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by Silvia Muzzioli

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CORRIDOR IMPLIED VOLATILITY AND THE VARIANCE RISK PREMIUM IN THE ITALIAN MARKET

Silvia Muzzioli[*],

Department of Economics and CEFIN, University of Modena and Reggio Emilia

Abstract

Corridor implied volatility introduced in Carr and Madan (1998) and recently implemented in Andersen and Bondarenko (2007) is obtained from model-free implied volatility by truncating the integration domain between two barriers. Corridor implied volatility is implicitly linked with the concept that the tails of the risk-neutral distribution are estimated with less precision than central values, due to the lack of liquid options for very high and very low strikes. However, there is no golden choice for the barriers levels', which will probably change depending on the underlying asset risk neutral distribution. The latter feature renders its forecasting performance mainly an empirical question.

The aim of the paper is twofold. First we investigate the forecasting performance of corridor implied volatility by choosing different corridors with symmetric and asymmetric cuts, and compare the results with the preliminary findings in Muzzioli (2010b). Second, we examine the nature of the variance risk premium and shed light on the information content of different parts of the risk neutral distribution of the stock price, by using a model-independent approach based on corridor measures. To this end we compute both realised and model-free variance measures which accounts for drops versus increases in the underlying asset price. The comparison is pursued by using intra-daily synchronous prices between the options and the underlying asset.

Keywords: corridor implied volatility, variance swap, corridor variance swap, variance risk premium.

JEL classification: G13, G14.

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[*] Department of Economics and CEFIN, University of Modena and Reggio Emilia, Viale Berengario 51, 41121 Modena (I), Tel. +390592056771 Fax +390592056947, e-mail: silvia.muzzioli@unimore.it. The author thanks participants of the first FEBS2011 conference for helpful comments and suggestions. Usual disclaimer applies.

1. Introduction

Volatility modelling and forecasting is essential for asset pricing models, option pricing and hedging and risk management. In order to measure and forecast volatility we can resort either on the past history of the underlying asset (historical volatility) or on the information embedded in option prices which reflect the investors' opinion about the future underlying asset evolution (implied volatility). Focusing on implied volatility, how to extract the relevant information from the cross-section of option prices is still an open debate.

Black-Scholes implied volatility is a model dependent forecast, which relies on the strict assumptions of the Black-Scholes option pricing model about the asset price evolution (Brownian motion) and the constancy of the volatility parameter. Several papers have empirically shown the discrepancy between the assumptions of the model and the reality of financial markets (volatility varies with the strike price of the option, the time to maturity and the option type). Model free implied volatility, introduced by Britten-Jones and Neuberger (2000), represents a valid alternative to Black-Scholes implied volatility, since it does not rely on a particular option pricing model, being consistent with several underlying asset price dynamics. A drawback of model free implied volatility is that it requires a continuum of option prices in strikes, ranging from zero to infinity, assumption which is not fulfilled in the reality of financial markets. Corridor implied volatility introduced in Carr and Madan (1998), and recently implemented in Andersen and Bondarenko (2007), is obtained from model-free implied volatility by truncating the integration domain between two barriers. Corridor implied volatility is implicitly linked with the concept that the tails of the risk-neutral distribution are estimated with less precision than central values, due to the lack of liquid options for very high and very low strikes. For example, the computation of market volatility indexes (see e.g. the VIX index for the Chicago Board Options Exchange, or the V-DAX New for the German stock market), which are closely followed by market participants, is done by operating a truncation of the domain of strike prices once two consecutive strikes with zero bid prices are observed.

The empirical literature about the forecasting performance of corridor implied volatility is very little and mixed: most studies are based on closing prices and investigate only symmetric corridor measures. Andersen and Bondarenko (2007), by using options on the S&P500 futures market, find that narrow corridor measures, closely related to Black Scholes implied volatility are more useful for volatility forecasting than broad corridor measures, which tend to model-free implied volatility as the corridor widens. A similar finding is obtained in Muzzioli (2010b) who finds that the best forecast for the Italian index options market is the one which operates a 50% cut of the risk neutral distribution. On the other hand, Tsiaras (2009), by using options on the 30 components of the DJIA index, concludes that CIV forecasts are increasingly better as long as the corridor width enlarges. To sum up, the empirical evidence suggests that there is no golden choice for the barriers levels', which will probably change depending on the underlying asset risk neutral distribution. The latter feature renders the forecasting performance of corridor implied volatility mainly an empirical question.

The investigation of corridor implied volatility has also important implications for the analysis of the variance risk premium. It is widely recognized that exposure to variance carries a negative risk premium: investors are willing to pay high prices in order to be insured against spikes in market variance. As noted by Carr and Wu (2006) investors are not only averse to increases in the variance level, but also to increases in variance of the return variance. Carr and Wu (2009) report evidence about five stock indexes and 35 stocks in the US market and find it to be strongly negative and highly significant. Andersen and Bondarenko (2009) are the first to exploit the concept of corridor implied variance in order to slice the risk neutral distribution of the stock price into different intervals and use the latter in order to investigate the pricing of market variance for different asset classes. In particular, for the SPX, they find a large negative risk premium which is very asymmetric since it is much larger in the downside part of the distribution than in the upside.

The aim of the paper is twofold. First we thoroughly investigate the forecasting performance of corridor implied volatility by pursuing a sensitivity analysis of corridor implied volatility with

respect to the choice of the barrier. The analysis is motivated by the need to find an optimal cut for the Italian index options market, which can be found by analysing a grid of different corridors. To this end we investigate different corridors with both symmetric and asymmetric cuts and compare the results with the preliminary findings in Muzzioli (2010b), where only four cuts of the risk neutral distributions are explored (which correspond to a 5%, 10%, 20% and 50% overall cut). Second, we examine the nature of the variance risk premium and shed light on the information content of different parts of the risk neutral distribution of the stock price, by using a model-independent approach based on corridor measures. To this end we compute both realised and model-free variance measures which accounts for drops versus increases in the underlying asset price. The comparison is pursued by using intra-daily synchronous prices between the options and the underlying asset.

As for the sensitivity analysis to the choice of the barriers, the results substantially complement and corroborate the preliminary findings in Muzzioli (2010b), by finding an optimal cut around 50%-60% of the risk neutral distribution. Moreover, the use of asymmetric cuts highlights a weak evidence of superiority of the corridor measure which rely more on put prices on the one which relies more on call prices.

As for the analysis of the variance risk premium, we find it to be large and negative: investors are willing to pay sizable premiums and experience a loss on average in order to be hedged against peaks of variance. The results are consistent with previous empirical literature (Carr and Wu (2006 and 2009), Andersen and Bondarenko (2009)). Upside and Downside risk premia are negative and sizeable and both statistically significant. Downside risk premia are overwhelmingly higher than upside risk premia (more than two times higher). This means that investors heavily price downside risk: downside risk premium is the main component of the overall risk premium. The results are consistent with different computation methodologies of realised semi-variance.

The paper proceeds as follows. Sections 2 and 3 recall the basic features of variance swaps and corridor variance swaps. Section 4 presents the data set used. Section 5 provides the details for

the computation of the volatility and variance measures. Section 6 illustrates the computation of the variance risk premium and corridor variance risk premium. Sections 7 presents the results for the forecasting performance of corridor implied volatility and Section 8 the analysis of the variance risk premium. The last section concludes.

2. Variance swaps.

Variance swaps provide investors with a simple way in order to have a pure exposure to the future level of variance. Variance swaps are traded over the counter. As they require a single payment at maturity, they are forward contracts on future realised variance. At maturity the long side pays a fixed rate (the variance swap rate) and receives a floating rate (the realised variance). A notional dollar amount is multiplied by the difference between the two rates. The payoff at maturity is:

$$N(\sigma_R^2 - VSR) \tag{1}$$

where N is a notional dollar amount, σ_R^2 is the realised variance, and VSR is the fix variance swap rate.

Assume that the stock price evolves as a diffusive process (no jumps allowed), as follows:

$$\frac{dS_t}{S_t} = \mu(t,...)dt + \sigma(t,...)dZ_t \tag{2}$$

Realized variance (also called integrated variance) in the period 0-T is given by:

$$\sigma_R^2 = \frac{1}{T}\int_0^T \sigma^2(t,...)dt \tag{3}$$

In practice, realised variance is monitored discretely (for example, the asset price could be observed each business day) and computed as:

$$\sigma_R^2 = \frac{1}{T}\sum_{i=1}^{n}\left(\ln\left(\frac{S_i}{S_{i-1}}\right)\right)^2 \tag{4}$$

where n is the number of observations during the period 0-T and the day-count convention could be different depending on the term sheet conditions: business days/252 or actual/365.

At the time of entry the contract has zero value. If we assume absence of arbitrage opportunities and the existence of a unique risk-neutral measure we can write:

$$E_q\left[e^{-rT}(\sigma_R^2 - VSR)\right] = 0 \tag{5}$$

where r is the risk-free discount rate corresponding to maturity T, and interest rates are assumed to be uncorrelated with realised variance. By no-arbitrage the variance swap rate should be equal to the risk-neutral expected value of realised variance over the life of the swap:

$$VSR = E_q\left[\sigma_R^2\right]. \tag{6}$$

Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000) show how to replicate the risk-neutral expectation of variance with a portfolio of options with strike price ranging from zero to infinity, as follows:

$$E_q\left[\sigma_R^2\right] = \frac{1}{T}E_q\left[\int_0^T \sigma^2(t,...)dt\right] = \frac{2e^{rT}}{T}\int_0^\infty \frac{M(K,T)}{K^2}dK \tag{7}$$

where *M(K,T)* is the minimum between a call or put option price, with strike price *K* and maturity *T*, i.e. only out-of-the-money options are used.

Equation (7) is also known as model-free implied variance, and its square root as model-free implied volatility, since, differently from Black-Scholes implied volatility it does not rely on any particular option pricing model. In fact, the definition is consistent with several underlying asset price dynamics: from diffusive to jump-diffusion process (Jiang and Tian (2005)). Relation (7) is exact if the underlying asset price evolves as a diffusive process and holds up to an approximation error when the underlying asset process displays jumps (Carr and Wu (2009)). Since in the reality of financial markets only a limited and discrete set of strike prices are quoted, both truncation and discretization errors arise in the computation of model free implied variance. Therefore, interpolation and extrapolation are needed in order to compute model-free implied variance (see

Jiang and Tian (2005) and (2007)). The CBOE volatility index (VIX) and the plethora of volatility indexes which have been introduced in various financial markets worldwide, are all based on the model free implied variance definition (which can be considered more precisely as a corridor implied variance because of the truncation of the domain of strike prices once two consecutive strikes with zero bid prices are observed). Therefore the volatility index squared represents a discretization of the 30-days variance swap rate, up to a discretization error and a jump induced error term.

3. Corridor variance swaps.

A corridor variance swap is a variant of variance swap which takes into account daily stock variations only when the underlying asset is in a specific corridor. At maturity the long side pays a fixed rate (the corridor variance swap rate) and receives a floating rate (the realised return variance which is accumulated only if the underlying lies in a pre-specified range). A notional dollar amount is multiplied by the difference between the two rates. The payoff at maturity is:

$$N(\sigma_{RC}^2 - CVSR) \tag{8}$$

where N is a notional dollar amount, σ_{RC}^2 is the realised variance in the corridor, and CVSR is the corridor variance swap rate.

Corridor realised variance in the period 0-T, can be defined as follows:

$$\sigma_{RC}^2 = \frac{1}{T}\int_0^T \sigma^2(t,...)1_{S_t\in[B_1,B_2]}dt \tag{9}$$

where $1_{S_t\in[B_1,B_2]}$ is the indicator function that is equal to 1 if the underlying is inside the two barriers and determines if variance is accumulated or not. In practice, realised corridor variance is monitored discretely and computed as:

$$\sigma_{RC}^2 = \frac{1}{T}\sum_{i=1}^n \left(\ln\left(\frac{S_i}{S_{i-1}}\right)\right)^2 1_{S_t\in[B_1,B_2]} \tag{10}$$

where $1_{S_t \in [B_1, B_2]}$ is the indicator function, which takes value 1 only if the underlying asset lies between the two barriers, n is the number of observations during the period 0-T and the day-count convention could be different depending on the term sheet conditions: business days/252 or actual/365.

Carr and Madan (1998) and Andersen and Bondarenko (2007) show that it is possible to compute the expected value of corridor variance under the risk-neutral probability measure (the corridor variance swap rate), by using a portfolio of options with strikes ranging from B_1 to B_2, as follows:

$$CVSR = E_q(\sigma_{RC}^2) = E_q \left[\frac{1}{T} \int_0^T \sigma^2(t,...) 1_{S_t \in [B_1, B_2]} dt \right] = \frac{2e^{rT}}{T} \int_{B_1}^{B_2} \frac{M(K,T)}{K^2} dK \qquad (11)$$

Equation (11) is known as corridor implied variance and its square root as corridor implied volatility. If $B_1=0$ and $B_2=\infty$, then corridor variance degenerates into model-free implied variance. It follows that a corridor variance swap is cheaper than a variance swap, since it enables investors to bet on possible patterns of the stock price.

Upside and downside variance swaps are a variant of corridor variance swaps which have the following payoffs:

$$N(\sigma_{RUC}^2 - VSRU) \qquad (12)$$

$$N(\sigma_{RDC}^2 - VSRD) \qquad (13)$$

where N is a notional dollar amount, VSRU and VSRD are the strike prices obtained by using formula (11) with barriers ($B_1=0$, $B_2=B$) and ($B_1=B$, $B_2=\infty$), respectively; upside (downside) corridor realised volatility is defined as:

$$\sigma_{RUC}^2 = \frac{1}{T} \sum_{i=1}^n \left(\ln\left(\frac{S_i}{S_{i-1}}\right) \right)^2 1_{S_i > B} \qquad (14)$$

$$\sigma_{RDC}^2 = \frac{1}{T} \sum_{i=1}^n \left(\ln\left(\frac{S_i}{S_{i-1}}\right) \right)^2 1_{S_i < B} \qquad (15)$$

and accumulated only if the underlying lies above (below) the barrier B.

Note that equations (14) and (15) recognize the full square of the return that ends in the corridor, other alternative contract specifications may treat the movements across the barrier recognizing only a fraction of the total move (see e.g. Carr and Lewis (2004).

The sum of a downside corridor variance swap and an upside corridor variance swap yields a standard variance swap. An investor can be interested in an upside (downside) variance swap if she is bullish (bearish) on the underlying asset, or if the volatility skew is too steep making down-variance too expensive relative to up-variance.

4. The Data Set

The data set consists of intra-daily data on FTSE MIB-index options (MIBO), recorded from 1 June 2009 to 30 November 2009. Each record reports the strike price, expiration month, transaction price, contract size, hour, minute, second and centisecond. MIBO are European options on the FTSE MIB index, which is a capital weighted index composed of 40 major stocks quoted on the Italian market. FTSE MIB options quote in index points, representing a value of 2.5 €, with 10 different expirations (the 4 three-monthly expiries in March, June, September and December, the 2 nearest monthly expiry dates, the 4 six-month maturities (June and December) of the two years subsequent the current year, the 2 annual maturities (December) of the third and fourth years subsequent the current year). The contract expires on the third Friday of the expiration month at 9.05 am. If the Exchange is closed that day, the contract expires on the first trading day preceding that day. For each maturity up to twelve months (monthly and three-month maturities), exercise prices are generated at intervals of 500 index points. At least 15 exercise prices are quoted for each expiry: one at-the-money, seven in-the-money and seven out-of-the-money strikes. The daily closing price is established by the clearing and settlement organisation Cassa di Compensazione e Garanzia.

As for the underlying asset, intra-daily prices of the FTSE MIB-index recorded from 1 June 2009 to 31 December 2009 are used. The FTSE MIB is the primary benchmark Index for the Italian equity market and seeks to replicate the broad sector weights of the Italian stock market. It is adjusted for stocks splits, changes in capital and for extraordinary dividends, but not for ordinary dividends. Therefore, the daily dividend yield is used in order to compute the appropriate value for the index, as follows:

$$\widehat{S}_t = S_t e^{-\delta_t \Delta t}$$

(16)

where S_t is the FTSE MIB value at time t, δ_t is the dividend yield at time t and Δt is the time to maturity of the option.

As a proxy for the risk-free rate, Euribor rates with maturities one week, one, two and three months are used. Appropriate yields to maturity are computed by linear interpolation. The data-set for the FTSE MIB index and the MIBO is kindly provided by Borsa Italiana S.p.A, Euribor rates and dividend yields are obtained from Datastream.

Several filters are applied to the option data set. First, in order not to use stale quotes, we eliminate options with trading volumes of less than one contract. Second, we eliminate options near to expiry which may suffer from pricing anomalies that might occur close to expiration (in order to be consistent with the computation methodology of quoted volatility indexes, we choose to use the most conservative filter that eliminates options with time to maturity of less than 8 days). Third, following Ait-Sahalia and Lo (1998) only at-the-money and out-of-the-money options are retained (call options with moneyness K/S > 0.97 and put options with moneyness K/S < 1.03). Fourth, option prices violating the standard no-arbitrage constraints are eliminated:

$$P \geq \max(Ke^{-r(T-t)} - Se^{-\partial(T-t)}, 0), \quad C \geq \max(Se^{-\partial(T-t)} - Ke^{-r(T-t)}, 0).$$

(17)

Finally, in order to reduce computational burden, we only retain options that are traded in the last hour of trade, from 16:40 to 17:40 (the choice is motivated by the high level of trading activity in

this interval). Option prices and the underlying index prices are then matched in a one minute window in order to obtain implied volatilities from synchronous prices.

5. The Computation of the Volatility and Variance Measures

In the following volatility measures are taken as the square root of variance measures defined in Sections 2 and 3. We compute two volatility measures: corridor implied volatility (σ_{CIV}) and realised volatility (σ_R). As for corridor implied volatility, each day of the sample, we divide quoted option prices in two sets: near term and next term options and we follow the procedure described below, which consists of three steps (repeated for both near and next term options): fitting the smile function; obtaining the risk neutral distribution of the underlying asset; computing corridor implied volatility. Last we interpolate between near and next term measures in order to have a 30-day measure.

As for the first step, we recover the Black-Scholes implied volatilities by using synchronous prices for the option and the underlying asset that are matched in one minute interval. These implied volatilities are averaged for each strike in the hour of trades resulting in a matrix of quoted strike prices and corresponding implied volatilities. Second, as only a discrete number of strikes are available, we need to interpolate and extrapolate option prices in order to generate the missing prices. As for the interpolation, following Campa et al. (1998), we use cubic splines to interpolate implied volatilities between strike prices. We extrapolate volatilities outside the listed strike price range by using a linear function that matches the slope of the smile function at K_{min} and K_{max}. This methodology has the advantage that the smile function remains smooth at K_{min} and K_{max}. As this latter methodology may generate implied volatilities that are artificially too high (in case the slope is positive) or too low (in case the slope is negative), we have imposed both a lower and an upper bound to implied volatilities equal to 0.001 and 0.999 respectively. Finally, we use the Black and Scholes formula in order to convert implied volatilities into call prices.

As for the second step, in order to obtain the risk neutral distribution of the underlying asset we resort to a non parametric method already tested in Muzzioli (2010b). In particular, we use the Derman and Kani (1994) algorithm with the modifications proposed in Moriggia et al. (2009), (the so called Enhanced Derman and Kani implied tree (EDK)) which are fundamental both to avoid arbitrage opportunities and to correctly model the tails of the distribution. The advantages of the proposed methodology are at least three. First, it is a methodology that fits the data well without imposing a rigid parametric structure. Second, it does not require any costly estimation of the risk-neutral probability by minimization of a loss function which may lead to different results given the subjective choice of the loss function used. Last, it ensures positivity of the risk-neutral probabilities.

As for the third step, corridor implied volatility (σ_{CIVi}) is computed both for near term (i=1) and for next term maturities (i=2), as a discrete version of the square root of equation (11):

$$\sigma_{CIVi} = \sqrt{\frac{2e^{rT}}{T}\int_{B_1}^{B_2}\frac{M(K,T)}{K^2}dK} \approx \sqrt{\frac{e^{rT}}{T}\sum_{i=1}^{m}\left[g(T,K_i)+g(T,K_{i-1})\right]\Delta K} \qquad (18)$$

where:

$$\Delta K = (K_{max} - K_{min})/m, \qquad (19)$$

m is the number of strikes used,

$$K_i = K_{min} + i\Delta K, \quad 0 \leq i \leq m, \qquad (20)$$

$$g(T,K_i) = [\min(C(T,K_i), P(T,K_i))]/K_i^2, \qquad (21)$$

$$B_1 = H_0^{-1}(p_1) \text{ and } B_2 = H_0^{-1}(1-p_2) \qquad (22)$$

The barriers B_1 and B_2 are computed by looking at the risk-neutral distribution obtained by fitting an implied binomial tree with 100 levels to quoted option prices. As the implied tree yields a discrete cumulative distribution,

$$H(x) = P(X \leq x) = \sum_{t \leq x} p(t), \qquad (23)$$

the barrier level x has been chosen to be the average between x_1 and x_2, where x_1 and x_2 are the barrier levels such that $P(X \leq x_1)$ and $(P(X \leq x_2))$ are the closest to the desired p.

We compute a total of eight corridor measures: four with symmetric cuts and four with asymmetric cuts. The four symmetric corridor measures are CIV0.2, CIV0.25, CIV0.3, CIV0.4, which correspond to p = 0.2, 0.25, 0.3, 0.4 respectively. From CIV0.2 to CIV0.4 we explore narrower corridor implied volatility measures. In order to assess if the lower part of the risk neutral distribution is more informative about future realised volatility than the upper part, we also compute corridor measures with asymmetric cuts of the risk neutral distribution: CIV(0.1-0.3) cuts 0.1 in the upper part and 0.3 in the lower part ($B_1 = H_0^{-1}(0.3)$ and $B_2 = H_0^{-1}(0.9)$) while CIV(0.3-0.1) cuts 0.3 in the upper part and 0.1 in the lower part ($B_1 = H_0^{-1}(0.1)$ and $B_2 = H_0^{-1}(0.7)$), therefore CIV(0.1-0.3) relies more on call prices than on put prices, while CIV(0.3-0.1) relies more on put prices than on call prices. Moreover, in order to separate the effect of out of the money call and put prices, which are sensitive to increases or decreases in the underlying asset, we compute upside and downside corridor measures CIVUP (CIVDW) with barriers B_1=F and B_2= K_{max} (B_1=K_{min} and B_2=F) respectively, where F is the forward price.

Last, in order to have a constant 30-day measure, each corridor implied volatility measure is computed by linear interpolation of the two corridor volatility measures computed with option prices which are the nearest to the remaining time of expiry of 30 days (σ_1 is corridor implied volatility for the near-term maturity and σ_2 for the next-term), as follows:

$$\sigma_{CIV} = \sqrt{\left\{ \frac{T_1}{365} \sigma_{CIV_1}^2 \left[\frac{T_2 - 30}{T_2 - T_1} \right] + \frac{T_2}{365} \sigma_{CIV_2}^2 \left[\frac{30 - T_1}{T_2 - T_1} \right] \right\} \times \frac{365}{30}} \qquad (24)$$

where:

T_i= number of days to expiry of the i-th maturity index option, $i=1,2$.

In order to investigate the variance swap rate and the variance risk premium, variance measures are taken as the square of volatility measures. In particular we compute three implied

variance measures: upside corridor variance swap rate ($VSRU = \sigma^2_{CIVUP}$), downside corridor variance swap rate ($VSRD = \sigma^2_{CIVDW}$), variance swap rate (VRS = VRSD +VSRU).

Let us turn to the computation of realised variance measures. We compute a total of five realised variance measures: realised variance (σ^2_R), upside realised variance (σ^2_{RU}), downside realised variance (σ^2_{RD}) , upside corridor realised variance (σ^2_{RUC}), downside corridor realised variance (σ^2_{RDC}), which are defined in the following.

Realised variance (σ^2_R) is computed, in annual terms, as the sum of five-minute frequency squared index returns over the next 30 days:

$$\sigma^2_R = \sum_{i=1}^{n} \left(\ln \left(\frac{S_i}{S_{i-1}} \right) \right)^2 * \frac{365}{30} \qquad (25)$$

where n is the number of index prices spaced by five minutes in the 30 days period. The choice of using five-minute frequency is made following Andersen and Bollerslev (1998) and Andersen et al. (2001) who showed the importance of using high frequency returns in order to measure realised volatility and point out that returns at a frequency higher than five minutes could be affected by serial correlation. Realised volatility (σ_R) is taken as the square root of realised variance.

Realised variance squares the returns, thus overlooking the information content of positive versus negative returns. Downside risk, i.e. the risk of experiencing a downward move of the underlying asset, has been measured by semi-variance (see e.g. Ang, Chen and Xing (2006)) or value at risk or expected shortfall by using daily data. A high frequency measure of downside risk, called Realised Semi-variance, has been proposed by Bandorff-Nielsen, Kinnebrock and Shephard (2010). Bandorrf-Nielsen and Shephard (2004) show that it is possible to separate two components in the quadratic variation process: the continuous evolution and the jump components. In this setting, realised semi-variance measures the variation of asset prices falls.

Downside realised variance is defined as:

$$\sigma^2_{RD} = \sum_{i=1}^{n} \left[\ln\left(\frac{S_i}{S_{i-1}}\right) \right]^2 1_{S_i - S_{i-1} < 0} \tag{26}$$

Symmetrically, upside realised variance is defined as:

$$\sigma^2_{RU} = \sum_{i=1}^{n} \left[\ln\left(\frac{S_i}{S_{i-1}}\right) \right]^2 1_{S_i - S_{i-1} > 0} \tag{27}$$

such that:

$$\sigma^2_R = \sigma^2_{RU} + \sigma^2_{RD}. \tag{28}$$

Only negative (positive) returns are used in order to compute downside (upside) realised variance.

As upside and downside corridor measures are the risk neutral expectation of realised variance conditional on the underlying asset price being higher or lower than the barrier, we also compute the corresponding realised semi-variance corridor measures, as defined in equations (14) and (15). Corridor downside realised variance is defined as:

$$\sigma^2_{RDC} = \sum_{i=1}^{n} \left[\ln\left(\frac{S_i}{S_{i-1}}\right) \right]^2 1_{S_i \leq F} , \tag{29}$$

and symmetrically, corridor upside realised variance is defined as:

$$\sigma^2_{RUC} = \sum_{i=1}^{n} \left[\ln\left(\frac{S_i}{S_{i-1}}\right) \right]^2 1_{S_i > F} , \tag{30}$$

where F is the forward price.

The asymmetric treatment of the case S=F in the corridor upside and downside realised variance is necessary in order to have:

$$\sigma^2_R = \sigma^2_{RUC} + \sigma^2_{RDC}. \tag{31}$$

Consistently with realised variance, semi-variance measures are annualised by means of the factor 365/30.

6. Variance risk premium and corridor variance risk premia.

The variance risk premium is the difference between the ex-post realised variance (over the lifetime of the swap) and the variance swap rate. Following Carr and Wu (2009) the variance risk premium is measured both in Euro terms as the Euro payoff of a long position in a variance swap with notional amount N=100 Euro, held up to expiry:

$$\text{EVRP} = N(\sigma_R^2 - VSR) \tag{32}$$

and in log returns terms as the continuously compounded excess return given by:

$$\text{LVRP} = \log(\sigma_R^2 / VSR). \tag{33}$$

The average variance risk premium in Euro term and in log returns term is the sample average of (32) and (33) respectively.

The upside and downside variance risk premium is the difference between the ex-post upside and downside realised variance (over the lifetime of the swap) and the upside and downside corridor variance swap rate. As upside and downside realised variance is measured in two different ways (by separating either positive and negative returns or realisations higher or lower than a threshold), and the risk premiums are computed either in Euro terms or in log-return terms, we have a total of eight different cases, as follows. The upside variance risk premium in Euro terms is the Euro payoff of a long position in an upside variance swap with notional amount N=100 Euro, held up to expiry:

$$\text{EVRPU} = N(\sigma_{RU}^2 - VSRU) \tag{34}$$

and in log returns terms it is measured as the continuously compounded excess return given by:

$$\text{LVRPU} = \log(\sigma_{RU}^2 / VSRU). \tag{35}$$

Similarly are defined the downside variance risk premium in Euro terms:

$$\text{EVRPD} = N(\sigma_{RD}^2 - VSRD) \tag{36}$$

and in log returns terms:

$$LVRPD = \log(\sigma_{RD}^2 / VSRD).\qquad(37)$$

Upside and downside corridor variance risk premium (VRPUC and VRPDC) are obtained by using upside and downside corridor realised variance (σ_{RUC}^2, σ_{RDC}^2), instead of upside and downside realised variance in equations (34 - 37).

7. The Results for the volatility measures.

Descriptive statistics for the volatility series are reported in Table 1. The volatility series in our sample period are plotted in Figures 1 and 2. Wide corridor CIV measures are higher than realised volatility; CIV measures become lower on average as the corridor width shrinks. Asymmetric cut CIV measures are higher on average than realised volatility; CIVUP is lower than CIVDW, reflecting the higher implied volatility of out of the money put options with respect to out of the money call options. Wide corridor CIV measures (CIV0.2 and CIV0.25) display positive skewness (the opposite holds for narrower corridor measures). Excess kurtosis affects most of the corridor measures. Given the presence of extreme movements in volatility, the hypothesis of a normal distribution is rejected for most volatility measures.

In order to compare the results with Muzzioli (2010b) we gauge the forecasting performance of the different volatility measures, by resorting to the same metrics[1] widely used in the literature (see e.g. Poon and Granger (2003)). In particular, we use the MSE, the RMSE, the MAE, the MAPE and the QLIKE, defined as follows:

$$MSE = \frac{1}{m}\sum_{i=1}^{m}(\sigma_i - \sigma_R)^2\qquad(38)$$

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[1] Mincer-Zarnowitz regressions are also used in the literature in order to assess the unbiasedness and efficiency (with respect some historical measure of volatility) of the volatility forecasts. In order to avoid the telescoping overlap problem described in Hansen et al. (2001) forecasts are usually sampled at a monthly frequency (see e.g. Jiang and Tian, 2005). Given the limited sample at our disposal, we leave the investigation of the unbiasedness and efficiency of the volatility forecasts for future research.

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}((\sigma_i)-(\sigma_R))^2} \qquad (39)$$

$$MAE = \frac{1}{m}\sum_{i=1}^{m}|\sigma_i-\sigma_R| \qquad (40)$$

$$MAPE = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{\sigma_i-\sigma_R}{\sigma_R}\right| \qquad (41)$$

$$QLIKE = \frac{1}{m}\sum_{i=1}^{m}\left(\ln(\sigma_i)+\frac{\sigma_R}{\sigma_i}\right) \qquad (42)$$

where σ_i is the volatility forecast (*i=CIV0.2, CIV0.25, CIV0.3, CIV0.1-0.3, CIV0.3-0.1, CIVU, CIVDW)*, σ_R is the subsequent realised volatility, *m* is the number of observations. The MSE, RMSE and the MAE are indicators of absolute errors, while the MAPE indicates the percentage error. The QLIKE discriminates between positive and negative errors by assigning a larger penalty if the forecast underestimate realised volatility. Since a higher volatility is usually associated with negative market returns, the QLIKE function considers more important the correct estimation of volatility peaks than volatility minima.

The evaluation measures for the volatility forecasts are reported in Table 2. In order to see if the differences in forecasting performance are significant from a statistical point of view, we compare the predictive accuracy of the forecasts by computing the Diebold and Mariano test statistic (for more details see Diebold and Mariano (1995)) by using the MSE function which is considered as robust to the presence of noise in the volatility proxy (Patton (2010)). The pair-wise comparisons are reported in Table 3 (t-statistics along with the p-values). Note that a negative (positive) t-statistic indicates that the row model produced smaller (larger) average loss than the column model. The Diebold and Mariano test statistic under the null of equal predictive accuracy is distributed as a *N(0,1)*.

According to all the indicators CIV0.25 and CIV0.3 obtain the best performance even if they are not distinguishable according to the Diebold and Mariano test. CIV0.2 performs better than

CIV0.4. The forecasting performance improves when the corridor shrinks (from an overall 40% cut of CIV0.2 to an overall 50%-60% cut of CIV0.25 and CIV0.3) but then deteriorates if the cut is too heavy (an 80% cut of CIV0.4), indicating the presence of an "optimal" cut around 50%-60%. Among asymmetric measures, the one which relies more on call prices (CIV0.1-0.3) performs better than the one which relies more on put prices (CIV0.3-0.1), but looking at the Diebold and Mariano tests of equal predictive accuracy the two measures are not distinguishable. Both upside and downside corridor measures obtain a poor performance, however CIVDW, which focus on put option prices performs better than CIVUP, which focus on call option prices.

In order to further investigate the robustness of the results, it is useful to look at the correlations between the implied volatility measures and subsequent realised volatility, which are reported in Figure 3. As we can see, CIV0.25 has the highest correlation with realised volatility, while CIV0.3 has a markedly lower correlation. Therefore the results point to an overall low degree of information of out-of-the-money options and substantially confirm the preliminary results of Muzzioli (2010b) which point to an overall 50% cut of the risk neutral distribution. Among corridor measures with asymmetric cuts, the one which uses more put prices than call prices (CIV0.3-0.1) has a strikingly higher correlation with future realised volatility than the one which uses more call prices than put prices. The result is the opposite than the one obtained by looking at the performance evaluation based on the ranking functions, where CIV0.1-0.3 obtains a little better performance than CIV0.3-0-1, which however is not statistically significant according to the Diebold and Mariano test. Looking at the different performance obtained by CIVUP and CIVDW, the results based both on the Diebold and Mariano test and on the correlation with realised volatility point to a better performance of CIVDW. Therefore we can say that overall, the asymmetric cut that trims the risk neutral distribution in the upper part more than in the lower part is more informative about future realised volatility. Put prices which carry information on the probability of a downturn move of the underlying asset convey better information about future realised volatility.

8. The results for the variance risk premium.

The investigation of the variance risk premium is pursued into two steps, first we investigate the overall variance risk premium; second we use upside and downside realised and implied variance measures in order to investigate the risk premium in the upper and lower part of the risk neutral distribution.

In Table 4 are reported the descriptive statistics for realised variance (σ_R^2), the variance swap rate (VSR) and the risk premium measured both in Euro terms (EVRP) and in log returns terms (LVRP). As we can see the average swap rate is overwhelmingly higher than the average realised variance, reflecting the presence of a substantial variance risk premium. The variance swap rate is more volatile than realised variance, since it has been computed with the prices of different options series which differ in strike price and time to maturity. Overall, the variance risk premium is found to be large and negative: the average risk premium in Euro terms is equal to -2.472, and the average risk premium in log terms is -0.417. This means that investors do considerably price variance risk: the average Euro loss for each notional amount of 100 Euro invested in a long variance swap is -2.472 Euro. The average continuously compounded excess return of being long the variance swap and holding it to maturity is -41.7%. The negative average returns are both statistically different from zero (the t-statistics are adjusted for serial dependence, according to Newey-West). Therefore investors are willing to accept a strongly negative return being long in a variance swap, in order to be hedged against peaks in volatility.

In Table 5 are reported the descriptive statistics for upside and downside realised variance (σ_{RU}^2, σ_{RD}^2) , upside and downside corridor realised variance (σ_{RUC}^2, σ_{RDC}^2) and upside and downside variance swap rate (*VSRU* , *VSRD*). Upside and downside measures computed either by accumulating positive and negative returns (σ_{RU}^2, σ_{RD}^2) or by separating realisations higher or lower than the barrier equal to the forward price (σ_{RUC}^2, σ_{RDC}^2) are very similar. On average, upside realised variance measures are also not very different from downside realised variance measures

(upside corridor realised variance is a little higher than downside corridor realised variance). Upside realised variance measures display negative skewness, while downside realised variance measures display positive skewness. The downside variance swap rate is much more higher than the upside variance swap rate, meaning that investors price more negative realisations of the underlying than positive ones.

In Table 6 are reported the descriptive statistics for upside and downside risk premia computed both in Euro terms and in log-return terms and by using both upside and downside realised variance (σ_{RU}^2, σ_{RD}^2) and upside and downside corridor realised variance (σ_{RUC}^2, σ_{RDC}^2). Upside and Downside risk premia, computed with both methodologies, are negative and sizeable and all statistically significant (the t-statistics are adjusted for serial dependence, according to Newey-West). The average Euro loss for each notional amount of 100 Euro invested in a long downside corridor variance swap is -1.76 Euro, while for an upside corridor variance swap is -0.74 Euro. If realised semi-variance is measured by positive and negative returns, for a downside corridor variance swap the average Euro loss is -1.62 and for an upside corridor variance swap the average Euro loss is -0.89.

It follows that downside risk premia are overwhelmingly higher than upside risk premia. If realised semi-variance is measured by positive and negative returns, the downside risk premium is almost twice than the upside risk premium, if realised semi-variance is measured with returns higher or lower than the forward value of the underlying, then the downside risk premium is between two and three times the upside risk premium. This means that investors highly price downside risk (more than two times than upside risk).

9. Conclusions

In this paper we have exploited the concept of corridor implied volatility in order to analyse the sensitivity of the forecasting performance of corridor measures to the choice of the barriers and shed light on the information content of different parts of the risk neutral distribution of the stock price. As for the first goal of our study, different corridor measures with symmetric and asymmetric cuts have been analysed and the difference in the forecasting performance has been statistically scrutinised on the basis of the Diebold and Mariano test of equal predictive accuracy with the use of a robust loss function. As for the second goal of our study, upside and downside corridor measures have been used in order to understand the nature of the variance risk premium.

As for the sensitivity analysis to the choice of the barriers, the results substantially corroborates the preliminary findings in Muzzioli (2010b), by finding an optimal cut around 50%-60% of the risk neutral distribution. Moreover, the use of asymmetric cuts does not ameliorate the performance. There is a weak evidence of superiority of the corridor measure which rely more on put prices on the one which relies more on call prices, highlighting that the left part of the risk neutral distribution (the one which accounts for drops in the underlying asset price) is the most important in the derivation of the volatility measure.

In the Italian market the variance risk premium is found to be large and negative. Investors are willing to pay sizable premiums and experience a loss on average, in order to be hedged against peaks of variance. Upside and Downside risk premia are negative and sizeable and both statistically significant. Downside risk premia are overwhelmingly higher than upside risk premia (more than two times higher). This means that investors heavily price downside risk. Downside risk premium is the main component of the overall risk premium. The results are consistent across different measures of semi-variance (which discriminate between positive and negative returns, or realisations higher or lower than a threshold).

The present paper lends itself to be extended in many directions. High on the research agenda is the use of a longer dataset in order to investigate the unbiasedness and efficiency of the different volatility forecasts. Moreover, we would like also to investigate if the variance risk

premium composition (downside versus upside) changes in periods of high-low volatility. Another important question regards the robustness of the results to the change of the proxy for realised volatility. Leaving for future research the question, we are confident that the substitution of a different proxy would not change the ranking, since the computation methodology which exploits intra-daily five-minutes squared returns, provides large gains in terms of consistent ranking with respect to other more noisy volatility proxies (Patton and Sheppard (2009), Hansen and Lunde (2006)). Moreover, the results for the variance risk premium decomposition do not change if different measures of realised semi-variance (which discriminate between positive and negative returns, or realisations higher or lower than a threshold) are used.

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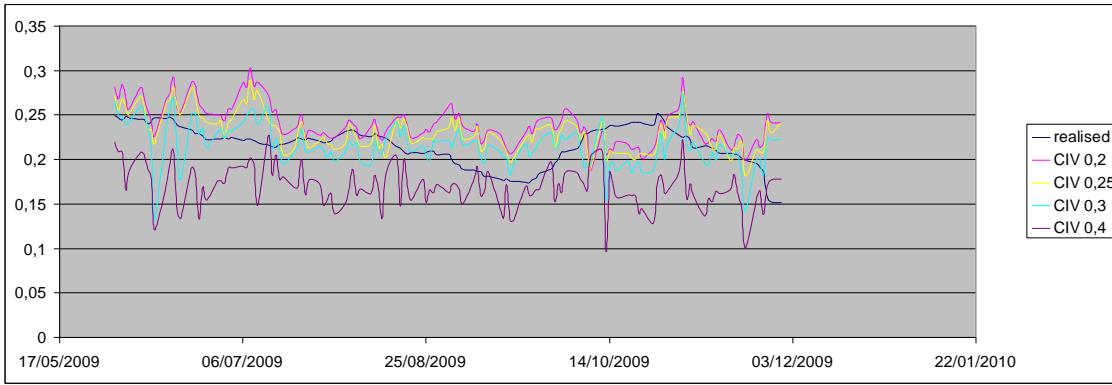
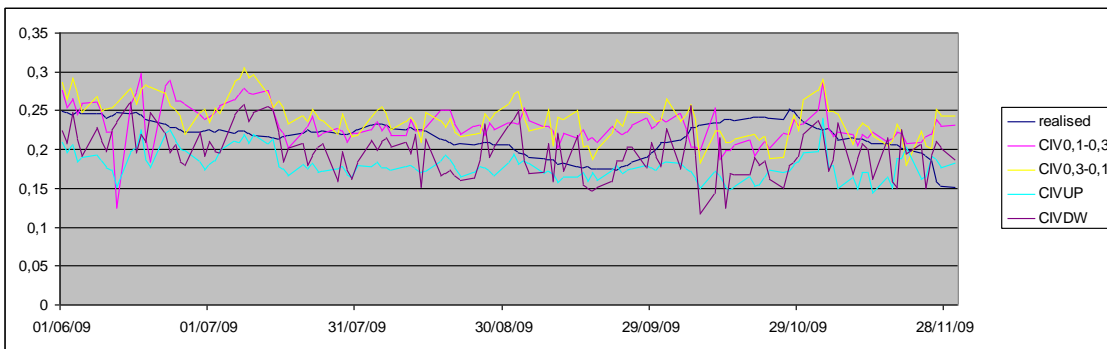Figure 1. Realised volatility and corridor implied volatility with symmetric cuts.

Figure 2. Realised volatility and corridor implied volatility with asymmetric cuts.
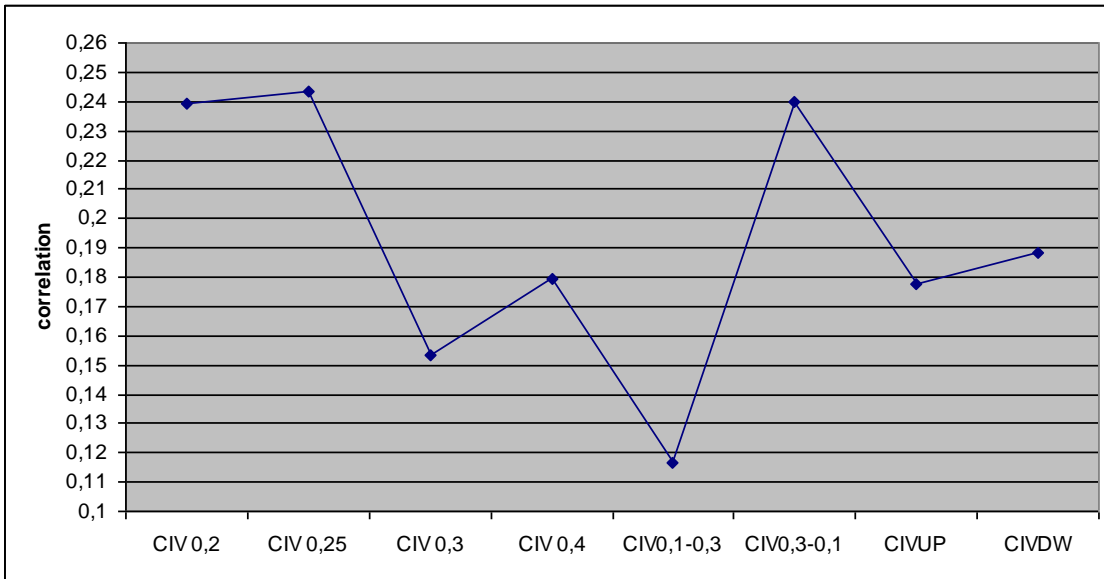
Figure 3. Correlation of corridor implied volatility measures with realised volatility.

Table 1. Descriptive statistics for the volatility series.

Statistic	σ_R	$\sigma_{CIV0.2}$	$\sigma_{CIV0.25}$	$\sigma_{CIV0.3}$	$\sigma_{CIV0.4}$	$\sigma_{CIV0.1-0.3}$	$\sigma_{CIV0.3-0.1}$	σ_{CIVUP}	σ_{CIVDW}
Mean	0.22	0.24	0.23	0.22	0.17	0.23	0.24	0.18	0.20
Std dev	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.02	0.03
Skewness	-0.70	0.55	0.48	-0.25	-0.18	-0.05	0.12	0.76	0.00
Kurtosis	3.00	3.08	2.89	4.42	3.59	5.39	2.95	3.82	2.63
Jarque Bera	10.84	6.53	5.09	12.39	2.66	31.30	0.37	16.25	0.75
p-value	0.00	0.04	0.08	0.00	0.26	0.00	0.83	0.00	0.69

The Table presents the descriptive statistics for the volatility series used in the sensitivity analysis: σ_r = realised volatility, σ_{CIV} = corridor implied volatility ($p = 0.2, 0.25, 0.3, 0.4$ respectively for CIV0.2, CIV0.25, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.3 and lower cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility.

Table 2. Predictive accuracy of the different volatility measures.

	$\sigma_{CIV0.2}$	$\sigma_{CIV0.25}$	$\sigma_{CIV0.3}$	$\sigma_{CIV0.4}$	$\sigma_{CIV0.1-0.3}$	$\sigma_{CIV0.3-0.1}$	σ_{CIVUP}	σ_{CIVDW}
MSE	0.0013	0.0009	0.0009	0.0030	0.0012	0.0014	0.0020	0.0015
RMSE	0.0362	0.0306	0.0294	0.0550	0.0339	0.0377	0.0451	0.0390
MAE	0.0299	0.0251	0.0226	0.0482	0.0272	0.0309	0.0387	0.0323
MAPE	0.1458	0.1221	0.1072	0.2176	0.1313	0.1500	0.1733	0.1486
QLIKE	-0.5238	-0.5267	-0.5252	-0.4849	-0.5235	-0.5230	-0.5072	-0.5138

The Table presents the indicators of the goodness of fit of the volatility series used in the study for the 30-day horizon:

$$MSE = \frac{1}{m}\sum_{i=1}^{m}(\sigma_i - \sigma_R)^2, \quad RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}((\sigma_i)-(\sigma_R))^2}, \quad MAE = \frac{1}{m}\sum_{i=1}^{m}|\sigma_i - \sigma_R|, \quad MAPE = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{\sigma_i - \sigma_R}{\sigma_R}\right|,$$

$$QLIKE = \frac{1}{m}\sum_{i=1}^{m}\left(\ln(\sigma_i) + \frac{\sigma_R}{\sigma_i}\right),$$ where σ_i is the volatility forecast, σ_R is the subsequent realised volatility, m is the

number of observations. , σ_{CIV} = corridor implied volatility ($p = 0.2, 0.25, 0.3, 0.4$ respectively for CIV0.2, CIV0.25, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.3 and lower cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility.

Table 3. Diebold and Mariano tests: pair-wise comparisons (MSE).

	$\sigma_{CIV0.25}$	$\sigma_{CIV0.3}$	$\sigma_{CIV0.4}$	$\sigma_{CIV0.1-0.3}$	$\sigma_{CIV0.3-0.1}$	σ_{CIVUP}	σ_{CIVDW}
$\sigma_{CIV0.2}$	4.13	2.72	-2.97	1.01	-1.52	-1.39	-0.49
	0.00	0.01	0.00	0.32	0.13	0.17	0.62
$\sigma_{CIV0.25}$		0.35	-4.20	-1.84	-3.79	-2.53	-1.72
		0.72	0.00	0.07	0.00	0.01	0.09
$\sigma_{CIV0.3}$			-5.93	-1.99	-2.05	-3.86	-2.59
			0.00	0.05	0.04	0.00	0.01
$\sigma_{CIV0.4}$				3.95	2.70	4.07	4.57
				0.00	0.01	0.00	0.00
$\sigma_{CIV0.1-0.3}$					-1.35	-2.05	-1.01
					0.18	0.04	0.31
$\sigma_{CIV0.3-0.1}$						-1.15	-0.23
						0.25	0.82
σ_{CIVUP}							2.31
							0.02

The Table reports the t-statistic and associated p-value for the Diebold and Mariano test of equal predictive accuracy for each couple of forecasts. The loss function used is the MSE$= \dfrac{1}{m} \sum_{i=1}^{m} (\sigma_i - \sigma_R)^2$, where σ_i is the volatility forecast, σ_R is the subsequent realised volatility, m is the number of observations, σ_{CIV} = corridor implied volatility ($p = 0.2, 0.25, 0.3, 0.4$ respectively for CIV0.2, CIV0.25, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.3 and lower cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility.

Table 4. Descriptive statistics for the variance measures and the risk premium in Euro terms and in log returns terms.

Statistic	σ^2_R	VSR	EVRP	LVRP
Mean	0.047	0.072	-2.472	-0.417
Std. Dev.	0.009	0.016	1.680	0.278
Skewness	-0.469	0.514	-0.131	0.097
Kurtosis	2.606	3.022	3.032	3.491
Jarque-Bera	5.668	5.774	0.380	1.525
Probability	0.059	0.056	0.827	0.466
t-stat			-9.738	-9.482
p-value			0.000	0.000

The Table presents the descriptive statistics for realised variance, the variance swap rate and the risk premium, computed both in Euro terms $\text{EVRP} = N(\sigma^2_R - VSR)$ and in log-return terms $\text{LVRP} = \log(\sigma^2_R / VSR)$, where σ^2_R is realised variance and VSR is the variance swap rate.

Table 5. Descriptive statistics for upside and downside variance measures and swap rates.

	σ^2_{RU}	σ^2_{RD}	σ^2_{RUC}	σ^2_{RDC}	VSRU	VSRD
Mean	0.024	0.023	0.025	0.022	0.032	0.040
Std. Dev.	0.004	0.005	0.015	0.019	0.007	0.012
Skewness	-0.599	0.510	-0.122	0.498	1.080	0.363
Kurtosis	2.598	2.891	1.808	1.888	4.518	2.648
Jarque-Bera	8.718	5.745	8.077	12.165	38.028	3.559
Probability	0.013	0.057	0.018	0.002	0.000	0.169

The Table presents the descriptive statistics for upside and downside realised variance (σ^2_{RU}, σ^2_{RD}), upside and downside corridor realised variance (σ^2_{RUC}, σ^2_{RDC}), upside and downside variance swap rates (VSRU, VSRD).

Table 6. Descriptive statistics for upside and downside risk premia.

	EVRPD	EVRPDC	EVRPU	EVRPUC	LVRPD	LVRPDC	LVRPU	LVRPUC
Mean	-1.62	-1.76	-0.89	-0.74	-0.50	-1.39	-0.32	-0.66
Std. Dev.	1.26	2.33	0.67	1.50	0.37	1.81	0.23	1.20
Skewness	-0.17	0.38	-0.26	-0.07	0.45	-0.94	0.26	-1.75
Kurtosis	3.08	2.60	3.19	2.23	3.89	2.93	2.35	5.84
Jarque-Bera	0.65	3.96	1.72	3.36	8.76	19.13	3.83	110.65
Probability	0.72	0.14	0.42	0.19	0.01	0.00	0.15	0.00
t-stat	-8.38	-4.45	-7.59	-2.14	-9.02	-4.40	-8.39	-3.23
p-value	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00

The Table presents the descriptive statistics for upside and downside risk premia, computed both in Euro terms and in log-return terms and by using both upside and downside realised variance (σ^2_{RU}, σ^2_{RD}) and upside and downside corridor realised variance (σ^2_{RUC}, σ^2_{RDC}), as follows: EVRPU = $N(\sigma^2_{RU} - VSRU)$, LVRPU = $\log(\sigma^2_{RU}/VSRU)$, EVRPD = $N(\sigma^2_{RD} - VSRD)$, LVRPD = $\log(\sigma^2_{RD}/VSRD)$, EVRPUC = $N(\sigma^2_{RUC} - VSRU)$, LVRPUC = $\log(\sigma^2_{RUC}/VSRU)$, EVRPDC = $N(\sigma^2_{RDC} - VSRD)$, LVRPDC = $\log(\sigma^2_{RDC}/VSRD)$.

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