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Property Rights, Warfare and the Neolithic Transition

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Abstract

This paper explains the multiple adoption of agriculture around ten thousand years ago, in spite of the fact that the first farmers suffered worse health and nutrition than their hunter gatherer predecessors. If output is harder for farmers to defend, adoption may entail increased defense investments, and equilibrium consumption levels may decline as agricultural productivity increases over a significant range, before eventually increasing thereafter. Agricultural adoption may have been a prisoners' dilemma in that adoption was individually attractive even though all groups would have been better off committing not to adopt while the initial productivity advantage of agriculture remained low.

Keywords: agriculture, defense, property rights, contest functions, Neolithic transition.

JEL codes: D74, N30, N40, O12, O40, Q10

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1 Introduction

One of the great puzzles of prehistory is why agriculture caught on so fast when archaeological evidence suggests those who adopted it had worse health and nutrition than their hunter gatherer predecessors and contemporaries.

Agriculture seems to have been independently adopted at least seven times: in Anatolia, Mexico, the Andes of South America, northern China, southern China, the Eastern United States, and in sub-Saharan Africa at least once and possibly up to four times. It spread from the sites of its original adoption at a speed which seems slow to the modern traveller but is remarkable by the standards of earlier innovations in prehistory - around one kilometer per year westward across the European continent, for example. You might think that, once the idea appeared and the climate made it possible, the answer was obvious: why sweat going out to hunt and gather when you can sit and watch the grass grow? An overwhelming productivity advantage of the new technology would seem quite enough to explain its evident appeal to our ancestors. And yet this suggestion is inconsistent, at least on the face of it, with some puzzling evidence from the skeletons of the first farmers (Weisdorf, 2005, and Bowles, 2009, provide valuable overviews). Studies of the bones and teeth of some of the earliest agricultural communities of the Near East show that farmers had worse health (due to poorer nutrition) than the hunter-gatherers who preceded them. Increases in agricultural productivity in later millennia more thanmade up for this eventually, but even so, the puzzle remains: what prompted agriculture to be adopted so quickly and often within a comparatively short space of time, if it did not achieve the one thing that a new agricultural technology surely ought to achieve - to leave people better fed than they were before?

As we discuss below, some significant difficulties arise in interpreting this evidence, but there is a strong prima facie case that agricultural adoption actually caused living standards to fall. There has been a large literature attempting to explain this paradox, which we review in section 2. This literature has principally focussed on reasons why a new technology that initially raised living standards might subsequently have had unintended and unforeseen negative consequences: reduced vitamin intake due to less variety in the diet, for instance, or increased disease due to crowding in settlements, or a variety of broadly Malthusian explanations in which any initial productivity advantage was offset by subsequent population growth. Each of these explanations makes the decision to adopt agriculture one that had negative consequences, at least for a time, for the hunter gatherer groups that made those adoption decisions. Had these consequences been foreseen, the adoption decision might not have seemed so attractive. In this paper we want instead to suggest to describe a process under which adoption of agriculture might have been a rational decision for the adopting groups but have had adverse consequences for society as a whole. We set out a formal model with two groups in which each group can be better off adopting agriculture than if it had not done so, given the decision of the other group, but both groups can be worse off after the adoption than they were before. We show therefore that there is no mystery in the idea that a technology which is attractive to each group might make all groups worse off; we also show that the model is consistent with some known features of the early Neolithic era.

Our model is based on the idea that agricultural communities needed to devote substantially more resources to defense than their hunter gatherer predecessors. Sitting and watching the grass grow is not the idyll it seems, for those who are sedentary are also vulnerable. When enemies attack, farmers have much more to lose than hunter-gatherers, who can melt into the forest without losing earthworks, houses, chattels and stores of food. So farmers not only face high risks, but they also need to spend time, energy and resources defending themselves, building walls, manning watchtowers, guarding herds, patrolling fields. This means less time and energy, fewer resources, devoted to making food. It could even happen that the greater productivity of the hours they spend growing and raising food is outweighed by the greater time they must spend defending themselves and the food they have grown - meaning that they produce less food in all. Almost certainly the end of the last ice age substantially improved the productivity of agriculture compared with the hostile conditions beforehand. But what would that have mattered if all, or more than all, of the additional benefits of the new farming technology ended up being spent on defense? Such additional defense expenditures could be highly costly even if, in equilibrium, fighting was no more frequent than it had been before the change.

On its own this story cannot resolve the paradox with which we began, since it explains the poor nutrition of the first farmers only at the price of making it mysterious why they should have adopted agriculture at all, let alone why this new technology should have spread with such rapidity. Stunted farmers would hardly have been good advertisements to their hunter gatherer neighbors of the qualities of their new wonder diet. What makes the difference, we suggest, is a crucial externality in the technology of defense. Once the very first farming communities began systematically to defend themselves, the fact that they could do so began to make them a threat to their neighbors, including communities who were on the margins of adopting agriculture themselves. For there is no such thing as a purely defensive technology. Even walls around a town can make it easier for attacking parties to travel out to raid nearby communities in the knowledge they have a secure retreat. The club that prehistoric man used to ward off attackers was the same club he used to attack others. Once a community has invested in even a modest army, whether of mercenaries or of its own citizens, the temptation to encourage that army to earn its keep by preying on weaker neighbors can become overwhelming. So, even if the first farming communities were not necessarily any better off than they would have been if no-one had adopted agriculture, once the process had started many communities had an interest in joining in. These interactions could lead each to act ineluctably against the collective interests of all.

In this paper we develop a formal model to show that it is entirely consistent with rational behavior for the discovery and adoption of a more productive technology to result in lower consumption. In fact we show that over a certain range of technological improvement, and a fairly wide range of parameter values, the more productive is the new technology the lower is consumption. In this case, consumption as a function of advances in technology follows a U-shape. A technological improvement in agriculture induces increased expenditure on defense whose impact on consumption initially outweighs the benefits of more advanced technology, before eventually the benefits of new technology become so important that consumption recovers. We perform a range of simulations that suggest that such a mechanism could have had effects important enough to be consistent with the prehistoric evidence. We also tackle the question of why communities would have adopted the technology in the first place. It might seem that communities might have been better off committing themselves not to adopt the "superior" technology, but we show that such a commitment would be difficult to enforce, because even when the net impact of the new technology is to lower living standards, each community may be better off adopting it independently of what its neighbors do.

Our model may therefore contribute to explaining an important transition in economic history that, quite plausibly, set economic growth in motion for the very first time. Agriculture and the associated social changes, notably the adoption of a sedentary way of life, made it possible for societies to accumulate economic resources in a way that hunter-gatherer societies were quite unable to do. If agriculture had been a technology that unambiguously improved the lot of humanity it would hardly be surprising that it had been adopted as soon as it became feasible. That is why the evidence about early Neolithic living standards has been such a puzzle - and perhaps adds substance to the eternal appeal that myths of the noble savage have had throughout human history, since such myths have seemed to suggest, counter-intuitively, that economic development since the time of the alleged fall has been both inevitable and regrettable.

In addition, the argument we set out here has implications for a broader set of questions that have preoccupied economists and economic historians. The first is the important role played in the determination of living standards by changes in the extent to which the output of production can be protected from theft. It is nowadays commonplace to acknowledge the importance of transactions costs in economic growth and development. North and Thomass (1973) place at the heart of economic history the notion of transactions costs, and the idea that institutions evolve to economize on transactions costs and in turn have an impact on economic outcomes. Acemoglu et.al. (2001) use similar arguments in their more systematic cross-country econometric investigation. Central to the transactions cost literature is the idea that economic endowments and technologies are not by themselves enough to yield economic growth. Individual agents still have to undertake transactions for the possibilities afforded by endowments and technology to be fully realized. However, agents may be inhibited from doing so by the cost of these transactions. Our argument carries a similar message. Even output that can be produced without the need for transactions as such must still be protected against theft before it can contribute to human welfare. Put like that the point may seem obvious - what is less obvious is that variations in the extent to which output can be protected may have played an important part in the dynamics of growth during prehistory as well as in historical times. Furthermore, increases in the vunerability of certain kinds of output to theft may not only have led to an increase in the incidence of theft, but more importantly to a diversion of societyís resources away from production and towards both aggressive and defensive investments, with a consequent impact on productive investment and growth.

Our argument thus demonstrates a reason why productivity and consumption do not necessarily move together, even when productivity growth is driven by a rational choice of technology. We have known since Malthus that productivity and consumption per capita may not move together, because population growth may outweigh changes in productivity, thereby bringing consumption per capita homeostatically back towards a certain long-run level. Our model demonstrates a simple and plausible mechanism, completely independent of population growth, whereby changes in productivity may induce behavioral changes that have a negative impact on living standards, which over a certain range is large enough to outweigh the potential benefit of technical progress.

Our paper is organized as follows. Section 2 summarizes briefly what we know about agricultural adoption, and reviews some explanations for the fact that it occurred in the way it did. Section 3 sets out our formal model of agricultural adoption by two communities. Its purpose is to show carefully that agricultural adoption can both improve the nutrition of any one community (relative to non-adoption and taking as given the behaviour of the other), while also worsening the nutrition of both communities relative to the status quo ante. Section 4 concludes.

2 Agricultural Adoption and Its Context

Evidence about the existence of at least seven and possibly as many as ten independent adoptions of agriculture is summarized in Richerson, Boyd and Bettinger (2001). Evidence for independent adoption has been found in Anatolia, Mexico, the Andes of South

America, northern China, southern China, the Eastern United States, and in sub-Saharan Africa at least once and possibly up to four times. The subsequent rapid spread of agriculture around the world is documented in Bellwood (1996) and Cavalli-Sforza, Menozzi and Piazza (1994). As Diamond (1995, especially chapter 6) emphasizes, it was not a discrete choice: most communities continued to combine hunting and farming to varying degrees for a very long time. Barker (2006) provides an encyclopaedic overview of what is known about this process.

In this section we review evidence relating to three questions. First, what is the evidence that farmers were worse off on average after adoption than before? Secondly, why might have adoption of agricultural at the margin have become attractive around this time when it had evidently not been sufficiently attractive before? Thirdly, what is the evidence that agriculture led to increased investment in defensive technology, as it must have done for the process specified in our model to have been at work?

There is substantial evidence of agricultural communities whose skeletons show marks of poorer nutrition than forager communities in similar areas at similar or slightly earlier dates The studies presented in Cohen & Armelagos (1984) provided the first main evidence for this; substantially updated evidence is presented in Cohen and Crane-Kramer (2007). Steckel and Rose (2002) have provided the most comprehensive attempt to standardize methodologies for comparing evidence across sites and regions. Other summaries of the evidence are presented in Weisdorf (2005) and Bowles (2009). The extent of this evidence for the existence of poorer nutrition among hunter gatherer than among roughly contemporaneous forager communities is not in serious dispute, but questions of causality are controversial. Two kinds of problem arise in concluding that agricultural adoption made foragers worse off.

First, nutrition is not a perfect indicator of well-being: agricultural adopters might consciously have traded other benefits, such as physical security, for somewhat worse nutrition; while there is limited positive evidence in favor of this argument it is almost impossible to rule out. More subtly, many cereals are substantial sources of energy while being deficient in some important nutrients, as a number of studies in Cohen and Armelagos (1984) and Cohen and Crane-Kramer (2007) discuss. Such a shift in the balance of nutrition towards calories and away from proteins and vitamins might have been attractive to the individuals but have led to their skeletons displaying greater evidence of malnutrition.

Secondly, we cannot know that in comparing farmers to foragers we are holding other factors constant. There are some reasons to think that the nutritional environment was deteriorating for hunter gatherers due to over-hunting, a hypothesis advanced by Winterhalder & Lu (1995) and strongly corroborated by data in Stutz, Munro and Bar-Oz

(2009). It is also plausible that even if initial farming adoption improved nutrition this might have been offset by subsequent population growth, as argued by Bar-Yosef and Belfer-Cohen (1989), Weisdorf (2008), Guzman and Weisdorf (2008) and Robson (2010). In particular, the mechanism proposed by Robson is that a shift from hunter-gathering to agriculture increased total factor productivity and led to a larger equilibrium population, causing the overall land-labour ratio to fall. Using evolutionary psychology, he suggests that farmers responded to land scarcity by having more children but investing less per child. Lower investment in somatic capital explains why farmers were less robust than hunter-gatherers.This, plus the associated population growth, might not only have lowered per capita nutrition but also caused disease due to over-crowding.

Given dating uncertainty, we obviously cannot be sure whether the poor nutrition of farmers documented by archaeologists represents the immediate aftermath of agricultural adoption or the impact after a period of induced population growth. However, there are enough independent sites showing poorer nutrition of farmers (beginning with those reported by Cohen & Armelagos, 1984) for the hypothesis that that agricultural technology made them worse off to be worth taking very seriously.

What reason might we have nevertheless to think that agricultural technology became sufficiently attractive to be adopted at the margin although it had not been so before? First, the global warming associated with the end of the Upper Paleolithic opened technological possibilities that had not existed before, even in previous interglacial periods (see Richerson, Boyd and Bettinger, 2001; Dow, Olewiler & Reed, 2006). Barker (2006) shows that many communities had adopted over time practices that made agriculture more attractive - a point also made by Tudge (1998). These practices made it possible to sedentarize gradually - by returning regularly to previous foraging grounds, for example, and tending to the plants and trees there. Of course, it is important not to exaggerate the obstacles to adoption in previous interglacial periods. As Mithen (1996) points out, earlier humans had sophisticated biological knowledge of both animals and plants, so that it does not seem likely that the problem lay in lack of the kinds of skill that agriculture would have required. Also, as Ofek (2006) argues, earlier hominid evolution had seen a number of powerful social and economic innovations including the hunter-gatherer lifestyle itself. One hypothesis (due to Cohen, 1977) suggests that there was a late Pleistocene food crisis caused by population pressure which more or less forced agricultural adoption as a way out. This is problematic, however, because of evidence that hunter-gatherers were able to control population growth through various measures including infanticide. The overall conclusion to be drawn from the evidence in Barker (2006) is therefore probably that the combination of earlier innovations and the magnitude of the global warming pushed agriculture over the threshold of productivity to make it attractive enough to adopt at the margin.

What is the evidence that agricultural adoption led to an increase in investments in defensive technology? Note that this is not the same as the claim that they led to an increase in the incidence of warfare, for which there is some evidence (Gat 1999; Baker, Bulte and Weisdorf, 2010). Increased warfare is compatible with but in no way implied by our model. Investments in defensive technology would have included the time spent in guarding herds and patrolling lands and settlements, an investment that leaves few archaeological traces. But the most lasting traces are those left by weaponry and fortifications (Gat 2006, pp.167-173). It is possible that some investments in weaponry might be due to costly sexual signaling independent of any need to the community (Seabright, 2010, p. 62); the greater elaborateness of weaponry found among agricultural communities might be a sign of greater prosperity rather than greater need. But the evidence from fortifications is harder to argue away—men don't build walls and dig ditches to impress women unless walls and ditches are what their women really want. The first village settlement at Jericho, for instance, has been dated to before 9000 b.c.e., and within a thousand years it had grown to a substantial settlement of severalhectares of mud-brick houses with thick walls. The first evidence of the famous city walls comes from the early eighth century b.c., and the presence of great water tanks, probably for irrigation, is attested from the seventh century. And a massive ditch, thirty feet deep and ten feet wide, was dug into the rock without metal tools. A similarly massive ditch, all the more impressive because of the small size of the community that built it, is recorded at Banpo neolithic village in central China (Seabright, 2010, p. 55).

While none of these three strands of evidence is free of difficulty, we feel confident in concluding that there is strong prima facie evidence that the adoption of agriculture was a) productive enough for adoption at the margin to become attractive to forager communities in the Holocene era, b) insufficiently productive to raise living standards on average where a large proportion of communities adopted it, and c) accompanied by substantial increases on investments in defensive technology, particularly in fortifications. The purpose of our model is to show how a) and b) are compatible, and the answer lies in the presence of c). The later dynamic consequences of agricultural adoption (notably those of increased population growth) are entirely compatible with the model but are not treated within it.

Theories that appeal to the adverse consequences of population growth following a productivity improvement are inevitably reminiscent of the arguments of Thomas Malthus (1798, II.25)1. However, with the exception of Robson (2010) cited above, Malthusian theory would predict that an initial increase in living standards above subsistence level would be followed by a subsequent reversion to the subsistence level as population growth caught up with the increased productivity of the fixed factor land. It does not explain the evidence we have cited here, which is of an initial reduction in per capita consumption which lasted for a significant time before it was eventually reversed. This is the purpose of our model.

The model is related to two strands of economic literature. The first, which models in various ways the choice of economic agents to devote resources to protecting their property and (often simultaneously) encroaching on the property of others, is surveyed in Dixit (2004); an interesting contribution is Grossman (1998). Gonzalez (2005) and Gonzalez & Neary (2008) develop the idea that an increase in the resources used for conflict may be induced by an increase in productivity, in an endogenous growth framework in which productive capital and conáict capital are complements. Unlike us, they focus on technological backwardness induced by anticipation of future conflict rather than on the possible welfare-reducing consequences of the adoption of more advanced technology. The second consists of models embodying contest functions (Becker 1983, Dixit 1987, Hirshleifer 1989, Nitzan 1994, Aidt 2002, Hwang, 2009), where the investments of one agent in some process that changes changes resource allocations in that agentís favour (lobbying, for instance) can be offset by the investments of a rival agent. Finally, a long-standing literature in political theory, going back to Ibn Khaldun (1377) and Ferguson (1774), and excellently discussed by Ernest Gellner (1994), considers the need to raise a surplus for defense as constituting the foundation of the division of labor in modern society, and as giving rise to some of the most intractable problems of political organization.

3 A Simple Model: Production and Theft

There are two groups, $i = 1, 2$ of equal size, each endowed with one unit of labor[1]. Each group allocates labor l_i^H to hunting, l_i^F to farming and l_i^W to warfare, with $l_i^H + l_i^F + l_i^W = 1$.

Both hunting and farming are forms of production, while warfare is an activity devoted to the theft of resources from others and defense against such theft. We begin with production. We assume that the outputs of hunting and farming are measured in such a way that single units of each are nutritionally equivalent. Labor is used to produce ouput in either hunting or farming, subject to the following strictly concave production technologies.

[1]Band size is therefore exogenous, unlike in Marceau & Myers (2006). Marceau & Myers explain post-adoption food crises by splintering of previously cooperative groups. Such mechanisms are of course compatible with the one discussed here - however, predicting when cooperation will occur and when it will break down is notoriously difficult, and the present model requires no such mechanism for food crises to occur.

$$\text{Hunting: } H_i = H\left(l_i^H\right) = \frac{\left(l_i^H\right)^{1-\eta}}{1-\eta} \quad \text{for } 0 < \eta < 1 \tag{1}$$

$$\text{Farming: } F_i = F\left(l_i^F\right) = \frac{\left(l_i^F\right)^{1-\eta} f_i}{1-\eta}$$

The above specification captures a range of concavity assumptions ranging from near linearity to highly inelastic production. If $f_i < 1$ then farming is less productive than hunting for the same level of labor input.

The marginal product of labor is given by

$$H'\left(l_i^H\right) = \left(l_i^H\right)^{-\eta} \text{ and } F'\left(l_i^F\right) = \left(l_i^F\right)^{-\eta} f_i \tag{2}$$

These functions satisfy the Inada conditions

$$H'(0), F'(0) = \infty. \tag{3}$$

Aggregate output is maximised when no labor is devoted to warfare and all labor is efficiently allocated between hunting and farming. In this case, output is given by

$$Y_i^{potential} = \frac{\left(1 + f_i^{\frac{1}{\eta}}\right)^{\eta}}{1-\eta}$$

This is an increasing function of f_i. Thus, whatever the value of f_i, mixed hunting and farming always yields a higher potential output than either hunting or farming alone. This is due to diminishing returns in each type of production. If f_i is small it would be grossly inefficient to specialise in farming, but there is some gain to be had from farming on a small scale as a supplement to hunting.

Next we consider warfare. The allocation of labor to warfare is valuable for each group because it enables this group to appropriate the food of its rival, and to resist similar attempts by the other group. In the model warfare results purely in theft, never in the destruction of resources; this simplification seems reasonable since if we were to take resource-destruction into account, it would be even easier to demonstrate the possibility of Pareto-inferior adoption equilbria.

Let γ_{ij} denote the proportion of group i's hunting output, and ϕ_{ij} the proportion of its farming output, that is transferred to group j. In equilibrium there will typically be transfers in both directions: each group will steal to some extent from the other. The total income of a group is equal to the hunting and farming output that it succeeds in keeping safe from theft, plus the hunting and farming output that it steals from the rival group:

$$C_i = (1-\gamma_{ij})H(l_i^H) + (1-\phi_{ij})F(l_i^F) + \gamma_{ji}H(l_j^H) + \phi_{ji}F(l_j^F) \quad (4)$$

It will be convenient to write this in the alternative form:

$$\begin{aligned} C_i &= (1-\gamma_{ij})H\left(\frac{1-l_i^W}{1+r_i}\right) + (1-\phi_{ij})F\left(\frac{r_i\left(1-l_i^W\right)}{1+r_i}\right) \\ &\quad +\gamma_{ji}H\left(\frac{1-l_j^W}{1+r_j}\right)) + \phi_{ji}F\left(\frac{r_j\left(1-l_j^W\right)}{1+r_j}\right) \end{aligned} \quad (5)$$

where $r_i = l_i^F/l_i^H$ is the ratio of farming labor to hunting labor and $1-l_i^W$ is is the total amount of labor devoted to these activities.

The proportions of output that are transferred depend on the amounts of labor devoted to warfare by the two groups and are given by the following logistic functions:

$$\begin{aligned} \gamma_{ij} &= \frac{2\gamma}{1+e^{-\alpha l}} \text{ and } \gamma_{ji} = \frac{2\gamma}{1+e^{\alpha l}} \\ \phi_{ij} &= \frac{2\phi}{1+e^{-\beta l}} \text{ and } \phi_{ji} = \frac{2\phi}{1+e^{\beta l}} \end{aligned} \quad (6)$$

where $\gamma, \phi > 0$ and $l = l_j^W - l_i^W$. We shall assume that $2\gamma < 1 + e^{-\alpha}$ and $2\phi < 1 + e^{-\beta}$. Since $l \in [-1,1]$ this ensures that $\gamma_{ij}, \gamma_{ji}, \phi_{ij}, \phi_{ji} < 1$ and hence each group retains a non-zero fraction of its own hunting and farming outputs. Note that the argument of both transfer functions is the absolute difference in military strength of the two groups, not the ratio of their strengths. Using the ratio of their strengths would not effect the main results of this paper.[2].

When the two groups are evenly matched $l = l_j^W - l_i^W = 0$ and

$$\begin{aligned} \frac{\partial \phi_{ij}}{\partial l} &= \frac{\beta\phi}{\alpha\gamma}\frac{\partial \gamma_{ij}}{\partial l} \\ \frac{\partial \phi_{ji}}{\partial l} &= \frac{\beta\phi}{\alpha\gamma}\frac{\partial \gamma_{ji}}{\partial l} \end{aligned}$$

[2] The equivalent ratio formulae would be as follows:

$$\frac{2\gamma}{1+\left(l_j^W/l_i^W\right)^\alpha}$$

For an application of this alternative approach see Rowthorn et al (2009).

Thus, the ratio $\dfrac{\beta\phi}{\alpha\gamma}$ indicates the relative vulnerability of farming and hunting with respect to changes in the balance of military power. If this ratio is large then an adverse shift in the balance of power will have a much greater impact on farming income than on huntng income.

We assume that each group chooses its own labor allocation so as to maximise consumption taking as given the labor allocation of the other group. The resulting solution will be a Nash equilibrium[3]. It is useful in the present context to distinguish between two kinds of Nash equilibrium. A solution is a *local* Nash equilibrium if neither group can increase its consumption through a *marginal* change in its own labor allocation. It is a *global* Nash equilibrium if there is no feasible labor re-allocation, however large, that will yield either group a higher level of consumption, taking the allocation of the other group as given. The following are necessary, although not sufficient, conditions for a Nash equilibrium of either kind:

$$\frac{\partial C_i}{\partial r_i} \leq 0, r_i \geq 0 \text{ and } r_i \frac{\partial C_i}{\partial r_i} = 0 \tag{7}$$

$$\frac{\partial C_i}{\partial l_i^W} \leq 0, l_i^W \geq 0 \text{ and } l_i^W \frac{\partial C_i}{\partial l_i^W} = 0 \tag{8}$$

Given that $\gamma_{ij}, \phi_{ij}, < 1$,the Inada conditions (3) together with (7) imply that optimal values of l_i^H and l_i^F are non-zero and hence $l_i^W < 1$. Thus, some labour is always devoted to both hunting and farming. Moreover,

$$(1-\gamma_{ij})H'(l_i^H) = (1-\phi_{ij})F'(l_i^F) \tag{9}$$

The intuitive meaning of equation (9) is simple. The total amount of labor that is available for production should always be allocated so as to equalize the marginal products in hunting and farming, net of any loss from theft.

3.1 Mixed Hunting and Farming: Symmetrical Solution when $f_i = f_j = f$

Suppose that both groups use the same farming technologies so that $f_i = f_j = f$. At a symmetrical equilibrium, the labor allocations in the two groups are identical. Thus, $l_i^F = l_j^F = l^F$, $l_i^H = l_j^H = l^H$, $l_i^W = l_j^W = l^W$ and $r_i = r_j = r$. The following proposition is proved in Appendix 2.

[3] We consider only pure strategies in whih groups do not randomise their behaviour. Throughout this paper the term "Nash equilibrium" is used to denote an equilibrium in which both groups use only pure strategies.

Proposition 1: *Equations (7) and (8) have a unique symmetric solution, which is given as follows*

$$\begin{aligned} \frac{l^F}{l^H} &= r = \left(\frac{(1-\phi)f}{1-\gamma}\right)^{1/\eta} && (10) \\ l^W &= \min(0, 1 - P/Q) && (11) \end{aligned}$$

where

$$\begin{aligned} P &= (1-\eta)(1+r) > 0 \\ Q &= \frac{\alpha\gamma}{1-\gamma} + \frac{\beta\phi r}{1-\phi} > 0 \end{aligned}$$

The symmetric solution given by equations (10) and (11) is a local Nash equilibrium if and only if it satisfies one of the following conditions:

(i) $P > Q$ *or*

(ii) $P \leq Q$ *and*

$$r^2 - 2\left(\frac{(1-\eta)^2}{\eta}\frac{(1-x)}{8} - x\right)r + x^2 \geq 0 \qquad (12)$$

where

$$x = \frac{\left(\frac{\alpha\gamma}{1-\gamma}\right)}{\left(\frac{\beta\phi}{1-\phi}\right)}$$

Proof. See Appendix 2.

Corollary 2 *Provided* η *is sufficiently close to* 1 *condition (12) is satisfied for all values of* f*.Thus, if there are strongly diminishing returns in production,the symmetric solution is always a local Nash equilbrium.*

Corollary 3 *If* $\frac{\alpha\gamma}{1-\gamma} \neq \frac{\beta\phi}{1-\phi}$ *and* η *is sufficiently close to zero, there are values of* f *for which condition (12) is not satisfied. Thus, if the production function is almost linear, the symmetric solution may not be a local Nash equilbrium.*

■

If condition (12) is satisfied then, taking the behavior of the other group as given, neither group can improve its situation through a marginal change in its own behavior.

This leaves open the possibility that a group might do better through a non-marginal change in behavior. It also leaves open the question of asymmetric Nash equilibria in which the two groups choose different strategies. We were not able to resolve these issues by analytic means so we explored the properties of the model by simulation. Using a wide variety of different combinations of f and the other parameters (8,000 in all) we found that almost 90% of these combinations give rise to a symmetric global Nash equilibrium. These are the only global Nash equilibria that we could find. In all of the numerical examples considered later in this paper, the symmetric solution satisfies the conditions given in Proposition 1 and our simulations indicate that it is also a global Nash equilibrium. Details of these simulations are given in Appendix 1.

3.2 When does farming reduce consumption?

With fixed expenditure on warfare, consumption is always higher if some labor is devoted to farming. However, this ignores the possibility that the introduction of farming may lead to additional warfare expenditure on such a scale that the amount left over for consumption actually falls. We shall now specify the parameter values for which this will occur.

Consider a modified version of the above model in which all parameters are the same, but each group is constrained to allocate zero labor to farming. In this case, $l_i^F, l_j^F = 0$ and consumption is given by the following version of equation (4):

$$\text{hunting only: } C_i = (1 - \gamma_{ij})H(l_i^H) + \gamma_{ji}H(l_j^H) \tag{13}$$

Each group chooses the allocation of labor between warfare and hunting that maximises its own consumption, taking the labour allocation of the other group as given

.

Proposition 4: *There is at most one symmetric Nash equilibrium in the hunting only version. If it exists it is given by the following equations:*

$$\text{hunting only: } l^W = \min\left(0, 1 - \frac{(1-\gamma)(1-\eta)}{\alpha\gamma}\right) \tag{14}$$

Proof: *The hunting only solution can be be derived by setting $r = 0$ in the equation (11) for mixed hunting and farming.*

Proposition 5: *At an internal symmetric equilibrium (where $l^W > 0$) consumption in the mixed hunting and farming economy is lower than in the hunting only economy if and only if the following inequality is satisfied:*

$$1+\left(\frac{1-\phi}{1-\gamma}\right)^{\frac{1-\eta}{\eta}} f^{\frac{1}{\eta}} < \left(1+\frac{\beta\phi}{\alpha\gamma}\left(\frac{1-\phi}{1-\gamma}\right)^{\frac{1-\eta}{\eta}} f^{\frac{1}{\eta}}\right)^{1-\eta}$$

Proof: See Appendix 2.

For sufficiently large values of f this inequality does not hold and consumption is therefore higher in the mixed economy than in the hunting only economy. Note that for small values of f the above inequality can be approximated as follows:

$$\frac{1}{1-\eta} < \frac{\beta\phi}{\alpha\gamma}$$

If this condition is not satisfied, there can be no value of f for which the introduction of farming leads to a fall in consumption. Other things being equal, the larger is the ratio $\frac{\beta\phi}{\alpha\gamma}$ the larger is the range of f over which the shift to agriculture reduces consumption.

3.3 The effects of agricultural innovation

With fixed expenditure on warfare, an agricultural innovation that increases farming productivity will lead to higher consumption. However, the shift to more advanced farming may involve so much extra warfare expenditure that the amount left over for consumption falls. The following proposition specifies the parameter values for which this will be the case.

Proposition 6: *At an internal symmetric Nash equilibrium* (where $l^W > 0$), *both groups will be worse off following a marginal increase in f if and only if the following conditions are satisfied*

$$\begin{aligned} \frac{1}{1-\eta} &< \frac{\beta\phi}{\alpha\gamma} \\ f &< \left(1-\eta-\frac{\alpha\gamma}{\beta\phi}\right)^{\eta}\left(\frac{1}{\eta}\right)^{\eta}\left(\frac{1-\gamma}{1-\phi}\right)^{1-\eta} \end{aligned}$$

Proof: See Appendix 2.

Since $0 < \eta < 1$ the first condition requires that $\frac{\beta\phi}{\alpha\gamma} > 1$. This is possible only if farming is more vulnerable than hunting to shifts in the balance of military power. When both conditions are satisfied, consumption will decline as productivity increases because the resulting shift of labor out of production into warfare outweighs the greater productivity of the labor that remains in production. Other things being equal, the larger is the value of $\frac{\beta\phi}{\alpha\gamma}$ the larger is the range of f for which the gains of higher agricultual productivity are outweighed by the resulting diversion of labor into warfare.

3.4 Some Numerical Examples

As an illustration, we shall now consider some numerical examples. In each example, simulation indicates that the symmetric solution is internal and is a global Nash equilibrium.

Figure 1 here

Figure 1 plots consumption against agricultural productivity (f) for four different combinations of the main parameters. Along each curve rising agricultural productivity is accompanied by a gradual shift of labor from hunting into farming. At the same time an increasing share of the labor force is devoted to warfare. Along curve C1, consumption rises uniformly as the benefits of technical progress always outweigh the costs of higher warfare expenditure. Along the other curves there is a phase during which the additional costs of warfare exceed the gains from technical progress, with the result that consumption falls. Along curve C2 this phase is of short duration and the fall in consumption is modest. Along curves C3 and C4, the fall in consumption is greater and lasts for longer. The values of f for which consumption is at a minimum along C2, C3 and C4 are 1.08, 1.21, 1.36, respectively. The corresponding values of l^w are 22%, 33% and 55%. A figure of 55% for the proportion of labor devoted to warfare is rather extreme, but the lower figures of 22% and 33% are more realistic. They indicate that for certain parameter values our model can generate a plausible explanation of how the extra warfare costs associated agriculture may outweigh its technical advantages.

Figure 2 here

Figure 2 compares consumption under two kinds of social arrangement. In one case, there is no agriculture at all and both communities engage exclusively in hunting. They allocate their labor between hunting and warfare as analyzed above and their level of consumption is determined by the Nash equilibrium given in equation (14). In the other case, both communities engage in mixed hunting and farming and their consumption is determined by the Nash equilibrium given in equations (10) and (11). Comparing the two curves there is a prolonged range over which the mixed hunting and farming communities have lower consumption than the specialized hunters. They would have been better off to have remained as pure hunters. Despite its potential benefits, the development of agriculture has made them poorer because it is associated with greater expenditure on warfare. For the same reason, even in communities where agriculture is already partially established, further technical progress in this sector may make them worse off. This phase does not last forever and consumption eventually recovers as technical progress continues.

We should emphasize that what we call "warfare" costs are not the costs of resource destruction, let alone loss of life, associated with warfare as it has been known historically in all societies. If we were to take resource destruction into account the likelihood that agriculture could lower consumption would be even higher than it is in our model. Our model captures only the costs of preparing for war - the costs of policing settlements and protecting their foodstocks. The fact that agricultural societies had to divert considerable resources into such policing activities - over and above any destruction of resources that resulted - could important part of the explanation for the drop in consumption that followed the Neolithic revolution.

3.5 Prisoners' Dilemma: agricultural innovation is difficult to resist even if it lowers consumption for everyone.

Proposition 6 implies that both groups might be better off committing themselves to inhibit agricultural innovation, but says nothing about how difficult or easy that commitment might be. Here we show that it may be difficult to make such a commitment, since it may be in each group's individual interest to adopt new agricultural techniques even though it is not in their collective interest.

Consider a system that is in symmetric equilibrium and suppose that a small improvement in agricultural productivity from f to $f+\Delta f$ becomes available. Suppose also that i adopts the new technology and the other group j does not. The effect on the consumption of i will depend on what happens to labor allocation in j. If there is no alteration in j's labor allocation, then i will experience the following change in consumption

$$\Delta C_i = \left[\frac{\partial C_i}{\partial f_i} + \frac{\partial C_i}{\partial l_i^W}\frac{dl_i^W}{f_i} + \frac{\partial C_i}{\partial r_i}\frac{dr_i}{f_i}\right]\Delta f$$

Since the system was initially in equilibrium, the second and third terms in parentheses are zero and hence

$$\begin{aligned}\Delta C_i &= \frac{\partial C_i}{\partial f_i}\Delta f \\ &= \frac{(1-\gamma_{ij})\left(l_i^F\right)^{1-\eta}}{1-\eta}\Delta f \\ &> 0\end{aligned}$$

Alternatively, j may adjust its labor allocation in response to what happens in i. In this case, the above formula must be modified as follows

$$\Delta C_i = \frac{\partial C_i}{\partial f_i}\Delta f + \left[\frac{\partial C_i}{\partial l_j^W}\frac{dl_j^W}{\partial f_i} + \frac{\partial C_i}{\partial r_j}\frac{dr_j}{\partial f_i}\right]\Delta f \tag{15}$$

We were unable to determine analytically the sign of the right hand expression, but extensive simulations suggest that it is normally positive (see Appendix 1). An analogous equation applies to group j if it chooses to follow the example of i and adopt the new technology.

The above discussion implies that it will normally pay either group to adopt the new technology unilaterally. It will also pay the other group to follow suit. However, under the conditions specified in Proposition 6, both groups will be worse off if they both adopt the new technology. In this case, they will be trapped in a prisoners' dilemma.

This is the situation on the downward sloping sections of the curves shown in Figure 1. The nature of the dilemma is illustrated in Table 1, which is based on equation (15). The table shows the implications of a small change in technology, Δf, at the symmetric equilibrium for $f = 1.1$ on curve C4.

Table 1: Prisoners' Dilemma Example

		Column Player	
		Do Not Adopt	Adopt
Row	Do Not Adopt	0, 0	$-2.87\Delta f$, $2.51\Delta f$
Player	Adopt	$2.51\Delta f$, $-2.87\Delta f$	$-0.36\Delta f$, $-0.36\Delta f$

If one group adopts the new technology unilaterally it makes a gain for itself but imposes an even larger loss on the other group. This loss is only partly recouped if the other group also adopts the new technology. Similar matrices can be computed throughout the downward sloping sections of curves C2, C3 and C4.

4 Conclusions

In this paper we have shown that the adoption of a superior production technology can make its adopters worse off if it requires them to devote more resources to stealing and to protecting their output from each other's aggression. They may nevertheless find adoption difficult to resist since it may be in each party's interest to adopt the new technology whatever the other does - in other words, adoption may be a prisoners' dilemma resulting in the collective harm via the pursuit of individual advantage.

We have set out a model in which a potentially superior technology produces output which is easier for others to steal, in a sense we have made analytically precise. We have demonstrated that at most one symmetric pure strategy equilibrium of our model exists and have given it a characterization. In simulations over a wide range of plausible parameter estimates we have shown that a pure strategy equilibrium exists in a a large majority of cases. In a substantial subset of these cases, consumption initially falls as the productivity of the superior technology increases, although it eventually recovers when the productivity advantage of the new technology becomes sufficiently great. Furthermore, in all these cases consumption falls in equilibrium even though each group benefits through unilateral adoption given the behaviour of the other group. Under these conditions a commitment by all groups to refrain from adopting the new technology would be difficult to achieve even though such an agreement would make them all better off.

Our model may contribute to explaining the puzzle that the spread of agriculture in the Neolithic revolution was accompanied by an apparent decline in human nutrition. If agriculture made people worse off why did it spread so dramatically throughout the world? The answer, we have argued, has two elements. First, the rapid spread of agriculture is explained by the fact that, after the end of the ice age, agriculture became productive enough to be attractive to many individual groups, given the choices of other groups with whom they came into contact. This improvement in productivity compared to the alternative of hunting and gathering was due principally to climate change. However, the evolution of human cognitive capacities as described by Steven Mithen may also have played a part: the early neolithic phase of global warming was far from being the only comparable one in human prehistory, but it may have been the only one to occur after human beings were cognitively ready to take advantage of it.

The second element in our answer helps to explain the poor nutrition of the first farmers. Many mechanisms have been proposed to explain why agricultural adoption might have had unforeseen and unintended consequences that lowered the nutrition of adopters, many of them the effects of population growth. We suggest that some consequences of adoption might make all adopters collectively worse off even if no adopter was individually worse off than without the adoption. In particular, increased investments in defense that were induced in equilibrium by the adoption of agriculture could impose a negative externality on neighbors: each group had to invest more in defense both because of its own decision to adopt and because of the adoption decisions of its neighbors.

Paradoxically, too, the higher the proportion of adopters among a group's neighbors, the stronger the incentive for the group itself to adopt, even though this would also require it to spend more on defense. Together with the population growth effect described by Bar-Josef and Belfer-Cohen and the depletion of game described by Winterhalder and Lu, this would have created a ratchet effect of adoption that goes a long way towards explaining the speed with which the technology spread.

In conclusion, there is no need to appeal to unanticipated consequences to understand why the higher productivity of agricultural labor over hunting and gathering did not initially lead to an improvement in living standards for the adopters. Even anticipated consequences could have had this result.To the extent that adoption could have been to the advantage of each individual group while being to the collective disadvantage of all, the paradox that agricultural adoption appears to have reduced living standards becomes less difficult to explain.

Although we have interpreted our model as being about the adoption of agriculture, in principle it is applicable to any technology that induces increased expenditure on security. Arms races in more modern periods of history undoubtedly owe something to this logic. It may also have a lesson for more mundane improvements in technology that are accompanied by increased opportunities for theft (such as the miniaturization of electronic goods). It is worth emphasizing also that although we have interpreted the players in our model as small groups initially devoted to hunting and gathering, they can also be interpreted as individuals or as groups of any size including nation states.

At the heart of the story is a fundamental externality from defense – activities that make one community more secure make its neighbors less secure. That externality - crucial as it was to the Neolithic revolution - has been of continued importance throughout human history right up to our own day.

References

Acemoglu, D., S. Johnson and J. Robinson (2001): "The Colonial Origins of Comparative Development: An Empirical Investigation", *American Economic Review*, 91(5).

Aidt, Toke (2002): "Strategic Political Participation and Redistribution", *Eco-*

nomics & Politics, 14, pp.19-40.

Baker, M. E. Bulte and J. Weisdorf (2010): "The origins of governments : from amorphy to anarchy and hierarchy", *Journal of Institutional and Theoretical Economics*, 6(2), 215-242.

Barker, Graeme (2006): *The Agricultural Revolution in Prehistory: Why Did Foragers Become Farmers?* Oxford, Oxford University Press.

Bar-Yosef, O. & A. Belfer-Cohen (1989): "The origins of sedentism and farming communities in the Levant", *Journal of World Prehistory*, 3, 447-97.

Becker, Gary (1983): "A Theory of Competition Among Pressure Groups for Political Influence", *Quarterly Journal of Economics*, 98, pp. 371-400.

Bellwood, P. (1996): "The Origins and Spread of Agriculture in the Indo-Pacific Region", in Harris (ed) *The Origins and Spread of Agriculture and Pastoralism in Eurasia*, Washington, D.C., The Smithsonian Institution.

Bowles, Samuel (2009): "Did the domestication of plants raise labor productivity?", Santa Fe Institute, mimeo.

Cohen, M.N. (1977): *The Food Crisis in Prehistory*, New Haven, Yale University Press.

Cohen, M.N. & G.J. Armelagos (1984): *Paleopathology at the Origins of Agriculture*, New York, Academic Press.

Cohen, Mark and Gillian Crane-Kramer (2007) : *Ancient Health : Skeletal Indicators of Agricultural and Economic Intensification*, University Press of Florida.

Coleman, David. & Schofield, Roger. (1986). The state of population theory: forward from Malthus, Oxford: Basil Blackwell.

Diamond, Jared (1997): *Guns, Germs and Steel: The Fates of Human Societies*, New York, W.W. Norton.

Dow, Gregor, Nancy Olewiler & Clye Reed (2006): "The Transition to Agriculture: Climate Reversals, Population Density and Technical Change", downloadable from http://www.econ.ku.dk/eed/programme_thursday.htm

Dixit, Avinash (1987): "Strategic Behavior in Contests", *American Economic Review*, 77, 891-898.

Dixit, Avinash (2004): *Lawlessness and Economics*, Princeton: Princeton University Press.

Ferguson, Adam (1996, originally 1773): *An Essay on the Origin of Civil Society*, Cambridge, Cambridge University Press.

Gat, Azar (1999) : "The Pattern of fighting in simple small-scale prestate societies", *Journal of Anthropological Research*, 55, 563-83.

Gat, Azar (2006): *War and Human Civilization*, Oxford, Oxford University Press.

Gellner (1994): *Conditions of Liberty: civil society and its rivals*, London, Hamish Hamilton.

Gonzalez, Francisco (2005): "Insecure Property and Technological Backwardness", *Economic Journal*, 115 (July), pp. 703-721.

Gonzalez, Francisco & Hugh Neary (2008): "Prosperity Without Conflict", *Journal of Public Economics*, 92(10-11), pp. 2170-2181.

Grossman, Herschel (1998): "Producers and Predators", *Pacific Economic Review*, vol. 3, no. 3, pp. 169-187

Guzmán, Ricardo A. & Jacob L.Weisdorf (2008): "The Neolithic Revolution from a price-theoretic perspective", Pontificia Universidad Católica de Chile, mimeo.

Hirshleifer, Jack (1989): "Conflict and rent-seeking success functions: ratio vs. difference models of relative success", *Public Choice*, 63, pp. 101-112.

Hwang, Sung-Ha (2009): "Contest Success Functions: Theory and Evidence", University of Massachussetts at Amherst, mimeo.

Ibn Khaldun, Abu Zaid (1969, originally 1377), *The Muqadimmah*, translated by Franz Rosenthal, Princeton, N.J., Princeton University Press.

Malthus, Thomas, Robert. 1798, An Essay on the Principle of Population, London, Johnson;

full text available at http://www.econlib.org/library/Malthus/malPop.html

Mithen, Steven (1996): *The Prehistory of the Mind*, London, Thames & Hudson.

Nitzan, S. (1994): "Modelling rent-seeking contests", *European Journal of Political Economy*, 10, pp.41-60.

North, Douglass & Robert Paul Thomas (1973): *The Rise of the Western World: A New Economic History*, Cambridge, Cambridge University Press.

Ofek, H. (2006). "Ape to Farmer in Five Uneasy Steps: An Economic Synopsis of Prehistory." A paper presented at the First Conference on Early Economic Developments, The University of Copenhagen. Copenhagen, Denmark. Downloadable from http://www.econ.ku.dk/eed/programme_friday.htm

Richerson, Peter, Robert Boyd & Robert Bettinger (2001): “Was Agriculture Impossible During the Pleistocene but Mandatory during the Holocene? A climate change hypothesis”, *American Antiquity*, 66(3).

Robson, Arthur (2010): “A ‘Bioeconomic View of the Neolithic Transitions to Agriculture”, *Canadian Journal of Economics/Revue canadienne d’économique*, Vol. 43, No. 1, pp. 280-300, February/fevrier

Steckel, Richard H. and Jerome C. Rose (2002) : *The Backbone of History : Health and Nutrition in the Western Hemisphere,* Cambridge, Cambridge University Press.

Stutz, Aaron, Natalie D. Munro and Guy Bar-Oz : "Increasing the resolution of the broad spectrum revolution in the Southern Levantine Epipaleolithic (19-12ka)", *Journal of Human Evolution*, 56, 294-306.

Tudge, Colin (1998) : *Neanderthals, Bandits and Farmers : How Agriculture Really Began*, London, Weidenfeld and Nicolson.

Weisdorf, Jacob L. (2005), "From Foraging to Farming: Explaining the Neolithic Revolution", *Journal of Economic Surveys*, Vol. 19, No. 4, pp. 561-586

Weisdorf, Jacob L. (2008): "Why Did the First Farmers Toil? Human Metabolism and the Origins of Agriculture", Discussion Paper no 08-15, Department of Economics, University of Copenhagen.

Winterhalder, B. & F. Lu (1995): “A Forager-Resource Population Ecology Model and Implication for Indigenous Conservation”, *Conservation Biology*, 11, 1354-64.

5 Appendix 1: Simulations

This appendix describes the simulations that were used to identify the Nash equilibria of the games presented in the text.

Programs. A strategy for group i can be specified by three variables, l_i^H, l_i^F, l_i^W, which denote the amount of labor devoted to hunting, farming and war, respectively. The total amount of labor is fixed such that $l_i^H + l_i^F + l_i^W = 1$. Suppose that group j devotes a given amount of labor $\bar{l}_j^W$ to war. What is the best response of group i to this behavior? To find the answer one can search over a grid specifying the values of both l_i^H and l_i^F. However. the following procedure is more economical. Consider an arbitrary strategy under which group i devotes l_i^W to war. Given this decision and the fact that j is devoting $\bar{l}_j^W$ to war, the highest payoff to i is achieved when l_i^F/l_i^H satisfies equation

(**??**) of the text. Thus,

$$
\begin{aligned}
l_i^F/l_i^H &= r_i = \left(\frac{(1-\phi_{ij})f}{1-\gamma_{ij}}\right)^{1/\eta} \\
\text{where } \gamma_{ij} &= \frac{2\gamma}{1+e^{-\alpha l}} \\
\phi_{ij} &= \frac{2\phi}{1+e^{-\beta l}} \\
l &= l_j^W - l_i^W
\end{aligned}
$$

The implied values of l_i^H and l_i^F are as follows,

$$
\begin{aligned}
l_i^H &= (1-l_i^W)/(1+r_i) \\
l_i^F &= r_i(1-l_i^W)/(1+r_i).
\end{aligned}
$$

For any given value of l_i^W these are the only values of l_i^H and l_i^F that could possibly be a best response to the given strategy of j, and there is no point in examining other strategies. In searching for a best response we therefore can restrict ourselves to a limited set of strategies that is indexed by the variable l_i^W and satisfy the above equations. There are many other strategies available to i but they cannot be optimal since their l_i^F/l_i^H ratio is inappropriate. By ignoring these non-starters we greatly reduce the amount of memory and computing time that is required for simulation.

On the basis of the above observations the following programmes were written in Matlab R14. One programme checks to see whether a particular pair of candidate strategies is a Nash equilibrium. This is done by comparing the payoffs to i from various strategies on the assumption that j plays the specified candidate strategy. The comparison is based on $10,001$ distinct strategies for group i which are specified by values of l_i^W distributed uniformly between 0 and 1. A second programme uses a 501×501 grid of strategies indexed by l_i^W and l_j^W to search for Nash equilibria. This involves searching through a matrix containing more than $250,000$ entries for each group. Both of these programmes consider only strategies for which the ratios l_i^F/l_i^H and l_j^F/l_j^H are optimal for the groups concerned. As a cross-check, searches were also done in which non-optimal values of l_i^F/l_i^H and l_j^F/l_j^H were allowed. These searches confirmed the results of the main programs. The main programmes were then modified to allow for groups who are constrained to specialize in hunting only or farming only.

Results. The existence of Nash equilibria was investigated for $8,000$ different parameter sets obtained by combining the following parameter values

Parameter values					
η	0.05	0.2	0.35	0.5	..
α	0.5	1	2	4	6
γ	0.05	0.2	0.35	0.5	..
β	0.5	1	2	4	6
ϕ	0.05	0.2	0.35	0.5	..
f	0.3	1	2	3	4

This investigation revealed that

- All Nash equilibria are symmetric.
- All Nash equilibria are unique.
- All Nash equilibria of the mixed hunting and farming model satisfy equations (10) and (11) of the text.
- 11% of parameter sets have no Nash equilibrium in the case where mixed hunting and farming are allowed.
- 5% of parameter sets have no Nash equilibrium in the case where the two groups are constrained to specialize in the same activity (hunting or fishing).

We also investigated the existence of equilibria for the examples shown in Charts 1 to 4. All of the parameter sets used for these diagrams have unique Nash equilibria that satisfy equations (10) and (11) of the text.

Finally, the value of $\frac{dC_i}{df_i}$ at a symmetric solution, as defined in Proposition 1, was evaluated for every combination of the following parameters:

$\eta = (0.15, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$;

$\alpha = (0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4)$;

$\gamma = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6)$;

$\beta = (0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4)$;

$\phi = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6)$;

$f = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2,$

$1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.5, 3.0, 3.5, 4.0)$.

There were 372,926 of these combinations for which $P/Q \leq 1$. This condition indicates that the symmetric solution is either interior or just on the boundary. For all such combinations$\frac{dC_i}{df_i} > 0$. We also investigated what happens when η is small (< 0.15).In a few cases the computed sign of $\frac{dC_i}{df_i}$was positive. This may reflect rounding errors due to the fact production functions are almost linear.

6 Appendix 2. Derivations and Proofs

6.1 Proof of Proposition 1

6.1.1 First Order Conditions

Differentiating (5) yields

$$\frac{\partial C_i}{\partial r_i} = -\frac{(1-\gamma_{ij})(1-l_i^W)}{(1+r_i)^2}H_i' + \frac{(1-\phi_{ij})(1-l_i^W)}{(1+r_i)^2}F_i' \tag{16}$$

$$\begin{aligned}\frac{\partial C_i}{\partial l_i^W} &= \frac{-(1-\gamma_{ij})H_i'}{1+r_i} - \frac{r_i(1-\phi_{ij})F_i'}{1+r_i} \\ &\quad -\frac{\partial \gamma_{ij}}{\partial l_i^W}H_i - \frac{\partial \phi_{ij}}{\partial l_i^W}F_i + \frac{\partial \gamma_{ji}}{\partial l_i^W}H_j + \frac{\partial \phi_{ji}}{\partial l_i^W}F_j\end{aligned} \tag{17}$$

At a symmetrical solution these equations imply that

$$\frac{\partial C_i}{\partial r_i} = -\frac{(1-\gamma)(1-l^W)\left(l^H\right)^{-\eta}}{(1+r)^2} + \frac{(1-\phi)(1-l^W)f\left(l^F\right)^{-\eta}}{(1+r)^2} \tag{18}$$

$$\frac{\partial C_i}{\partial l_i^W} = -\left(\frac{(1-\gamma)\left(l^H\right)^{-\eta} + (1-\phi)rf\left(l^F\right)^{-\eta}}{1+r}\right) + \frac{\alpha\gamma\left(l^H\right)^{1-\eta} + \beta\phi f\left(l^F\right)^{1-\eta}}{1-\eta} \tag{19}$$

To be a local or global Nash equilibrium, a symmetric solution must satisfy the complementary slack conditions:

$$\frac{\partial C_i}{\partial r_i} \leq 0, r \geq 0 \text{ and } r\frac{\partial C_i}{\partial r_i} = 0 \tag{20}$$

$$\frac{\partial C_i}{\partial l_i^W} \leq 0, l^W \geq 0 \text{ and } l^W\frac{\partial C_i}{\partial l_i^W} = 0 \tag{21}$$

We shall now show that there is at most one symmetric solution that satisfies these conditions. Hence there can be at most one symmetric Nash equilibrium.

The Inada conditions $H'(0) = F'(0) = \infty$ ensure that $1 - l^W, l^H, l^F, r > 0$.Hence $\frac{\partial C_i}{\partial r_i} = 0$ and from (18) it follows that

$$\frac{l^F}{l^H} = r = \left[\frac{f(1-\phi)}{1-\gamma}\right]^{1/\eta}$$

After rearrangement and noting that $l^H + l^F = 1 - l^W$ we can write (19) as follows

$$\left(l^H\right)^\eta (1+r)\frac{\partial C_i}{\partial l_i^W} = -P + \ Q(1 - l^W)$$

where

$$\begin{aligned} P &= (1-\eta)(1+r) \\ Q &= \frac{\alpha\gamma}{(1-\gamma)} + \frac{\beta\phi r}{(1-\phi)} \end{aligned}$$

In a Nash equilibrium it must be the case that l^H, $l^F > 0$. Hence r is finite and $P, Q > 0$. Two cases arise.

Case 1: $Q \geq P$. In this case, $\partial C_i / \partial l_i^W = 0$ if and only if $l^W = 1 - P/Q$. This is the only symmetric solution.

Case 2: $Q < P$.In this case, $l^W = 0$ and $\partial C_i / \partial l_i^W < 0$.

In each case, the complementary slack conditions (20) and (21) are satisfied. Thus, there is at most one symmetric solution. If it exists it is given by the following equations

$$\begin{aligned} \frac{l^F}{l^H} &= r = \left[\frac{f(1-\phi)}{1-\gamma}\right]^{1/\eta} \\ l^W &= \min(0, 1 - P/Q) \end{aligned}$$

6.1.2 Second Order Conditions

If $P < Q$ the differential $\partial C_i / \partial l_i^W$ is strictly negative. This is sufficient for a local maximum. If $P = Q$ the differential $\partial C_i / \partial l_i^W$ is equal to zero. A sufficient condition for a local maximum is that C_i is a strictly concave function of l_i^w and r_i for $i = 1, 2$.

Holding r_i constant and differentiating (17) with respect to l_i^w yields

$$\begin{aligned}\frac{\partial^2 C_i}{\partial \left(l_i^W\right)^2} &= \frac{(1-\gamma_{ij})H_i''}{(1+r_i)^2} + \frac{(1-\phi_{ij})r_i^2 F_i''}{(1+r_i)^2} \\ &+2\frac{\partial \gamma_{ij}}{\partial l_i^W}\frac{H_i'}{1+r_i} + 2\frac{\partial \phi_{ij}}{\partial l_i^W}\frac{r_i F_i'}{1+r_i} \\ &-\frac{\partial^2 \gamma_{ij}}{\partial \left(l_i^W\right)^2}H_i - \frac{\partial^2 \phi_{ij}}{\partial \left(l_i^W\right)^2}F_i \\ &+\frac{\partial^2 \gamma_{ji}}{\partial \left(l_i^W\right)^2}H_j + \frac{\partial^2 \phi_{ji}}{\partial \left(l_i^W\right)^2}F_j\end{aligned} \tag{22}$$

At a symmetric equilibrium

$$\begin{aligned}\frac{\partial \gamma_{ij}}{\partial l_i^W} &= -\frac{\alpha\gamma}{2}, \frac{\partial \phi_{ij}}{\partial l_i^W} = -\frac{\beta\phi}{2} \\ \frac{\partial^2 \gamma_{ij}}{\partial \left(l_i^W\right)^2} &= \frac{\partial^2 \phi_{ij}}{\partial \left(l_i^W\right)^2} = \frac{\partial^2 \gamma_{ji}}{\partial \left(l_i^W\right)^2} = \frac{\partial^2 \phi_{ji}}{\partial \left(l_i^W\right)^2}\end{aligned}$$

Hence

$$\begin{aligned}\frac{\partial^2 C_i}{\partial \left(l_i^W\right)^2} &= \frac{(1-\gamma)H''}{(1+r)^2} + \frac{(1-\phi)r^2 F''}{(1+r)^2} \\ &-\frac{\alpha\gamma H'}{1+r} - \frac{\beta\phi r F'}{1+r}\end{aligned} \tag{23}$$

Holding l_i^w constant and differentiating (16) with respect to r_i yields

$$\frac{2(1+r_i)}{1-l_i^W}\frac{\partial C_i}{\partial r_i} + \frac{(1+r_i)^2}{1-l_i^W}\frac{\partial^2 C_i}{\partial (r_i)^2} = \frac{(1-\gamma_{ij})(1-l_i^W)H_i''}{(1+r_i)^2} + \frac{(1-\phi_{ij})(1-l_i^W)F_i''}{(1+r_i)^2} \tag{24}$$

At a symmetric equilibrium $\frac{\partial C_i}{\partial r_i} = 0$ and hence

$$\frac{\partial^2 C_i}{\partial (r_i)^2} = \frac{(1-l^W)^2}{(1+r)^4}\left[(1-\gamma)H'' + (1-\phi)F''\right] \tag{25}$$

Holding r_i constant and differentiating (16) with respect to l_i^w yields

$$\begin{aligned}\frac{(1+r_i)^2}{(1-l_i^W)^2}\frac{\partial C_i}{\partial r_i} + \frac{(1+r_i)^2}{1-l_i^W}\frac{\partial^2 C_i}{\partial r_i \partial l_i^W} &= \frac{(1-\gamma_{ij})H_i''}{(1+r_i)} - \frac{(1-\phi_{ij})r_i F_i''}{(1+r_i)} \\ &+\frac{\partial \gamma_{ij}}{\partial l_i^W}H_i' - \frac{\partial \phi_{ij}}{\partial l_i^W}F_i'\end{aligned} \tag{26}$$

At a symmetric equilibrium $\frac{\partial C_i}{\partial r_i} = 0$ and hence

$$\frac{\partial^2 C_i}{\partial r_i \partial l_i^W} = \frac{(1-l^W)}{(1+r)^2}\left[\frac{(1-\gamma)H''}{(1+r)} - \frac{(1-\phi)rF''}{(1+r)} - \frac{\alpha\gamma}{2}H' + \frac{\beta\phi}{2}F'\right] \tag{27}$$

For C_i to be strictly concave function of l_i^w and r_i the following conditions must hold

$$\begin{aligned}\frac{\partial^2 C_i}{\partial \left(l_i^W\right)^2} &< 0 \\ \frac{\partial^2 C_i}{\partial \left(r_i\right)^2} &< 0.\end{aligned}$$

Equations (23) and (25) imply that the above inequalities hold since $H'', F'' < 0$ and $H', F' > 0$.It must also be the case that

$$\frac{\partial^2 C_i}{\partial \left(l_i^W\right)^2}\frac{\partial^2 C_i}{\partial \left(r_i\right)^2} - \left(\frac{\partial^2 C_i}{\partial r_i \partial l_i^W}\right)^2 > 0. \tag{28}$$

Let

$$Z = \frac{r\left(l^H\right)^{2\eta}(1+r)^4}{(1-\gamma)^2(1-l^W)^2}\left[\frac{\partial^2 C_i}{\partial \left(l_i^W\right)^2}\frac{\partial^2 C_i}{\partial \left(r_i\right)^2} - \left(\frac{\partial^2 C_i}{\partial r_i \partial l_i^W}\right)^2\right]$$

Since $l^W < 1$ and $r > 0$, it is clear that (28) is satisfied if and only if $Z > 0$. From (23), (25) and (28) it follows that

$$Z = \frac{r\left(l^H\right)^{2\eta}}{(1-\gamma)^2} \times \left\{ \begin{array}{c} \left(\frac{(1-\gamma)H''}{(1+r)^2} + \frac{(1-\phi)r^2F''}{(1+r)^2} - \frac{\alpha\gamma H'}{1+r} - \frac{\beta\phi r F'}{1+r}\right) \times ((1-\gamma)H'' + (1-\phi)F'') \\ -\left(\frac{(1-\gamma)H''}{(1+r)} - \frac{(1-\phi)rF''}{(1+r)} - \frac{\alpha\gamma}{2}H' + \frac{\beta\phi}{2}F'\right)^2 \end{array} \right\}$$

After re-arrangement this can be written as follows

$$Z = L + M - N$$

where

$$\begin{aligned} L &= \frac{r\left(l^H\right)^{2\eta}}{(1-\gamma)^2}(1-\gamma)(1-\phi)H''F'' > 0 \\ M &= \frac{r\left(l^H\right)^{2\eta}}{(1-\gamma)^2}(-\alpha\gamma(1-\phi)F''H' - \beta\phi(1-\gamma)H''F') > 0 \\ N &= \frac{r\left(l^H\right)^{2\eta}}{(1-\gamma)^2}\left(\frac{\alpha\gamma}{2}H' - \frac{\beta\phi}{2}F'\right)^2 \geq 0 \end{aligned}$$

The above inequalities enable us to state alternative sufficient conditions to ensure that $Z > 0$.

The first condition is derived by noting that

$$\begin{aligned}
L &= \frac{r\left(l^H\right)^{2\eta}}{(1-\gamma)^2}(1-\gamma)(1-\phi)\eta^2\left(l^H\right)^{-2(n+1)} f\left(\frac{l^F}{l^H}\right)^{-(\eta+1)} \\
& \frac{r\left(l^H\right)^{2\eta}}{(1-\gamma)^2}(1-\gamma)(1-\phi)\eta^2\left(l^H\right)^{-2(n+1)} f\frac{(1-\gamma)}{(1-\phi)f}\left(\frac{l^F}{l^H}\right)^{-1} \\
&= \eta^2\left(l^H\right)^{-2} \\
M &= \frac{r\left(l^H\right)^{2\eta}}{(1-\gamma)^2}\eta\left(l^H\right)^{-(2\eta+1)}\left(\alpha\gamma(1-\phi)f\left(\frac{l^F}{l^H}\right)^{-(\eta+1)} + \beta\phi(1-\gamma)f\left(\frac{l^F}{l^H}\right)^{-\eta}\right) \\
&= \frac{r\left(l^H\right)^{2\eta}}{(1-\gamma)^2}\eta\left(l^H\right)^{-(2\eta+1)}\left(\alpha\gamma(1-\phi)f\frac{(1-\gamma)}{(1-\phi)f}\left(\frac{l^F}{l^H}\right)^{-1} + \beta\phi(1-\gamma)f\frac{(1-\gamma)}{(1-\phi)f}\right) \\
&= \eta\left(l^H\right)^{--1}\left(\frac{\alpha\gamma}{1-\gamma}+\frac{\beta\phi r}{1-\phi}\right) \\
N &= \frac{r\left(l^H\right)^{2\eta}}{(1-\gamma)^2}\frac{1}{4}\left(\alpha\gamma-\beta\phi f\left(\frac{l^F}{l^H}\right)^{-\eta}\right)^2 \\
&= \frac{r\left(l^H\right)^{2\eta}}{(1-\gamma)^2}\frac{1}{4}\left(\alpha\gamma-\beta\phi f\frac{(1-\gamma)}{(1-\phi)f}\right)^2 \\
&= \frac{r}{4}\left(\frac{\alpha\gamma}{1-\gamma}-\frac{\beta\phi}{1-\phi}\right)^2
\end{aligned}$$

By definition,

$$l^H = \frac{1-l^W}{1+r}$$

Combining this with the first order condition (11) for an internal equilibrium yields

$$\frac{\alpha\gamma}{1-\gamma}+\frac{\beta\phi r}{1-\phi} = (1-\eta)\left(l^H\right)^{-1}$$

Subtituting in the formulae for L and M yields

$$\begin{aligned}
L+M &= \eta^2\left(l^H\right)^{-2} + \eta(1-\eta)\left(l^H\right)^{-2} \\
&= \eta\left(l^H\right)^{-2}
\end{aligned}$$

Hence

$$\begin{aligned}
Z &= L+M+N \\
&= \eta\left(l^H\right)^{-2} - \frac{r}{4}\left(\frac{\alpha\gamma}{1-\gamma}-\frac{\beta\phi}{1-\phi}\right)^2
\end{aligned}$$

Replacing l^H yields the following

$$Z = \frac{\eta}{(1-\eta)^2}\left(\frac{\alpha\gamma}{1-\gamma} + \frac{\beta\phi r}{1-\phi}\right)^2 - \frac{r}{4}\left(\frac{\alpha\gamma}{1-\gamma} - \frac{\beta\phi}{1-\phi}\right)^2 \tag{29}$$

where

$$r = \left(\frac{(1-\phi)f}{1-\gamma}\right)^{1/\eta}$$

Define $\delta < 1$ as follows

$$\frac{\alpha\gamma}{1-\gamma} = (1-\delta)\frac{\beta\phi}{1-\phi}$$

Also define

$$Y = \frac{(1-\eta)^2}{\eta}\left(\frac{\beta\phi}{1-\phi}\right)^{-2} Z \tag{30}$$

From (29) it follows that

$$\begin{aligned} Y &= ((1-\delta)+r)^2 - \frac{r}{4}\frac{(1-\eta)^2}{\eta}\delta^2 \\ &= r^2 - 2\left(\frac{(1-\eta)^2}{\eta}\frac{\delta^2}{8} - (1-\delta)\right) r + (1-\delta)^2 \end{aligned} \tag{31}$$

From(30), and (31) it follows that $Z > 0$ if and only if

$$r^2 - 2\left(\frac{(1-\eta)^2}{\eta}\frac{\delta^2}{8} - (1-\delta)\right) r + (1-\delta)^2 > 0$$

This is the condition for C_i to be strictly concave in l_i^W and r_i.

6.2 Proof of Proposition 2

Assuming that the system is originally in an interior symmetrical equilibrium, the effect of a simultaneous and equal change in f_i and f_j can be derived as follows

$$\begin{aligned} (1-\eta)C &= \left(l^H\right)^{1-\eta} + \left(l^F\right)^{1-\eta} f \\ &= \left(l^H\right)^{1-\eta}\left(1 + r^{1-\eta} f\right) \\ &= \left(\frac{P}{(1+r)Q}\right)^{1-\eta}\left(1 + r^{1-\eta} f\right) \\ &= \left(\frac{(1-\eta)(1-\gamma)}{\alpha\gamma + \beta\phi r^{1-\eta} f}\right)^{1-\eta}\left(1 + r^{1-\eta} f\right) \end{aligned} \tag{32}$$

Let

$$
\begin{aligned}
g &= r^{1-\eta} f \\
&= \left(\frac{(1-\phi)f}{1-\gamma}\right)^{\frac{1-\eta}{\eta}} f \\
&= \left(\frac{1-\phi}{1-\gamma}\right)^{\frac{1-\eta}{\eta}} f^{\frac{1}{\eta}}
\end{aligned}
$$

Then

$$\frac{dg}{df} = \frac{g}{\eta f} > 0$$

and

$$\ln C = Const + \ln(1+g) - (1-\eta)\ln(\alpha\gamma + \beta\phi g)$$

and

$$
\begin{aligned}
\frac{dC}{df} &= \left(\frac{1}{1+g} - \frac{(1-\eta)\beta\phi}{\alpha\gamma+\beta\phi g}\right)\frac{dg}{df} \\
&= C\left(\frac{\alpha\gamma - (1-\eta)\beta\phi + \eta\beta\phi g}{(1+g)\,(\alpha\gamma+\beta\phi g)}\right)\frac{g}{\eta f}
\end{aligned}
$$

Note that $\frac{dC}{df} < 0$ if and only if

$$\alpha\gamma < (1-\eta)\beta\phi \tag{33}$$

$$g < \frac{(1-\eta)\beta\phi - \alpha\gamma}{\eta\beta\phi} \tag{34}$$

The second of these inequalities can be written as follows

$$f < \left(\frac{(1-\eta)\beta\phi - \alpha\gamma}{\eta\beta\phi}\right)^{\eta}\left(\frac{1-\gamma}{1-\phi}\right)^{1-\eta} \tag{35}$$

The inequalities (33) and (35) are the conditions given in Proposition 2 of the text.

6.3 Proof of Proposition 4

Consumption in the mixed economy is given by equation (32)

$$(1-\eta)C = \left(\frac{(1-\eta)(1-\gamma)}{\alpha\gamma+\beta\phi r^{1-\eta} f}\right)^{1-\eta}\left(1+r^{1-\eta} f\right) \tag{36}$$

where

$$r = \left(\frac{(1-\phi)f}{1-\gamma}\right)^{1/\eta}$$

Consumption in the hunting only society can be derived by setting $f = 0$ in this equation:

$$(1-\eta)C^H = \left(\frac{(1-\eta)(1-\gamma)}{\alpha\gamma}\right)^{1-\eta}$$

Hence

$$\frac{C^H}{C} = \left(1 + \frac{\beta\phi r^{1-\eta} f}{\alpha\gamma}\right)\frac{1}{1+r^{1-\eta} f}$$

Eliminating r yields proposition 4.

6.4 Unilateral increase in agricultural productivity

To derive the effect of a unilateral change in technology by one group alone, we proceed as follows. Consumption for the two groups is given by

$$C_i = (1-\gamma_{ij})H_i + (1-\phi_{ij})F_i + \gamma_{ji}H_j + \phi_{ji}F_j \quad (37)$$

$$C_j = (1-\gamma_{ji})H_j + (1-\phi_{ji})F_j + \gamma_{ij}H_i + \phi_{ij}F_i \quad (38)$$

Differentiating (37) and (38) yields the conditions for an internal best response by each group

$$\begin{aligned}\frac{\partial C_i}{\partial l_i^W} &= \frac{-(1-\gamma_{ij})H_i'}{1+r_i} - \frac{r_i(1-\phi_{ij})F_i'}{1+r_i} \\ &\quad - \frac{\partial \gamma_{ij}}{\partial l_i^W}H_i - \frac{\partial \phi_{ij}}{\partial l_i^W}F_i + \frac{\partial \gamma_{ji}}{\partial l_i^W}H_j + \frac{\partial \phi_{ji}}{\partial l_i^W}F_j \\ &= 0\end{aligned} \quad (39)$$

$$\begin{aligned}\frac{\partial C_j}{\partial l_j^W} &= \frac{-(1-\gamma_{ji})H_j'}{1+r_j} - \frac{r_j(1-\phi_{ji})F_j'}{1+r_j} \\ &\quad - \frac{\partial \gamma_{ji}}{\partial l_j^W}H_j - \frac{\partial \phi_{ji}}{\partial l_j^W}F_j + \frac{\partial \gamma_{ij}}{\partial l_j^W}H_i + \frac{\partial \phi_{ij}}{\partial l_j^W}F_i \\ &= 0\end{aligned} \quad (40)$$

The above makes use of some of the following

$$
\begin{aligned}
l_i^H &= \frac{1-l_i^w}{1+r_i}, l_i^F = \frac{r_i(1-l_i^w)}{1+r_i} \\
l_j^H &= \frac{1-l_j^w}{1+r_j}, l_j^F = \frac{r_j(1-l_j^w)}{1+r_j} \\
\frac{\partial l_i^H}{\partial l_i^W} &= \frac{-1}{1+r_i}, \frac{\partial l_i^F}{\partial l_i^W} = \frac{-r_i}{1+r_i} \\
\frac{\partial l_i^H}{\partial r_i} &= \frac{-(1-l_i^w)}{(1+r_i)^2}, \frac{\partial l_i^F}{\partial r_i} = \frac{(1-l_i^w)}{(1+r_i)^2} \\
\frac{\partial l_j^H}{\partial l_j^W} &= \frac{-1}{1+r_j}, \frac{\partial l_j^F}{\partial l_j^W} = \frac{-r_j}{1+r_j} \\
\frac{\partial l_j^H}{\partial r_j} &= \frac{-(1-l_j^w)}{(1+r_j)^2}, \frac{\partial l_j^F}{\partial r_i} = \frac{(1-l_j^w)}{(1+r_j)^2}_i
\end{aligned}
$$

Differentiating (39) with respect to f_i yields

$$
\begin{aligned}
0 = &\left[-\frac{(1-\gamma_{ij})}{1+r_i}H_i''\frac{\partial l_i^H}{\partial f_i} + \frac{(1-\gamma_{ij})}{(1+r_i)^2}H_i'\frac{\partial r_i}{\partial f_i} + \frac{1}{1+r_i}\frac{\partial \gamma_{ij}}{\partial l}H_i'\frac{\partial l}{\partial f_i}\right] \\
&+\left[-\frac{r_i(1-\phi_{ij})}{1+r_i}F_i''\frac{\partial l_i^F}{\partial f_i} - \frac{r_i(1-\phi_{ij})}{(1+r_i)}\frac{\partial F_i'}{\partial f_i} - \frac{(1-\phi_{ij})F_i'}{(1+r_i)^2}\frac{\partial r_i}{\partial f_i} + \frac{r_i}{1+r_i}\frac{\partial \phi_{ij}}{\partial l}F_i'\frac{\partial l}{\partial f_i}\right] \\
&-\frac{\partial \gamma_{ij}}{\partial l_i^W}H_i'\frac{\partial l_i^H}{\partial f_i} - \left[\frac{\partial \phi_{ij}}{\partial l_i^W}F_i'\frac{\partial l_i^F}{\partial f_i} + \frac{\partial \phi_{ij}}{\partial l_i^W}\frac{\partial F_i}{\partial f_i}\right] + \frac{\partial \gamma_{ji}}{\partial l_i^W}H_j'\frac{\partial l_j^H}{\partial f_i} + \frac{\partial \phi_{ji}}{\partial l_i^W}F_j'\frac{\partial l_j^F}{\partial f_i} \\
&-\frac{\partial^2 \gamma_{ij}}{\partial l \partial l_i^W}\frac{\partial l}{\partial f_i}H_i - \frac{\partial^2 \phi_{ij}}{\partial l \partial l_i^W}\frac{\partial l}{\partial f_i}F_i + \frac{\partial^2 \gamma_{ji}}{\partial l \partial l_i^W}\frac{\partial l}{\partial f_i}H_j + \frac{\partial^2 \phi_{ji}}{\partial l \partial l_i^W}\frac{\partial l}{\partial f_i}F_j
\end{aligned} \tag{41}
$$

Let

$$
\begin{aligned}
\frac{\partial l_i^W}{\partial f_i} &= x_i, \frac{\partial l_j^W}{\partial f_i} = x_j \\
\frac{\partial r_i}{\partial f_i} &= y_i, \frac{\partial r_j}{\partial f_i} = y_j
\end{aligned}
$$

Then

$$\begin{aligned}
\frac{\partial l_i^H}{\partial f_i} &= -\frac{1}{1+r_i}x_i - \frac{(1-l_i^w)}{(1+r_i)^2}y_i \\
\frac{\partial l_i^F}{\partial f_i} &= -\frac{r_i}{1+r_i}x_i + \frac{(1-l_i^w)}{(1+r_i)^2}y_i \\
\frac{\partial l_j^H}{\partial f_i} &= -\frac{1}{1+r_j}x_j - \frac{(1-l_j^w)}{(1+r_j)^2}y_j \\
\frac{\partial l_j^F}{\partial f_i} &= -\frac{r_j}{1+r_j}x_j + \frac{(1-l_j^w)}{(1+r_j)^2}y_j \\
\frac{\partial l}{\partial f_i} &= x_j - x_i \\
\frac{\partial F_i}{\partial f_i} &= \frac{F_i}{f_i} \\
\frac{\partial F_i'}{\partial f_i} &= \frac{F_i'}{f_i}
\end{aligned}$$

Substituting in (41) yields

$$\begin{aligned}
0 = &\frac{-(1-\gamma_{ij})}{1+r_i}H_i''\left[-\frac{1}{1+r_i}x_i - \frac{(1-l_i^w)}{(1+r_i)^2}y_i\right] \\
&-\frac{r_i(1-\phi_{ij})}{1+r_i}F_i''\left[-\frac{r_i}{1+r_i}x_i + \frac{(1-l_i^w)}{(1+r_i)^2}y_i\right] \\
&+\frac{1}{(1+r_i)^2}\left[(1-\gamma_{ij})H_i' - (1-\phi_{ij})F_i'\right]y_i \\
&+\frac{1}{1+r_i}\left[\frac{\partial \gamma_{ij}}{\partial l}H_i' + r_i\frac{\partial \phi_{ij}}{\partial l}F_i'\right](x_j - x_i) \\
&-\frac{r_i(1-\phi_{ij})F_i'}{(1+r_i)\,f_i} - \frac{\partial \phi_{ij}}{\partial l_i^W}\frac{F_i}{f_i} \\
&-\frac{\partial \gamma_{ij}}{\partial l_i^W}H_i'\left[-\frac{1}{1+r_i}x_i - \frac{(1-l_i^w)}{(1+r_i)^2}y_i\right] \\
&-\frac{\partial \phi_{ij}}{\partial l_i^W}F_i'\left[-\frac{r_i}{1+r_i}x_i + \frac{(1-l_i^w)}{(1+r_i)^2}y_i\right] \\
&+\frac{\partial \gamma_{ji}}{\partial l_i^W}H_j'\left[-\frac{1}{1+r_j}x_j - \frac{(1-l_j^w)}{(1+r_j)^2}y_j\right] \\
&+\frac{\partial \phi_{ji}}{\partial l_i^W}F_j'\left[-\frac{r_j}{1+r_j}x_j + \frac{(1-l_j^w)}{(1+r_j)^2}y_j\right] \\
&-\frac{\partial^2 \gamma_{ij}}{\partial l \partial l_i^W}\frac{\partial l}{\partial f_i}H_i - \frac{\partial^2 \phi_{ij}}{\partial l \partial l_i^W}\frac{\partial l}{\partial f_i}F_i + \frac{\partial^2 \gamma_{ji}}{\partial l \partial l_i^W}\frac{\partial l}{\partial f_i}H_j + \frac{\partial^2 \phi_{ji}}{\partial l \partial l_i^W}\frac{\partial l}{\partial f_i}F_j
\end{aligned} \tag{42}$$

In symmetrical equilibrium all second differentials $\frac{\partial^2\gamma_{ij}}{\partial l \partial l_i^W}$ etc. are zero. Moreover,

$$\begin{aligned}
\frac{\partial\gamma_{ij}}{\partial l} &= \frac{\alpha\gamma}{2}, \frac{\partial\phi_{ij}}{\partial l_j^W} = \frac{\beta\phi}{2} \\
\frac{\partial\gamma_{ij}}{\partial l_i^W} &= -\frac{\alpha\gamma}{2}, \frac{\partial\phi_{ij}}{\partial l_i^W} = -\frac{\beta\phi}{2} \\
\frac{\partial\gamma_{ji}}{\partial l_i^W} &= \frac{\alpha\gamma}{2}, \frac{\partial\phi_{ji}}{\partial l_i^W} = \frac{\beta\phi}{2}
\end{aligned}$$

Assuming symmetrical equilibrium, substituting in (42) and rearranging yields

$$\begin{aligned}
0 = &\frac{-(1-\gamma)}{1+r}H''\left[-\frac{1}{1+r}x_i - \frac{(1-l^w)}{(1+r)^2}y_i\right] \\
&-\frac{r(1-\phi)}{1+r}F''\left[-\frac{r}{1+r}x_i + \frac{(1-l^w)}{(1+r)^2}y_i\right] \\
&+\frac{1}{(1+r)^2}\left[(1-\gamma)H' - (1-\phi)F'\right]y_i \\
&+\frac{1}{1+r}\left[\frac{\alpha\gamma}{2}H' + r\frac{\beta\phi}{2}F'\right](x_j - x_i) \\
&-\frac{r(1-\phi)F'}{(1+r)f} + \frac{\beta\phi}{2}\frac{F}{f} \\
&+\frac{\alpha\gamma}{2}H'\left[-\frac{1}{1+r}x_i - \frac{(1-l^w)}{(1+r)^2}y_i\right] \\
&+\frac{\beta\phi}{2}F'\left[-\frac{r}{1+r}x_i + \frac{(1-l^w)}{(1+r)^2}y_i\right] \\
&+\frac{\alpha\gamma}{2}H'\left[-\frac{1}{1+r}x_j - \frac{(1-l^w)}{(1+r)^2}y_j\right] \\
&+\frac{\beta\phi}{2}F'\left[-\frac{r}{1+r}x_j + \frac{(1-l^w)}{(1+r)^2}y_j\right]
\end{aligned}$$

This can be written in the form

$$Ax_i + By_i + Dy_j = \frac{r(1-\phi)F'}{(1+r)f} - \frac{\beta\phi}{2}\frac{F}{f} \tag{43}$$

where

$$\begin{aligned}
A &= \frac{1}{(1+r)^2}\left[(1-\gamma)H'' + r^2(1-\phi)F''\right] \\
&\quad -\frac{1}{(1+r)}\left[\alpha\gamma H' + r\beta\phi F'\right]
\end{aligned}$$

$$B = \frac{(1-l^w)}{(1+r)^3}\left[(1-\gamma)H'' - r(1-\phi)F''\right]$$
$$+\frac{1}{(1+r)^2}\left[(1-\gamma)H' - (1-\phi)F'\right]$$
$$+\frac{(1-l^w)}{(1+r)^2}\left[-\frac{\alpha\gamma}{2}H' + \frac{\beta\phi}{2}F'\right]$$

$$D = \frac{(1-l^w)}{(1+r)^2}\left[-\frac{\alpha\gamma}{2}H' + \frac{\beta\phi}{2}F'\right]$$

Differentiating (40) with respect to f_i yields

$$0 = \left[-\frac{(1-\gamma_{ji})}{1+r_j}H_j''\frac{\partial l_j^H}{\partial f_i} + \frac{(1-\gamma_{ji})}{(1+r_j)^2}H\frac{\partial r_j}{\partial f_i} + \frac{1}{1+r_j}\frac{\partial \gamma_{ji}}{\partial l}H_j'\frac{\partial l}{\partial f_i}\right]$$
$$+ \quad \left[-\frac{r_j(1-\phi_{ji})}{1+r_j}F_j''\frac{\partial l_j^F}{\partial f_i} - \frac{(1-\phi_{ji})}{(1+r_j)^2}F_j'\frac{\partial r_j}{\partial f_i} + \frac{r_j}{1+r_j}\frac{\partial \phi_{ji}}{\partial l}F_j'\frac{\partial l}{\partial f_i}\right]$$
$$-\frac{\partial \gamma_{ji}}{\partial l_j^W}H_j'\frac{\partial l_j^H}{\partial f_i} - \frac{\partial \phi_{ji}}{\partial l_j^W}F_j'\frac{\partial l_j^F}{\partial f_i} + \frac{\partial \gamma_{ij}}{\partial l_j^W}H_i'\frac{\partial l_i^H}{\partial f_i} + \left[\frac{\partial \phi_{ij}}{\partial l_j^W}F_i'\frac{\partial l_i^F}{\partial f_i} + \frac{\partial \phi_{ij}}{\partial l_j^W}\frac{\partial F_i}{\partial f_i}\right]$$
$$-\frac{\partial l}{\partial f_i}H_j - \frac{\partial^2 \phi_{ji}}{\partial l \partial l_j^W}\frac{\partial l}{\partial f_i}F_j + \frac{\partial^2 \gamma_{ij}}{\partial l \partial l_j^W}\frac{\partial l}{\partial f_i}H_i + \frac{\partial^2 \phi_{ij}}{\partial l \partial l_j^W}\frac{\partial l}{\partial f_i}F_i$$

In symmetrical equilibrium all second differentials $\frac{\partial^2 \gamma_{ji}}{\partial l \partial l_j^W}$ etc. are zero. Moreover,

$$\frac{\partial \gamma_{ji}}{\partial l} = -\frac{\alpha\gamma}{2}, \frac{\partial \phi_{ji}}{\partial l_j^W} = -\frac{\beta\phi}{2}$$
$$\frac{\partial \gamma_{ji}}{\partial l_j^W} = -\frac{\alpha\gamma}{2}, \frac{\partial \phi_{ji}}{\partial l_j^W} = -\frac{\beta\phi}{2}$$
$$\frac{\partial \gamma_{ij}}{\partial l_j^W} = \frac{\alpha\gamma}{2}, \frac{\partial \phi_{ij}}{\partial l_j^W} = \frac{\beta\phi}{2}$$

In symmetric equilibrium the second differentials $\frac{\partial^2 \gamma_{ji}}{\partial l \partial l_j^W}$ etc. are zero and above equation can be written after rearrangement as follows

$$0=-\frac{(1-\gamma)}{1+r}H''\left[-\frac{1}{1+r}x_j-\frac{(1-l^w)}{(1+r)^2}y_j\right]$$
$$-\frac{r(1-\phi)}{1+r}F''\left[-\frac{r}{1+r}x_j+\frac{(1-l^w)}{(1+r)^2}y_j\right]$$
$$+\frac{1}{(1+r)^2}\left[(1-\gamma)H'-(1-\phi)F'\right]y_j$$
$$\frac{1}{1+r}\left[-\frac{\alpha\gamma}{2}H'-\frac{r\beta\phi}{2}F'\right](x_j-x_i)$$
$$\frac{\alpha\gamma}{2}H'\left[-\frac{1}{1+r}x_j-\frac{(1-l^w)}{(1+r)^2}y_j\right]$$
$$\frac{\beta\phi}{2}F'\left[-\frac{r}{1+r}x_j+\frac{(1-l^w)}{(1+r)^2}y_j\right]$$
$$+\frac{\alpha\gamma}{2}H'\left[-\frac{1}{1+r}x_i-\frac{(1-l^w)}{(1+r)^2}y_i\right]$$
$$+\frac{\beta\phi}{2}F'\left[-\frac{r}{1+r}x_i+\frac{(1-l^w)}{(1+r)^2}y_i\right]+\frac{\beta\phi}{2}\frac{F}{f}$$

This can be written in the form

$$Ax_j+Dy_i+By_j=-\frac{\beta\phi}{2}\frac{F}{f} \tag{44}$$

where A, B and D are defined above.

In equilibrium

$$r_i = \left(\frac{f_i(1-\phi_{ij})}{(1-\gamma_{ij})}\right)^{1/\eta}$$
$$r_j = \left(\frac{f_j(1-\phi_{ji})}{(1-\gamma_{ji})}\right)^{1/\eta}$$

Thus,

$$\eta\ln r_i = \ln f_i+\ln(1-\phi_{ij})-\ln\left(1-\gamma_{ij}\right)$$
$$\eta\ln r_j = \ln f_j+\ln(1-\phi_{ji})-\ln\left(1-\gamma_{ji}\right)$$

and

$$\frac{\eta}{r_i}\frac{\partial r_i}{\partial f_i} = \left[\frac{-1}{(1-\phi_{ij})}\frac{\partial\phi_{ij}}{\partial l}+\frac{1}{(1-\gamma_{ij})}\frac{\partial\gamma_{ij}}{\partial l}\right]\frac{\partial l}{\partial f_i}+\frac{1}{f_i}$$
$$\frac{\eta}{r_j}\frac{\partial r_j}{\partial f_i} = \left[\frac{-1}{(1-\phi_{ji})}\frac{\partial\phi_{ji}}{\partial l}+\frac{1}{(1-\gamma_{ji})}\frac{\partial\gamma_{ji}}{\partial l}\right]\frac{\partial l}{\partial f_i}$$

In symmetric equilibrium these yield

$$\frac{\eta}{r}y_i = -\left[\frac{1}{(1-\phi)}\frac{\beta\phi}{2} - \frac{1}{(1-\gamma)}\frac{\alpha\gamma}{2}\right](x_j - x_i) + \frac{1}{f} \tag{45}$$

$$\frac{\eta}{r}y_j = \left[\frac{1}{(1-\phi)}\frac{\beta\phi}{2} - \frac{1}{(1-\gamma)}\frac{\alpha\gamma}{2}\right](x_j - x_i) \tag{46}$$

which can be written,after addition, as follows

$$\left[\frac{\beta\phi}{(1-\phi)} - \frac{\alpha\gamma}{(1-\gamma)}\right]x_i - \left[\frac{\beta\phi}{(1-\phi)} - \frac{\alpha\gamma}{(1-\gamma)}\right]x_j + \frac{2\eta}{r}y_j = 0 \tag{47}$$

$$y_i + y_j = \frac{r}{\eta f} \tag{48}$$

The system of linear equations (43), (44), (47) and (48) can be explicitly solved, but the resulting formulae are very complicated. However, they can be easily solved by standard numerical methods in particular cases.

To find the effect on consumption for group i note that

$$\frac{dC_i}{df_i} = \frac{\partial C_i}{\partial f_i} + \frac{\partial C_i}{\partial l_i^W}\frac{\partial l_i^W}{\partial f_i} + \frac{\partial C_i}{\partial r_i}\frac{\partial r_i}{\partial f_i} + \frac{\partial C_i}{\partial l_j^W}\frac{\partial l_j^W}{\partial f_i} + \frac{\partial C_i}{\partial r_j}\frac{\partial r_j}{\partial f_i}$$

For an internal equilibrium $\frac{\partial C_i}{\partial l_i^W} = 0$ and $\frac{\partial C_i}{\partial r_i} = 0$. Hence

$$\begin{aligned}\frac{dC_i}{df_i} &= \frac{\partial C_i}{\partial f_i} + \frac{\partial C_i}{\partial l_j^W}\frac{\partial l_j^W}{\partial f_i} + \frac{\partial C_i}{\partial r_j}\frac{dr_j}{\partial f_i} \\ &= \frac{\partial C_i}{\partial f_i} + \frac{\partial C_i}{\partial l_j^W}x_j + \frac{\partial C_i}{\partial r_j}y_j\end{aligned}$$

Also, from (37)

$$\begin{aligned}\frac{\partial C_i}{\partial f_i} &= \frac{(1-\phi_{ij})F_i}{f_i} \\ \frac{\partial C_i}{\partial l_j^W} &= -\frac{\partial \gamma_{ij}}{\partial l_j^W}H_i - \frac{\partial \phi_{ij}}{\partial l_j^W}F_i \\ &\quad + \frac{\partial \gamma_{ji}}{\partial l_j^W}H_j + \frac{\partial \phi_{iji}}{\partial l_j^W}F_j \\ &\quad + \gamma_{ji}H_j'\frac{\partial l_j^H}{\partial l_j^W} + \phi_{ji}F_j'\frac{\partial l_j^F}{\partial l_j^W} \\ \frac{\partial C_i}{\partial r_j} &= \gamma_{ji}H_j'\frac{\partial l_j^H}{\partial r_j} + \phi_{ji}F_j'\frac{\partial l_j^F}{\partial r_j}\end{aligned}$$

At symmetric equilibrium

$$
\begin{aligned}
\frac{\partial C_i}{\partial f_i} &= \frac{(1-\phi)F}{f} \\
\frac{\partial C_i}{\partial l_j^W} &= -\alpha\gamma H - \beta\phi F - \frac{1}{1+r}\left(\gamma H' + r\phi F'\right) \\
\frac{\partial C_i}{\partial r_j} &= \frac{1-l^W}{(1+r)^2}(-\gamma H' + \phi F')
\end{aligned}
$$

Thus

$$
\begin{aligned}
\frac{dC_i}{df_i} &= \frac{(1-\phi)F}{f} - \left[-\alpha\gamma H - \beta\phi F - \frac{1}{1+r}\left(\gamma H' + r\phi F'\right)\right] x_j \\
&\quad + \frac{1-l^W}{(1+r)^2}(-\gamma H' + \phi F')y_j
\end{aligned} \tag{49}
$$

Substituting the previously obtained values of x_j and y_j yields the required differential.

Figure 1. Agricultural Productivity and Consumption: Mixed Farming and Hunting

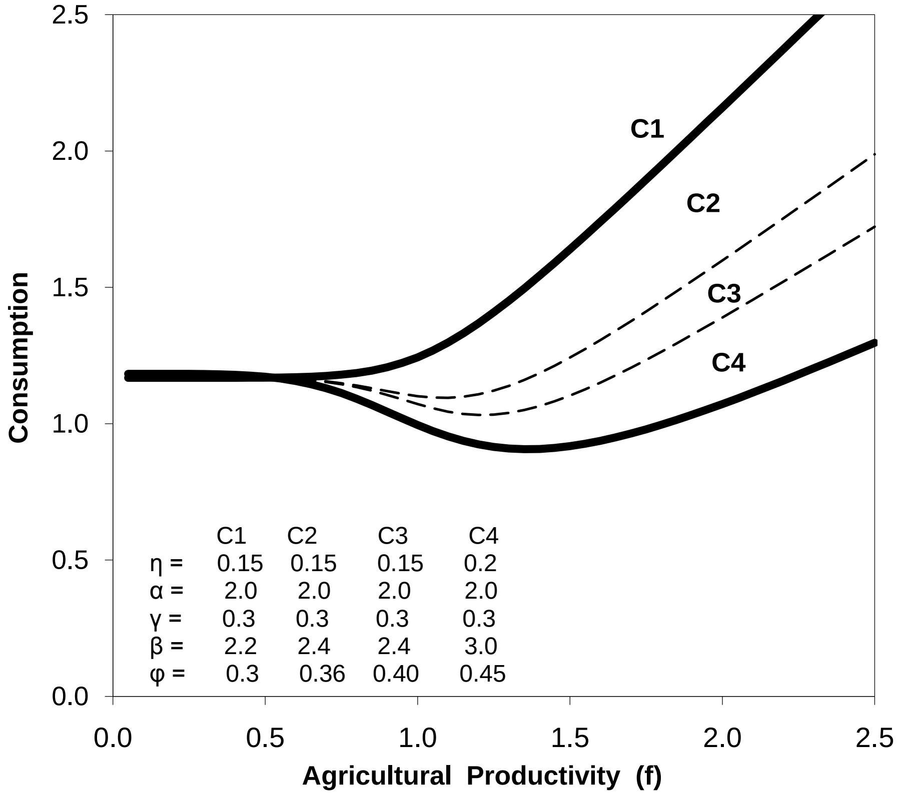

Figure 2 . Comparison of Mixed Economy with Specialized Hunting (Parameters as in Curve C4 of Figure 1)