A note on Common Agency models of moral hazard

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Abstract

We consider Common Agency games of moral hazard and we suggest that there is only a very weak support for the standard restriction to take-it or leave-it contracts.

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1 Introduction

When Common Agency games are considered, it is well known that restricting principals to the use of simple take-it or leave-it offers implies a loss of generality, even in a complete information scenario. If principals were allowed to propose menus over the relevant alternatives, there could be equilibrium outcomes that could not be supported by simple offers.[1]

In particular, Peters [2003] identifies a set of restrictions on players' preferences, called "no-externalities assumption", guaranteeing that every pure strategy equilibrium allocation of a Common Agency game with menus can in fact be supported at (pure strategy) equilibrium in a simpler game where principals' strategies are restricted to be take-it or leave-it offers. Interestingly, these conditions are satisfied by a large class of games.[2]

In a related work, Attar et al. [2006] argue that these conditions are in fact not sufficient in a moral hazard framework. The present paper proposes an alternative set of conditions to establish the result. These conditions are quite restrictive, since they require the agent's best-reply to be always single-valued. Nonetheless, they cannot be easily weakened as we show with a counter-example.

Many recent researches consider a framework where multiple principals compete in the presence of moral hazard: Bernheim and Whinston [1986], Bisin and Guaitoli [2004], Ishiguro [2005], Martimort

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[1]See, among many others, the two influential works of Peters [2001] and Martimort and Stole [2002]. These papers also show that menus can efficiently replace any sort of *indirect* communication mechanism (this is usually referred to as the *Delegation Principle*).

[2]Peters [2003] provides several examples.

[2004], Parlour and Rajan [2001], Attar et al. [2005]. Most of these papers limit the analysis to take-it or leave-it offers strategies. Our research suggests that this restriction may be problematic.[3]

The note is organized as follows. Section 2 presents the reference model, Section 3 provides our Theorem and the related counter-example.

2 Common Agency under moral hazard

We refer to a scenario where there are a number of principals (indexed by $j \in N = \{1,...,n\}$) contracting with one agent (denoted by index, 0). We assume that the agent takes an effort e from the set E and obtains an allocation $y_j \in Y_j$ from principal j. The effort choice is not contractible.

The payoff to principal $j \in \{1,\dots n\}$ is represented by the von Neumann-Morgenstern utility function

$$V_j : \prod_{k \in N} Y_k \times E \to \mathbb{R}_+.$$

For the agent the payoff is represented by the function

$$U : \prod_{k \in N} Y_k \times E \to \mathbb{R}_+.$$

Principals compete through menus. A menu for principal j, say M_j, is a subset of the set of feasible offers Y_j.

We denote by Σ_j the set of all menus available to principal j, and by Σ the collection of all the Σ_j sets. The agent chooses an item y_j from each menu and an effort e. For every collection of menus $(M_1, M_2, ..., M_N)$, the strategy for the agent is hence a map $s_0 : M \to M \times E$, with $M = \times_{i \in N} M_i$, and S_0 is the corresponding agent's strategy set. The Common Agency game with moral hazard we refer to is given by:

$$\Gamma = \left\{\Sigma, S_0, u(.,e), (v_j)_{j \in N}\right\}.$$

We assume that all the relevant spaces satisfy standard regularity assumptions (for a general discussion, the reader can refer to Peters [2001] and Peters [2003]). We focus our attention on the *pure strategy* Subgame Perfect Equilibria (SPE) of the game Γ.

We now consider the corresponding game where principal strategies are restricted to be take-it or leave-it offers: the menu offered by every principal is a singleton. A strategy for principal j in this simpler game is the offer $\tilde{y}_j \in Y_j$; we let $\tilde{\Sigma}_j$ be the strategy space for principal j and $\tilde{\Sigma} = \times_{j \in N} \tilde{\Sigma}_j$.
The strategy of the agent is then a map $\tilde{s}_0 : \tilde{\Sigma} \to E$, and $\tilde{S}_0$ denotes the collection of all such maps.
The Common Agency game induced by take-it or leave-it offers is then:

$$\tilde{\Gamma} = \left\{\tilde{\Sigma}, \tilde{S}_0, U(.), (V_j(.))_{j \in N}\right\}.$$

Peters [2003] identifies a set of sufficient conditions on players preferences guaranteeing that every equilibrium outcome of the menu game Γ can be supported as an equilibrium in the game $\tilde{\Gamma}$. In the pure

[3]The same remark could also be raised in a larger class of complete information Common Agency models, if one interprets the participation decision as a non contractible action.

moral hazard framework that we introduced here, these conditions (referred to as the "no-externalities assumption") can be stated in the following way:

For every principal $j \in N$:

A. There exists a function $W_j : Y_j \times E \to \mathbb{R}_+$ such that for all $(y_1, y_2, ..., y_n) \in Y$, and for all $e \in E$

$$V_j(y_1, y_2, ..., y_n, e) = W_j(y_j, e)$$

B. For any closed subset $B \subset Y_j$ there is a $y_j \in B$ such that

$$U(y_j, y_{-j}, e) \geq U(y'_j, y_{-j}, e)$$

for all $y'_j \in B$, $y_{-j} \in Y_{-j}$, and $e \in E$.

That is: each principal's utility only depends on his own action and on the agent's effort; in addition, the agent has a weak preference ordering over the actions of every single principal that is independent of her effort choice and of the other principal's offers.

Unfortunately, these conditions turn out not to be sufficient if moral hazard is explicitly considered: Attar et al. [2006] show that whenever the single agent is indifferent over alternative outcomes, there can be equilibrium allocations of the menu game that cannot be sustained through simple offers even if the "no-externalities assumption" holds. If the single agent is indifferent among alternative outcomes, then her *strict* preference order over each principal's offers may depend on other principals' proposals, even though the *weak* preference order does not. Importantly, such a preference reversal can be exploited with menus in a way that is not available through direct mechanisms.

We propose here an alternative approach to establish the result, eliminating any source of indifference on the agent's side. By doing so, we are able to generalize the no-externalities assumption in one dimension, as it is no longer necessary to assume that there are no direct externalities among principals (part *A* of Peters' original condition).

3 A no-externality Theorem with moral hazard

We first of all restrict the analysis to a specific class of games, that we denote $\mathcal{G}$.

Definition 1 *A Common Agency game Γ belongs to the class $\mathcal{G}$ if, for every $j \in N$, for every $y_{-j} \in Y_{-j}$,*

$$U(y_j, y_{-j}, e) \neq U(y'_j, y_{-j}, e'), \quad \forall y_j \in Y, y'_j \in Y, e \in E, e' \in E, \text{ with } (y_i, e) \neq (y'_i, e')$$

Hence, everything else equal, the agent is never indifferent to a change in the offer made by any of the principals.

This condition is generically satisfied in the payoff space of the agent whenever finite games are considered.[4] Now, it turns out that for every Common Agency game belonging to $\mathcal{G}$, the original no-externality condition (B) is in fact sufficient to provide a proof:

[4]Importantly, when the relevant decision sets are not finite (as it is the case in many applications), indifference is a common feature of economic models (for more precise definitions of genericity in games, see Anderson and Zame [2001]).

Theorem 1 *Consider any game Γ in the class $\mathcal{G}$. If the preferences of the agent satisfy condition (B), then for every outcome that can be supported as a pure strategy SPE of the game Γ, there is a pure strategy equilibrium of the direct mechanism game $\tilde{\Gamma}$ that implements the same outcome.*

Proof. Take the game Γ. For every given array of menus (M_j, M_{-j}) we let the best reply of the agent be:

$$[y^*, e^*] = \underset{(y \in M, e \in E)}{\arg\max} \; U(y, e). \tag{1}$$

We now consider a *SPE* of the game Γ, that we denote $\left(\left(M_j^* \right)_{j \in N}, s_0^* \right)$, and we let (y^*, e^*) be its corresponding equilibrium outcome.

The candidate for equilibrium in the game $\tilde{\Gamma}$ will therefore be the array $\left(\left(\tilde{y}_j^* \right)_{j \in N}, \tilde{s}_0^* \right)$, where for every j:

$$\tilde{y}_j^* = y_j^* \tag{2}$$

and $\forall \, (\tilde{y}_j, \tilde{y}_{-j}) \in \tilde{\Sigma}$:

$$\tilde{s}_0^* = \underset{e \in E}{\arg\max} \; u(\tilde{y}_j, \tilde{y}_{-j}, e). \tag{3}$$

One should notice that this strategy profile induces the same outcome as that corresponding to $\left(\left(M_j^* \right)_{j \in N}, s_0^* \right)$ in the game Γ.

We then have to show that, if we consider the game $\tilde{\Gamma}$, none of the players has a profitable deviation. First, we look at the single agent: if principals are offering $\left(\tilde{y}_j^*, \tilde{y}_{-j}^* \right)$, then $\tilde{s}_0^* = e^*$ is by construction her best reply.

Consider now the behavior of principals in the game $\tilde{\Gamma}$: let assume that principal j deviates to $\tilde{y}_j'$. For the deviation to be profitable it should be:

$$V_j \left(\tilde{y}_j', \tilde{y}_{-j}^*, \tilde{e}' \right) > V_j \left(y_j^*, y_{-j}^*, e^* \right)$$

where $\tilde{e}'$ is such that:

$$\tilde{e}' = \underset{(e \in E)}{\arg\max} \; U\left(\tilde{y}_j', y_j^*, e \right).$$

Importantly, the deviation was already available in the original game Γ. Furthermore, the best reply in terms of effort to the deviation $\tilde{y}_j'$ is going to be the same in the game Γ and in $\tilde{\Gamma}$. If we denote

$$\left(y_{-j}^{'}, e^{'} \right) = \underset{(y_{-j} \in M_{-j}, e \in E)}{\arg\max} \; u\left(\tilde{y}_j', y_{-j}, e \right)$$

the best reply of the agent that, in the game Γ, is induced by the deviation $\tilde{y}_j'$, then it must be $\tilde{e}' = e^{'}$.

This last result can be shown by contradiction. Let us consider the game Γ and suppose that $\tilde{e}' \neq e^{'}$. Now, given separability, the agent will not modify her choices y_{-j}^* in the menus M_{-j} offered by the non-deviating principals. If $\tilde{e}' \neq e^{'}$, then one should have:

$$u\left(\tilde{y}_j', y_{-j}^*, \tilde{e}' \right) \neq u\left(\tilde{y}_j', y_{-j}^*, e^{'} \right)$$

that cannot be true, otherwise either $\tilde{e}'$ or e' is not a maximum. Hence, letting $\tilde{y}'_j$ be a profitable deviation in the game $\tilde{\Gamma}$ contradicts the assumption of (y^*, e^*) being a (pure strategy) equilibrium outcome of the menu game Γ. ■

Importantly, no restriction on principals' preferences has been explicitly introduced. That is, once we restrict the analysis to Common Agency games in the class $\mathcal{G}$, condition (A) is not needed to prove the theorem.

The conditions we have introduced are indeed quite strong and they are not satisfied in any of the existing works of Common Agency under moral hazard. [5] Trying to weaken them is anyway not a trivial exercise. In particular, we introduce a counter-example showing that taking the agent's effort choice to be single-valued is not enough to get the result. In other words, the restriction to the class $\mathcal{G}$ turns out to be critical.

We provide in what follows an example of a Common Agency game with moral hazard where no-externality conditions (A) and (B) are satisfied and the agent optimal effort choice is always single-valued. We show that even in this case our Theorem 1 cannot be established.

That is, we consider games in the class Γ':

Definition 2 *A Common Agency game Γ belongs to the class Γ' if, for every $j \in N$,*

$$\forall (y_j, y_{-j}) \in Y, \forall e \in E, \forall e' \in E, \quad e \neq e' \Longrightarrow U(y_j, y_{-j}, e) \neq U(y_j, y_{-j}, e').$$

Common agency games in the class Γ' exhibit the property that the agent's effort choice is unique for every array of principals' offers. Now, consider the following example, where two principals (P1 and P2) contract with one agent in a pure moral hazard scenario. P1 can take three decisions *a*, *b* or *c* and P2 can choose between *A*, *B* and *C*. The agent can select her non-contractible level of effort in the set $\{e', e''\}$.

If the agent chooses effort e', the corresponding payoffs are given by the matrix:

	A	B	C
a	$(1,1,0)$	$(1,2,0)$	$(1,0,0)$
b	$(4,1,1)$	$(4,2,5)$	$(4,0,5)$
c	$(3,1,5)$	$(3,2,5)$	$(3,0,5)$

where the first element in each cell refers to the payoff of P1, the second one to the payoff of P2 and the last one to that of the agent. Payoffs associated to the choice of e'' are:

	A	B	C
a	$(0,3,4)$	$(0,1,4)$	$(0,0,4)$
b	$(1,3,4)$	$(1,1,4)$	$(1,0,4)$
c	$(2,3,4)$	$(2,1,4)$	$(2,0,4)$

The payoffs corresponding to the optimal effort choice of the agent are summarized in the matrix:

[5] We essentially require that the optimal effort selected by the agent should be independent of the allocations proposed by principals. In addition, no indifference is allowed. This implies that the effort choice cannot alter in any relevant way the payoffs' distribution. We should also remark that M. Peters has recently proposed the introduction of a further restriction to the original no-externalities assumption (see Peters [2005]). He argues that taking the *set* of optimal choices of the agent from the menu of each principal to be independent from her effort choice and from the other principals' offers is a sufficient condition to reestablish the result. Unfortunately, even this last condition can be hardly applied to standard moral hazard environments.

	A	B	C
a	$(0,3,4)$	$(0,1,4)$	$(0,0,4)$
b	$(1,3,4)$	$(4,2,5)$	$(4,0,5)$
c	$(3,1,5)$	$(3,2,5)$	$(3,0,5)$

This game belongs to the class Γ', and (A) and (B) are satisfied. However, the outcome $(4,2,5)$ cannot be sustained as an equilibrium if principals are restricted to the use of take-it or leave-it offers. If Principal 1 plays b, then Principal 2 will always prefer to play A rather than B.

The outcome $(4,2,5)$ can indeed be supported at equilibrium in a game where principals are allowed to use menus. In particular, the menus $M_1^* = \{b,c\}$ for $P1$ and $M_2^* = \{B\}$ for $P2$ lead to a pure strategy equilibrium where (4,2,5) is implemented. If $P1$ offers the menu $\{b,c\}$ and $P2$ offers the degenerate menu $\{B\}$, then the agent will pick the element $\{b\}$ from the first menu and exert the effort e'. It is straightforward to show that there are no other menus giving P1 a higher payoff. It is also clear that P2 has no incentive to play the degenerate menus $\{A\}$ and $\{C\}$: as a consequence we cannot find any profitable deviation for P2 either.

Hence, the standard approach to model strategic competition in the presence of a single agent who is taking a non-contractible effort appears to be significatively limited. If principals are allowed to offer menus in a standard moral hazard environment, then one can typically generate new equilibrium outcomes. [6]

References

Robert M. Anderson and William R. Zame. Genericity with infinitely many parameters. *Advances in Theoretical Economics*, 1(1):1–62, 2001. URL `http://www.bepress.com/bejte/advances/vol1/iss1/art1`.

Andrea Attar, Eloisa Campioni, and Gwenaël Piaser. Multiple lending and constrained efficiency in the credit market. CORE DP 2005-31, 2005.

Andrea Attar, Gwenaël Piaser, and Nicolás Porteiro. Negotiation and take-it or leave-it in common agency with non-contractible actions. *Journal of Economic Theory*, forthcoming, 2006.

B. Douglas Bernheim and Michael D. Whinston. Common agency. *Econometrica*, 54(4):923–942, 1986.

Alberto Bisin and Danilo Guaitoli. Moral hazard with non-exclusive contracts. *Rand Journal of Economics*, 2:306–328, 2004.

Shingo Ishiguro. Competition and moral hazard. Mimeo, Osaka University, 2005.

David Martimort. Delegated common agency under moral hazard and the formation of interest group. mimeo, IDEI, 2004.

David Martimort and Lars A. Stole. The revelation and delegation principles in common agency games. *Econometrica*, 70(4):1659–1673, July 2002.

Christine A. Parlour and Uday Rajan. Competition in loan contracts. *American Economic Review*, 91(5):1311–1328, December 2001.

[6] One should notice that there is always a (weak) rationale for the use of take-it or leave-it offers. Theorem 1 in Peters [2003] shows that every equilibrium outcome of this simple game survives when principals are allowed to use menus.

Michael Peters. Common agency and the revelation principle. *Econometrica*, 69(5):1349–1372, September 2001.

Michael Peters. Negotiation and take-it-or-leave-it in common agency. *Journal of Economic Theory*, 111 (1):88–109, July 2003.

Michael Peters. Errata - negotiation and take-it or leave-it in common agency. mimeo University of British Columbia, 2005.