



**Lancaster University Management School**  
**Working Paper**  
**2006/024**

**On The Relationship between Nominal**  
**Exchange rates and domestic and foreign prices**

Ivan Paya and David Peel

The Department of Economics  
Lancaster University Management School  
Lancaster LA1 4YX  
UK

© Ivan Paya and David Peel  
All rights reserved. Short sections of text, not to exceed  
two paragraphs, may be quoted without explicit permission,  
provided that full acknowledgement is given.

The LUMS Working Papers series can be accessed at <http://www.lums.lancs.ac.uk/publications/>  
LUMS home page: <http://www.lums.lancs.ac.uk/>

# On The Relationship between Nominal Exchange Rates and Domestic And Foreign Prices

Ivan Paya<sup>a,1</sup>, David A. Peel<sup>b</sup>

<sup>a</sup>Departamento Fundamentos Analisis Economico, University of Alicante, 03080  
Alicante, Spain.

<sup>b</sup>Lancaster University Management School, Lancaster, LA1 4YX, UK

## Abstract

A number of authors have found significant cointegrating relationships between spot exchange rates and domestic and foreign price levels for the major currencies where the magnitude of the coefficients makes economic interpretation of PPP cumbersome. Using theoretically well motivated nonlinear models for “artificially” created real exchange rates, this paper investigates the properties of two alternative cointegration procedures, namely the Johansen and Saikkonen methodologies. The latter procedure appears to outperform the former one in terms of finding the “true” cointegrating coefficients. The new weights obtained with the Saikkonen method are then used to estimate nonlinear ESTAR model for the real exchange rate. The “new” real exchange rates exhibit, in most cases, much lower half-life shocks than the ones predicted by the Rogoff (1996) puzzle.

Keywords: PPP, Johansen, Saikkonen, bootstrap.

JEL classification: F31, C22, C51

---

<sup>1</sup>**Corresponding author.** E-mail address: ivanpaya@merlin.fae.ua.es Tel.: +34 965 903614 Fax: +34 965 903898.

Ivan Paya acknowledges financial support from the IVIE.

# 1 Introduction

A number of authors have employed multivariate cointegration methodology to test for a long run relationship between exchange rates and foreign and domestic price levels in the recent floating exchange rate period. (see e.g., MacDonald, 1993; and Baum et al., 2001). The standard empirical findings employing these methods are that cointegration cannot be rejected but the assumption of proportionality between the nominal exchange rate and domestic and foreign prices is not supported by the data.<sup>2</sup>

Nevertheless the magnitude and instability of the reported coefficients are sometimes such that it is difficult to take them seriously. Table 1 presents some of the results reported in the literature for a sample of currencies. For instance, MacDonald(1993), employing Johansen's multivariate cointegration methodology, reports coefficients of 15.166 and -7.825 for the US and German consumer price levels in the cointegrating vector for the US Dollar/Mark exchange rate.<sup>3</sup> The reported coefficients employing wholesale prices for the same currency are 65.984 and -37.594. Employing the same cointegration method Baum et al. (2001), employing a longer span of data, report coefficients for consumer prices of 2.7 and -0.668 for the Dollar/Mark, and 62.11 and -36.466 for wholesale prices. A sample of other results are reported in Table 1.

The sample of results reported in Table 1 illustrates that the coefficients in

---

<sup>2</sup>If we assume that some goods are non-traded and that the consumer price index is a weighted average of traded and non-traded prices then it is, of course, well known that the real exchange rate in terms of consumer prices is given by

$$s_t = \frac{p_t}{\alpha} - \frac{p_t^*}{\beta} - \frac{(1-\alpha)p_t^{nt}}{\alpha} + \frac{(1-\beta)p_t^{*nt}}{\beta}$$

where  $s_t$  is the exchange rate,  $p_t, p_t^*$  are domestic and foreign consumer prices,  $p_t^{nt}, p_t^{*nt}$  are domestic and foreign non-traded prices,  $\alpha, \beta$  are the weight of traded goods in the domestic and consumer price index. Clearly cointegration between the exchange rate and domestic and foreign consumer prices requires that the prices of nontraded goods are stationary or cointegrated.

<sup>3</sup>Cheung and Lai(1993) report similar coefficients, 4.97 and -7.64.

the reported cointegrating vectors can be extremely large, with the coefficients on prices ranging wildly and also exhibiting the wrong sign.(see Baum et al. (2001) results for Dollar/DM).<sup>4</sup> They also appear to exhibit large instabilities as observations are added to the samples.(see the Dollar/Pound entry for consumer prices).<sup>5</sup>

Froot and Rogoff (1995) suggest that the interpretation of the cointegration results may not have a clear economic interpretation. They conjecture that due to serious small-sample bias the coefficients obtained in cointegration analysis can vary widely across country pairs making economic interpretation very difficult.

Subsequent to the conjecture of Froot and Rogoff we have seen a number of papers published which suggest that purchasing power deviations (PPP) can be parsimoniously described by a nonlinear adjustment process (see e.g., Michael, Nobay and Peel, 1997; Obstfeld and Taylor, 1997;; Taylor et al., 2001). These papers are empirical applications of the recent theoretical analysis of purchasing power deviations (see e.g., Dixit, 1989; Dumas, 1992; Uppal, 1993; Sercu et al., 1995; O’Connell, 1997; Ohanian and Stockman, 1997; and O’Connell and

<sup>4</sup>Baum et al. report results for several other countries where the coefficients are incorrectly signed. They do not comment on this.

<sup>5</sup>In this paper, we extend the sample on spot exchange rates and prices used in the MacDonald and Baum et al. studies from January 1973 to May 2001 for consumer prices. We then estimate the PPP hypothesis using the same multivariate cointegration methodology. There is no evidence for cointegration at 5% in the dollar/DM exchange rate.

Country	$k$	$\beta' = (S_t, P_t, P_t^*)$	$TR$
Germany	26	(1,-31.35,31.12)	21.09
Japan	7	(1,-15.18,36.64)	72.90*
UK	13	(1,-2.44,1.59)	50.73*
France	8	(1,-1.99,0.353)	35.22*
Netherlands	13	(1,-4.596,5.51)	44.58*

Wei, 1997), which demonstrate how transactions costs or the sunk costs of international arbitrage induce nonlinear adjustment of the real exchange rate to purchasing power parity. Whilst globally mean reverting this nonlinear process has the property of exhibiting near unit root behavior for small deviations from PPP, since small deviations from PPP are left uncorrected if they are not large enough to cover the transactions costs or the sunk costs involved in international arbitrage.

Two parametric nonlinear models, suggested by the theoretical literature, that captures the nonlinear adjustment process in aggregate data are the exponential smooth transition autoregression model (ESTAR), of Granger and Terasvirta (1993) in which transitions between regimes (small or large deviations) is assumed to occur smoothly or the threshold process of Tong (1990) in which adjustment is assumed to be abrupt. A smooth adjustment process is suggested in the analysis of Dumas (1992). Also, as noted by Terasvirta (1994), in aggregate data regime changes may appear to be smooth rather than discrete given that heterogeneous agents do not act simultaneously even if they make dichotomous decisions.<sup>6</sup>

A key property of some ESTAR models (also shared by some Threshold models) is that data simulated from them, although globally mean reverting, can appear to exhibit a unit root (Davutyan and Pippenger, 1985; Pippenger and Goering, 1993; Michael et al., 1997; Taylor et al., 2001). In Figure 1 we plot the deterministic relationship between changes in purchasing power deviations from equilibrium and lagged deviations obtained from the ESTAR model.

$$y_t = e^{\gamma(y_{t-1})^2} y_{t-1} \quad (1)$$

where  $y_t$  =deviation of purchasing power from equilibrium, assumed zero and  $\gamma$  is a negative constant.

---

<sup>6</sup>See also Anderson (1997) for an empirical application of agent heterogeneity and smooth transition in the bond market.

This ESTAR model has been found to provide a parsimonious fit in empirical work (see Taylor et al., 2001). It is clear from Figure 1 why data simulated from this ESTAR model could appear to exhibit a unit root. In the vicinity of equilibrium the process mimics a unit root process. As a consequence the test proposed in Froot and Rogoff (1995), namely that we impose unit coefficients and test directly, employing unit root tests, whether PPP deviations are mean reverting, can have low power if the true data generating process is nonlinear. From this perspective it is interesting that Sarantis (1999), Taylor et al (2001) impose unit coefficients and estimate nonlinear models for PPP deviations which appear parsimonious descriptions of the data process. Of particular interest are the results reported in Taylor et al., (2001) who employ the ESTAR specification. Nonlinear impulse response functions obtained from their estimated ESTAR models show that whilst the speed of adjustment for small shocks around equilibrium will be highly persistent, larger shocks mean-revert much faster than the “glacial rates” previously reported for linear models (Rogoff, 1996). In this respect, the nonlinear models provide some solution to the PPP puzzle outlined in Rogoff (1996).

In this paper we initially assume that adjustment to purchasing power parity (PPP) in the true data generating process can be captured by the ESTAR process given by (1). Using simulated data from such a model, in which proportionality,  $(1, -1)$ , is imposed, we examine the empirical results obtained when the Johansen method is employed to determine whether the spot exchange rate is cointegrated with domestic and foreign prices and whether proportionality can be rejected. Our empirical results show that the Johansen method produces poor estimates, on average, of the cointegrating vector, with a range of values that include those reported using this method in the literature.

We also analyse the asymptotically efficient estimator for cointegration regression introduced by Saikkonen (1991). This estimator is preferred over other single equation estimators such as Phillips and Hansen (1990) on the basis both

of asymptotic efficiency and that no initial estimates of coefficients are required. Using simulated data we find that the Saikkonen estimator produces estimates of the cointegrating weights which are much closer on average to their true values, with much smaller standard errors than the Johansen method.

Given this result we employ the Saikkonen estimator on three data sets and find that proportionality is rejected, though with coefficients that are a priori more sensible. We estimate ESTAR models employing these weights and find significant evidence of nonlinear adjustment employing non unit weights. We bootstrap these estimated models and obtain estimates of the bias. Employing bias corrected weights we examine the nonlinear impulse response functions obtained from these models and compare them with the nonlinear impulse responses obtained from ESTAR models in which unit coefficients are imposed. For the majority of the real exchange rates examined the nonlinear impulse responses show that the speed of adjustment is faster, sometimes dramatically faster than in the case where unit coefficients are imposed. Consequently the analysis provides a further explanation of the Rogoff puzzle.

## 2 Methodology

We assume the true data generating process for the purchasing power deviations ( $y_t$ ) has the simple form of ESTAR model reported in Taylor et al. (2001) and Venetis, Paya and Peel (2001) namely

$$y_t = \alpha + e^{\gamma(y_{t-1} - \alpha)^2} (y_{t-1} - \alpha) + u_t \quad (2)$$

$$y_t = s_t - p_t + p_t^*$$

where  $y_t$  is the real exchange rate,  $s_t$  is the logarithm of the spot exchange rate,  $p_t$  is the logarithm of the domestic price level and  $p_t^*$  the logarithm of the

foreign price level.  $\alpha$ , is the constant equilibrium level of the real exchange rate,  $\gamma$  is a positive constant- the speed of adjustment and  $u_t$  is a random disturbance term.

We obtained estimates of (2) for the Dollar/DM, Dollar/Yen, Dollar/Pound, Dollar/Franc and Dollar/Guilder real exchange rates over the period 1973-2001 employing the consumer price index.<sup>7</sup> The models are also estimated for the sub-periods for which MacDonald and Baum et al. report the Johansen cointegration results. The results are similar to those reported in Taylor et al. (2001) and are shown in Table 2. All of the estimated models would be accepted on the basis of standard residual tests though the residuals do exhibit significant non normality except for the smallest subsample (1974-1990). The nonlinear ESTAR model of purchasing power deviations, with unit coefficients imposed, thus appears to be a parsimonious representation of PPP deviations in the Post-War floating period for the three different currencies in the three different subsamples used by MacDonald (1993), Baum et al. (2001) and in this paper.

Given this empirical finding we next employed two methods to generate simulated data. Write (2) as

$$s_t = p_t - p_t^* + \alpha + e^{\gamma}(s_{t-1} - p_{t-1} + p_{t-1}^* - \alpha) + u_t \quad (3)$$

Given estimates of  $\alpha$  and  $\gamma$ , a given starting value of  $s_{t-1}$ , an error vector  $u_t$  and values of the prices we can simulate a “fake” series for  $s_t$ . For the first simulation method we employed the actual values of the prices for the three currencies. The residuals are obtained from bootstrapping, with replacement, the estimated residuals obtained from the ESTAR models reported for the whole

---

<sup>7</sup>Estimations of (2) were done for a number of other real exchange rates. To preserve space we concentrate on the five currencies mentioned above because results were qualitatively similar (for a full discussion, see Venetis et al., 2001).



period in Table 2. This gives us 339 observations. The bootstrapped residuals were centered on zero and scaled.<sup>8</sup> Since the first values of the logarithms of domestic and foreign prices in our sample of data are normalized to zero we let the starting value of  $s$  equal zero, so we simulate from an equilibrium starting point. We do this experiment 10,000 times. With the resulting 10,000 samples of 339 observations of  $s_t$  we investigate the cointegration properties between our simulated spot rates and the actual values of domestic and foreign prices. This simulated data has, by construction, the unit coefficients imposed on relative prices.

The second method we employed created artificial data purely from simulation of ARIMA process for domestic and foreign prices, calibrated on estimates for our data set, with normal or  $t$  distributions assumed for the residuals  $u_t$ , with the standard deviation set equal to that in the empirical estimates. The results from the two methods were qualitatively similar. However given the non-normality observed in the actual residuals in the estimates of (2) we prefer the bootstrap method, for which we report results.

### 3 Empirical results

#### 3.1 Johansen cointegration

We proceed to estimate the Johansen cointegration procedure between the “artificially” created spot rate,  $s_t$ , under the nonlinear set-up described above and the actual prices of domestic and foreign economies. That is, we estimate 10,000 cointegration relationships between  $s_t$  and the actual price series. Given the empirical results reported in the literature we fixed the VAR length at 8, 12, 13. The results were qualitatively similar for the different lag lengths. Table 3 shows the results of the cointegrating vector  $\beta'$  of spot rate (normalized to be one), and

---

<sup>8</sup>The scaling factor is  $(n/n-k)^{-0.5}$

domestic and foreign price levels, their corresponding standard deviations and the percentage of times that cointegration would be accepted according to the Johansen trace statistic.<sup>9</sup> The mean of the cointegrating vector differs a lot from the median cointegrating vector due to the extremely high standard deviations. Moreover, only in around twenty percent of the replications we would accept significant cointegration relationships between Dollar/DM and Dollar/Yen PPP relationships. The median of the estimated vector is very close to the theoretical values of (1,-1,1) except for the coefficient of the Japanese price level that is almost zero.

### 3.2 Saikkonen cointegration test

In this section we apply the asymptotically efficient estimator for cointegration regression introduced by Saikkonen (1991). The Saikkonen estimates of the cointegrating vector are obtained from the following least-squares regression

$$z_t = Ax_t + \sum_{j=-K}^K \Pi_j \Delta x_t + v_t \quad t = K + 1, \dots, T - K \quad (4)$$

where  $z_t$  is the “endogenous” variable, in our case the spot exchange rate, and  $x_t$  the “exogenous” variables, in this case, domestic and foreign prices. We regress the spot rate on the domestic and foreign price levels and on the change in price levels with  $K$  lags and leads. This procedure ensures that the estimated coefficients on the price levels divided by their standard deviations are standard normally distributed. The  $K$  term for number of lags and leads is bounded by the term  $T^{1/3}$ , in our case 7.

In Table 4. We report the results of applying the Saikkonen method to our 10,000 simulated data sets. The estimates of the cointegration vector are much

---

<sup>9</sup>The max-statistic was also computed in the test but the percentage of times that was significant at 5% was the same than with the trace statistic.

better behaved than those obtained with the Johansen method. The standard deviation of the estimates is dramatically lower whilst the average and median values are closer to their true values.

Given this result we applied the Saikkonen method to our five different exchange rates. Table 5 presents the results of applying the Saikkonen test to actual spot and price levels.

We report three residual-based tests depending on the null hypothesis. The Augmented-Dickey-Fuller (ADF) and Phillips-Perron (PP) test the null hypothesis of unit root. Both residual-based tests provide evidence for cointegration for the five spot exchange rates and corresponding price levels. The last column of table 5 presents the Shin (1994) test for cointegration. It is also a residual-based test where the null hypothesis is that of cointegration or stationary residuals on the Saikkonen regression.<sup>10</sup> In the case of the Dollar/DM, Dollar/Franc and Dollar/Guilder exchange rates no constant was introduced as it appeared to be

---

<sup>10</sup>The Shin test is a modification of the Kwiatkowski et al. (1992) (KPSS) test for stationarity where I(1) regressors are added in the cointegration regression as described in (4). The KPSS test uses the components model

$$y_t = \alpha + \delta t + X_t + v_t \quad X_t = X_{t-1} + u_t$$

where  $y_t$  is the sum of the deterministic trend  $t$ , a random walk  $X_t$ , and a stationary error  $v_t$ . Under the null hypothesis that  $y_t$  is trend stationary,  $v_t$  is assumed to be stationary and then we only need to test that  $\sigma_u^2 = 0$ . It is also possible to consider the case where  $y_t$  is stationary around a level  $\alpha$ , so we set the parameter  $\delta = 0$ . Following Shin, let  $S_t$  be the partial sum processes of the residuals  $v_t$  from equation (4). Let  $s^2(l)$  be the consistent semiparametric estimator of the long-run variance of the regression error using the Newey-West correction. The Shin statistic is then

$$Shin = T^{-2} \sum S_t^2 / s^2(l)$$

The critical values will depend on the number of regressors, in our case two, domestic and foreign price level, and on the deterministic components, constant, trend or none. The Shin statistic is then

insignificant and we cannot reject the null of residual stationarity at five percent for the Dollar/Franc and Dollar/Guilder and at one percent for the Dollar/DM. For the Dollar/Yen and Dollar/Pound rates, a constant was introduced in the Saikkonen cointegrating regressions. The null of cointegration can be rejected for the Dollar/Yen but not for the Dollar/Pound rate. Overall, the residual based tests for cointegration under different null hypothesis appear to indicate a significant long-run relationship between spot exchange rates and prices.

The symmetry and proportionality hypotheses cannot be rejected in the Dollar/DM case. Only the symmetry hypothesis cannot be rejected for the Dollar/Yen and Dollar/Franc. In the case of case of the Dollar/Pound and Dollar/Guilder rates, both hypothesis are rejected. However, given that theoretically the weights can differ from unity we proceed on the basis of the estimated values.

The Saikkonen methodology provides estimates of the cointegrating vector which are much closer, on average to their theoretical values than the multivariate Johansen methodology when the true data generating process is of the ESTAR form (1). Nevertheless the weights differ from their theoretical values in the bootstrapped simulations, reported in Table 4. Accordingly we used the initial estimates of the cointegrating vectors obtained from the Saikkonen method, reported in Table 5 to create the real exchange rate using those weights as the “true” ones. For instance, the real exchange rate Dollar/Pound would be  $q_t^{UK} = s_t^{UK} - 1.061P^{US} + 0.722P^{UK}$ . In the previous section we estimated the nonlinear ESTAR model for the real exchange rates with the proportionality hypothesis imposed. We now estimate the nonlinear model with the weights obtained from the Saikkonen procedure. The results presented in Table 6 show

---


$$Shin = T^{-2} \sum S_t^2 / s^2(l)$$

The critical values will depend on the number of regressors, in our case two, domestic and foreign price level, and on the deterministic components, constant, trend or none.

that the ESTAR model is accepted for all five currencies.

We then generate 10,000 replications of the real exchange rate for the five currencies using the weights for the dollar and foreign prices ( $w_s, w_s^*$ ) obtained using the Saikkonen method as they appear in table 5. The residual term  $u_t^b$  are the bootstrap residuals from the ESTAR model estimation of table 6. The lag length on the Saikkonen regressions are the same in each replication and we set them as four, six or seven for the different exchange rates according to Table 5.

$$s_t = \alpha + w_s p_t - w_s^* p_t^* + e^{\gamma(s_{t-1} - \alpha - w_s p_{t-1} + w_s^* p_{t-1}^*)^2} (s_{t-1} - \alpha - w_s p_{t-1} + w_s^* p_{t-1}^*) + u_t^b \quad (5)$$

The results of the bootstrap simulations are presented in Table 7. The residual-based test for cointegration is also calculated for each replication. In the table we show the percentage of times that we would reject the null of non-cointegration for the ADF and PP tests at one percent level. For the Shin test, we divide it in two cases. One where a constant would be included in the Saikkonen regression (demeaned), that is, when it appears to be significant, and the other case when it would not be included (standard).<sup>11</sup> The numbers under this statistic is the percentage of times that the null of stationarity cannot be rejected at one percent.

It is apparent from Table 7 that the Shin test is sensitive to inclusion of the constant. We can use the bootstrap estimations to asses the bias of any estimator  $\hat{\theta}$ . The bias will be the difference between the expectation of the estimator  $\hat{\theta}$  and the quantity  $\theta$  being estimated.<sup>12</sup> The bootstrap biased-corrected estimator

<sup>11</sup>For the Newey-West semiparametric corrections used in the Shin test to remove persistent serial correlation of the residual process we chose  $l = 12$  as the appropriate choice for the lag parameter.

<sup>12</sup>See Efron and Tibshirani (1993, ch.10) for full discussion of bias estimation under bootstrap.

is  $\tilde{\theta}^* = 2\hat{\theta} - \hat{\theta}^*$ , where  $\hat{\theta}^*$  is the average bootstrap estimation. So, the biased-corrected pair of price weights are (0.841,0.706), (2.034,2.236), (1.362,1.044), (1.357,1.538) and (2.171,2.963) for the Dollar/DM, Dollar/Yen, Dollar/Pound, Dollar/Franc and Dollar/Guilder respectively. We then use these bias corrected weights to estimate the nonlinear ESTAR model reported in Table 8. The models are significant and the residuals pass standard diagnostic criteria.

### 3.3 Nonlinear impulse response function

A number of properties of the impulse response functions of linear models do not carry over to the nonlinear models. In particular, impulse responses produced by nonlinear models are a) history dependent, so they depend on initial conditions b) they are dependent on the size and sign of the current shock and c) they depend on the future shocks as well. That is, nonlinear impulse responses critically depend on the “past”, “present” and the “future”.

The Generalized Impulse Response Function (GIRF) introduced by Koop, Pesaran and Potter (1996) successfully confronts the challenges that arise in defining impulse responses for nonlinear models. The impulse response is defined as the average difference between two realizations of the stochastic process  $\{y_{t+h}\}$  which start with identical histories up to time  $t - 1$  (initial conditions) but one realization is “hit” by a shock at time  $t$  while for the other (the benchmark profile) no shock occurs. In a context similar to ours, Taylor and Peel (2000) conduct GIRF analysis on the deviations of real exchange rates from monetary fundamentals and Taylor et al. (2001) use impulse response functions to gauge how long shocks survive in real exchange rate nonlinear models. The GIRF of Koop et al. (1996) is defined as,

$$GIRF_h(h, \delta, \omega_{t-1}) = E(y_{t+h}|u_t = \delta, \omega_{t-1}) - E(y_{t+h}|u_t = 0, \omega_{t-1}) \quad (6)$$

where  $h = 1, 2, \dots$ , denotes horizon,  $u_t = \delta$  is an arbitrary shock occurring at

time  $t$  and  $\omega_{t-1}$  defines the history set of  $y_t$ . Given that  $\delta$  and  $\omega_{t-1}$  are single realizations of random variables, (7) is considered to be a random variable. In order to obtain sample estimates of (7), we average out the effect of all histories  $\omega_{t-1}$  that consist of every set  $(y_{t-1}, \dots, y_{t-p})$  for  $t \geq p + 1$  where  $p$  is the autoregressive lag length and we also average out the effect of future shocks  $u_{t+h}$ . In particular, for each available history we use 300 repetitions<sup>13</sup> to average out future shocks, where future shocks are drawn with replacement from the models residuals, and then we average the result across all histories. Without loss of generality, the impulse response horizon is set to  $\max\{h\} = 48$  months in the future. We set  $\delta = i\hat{\sigma}_u$  where  $\hat{\sigma}_u$  is the residual standard deviation and  $i = 1, 3, 5$ . The particular choice of  $\delta$ 's would allow us to compare and contrast the persistence of very large and very small shocks. The residual standard error in our estimates in Table 3 and 9 is approximately 0.033, which also corresponds to those reported in the literature, (see e.g. Taylor et al). Consequently a  $1\hat{\sigma}_u$  shock implies a 3% shock on the log real exchange rate  $y_t$  (equal to  $\ln(1+k/100)$  with  $k = 3$ ) and  $3\hat{\sigma}_u, 5\hat{\sigma}_u$  correspond roughly to 10% and 15% shocks respectively. As in Taylor et al. (2001) we will report the half-lives of shocks, that is the time needed for  $GIRF_h < \frac{1}{2}\delta$ .

Table 9 presents the results of the half-life shocks for the ESTAR nonlinear models of the real exchange rates. The results in Panel A correspond to the real exchange rate with weights of (1,-1), while the results in Panel B correspond to the real exchange rate using the bias corrected weights obtained with Saikkonen methodology presented in Table 7. We observe that the speed of adjustment is marginally slower for the model with biased corrected weights for two of the currencies but much faster for the Dollar/Yen, Dollar/Guilder and moderately faster for the Dollar/Franc. Consequently non unit weights can make some contribution to explanation of the Rogoff puzzle.

---

<sup>13</sup>We found out that the difference with using 500 repetitions was quantitatively insignificant.

Of course if weights of unity are imposed, when the true weights are non unity, then measured PPP deviations will exhibit a unit root.

In Figures 2-6 we plot three measures of PPP deviations. These are with unit coefficients imposed and the two sets of Saikkonen weights (bias and non bias corrected). We observe that the qualitative behavior of the three definitions of the PPP deviations is similar and in four cases the correlations are very high.<sup>14</sup> This is one explanation of why the estimated ESTAR models appear to be parsimonious explanations of PPP deviations in our samples of data even though the weights differ.

---

<sup>14</sup>The correlations between the real exchange rates using unit coefficients (labelled as Unity), the Saikkonen weights obtained in Table 5 (labelled as Sa), and the Saikkonen weights bias-corrected (labelled as SaBC) for the five different exchange rates are presented in the table below.

	Germany			Japan			UK		
	Unity	Sa	SaBC	Unity	Sa	SaBC	Unity	Sa	SaBC
Unity									
Sa	0.99			0.73			0.36		
SaBC	0.99	0.99		0.74	0.99		0.62	0.95	
		France		Netherlands					
Unity									
Sa	0.92			0.93					
SaBC	0.77	0.96		0.89	0.95				



## 4 Conclusions

Recent empirical work is suggestive that the ESTAR model , with weights of unity imposed, provides a parsimonious explanation of PPP deviations. However a number of authors employing the Johansen methodology have reported cointegrating vectors that differ massively from unity.

Assuming that the true DGP for PPP deviations is described by an ESTAR model, which is consistent with recent theory, we employed simulated data from an ESTAR process, with unit weights imposed, to investigate the properties of the Johansenn and Saikkonen cointegration methods. We found that the Johansen method performed relatively poorly giving a wide range of parameter values in the cointegrating vector. These encompassed values observed in real data. The Saikkonen method had much better properties with average estimated values closer to the true and a much smaller standard deviation. Given that absence of unit weights is theoretically well motivated our analysis suggests that if the DGP is hypothesized to be of a nonlinear form then the Saikkonen cointegration method should be preferred to the Johanssen method.

Employing non unit bias corrected weights obtained from the Saikkonen method we estimated ESTAR models for five real exchange rates. The impulse response functions from these models were compared with those in which unit roots were imposed. The differences in speed of adjustment did not display a uniform pattern, though they were dramatically faster in the case of the Dollar/Yen real exchange rate where the Saikkonen weights were further way from unity. Non unit weights thus appears to be another component of an explanation of the Rogoff puzzle.

## REFERENCES

- Anderson, H.M., 1997. Transaction costs and nonlinear adjustment towards equilibrium in the US Treasury Bill market, *Oxford Bulletin of Economics and Statistics*, 59, 465-484.
- Baum, C.F., Barkoulas, J.T. and M. Caglayan (2001) "Nonlinear Adjustments to Purchasing Power Parity in the Post-Bretton Woods Era" *Journal of International Money and Finance*, 20, 379-399.
- Cheung Y-W and Lai,K (1993) "Long Run Purchasing Power Parity During The Recent Float" *Journal Of International Economics*, 34, 181-92
- Dauvutyan, N and J.Pippenger (1985) "Purchasing Power Parity did not collapse during the 1970's", *American Economic Review*, 75, 1151-1158.
- Dumas, B. 1992. Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World, *Review Of Financial Studies*, 5, 153-80.
- Dixit, A. (1989) "Hysteresis, import penetration, and exchange rate pass-through," *Quarterly Journal of Economics* 104, pp. 205-228.
- Effron, B. and R. Tibshirani (1993). *An Introduction to the Bootstrap*. Chapman & Hall, New York.
- Froot,K and K. Rogoff (1995) "Perspectives on PPP and Long-Run Real Exchange Rates" in Grossman G and Rogoff, K (eds) *The Handbook of International Economics*, vol3, Amsterdam,North Holland.
- Granger, C.W.J. and T. Terarsvirta (1993) *Modelling nonlinear economic relationships*, Oxford University Press, Oxford.
- Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin (1992) "Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?" *Journal of Econometrics* 54, pp. 159-178.
- Koop, G., Pesaran, M.H., Potter, S.M., 1996. Impulse response analysis in nonlinear multivariate models, *Journal of Econometrics* 74, 119-147

MacDonald, R. (1993) "Long-Run Purchasing Power Parity: Is It for Real", *Review of Economic and Statistics*, 75, 690-695.

Michael, P., Nobay, A.R., Peel, D.A., (1997) " Transactions costs and non-linear adjustment in real exchange rates: an empirical investigation", *Journal of Political Economy*, 105, (4), 862-879.

Newey, W., and K., West, 1987. A Simple Positive Semi-Definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica*, 55, 703-708.

Obstfeld, M. and A.M. Taylor., (1997) " Nonlinear aspects of goods-market arbitrage and adjustment : Hecksher's commodity points revisited", *Journal of the Japanese and International Economies* 11, pp. 441-479.

O'Connell, P. 1998. Market frictions and real exchange rates. *Journal of International Money and Finance* 17, pp. 71-95.

O'Connell, P. and Wei, S.J., 1997. The bigger they are, the harder they fall: how price differences between US cities are arbitrated. Discussion Paper, Department of Economics, Harvard University.

Ohanian, Lee E. and Alan C. Stockman, 1997. "Short-run independence of monetary policy under pegged exchange rates and effects of money on exchange rates and interest rates", *Journal of Money, Credit and Banking* 1997, vol. 29(4), pp. 783-806.

Pippenger, M.K. and G.Goering., (1993) "A Note on the Empirical Power of Unit Root Tests Under Threshold Processes", *Oxford Bulletin of Economics and Statistics*, 55,473-81.

Pittis, N. Christou, C. Hassapis, C. and S. Kalyvitis., (2001) "Purchasing Power Parity tests in the British Pound-US Dollar market under near-to-unit roots". Mimeo University of Pireus.

Phillips, P.C.B. and B. Hansen (1990) "Statistical inference in instrumental variables regression with I(1) processes", *Review of Economic Studies* 57, pp. 99-125.

Rogoff, K., 1996. The purchasing power parity puzzle. *Journal of Economic Literature* 34, 647-668.

Saikkonen, P. (1991) "Asymptotically efficient estimation of cointegration regressions" *Econometric Theory* 7, 1-21.

Sarantis, N. (1999) "Modeling nonlinearities in Real Effective Exchange Rates" *Journal of International Money and Finance* 18, 27-45.

Sercu, P., Uppal, R., Van Hull, C. 1995 The exchange rate in the presence of transactions costs: implications for tests of purchasing power parity. *Journal of Finance* 50, 1309-1319.

Shin, Yongheol (1994) "A residual-based test of the null of cointegration against the alternative of no cointegration", *Econometric Theory* 10, pp. 91-115.

Taylor, M.P., Peel, D.A. 2000. Nonlinear adjustment, long-run equilibrium and exchange rate fundamentals. *Journal of International Money and Finance*, 19, 33-53

Taylor, M.P., Peel, D.A., Sarno, L., 2001. Nonlinear mean-reversion in real exchange rates: Toward a solution to the purchasing power parity puzzles. *International Economic Review*, 42, (4), 1015-1042.

Teräsvirta, T. (1994) "Specification, estimation, and evaluation of Smooth Transition Autoregressive Models," *Journal American Statistics Association* 89, pp. 208-218.

Tong, H. (1990) *Nonlinear time series: A dynamical system approach*, Clarendon Press, Oxford.

Uppal, R. 1993 A General equilibrium model of international portfolio choice. *Journal of Finance* 48, 529-553.

Venetis, I.A., Paya, I., Peel, D.A., 2001. Do real exchange rate 'mean revert' to productivity? A nonlinear approach. Manuscript, Cardiff Business School.

Table 1. Johansen cointegration results for proportionality of PPP

	MacDonald (1993)		Baum et al. (2001)	
Consumer price index	Sample 1974:01-1990:06		Sample 1973:08-1995:12	
Country	$k$	$\beta' = (S_t, P_t, P_t^*)$	$k$	$\beta' = (S_t, P_t, P_t^*)$
Germany	12	(1,-15.166,7.825)	10	(1,2.700,-0.668)
Japan	12	(1,-4.268,3.992)	13	(1,-5.127,4.168)
UK	12	(1,-0.712,0.180)	13	(1,-18.938,26.080)
France	12	(1,-4.454,-5.022)	8	(1,-2.566,2.566)
Netherlands			13	(1,-5.447,4.186)
Wholesale price index				
Germany	12	(1,-65.984,37.594)	8	(1,-62.11,36.466)
Japan	12	(1,-2.403,1.753)	13	(1,-2.488,1.808)
UK	12	(1,-0.403,1.353)	13	(1,-0.421,1.474)
France	12	(1,-1.211,0.799)		
Netherlands			5	(1,87.865,-47.796)

Notes: The vector  $\beta'$  denotes the cointegrating vector for the variables  $(S_t, P_t, P_t^*)$ , where  $S_t$  is the logarithm of the domestic price of foreign currency at time  $t$  and  $P_t$  and  $P_t^*$  are the logarithms of the domestic and foreign levels of prices.  $k$  indicates the order of the vector error correction model, VECM

Table 2(a). ESTAR model of real exchange rate.

Germany							
$\hat{\alpha}$	$\hat{\gamma}$	<i>s.e</i>	<i>JB</i>	<i>Q</i> (1)	<i>Q</i> (4)	<i>A</i> (1)	<i>A</i> (4)
Sample 1973:01-2001:05							
-0.018	-0.29	0.0331	0.01	0.27	0.34	0.22	0.32
(0.048)	(0.14)						
Sample 1973:08-1995:12							
-0.014	-0.29	0.0338	0.02	0.58	0.42	0.23	0.64
(0.051)	(0.14)						
Sample 1974:01-1990:06							
-0.014	-0.27	0.0337	0.43	0.82	0.30	0.04	0.18
(0.058)	(0.151)						

Notes: Numbers in parentheses are standard error estimates.

*s* denotes the residuals standard error. *JB* is the Jarque-Bera statistic p-value of the null of normality in the residuals. *Q*(*l*) is the p-value of the Ljung-Box statistic for residual autocorrelation for lag *l*. *A*(*l*) is the p-value of the LM statistic for ARCH in the residuals with lag *l*

Table 2(b). ESTAR model of real exchange rate.

Japan							
$\hat{\alpha}$	$\hat{\gamma}$	<i>s.e</i>	<i>JB</i>	<i>Q</i> (1)	<i>Q</i> (4)	<i>A</i> (1)	<i>A</i> (4)
Sample 1973:01-2001:05							
0.486	-0.15	0.0339	0.00	0.07	0.15	0.07	0.17
(0.069)	(0.07)						
Sample 1973:08-1995:12							
0.502	-0.13	0.0335	0.07	0.06	0.18	0.24	0.12
(0.074)	(0.07)						
Sample 1974:01-1990:06							
0.397	-0.42	0.0339	0.23	0.14	0.13	0.19	0.13
(0.040)	(0.19)						

Table 2(c). ESTAR model of real exchange rate.

UK							
$\hat{\alpha}$	$\hat{\gamma}$	<i>s.e</i>	<i>JB</i>	<i>Q</i> (1)	<i>Q</i> (4)	<i>A</i> (1)	<i>A</i> (4)
Sample 1973:01-2001:05							
0.111	-0.50	0.0315	0.01	0.11	0.61	0.00	0.00
(0.061)	(0.27)						
Sample 1973:08-1995:12							
0.111	-0.50	0.0338	0.22	0.08	0.52	0.01	0.02
(0.066)	(0.27)						
Sample 1974:01-1990:06							
0.12	-0.43	0.0339	0.46	0.41	0.83	0.62	0.15
(0.07)	(0.23)						

Table 2(d). ESTAR model of real exchange rate.

France							
$\hat{\alpha}$	$\hat{\gamma}$	<i>s.e</i>	<i>JB</i>	<i>Q</i> (1)	<i>Q</i> (4)	<i>A</i> (1)	<i>A</i> (4)
Sample 1973:01-2001:05							
-0.022	-0.34	0.0318	0.00	0.60	0.57	0.46	0.81
(0.055)	(0.18)						
Sample 1973:08-1995:12							
-0.011	-0.34	0.0328	0.00	0.87	0.49	0.39	0.67
(0.055)	(0.19)						
Sample 1974:01-1990:06							
-0.007	-0.31	0.0326	0.02	0.50	0.16	0.23	0.58
(0.062)	(0.18)						

Table 2(e). ESTAR model of real exchange rate.

Netherlands							
$\hat{\alpha}$	$\hat{\gamma}$	<i>s.e</i>	<i>JB</i>	<i>Q</i> (1)	<i>Q</i> (4)	<i>A</i> (1)	<i>A</i> (4)
Sample 1973:01-2001:05							
0.048	-0.28	0.0333	0.09	0.34	0.45	0.04	0.25
(0.064)	(0.16)						
Sample 1973:08-1995:12							
0.056	-0.28	0.0346	0.05	0.66	0.46	0.05	0.24
(0.064)	(0.15)						
Sample 1974:01-1990:06							
0.062	-0.26	0.0341	0.31	0.83	0.10	0.00	0.04
(0.068)	(0.15)						



Table 3. Johansen cointegration results on ESTAR generated real exchange rate

Country	$k$	Mean $\beta'$	Median $\beta'$	Std. dev. $(p_t, p_t^*)$	$TR$
Germany	12	(1,4.23,-15.62)	(1,-1.12,1.55)	(413,1314)	22%
Japan	13	(1,-3.59,55.30)	(1,-0.95,-0.12)	(268,5453)	20.5%
UK	13	(1,-4.68,9.07)	(1,-0.90,1.02)	(297,675)	100%
France	8	(1,-0.91,0.70)	(1,-1.01,0.97)	(15,86)	45%
Netherlands	13	(1,-1.18,2.19)	(1,-1.03,1.25)	(19,114)	99%

Notes: The vector  $\beta'$  denotes the cointegrating vector for the variables  $(S_t, P_t, P_t^*)$ , where  $S_t$  is the logarithm of the domestic price of foreign currency at time  $t$  and  $P_t$  and  $P_t^*$  are the logarithms of the domestic and foreign levels of prices.  $k$  indicates the order of the vector error correction model, VECM. Std. dev.  $(p_t, p_t^*)$  denotes the standard deviation of the coefficients of domestic and foreign prices in the cointegrating relationship.  $TR$  denotes the percentage of times that the Trace Statistic could not reject at least one cointegrating relationship at 5% according to the Osterwald-Lenum (1992) critical values.

Table 4. Saikkonen cointegration results on ESTAR generated real exchange rate

Country	$K$	Mean $\beta'$	Median $\beta'$	Std. dev. $(P, P^*)$
Germany	4	(1,0.992,-0.990)	(1,1.001,-1.023)	(1.38,2.52)
Japan	6	(1,0.971,-0.886)	(1,0.969,-0.883)	(0.65,1.26)
UK	7	(1,0.769,-0.799)	(1,1,0.772,-0.800)	(1.15,0.77)
France	6	(1,0.98,-0.98)	(1,0.96,-0.97)	(1.18,1.05)
Netherlands	7	(1,0.99,-0.98)	(1,0.98,-0.95)	(1.14,1.83)

$K$  is the number of lags and leads chosen in the Saikkonen regression (3) for each country. This number is chosen so that for a number greater than  $K$  the coefficient matrices  $\Pi_j$  are zero for  $j > K$ . The vector  $\beta'$  represents the weight for domestic and foreign price levels obtained from the estimated matrix  $A$  in (3). Std. dev.  $(P_t, P_t^*)$  denotes the standard deviation of the coefficients of domestic and foreign prices in the cointegrating relationship

Table 5. Saikkonen cointegration test

Country	$K$	$\beta' = (P, P^*)$	$H_{sym}$	$H_{pro}$	$ADF$	Residual tests	
						$PP$	$Shin$
Germany	3	(0.837,-0.708)	0.436	0.733	-2.32*	-2.18*	1.26 <sup>a</sup>
Japan	6	(2.012,-2.178)	0.585	0.000	-2.52*	-2.47*	0.97*
UK	7	(1.061,-0.722)	0.000	0.000	-3.66**	-3.69**	0.211
France	6	(1.267,-1.361)	0.290	0.010	-1.63 <sup>b</sup>	-1.95*	0.41
Netherlands	7	(2.122,-2.683)	0.000	0.000	-2.66**	-2.67**	0.73

$K$  is the number of lags and leads chosen in the Saikkonen regression (3) for each country.

This number is chosen so that for a number greater than  $K$  the coefficient matrices  $\Pi_j$  are zero for  $j > K$ . The vector  $\beta'$  represents the weight for domestic and foreign price levels obtained from the estimated matrix  $A$  in (3).  $H_{sym}$  tests the hypothesis of symmetry and  $H_{pro}$  tests the hypothesis of proportionality. The residual tests are the Augmented .

Dickey-Fuller (ADF), the Phillips-Perron (PP) and the Shin tests.

<sup>a</sup> We do not reject the null of stationarity at 1%. <sup>b</sup> We reject the null of stationarity at 10%.

Table 6. Nonlinear ESTAR model for real exchange rates  
obtained from Saikkonen estimations

Sample 1973:01-2001:05							
$\hat{\alpha}$	$\hat{\gamma}$	<i>s.e</i>	<i>JB</i>	<i>Q</i> (1)	<i>Q</i> (4)	<i>A</i> (1)	<i>A</i> (4)
Germany							
-0.027	-0.29	0.0335	0.00	0.24	0.44	0.31	0.60
(0.060)	(0.15)						
Japan							
-0.527	-0.48	0.0361	0.00	0.01	0.05	0.11	0.24
(0.043)	(0.18)						
UK							
-0.279	-0.23	0.0313	0.00	0.13	0.66	0.00	0.00
(0.054)	(0.12)						
France							
0.161	-0.47	0.0323	0.00	0.67	0.48	0.52	0.87
(0.042)	(0.23)						
Netherlands							
0.278	-0.28	0.0356	0.36	0.48	0.60	0.08	0.48
(0.052)	(0.13)						

Table 7. Saikkonen cointegration test in bootstrap simulation

Country	Mean $w_s$	S.D. $w_s$	Mean $w_s^*$	S.D. $w_s^*$	$ADF$	$PP$	<i>Shin</i>	
							<i>Standard</i>	<i>Demeaned</i>
Germany	0.85	1.39	-0.74	2.52	82%	88%	97%	28%
Japan	1.99	0.66	-2.12	1.27	76%	89%	97%	30%
UK	0.73	1.41	-0.40	0.93	67%	75%	90%	26%
France	1.18	1.08	-1.18	0.95	89%	94%	100%	83%
Netherlands	2.07	1.16	-2.40	1.86	93%	97%	99%	44%

Table 8. ESTAR model for real exchange rates with  
Saikkonen bias corrected weights on prices

Sample 1973:01-2001:05							
$\hat{\alpha}$	$\hat{\gamma}$	<i>s.e</i>	$JB$	$Q(1)$	$Q(4)$	$A(1)$	$A(4)$
Germany							
-0.03	-0.28	0.0335	11.04	0.24	0.44	0.31	0.60
(0.061)	(0.15)		(0.00)				
Japan							
-0.551	-0.48	0.0362	14.82	0.01	0.06	0.11	0.25
(0.042)	(0.17)		(0.00)				
UK							
-0.132	-0.31	0.0317	17.57	0.12	0.65	0.00	0.00
(0.052)	(0.16)		(0.00)				
France							
0.315	-0.52	0.0322	0.00	0.66	0.45	0.39	0.78
(0.035)	(0.21)						
Netherlands							
0.416	-0.52	0.0356	0.54	0.36	0.51	0.09	0.38
(0.035)	(0.18)						

Table 9. Estimated half-lives of shocks measured in months

Panel A	Real exchange rate unit coefficients				
	$1\hat{\sigma}_u$	$3\hat{\sigma}_u$	$5\hat{\sigma}_u$	$10\hat{\sigma}_u$	$\gamma$
Germany	28	25	22	14	-0.29
Japan	41	40	38	33	-0.15
UK	28	24	21	12	-0.50
France	26	24	21	14	-0.34
Netherlands	25	22	18	11	-0.28
Panel B	Real exchange rate Saikkonen bias-corrected coefficients				
Germany	29	26	23	14	-0.28
Japan	24	23	21	15	-0.48
UK	39	37	34	23	-0.31
France	23	20	18	11	-0.52
Netherlands	20	17	13	5	-0.52

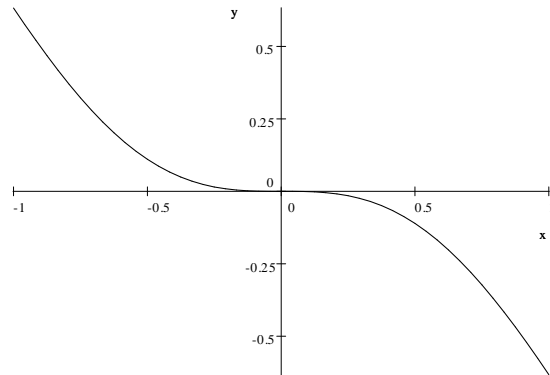


Figure 1: Deterministic plot of  $\Delta y, y_{t-1}$  from ESTAR with  $\gamma = 1$ .

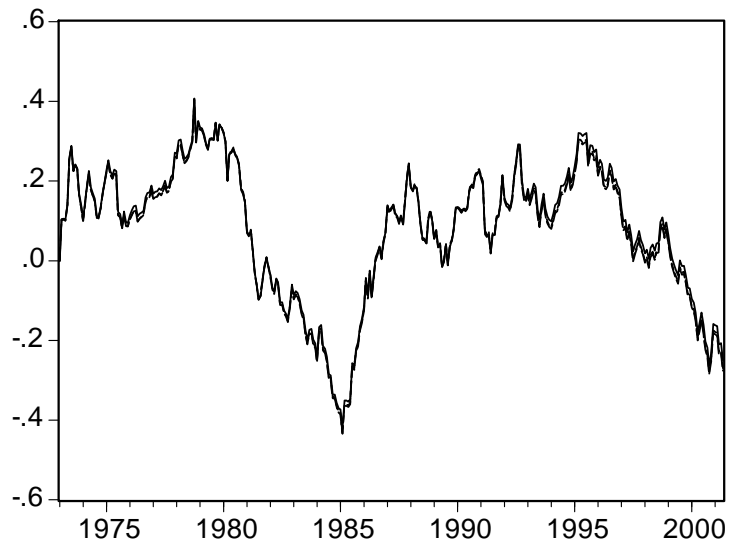


Figure 2: Dollar/DM. Solid line: Real exchange rate (unit coefficients). Short-dotted line: Real exchange rate (Saikkonen weights). Long-dotted line: Real exchange rate (Saikkonen bias-corrected weights).

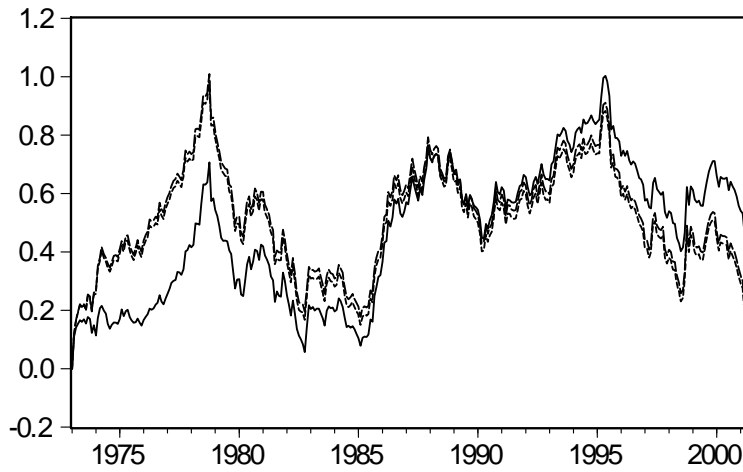


Figure 3: Dollar/Yen. Solid line: Real exchange rate (unit coefficients). Short-dotted line: Real exchange rate (Saikkonen weights). Long-dotted line: Real exchange rate (Saikkonen bias-corrected weights).

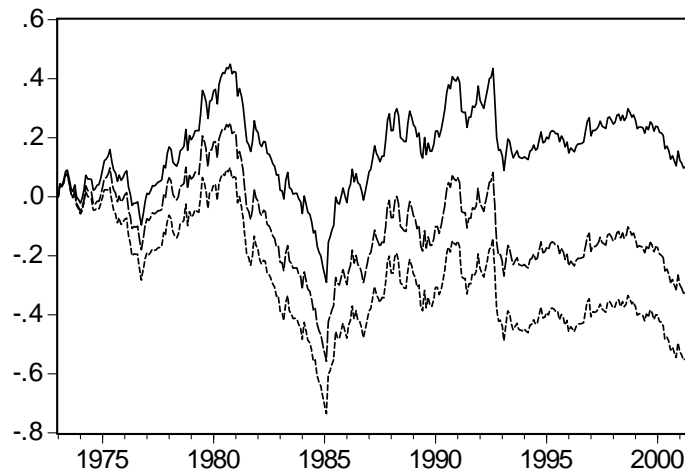


Figure 4: Dollar/Pound. Solid line: Real exchange rate (unit coefficients). Short-dotted line: Real exchange rate (Saikkonen weights). Long-dotted line: Real exchange rate (Saikkonen bias-corrected weights).

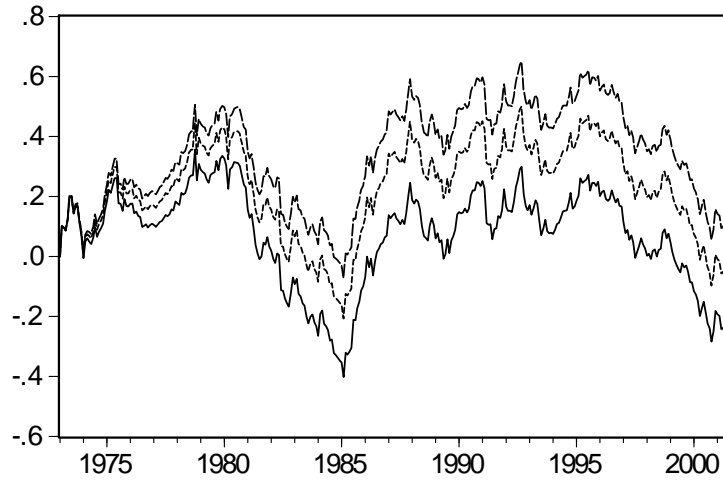


Figure 5: Dollar/Franc. Solid line: Real exchange rate (unit coefficients). Short-dotted line: Real exchange rate (Saikkonen weights). Long-dotted line: Real exchange rate (Saikkonen bias-corrected weights).

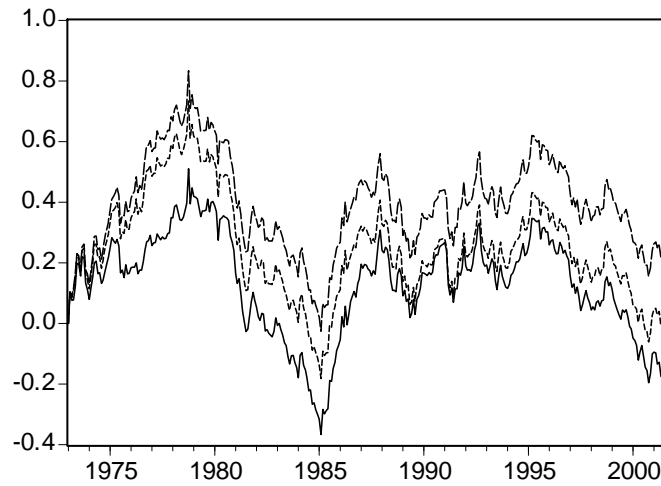


Figure 6: Dollar/Guilder. Solid line: Real exchange rate (unit coefficients). Short-dotted line: Real exchange rate (Saikkonen weights). Long-dotted line: Real exchange rate (Saikkonen bias-corrected weights).