Multiple Lending and Constrained Efficiency in the Credit Market

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Abstract

This paper studies the relationship between competition and incentives in an economy with financial contracts. We concentrate on non-exclusive credit relationships, that is, an entrepreneur can simultaneously accept more than one contractual offer. Several homogeneous lenders compete on the contracts they offer to finance the entrepreneur's investment project. We model a common agency game with moral hazard, and we characterize its equilibria. As expected, notwithstanding the competition among the principals (lenders), non-competitive allocations can be supported. In particular, positive profit equilibria are pervasive. We then provide a complete characterization of the constrained utility possibility set, and observe that all the equilibrium payoffs of the decentralized game belong to the frontier of the set. Hence, all credit market equilibria turn out to be constrained Pareto efficient.

JEL Classification:D4, D6, G2.

1 Introduction

The paper is devoted to the analysis of credit markets where several lenders strategically compete over the contracts they offer to entrepreneur-borrowers. At the stage of contracting, the decision of the unique borrower crucially depends on the loans she is simultaneously receiving from all the lenders of the economy. We consider a set-up where the contracts are *non-exclusive*, i.e. the borrower is allowed to accept more than one contract at a time. The main aim of the work is to emphasize the interplay of contractual externalities in determining the welfare properties of the equilibria arising in such framework.

Exclusivity clauses are not explicitly imposed in several financial relationships. It is recognized that many firms have access to multiple credit sources, as shown by Petersen and Rajan (1995) for the small firms in the U.S., or by Detragiache, Garella, and Guiso (2000) for the Italian case. The credit card market has been proposed as an example of non-exclusive dealings, as Bizer and DeMarzo (1992) and Parlour and Rajan (2001) have already explained. Examples of financial interactions with non-exclusive contracting which aim at clarifying the relationship between incentives and competition are very recent: the main general results and implications are discussed by Segal and Whinston (2003).

The relevant finding of the literature on non-exclusivity can be summarized as follows: the contractual externalities emerging when several principals interact with one common agent, can be responsible for existence of *second-best* inefficient equilibria. In other words, a social planner who is subject to incentive constraints and feasibility can achieve outcomes that Pareto dominate the equilibrium outcomes of players' interactions.

Constrained inefficient equilibria have mostly been analyzed in insurance set-ups (Arnott and Stiglitz (1993), Hellwig (1983), Kahn and Mookherjee (1998) and Bisin and Guaitoli (2004)). The present essay proposes an investigation on the welfare properties of equilibria in the credit market, where strategic competition is over financial contracts.

The theoretical contributions examining credit market imperfections mostly use the principal-agent model as a tool to represent credit relationships. The solution to the principal-agent model is equivalent to the outcome of a planner's maximization problem under exclusive contracting and subject to the same asymmetry of information, given appropriate welfare weights. Hence, every equilibrium of the principal-agent contracting game turns out to be second-best efficient.

Enriching the analysis of competition in the credit market using the common agency approach to financial contracting, we believe is a necessary step to analyze competition on financial markets and the welfare properties of the equilibria of the new game. This work could then be regarded as part of a research project on welfare foundations for policy intervention, in particular along the lines of the credit channel of monetary policy.

We propose a simple, static and partial equilibrium model of the credit market to study multiple credit relationships. We model competition between an arbitrarily large and finite number of lenders who offer credit lines to a single borrower. The borrower has to take some non-observable action; she can choose not to perform the hidden action and

to divert resources for private use, in which case she will appropriate a fixed proportion of the received liquidity.[1] By exerting effort in the investment project, she will get some stochastic returns from the production technology. None of the lender is irrelevant, in the sense that every proposed loan affects the entrepreneur on the effort she will choose.

The possibility to sign several contracts simultaneously generates externalities among the financiers. These externalities are responsible for the emergence of equilibrium results which are not in line with Bertrand theory of competition. If the agency costs are high enough, competition among financiers delivers non-competitive results in the forms of credit rationing and of positive extra-profits at equilibrium.

The model is closely related to the Parlour and Rajan (2001)'s work on the credit cards market, which we regard as a useful benchmark to provide insights on the emergence of positive profit equilibria. We modified their analysis, generalizing their incentive structure. As a result, we are able to provide (in a richer set-up) the same equilibrium characterization as Parlour and Rajan (2001).

The main contribution of this work relies, though, on the welfare analysis of the credit market equilibria. Examining this issue, we show that every positive-profit equilibrium is constrained Pareto efficient, i.e. it would be the outcome of the decision of a central authority subject to the same informational constraints. The result is not in line with the main findings of the existing literature on competition and incentives, which has always emphasized that non-exclusive contracting generates those externalities sustaining constrained inefficient outcomes.[2]

Our simple example clarifies how could the incentive structure eliminate the (constrained) inefficiency result. The crucial element turns out to be the existence of a binding Incentive Compatibility constraint in the credit market equilibrium. This guarantees that every equilibrium outcome can in fact be obtained as the solution of a planner's problem. In particular, the feasible sets of the two problems result to be the same.

Surprisingly, the same argument applies to the recent literature on insurance markets with non-exclusive contracting. Removing the assumption of risk-averse agents in the framework of Bisin and Guaitoli (2004) and Kahn and Mookherjee (1998) implies that their positive profits equilibria (which were the inefficient ones) will correspond to binding incentive constraints, and collapse to allocations on the second-best frontier. While in insurance contexts it is widely accepted that the customer be risk-averse, in a lender-borrower interaction this assumption has often been relaxed.[3]

The discussion is organized in the following way: Section 2 presents the features of our set-up. Section 3 characterizes the credit market equilibria of a simplified version of the model. Section 4 provides the welfare analysis and the results on the efficiency of the positive-profit equilibria as compared to those obtained in the insurance set-ups. Section 5 presents a discussion of our results. The Appendix presents the general version of our credit economy and contains all the proofs.

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[1]This model is indeed compatible with the literature on strategic default. For a detailed justification of such an approach to strategic default, refer to Parlour and Rajan (2001).

[2]See Arnott and Stiglitz (1993), Kahn and Mookherjee (1998), Bisin and Guaitoli (2004), Segal and Whinston (2003).

[3]See for example, Parlour and Rajan (2001) or Holmstrom and Tirole (1997).

2 The model

Credit relationships are represented in a simple way. In this economy there are $N \geq 2$ lenders (indexed by $i \in N = \{1, 2, ..., N\}$) who compete over the loan contracts to finance a single borrower.[4] The entrepreneur is penniless though she has access to the technology for the production of the only existing good. Contractual offers are simultaneous. Having received all the contract's proposals, the borrower decides which of them to sign taking into account that she can accept any subset of them.[5]

The production process is stochastic and the probability distribution over the random outcomes is determined by the entrepreneur's choice on a non-contractible action (effort). The entrepreneur's effort space is made of two elements: e^H and e^L, with $e^H > e^L$. If the high effort e^H is chosen, production successfully yields $G(I)$ for every I invested with probability p and 0 with probability $1 - p$. If, on the contrary, the low effort e^L is taken, the entrepreneur's activity is unsuccessful with probability 1.

Exerting effort implies some disutility for the borrower. We chose to represent this feature using a private benefit function $\psi(e)$, which takes values $\psi(e^H) = 0$ and $\psi(e^L) = B(I)$. The private benefit function $B(I)$ is taken to be continuous, increasing and convex:

$$B(0) = 0, \ B'(I) > 0, \ B''(I) \geq 0,$$

and such to satisfy the Inada conditions:

$$\lim_{I \to 0} B'(I) = 0, \ \text{and} \ \lim_{I \to \infty} B'(I) = +\infty.$$

Once should hence notice that we are considering the set of outcomes $Y = \{G(I), 0\}$. The private choice of effort affects the probability distribution of these realizations. In particular, if the entrepreneur exerts the high effort e^H, the probability vector is given by the array: $\{p, 1 - p\}$ with $p > 0$. If instead the entrepreneur shirks, i.e. chooses low effort e^L, the lottery is degenerate and equal to: $\{0, 1\}$. The production function $G(I)$ is continuous, increasing and strictly concave in I. Inada conditions are also satisfied.

Let us describe the normal form of the game we are considering. Lenders strategically compete over financial contracts. The strategy of each lender i is the choice of the contract C_i. The contract offer C_i prescribes a repayment line R_i and a loan amount I_i, i.e.

$$C_i = (R_i, I_i) \in \mathcal{C}_i \subseteq \mathbb{R}^2_+$$

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[4] Abusing notation we indicate the set of lenders and its cardinality with N.
[5] This defines a scenario of *delegated* common agency Martimort and Stole (2003)

where $\mathcal{C}_i$ is the set of feasible contract offers for each lender i. We also denote $\mathcal{C} = \times_i \mathcal{C}_i$ the aggregate set of offered contracts.[6] The borrower's strategy is therefore given by the map:

$$s_b : \mathcal{C} \to \{0,1\}^N \times \{e^H, e^L\}$$

With a small abuse of notation, we also define the generic element of the set $\{0,1\}^N$ as the array $a_b = \left(a_b^1, a_b^2, ..., a_b^N\right)$, where $a_b^i = \{0,1\}$ is the borrower's decision of rejecting or accepting lender i's offer. The choice of the array a_b defines the set of accepted contracts $\mathcal{A}$:

$$\mathcal{A} = \left\{i \in N : a_b^i = 1\right\}$$

In other words, the borrower decides on the choice of the effort level and on the relevant set of offers to accept $\mathcal{A}$. Her strategy set will be denoted as $\mathcal{S}_b$, so that $s_b \in \mathcal{S}_b$.

Let us now consider payoffs. The borrower's payoff is given by:

$$\pi_b = \begin{cases} p\left[G(I) - R\right] & \text{if } e^H \text{ is chosen} \\ BI & \text{if } e^L \text{ is chosen} \end{cases}$$

where R and I denote the aggregate repayment and investment, respectively:

$$R = \sum_{i \in \mathcal{A}} R_i \ \text{ and } \ I = \sum_{i \in \mathcal{A}} I_i.$$

Lender i's payoff is given by:

$$\pi_i = \begin{cases} pR_i - (1+r)I_i & \text{if } C_i \text{ is accepted} \\ 0 & \text{otherwise} \end{cases}$$

where $r \in \mathbb{R}_+$ is the lender's cost of collecting deposits.[7]

One has also to notice that lender i's payoff does not directly depend on lender j's strategies. Existence of contractual externalities among lenders is originated by the borrower's behavior only: at the stage of contracting with lender i, the action chosen by the borrower depends on the contractual offer she is receiving from lender j.[8]

If low effort is chosen, the limited liability assumption guarantees that the entrepreneur will have no appropriable resources to repay the loan. The only way to give him a

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[6]One should notice that lenders' strategy are restricted to be take-it or leave-it offers. When Common Agency games of *complete* information are considered, it is well known that this restriction on principals' strategy spaces involves a loss of generality. If principals are allowed to offer menus over the relevant alternatives, there can be equilibrium outcomes that could not be supported by simple offers (see Peters (2001) and Martimort and Stole (2002). There is, anyway, always a rationale for the use of take-it or leave-it offers: Peters (2003) shows that every equilibrium outcome of this simple game continues to be an equilibrium outcome of the extended game where principals are competing over menus.

[7]Lenders here rely on the deposit market to finance entrepreneurial activity.

[8]This is usually referred to as the absence of *direct externalities* among principals. Most common agency models have been developed in such a simplified scenario. Examples of recent researches where direct externalities among principals are considered include Bernheim and Whinston (1998) and Martimort and Stole (2003).

positive repayment will be to induce the entrepreneur to perform high effort. In particular, whenever the low effort is chosen, each lender i will earn the negative profit:

$$\pi_i = -(1+r)I_i$$

We model credit market interactions as a sequential game, with a first stage where several lenders play a simultaneous move game and a second stage where the borrower decides on acceptance/rejection of each offer and finally exerts effort. In formal terms, loan relationships are represented by the following complete information common agency game Γ:

$$\Gamma = \{(\pi_i)_{i \in N}, \pi_b, \mathcal{C}, \mathcal{S}_b\}.$$

3   Credit market equilibria

This section discusses the properties of the Subgame Perfect Nash Equilibria of the game Γ. The equilibria match those discussed by Parlour and Rajan (2001). Importantly, we are able to show that the specific characterization obtained in Parlour and Rajan (2001) can in fact be reproduced in a more general setting, where the private benefit earned by the single agent is taken to be a non-linear function of the amount borrowed I.

The discussion of this general case is proposed in the Appendix (that contains all the proofs), while the paragraphs present the simpler case with $B(I) = BI$ with $B > 0$. This scenario indeed simply reformulates the original Parlour and Rajan (2001) model in a moral hazard setting and introduces a positive interest rate on deposits. Having presented the equilibria of the credit market we then characterize the constrained Pareto frontier of the economy, and to show that competition among lenders sustains here only second-best efficient allocations (equilibria). We start by remarking that whenever the effort e^L chosen, then every active lender earns negative profits. As a consequence, at equilibrium only the high level of effort will be implemented. Furthermore, since the entrepreneur's private benefit is taken to be monotonically increasing in the aggregate lending I, then the borrower always has the incentive to accept the whole array of offered contracts when e^L is selected. This greatly simplifies the incentive analysis.

We choose Subgame Perfection as the relevant equilibrium concept for the game Γ.

Definition 1 *A (pure strategy) equilibrium of the game Γ is an array* $\left[\left(\tilde{R}_i, \tilde{I}_i \right)_{i \in N}, (\tilde{a}_b^i)_{i \in N}, p \right]$ *such that:*

- *the borrower is optimally choosing the set of accepted contracts $\mathcal{A}$ (i.e. she is choosing her optimal array $a_b \in \{0,1\}^N$) and implementing the high level of effort;*

- *for every lender $i \in N$, the pair $\left(\tilde{R}_i, \tilde{I}_i \right)$ is a solution to the following problem:*

$$\max_{R_i, I_i} \quad p R_i - (1+r)I_i$$

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$s.t.$

$$p \left[G \left(\sum_{j \neq i} \tilde{I}_j \tilde{a}_b^j + I_i \tilde{a}_b^i \right) - \left(\sum_{j \neq i} \tilde{R}_j \tilde{a}_b^j + R_i \tilde{a}_b^i \right) \right] \geq B \left(\sum_{j=1}^{N} \tilde{I}_j \tilde{a}_b^j + I_i \tilde{a}_b^i \right) \qquad (1)$$

The inequality (1) is the borrower's Incentive Compatibility (IC) constraint and it is formulated in terms of aggregate investment and aggregate revenues. The borrower has no endowment, her exogenous reservation utility is taken to be zero and thus her participation decision will always be satisfied. The constraint (1) hence defines lender i's set of feasible contracts under non-exclusivity.

We also remark that the aggregate surplus corresponding to the choice of e^H corresponds to $S = \pi_b + \sum_{i \in N} \pi_i$. The first-best investment level that maximizes S will be referred to as I^*:

$$I^* = \underset{I}{argmax} \; S \equiv \underset{I}{argmax} \; pG(I) - (1+r)I$$

where I^* is such that $pG'(I^*) = 1 + r$ and that the corresponding surplus is positive.[9]

We characterize equilibrium allocations in terms of the incentive parameter B. As a starting point, we introduce the threshold value B_z, which defines the lowest level of incentives compatible with the *first-best* level of investment:

$$B_z = \frac{pG(I^*) - (1+r)I^*}{I^*} \qquad (2)$$

Fig.1 identifies B_z using the total surplus hump-shaped curve, $S = pG(I) - (1+r)I$, and straight lines starting from the origin with slope equal to B. Equation (2) implies that B_z is the slope of the ray that cuts the surplus' curve at its maximum.

If $B = B_z$, then for the first-best investment I^* to be feasible, the (IC) constraint should bind. From equation (2) it follows that in every economy where $B = B_z$, whenever I^* is implemented, the borrower gets the entire surplus. Lenders' profits are equal to zero in the aggregate and the corresponding aggregate repayment will be R^* such that $pR^* - (1+r)I^* = 0$.[10]

Whenever $B > B_z$ allocations giving zero-profits to the lenders are feasible only if the level of debt is taken to be lower than I^*.[11] We denote this level of aggregate investment $\bar{I}(B)$:

$$\bar{I}(B) : pG(\bar{I}(B)) - (1+r)\bar{I}(B) = B\bar{I}(B),$$

and $\bar{I}(B) < I^*$ holds . If, on the contrary, $B < B_z$, it is then possible to achieve I^* and to leave some extra-surplus to lenders. One should also notice that for every $B < B_z$ it would be feasible to implement the first best level of investment I^* in a monopolistic environment.

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[9]Inada conditions guarantee the existence of such an I^*.

[10]This of course implies that every lender earns zero profit, given that their condition is symmetric and limited liability holds.

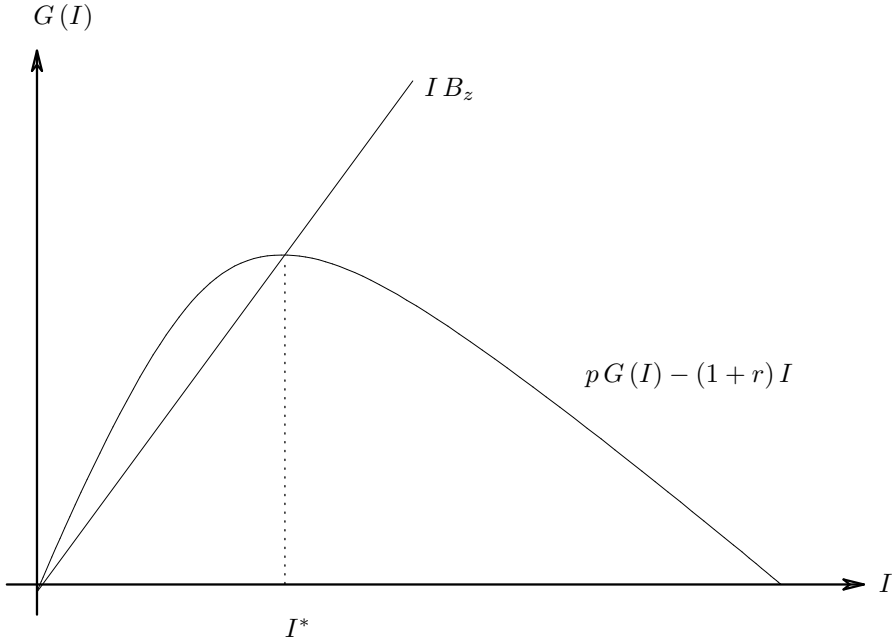[11]That is, the first-best investment level I^* does not belong to the feasible set defined by (1).

Figure 1: Graphical representation of B_z

3.1 Characterization

We first consider the scenario where credit relationships take place under the additional assumption of exclusive contracting, i.e. the borrower can only accept at most one contract at a time. Thus lenders compete à la Bertrand over contracts; at equilibrium they will always receive zero profits and the borrower will appropriate the whole surplus.[12] Importantly, competition under exclusivity delivers the first-best level of credit I^* only for those economies where incentive problems are very mild ($B < B_z$). If the incentive to shirk is high enough, say $B \geq B_z$, then zero-profit equilibria will be associated to the constrained amount of credit $\bar{I}(B)$.

If we allow for non-exclusive contracting, then we formally enter into a common agency set-up. Given the high degree of externalities involved in the analysis, positive profits equilibria and low levels of aggregate investment are a typical feature of the analysis. In our model, we can sustain zero-profit equilibria with competition among lenders offering non-exclusive contracts for those parameter values which make the moral hazard problem very mild. As will be clear from the following discussion, equilibria exhibit different features according to the magnitude of the parameter B relative to the level B_z. To provide a full description of equilibrium outcomes we also introduce the threshold:

$$B_c = \frac{p\,G\,(I^*) - I^*(1+r)}{2I^*}$$

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[12]The mechanism of undercutting on each opponent's proposal squeezes any possible rent for the lenders. There cannot be an equilibrium with positive profits for the lenders, given they are all identical and the entrepreneur chooses only one proposal.

One should notice that whenever $B < B_c < B_z$, then the outcome I^* can be sustained also in the presence of two lenders. In addition, we let B_m be the minimal incentive level sufficient to sustain the monopolistic outcome I_m:

$$I_m = \arg\max_I \ \pi_m = \arg\max_I \ pG(I) - BI - (1+r)I.$$

Hence, if we consider

$$B_l = \frac{p\, G\, (I_m) - I_m(1+r)}{I^* + I_m},$$

then, for every $B < B_l < B_z$ it is feasible to implement the monopolistic outcome even if the supplementary amount I^* were offered.

We are now ready to provide a simple characterization of equilibria in terms of the incentive parameter B.

This is done in the following:

Proposition 1 *Whenever $B < B_z$, then the outcome (R^*, I^*) can be supported as a (pure strategy) equilibrium of the game Γ. In particular:*

i) *If $B \leq B_c$, then (R^*, I^*) is an equilibrium outcome for any N.*

ii) *If $B \in (B_c, B_z]$, then there exists a critical number of lenders N_B such that for all $N > N_B$ the aggregate allocation (R^*, I^*) is an equilibrium outcome.*

iii) *For every $B \in (B_c, B_l]$, there exists an equilibrium where only one contract is bought. The contract guarantees a positive profit to lender i. Furthermore, there is a second lender who offers a zero profit contract that is not accepted. All other lenders are not active.*

The intuition for the result (i) is the following: consider a scenario where $(N - 2)$ lenders are not active, while each of the remaining two can offer a contract associated to a level of debt equal to I^*. Notice that by definition, $2B_c = B_z$: this guarantees that two lenders are enough to sustain the zero-profit equilibrium. If $B = B_c$, then the borrower is indifferent between accepting any of the two contracts exerting the desired effort, and accepting both of them taking zero effort. As long as any single lender $k = i, j$ offers a contract different from the zero-profit one $\left(R_k \neq \frac{I^*(1+r)}{p}, I_k \neq I^*\right)$, a Bertrand argument applies: the two-lenders competition generates undercutting on each other's offers until the marginal cost of funds meets marginal revenues.

One should note that the competitive result emerging in a scenario of exclusivity can hence be implemented even without imposing exclusivity clauses. If the incentive to shirk is low, then, despite the high amount of externalities associated to competition under non-exclusivity, the first best outcome can still be reached.

If the incentive to take the low action falls between B_c and B_z, then zero-profits equilibria may arise only if N is large enough. This is stated in (ii); in this case, there is no room for a single lender to offer the contract (R^*, I^*) and trigger the low effort choice. The borrower will always have an incentive to accept it in conjunction with others zero

profit contracts and shirk. We hence consider a scenario where all lenders offer the contract (R_i, I_i), where $I_i = \frac{I^*}{N-1}$ and R_i is the repayment level that guarantees zero-profits to the i-th lender when offering the loan amount $\frac{I^*}{N-1}$, i.e. $R_i = \frac{I^*(1+r)}{(N-1)p}$. The borrower accepts $(N-1)$ offers and selects $e = e^H$. All active lenders enjoy zero-profit, but they cannot have an incentive to deviate given the existence of the inactive one. It is then possible to find a level of N high enough such that this last lender could not profitably deviating without inducing low effort.

Finally, (iii) identifies a situation where positive profit may emerge even though the incentive to take low action is relatively small. In such a case, the first-best level of investment I^* will be achieved but the distribution of the total surplus will be rather favorable to the lenders. The equilibrium is sustained by latent contracts, i.e. contracts which are not bought at equilibrium and are used to deter entry. The analysis of these sort of equilibria has been introduced in Arnott and Stiglitz (1993) and developed by Hellwig (1983) and Bisin and Guaitoli (2004).

If we consider the case $B \geq B_z$, then positive profits equilibria are a general feature of the analysis. The following proposition provides a full characterization of equilibria in this region.

Proposition 2 *If $B \geq B_z$, then no allocation guaranteeing zero-profit for lenders can be sustained at equilibrium. In addition:*

iv) If $B \in [B_z, B_m)$, then there is a critical number of lenders N_B such that for every $N \geq N_B$, there exist a positive profit equilibrium. The equilibrium outcome $\left(N\tilde{R}, N\tilde{I}\right)$ is characterized by the following set of equations:

$$p\left[G\left(N\tilde{I}\right) - N\tilde{R}\right] = p\left[G\left((N-1)\tilde{I}\right) - (N-1)\tilde{R}\right] \tag{3}$$

$$p\left[G\left(N\tilde{I}\right) - N\tilde{R}\right] = BN\tilde{I} \tag{4}$$

and exhibits the feature that:

$$(N-1)\tilde{I} > I_m \tag{5}$$

v) If $B \geq B_m$, then the monopoly outcome can be supported at equilibrium for any N.

If the incentive problem is relatively strong, given the concavity of the $G(.)$ function, then any equiproportional reduction in both the repayment and the credit offered by a single firm satisfies the IC constraint as a strict inequality. Hence, there is room for a profitable deviation to break any zero-profit equilibrium.

One should also note that (iv) refers to an equilibrium where all the N existing lenders are active and the borrower is indifferent between accepting $(N-1)$ or N contracts while

exerting high effort, as shown by (3). This *no-side-contracting* condition is crucial to establish existence of equilibria with positive profits in several works on moral hazard in insurance economies because it prevents additional purchases of insurance.[13] Furthermore, when the borrower accepts N contracts, her Incentive Compatibility constraint (4) will bind. Finally, the aggregate level of credit issued by $(N-1)$ lenders is strictly greater than I_m that corresponds to the investment chosen by one monopolistic lender (5). At equilibrium, every lender is active in the market, though the aggregate investment level turns out to be strictly lower than I^*. Competition over financial contracts and moral hazard determine rationing in credit supply and redistribution towards the intermediation sector. Whenever $B > B_z$, we are in the increasing part of the social surplus function $S = pG(I) - (1+r)I$ represented in Fig. 1. As a consequence, a single lender i offering a zero-profit contract can profitably deviate if all the others are playing a zero-profit strategy: a Bertrand outcome cannot hence be sustained.

Finally, when the moral hazard problem is very relevant, say $B \geq B_m$, then any credit level different from the monopoly one induces shirking.

The main concern of this paper is to characterize the welfare properties of credit market equilibria when multiple lenders compete over loan contracts. This is the task of the next section.

4 Welfare analysis

We will provide here a description of the economy's feasible set, that is the set of players' payoffs corresponding to the allocations implementable by a social planner. We introduce the notion of social planner and the related concept of *constrained* efficiency in the same way as it is done in the literature on incentives in competitive markets (see for instance Bisin and Guaitoli (2004)), but we manage to characterize the whole constrained Pareto frontier. The social planner will choose the aggregate investment level I and the aggregate repayment R to maximize his preferences over the aggregate feasible set that is usually denoted as the *utility possibility set*.

We will henceforth denote π_L the payoff earned by lenders in the aggregate credit sector and π_b the corresponding borrower's payoff. Let us start considering the first-best situation, where the relevant constraints faced by the planner are those imposed by technology and resources (together with limited liability requirements). The corresponding utility possibility set is:

$$\mathcal{F}(\pi_L, \pi_b) = \left\{ (\pi_L, \pi_b) \in \mathbb{R}_+^2 : \pi_L + \pi_b \leq pG(I^*) - I^*(1+r) \right\} \tag{6}$$

The frontier of the set $\mathcal{F}$ is referred to as the first-best Pareto frontier. All the arrays (π_L, π_b) belonging to this Pareto frontier are such that there does not exist a pair $(\pi_L', \pi_b') \in \mathcal{F}$ with $\pi_L' \geq \pi_L$ and $\pi_b' > \pi_b$ or $\pi_L' > \pi_L$ and $\pi_b' \geq \pi_b$.

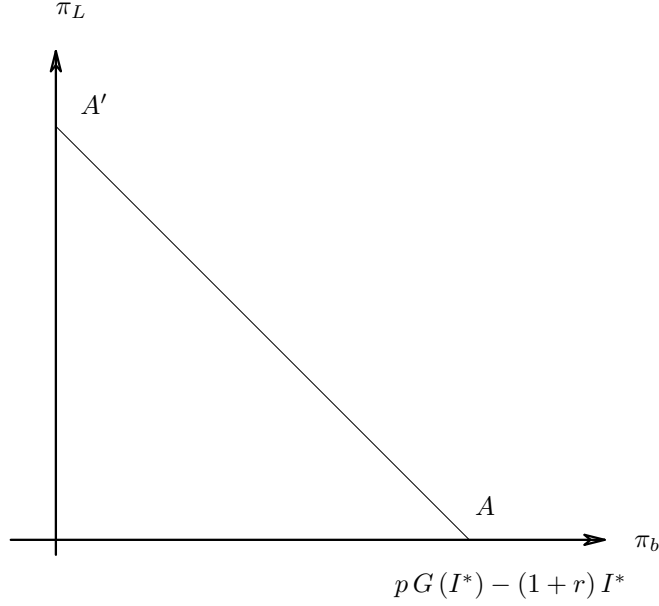[13]See Bisin and Guaitoli (2004) and Kahn and Mookherjee (1998).

Figure 2: The first-best Pareto frontier

Observe that the payoffs functions $\pi_L(R, I)$ and $\pi_b(R, I)$ evaluated at the high level of effort are both linear in the aggregate repayment R. As a consequence, the first-best Pareto frontier will be a downward-sloping 45-degree line. By using the variable pR as a transfer, we can draw the first-best Pareto frontier $\pi_L^*(\pi_b^*)$ as the line depicted in Figure 2.

Every point on the first-best Pareto frontier corresponds to the optimal investment level I^*. In particular, point A identifies a situation where the whole surplus is distributed to the borrower, $\pi_b^* = pG(I^*) - (1+r)I^*$, so that $pR = (1+r)I^*$, i.e. $\pi_L^* = 0$. If we consider the opposite case $\pi_b^* = 0$, then from (6) we get $pR = pG(I^*)$, i.e. lenders are receiving everything and the borrower is left at her reservation utility of zero (point A').

Let us now define the second-best allocations, i.e. those implementable by a planner who is facing informational constraints. The *constrained utility possibility set* is the set of outcomes (π_L, π_b) such that:

$$\mathcal{F}'(\pi_L, \pi_b) = \left\{(\pi_L, \pi_b) \in \mathbb{R}_+^2 : \pi_L \leq \pi_L^{**}(\pi_b^{**}, B), \quad \pi_b \leq \pi_b^{**} \text{ for every } \pi_b^{**} \in [0, pG(I^*) - I^*(1+r)]\right\}$$

where for every given π_b^{**}, $\pi_L^{**}(\pi_b^{**}, B)$ is such that:

$$\pi_L^{**}(\pi_b^{**}, B) = \max_{R,I} pR - (1+r)I \tag{7}$$

s.t.

$$pR - (1+r)I + \pi_b^{**} \leq pG(I) - (1+r)I \tag{8}$$

$$\pi_b^{**} \geq BI \tag{9}$$

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With respect to the first-best problem, we have introduced here the Incentive Compatibility requirement appearing in equation (9). Observe that for a given π_b^{**}, the lender's objective function is monotone in R, hence equation (8) will bind at the optimum. We can therefore substitute the expression for pR obtained in (8), in the objective function. The system (7)-(9) can be rewritten as:

$$\pi_L^{**}\left(\pi_b^{**}, B\right) = \max_I p\,G(I) - \pi_b^{**} - (1+r)I \tag{10}$$

s.t.

$$\pi_b^{**} \geq BI \tag{11}$$

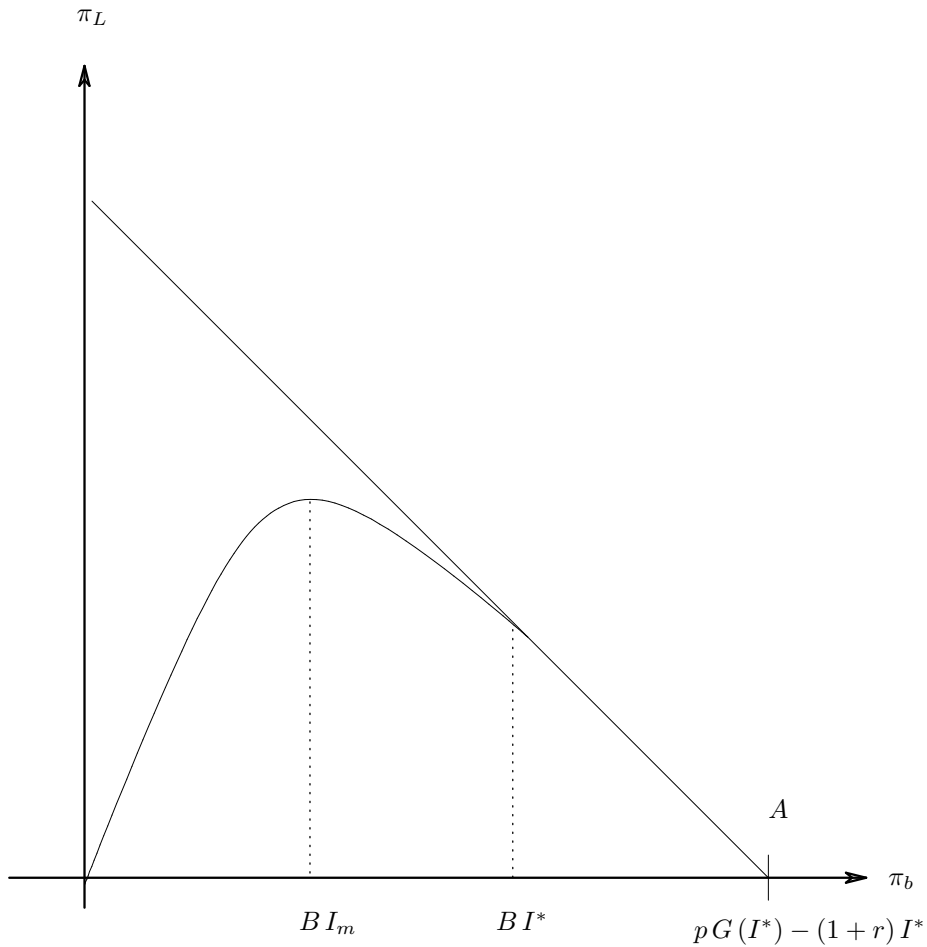Figure 3: The first and second-best Pareto frontiers for $B < B_z$

One should notice that the *constrained utility possibility set* and the second-best Pareto frontier are parameterized to the given incentive structure B. Recall that we defined B_z as the level of the incentive parameter such that:

$$pG\left(I^*\right) - (1+r)I^* = B_z I^*$$

implying that $pR = (1+r)I^*$, i.e. lenders make zero profits. Hence,

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$$\forall\, B < B_z \quad pG\left(I^*\right) - (1+r)I^* < BI^*$$

That is, constraint (11) is slack and the first-best is feasible in the second-best problem. In particular, the point $(\pi_b^{**}, \pi_L^{**}) = (pG\left(I^*\right) - (1+r)I^*, 0)$ belongs to the second-best Pareto frontier (point A in Figure 3). Hence given $B < B_z$, there is room to reduce π_b^{**} without making the constraint (11) binding. There will therefore be an interval of entrepreneur's utilities, i.e. $\pi_b^{**} \in [BI^*, pG\left(I^*\right) - I^*(1+r)]$, such that the second-best Pareto frontier $\pi_L^{**}\left(\pi_b^{**}, B\right)$ coincides with the first-best one $\pi_L^*(\pi_b^*)$ (Figure 3). By further reducing the entrepreneur's payoff we get to $\pi_b^{**} = BI^*$ and $\pi_L^{**} = pG\left(I^*\right) - I^*(1+r) - BI^*$. Every further reduction in π_b^{**} will imply a decrease in the investment level.

If we consider the case $B > B_z$, equation (11) will always be binding at the optimum level of investment, hence it is not possible to sustain the first-best investment level I^*. As a consequence, for every $B > B_z$ the second-best frontier $\pi_L^{**}(\pi_b^{**}, B)$ will always lie below the first-best one, as it is depicted in Figure 4.
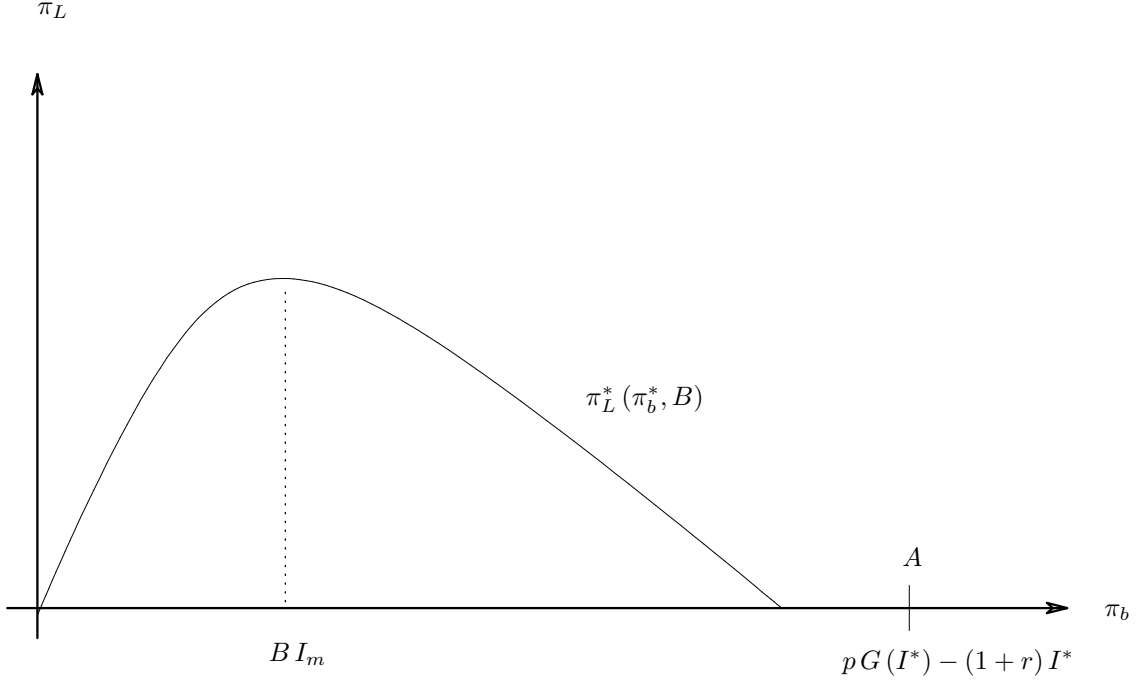
π_L

Figure 4: The second-best Pareto frontier for $B > B_z$

For the cases of relatively mild incentive problem the second-best frontier has a linear part, that corresponds to the implementation of the first-best level of investment; whenever the moral hazard becomes harsher, then the frontier contracts inwards.

No matter the value of B, the highest possible payoff for the lending sector corresponds to the monopolistic allocation, when the entrepreneur is squeezed to a payoff of $\pi_b^{**} = BI_m$ and the lenders appropriate all the rest.[14] Whenever $\pi_b^{**} < BI_m$ every reduction in π_b^{**} calls for a reduction in π_L^{**}. In the limit the only way to set $\pi_b^{**} = 0$ is to fix an investment

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[14]Recall that every monopolistic investment depends on the value of the incentive parameter, hence it should be written $I_m(B)$.

level equal to zero, so that there will not be anything left for the lenders, either. Finally, we argue that the concavity of $G(I)$ induces a concavity in the second-best Pareto frontier (Figure 3 and Figure 4).

Lemma 1 *Take any $B \in [0,1]$ then for every $\pi_b^{**} \leq BI^*$ the frontier $\pi_L^{**}(\pi_b^{**}, B)$ is a concave curve. In particular, $\pi_L^{**}(\pi_b^{**}, B)$ has a maximum in $\pi_b^{**} = BI_m$. For every $\pi_b^{**} < BI_m$, $\pi_L^{**}(\pi_b^{**}, B)$ is monotonically increasing.*

Proof. If (11) is not binding, we are back to the linear part of the frontier, which is trivially concave. The interesting case is that of a binding incentive compatibility constraint (11). Given π_b^{**} and B, then I is uniquely determined and given by $I = \frac{\pi_b^{**}}{B}$. As a consequence, we get:

$$\pi_L^{**}(\pi_b^{**}, B) = pG\left(\frac{\pi_b^{**}}{B}\right) - \frac{\pi_b^{**}}{B}(1+r) - \pi_b^{**}$$

that is a strictly concave function of π_b^{**}. In particular, for $B > B_z$ the second-best Pareto frontier is strictly concave. ∎

Defining the constrained Pareto frontier of the economy gives us more intuitions about the welfare implications of competition over loan contracts. The existence of positive profits equilibria and some form of rationing in credit markets, where an arbitrarily large number of homogeneous lenders is competing, turn out to be the by-product of the competitive process itself under asymmetric information. In such circumstances, a planner facing the same informational constraints as the lenders, cannot implement Pareto-dominant allocations with respect to the equilibrium outcomes of the strategic interactions between N lenders and a single borrower. A formal proof will be provided in the Appendix. Here we remark that the equilibria characterized in Proposition 1 involve either a competitive outcome (see (i) and (ii)) where lenders earn zero-profit or the existence of some latent contracts (iii). It is quite intuitive that the competitive equilibria (i) and (ii) will lie on the linear part of the frontier (they correspond to point A in Figure 1). The equilibria with positive profits and latent contracts (iii) fall in the region $B < B_z$ where it is always possible to sustain the first-best level of investment I^* together with $\pi_b^{**} > BI^*$. Hence, the latent contracts are just a device for a different sharing of the surplus. The equilibrium level of investment would be the same that a social planner would choose when solving (10)-(11) with a slack incentive compatibility constraint. This equilibrium allocation would correspond to a point on the linear part of the second-best Pareto frontier $\pi_L^{**}(\pi_b^{**}, B)$ where it coincides with the first-best one.

The equilibria described in Proposition 2 satisfy the same property. Their crucial feature turns out to be that they satisfy the (IC) constraint as an equality. As a consequence, to every payoff earned by the entrepreneur/borrower will correspond the same level of credit issuance, both at equilibrium and at the optimum. That is, the payoff earned by the lending sector will also be the same. This sustains the (constrained) optimality result.[15]

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[15]See Proposition ... in the Appendix.

we state the following:

Proposition 3 *Take a $B > B_z$ and consider the positive profits equilibrium defined in part (iv) of Proposition 2. Then, if we denote as $\tilde{\pi}_b$ and $\tilde{\pi}_L$ the payoffs earned by the single borrower and by all the lenders, respectively, we have that the pair $(\tilde{\pi}_b, \tilde{\pi}_L)$ belongs to the constrained Pareto frontier $\pi_L^{**}(\pi_b^{**}, B)$.*

Proof. We first introduce a useful definition. Assume that the borrower earns $\tilde{\pi}_b$ in the positive-profits equilibrium, we denote $\tilde{\pi}_L(\tilde{\pi}_b)$ the lenders' payoff induced by $\tilde{\pi}_b$ at equilibrium.

Let us now take $\pi_b^{**} = \tilde{\pi}_b$ and construct the equilibrium relationship $\tilde{\pi}_L(\tilde{\pi}_b)$. In the positive-profits equilibrium defined by (3)-(5) each lender offers the same contract $(\tilde{I}, \tilde{R})$ and in the aggregate the borrower buys all contracts and exerts high effort. The borrower is indifferent between accepting N or $(N-1)$ contracts. Let us call I^A the amount of credit issued and pR^A the expected revenues of the lenders. Given that the Incentive Compatibility constraint is binding in this equilibrium, we then have $\pi_b^{**} = \tilde{\pi}_b = BN\tilde{I}$, that implies:

$$I^A = \frac{\tilde{\pi}_b}{B} = \frac{\pi_b^{**}}{B} \tag{12}$$

where we denoted $I^A = N\tilde{I}$.

Given the borrower payoff $\tilde{\pi}_b$ and the incentive parameter B, the aggregate investment level I^A that supports $\tilde{\pi}_b$ at equilibrium is uniquely determined. In particular, the Incentive Compatibility constraint of the equilibrium defines the same level of aggregate investment of the second-best problem. This investment level I^A determines the aggregate surplus of the economy as:

$$S^A = pG(I^A) - (1+r)I^A \tag{13}$$

and the lenders' payoff once deduced the borrower's utility π_b^{**}:

$$\pi_L^{**}(\pi_b^{**}) = S^A - \pi_b^{**} = pG(I^A) - (1+r)I^A - BI^A \tag{14}$$

Notice that the payoff the credit sector earns is strictly positive:

$$\pi_L^{**}(\pi_b^{**}) = pR^A - (1+r)I^A > 0 \tag{15}$$

In particular, the system of equations (12)-(14) identifies a pair (π_L^{**}, π_b^{**}) belonging to the frontier of the *constrained utility possibility set* $\mathcal{F}'(\pi_b, \pi_L)$. ∎

In this simple framework, the main result can be summarized as follows: the common agency interactions in the market for loans deliver constrained Pareto efficient equilibria, despite the externalities due to strategic competition over financial contracts. The next section investigates in greater detail the sources of this finding.

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5 Discussion

We have proposed a simple characterization of the constrained Pareto frontier. At the optimum, either the first best outcome is implemented, or the Incentive Compatibility constraint binds as an equality.

The equilibrium outcomes described in Proposition 1 are such that the investment level I^* is achieved, whereas those characterized in Proposition 2 imply a binding Incentive constraint. This is the key to understand the efficiency result: whenever the IC constraint binds, there is a one-to-one relationship between the entrepreneur payoff and the amount of credit issued at equilibrium (and at the optimum). As a consequence, the overall payoff for lenders is determined as well. In particular, the no-side-contracting condition (5) only serves as a redistributive rule within the lending sector.

In our context, whenever the IC constraint is not binding, there is always room for an additional lender to offer a profitable contract that will be accepted without inducing the borrower to select the low effort.

This intuition provides useful insights to further discuss the literature on competitive markets under asymmetric information. In particular, we can relate our findings to the studies on competition in insurance and financial markets: Arnott and Stiglitz (1993), Hellwig (1983), Bizer and DeMarzo (1992), Kahn and Mookherjee (1998), Bisin and Guaitoli (2004), and many others. These researches emphasized how asymmetric information in non-exclusive markets may give rise to the the so-called "non-price equilibria", that typically fail to be (constrained) Pareto efficient. If one modifies the basic description of the economy so to get a binding Incentive Compatibility constraint at equilibrium, then efficiency can be reestablished.

Take as an example the recent work of Bisin and Guaitoli (2004), where the inefficient equilibria are those supported by latent policies. Introducing in this framework a risk-neutral agent reduces the (endogenous) reservation utility of the low effort at equilibrium and leads to a binding IC constraint. It is then straightforward to verify the efficiency property of every equilibrium outcome.[16] Even though risk aversion is a natural assumption in insurance models (as the one considered in Bisin and Guaitoli (2004)), most of the banking literature typically considers risk-neutral entrepreneurs/borrowers.

Our investigation, then, contributes to improve the understanding of the welfare properties of strategic competition among intermediaries in the presence of a single agent. Intuition seems to suggest that whenever common agency games with complete information are considered, then efficient outcomes can be supported at equilibrium as long as the Incentive constraint binds.[17] **INCLUDE SOME COMMENTS ON ELOISA'S WORK HERE ABOUT THE RELATIONSHIP BETWEEN A BINDING IC**

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[16]With reference to Bisin and Guaitoli (2004), if a risk-neutral agent is considered, then their equation (7) coincides with the IC constraint. In addition, examining equations $(8) - (12)$ that define the planner's problem, one can check that the IC binds also at the optimum and that the second best can be decentralized.

[17]Importantly, we have not restricted in any way participation decisions. That is, our results apply to the so-called "delegated" scenario.

AND THE INTRODUCTION OF A FURTHER EXTERNALITY DUE TO COURNOT COMPETITION IN MONITORING...

Finally, it is important to remark that our analysis is focused on the simple scenario where lenders' strategies are restricted to be take-it or leave-it offers. Investigating the welfare implications of competition through more complex mechanisms is still a very open issue. This is a relevant issue for policy issues and will be a major topic in future researches.

6   Appendix

To show that the results are robust to more general preference structures for the entrepreneur, in the proofs we will consider the situation in which the entrepreneur's private benefit is represented by the non-linear function $B(I)$, which properties have been defined in Section 2. The specification BI makes the discussion clearer in the paper and turns out to be unrestrictive.

We hence consider a total private benefit function $B(I)$ that is continuous, increasing and convex in I:

$$B(0) = 0, \; B'(I) > 0, \; B''(I) \geq 0,$$

and such that the Inada conditions hold:

$$\lim_{I \to 0} B'(I) = 0, \; \text{and} \; \lim_{I \to \infty} B'(I) = +\infty.$$

Everything else remaining unchanged, we now characterize the equilibrium allocations in terms of the incentive function $B(.)$. More precisely, we introduce the sets $\mathcal{B}_z$, $\mathcal{B}_c$ and $\mathcal{B}_l$ corresponding to the parameters B_z, B_c and B_l considered in the text.

Definition 2 *We take $\mathcal{B}_z$ to be the set of functions $B(.)$ such that:*

$$B(I^*) \leq pG(I^*) - (1+r)I^*; \tag{A.1}$$

$\mathcal{B}_c$ is the set of functions $B(.)$ satisfying:

$$B(2I^*) \leq pG(I^*) - I^*(1+r); \tag{A.2}$$

finally, we refer to the set $\mathcal{B}_l$ as to the set of $B(.)$ such that:

$$B(I^* + I_m) \leq pG(I_m) - (1+r)I_m. \tag{A.3}$$

Under our assumptions on $B(.)$, it holds that if $B(.) \in \mathcal{B}_z$ then $B(.) \notin \mathcal{B}_c$.

We now restate the propositions included in the text as relative to the sets of functions $B(.)$.

Proposition 4 *Whenever $B \notin \mathcal{B}_z$, then the outcome (R^*, I^*) can be supported as a (pure strategy) equilibrium of the game Γ. In particular,*

  i) *Take any $B(.) \in \mathcal{B}_c$, then (R^*, I^*) is an equilibrium outcome for any given N;*
 ii) *If $B(.) \in \mathcal{B}_z$, and $B(.) \notin \mathcal{B}_c$ then there exists a critical number of lenders $\bar{N}_B$ s.t. for all $N > \bar{N}_B$ the aggregate allocation (R^*, I^*) is an equilibrium outcome;*
iii) *For every $B(.) \in \mathcal{B}_l$, there exists an equilibrium where only one contract (say, $\mathcal{C}_i$) is bought. The contract guarantees positive profits to lender i. Furthermore, there is a second lender (say, lender j) who offers a zero-profit contract that is not accepted. All other lenders are not active.*

Proof of part (i). We consider the following array of offered contracts:

$$\{(R_i, I_i) = (R_j, I_j) = (R^*, I^*) \ \text{ for } i \neq j; \ (R_k, I_k) = (0,0) \ \forall \ k \neq i,j\}.$$

That is, there are two lenders, say lender i and lender j who offer the first-best allocation, while all other lenders are offering the null contract $(0,0)$. The borrower is indifferent between accepting the $i-th$ and the $j-th$ contract; given that $B(.) \in \mathcal{B}_c$, accepting all contracts and choosing low action is never a best reply.

In such a scenario, no lender has a profitable deviation given that the first-best outcome is implemented and the borrower's profit is maximized. ∎

Proof of part (ii).

Consider the case of $B(.) \in \mathcal{B}_z$ and $B(.) \notin \mathcal{B}_c$. If every lender offers the contract $(R', I') = \left(\frac{R^*}{N-1}, \frac{I^*}{N-1}\right)$, it is incentive compatible for the borrower to accept $(N-1)$ out of N contracts, provided that the high effort is chosen.

By doing so, the first-best aggregate level of investment would be implemented, the borrower would achieve her maximum expected payoff with each single lender getting zero profits. Let us evaluate if there exist profitable deviations for lenders.

Given what his opponents offer, lender i can never propose a loan that the borrower will accept and guarantees herself positive profits. When all $j \neq i$ lenders offer (R', I'), whatever lender i proposes, the borrower can always buy the remaining $(N-1)$ contracts and achieve her maximum payoff. Hence, it is a best response for lender i to offer (R', I') when all other lenders offer (R', I').

Finally, we have to show that the borrower has no incentive to shirk:

$$pG\left(\frac{N-1}{N-1}I^*\right) - (N-1)\frac{I^*(1+r)}{N-1} \geq B\left(I^*\frac{N}{N-1}\right); \tag{A.4}$$

that is, we want that the utility she gets from buying all N contracts and exerting low action be lower than the first-best payoff:

$$pG(I^*) - I^*(1+r) \geq B\left(I^*\frac{N}{N-1}\right). \tag{A.5}$$

As the function $B(.)$ belongs to the set $\mathcal{B}_z$, then it satisfies the condition:

$$pG(I^*) - I^*(1+r) \geq B(I^*). \tag{16}$$

Thus, if N is high enough, condition (A.5) is satisfied. Hence, for every $B(.) \in \mathcal{B}_z$ and $B(.) \notin \mathcal{B}_c$ and N high enough, the borrower has no incentive to deviate from buying $(N-1)$ contracts and choosing high effort. There does not exist any contract for any lender i that gives him positive profits and is accepted by the borrower. Hence, (R', I') for each lender, and the borrower accepting $(N-1)$ contracts and exerting high effort constitute an equilibrium. ∎

Proof of part (iii).

Take any $B \in \mathcal{B}_l$ and consider the function $x(I) = pG(I) - I(1 + r) - B(I + I^*)$; by continuity there exists an investment level I' such that $x(I') = 0$.

The equilibrium is defined by one lender, say lender i, offering the contract $C_i = (I_i, R_i)$ with $I_i = I^*$ and R_i s.t. $pR_i = pG(I^*) - I^*(1 + r) - (pG(I') - I'(1 + r))$, hence making positive profits. A second lender, say lender j offers the zero-profit contract $C_j = (I_j, R_j)$ such that $I_j = I'$ and $pR_j = (1 + r)I'$. All other lenders $k \neq i, j$ are offering the null contract $C_k = (0, 0)$. The borrower is accepting C_i, only.

Given the behavior of the other players, lender i must offer the borrower at least a payoff of $pG(I') - I'(1 + r)$ in order for his contract to be bought. Hence, he has the incentive to set the investment level at I^* so to realize the maximum amount of profits $p R_i$.

Let us now consider lender j: he cannot profitably deviate from the level of investment $I_j = I'$ and be guaranteed that his offer is accepted, without inducing the borrower to shirk to the low action.

Given the existence of the latent contract j, no contract offering positive investment level proposed by any of the inactive lenders will be accepted at equilibrium.

Finally, the borrower is indifferent between accepting either contract i or j in isolation and choosing high effort, and buying both contracts and choosing low action. That is, accepting i only is a best reply. ∎

To complete the characterization of all relevant equilibria, we need to define the set of incentive functions related to the implementation of the monopolistic outcome, i.e. $\mathcal{B}_m$.

Definition 3 *We refer to $\mathcal{B}_m$ as to the set of functions $B(.)$ such that:*

$$B\left(I_m + I''\right) \leq pG\left(I''\right) - (1 + r)I'' \tag{A.6}$$

where $I'' = \underset{I}{argmax} \ pG(I) - (1 + r)I - B(I_m + I)$.

Proposition 5 *If $B \notin \mathcal{B}_z$, then no allocation guaranteeing zero profits to the lenders can be sustained at equilibrium. In addition, iv) If $B(.) \in \mathcal{B}_m$, then there is a critical number of lenders N_B such that for every $N \geq N_B$, there exists a positive profit equilibrium. The equilibrium outcome $\left(N\tilde{R}, N\tilde{I}\right)$ is characterized by the following set of equations:*

$$p\left[G\left(N\tilde{I}\right) - N\tilde{R}\right] = p\left[G\left((N - 1)\tilde{I}\right) - (N - 1)\tilde{R}\right] \tag{A.7}$$

$$p\left[G\left(N\tilde{I}\right) - N\tilde{R}\right] = B\left(N\tilde{I}\right) \tag{A.8}$$

and exhibits the feature that:

$$(N - 1)\tilde{I} > I_m \tag{A.9}$$

v) If $B \notin \mathcal{B}_m$, then the monopoly outcome can be supported at equilibrium for any given N.

Proof.

Assume by contradiction that $B \notin \mathcal{B}_z$ and that there exists a zero-profit equilibrium. At equilibrium, it should be $\sum_i I_i \leq \bar{I}(B)$, otherwise the borrower will strictly prefer to accept all loans and shirk.[18] Now, there must be at least two lenders offering a positive loan amount, otherwise monopoly would be implemented breaking the conjectured zero-profit equilibrium. Let us call these lenders i and j and let $I_i > 0$ and $I_j > 0$ be the respective amount of loan each of them offers.

It must be that: $\bar{I} - I_j \geq I_i > 0$, and since we conjectured a zero-profit equilibrium with two strictly positive contractual offers: $pG(I_j) - (1+r)I_j > B(I_j)$.

Hence, there should exist an $\epsilon > 0$ such that:

$$pG(I_j + \epsilon) - (1+r)(I_j + \epsilon) > B(I_j + \epsilon)$$

and given we are in the increasing part of the surplus function,

$$pG(I_j + \epsilon) - (1+r)(I_j + \epsilon) > pG(I_j) + (1+r)I_j.$$

But then, there exists a $\delta > 0$ such that:

$$pG(I_j + \epsilon) - (1+r)(I_j + \epsilon) - \delta > \max\left\{pG(I_j) + (1+r)I_j, B(I_j + \epsilon)\right\}.$$

That is, there exists a profitable deviation for lender i when offering (ϵ, δ). ∎

Proof of part (iv).

The proof is organized in two steps. First, we show that there is an aggregate contract $\left(N\tilde{R}, N\tilde{I}\right)$ which is a solution of the system (A.7)-(A.8) and satisfies (A.9). In a next step we show that the strategy profile $(R_i, I_i) = \left(\tilde{R}, \tilde{I}\right)$ for every lender $i \in N$ together with the borrower decision of accepting all contracts and choosing the high level effort is a subgame perfect equilibrium.

Considering (A.7) and (A.8) together we get:

$$p\left[G\left((N-1)\tilde{I}\right) - \left(1 - \frac{1}{N}\right)G\left(N\tilde{I}\right)\right] - \frac{B\left(N\tilde{I}\right)}{N} = 0. \tag{A.10}$$

We define $f(I) = p\left[G((N-1)I) - \left(1 - \frac{1}{N}\right)G(NI)\right] - \frac{B(NI)}{N}$.

One should notice that the aggregate investment level belongs to the interval $\left[I_m, \bar{I}(B)\right]$, where $\bar{I}(B)$ is the level of investment such that

$$pG\left(\bar{I}(B)\right) - B\left(\bar{I}(B)\right) - (1+r)\bar{I}(B) = 0$$

and I_m represents the monopolistic level of investment corresponding to the relevant $B(.) \in \mathcal{B}_m$. It is easy to check that $\bar{I}(B) > I_m$. Now, let us denote $\bar{I}(B)^o = \frac{\bar{I}(B)}{N}$. Evaluating the function $f(I)$ at $\bar{I}(B)^o$, we obtain:

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[18]Paralleling the discussion in the text, we take $\bar{I}(B)$ to be the credit level that guarantees the full appropriation of surplus by the borrower, given the incentive structure B.

$$f\left(\bar{I}(B)^o\right) = pG\left(\frac{(N-1)}{N}\bar{I}(B)\right) - pG\left(\bar{I}(B)\right) + \frac{1+r}{N}\bar{I}(B). \tag{A.11}$$

Given that the function $G(.)$ is concave and recalling that $pG'\left(\bar{I}(B)\right) > 1+r$, we have that

$$pG(\bar{I}(B)) - \frac{(1+r)}{N}\bar{I}(B) > p\left[G\left(\frac{(N-1)\bar{I}(B)}{N}\right)\right] \tag{A.12}$$

and $f\left(\bar{I}(B)^o\right) < 0$.

Using a similar argument, and recalling the definition of $\mathcal{B}_m$ we can check that for every $B \in \mathcal{B}_m$ there exist an N_B large enough such that $\forall N \geq N_B$ we get:

$$f\left(\frac{I_m}{N-1}\right) > 0. \tag{A.13}$$

By continuity of $f(.)$, for every $N \geq N_B$ there exists a value $\tilde{I}(B,N)$ such that $f\left(\tilde{I}\right) = 0$. Given $\tilde{I}$, the value of R satisfying (A.7)-(A.8) can hence be defined in a direct way.

Now, we have to show that at equilibrium every lender will offer the contract $\left(\tilde{R}, \tilde{I}\right)$ and that the borrower will always have an incentive to accept all contracts and to select the high action.

Let us start with the borrower's behavior: if each lender is playing $(\tilde{R}, \tilde{I})$, then the borrower's strategy of accepting N contracts and exerting high effort is a best reply. Equations (A.7) and (A.8) guarantee that when $\left(N\tilde{R}, N\tilde{I}\right)$ is offered in the aggregate, then the borrower cannot deviate by accepting $(N-1)$ contracts. No reductions in the number of accepted contracts will be profitable.

Let us consider now the behavior of the lenders. Suppose all $(N-1)$ lenders except lender i offer $\left(\tilde{R}, \tilde{I}\right)$ and consider lender i's best response. Assume lender i offers (R_i, I_i), his best payoff can be measured with respect to the aggregate amount of loans the borrower takes up:

$$\begin{aligned}
\pi_i &= pR_i - (1+r)I_i = \\
&= pG\left(k\tilde{I} + I_i\right) - (1+r)I_i - kp\tilde{R} \\
&\quad - \max\left\{pG\left((N-1)\tilde{I}\right) - p(N-1)\tilde{R}, B\left((N-1)\tilde{I} + I_i\right)\right\},
\end{aligned} \tag{A.14}$$

where π_i is lender i's payoff as a function of (R_i, I_i) and $k = \{0, 1, 2, ..., N-1\}$ is the number of contracts the borrower buys together with the i-th. On the right hand side of equation (A.14) we represented the surplus at the aggregate amount of investment $\left(k\tilde{I} + I_i\right)$ net of the reimbursements of the k lenders offering $\left(\tilde{R}, \tilde{I}\right)$ and of the borrower's utility. The borrower's payoff cannot fall below the minimum between the amount she obtains accepting the $(N-1)$ contracts and exerting high effort, i.e. $pG\left((N-1)\tilde{I}\right) - p(N-1)\tilde{R}$, and the utility from shirking, i.e. $B\left((N-1)\tilde{I} + I_i\right)$.

We remark that using equations (A.7) and (A.8), the repayment of each of the $(N-1)$ lenders is given by:

$$pR = p\left[G\left(N\tilde{I}\right) - G\left((N-1)\tilde{I}\right)\right]. \tag{A.15}$$

Now, let us examine the choice of I_i by the i-th principal.

There can be two cases: either $I_i \leq \tilde{I}$ or $I_i > \tilde{I}$.

In the first case, from the definition of the equilibrium the borrower will have to be guaranteed at least $pG\left((N-1)\tilde{I}\right) - p(N-1)\tilde{R}$ that is greater then $B\left((N-1)\tilde{I} + I_i\right)$ for every $I_i \leq \tilde{I}$.

In addition, given $I_i \leq \tilde{I}$ and the concavity of $G(.)$, the borrower will buy all the $(N-1)$ contracts together with the i-th. When choosing I_i the lender anticipates how his proposal affects the entrepreneur's choice on the number of contracts k she purchases from the opponents. We show that for every $I_i \leq \tilde{I}$, the entrepreneur's payoff is in fact increasing in k up to $k = N - 1$. Take the entrepreneur's payoff in the case she buys $k\tilde{I} + I_i$ and compare it with buying $(N-1)\tilde{I} + I_i$, it's easy to see that: $\quad \forall k = 0, 1, ..., N-2$

$$pG((N-1)\tilde{I} + I_i) - p(N-1)\tilde{R} - pR_i > pG(k\tilde{I} + I_i) - pk\tilde{R} - pR_i \tag{A.16}$$

Hence, the borrower tries to implement an aggregate loan amount of $(N-1)\tilde{I}$ or $N\tilde{I}$. This remains true even in the case when $I_i > \tilde{I}$. If lender i chooses to offer $I_i > \tilde{I}$ and the number of lenders N is sufficiently high, the concavity of $G(.)$ implies that the borrower selects the number of accepted contracts k from the non-deviating lenders according to the following condition:

$$pG'(k\tilde{I} + I_i) \approx \frac{pG(N\tilde{I}) - pG((N-1)\tilde{I})}{\tilde{I}} \approx pG'((N-1)\tilde{I}) \tag{A.17}$$

Any increase in I_i must hence be accompanied by a reduction in k. Conditional on choosing high effort, the previous discussion completely characterizes the borrower's choice of k, i.e. her best reply in terms of the number of accepted contracts.[19]

We remark that whenever $I_i > \tilde{I}$, implementing high effort and choosing the associated k contracts together with I_i is always dominated by accepting all offers and shirking. That is, for every $I_i > \tilde{I}$ we have:

$$B((N-1)\tilde{I} + I_i) > pG(k\tilde{I} + I_i) - pR_i - kp\tilde{R} \tag{A.18}$$

To show that the inequality (A.18) holds for every possible choice of I_i, consider that the i-th lender must earn at least what he could earn offering the contract $(\tilde{I}, \tilde{R})$, i.e. $pR_i - (1+r)I_i \geq p\tilde{R} - (1+r)\tilde{I}$. Let us take the most favorable case for the borrower, that is $pR_i - (1+r)I_i = p\tilde{R} - (1+r)\tilde{I}$. In addition, according to equation (A.17), the choice of k satisfies $k\tilde{I} = (N-1)\tilde{I} - I_i$. We can therefore rewrite the inequality (A.18) as:

$$B\left((N-1)\tilde{I} + I_i\right) > pG((N-1))\tilde{I} - pR_i - \left((N-1) - \frac{I_i}{\tilde{I}}\right)p\tilde{R} \tag{A.19}$$

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[19]Notice that whenever $I_i > \tilde{I}$ the best reply of the borrower is multivalued. She is indifferent between buying the i-th contract together with $k < N - 1$ non-deviating ones, and rejecting the offer I_i and accepting all other offers.

Since the equilibrium conditions guarantee: $B\left(N\tilde{I}\right) = pG((N-1)\tilde{I}) - (N-1)p\tilde{R}$, (A.19) can hence be rewritten as:

$$B\left((N-1)\tilde{I} + I_i\right) - B\left(N\tilde{I}\right) > p\tilde{R}\frac{I_i}{\tilde{I}} - (1+r)I_i \tag{A.20}$$

which becomes:

$$\frac{B\left((N-1)\tilde{I} + I_i\right) - B\left(N\tilde{I}\right)\left(I_i - \tilde{I}\right)}{\left(I_i - \tilde{I}\right)} > \frac{pG(N\tilde{I}) - pG((N-1)\tilde{I})}{\tilde{I}}(I_i - \tilde{I}) - (1+r)((I_i - \tilde{I}))$$

and, considering derivatives, we get:

$$B'\left(N\tilde{I}\right) + (1+r) > pG'(N\tilde{I}). \tag{A.22}$$

Given that $B'(I_m) + (1+r) = pG'(I_m)$ and taking into account the convexity of $B(.)$, the inequality $(A.22)$ holds.

For every $I_i > \tilde{I}$ equation (A.22) is true, since $(N-1)\tilde{I} > I_m$. Therefore, whenever the i-th lender tries to offer a higher amount of loans $I_i > \tilde{I}$ he cannot prevent shirking. This clarifies that none of the lenders can profitably deviate from the offer $(\tilde{I}, \tilde{R})$. ∎

Proof of (v).

We want to prove that, at equilibrium, there is one lender, say lender i, offering $C_i = (I_m, R_m)$ while the remaining $N-1$ ones are offering $(0,0)$. For lender i, C_i is clearly a best response, since he attains the maximum available profit. Let us consider if there exist profitable deviations for any of the non-active $N-1$ lender, say lender j. To have a profitable deviation, lender j should be able to offer some positive amount of loan $I_j > 0$ and having his contract accepted either together with the incumbent's loan I_m or in competition with him.

If I_j is bought together with I_m, then the profit the deviating lender could get is given by:

$$\pi_j = pG(I_m + I_j) - (1+r)I_j - pR_m - B(I_m + I_j), \tag{A.23}$$

where $pR_m = pG(I_m) - B(I_m)$ from the maximization problem of a monopolist. We can rewrite equation (A.23) as:

$$\pi_j = pG(I_m + I_j) - pG(I_m) - (1+r)I_j - [B(I_m + I_j) - B(I_m)]. \tag{A.24}$$

The concavity of $G(.)$ and the convexity of $B(.)$ imply that:

$$\pi_j < pG'(I_m)I_j - (1+r)I_j - B(I_m + I_j) + B(I_m) < pG'(I_m)I_j - (1+r)I_j - B'(I_m)I_j = 0. \tag{A.25}$$

Hence, lender j cannot profitably offer a loan that is taken up together with the i-th and gives him positive profits. In case lender j tries to compete with the incumbent, the best he can earn is given by:

$$\pi_j = pG(I_j) - (1+r)I_j - B(I_m + I_j). \tag{A.26}$$

Now, the best lender j can do is to offer the amount of investment that solves: $pG'(I_j) = (1+r) + B'(I_m + I_j)$. But, given that $B(.) \notin \mathcal{B}_m$, by doing so the entrepreneur will always strictly prefer to buy both contracts and shirk. As a consequence, lender j does not have any profitable deviation from offering $(0,0)$. ∎

We now analyze welfare properties of equilibria. The features of constrained Pareto optimal allocations derived in Section 2 naturally extend to a scenario where private benefits are a non-linear function of the borrowed amount I. First, it is possible to show that the convexity of the $B(I)$ function preserves the concavity of the Pareto frontier:

Lemma 2 *For every $\pi_b^{**} \leq B(I^*)$ the second-best Pareto frontier $\pi_L^{**}(\pi_b^{**}, B)$ is a concave curve. In particular, $\pi_L^{**}(\pi_b^{**}, B)$ has a maximum in $\pi_b^{**} = B(I_m)$. For every $\pi_b^{**} < BI_m$, $\pi_L^{**}(\pi_b^{**}, B)$ is monotonically increasing.*

Proof.

If $\pi_b^{**} \leq B(I^*)$ is not binding, we are back to the linear part of the frontier, which is trivially concave. The interesting case is that of a binding incentive compatibility constraint, i.e. $\pi_b^{**} = B(I^*)$. Given π_b^{**} and B, then I is uniquely determined and given by $I = B^{-1}(\pi_b^{**})$. As a consequence, we get:

$$\pi_L^{**}(\pi_b^{**}) = pG(B^{-1}(\pi_b^{**})) - (1+r)B^{-1}(\pi_b^{**}) - \pi_b^{**} \tag{A.27}$$

that is a strictly concave function of π_b^{**}:

$$\frac{\partial \pi_L^{**}}{\partial \pi_b^{**}} = pG'(B^{-1}(\pi_b^{**})) B^{-1\,\prime}(\pi_b^{**}) - (1+r)B^{-1\,\prime}(\pi_b^{**}) - 1 \tag{A.28}$$

$$\frac{\partial^2 \pi_L^{**}}{\partial (\pi_b^{**})^2} = pG''(B^{-1}(\pi_b^{**})) (B^{-1\,\prime}(\pi_b^{**}))^2 + pG'(B^{-1}(\pi_b^{**})) B^{-1\,\prime\prime}(\pi_b^{**}) - (1+r)B^{-1\,\prime\prime}(\pi_b^{**}) \tag{A.29}$$

Since $B'' \geq 0$, we have $B^{-1\,\prime\prime} \leq 0$ and we can conclude that $\dfrac{\partial^2 \pi_L^{**}}{\partial (\pi_b^{**})^2} \leq 0$. ∎

Then, we argue that the equilibrium outcomes characterized through Proposition 5 and Proposition 6 identify (constrained) efficient allocations. Importantly, all these equilibria are such that the relevant (IC) constraint turns out to be binding. One can in fact show the following:

Proposition 6 *Consider any pure strategy equilibrium $\left[\left(\hat{C}_i\right), (\hat{a}_b^i), p\right]_{i \in N}$ of the game Γ. If the allocation implemented by $\left[\left(\hat{C}_i\right), (\hat{a}_b^i), p\right]_{i \in N}$ induces a binding Incentive Compatibil-*

ity constraint, then the equilibrium outcome is constrained Pareto efficient. In particular, the equilibrium outcomes

Proof.

Let $\hat{\pi}_b$ be the payoff that the borrower enjoys at equilibrium. We take $\hat{\pi}_L(\hat{\pi}_b)$ to be the induced aggregate payoff to lenders. We then consider $\pi_b^{**} = \hat{\pi}_b$ and construct the equilibrium relationship $\hat{\pi}_L(\hat{\pi}_b)$. We also take $\hat{I}$ and $p\hat{R}$ to be the amount of credit issued and the aggregate revenues for lenders, respectively. Given that the Incentive Compatibility constraint is binding at , we then have $\pi_b^{**} = \tilde{\pi}_b = B(N\tilde{I})$, that implies:

$$I^A = B^{-1}(\tilde{\pi}_b) = B^{-1}(\pi_b^{**}) \tag{A.30}$$

where we denoted $I^A = N\tilde{I}$.

Given the borrower payoff $\tilde{\pi}_b$, the aggregate investment level I^A that supports $\tilde{\pi}_b$ at equilibrium is uniquely determined. In particular, the Incentive Compatibility constraint of the equilibrium defines the same level of aggregate investment of the second-best problem. This investment level I^A determines the aggregate surplus of the economy as:

$$S^A = pG(I^A) - (1+r)I^A \tag{A.31}$$

and the lenders' payoff once deduced the borrower's utility π_b^{**}:

$$\pi_L^{**}(\pi_b^{**}) = S^A - \pi_b^{**} = pG(I^A) - (1+r)I^A - B(I^A) \tag{A.33}$$

Notice that the payoff the credit sector earns is strictly positive:

$$\pi_L^{**}(\pi_b^{**}) = pR^A - (1+r)I^A > 0 \tag{A.34}$$

In particular, the system of equations (A.30)-(A.33) identifies a pair (π_L^{**}, π_b^{**}) belonging to the frontier of the constrained utility possibility set $\mathcal{F}'(\pi_b, \pi_L)$. $\blacksquare$

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