] dF(\theta_1)\, dF(\theta_2) = 2\int_{\underline{\theta}}^{\overline{\theta}} T(\theta)\, dF(\theta)$$

the monopolist's problem is given by

$$\max_{q,T} 2\int_{\underline{\theta}}^{\overline{\theta}} T(\theta)\, dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} c\left(q(\theta_1) + q(\theta_2)\right) f(\theta_1) f(\theta_2)\, d\theta_1 d\theta_2$$

subject to

$$\textit{(IR)}\ u(\theta) \equiv u(\theta, \theta) \geq 0$$

and

$$\textit{(IC)}\ u(\theta) \geq u(\theta, \theta').$$

IC can be written as

$$h(\theta)\, q(\theta) - T(\theta) \geq h(\theta)\, q(\theta') - T(\theta').$$

IC constraint implies the following first-order condition:

$$T'(\theta) - h(\theta)\, q'(\theta) = 0$$

Integration of the above equality and then integration by parts gives us

$$\begin{aligned} T(\theta) &= \int_{\underline{\theta}}^{\theta} h(\tau)\, q'(\tau)\, d\tau + T(\underline{\theta}) \\ &= h(\theta)\, q(\theta) - h(\underline{\theta})\, q(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} h'(\tau)\, q(\tau)\, d\tau + T(\underline{\theta}) \\ &= h(\theta)\, q(\theta) - \int_{\underline{\theta}}^{\theta} h'(\tau)\, q(\tau)\, d\tau + g(\underline{\theta}) - u(\underline{\theta}) \end{aligned}$$

Since at the optimal menu, we should have $u(\underline{\theta}) = 0$, and (by the standard techniques) and also $g(\underline{\theta}) = 0$,

$$\begin{aligned} \int_{\underline{\theta}}^{\overline{\theta}} T(\theta)\, f(\theta)\, d\theta &= \int_{\underline{\theta}}^{\overline{\theta}} \left[h(\theta)\, q(\theta) - \int_{\underline{\theta}}^{\theta} h'(\tau)\, q(\tau)\, d\tau\right] dF(\theta) \\ &= \int_{\underline{\theta}}^{\overline{\theta}} h(\theta)\, q(\theta)\, dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} h'(\tau)\, q(\tau)\, d\tau dF(\theta) \end{aligned}$$

and by integration by parts, we obtain

$$\begin{aligned} -\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} h'(\tau)\, q(\tau)\, d\tau f(\theta)\, d\theta &= (1 - F(\theta)) \int_{\underline{\theta}}^{\theta} h'(\tau)\, q(\tau)\, d\tau \Big|_{\theta=\underline{\theta}}^{\theta=\overline{\theta}} - \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)\, h'(\theta)\, (1 - F(\theta))\, d\theta \\ &= -\int_{\underline{\theta}}^{\overline{\theta}} q(\theta)\, h'(\theta)\, (1 - F(\theta))\, d\theta \end{aligned}$$

Hence, we obtain

$$\int_{\underline{\theta}}^{\overline{\theta}} T(\theta) f(\theta) d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{h(\theta)}{h'(\theta)} - \frac{1 - F(\theta)}{f(\theta)} \right) h'(\theta) q(\theta) f(\theta) d\theta$$

From envelope theorem, we obtain $h'(\theta) = F(p(\theta))$ and

$$\left(\frac{h(\theta)}{h'(\theta)} - \frac{1 - F(\theta)}{f(\theta)} \right) h'(\theta) = h(\theta) - \frac{1 - F(\theta)}{f(\theta)} F(p(\theta)) \equiv \varphi(\theta)$$

The monopolist's problem can be rewritten as

$$\max_{q(\theta)} \left[2 \int_{\underline{\theta}}^{\overline{\theta}} \varphi(\theta) q(\theta) f(\theta) d\theta - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} c(q(\theta_1) + q(\theta_2)) f(\theta_1) f(\theta_2) d\theta_1 d\theta_2 \right]$$

subject to

$$q'(\theta) \geq 0$$

The constraint $q'(\theta) \geq 0$ will hold as long as $\varphi(\theta)$ is nondecreasing.

4.3 Revenue Superiority of With Resale Model

The revenue superiority of with resale model is apperant from the conditions

$$T^R(\theta) = \int_{\underline{\theta}}^{\theta} h(\tau) q^{R\prime}(\tau) d\tau + T^R(\underline{\theta})$$

and

$$T^N(\theta) = \int_{\underline{\theta}}^{\theta} \tau q^{N\prime}(\tau) d\tau + T^N(\underline{\theta})$$

Since $h(\theta) > \theta$, we conclude that for the revenue maximizing q^N in without resale model, we could find a $\widetilde{T}^R$ which is greater than T^N so that the menu $\{q^N, \widetilde{T}^R\}$ would give more revenue in with resale model. We hence obtain the following proposition.

Proposition 2 *If the monopolist has a convex cost function, and the consumers have linear utility functions, then the maximal revenue achievable in an environment with resale is more than the maximal revenue achievable in an environment without resale.*

5 Conclusion

In this paper, we have found conditions for the optimal mechanisms for a monopolist who expects that consumers would resell in a secondary market. Abstaining from the possible disclosure policy of the monopolist, we focused on the nonlinear pricing menus that the monopolist would be able to implement. We have worked on the continuous distributions of the values of the consumers, which has never done before for the monopoly with resale problem. We have shown that the profitability of banning the resale depends on the specifics of the environment.

This paper shows that resale opportunities significantly effects the optimal menu of the monopolist and the textbook result "resale is harmful for the monopolist" is not always correct. Solving for the optimal menu of the monopolist with resale is yet to be done.

References

[1] Bolton, P., and M. Dewatripont (2005) "Contract Theory." Cambridge, Mass.: MIT Press.

[2] Calzolari, G., and A. Pavan (2006) "Monopoly with Resale." RAND Journal of Economics, Volume 37, No. 2

[3] Laffont, J. and D. Martimort (2002) "The Theory of Incentives: the Principal-Agent Model." Princeton University Press, Princeton.

[4] Maskin, E. and J. Riley (1984) "Monopoly with Incomplete Information." RAND Journal of Economics, Vol. 15, 171-196.

[5] Mirrlees, J. (1971) "An Exploration in the Theory of Optimum Income Taxation." Review of Economic Studies, 38, 175-208.

[6] Myerson, R. (1981) "Optimal Auction Design," Mathematics of Operations Research, 6(1): 58–73.

[7] Mussa, M., and S. Rosen (1978) "Monopoly and Product Quality." Journal of Economic Theory, 18, 301-317.

[8] Wilson, R. (1993) "Nonlinear Pricing." Oxford University Press, New York