

# Hedging and Vertical Integration in Electricity Markets <sup>\*</sup>

RENÉ AÏD<sup>†</sup>   GILLES CHEMLA<sup>‡</sup>   ARNAUD PORCHET<sup>§</sup>   NIZAR TOUZI<sup>¶</sup>

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## Abstract

This paper analyzes the interactions between vertical integration and (wholesale) spot, forward and retail markets in risk management. We develop an equilibrium model that fits electricity markets well. We point out that vertical integration and forward hedging are two separate levers for demand and spot price risk diversification. We show that they are imperfect substitutes as to their impact on retail prices and agents' utility because the asymmetry between upstream and downstream segments. While agents always use the forward market, vertical integration may not arise. In addition, in presence of highly risk averse downstream agents, vertical integration may be a better way to diversify risk than spot, forward and retail markets. We illustrate our analysis with data from the French electricity market.

## 1 Introduction

Corporate risk management has long been viewed as a prominent motive for vertical integration <sup>1</sup>. In particular, the (supply or demand) insurance rationale for vertical integration

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<sup>1</sup>The abundant literature that has developed in the past few decades tends to see vertical integration as a response to problems caused by contractual incompleteness (Williamson (1971), Grossman and Hart (1986))), for example as ways to acquire valuable private information about the production process (Arrow

is the one that managers cite as the most important (Bolton and Whinston, 1993). In the 1970s, for example, oil refining industries produced up to 90% of their crude oil needs in order to avoid the significant costs that a stop in deliveries would entail (Teece, 1976). More generally, uncertainty in demand (Carlton (1979)), lack of market flexibility and risk aversion may all provide rationale for vertical integration (Hendrikse and Peters (1989), Perry (1989), Emons (1996), Sekkat (2006)). Of course there are other means of managing risk, such as operational hedging and financial instruments that have developed tremendously in the past few decades (Hull, 2003). In this paper, we examine the extent to which the development of the financial instruments often used for hedging purposes affects the incentive for vertical integration and the way industry structure affects the financial markets aforementioned and their prices. Our objective is to clarify and to quantify the effects of vertical integration and forward markets in risk management and their interactions. These issues, which are of particular importance in commodity and energy markets and which are subject to ongoing debate in regulatory reforms in these markets, are analyzed with a model that fits electricity markets well, and illustrated with French electricity data.

Specifically, we study retail markets, wholesale spot markets and forward markets, and the relationship between equilibrium prices on these markets and vertical integration. To focus on risk, we abstract from considerations related to strategic behavior or market power, and instead we assume price-taking firms that disregard any influence they could have on the equilibrium price or on the other agents' decisions <sup>2</sup>. We develop a two-date equilibrium model of retail, (wholesale) spot and forward markets for a non-storable good. Non-storability prevents firms from benefiting from yet another possibility of managing risk, which in our model is a central feature of both vertical integration and forward markets. At an initial date, downstream firms (or downstream subsidiaries of integrated firms) choose the number of accounts to open, which in a market of fixed size boils down to choosing their retail market shares, and forward positions under demand (and price) uncertainty. After uncertainty is realized, producers produce the good, sell it to retailers on the wholesale spot market, and retailers sell the good to consumers. Since the good

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(1975)), to avoid rationing (Teece (1976), Green (1986), Bolton and Whinston (1993)), to weaken rivals (Bolton and Whinston (1993), Rey and Tirole (2006)) or as a bargaining tool with vertically-related segments (Chemla (2003)). On vertical integration, see also, among many others, Carlton (1979), Perry (1989), Joskow (2005), Lafontaine and Slade (2007) and Sekkat (2006).

<sup>2</sup>We also ignore other forms of vertical restraints or profit sharing rules.

is non-storable, no production can occur at the initial date to serve the market later on. Agents have preferences over random profits defined by a mean-variance utility function, which can be thought of as a reduced form of traditional motives for risk management policy <sup>3</sup>.

We derive the equilibrium prices and exchanged quantities on the three markets in closed forms. We show that vertical integration and forward hedging are two mechanisms that are substitutes in the way they achieve risk diversification. Specifically, the reduction in risk achieved through either means encourages firms to open more client accounts at the initial date, thus leading to greater supply and lower retail prices after generation occurs. Hence, their first two effects are to decrease the retail price and to enable agents with low generation capacity to obtain larger market shares. In addition, they both tend to decrease downstream firms' utility when upstream firms are only partially integrated: Because of industry-wide risk diversification, downstream firms face lower retail prices, which reduces their profit.

We further show that they are imperfect substitute risk management mechanisms because of a significant asymmetry between upstream and downstream segments: Retailers have to open accounts and, hence, choose their market shares under uncertainty, while producers choose production after demand uncertainty is revealed. Therefore, downstream firms are more exposed to demand risk. First, vertical integration eliminates this asymmetry while forward hedging does not. Second, vertical integration is more robust to high risk aversion than forward markets. Third, vertical integration can also increase downstream firms' utility provided that they have sufficiently high risk aversion. Fourth, a non-integrated economy can be a stable equilibrium whereas a situation where no agents trade forward contracts is almost never a stable equilibrium. Finally, we prove that our conclusions are robust to the inelasticity of demand to retail price.

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<sup>3</sup> Mean-variance utility functions are widely used in asset pricing. We do not specify the market frictions that may prompt agents to be eager to hedge risk. In the absence of market frictions such as taxes or bankruptcy costs, firms may be indifferent about their hedging policy. See, for instance, Smith and Stulz (1984), Froot, Scharfstein and Stein (1993) and Grinblatt and Titman (2002). This corporate risk management literature (see also Mello and Ruckes (2007)) also sometimes relies on risk-neutral settings with constraints or frictions in order to work with implied risk aversion, but this makes it more difficult to work with fully-fledged asset price equilibria

Apart from our contribution to the risk management features of vertical integration, our paper is related to Allaz's (1992) and Bessembinder and Lemmon's (2002) equilibrium models of forward markets. We add a retail activity to their settings, and we consider jointly forward hedging and vertical integration as tools to manage risk. We also contribute to the recent literature on risk management through market versus non-market mechanisms. While markets enable investors to diversify easily (Doherty and Schlesinger (2002) and they are less sensitive to moral hazard (Doherty (1997))), non-market mechanisms such as reinsurance companies keep an important role. In Gibson, Habib and Ziegler (2007), the importance of non-market mechanisms stems from excessive information gathering from investors in financial markets. In our setting, the scope for vertical integration arises as complementary to hedging, and vertical integration is desirable because of the asymmetry between downstream and upstream firms and because of investors' high levels of risk aversion.

The paper is organized as follows. We develop our setting and we present the equilibrium problem in Section 2. We then compare two different environments. First, in Section 3, we examine the equilibrium and the effect of vertical integration in the absence of a forward market. In Section 4, we derive the equilibrium in the presence of the forward market and we analyze the effect of the forward market and vertical integration on both the industrial markets and financial markets. We then illustrate our results in Section 5 through the French electricity market. Section 6 discusses further empirical predictions. Section 7. Appendix A contains the proofs and Appendix B shows that our main results remain in presence of an elastic demand curve.

## 2 The model

In this section we first describe price-taking retail, (wholesale) spot and forward markets for a non-storable good. Then, we define an equilibrium on these markets.

## 2.1 The markets

We consider a set  $\mathcal{P}$  of producers that produce a homogenous, non-storable good that they can sell to a set  $\mathcal{R}$  of retailers. After sourcing on the wholesale markets, retailers compete in a market for consumers whose demand  $D$  is random. Demand  $D$  is described by a random variable on a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . For simplicity, we assume throughout most of the paper that demand is inelastic. Section 5 illustrates that this assumption is broadly consistent with electricity markets. However, Section B shows that our main results obtain when demand is allowed to be elastic. We assume that all firms are price-takers, i.e. all agents compete disregarding any influence they could have on equilibrium prices, or on the other agents' behavior <sup>4</sup>.

For simplicity, all agents have access to wholesale markets. All agents are allowed to trade, including speculative agents that play no role in production or retail segments. We denote by  $\mathcal{K}$  the set of all agents: Producers, retailers and traders. Agents are not necessarily specialized in a single segment. Hence the subsets  $\mathcal{P}$  and  $\mathcal{R}$  of  $\mathcal{K}$  are possibly intersecting, leading to four different types of agents:

- Upstream firms, i.e. producers who produce and sell on the wholesale markets;
- Downstream firms, i.e. retailers who buy on the wholesale markets and sell goods to consumers;
- Integrated firms who produce, trade on the markets and deliver outputs to consumers;
- Traders who speculate on all markets.

Retailers can obtain goods from three sources: Wholesale markets, production if they are integrated firms, and forward markets where they agree to buy or sell units of good at the next date for price  $q$ . In particular, forward markets turn out to be linear contracts.

In our setting, vertical integration can be thought of as a profit-sharing scheme as

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<sup>4</sup>This enables us to abstract from anti-competitive motives for vertical integration surveyed in Rey and Tirole (2006).

in Rey and Tirole (2006) and Chemla (2003) <sup>5</sup>. It is also a mechanism that enables agents to diversify industry-specific risk that will dominate other forms of mergers, and in particular horizontal integration, in diversifying risk that channels throughout vertically-related segments.

There are two dates:

- At  $t = 0$ , retailers open accounts and thereby commit to supply a fixed number (out of a publicly known total number) of consumers at a later date. This boils down to allowing downstream firms to choose market shares  $\alpha_k \in [0, 1]$ ,  $k \in \mathcal{R}$ . In addition, agents take forward positions  $f_k$ ,  $k \in \mathcal{K}$  (where  $f_k > 0$  represents a purchase).
- At  $t = 1$ , demand uncertainty is revealed. Agents take positions  $S_k$ ,  $k \in \mathcal{K}$ , on the wholesale spot market (where  $S_k > 0$  denotes a purchase) and producers also choose their generation levels  $G_k$ ,  $k \in \mathcal{P}$ . Since the good is non-storable, production can only occur at that time  $t = 1$  when the demand uncertainty is observed and in which consumers buy the good <sup>6</sup>.

It should be noted that our setting, in which downstream firms choose the number of accounts that they open and thereby commit to provide clients with their future demand of a non-durable good, fits well with an electricity market. This, combined with risk aversion, is one mechanism through which firms bear risk that they may be willing to manage that also enables us to combine asset pricing, industrial organization and corporate finance in our model <sup>7</sup>.

At  $t = 1$ , the market-clearing constraint requires that demand be satisfied:

$$1 = \sum_{k \in \mathcal{R}} \alpha_k . \quad (2.1)$$

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<sup>5</sup>In these papers and many of those summarized in Rey and Tirole (2006), vertical integration arises as a response to contractual incompleteness where output and profits are difficult to contract upon.

<sup>6</sup>Our results are robust to equilibrium definitions where decisions on the retail and forward markets are not taken simultaneously.

<sup>7</sup>Specific market frictions with otherwise risk-neutral environments have also widely been and elegantly used in the economics literature, e.g. in Froot, Scharfstein and Stein (1993) and Mello and Ruckes (2007). Such settings are better tailored to examine corporate finance problems without as much emphasis on market equilibria as we put in this paper

In addition, generation levels must meet demand  $D$ :

$$D = \sum_{k \in \mathcal{P}} G_k . \quad (2.2)$$

The market-clearing constraints on the wholesale spot and the forward markets can be written

$$0 = \sum_{k \in \mathcal{K}} S_k . \quad (2.3)$$

and

$$0 = \sum_{k \in \mathcal{K}} f_k . \quad (2.4)$$

Agent  $k$  obtains a total payoff from its activity on the retail, forward and spot markets (net its production costs):

$$\pi_k = p\alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} - qf_k - wS_k - c_k(G_k) \mathbf{1}_{\{k \in \mathcal{P}\}} .$$

where  $p$ ,  $w$ , and  $q$  denote the retail price, the wholesale spot price and the forward price, respectively, and  $c_k$  is the cost function to producer  $k \in \mathcal{P}$ . The cost function is defined on  $\mathbb{R}_+$ , it is continuously differentiable, strictly convex, and it satisfies the Inada conditions  $c'_k(0+) = 0$ ,  $c'_k(+\infty) = +\infty$ .

Non-storability imposes that the net volume of good bought, sold or produced by agent  $k$  at  $t = 1$  is zero:

$$0 = \alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} - f_k - S_k - G_k \mathbf{1}_{\{k \in \mathcal{P}\}} ,$$

This allows us to discard variable  $S_k$  and to write:

$$\pi_k = (p - w)\alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} + (w - q)f_k + (wG_k - c_k(G_k)) \mathbf{1}_{\{k \in \mathcal{P}\}} .$$

The payoff has three possible ingredients: The payoff to a retailer that satisfies demand  $\alpha_k D$  at retail price  $p$  by sourcing on the wholesale spot market at price  $w$ ; the payoff to a trader buying a volume  $f_k$  on the forward market at price  $q$  and selling it on the spot market at price  $w$ ; and the profit made by a producer who generates a volume  $G_k$  at cost  $c_k(G_k)$  and sells it on the spot market at price  $w$ .

We further assume that agent  $k$ 's preferences are described by a mean-variance utility function. The risk aversion coefficient  $\lambda_k$  can also be interpreted as the cost due to frictions that are necessary for firms to care about corporate risk management, such as bankruptcy costs and lost tax shields (Grinblatt and Titman, 2002, Chapter 21, Smith and Stulz, 1984)<sup>8</sup>. The utility function is denoted:<sup>9</sup>

$$\text{MV}_{\lambda_k}[\xi] := \mathbb{E}[\xi] - \lambda_k \text{Var}[\xi] .$$

## 2.2 Spot market equilibrium

We proceed by backward induction and we start our analysis by determining the spot market equilibrium at  $t = 1$ . At that time, when they enter the spot market, agents know the realization of demand uncertainty  $D$ , and decisions on the retail and forward markets have already been made. The outcome turns out to be independent of any decision taken at time  $t = 0$ . Producer  $k$ 's generation payoff,  $wG_k - c_k(G_k)$ , leads to an equilibrium spot price:

$$w^* = C'(D), G_k^* = (c'_k)^{-1}(w^*) , \quad (2.5)$$

where the superscript  $-1$  denotes the inverse function, and the aggregate cost function  $C$  is defined as

$$C(x) := \sum_{k \in \mathcal{P}} c_k \circ (c'_k)^{-1} \circ \left( \sum_{k \in \mathcal{P}} (c'_k)^{-1} \right)^{-1} (x) .$$

Hence,  $C'(x) = (\sum_{k \in \mathcal{P}} (c'_k)^{-1})^{-1}(x)$ , so that the random variable

$$C(D) = \sum_{k \in \mathcal{P}} c_k(G_k^*(w^*))$$

is the sum of the production costs over the entire upstream industry.

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<sup>8</sup>Debt and non-debt tax shields can be lost when cash flows are volatile (Graham, 2000, Grinblatt and Titman, 2002, Chapter 14). In addition, although bankruptcy costs are often assumed to be a fraction of firm value at the time of bankruptcy (Gilson, 1997), cash flow volatility is a well-known determinant of capital structure and is widely perceived as a significant cost of debt (Titman and Wessels, 1988, Frank and Goyal, 2007).

<sup>9</sup>Such utility functions are widely used in asset pricing (Elton and Gruber, 1995). Although  $\text{MV}_{\lambda_k}$  is not monotonic, which implies possibly negative equilibrium prices, it can be seen as a second order expansion of a monotonic Von Neumann-Morgenstern utility function (Markowitz, 1979).



In equilibrium upstream firms produce up to the point where the spot price equals the marginal cost. This is consistent with both perfectly competitive markets (Green, Mas Colell, and Whinston, 2005) and regulated industries in which prices are set at marginal cost by the regulator (Laffont and Tirole, 1993).

The spot market equilibrium only depends on (exogenous) demand  $D$  and is therefore independent of any other equilibrium prior to time  $t = 1$ . This results from the non-storability condition and the inelasticity assumption on  $D$ <sup>10</sup>.

Note that both the equilibrium spot price  $w^*$  and the generation payoff

$$\Pi_k^g := (w^* G_k^* - c_k(G_k^*)) \mathbf{1}_{\{k \in \mathcal{P}\}} \quad (2.6)$$

are exogenous random variables, the distribution of which is assumed to be known by all agents. We can then substitute  $w^*$  and  $G_k^*$  for variables  $w$  and  $G_k$ , and we define the payoff to agent  $k$  as

$$\Pi_k(p, q, \alpha_k, f_k) := \Pi_k^r(p, \alpha_k) + \Pi_k^t(q, f_k) + \Pi_k^g, \quad (2.7)$$

where  $\Pi_k^g$  is defined in (2.6) and

$$\begin{aligned} \Pi_k^r(p, \alpha_k) &:= (p - w^*) \alpha_k D \mathbf{1}_{\{k \in \mathcal{R}\}} \\ \Pi_k^t(q, f_k) &:= (w^* - q) f_k. \end{aligned}$$

Here,  $\Pi_k^r$  is the net retail payoff derived from supplying a retail demand by sourcing on the spot market, and  $\Pi_k^t$  is the net trading profit earned by buying  $f_k$  units of goods on the forward market and selling them on the spot market. Finally,  $\Pi_k^g$  is the net generation payoff obtained by producing  $G_k$  and selling it on the spot market<sup>11</sup>.

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<sup>10</sup> Note that this situation is different from Allaz (1992), where the demand elasticity to spot price implies a dependency of the spot price to forward positions and a reduction of the market power of the producers.

<sup>11</sup> Our results obtain with elastic demand as long as the equilibrium spot market depends only on the retail price. Nonetheless, for the sake of clarity, we carry the bulk of our analysis with inelastic demand and we address possible generalizations in Section B.

## 2.3 Competitive Equilibrium

In order to define an equilibrium, we introduce the following two sets:

$$\mathbf{A} := \left\{ (\alpha_k)_{k \in \mathcal{K}} \in [0, 1]^{|\mathcal{K}|} : \forall k \notin \mathcal{R}, \alpha_k = 0 \text{ and } \sum_{k \in \mathcal{K}} \alpha_k = 1 \right\}$$

$$\mathbf{F} := \left\{ (f_k)_{k \in \mathcal{K}} \in \mathbb{R}^{|\mathcal{K}|} : \sum_{k \in \mathcal{K}} f_k = 0 \right\}.$$

**Definition 2.1.** *An equilibrium of the retail-forward equilibrium problem is a quadruple  $(p^*, q^*, \alpha^*, f^*) \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbf{A} \times \mathbf{F}$  such that:*

$$(\alpha_k^*, f_k^*) = \operatorname{argmax}_{\alpha_k, f_k} \operatorname{MV}_{\lambda_k} [\Pi_k(p^*, q^*, \alpha_k, f_k)], \quad \forall k \in \mathcal{K}.$$

This defines a simultaneous competitive equilibrium on both markets. Each agent submits a supply function that specifies his position on the forward market and his market share for each price level. Each agent chooses his supply function taking prices as given. Then, the auctioneer collects all supply functions and sets prices that ensure market clearing and demand satisfaction <sup>12</sup>.

## 3 Analysis in the absence of a forward market

In this section, we focus on equilibria in the absence of a forward market. We derive the equilibrium and we analyse the results. In the absence of a forward market, we define the profit function without a forward position:

$$\Pi_k^0(p, \alpha_k) := \Pi_k(p, 0, \alpha_k, 0) = \Pi_k^r(p, \alpha_k) + \Pi_k^g.$$

In this simplified setting, Definition 2.1 reduces to:

**Definition 3.1.** *An equilibrium of the retail equilibrium problem is a pair*

*$(p^*, \alpha^*) \in \mathbb{R}_+ \times \mathbf{A}$  such that:*

$$\alpha_k^* = \operatorname{argmax}_{\alpha_k} \operatorname{MV}_{\lambda_k} [\Pi_k^0(p^*, \alpha_k)], \quad \forall k \in \mathcal{K}.$$

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<sup>12</sup>Our results are robust to alternative definitions that allow for sequentiality between investment and forward and retail markets.

### 3.1 Characterization of the equilibrium

Let

$$\Pi_I^g := \sum_{k \in \mathcal{R} \cap \mathcal{P}} \Pi_k^g$$

be the aggregate generation profit realized by all integrated firms, i.e. the firms that run both generation and supply units, and let

$$\Pi^r := \sum_{k \in \mathcal{R}} \Pi_k^r(p^*, \alpha_k^*) = (p^* - w^*)D$$

be the aggregate retail profit to all retailers. We also define

$$\Lambda := \left( \sum_{k \in \mathcal{K}} \lambda_k^{-1} \right)^{-1}, \quad \Lambda_{\mathcal{R}} := \left( \sum_{k \in \mathcal{R}} \lambda_k^{-1} \right)^{-1}, \quad (3.1)$$

the aggregate risk aversion coefficients for the set of all agents and for the set of all retailers, respectively. Parameter  $\lambda_k^{-1}$  corresponds to Agent  $k$ 's risk tolerance, as defined in Gollier (2004)<sup>13</sup>. We only focus on interior equilibria, i.e. equilibria where constraints  $\alpha^* \in [0, 1]$  and  $p^* \geq 0$  are not binding, by discarding cases where some retailers in  $\mathcal{R}$  have zero market shares. The equilibrium is then characterized by the following Proposition.

**Proposition 3.1.**  $(p^*, \alpha^*) \in \mathbb{R}_+^* \times \text{int}(\mathbf{A})$  defines an equilibrium of the retail problem without a forward market iff:

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \frac{\text{Cov}[\Pi^r, \Pi_I^g]}{\text{Var}[\Pi^r]} - \frac{\text{Cov}[\Pi^r, \Pi_k^g]}{\text{Var}[\Pi^r]}, \quad (3.2)$$

and  $p^*$  solves the second order polynomial equation:

$$0 = \mathbb{E}[(p^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Cov}[(p^* - w^*)D, (p^* - w^*)D + \Pi_I^g]. \quad (3.3)$$

**Proof.** See Appendix A  $\square$

Intuitively, firm  $k$  chooses its market share in order to maximize its expected return while keeping variance as low as possible. In other words, the market shares have a home-made risk-management feature. Firm  $k$ 's market share increases with its risk tolerance relative to that of the retail market and with the covariance between the aggregate generation profit

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<sup>13</sup>We follow in this Wilson (1979), where an aggregate risk tolerance is defined by summing over the risk tolerances of the syndicate members, as in (3.1).

realized by integrated firms and the aggregate retail profit relative to the variance of the aggregate retail profit, but it decreases with the covariance between firm  $k$ 's generation profit and the aggregate retail profit relative to the variance of the aggregate profit. This is because the lower that covariance, the greater the diversification benefits that firm  $k$  obtains through production. Hence, firm  $k$  can increase its expected profit through an increase in its market share without increasing risk much.

### 3.2 The retail price

We now examine retail price properties. From Equation (3.3), neither existence nor uniqueness is granted <sup>14</sup>. We argue that, if there exist two solutions, only one is relevant.

The risk-neutral price, i.e. the price when some retailers are risk neutral, boils down to:

$$p^0 = \frac{\mathbb{E}[w^*D]}{\mathbb{E}[D]}, \quad (3.4)$$

Since  $w^* = C'(D)$  is a non-decreasing function of  $D$ ,  $w^*$  and  $D$  are positively correlated, and the risk neutral retail price is greater than the expected spot price:  $p^0 \geq \mathbb{E}[w^*]$ .

When all retailers are risk-averse, we expect the equilibrium retail price to tend to the risk neutral price when the risk aversion coefficient of at least one retailer tends to zero. This leads to the following result

**Proposition 3.2.** *Only one equilibrium retail price can be economically relevant.*

**Proof.** See Appendix A  $\square$

We now examine the impact of vertical integration on the equilibrium retail price.

**Proposition 3.3.** *When  $\Pi^r$  and  $\Pi_I^g$  are negatively correlated, the presence of integrated producers decreases the retail price.*

**Proof.** See Appendix A  $\square$

The correlation between  $\Pi^r$  and  $\Pi_I^g$  tends to be negative. Since  $w^* = C'(D)$ , profit  $\Pi_k^g$  is an increasing function of  $D$ . In contrast, the retail profit  $\Pi^r = (p^* - w^*)D$  is likely to

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<sup>14</sup> This is a common feature in such mean-variance settings.

be a decreasing function of  $D$ : Since  $p^*$  is fixed while  $w^*$  increases with  $D$ ,  $\Pi^r$  will likely decrease with  $D$ . It should be noted that the higher the slope of C, the more negative we expect this correlation, and the more powerful the effect of vertical integration on retail prices. In the numerical application in Section 5, this correlation is indeed negative.

Intuitively, vertical integration by at least one firm leads to a decrease in the retail price because it facilitates risk diversification between the upstream segment and the downstream segment. Hence, firms can charge a retail price that is lower than in the absence of vertical integration, which their price-taking behavior will prompt them to do.

### 3.3 Market shares

We turn to the properties of market shares. First, note that when there are some risk-neutral retailers, the market shares held by the risk averse retailers are given by:

$$\alpha_k^0 = -\frac{\text{Cov}[\Pi^r, \Pi_k^g]}{\text{Var}[\Pi^r]}, \quad (3.5)$$

while the remaining demand is split among the risk neutral retailers. In particular, a risk averse retailer who has no generation unit ends up with a zero market share.

Notice that, in order to satisfy the non-negativity condition of the market shares, equation (3.5) implies  $\text{Cov}[\Pi^r, \Pi_k^g] \leq 0$ . This provides us with yet another justification for the assumption required for vertical integration to have a negative effect on the retail price.

In the absence of integrated producers and risk-neutral suppliers, the equilibrium market shares are given by:

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k}.$$

The market shares are distributed proportionally to the risk tolerances, and only depend on these parameters.

**Proposition 3.4.** *When  $\Pi^r$  is negatively correlated with  $\Pi_I^g$ , vertical integration prompts a retailer to increase its market share.*

If integrated firms enter the market, the retailers see their equilibrium market shares

move to:

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \frac{\text{Cov}[\Pi^r, \Pi_I^g]}{\text{Var}[\Pi^r]},$$

while the integrated firms have market shares:

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \frac{\text{Cov}[\Pi^r, \Pi_I^g]}{\text{Var}[\Pi^r]} - \frac{\text{Cov}[\Pi^r, \Pi_k^g]}{\text{Var}[\Pi^r]}.$$

The downstream firms see their market shares decrease while the integrated agents increase theirs. Indeed, the latter will decrease their risk by increasing their investment in the retail market. In addition, although the market shares are different from the previous case, the relative market shares among the set of downstream firms remain unchanged:

$$\frac{\alpha_i^*}{\alpha_j^*} = \frac{\lambda_j}{\lambda_i}.$$

## 4 Analysis with a forward market

We now examine the setting in the presence of a forward market. We first characterize the equilibria. Then, we analyse retail and forward prices and positions.

### 4.1 Characterization of the equilibria

Let

$$\Pi^e := \sum_{k \in \mathcal{K}} \Pi_k(p^*, q^*, \alpha_k^*, f_k^*) = p^* D - C(D),$$

be the aggregate profit to both the upstream and the upstream segment, which coincides with the social surplus in our setting.

We still focus on interior equilibria, when the constraints are not binding. The equilibrium with a forward market is characterized by the following proposition.

**Proposition 4.1.**  *$(p^*, q^*, \alpha^*, f^*) \in \mathbb{R}_+^* \times \mathbb{R}_+^* \times \text{int}(\mathbf{A}) \times \mathbf{F}$  defines an equilibrium of the retail-forward equilibrium problem iff:*

$$f_k^* = \frac{\Lambda}{\lambda_k} \frac{\text{Cov}[w^*, \Pi^e]}{\text{Var}[w^*]} - \frac{\text{Cov}[w^*, \Pi_k^g]}{\text{Var}[w^*]} - \alpha_k^* \frac{\text{Cov}[w^*, \Pi^r]}{\text{Var}[w^*]} \quad (4.1)$$

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\text{Cov}[w^*, \Pi^r]}{\Delta} \text{Cov}\left[w^*, \Pi_k^g - \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \Pi_I^g\right] - \frac{\text{Var}[w^*]}{\Delta} \text{Cov}\left[\Pi^r, \Pi_k^g - \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \Pi_I^g\right] \quad (4.2)$$

$$q^* = \mathbb{E}[w^*] - 2\Lambda \text{Cov}[w^*, p^* D - C(D)], \quad (4.3)$$

and  $p^*$  is a root of the second order polynomial equation

$$0 = \mathbb{E}[(p^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Cov}[(p^* - w^*)D, (p^* - w^*)D + \Pi_I^g] \quad (4.4)$$

$$+ 2\Lambda_{\mathcal{R}} \frac{\text{Cov}[w^*, (p^* - w^*)D]}{\text{Var}[w^*]} \text{Cov} \left[ w^*, (p^* - w^*)D + \Pi_I^g - \frac{\Lambda}{\Lambda_{\mathcal{R}}} (p^* D - C(D)) \right] ,$$

where

$$\Delta := \text{Var}[w^*] \text{Var}[\Pi^r] - \text{Cov}^2[w^*, \Pi^r] . \quad (4.5)$$

**Proof.** See Appendix A.  $\square$

## 4.2 The forward price

From 4.3, the forward price equals the expected spot price corrected by a risk premium term that accounts for the correlation between the spot price and the aggregate profit  $\Pi^e$ , and the market aggregate risk aversion.<sup>15</sup>

We can rewrite  $q^*$  as:

$$q^* = \mathbb{E}[Zw^*] \text{ with } Z := 1 - 2\Lambda(\Pi^e - \mathbb{E}[\Pi^e]) .$$

If  $\Lambda$  is sufficiently small to ensure that  $Z$  is always strictly positive,  $Z$  defines a change in probability and  $q^*$  is given by the expected  $w^*$  under a risk-neutral probability.

The forward price only depends on retail and spot prices. It is independent of the distribution of market shares and of that of generation assets. Moreover, the risk-neutral forward price, i.e. the price if some traders are risk neutral, boils down to the expected spot price  $q^0 = \mathbb{E}[w^*]$ .

When the cost functions are quadratic, e.g.  $c_k(x) := \frac{1}{2}x(a_k x + b_k)$ ,  $a_k, b_k > 0$ ,  $q^*$  can be written:

$$q^* = \mathbb{E}[w^*] - \frac{2\Lambda}{a} \text{Var}[w^*](p^* - \mathbb{E}[w^*]) + \frac{\Lambda}{a} \text{Var}^{\frac{3}{2}}[w^*] \text{Skew}[w^*] ,$$

where  $a^{-1} := \sum_{k \in \mathcal{P}} a_k^{-1}$ , as in Bessembinder and Lemmon (2002).

The forward price increases with spot price skewness and, in the case where the retail price is higher than the expected spot price, the forward price decreases with spot price

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<sup>15</sup> This equation has a form that is similar to that of other equilibrium models in a mean-variance setting, as shown in Allaz (1992).

volatility. This shows that, in the case of electricity, where spot price volatility is high, forward prices that are lower than the expected spot price are common. Nevertheless, forward prices that are greater than the expected spot price can occur when spot price skewness is large and positive, i.e. when large upward peaks are possible.

In addition, the forward market equilibrium leads to the following relationship between retail and forward prices:

$$p^* = \frac{\mathbb{E}[w^*] - q^*}{2\Lambda \text{Cov}[w^*, D]} + \frac{\text{Cov}[w^*, C(D)]}{\text{Cov}[w^*, D]} .$$

In particular, higher forward prices correspond to lower retail prices and conversely.

### 4.3 The retail price

In the presence of a forward market, the equation for  $p^*$  is similar to that found in the absence of a forward market, with an extra term

$$2\Lambda_{\mathcal{R}} \frac{\text{Cov}[w^*, (p^* - w^*)D]}{\text{Var}[w^*]} \text{Cov} \left[ w^*, (p^* - w^*)D + \Pi_I^g - \frac{\Lambda}{\Lambda_{\mathcal{R}}} (p^* D - C(D)) \right] .$$

This term corresponding to the hedging property of the forward market.

As in Section 3, the risk neutral retail price boils down to  $p^0 = \frac{\mathbb{E}[w^* D]}{\mathbb{E}[D]}$ . This is not surprising since in that case risk management is irrelevant. We can still write the Taylor expansion around  $\Lambda_{\mathcal{R}} = 0$  and show that only one root for  $p^*$  is relevant, ensuring the uniqueness of the equilibrium retail price.

We can also exhibit the following properties of equilibrium retail prices:

- **The price in a fully integrated economy.** If all producers are integrated, i.e.  $\mathcal{P} \subset \mathcal{R}$ , we obtain:

$$\begin{aligned} 0 &= \mathbb{E}[(p^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Cov}[(p^* - w^*)D, p^* D - C(D)] \\ &\quad + 2\Lambda_{\mathcal{R}} \left( 1 - \frac{\Lambda}{\Lambda_{\mathcal{R}}} \right) \frac{\text{Cov}[w^*, (p^* - w^*)D]}{\text{Var}[w^*]} \text{Cov}[w^*, p^* D - C(D)] . \end{aligned}$$

In particular, in the absence of non-integrated traders,  $\mathcal{R} = \mathcal{K}$  and  $\Lambda = \Lambda_{\mathcal{R}}$ , so that the above equation boils down to (3.3), i.e. the forward market has no impact on the retail price.



To see this, consider the following example: Suppose there are only  $N$  integrated producers, with the same cost function and the same risk aversion coefficient. By symmetry,  $S_k^* = \frac{D}{N}$ ,  $\alpha_k^* = \frac{1}{N}$  and  $f_k^* = 0$  for all  $k$ . The agents do not take any position on the forward market. The retail price should therefore not be affected.

This result highlights the substitution effect between forward hedging and vertical integration. When the industry can diversify demand risk through vertical integration, forward hedging becomes irrelevant. Conversely, Subsection 5.2.3 illustrates that in the presence of forward hedging, vertical integration has little impact on the retail price.

- **The effect of forward trading.** In a partially integrated economy,  $p_F^* \leq p_{NF}^*$ , i.e. the forward market reduces the retail price, iff

$$0 \leq \left(1 - \frac{\Lambda}{\Lambda_R}\right) \text{Cov}^2[w^*, (p_{NF}^* - w^*)D] \quad (4.6)$$

$$+ \text{Cov}[w^*, (p_{NF}^* - w^*)D] \text{Cov}[w^*, \Pi_I^g - \frac{\Lambda}{\Lambda_R} \Pi^g] . \quad (4.7)$$

In particular, this is verified if no producer is integrated and the retail income without a forward market is negatively correlated to the spot price. This is also verified if no retailer is integrated and  $\Lambda = 0$  (e.g. if there is a risk-neutral trader).

In Subsection 5.2, forward hedging will be shown to reduce the retail price, but this effect decreases with the degree of integration in the industry.

- **The effect of integration.** Let  $p_{NI}^*$  be the equilibrium retail price in the absence of integration. In order to compare this price with the equilibrium retail price in the presence of integrated firms, we substitute  $p_{NI}^*$  for  $p^*$  in the right hand side of (4.4) and we examine its sign, i.e. the sign of

$$\frac{\text{Cov}[w^*, (p_{NI}^* - w^*)D]}{\text{Var}[w^*]} \text{Cov}[w^*, \Pi_I^g] - \text{Cov}[(p_{NI}^* - w^*)D, \Pi_I^g]. \quad (4.8)$$

In particular, with quadratic cost functions, we obtain:

$$\begin{aligned} & \frac{\text{Cov}[w^*, (p_{NI}^* - w^*)D]}{\text{Var}[w^*]} \text{Cov}[w^*, \Pi_k^g] - \text{Cov}[(p_{NI}^* - w^*)D, \Pi_k^g] \\ &= \frac{a^3}{2a_k} \left( \text{Var}[D^2] - \frac{\text{Cov}^2[D^2, D]}{\text{Var}[D]} \right) , \end{aligned}$$

which is always positive, so that the right side of (4.4) is positive for  $p_{NI}^*$ , i.e. partial vertical integration reduces the equilibrium retail price.

We will see in Subsection 5.3 that the presence of vertically integrated producers always reduces the retail price. This effect is nonetheless drastically reduced in comparison to the case without a forward market.

We summarize our main results in the following proposition:

**Proposition 4.2.** *The forward market and partial vertical integration reduce the retail price if and only if 4.7 is satisfied and the sign of 4.8 is positive, respectively.*

#### 4.4 Positions on the forward market

Equation 4.1 shows that in contrast to the forward price, forward positions depend on both  $p^*$  and  $\alpha^*$ . Agent  $k$ 's forward position has three components. The first term, i.e. the fraction  $\frac{\Lambda}{\lambda_k}$  of a constant term that involves the correlation between the global profit and the spot price, can thus be interpreted as the trading component.<sup>16</sup> The second term, i.e. the fraction  $\alpha_k^*$  of the correlation between the global retail profit and the spot price is the retail component. If the retail market revenue is negatively correlated with the spot price, as we argued in the previous section, then retailers will take long positions on the forward market to hedge against high spot prices. Finally, the last term corresponds to the generation component, which takes the form of the correlation between the generation payoff and the spot price. As generation profits are positively correlated to the spot price, producers will take short forward positions to hedge against low spot prices.

#### 4.5 Positions on the retail market

We now turn to the market shares characterized in 4.2. When  $\Pi_k^g = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \Pi_I^g$  for all  $k \in \mathcal{R}$ , the market shares become  $\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k}$ , as in a non-integrated economy without a forward market. This obtains, for instance, when there are no integrated firms, or all producers are integrated and generation profits are proportional to risk tolerances. Another formulation for  $\alpha_k^*$  is:

$$\alpha_k^* = \alpha_k^0 + \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \left( 1 - \frac{\text{Cov}[w^*, \Pi^r]}{\Delta} \text{Cov}[w^*, \Pi_I^g] + \frac{\text{Var}[w^*]}{\Delta} \text{Cov}[\Pi^r, \Pi_I^g] \right),$$

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<sup>16</sup> To see this, note that if  $k$  is a speculator only, then the last two terms are zero.

where

$$\alpha_k^0 = \frac{\text{Cov}[w^*, \Pi^r]}{\Delta} \text{Cov}[w^*, \Pi_k^g] - \frac{\text{Var}[w^*]}{\Delta} \text{Cov}[\Pi^r, \Pi_k^g]$$

is retailer  $k$ 's risk neutral market share. This enables us to analyse the deviation of market shares from the risk neutral equilibrium.

In Section 5, we will observe two important characteristics of market shares. First, the presence of a forward market is a means for downstream firms to obtain larger market shares in the downstream segment. Second, the higher the level of integration of an integrated producer, the higher its market share.

#### 4.6 Utility functions and the strong asymmetry between downstream and upstream

The asymmetry relative to risk between retailers and producers is due to several differences. First, in the absence of a forward market and vertical integration, retailers have to make market share decisions under uncertainty, while producers know the realization of demand when production takes place. Second, if final demand is inelastic to the retail price, upstream profits are independent of the retail price, while downstream revenues depend on both retail and spot prices. This asymmetry is central to our analysis.<sup>17</sup>

Hence, an upstream firm always benefits from trading forward contracts. Since, in our setting, the generation profit  $\Pi_k^g$  boils down to an exogenous random variable, when an upstream firm chooses  $\bar{f}_k(q) = 0$  for all forward price  $q$ , it is guaranteed to receive a utility  $\text{MV}_{\lambda_k}[\Pi_k^g]$ , i.e. the utility in the absence of a forward market. Hence, the presence of a forward market always increases the utility to upstream firms because the strategy  $\bar{f} = 0$  is admissible and yields the same utility as in the absence of a forward market.

In contrast, downstream firms do not necessarily obtain a higher utility when forward contracts become available. Indeed, the retail profit  $\Pi_k^r$  depends on  $p^*$ , and thus on the other agents' decisions. If the retail price  $p^*$  in the presence of forward trading is different from the retail price without forward trading, agent  $k$ 's retail profit will also be different.

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<sup>17</sup>One illustration of this asymmetry is the Californian electricity crisis, in which retailers suffered large losses while producers could take advantage of high spot prices.

Not taking a position on the forward market will not guarantee agent  $k$  to obtain the same utility as in the absence of a forward market.

**Proposition 4.3.** *Forward markets benefits upstream firms, but not necessarily downstream firms, even though downstream firms benefit from trading forward contracts.*

In Subsection 5.2, the availability of forward contracts decreases the utility to downstream firms. When forward markets are available, each retailer is individually better off taking positions on them. Hence, all retailers trade forward contracts and they can offer lower retail prices. Nevertheless, the decrease in expected profits offsets that in variance and this risk hedging mechanism implies a decrease in utility in comparison to the environment without forward contracts.

The effect of vertical integration on the agents' utility depends on the utility of vertically integrated firm. It turns out that our results are robust to various ways of handling this problem, in particular Wilson's (1979) approach to sum the risk tolerance of the integrated firms or an approach that would assume that all risk aversion coefficients, including those to vertically integrated firms, are equal.<sup>18</sup> In Subsection 5.3, we observe that, like forward hedging, vertical integration decreases the agents' utility because of its negative effect on retail prices. Nevertheless, for large risk aversion coefficients this effect can be reversed and the gain from risk diversification through vertical integration can be higher than the loss in expected profit.

## 5 Application to the electricity industry

We illustrate our analysis with data from the French electricity market. Electricity is a non-storable good with broadly inelastic demand and price-taking firms, so this industry fits our theoretical setting well. This market is characterized by the presence of a regulated dominant agent and recently entered competitors.

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<sup>18</sup>If we were to model the integration stage rather than focusing on the market structure, the change in utilities due to integration would also depend on the takeover game (see Tirole, 2006, Chapter 11, and Grinblatt and Titman, 2002, Chapter 20).

## 5.1 Methodology

We compute the retail and forward equilibria using data from the French electricity market. Only spot prices and demand levels are publicly available. To estimate  $C$ , we invert the spot price  $w^* = C'(D)$ . Demand and spot price hourly data from December 2004 to March 2005 are available on web sites [www.rte-france.com](http://www.rte-france.com) and [www.powernext.fr](http://www.powernext.fr). They provide us with reliable estimates for  $D$  and  $w^*$ .

This winter was generally mild, but it was followed by a cold wave in March. The recordings are shown on Figure 5.1-left. The circles correspond to values in March. These are indicative of high market volatility. Nonetheless they remain strongly heterogeneous since many generation units were unavailable during the March cold wave and that likely had an impact on the generation costs. To address this heterogeneity, we have added a constant to the demand sample in March 2005. We have then regressed function  $C$  so that  $P^* = C'(D)$  (Figure 5.1-right). The data for  $D$  and  $P^*$ , the risk aversion coefficients and

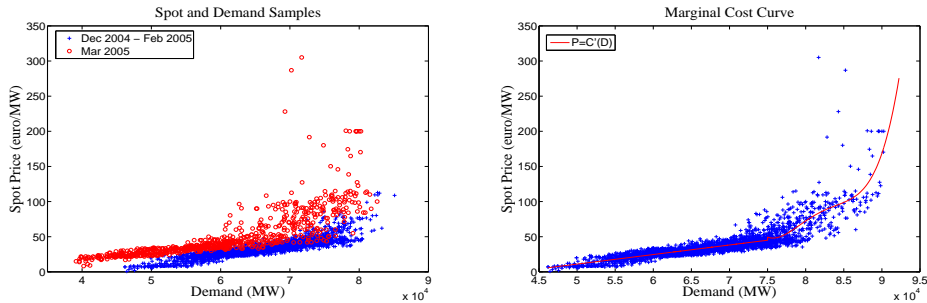


Figure 5.1: Demand and spot price samples (left). Processed and interpolated data (right).

the regressed cost function  $C$  enable us to compute the equilibrium.

Without loss of generality, the analysis can be performed with two agents. Indeed, equations (4.1) and (4.2) are linear in  $\lambda_k^{-1}$  and  $\Pi_k^g$  while (4.4) and (4.3) only involve the aggregate risk aversion coefficients  $\Lambda$  and  $\Lambda_{\mathcal{R}}$ .

## 5.2 The effect of the forward market

In this section, we compare the equilibria with and without a forward market under different environments. We consider two agents in the following scenarios:

1. Agent 1 is an integrated producer that competes with a downstream firm
2. Agent 1 is an upstream firm, while Agent 2 is a downstream firm
3. Agent 1 is integrated and competes with an upstream firm
4. Agent 1 and Agent 2 are both integrated.

### 5.2.1 Unbundling of the retail activity

An integrated firm competes with a downstream firm. We examine whether or not retail competition with an integrated monopolist is viable.

In the absence of a forward market, the downstream firm has to buy input from the integrated producer on the spot market. Figure 5.2-left shows Agent 1's market share as a function of both agents' risk aversion coefficients. Agent 2 is limited to a very small market share, less than 2%, for any risk aversion coefficient. Agent 2 is subject to high financial risk because it is exposed to the high spot price volatility. Agent 2 has limited

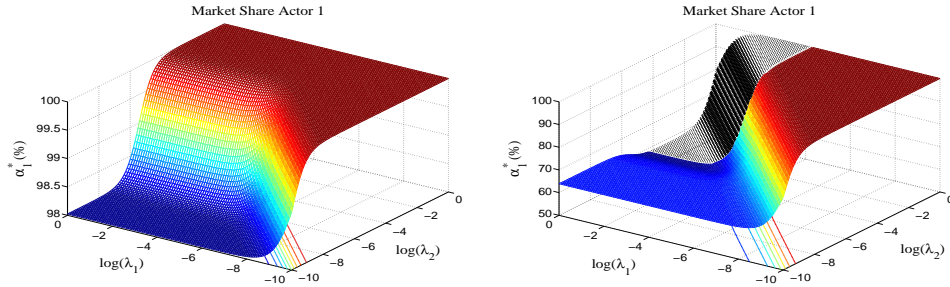


Figure 5.2: Agent 1's market share without (left) and with (right) the forward market as functions of the logarithm of risk aversion coefficients.

possibilities to enter the retail market, all the more limited as its risk aversion is large.

Figure 5.2-right shows Agent 1's market share with a forward market. Dotted mesh regions represent zones where there is no equilibrium ( $q^* < 0$ ). In contrast to the previous

case, if Agent 2 is less risk averse than Agent 1, then it can enter the retail market and obtain a larger market share, up to 40%. The retail price remains unchanged, as shown on

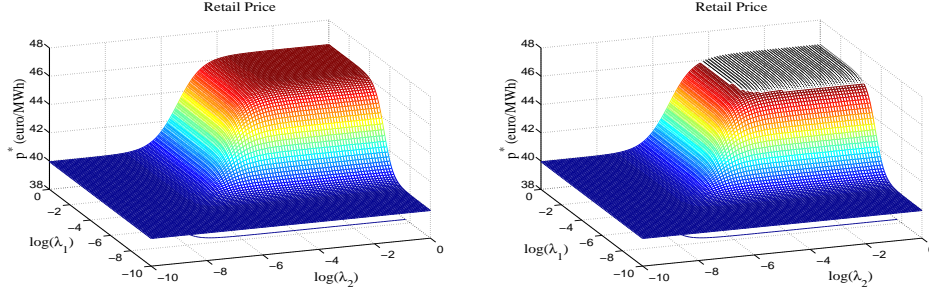


Figure 5.3: Retail price without (left) and with (right) the forward market as functions of the logarithm of risk aversion coefficients.

Figure 5.3. This is because all producers are integrated (see Section 4.3). The upstream risk is already diversified via the producers integrated structure.

The forward market changes the risk allocation between retailers but it does not enhance risk diversification. When both agents are highly risk averse, they do not agree on exchanging forward contracts: There is no forward market equilibrium (dotted mesh zones in Figure 5.4). When the equilibrium exists, the forward price is almost always greater than the expected spot price ( $E[P^*] = 37.9518$ ), at least for sufficiently low risk aversion coefficients. Agent 2's forward position is in the range of its expected demand:  $f_2 \simeq 1.1 \alpha_2 \mathbb{E}[D]$ . Agent 2 hedges its retail demand by 10% above the expected demand, whatever its risk aversion coefficient. That is, the integrated producer is better off being short even if it has to buy back part of the previously sold volumes on the spot market. Finally, Figure 5.5 shows that the forward market increases both agents utilities. This figure shows the gain in the agents utility  $\Delta U = U^F - U^{NF}$  due to the forward market. For convenience, we plotted the monotonic transform  $\phi(\Delta U)$ , where  $\phi(x) := \text{sgn}(x) \log(1 + |x|)$ , in order to show both the logarithm of  $\Delta U$  and its sign. Both agents benefit from the forward market.

For a better understanding of the effect of forward hedging, Table 1 reports the relative gains in utility, average profit and variance ("Risk") with  $\lambda_1 = \lambda_2 = 10^{-6}$ . We also computed the ratio "excess average profit" over "excess risk", denoted "Profit vs. Risk",

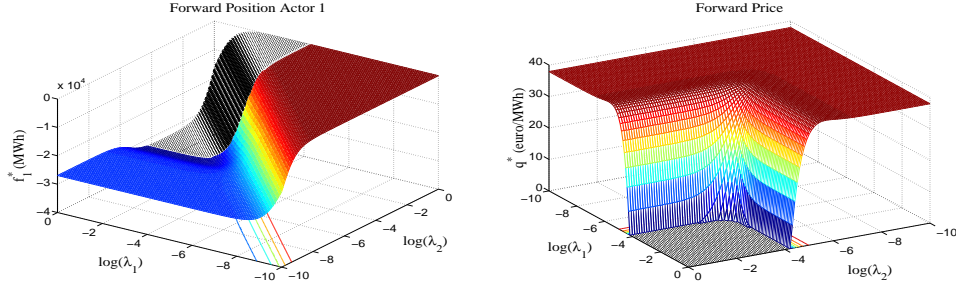


Figure 5.4: Agent 1's forward position (left) and forward price as functions of the logarithm of risk aversion coefficients.

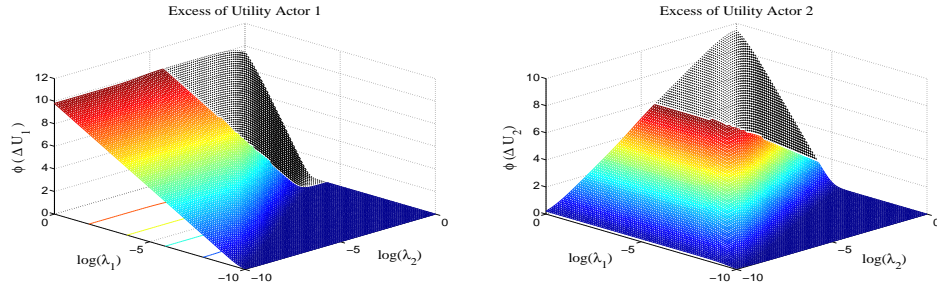


Figure 5.5: Increase in utility for Agent 1 (left) and Agent 2 (right) as functions of the logarithm of risk aversion coefficients.

and a performance measure, namely the ratio “expected profit over risk”. It appears that forward trading has a higher effect on Agent 2's utility than on Agent 1's. Agent 1 uses forward contracts mostly for hedging purposes, thus reducing the variance of its profits by 59 %. The relative loss in average profit associated to this hedging position is low, 0.67 times smaller than the decrease in the variance, resulting in a slight increase in utility. In contrast, Agent 2 increases both risk and its average profit by 793 %. The increase in average profit is twice the increase in risk, so its utility also increases. Forward trading has a higher effect on Agent 2's business. Nevertheless, the ratio expected gain over risk increases more for Agent 1 than for Agent 2.



		Agent 1	Agent 2
Utility:	$\frac{\Delta U}{ U }$	$8.8 \cdot 10^{-2} \%$	793 %
Av. Profit:	$\frac{\Delta \mathbb{E}}{ \mathbb{E} }$	-0.17 %	793 %
Risk:	$\frac{\Delta \text{Var}}{\text{Var}}$	-59 %	793 %
Profit vs. Risk:	$\frac{\Delta \mathbb{E}}{\lambda \Delta \text{Var}}$	0.67	2
Performance:	$\frac{\lambda \text{Var}}{ \mathbb{E} } \times \Delta \frac{\mathbb{E}}{\lambda \text{Var}}$	146 %	$\approx 0$

Table 1: The impact of forward trading on utility, profit and risk, when  $\lambda_1 = \lambda_2 = 10^{-6}$ .

### 5.2.2 Full unbundling

In this section, the generation activity is separated from the retail activity. Agent 1 is an upstream firm while Agent 2 is a downstream firm. In an unbundled economy, we proved that the forward market has no impact on the market shares, which only depend on the risk aversion coefficients (cf. Section 4.5). Nonetheless, the forward market affects the retail price. Figure 5.6 shows the retail price as a function of the risk aversion coefficients without (left) and with (right) the forward market (For the sake of clarity the zones with no-equilibrium are omitted). If Agent 2 is too risk-averse, there is no equilibrium even with the forward market. This is an illustration of the asymmetry between retailers and producers. Retail might not be sustainable in this model, whereas generation always is. In addition, the forward market decreases the retail price.

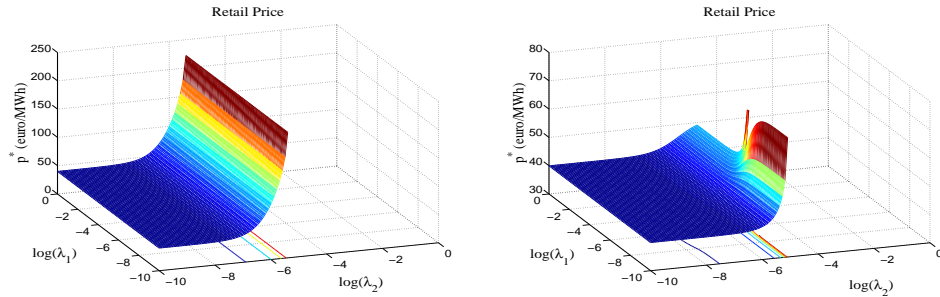


Figure 5.6: Retail price without (left) and with (right) the forward market as functions of the logarithm of risk aversion coefficients.

Figure 5.7 shows the logarithmic transform of the gain in utility ( $\phi(\Delta U)$ ) due to the forward market for both agents. The forward market increases the upstream's utility, while it decreases the downstream firm's, unlike the integrated economy in Section 5.2.1.

As argued in Section 4, the admissibility of strategy  $f_k = 0$  ensures that the forward market increases the upstream firms' utility. While strategy  $f_k = 0$  is also admissible for downstream firms, it is not profitable if the other retailers do trade forward contracts. The forward market does not necessarily increase their utility.

Once partially hedged on the forward market, the downstream firms can offer lower retail prices. In the meantime, market shares depend on risk tolerances, but not on the forward market.

The gain from hedging is half the expected loss on the retail market that is induced by the decrease in the retail price:  $U_2^F - U_2^{NF} \simeq \frac{1}{2}(p^F - p^{NF})\mathbb{E}[D]$ . This explains the decrease in the retail price and in the retailers utility. To illustrate this argument, we computed

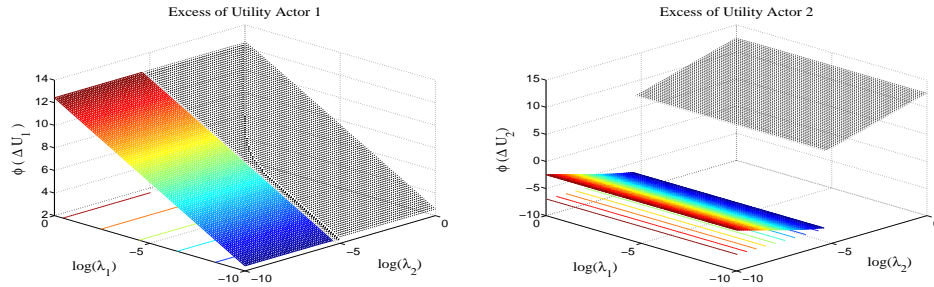


Figure 5.7: Gain in utility due to the forward market for Agents 1 and 2, as functions of the logarithm of risk aversion coefficients.

some indicators of risk and profit as in the previous subsection. The results are shown in Table 2. Agent 1 can now both increase its average profit by 0.88 % and decrease its risk by 99 %, hence increasing its utility by 326 %. In the meantime, Agent 2 decreases its risk, average profit and utility by 98 %, the decrease in average profit being twice the risk reduction, as above.

		Agent 1	Agent 2
Utility:	$\frac{\Delta U}{ U }$	326 %	−97 %
Av. Profit:	$\frac{\Delta E}{ E }$	0.88 %	−97 %
Risk:	$\frac{\Delta \text{Var}}{\text{Var}}$	−99 %	−97 %
Profit vs. Risk:	$\frac{\Delta E}{\lambda \Delta \text{Var}}$	−0.62 %	2
Performance:	$\Delta \frac{E}{\lambda \text{Var}}$	117.63	$\approx 0$

Table 2: The impact of forward trading on utility, profit and risk, when  $\lambda_1 = \lambda_2 = 10^{-6}$ .

### 5.2.3 Unbundling of the generation activity

Here, generation assets are split between integrated (Agent 1) and upstream (Agent 2) firms.

In addition to the risk aversion coefficients set at  $\lambda_1 = \lambda_2 = 10^{-6}$ , we need to specify the distribution of generation assets between the agents. We choose three different methods when allocating the distribution of generation assets between the two agents, each parameterized by a coefficient  $0 \leq x \leq 100$ . Method 1 allocates the first  $x\%$  of generation capacity with the lowest marginal cost to Agent 1. Method 2 allocates the first  $x\%$  of generation capacity with the highest marginal cost to Agent 1. In Method 3, the allocation is more "balanced": Agent 1 obtains the first  $x/2\%$  of generation capacity with the lowest marginal cost and the first  $x/2\%$  with the highest marginal cost.

When Agent 1 has no generation capacity, we are back to the previous subsection. In this setting, the retail price decreases with Agent 1's total capacity (see Figure 5.8) in both cases, with and without a forward market. It is thus minimal when Agent 1 owns all the generation assets. Nonetheless, with the forward market, the effect on the retail price is low. Unbundling the generation monopoly increases the retail price. In addition, the forward market has a major effect on the retail price, which decreases by up to 20%.

As in paragraph 5.2.2, the forward market always increases Agent 2's utility, while it always decreases that of Agent 1, the integrated firm (Figure 5.9). The magnitude of this decrease is nonetheless reduced with integration. Upstream firms prompt downstream firms to trade forward contracts and to decrease the retail price, thereby decreasing their

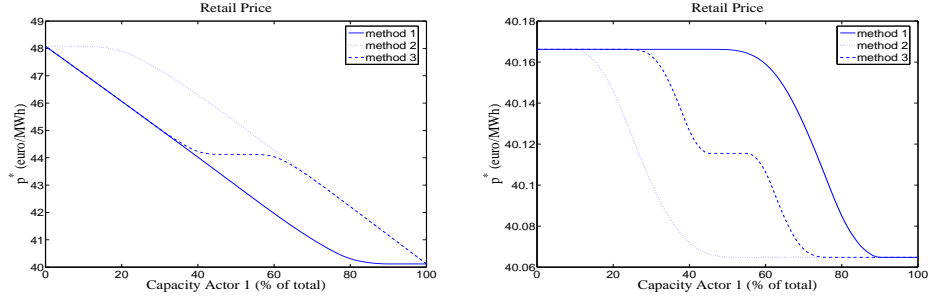


Figure 5.8: Retail price for  $\lambda_1 = \lambda_2 = 10^{-6}$ , without (left) and with (right) a forward market, as a function of Agent 1's proportion of total capacity.

utility.

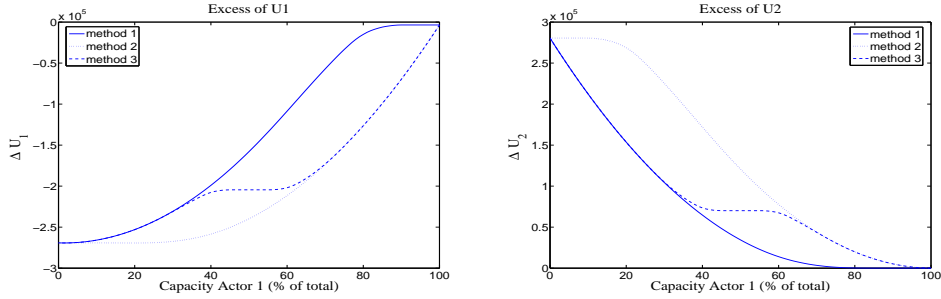


Figure 5.9: Gain in utility for Agent 1 and Agent 2 due to the forward market, as a function of Agent 1's proportion of total capacity.

We now let the risk aversion coefficients vary for a given distribution of generation assets. In the absence of a forward market, Figure 5.10 presents Agent 1 with 70 % of generation capacity on the left, and 60 % on the right. Comparing with Figure 5.6, an equilibrium always exists if the level of integration of Agent 1, i.e. its proportion of total capacity, is high enough. This proves that vertical integration can induce better risk diversification than forward trading when retailers are highly risk averse. Vertical integration is more robust to high risk aversion.

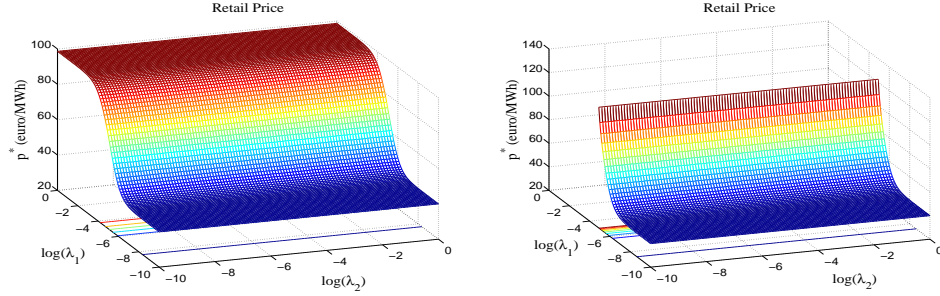


Figure 5.10: Retail price without a forward market, when Agent 1 owns 70 % (left) and 60 % (right) of production capacity, as a function of risk aversion coefficients.

#### 5.2.4 Competition between integrated producers

In a fully integrated market structure, the retail price is affected neither by the forward market nor by the distribution of the generation capacity. As before, we start by setting the risk aversion coefficients  $\lambda_1 = \lambda_2 = 10^{-6}$ , and we let the distribution of generation assets vary. As in paragraph 5.2.1, Figure 5.11 shows that the forward market enhances

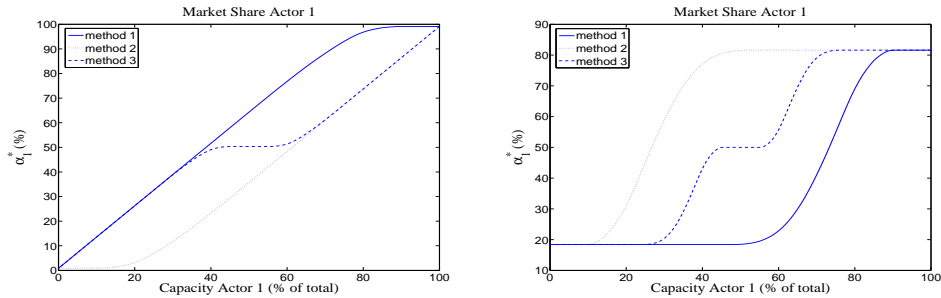


Figure 5.11: Agent 1's market share without (left) and with (right) the forward market, for  $\lambda_1 = \lambda_2 = 10^{-6}$ , as a function of Agent 1's proportion of total capacity.

the ability of an agent that owns little generation capacity to take a significant position on the retail market. Both agents utilities increase with the total capacity, both with and without a forward market, as Figure 5.12-left shows. Agents are symmetric, and they have the same behavior. In addition, the forward market increases both agents' utility (see Figure 5.12-right), although the increase is very small (in the range of 0.1%).

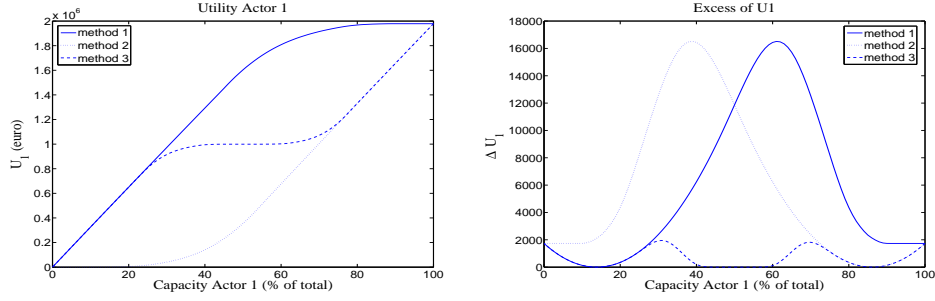


Figure 5.12: Utility without the forward market (left) and gain in utility due to the forward market (right) for Agent 1 with  $\lambda_1 = \lambda_2 = 10^{-6}$ .

### 5.3 The effect of vertical integration

We focus on the effect of vertical structure on the agents' utility.<sup>19</sup>

#### 5.3.1 Methodology

We consider two downstream firms  $R_1, R_2$ , and two upstream firms  $P_1, P_2$ . We assume that  $R_1$  and  $P_1$  decide to merge, hereby creating an integrated producer  $I_1$ . Then we allow  $R_2$  and  $P_2$  to merge, thereby creating another integrated producer  $I_2$ . We assume that  $R_1, R_2, P_1$  and  $P_2$  have the same risk aversion coefficient  $\lambda$ .<sup>20</sup>

In order to evaluate the benefit that agents  $R_1$  and  $P_1$  would obtain if they merge, we compare the utility  $MV_\lambda[\Pi_{I_1}]$  of the resulting entity to the utility  $MV_\lambda[\Pi_{R_1} + \Pi_{w_1}]$  to  $R_1$  and  $P_1$ . We refer to Agent 1 as either the pair  $R_1, P_1$  or  $I_1$  if integration took place.

<sup>19</sup>Results concerning retail prices and market shares are similar to those of the previous section and confirm our conclusions. They are available upon request.

<sup>20</sup>Integrated structures' risk aversion coefficient may depend on the synergies resulting from the merger, the cost reductions, the risk management policy in the new structure, etc. Without a forward market, the producers risk aversion coefficients do not affect the equilibrium. Hence, it seems reasonable to attribute a coefficient  $\lambda$  to  $I_1$  and  $I_2$ . This is no longer the case with the forward market, and the linearity of the equilibrium suggests that risk tolerances should be summed among agents of the same type. Nonetheless, when aggregating agents of different types, this argument is not clear. In the absence of a clear-cut answer, we choose to apply the same rule as in the absence of a forward market and attribute a coefficient  $\lambda$  to  $I_1$  and  $I_2$ . This is consistent with the idea that firms face the same frictions, e.g. the same tax code and bankruptcy costs.

### 5.3.2 Vertical integration without and with a forward market

Figure 5.13 shows that when there is no forward market a firm is worse off being integrated, whatever the type of its competitors, when  $\lambda = 10^{-6}$ . As with forward hedging, vertical integration induces a decrease in the retail price that reduces the expected profit more than the decrease in the variance. Since there is no effect that prompts the agents to vertically integrate although they suffer a decrease in utility, the equilibrium is unbundling. The

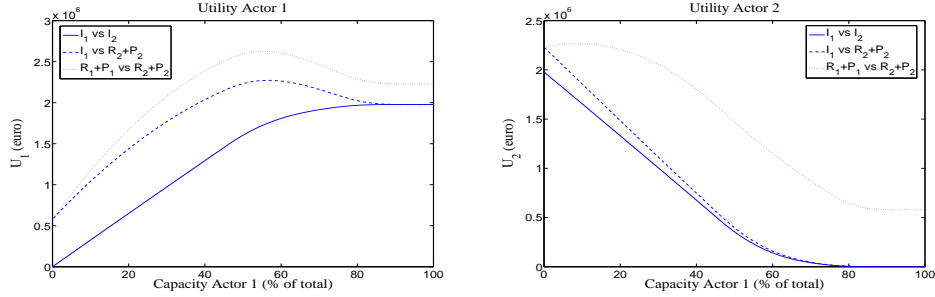


Figure 5.13: Agent 1's (left) and Agent 2's (right) utilities without a forward market, as functions of Agent 1's proportion of total capacity.

great difference between vertical integration and forward trading is that vertical integration can increase utility when agents are highly risk averse.

The same computation as in Figure 5.13 with a risk aversion coefficient  $\lambda = 10^{-5.1}$  is depicted in Figure 5.14. If  $R_1$  owns enough generation capacity, then  $I_1$ 's utility when facing  $R_2 + P_2$  is higher than that of  $R_1 + P_1$ . Hence, there is an incentive for vertical integration when other agents are non-integrated. Similarly, Figure 5.14-right shows that  $I_2$ 's utility when facing  $I_1$  is higher than that of  $R_2 + P_2$ , which confers  $I_2$  with an incentive for vertical integration when competing with integrated agents. The equilibrium is then full-integration.

In the presence of a forward market, when the risk aversion coefficients equal  $\lambda = 10^{-6}$ , the effect of vertical integration is drastically reduced (Figure 5.15). Integrating has almost no effect on utility. In addition, even for a larger risk aversion coefficient (ex:  $\lambda = 10^{-5.1}$ ), the forward market reduces the incentive for vertical integration.

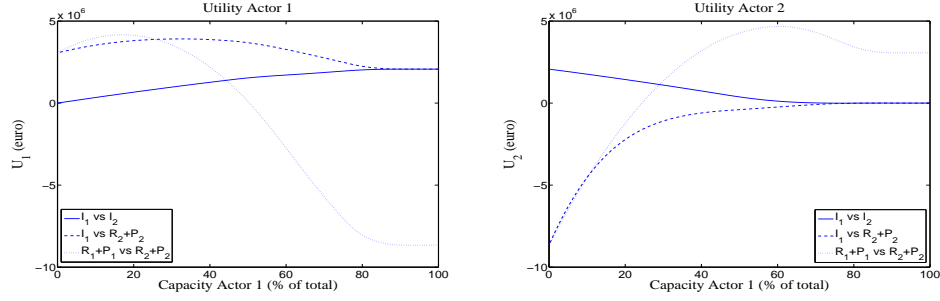


Figure 5.14: Agent 1's (left) and Agent 2's (right) utility without a forward market, as functions of Agent 1's proportion of total capacity.

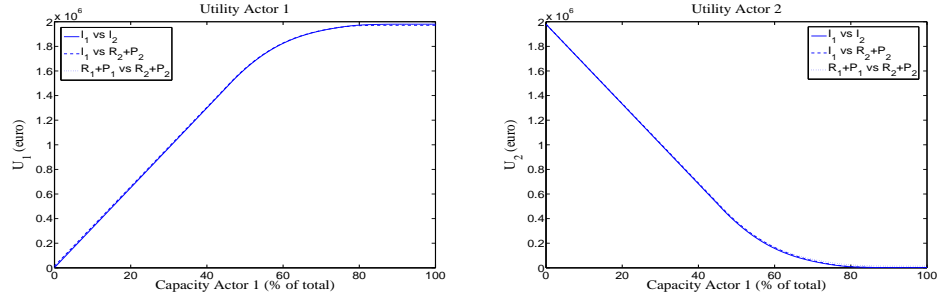


Figure 5.15: Agent 1's (left) and Agent 2's (right) utilities with a forward market, as functions of Agent 1's proportion of total capacity.

## 6 Further Empirical Predictions

We have shown and illustrated that forward markets reduce the prevalence of vertical integration in an electricity market. Specifically, forward markets enable retailers that have no generation capacity to grab a significant market share. Furthermore, both vertical integration and forward markets are risk-management mechanisms that lead to a decrease in retail prices.

More generally, our analysis predicts that there is a negative relationship between the development of forward markets and firms' incentives to merge with vertically related segments. Other things equal, we expect the prevalence of vertical integration to be higher in industries that are more subject to uncertainty and where other risk management mechanisms are less readily available. In particular, we expect that this would be the case



when hedging markets are less readily available.

This should also be true in industries where goods are non storable or where storage costs are higher. Since, when goods are durable, production timing can be adjusted so as to manage risk, forward markets and vertical integration should be less important than in industries that have non-durable goods.

In addition, be there developed forward markets or not, we expect vertical integration to be more widespread in industries that are subject to greater risk aversion, in particular through greater regulatory pressure, higher bankruptcy costs and higher corporate taxes.

## 7 Concluding Remarks

We have shown that in a risk management setting, vertical integration reduces retail prices. It exhibits properties that linear instruments such as forward contracts cannot achieve. It reduces the asymmetric risk structure between upstream and downstream segments. We have shown a number of mechanisms through which some level of integration is beneficial even in the presence of wholesale markets.

A next step would be to examine the possibility of large agents and market power. Allaz (1992) has pointed out that forward trading may reduce producers' market power. In contrast, vertical integration sometimes extends market power (Bolton and Whinston (1993), Chemla (2003), Rey and Tirole (2006)). It would be interesting to study the resulting effect when these two mechanisms coexist. Future developments in the field of market equilibria should include those elements into the risk management analysis. Our analysis could then serve as a benchmark to quantify the effects of imperfect competition.

Another avenue for future research may be to examine the decision of whether or not trading activities and operating activities should be under the same corporate roof. Such analysis would likely combine issues related to trading and arbitrage with transactions costs, e.g. financial constraints, as in Gromb and Vayanos (2004), with questions such as those discussed in this paper.

## A Appendix A: Proofs

### A.1 Proof of Proposition 3.1

Maximizing the mean-variance criteria over  $\alpha_k$  yields the following first order condition for agent  $k \in \mathcal{R}$ :

$$0 = \mathbb{E}[(p^* - w^*)D] - 2\lambda_k \text{Cov}[(p^* - w^*)D, (p^* - w^*)\alpha_k^* D + \Pi_k^g],$$

By convexity, the first-order condition is sufficient and gives:

$$\alpha_k^* = \frac{\mathbb{E}[(p^* - w^*)D]}{2\lambda_k \text{Var}[(p^* - w^*)D]} - \frac{\text{Cov}[(p^* - w^*)D, \Pi_k^g]}{\text{Var}[(p^* - w^*)D]}.$$

The coupling constraint  $\sum_{k \in \mathcal{R}} \alpha_k^* = 1$  gives the condition on  $p^*$ :

$$0 = \mathbb{E}[(p^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Var}[(p^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Cov}[(p^* - w^*)D, \Pi_I^g],$$

and allows us to derive (3.3).

### A.2 Proof of Proposition 3.2

Assume that some retailers are risk neutral, so that the aggregate risk aversion coefficient  $\Lambda_{\mathcal{R}}$  also tends to zero. Suppose equation (3.3) has two non-negative solutions  $p_- \leq p_+$ .

The Taylor expansion of these roots around  $\Lambda_{\mathcal{R}} = 0$  reads:

$$\begin{aligned} p_- &\simeq p^0 + \frac{2\Lambda_{\mathcal{R}}}{\mathbb{E}[D]\text{Var}[D]} \left( \text{Var}[D]\text{Cov}[wD, wD - \Pi_I^g] - \text{Cov}^2[D, wD - \frac{1}{2}\Pi_I^g] \right. \\ &\quad \left. + 2\text{Cov}^2[D, wD - \frac{1}{2}\Pi_I^g - p^0 D] \right) \\ p_+ &\simeq \frac{\mathbb{E}[D]}{2\Lambda_{\mathcal{R}}\text{Var}[D]}. \end{aligned}$$

This shows that  $p_-$  tends to  $p^0$  as  $\Lambda_{\mathcal{R}}$  tends to 0, while  $p_+$  tends to infinity. Hence,  $p_-$  is the economically relevant root, and the equilibrium is in fact uniquely characterized.

### A.3 Proof of Proposition 3.3

We denote by  $p_{NI}^*$  the smallest solution to (3.3) in the absence of vertical integration, and by  $p_I^*$  this solution when at least one producer is integrated with a retailer. Suppose

that  $\mathcal{R} \cap \mathcal{P} = \emptyset$ , i.e. there is no integrated producer who have both generation and retail activities. Equation (3.3) then reduces to:

$$0 = \mathbb{E}[(p_{NI}^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Var}[(p_{NI}^* - w^*)D] . \quad (\text{A.1})$$

As a consequence, it is also greater than the expected spot price.

When some agents are integrated, i.e.  $\mathcal{R} \cap \mathcal{P} \neq \emptyset$ , the equation that determines the retail price is

$$0 = \mathbb{E}[(p_I^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Cov}[(p_I^* - w^*)D, (p_I^* - w^*)D + \Pi_I^g] .$$

Suppose that one retailer, say  $i$ , chooses to vertically integrate with one producer. Then,  $\Lambda_{\mathcal{R}}$  is unchanged and  $\Pi_I^g = \Pi_i^g$ .

Since  $\Pi^r$  and  $\Pi_i^g$  are negatively correlated,

$$0 \geq \mathbb{E}[(p_I^* - w^*)D] - 2\Lambda_{\mathcal{R}} \text{Var}[(p_I^* - w^*)D] . \quad (\text{A.2})$$

Inequality (A.2), combined with (A.1), shows that  $p_I^*$  is lower than  $p_{NI}^*$ . Integrated firms then tend to decrease the retail price.

#### A.4 Proof of Proposition 4.1

If  $k \in \mathcal{R}$ , the first order condition to profit maximization can be written:

$$0 = M \begin{bmatrix} f_k^* \\ \alpha_k^* \end{bmatrix} - \begin{bmatrix} \frac{\mathbb{E}[w^* - q^*]}{2\lambda_k} - \text{Cov}[P^*, \Pi_k^g] \\ \frac{\mathbb{E}[(p^* - w^*)D]}{2\lambda_k} - \text{Cov}[(p^* - w^*)D, \Pi_k^g] \end{bmatrix} ,$$

where  $M$  is the variance-covariance matrix of vector  $[w^*, (p^* - w^*)D]$ . By inverting the system, we obtain  $f_k^*$  and  $\alpha_k^*$  in terms of  $p^*$  and  $q^*$ . If  $k \notin \mathcal{R}$ ,  $\Pi_k$  does not depend on  $\alpha_k$  and the first order condition reads:

$$0 = \mathbb{E}[w^* - q^*] - 2\lambda_k \text{Cov}[w^* - q^*, (w^* - q^*)f_k^* + \Pi_k^g] .$$

The market-clearing constraint (2.4) can be written:

$$\begin{aligned}
0 &= \sum_{k \in \mathcal{K}} f_k^* \\
&= \frac{\text{Var}[(p^* - w^*)D]}{\Delta} \left( \frac{\mathbb{E}[w^* - q^*]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[w^*, \sum_{k \in \mathcal{R}} \Pi_k^g] \right) \\
&\quad - \frac{\text{Cov}[w^*, (p^* - w^*)D]}{\Delta} \left( \frac{\mathbb{E}[(p^* - w^*)D]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[(p^* - w^*)D, \sum_{k \in \mathcal{R}} \Pi_k^g] \right) \\
&\quad + \frac{\mathbb{E}[w^* - q^*]}{2\text{Var}[w^*]} \left( \frac{1}{\Lambda} - \frac{1}{\Lambda_{\mathcal{R}}} \right) - \frac{\text{Cov}[w^*, \sum_{k \in \mathcal{R}} \Pi_k^g]}{\text{Var}[w^*]},
\end{aligned}$$

where  $\Delta$  is in fact the determinant of  $M$ , which leads to:

$$\begin{aligned}
0 &= \Delta \left( \frac{\mathbb{E}[w^* - q^*]}{2\Lambda} - \text{Cov}[w^*, \sum_{k \in \mathcal{K}} \Pi_k^g] \right) \\
&\quad - \text{Cov}[w^*, (p^* - w^*)D] \text{Var}[w^*] \left( \frac{\mathbb{E}[(p^* - w^*)D]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[(p^* - w^*)D, \sum_{k \in \mathcal{R}} \Pi_k^g] \right) \\
&\quad + \text{Cov}^2[w^*, (p^* - w^*)D] \left( \frac{\mathbb{E}[w^* - q^*]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[w^*, \sum_{k \in \mathcal{R}} \Pi_k^g] \right).
\end{aligned} \tag{A.3}$$

The load satisfaction constraint (2.1) reads:

$$\begin{aligned}
1 &= \sum_{k \in \mathcal{R}} \alpha_k^* \\
&= - \frac{\text{Cov}[w^*, (p^* - w^*)D]}{\Delta} \left( \frac{\mathbb{E}[w^* - q^*]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[w^*, \sum_{k \in \mathcal{R}} \Pi_k^g] \right) \\
&\quad + \frac{\text{Var}[w^*]}{\Delta} \left( \frac{\mathbb{E}[(p^* - w^*)D]}{2\Lambda_{\mathcal{R}}} - \text{Cov}[(p^* - w^*)D, \sum_{k \in \mathcal{R}} \Pi_k^g] \right),
\end{aligned}$$

which yields, using (A.3):

$$0 = \frac{\mathbb{E}[w^* - q^*]}{2\Lambda} - \text{Cov}[w^*, (p^* - w^*)D] + \sum_{k \in \mathcal{K}} \Pi_k^g.$$

As  $\sum_{k \in \mathcal{K}} \Pi_k^g = w^*D - C(D)$ , we obtain equation (4.3). Using this result to simplify (A.3), we derive the desired (4.4). Finally, from these two equations, we can re-arrange for  $f_k^*$  and  $\alpha_k^*$  to obtain (4.1) and (4.2).

## A.5 Equations for the retail price under elastic demand

The retail price,  $p^*$ , is then given by the smallest root of equation:

$$\begin{aligned}
0 = & (p^*)^2 \left\{ -\mu \left( 1 + \mu \frac{c}{N_P} \right) - 2\Lambda_{\mathcal{R}} \left[ 1 + 4\mu \frac{c}{N_P} \left( 1 + \mu \frac{c}{N_P} \right) - \mu \frac{c}{N_P} \frac{N_I}{N_P} \left( 1 + 2\mu \frac{c}{N_P} \right) \right] \text{Var}[D_0] \right\} \\
& + p^* \left\{ \left( 1 + 2\mu \frac{c}{N_P} \right) \mathbb{E}[D_0] + \mu p_0 \left( 1 + 2\mu \frac{c}{N_P} \right) \right\} \\
& + p^* \Lambda_{\mathcal{R}} \frac{c}{N_P} \left( 4 - \frac{N_I}{N_P} + 4\mu \frac{c}{N_P} \left( 2 - \frac{N_I}{N_P} \right) \right) (\text{Cov}[D_0, D_0^2] + 2\mu p_0 \text{Var}[D_0]) \\
& - \frac{c}{N_P} (\mathbb{E}[D_0^2] + 2\mu p_0 \mathbb{E}[D_0] + \mu^2 p_0^2) \\
& - \Lambda_{\mathcal{R}} \frac{c^2}{N_P^2} \left( 2 - \frac{N_I}{N_P} \right) (\text{Var}[D_0^2] + 4\mu p_0 \text{Cov}[D_0, D_0^2] + 4\mu^2 p_0^2 \text{Var}[D_0])
\end{aligned}$$

in the absence of a forward market, and by:

$$\begin{aligned}
0 = & (p^*)^2 \left\{ -\mu - \mu^2 \frac{c}{N_P} - 2\Lambda \left( 1 + 3\mu \frac{c}{N_P} + 2\mu^2 \frac{c^2}{N_P^2} \right) \text{Var}[D_0] \right\} \\
& + p^* \left\{ \left( 1 + 2\mu \frac{c}{N_P} \right) \mathbb{E}[D_0] + \mu p_0 \left( 1 + 2\mu \frac{c}{N_P} \right) \right\} \\
& + p^* \Lambda \frac{c}{N_P} \left( 3 + 4\mu \frac{c}{N_P} \right) (\text{Cov}[D_0, D_0^2] + 2\mu p_0 \text{Var}[D_0]) \\
& - \frac{c}{N_P} (\mathbb{E}[D_0^2] + 2\mu p_0 \mathbb{E}[D_0] + \mu^2 p_0^2) - \Lambda_{\mathcal{R}} \frac{c^2}{N_P^2} \left( 2 - \frac{N_I}{N_P} \right) \text{Var}[D_0^2] \\
& + 2\Lambda_{\mathcal{R}} \frac{c^2}{N_P^2} \left( 1 - \frac{\Lambda}{2\Lambda_{\mathcal{R}}} - \frac{N_I}{2N_P} \right) \frac{\text{Cov}^2[D_0, D_0^2]}{\text{Var}[D_0]} \\
& - 4\mu p_0 \Lambda \frac{c^2}{N_P^2} (\text{Cov}[D_0, D_0^2] + \mu p_0 \text{Var}[D_0])
\end{aligned}$$

in the presence of a forward market.

## B Appendix B: Elastic Demand Curve

If demand is elastic to spot price, our results are unchanged because the spot market equilibrium remains independent of retail and forward equilibria.<sup>21</sup> As mentioned in Remark ??, the model can also be solved with a demand curve that is elastic to the retail price. Suppose that the demand is a random function of retail price  $D(p)$ . We can solve the problem as in Sections 3 or 4 and equations (3.2) and (4.1)-(4.2)-(4.3) remain valid. The

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<sup>21</sup>This may not be true under imperfect competition.

only difference lies in the equation for  $p^*$ . In the presence of elasticity to retail prices, this equation becomes

$$0 = \mathbb{E}[(p^* - w^*(p^*))D(p^*)] \\ - 2\Lambda_{\mathcal{R}}\text{Cov}[(p^* - w^*(p^*))D(p^*), (p^* - w^*(p^*))D(p^*) + \Pi_I^g(p^*)]$$

in the absence of a forward market, and

$$0 = \mathbb{E}[(p^* - w^*(p^*))D(p^*)] - 2\Lambda_{\mathcal{R}}\text{Cov}[(p^* - w^*(p^*))D(p^*), (p^* - w^*(p^*))D(p^*) + \Pi_I^g(p^*)] \\ + 2\Lambda_{\mathcal{R}}\frac{\text{Cov}[w^*(p^*), (p^* - w^*(p^*))D(p^*)]}{\text{Var}[w^*(p^*)]}\text{Cov}[w^*(p^*), (p^* - w^*(p^*))D(p^*) + \Pi_I^g(p^*)] \\ - 2\Lambda\frac{\text{Cov}[w^*(p^*), (p^* - w^*(p^*))D(p^*)]}{\text{Var}[w^*(p^*)]}\text{Cov}[w^*(p^*), p^*D(p^*) - C(D(p^*))]$$

with a forward market. This non-linear equation may be hard to solve, especially if we cannot formulate the spot market equilibrium. Nevertheless, the equation simplifies in some cases, as we show in the following subsection.

### B.0.1 Quadratic cost functions

Consider quadratic and symmetric cost functions  $c_k(x) = \frac{c}{2}x^2$ ,  $\forall k \in \mathcal{K}$ . Suppose that demand is a linear function of the retail price:

$$D(p) = D_0 - \mu(p - p_0),$$

where  $D_0$  is an exogenous random variable,  $p_0$  is some non-negative reference price and  $\mu > 0$ . Then the spot market equilibrium can be written:

$$\begin{aligned} S_k^* &= \frac{1}{N_P}D(p^*), & w^* &= \frac{c}{N_P}D(p^*) \\ \Pi_k^g &= \frac{c}{2N_P^2}D^2(p^*), & \Pi_I^g &= \frac{cN_I}{2N_P^2}D^2(p^*) \\ \Pi^g &= \frac{c}{2N_P}D^2(p^*) \end{aligned} \tag{B.1}$$

where  $N_P$  is the number of producers and  $N_I$  the number of integrated firms. The retail price  $p^*$  is then given by the smallest root of a second order polynomial equation (cf. Appendix A).

### B.0.2 Examples

To illustrate the effect of demand elasticity, we compute the equilibrium found above in two cases: one downstream firm and one upstream firm, as we did in paragraph 5.2.2, and

one downstream firm and one integrated firm, as in paragraph 5.2.1. We use the demand samples in the previous section. Taking expectations on both sides of  $w^*$  in (B.1), we estimate the cost function coefficient  $c$  as:

$$c = N_P \frac{\mathbb{E}[w^*]}{\mathbb{E}[D]} \simeq N_P \times 5.6143 \cdot 10^{-4}.$$

We set  $p_0 = \mathbb{E}[w] \simeq 37.95$  as the expected spot price, and we write the elasticity coefficient  $\mu$  in percentage of expected demand  $\mathbb{E}[D]$ .

- **Downstream firm and upstream firm.** Denote the upstream firm as Agent 1 and the downstream firm as Agent 2. We set both risk aversion coefficients to  $\lambda_1 = \lambda_2 = 10^{-6}$ . Figure B.16-right shows the retail price with and without a forward market, as a function of  $\mu$ , in percentage of  $\mathbb{E}[D]$ . Again, the presence of

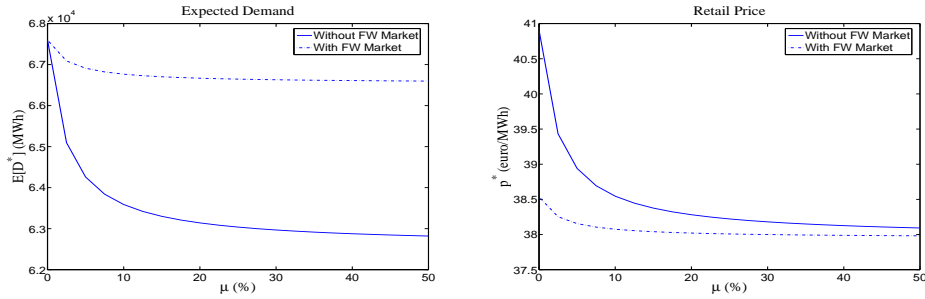


Figure B.16: Expected demand (left) and retail price (right) with and without a forward market as a function of  $\mu$ .

a forward market decreases the retail price. Nonetheless, this effect tends to shrink as demand elasticity increases. We also note that, in both cases, demand elasticity decreases the retail price. As consumers respond to the retail price, the retailers face low demand if they set a high retail price. They are thus led to decrease the retail price. Figure B.17 shows Agent 1's utility with and without a forward market. This utility increases in the presence of a forward market and the upstream firm's utility decreases with demand elasticity. The asymmetry that enables upstream firms to set the spot price and thus to affect downstream firms' profit, while downstream firms cannot affect upstream firms' profit. is reduced when demand is elastic to retail price, as downstream firms affect upstream firms' profit via the retail price.

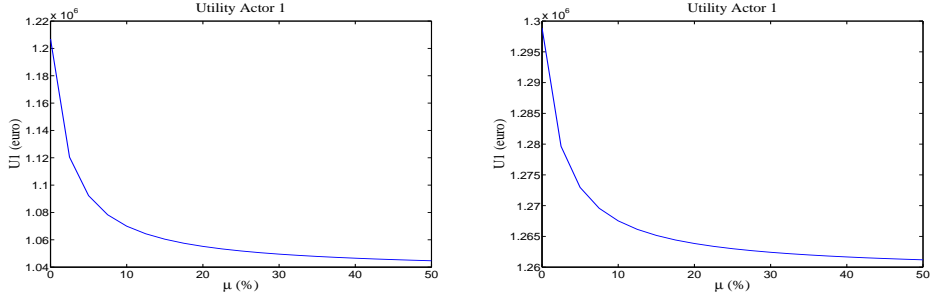


Figure B.17: Agent 1's utility with (right) and without (left) a forward market as a function of  $\mu$ .

Since the expected demand decreases with demand elasticity (see Figure B.16-left), so does the producers' utility. In addition, while the relationship between Agent

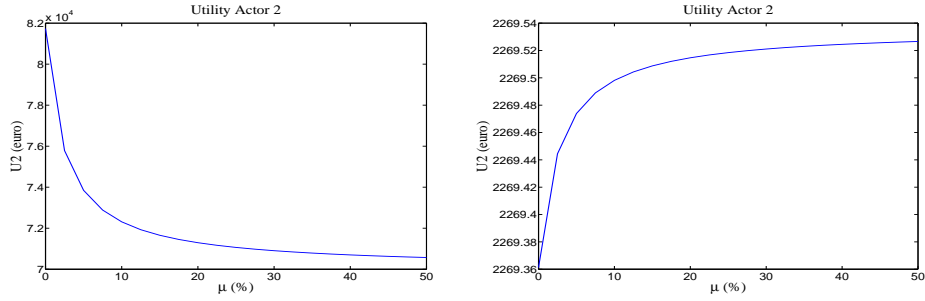


Figure B.18: Agent 2's utility with (right) and without (left) a forward market as a function of  $\mu$ .

2's utility and demand elasticity is negative when there is no forward market, it is positive in the presence of a forward market (Figure B.18). When there is no forward market, the downstream firm is affected by the decrease in both demand and the retail price. With a forward contract, the downstream firm can transfer more risk to the upstream firm and take advantage of demand elasticity. In particular the gain in Agent 2's utility due to the forward market increases with demand elasticity, as we suggested in Subsection 5.2.2.

- **Upstream firm and integrated firm.** Let Agent 1 be an integrated firm and Agent 2 be a downstream firm. The risk aversion coefficients to both agents are



set to  $\lambda_1 = \lambda_2 = 10^{-6}$ . Figure B.19-right shows Agent 1's market share with and

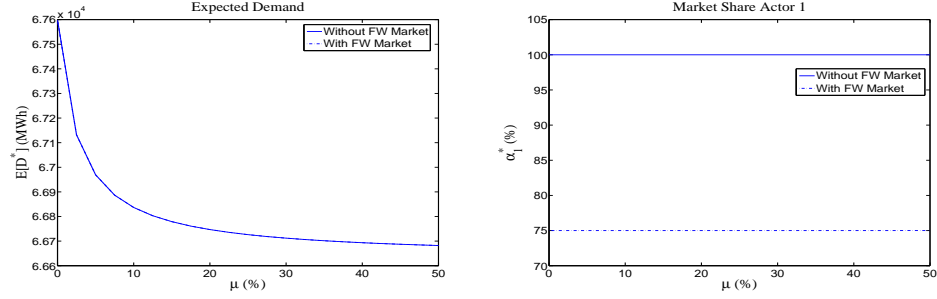


Figure B.19: Expected demand (left) and Agent 1's market share (right) with and without a forward market as a function of  $\mu$ .

without a forward market. As in Subsection 5.2.1, the downstream firm cannot compete with the integrated retailer in the absence of a forward market. In this example, both agents have the same risk aversion coefficient. Agent 2 cannot enter the market and it has a market share almost equal to zero. The forward market enables Agent 2 to obtain a market share of 25%. In addition, market shares and the benefit of integration are not affected by demand elasticity.

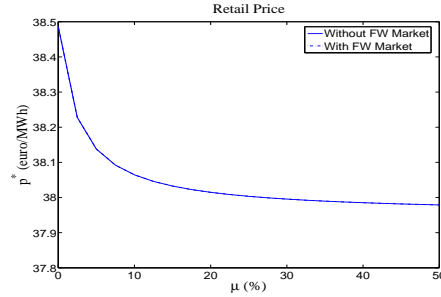


Figure B.20: Retail price as a function of  $\mu$ .

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