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by

### Sebastian Sienknecht

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# Persistence Endogeneity Via Adjustment Costs: An Assessment based on Bayesian Estimations ☆

Sebastian Sienknecht\*

Department of Economics, Friedrich-Schiller-University Jena Carl-Zeiss-Str. 3, 07743 Jena, Germany November 15, 2010

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### **Abstract**

This paper estimates a Dynamic Stochastic General Equilibrium (DSGE) model for the European Monetary Union using Bayesian techniques. A salient feature of the model is an extension of the typically postulated quadratic adjustment cost structure for the monopolistic choice of price variables. As shown by Sienknecht (2010b), the enlargement of the original formulation by Rotemberg (1982) and Hairault and Portier (1993) leads to structurally more sophisticated inflation schedules than in the staggering environment of Calvo (1983) with rule-of-thumb setters. In particular, a desired lagged inflation term arises always together with a two-period-ahead expectational expression. The two terms are linked by a novel structural parameter. We confront the price inflation relationship obtained by Sienknecht (2010b) with European data and compare its data description performance against the widespread extension of the Calvo setting with rule-of-thumb behavior.

JEL classification: C11; C15; E31; E32

Key words: Bayesian; Simulation; Indexation; Model Comparison

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<sup>\*</sup>Phone: ++49-3641-9-43213, Fax: ++49-3641-9-43212, Email: Sebastian. Sienknecht@uni-jena.de

# 1 Introduction

The Bayesian estimation methodology is the most common choice when it comes to evaluate a Dynamic Stochastic General Equilibrium (DSGE) model with empirical data. It allows for an estimation of model parameters taking all economic relationships simultaneously into consideration. Apart from this desirable full characterization of observed data, the Bayesian approach allows for an intuitive comparison between different models with respect to their empirical fit. We complement previous theoretical work by Sienknecht (2010b) using Bayesian techniques. The novelty of the model introduced in that paper is an extension of the quadratic adjustment cost structure originally postulated by Rotemberg (1982) and Hairault and Portier (1993). On the aggregate level, this modified cost structure leads to a lagged inflation term connected with an additional and unavoidable two-periodahead inflation expectation. This connection is direct, through a novel structural parameter. Therefore, Sienknecht (2010b) obtains more sophisticated hybrid New Keynesian Phillips curves than in the staggering environment of Calvo (1983) with rule-of-thumb setters. Naturally, the question about the plausibility of this inflation schedule structure arises because an additional two-period-ahead expectation term is rather unusual. The argument is clarified by Sienknecht (2010b). Accordingly, the importance of intertemporal adjustment cost amounts could be taken into consideration, along with a quarterly parameter calibration. The latter makes the two-period-ahead expectational time horizon not questionable at least on theoretical grounds. However, these theoretical foundations have to be complemented by sensible econometric results in order to verify the practical usefulness of the model. We pursue two main objectives by taking selected time series for the European Monetary Union into account. Firstly, a reduced version of the hybrid adjustment cost DSGE model by Sienknecht (2010b) is estimated using Bayesian methods in order to obtain empirical values for the newly introduced inflation persistence parameter. Secondly, two further models are estimated as a point of reference (as in Rabanal and Rubio-Ramírez (2005)). One of them is the framework with a purely forward-looking New Keynesian Phillips curve as postulated by Rotemberg (1982) and Hairault and Portier (1993). The remaining reference model entails a hybrid inflation schedule resulting from a standard Calvo environment with rule-of-thumb setters. We estimate the reference models in order to rank the adjustment cost

model by Sienknecht (2010b) against them. The ranking criterion is the ability of a model to fit the observed data series in terms of marginal likehoods.

The rest of the paper is organized as follows. Section 2 presents our linearized DSGE model versions. Section 3 describes the underlying dataset for the European Monetary Union and reviews the basic idea behind the Bayesian estimation methodology. Section 4 presents our estimation results. Section 5 concludes and points to further areas of research.

# 2 Linearized DSGE Models

This section outlines three different linear equation systems. Each collection of equations is characterized by a specific New Keynesian Phillips curve, while all relationships other than the inflation schedule remain identical across the models (as in Rabanal and Rubio-Ramírez (2005)). The baseline (BSAC) model contains a purely forward-looking inflation curve following Rotemberg (1982) and Hairault and Portier (1993). We compare the empirical performance of this baseline specification to the hybrid (HYAC) alternative postulated by Sienknecht (2010b). Most importantly, we pursue an empirical comparison of this hybrid inflation curve against the standard Calvo (1983) schedule with rule-of-thumb setters (HYCC). The latter is derived according to Galí et al. (2001). Note that the only disturbance considered by Sienknecht (2010b) is a stochastic shock to the interest rate. However, we have to increase the number of shocks at least to the number of data time series in order to rule out a stochastic singularity in the Bayesian estimation. Since we employ three time series of European data, the stochastic singularity problem is avoided by introducing three exogenous stochastic shocks.

# 2.1 Common Equations

Several relationships are general to the three models considered here and can be derived as in Sienknecht (2010a, 2010b). Concerning the representative household, we assume internal habit formation in consumption (Casares (2006)). The detailed optimization problem of the household can be inspected in the appendix. From the first-order conditions, we derive the linearized Euler equation for intertemporal consumption. Since aggregate demand is driven by consumption only  $(\hat{Y}_t = \hat{C}_t)$ , we

obtain

$$\hat{Y}_{t} = \Theta_{1} \hat{Y}_{t-1} + \Theta_{2} E_{t} [\hat{Y}_{t+1}] - \Theta_{3} E_{t} [\hat{Y}_{t+2}] - \Theta_{4} (\hat{R}_{t} - E_{t} [\hat{\pi}_{t+1}^{p}]). \tag{1}$$

Throughout the paper, a hat over a variable represents its logarithmic deviation from its steady state. Here,  $\hat{R}_t$  gives the gross interest rate of riskless nominal bonds, and  $\hat{\pi}_t^p$  represents the gross price inflation rate. The variables  $\Theta_1 \dots \Theta_7$  are functions of deep model parameters and can be examined in detail in the appendix. The same applies for  $\iota_1 \dots \iota_4$  when considering the household's marginal rate of substitution with an inverse real wage elasticity of labor supply  $\eta$ :

$$\widehat{MRS}_{t} = \eta \hat{N}_{t} + \iota_{1} \hat{Y}_{t} - \iota_{2} \hat{Y}_{t-1} - \iota_{3} E_{t} [\hat{Y}_{t+1}]. \tag{2}$$

Aggregate employment  $\hat{N}_t$  and real output are linked to one another through the following technology with decreasing returns to labor:

$$\hat{N}_t = \frac{1}{1 - \alpha} \left( \hat{Y}_t - \hat{\varpi}_{A,t} \right) \qquad , \qquad 0 < \alpha < 1.$$
 (3)

The real marginal cost schedule of the intermediate firm can be derived as

$$\widehat{MC}_t = \widehat{W}_t - \widehat{P}_t + \left(\frac{\alpha}{1-\alpha}\right)\widehat{Y}_t - \left(\frac{1}{1-\alpha}\right)\widehat{\varpi}_{A,t},\tag{4}$$

where  $\hat{W}_t$  is the aggregate nominal wage rate and  $\hat{P}_t$  is the aggregate price level. The variable  $\hat{\varpi}_{A,t}$  represents a technology shock which increases the marginal product of labor. Price inflation and nominal wage inflation rates are linked to one another by the following inflation identity:

$$\hat{W}_t - \hat{P}_t = \hat{W}_{t-1} - \hat{P}_{t-1} + \hat{\pi}_t^w - \hat{\pi}_t^p.$$
 (5)

The monetary policy authority is assumed to follow a Taylor rule (see Taylor (1993)) with interest rate smoothing:

$$\hat{R}_t = (1 - \phi) \left( \delta_\pi \ \hat{\pi}_t^p + \delta_y \ \left( \hat{Y}_t - \hat{Y}_t^{pot.} \right) \right) + \phi \ \hat{R}_{t-1} + \hat{\epsilon}_{R,t}, \tag{6}$$

where the shock variable  $\hat{\epsilon}_{R,t}$  captures an unsystematic deviation from the instrument rule and  $\phi$  denotes the degree of interest rate smoothing. In contrast to Sienknecht

(2010b), the output gap  $\hat{Y}_t - \hat{Y}_t^{pot}$  is taken into account and not simply real output  $\hat{Y}_t$ . The reason is the appearance of the technology shock variable  $\hat{\varpi}_{A,t}$ , which affects real output and its potential level  $\hat{Y}_t^{pot}$  simultaneously.

# 2.2 Baseline Adjustment Cost (BSAC) Model

The first model version is given by the equations declared so far together with the purely forward-looking New Keynesian Phillips curve. Nominal rigidities are due to costs of price adjustment for the monopolistic agent (Rotemberg (1982) and Hairault and Portier (1993)). Detailed derivations can be found for example in Sienknecht (2010a). The development of price inflation is derived from the first-order condition of the monopolistic intermediate firm. On the aggregate level, we obtain the following inflation schedule:

$$\hat{\pi}_t^p = \beta E_t \left[ \hat{\pi}_{t+1}^p \right] + \gamma \left( \widehat{MC}_t + \hat{\mu}_{p,t} \right), \tag{7}$$

where  $\beta$  is the household discount factor and  $\hat{\mu}_{p,t}$  denotes the time-varying monopolistic markup of the intermediate firm. It is inversely related to the price elasticity of demand  $\hat{\epsilon}_{p,t}$ , with  $\epsilon_p$  as its steady state counterpart:

$$\hat{\mu}_{p,t} = -\frac{1}{\epsilon_p - 1} \ \hat{\epsilon}_{p,t}. \tag{8}$$

Therefore, an unexpected decrease in  $\hat{\epsilon}_{p,t}$  implies an increase in the intermediate firm's monopolistic power. Thus, it represents an elasticity-driven cost-push shock. The degree of inflation reagibility  $\gamma$  is a function of deep parameters (see the appendix).

# 2.3 Hybrid Adjustment Cost (HYAC) Model

The second model version is given by the common equations and the hybrid New Keynesian Phillips curve for price inflation derived by Sienknecht (2010b). The latter results from a modification of the nonlinear adjustment cost structure given

by Rotemberg (1982) and Hairault and Portier (1993):

$$Q_t^x = \frac{\psi_x}{2} \left( \frac{X_t}{X_{t-1}} - \frac{X}{X} \right)^2 + \frac{\upsilon_x}{2} \left( \frac{X_t}{X_{t-1}} - \frac{X_{t-1}}{X_{t-2}} \right)^2 \qquad , \qquad \psi_x, \upsilon_x > 0, \tag{9}$$

where  $X_t$  is the choice variable of a monopolistic agent not explicitly indexed and X denotes the steady state level of this variable. The extension by Sienknecht (2010b) is given by the second term with a novel rigidity parameter  $v_x > 0$ . Whenever  $X_t$  is changed, adjustment costs arise if  $X_t \neq X_{t-1}$ . In addition, the change of these costs relative to the last period  $\left(\frac{X_t}{X_{t-1}} \neq \frac{X_{t-1}}{X_{t-2}}\right)$  is costly<sup>1</sup>. The agent is agnostic with respect to the last period's adjustment cost amount if  $v_x = 0$ . The representative intermediate firm maximizes its profits by choosing its own price taking the cost structure (9) and its specific product demand schedule into account (see the appendix). Aggregation and linearization in logarithms leads to the following hybrid New Keynesian Phillips curve for price inflation:

$$\hat{\pi}_{t}^{p} = \gamma_{1} \ \hat{\pi}_{t-1}^{p} + \gamma_{2} \ E_{t} \left[ \hat{\pi}_{t+1}^{p} \right] - \gamma_{3} \ E_{t} \left[ \hat{\pi}_{t+2}^{p} \right] + \gamma_{4} \ \left( \widehat{MC}_{t} + \hat{\mu}_{p,t} \right), \tag{10}$$

where  $\gamma_1 \dots \gamma_4$  are functions of deep parameters stated explicitly in the appendix. Our inflation equation (10) differs structurally from the baseline model since a lagged term arises in conjunction with a two-period-ahead expectational term. Both are directly linked through the (deep) parameter  $v_p$ . Setting this parameter equal to zero ( $v_p = 0$ ) eliminates the corresponding composite parameters  $\gamma_1$  and  $\gamma_3$  and gives a New Keynesian Phillips curve structurally equivalent to equation (7).

# 2.4 Hybrid Calvo (HYCC) Model

The most widespread approach of nominal rigidity modelling is the staggered pricesetting environment of Calvo (1983). As a common practice, purely forward-looking New Keynesian Phillips curves are transformed into hybrid versions by assuming rule-of-thumb setters. We formulate this behavior in the spirit of Galí et al. (2001). In our setting, the following hybrid New Keynesian Phillips curve for price inflation

The mechanism is clarified in detail by Sienknecht (2010b). Note that the adjustment costs stated in (9) vanish under flexible prices ( $Q^x = 0$ ).

is obtained $^2$ :

$$\hat{\pi}_{t}^{p} = \gamma_{1}^{c} \hat{\pi}_{t-1}^{p} + \gamma_{2}^{c} E_{t} [\hat{\pi}_{t+1}^{p}] - \gamma_{3}^{c} E_{t} [\hat{\pi}_{t+2}^{p}] + \gamma_{4}^{c} (\widehat{MC}_{t} + \hat{\mu}_{p,t}), \qquad (11)$$

where the superscript 'c' denotes reaction parameters under the Calvo pricing assumption. Their dependence on deep model parameters can be confirmed in the appendix. Note that in this well-known approach an expectational term of two periods ahead does not arise. We included the term  $\gamma_3^c E_t[\hat{\pi}_{t+2}^p]$  with  $\gamma_3^c = 0$  merely for the sake of comparability against the hybrid adjustment cost (HYAC) case.

### 2.5 Shock Processes

The elasticity  $\hat{\epsilon}_{p,t}$  and the technology shock variable  $\hat{\omega}_{A,t}$  are assumed to behave persistently by means of AR(1) processes with white noise disturbances<sup>3</sup>:

$$\hat{\epsilon}_{p,t} = \rho_p \ \hat{\epsilon}_{p,t-1} + e_{p,t} \quad , \quad \rho_p \in [0,1) \quad , \quad e_{p,t} \sim N\left(0, \sigma_p^2\right) \tag{12}$$

and

$$\hat{\varpi}_{A,t} = \rho_A \ \hat{\varpi}_{A,t-1} + e_{A,t} \ , \ \rho_A \in [0,1) \ , \ e_{A,t} \sim N(0,\sigma_A^2). \tag{13}$$

However, the interest rate shock variable is a pure white noise process:

$$\hat{\epsilon}_{R,t} = e_{R,t} \quad , \quad e_{R,t} \sim N\left(0, \sigma_R^2\right). \tag{14}$$

# 3 Data and Preliminaries

We proceed to estimate the presented theoretical models and to assess their empirical fit. The ranking strategy is closely related to Rabanal and Rubio-Ramírez (2005).

<sup>&</sup>lt;sup>2</sup> A detailed derivation can be found in Sienknecht (2010b).

<sup>&</sup>lt;sup>3</sup> The assumption of white noise disturbances is merely for the sake of generating impulse responses and will be relaxed later (see section 3.3)

That is, we compare the probabilities (likelihoods) of the models to describe the empirical data. Apart from presenting the set of time series used, the following sections explain the rationale behind our fixed parameter values and the parameter priors. They are crucial for the Bayesian estimation procedure, which is also briefly reviewed.

### 3.1 The Data

The underlying dataset includes three quarterly and seasonally adjusted series. All series are from the Area-Wide-Model (AWM) for the time period 1970Q2-2005Q4<sup>4</sup>:

- 1. Log of real consumption, seasonally adjusted (AWM code: PCR).
- 2. Inflation rate, annualized quarterly change of consumption deflator in percent, seasonally adjusted (AWM code: PCD).
- 3. Short-term nominal interst rate, in percent (AWM code: STN).

### 3.2 Estimation Methodology

Instead of reviewing the vast amount of literature on Bayesian estimation methods, we provide this section with the very essential idea. The interested reader is referred to core contributions such as Fernández-Villaverde and Rubio-Ramírez (2004) and An and Schorfheide (2007). Fernández-Villaverde (2009) and Almeida (2009) have provided recently very comprehensive explanations on the topic. Since we use the Matlab preprocessor Dynare for solving and estimating our three DSGE models, the following explanations rely heavily on the pertinent instruction manuals<sup>5</sup>. The Bayesian procedure estimates a subset of model parameters with a weighted Maximum Likelihood approach. More precisely, priors for the parameters to be estimated (mean, variance, and type of distribution) are prespecified and combined with the

<sup>&</sup>lt;sup>4</sup> The data was obtained from the website http://www.eabcn.org/area-wide-model. See Fagan et al. (2005) for an overview of the AWM and the underlying historical series. We extract the trend of real consumption by passing the series through a Hodrick-Prescott filter on a quarterly time basis.

<sup>&</sup>lt;sup>5</sup> We use Matlab version 7.11.0.584 (R2010b) and Dynare version 4.2.2. See for a reference manual Adjemian et al. (2011). More information on Dynare can be retrieved from the website http://www.dynare.org/.

model-specific likelihood function. This gives the target function to be maximized with an optimization routine<sup>6</sup>. As an outcome, one obtains the combination of posterior parameter estimates that renders the dataset and the imposed a-priori beliefs "most likely". The following lines should give a sensible idea of the Bayesian estimation procedure. All steps explained below are ultimately targeted towards a posterior density function of the form  $p(\theta|Y_T)$ , where  $\theta$  denotes the vector of parameters to be estimated and  $Y_T$  is the vector of observables up to period T. We start with general explanations on the computation of the rational expectations equilibrium and the likelihood function. Note first that any DSGE model is simply a collection of nonlinear first-order equilibrium conditions that takes the following general form:

$$E_t\left\{f\left(\boldsymbol{y}_{t+1},\boldsymbol{y}_t,\boldsymbol{y}_{t-1},\boldsymbol{e}_t\right)\right\} = 0$$
 ,  $\boldsymbol{e}_t \sim N\left(0,\Sigma_e\right)$  ,  $E_t\left(\boldsymbol{e}_t\boldsymbol{e}_s'\right) = 0$  ,  $t \neq s$ , (15)

where y is the vector of endogenous variables and e is the vector of shock innovations assumed to be Gaussian white noise processes. The set of endogenous variables  $y_{t-1}$  denotes predetermined variables while the remaining quantities are only known at time t. The solution to this system is called the policy function g(.). It is a set of equations relating variables in the current period to the past state of the system and to current shocks:

$$\mathbf{y}_{t} = g\left(\mathbf{y}_{t-1}, \mathbf{e}_{t}\right). \tag{16}$$

Our three models are systems of log-linearized equations. A first-order Taylor expansion in logarithms around the deterministic steady state of (15) and (16) yields the approximated system:

$$E_t \left\{ f_{v+1} \hat{\boldsymbol{y}}_{t+1} + f_v \hat{\boldsymbol{y}}_t + f_{v-1} \hat{\boldsymbol{y}}_{t-1} + f_{e+1} \boldsymbol{e}_{t+1} + f_e \boldsymbol{e}_t \right\} = 0 \tag{17}$$

A well-kwown advantage of Bayesian estimation is the avoidance of several problems connected with a stand-alone Maximum Likelihood estimation of medium-scale DSGE models. In particular, the reweighting of the likelihood function increases the curvature of the target function. This avoids common problems, such as a flat likelihood over large parameter subspaces and the so-called "dilemma of absurd parameter estimates".

and

$$\hat{\boldsymbol{y}}_t = g_{v-1}\hat{\boldsymbol{y}}_{t-1} + g_e \boldsymbol{e}_t, \tag{18}$$

where  $f_{y+1}$ ,  $f_y$ , and  $f_{y-1}$  are the matrix derivatives with respect to  $\mathbf{y}_{t+1}$ ,  $\mathbf{y}_t$ , and  $\mathbf{y}_{t-1}$ , evaluated at the steady state. The same reasoning applies for the remaining derivative terms  $g_{y-1}$  and  $g_e$ . Vectors denoted as  $\hat{\mathbf{y}}_{t+1}$ ,  $\hat{\mathbf{y}}_t$ , and  $\hat{\mathbf{y}}_{t-1}$  contain the logarithmic deviation of endogenous variables from their steady state. The system (17) and (18) can be rewritten in compact matrix notation as

$$A\hat{y}_t = B\hat{y}_{t-1} + Ce_t, \tag{19}$$

where the matrices A, B, and C contain matrix derivatives, evaluated at the steady state. The system can be solved with a generalized Schur decomposition. Additionally, the fulfillment of the Blanchard-Kahn condition (see Blanchard and Kahn (1980)) is checked, namely a number of generalized eigenvalues outside the unit circle equalized to the number of forward-looking variables. We need to establish a relationship between the observables  $\hat{y}^*$  in the set of observable variables  $Y_T$  and the model variables  $\hat{y}$ . This is done in terms of a measurement equation with a measurement error  $\tilde{e}$ . We can therefore rewrite the solution to a DSGE model as a system in the following manner:

$$\hat{\boldsymbol{y}}_{t}^{*} = \boldsymbol{F}\hat{\boldsymbol{y}}_{t} + \boldsymbol{G}\tilde{\boldsymbol{e}}_{t} \tag{20}$$

and

$$\hat{\boldsymbol{y}}_{t} = \boldsymbol{D}\hat{\boldsymbol{y}}_{t-1} + \boldsymbol{E}\boldsymbol{e}_{t}, \tag{21}$$

where the first equation is the measurement equation with a linking matrix  $\mathbf{F}$  and a weighting matrix  $\mathbf{G}$ . The measurement error vector  $\tilde{\mathbf{e}}$  is assumed to be a Gaussian white noise process in the same manner as  $\mathbf{e}$ . The second equation corresponds to (18). The log-likelihood of a model is retrieved by using the Kalman filter recursion<sup>7</sup>.

<sup>&</sup>lt;sup>7</sup> See for example Fernández-Villaverde (2009) and Hamilton (1994), ch. 13.

Its application leads to the expression:

$$\ln \mathcal{L}\left(\boldsymbol{Y}_{T}|\boldsymbol{\theta}\right) = -\frac{Tk}{2}\log\left(2\pi\right) - \frac{1}{2}\sum_{t=1}^{T}\log\left|\Sigma_{\hat{\boldsymbol{y}}_{t|t-1}^{*}}\right|$$

$$-\frac{1}{2}\sum_{t=1}^{T}\left(\hat{\boldsymbol{y}}_{t}^{*} - \hat{\boldsymbol{y}}_{t|t-1}^{*}\right)'\left(\Sigma_{\hat{\boldsymbol{y}}_{t|t-1}^{*}}\right)^{-1}\left(\hat{\boldsymbol{y}}_{t}^{*} - \hat{\boldsymbol{y}}_{t|t-1}^{*}\right),$$

$$(22)$$

where the vector  $\boldsymbol{\theta}$  contains the parameters to be estimated and, again,  $\boldsymbol{Y}_T$  gives the set of observable endogenous variables  $\hat{\boldsymbol{y}}_t^*$  in the measurement equation until period T (see Almeida (2009)). Note that  $\hat{\boldsymbol{y}}_{t|t-1}^*$  is a predictor of  $\hat{\boldsymbol{y}}_t^*$  using information up to t-1,  $\Sigma_{\hat{\boldsymbol{y}}_{t|t-1}^*}$  is a predictor of the variance-covariance matrix of  $\hat{\boldsymbol{y}}_t^*$  using information up to t-1, and both pedictors depend on the parameter vector  $\boldsymbol{\theta}$ . Given the likelihood function (22) and having defined a prior densitity  $p(\boldsymbol{\theta})$ , the computation of the posterior density  $p(\boldsymbol{\theta}|\boldsymbol{Y}_T)$  as an update of the priors is straightforward. Using the Bayes theorem, we can write the posterior density as

$$p(\boldsymbol{\theta}|\boldsymbol{Y}_T) = \frac{p(\boldsymbol{\theta};\boldsymbol{Y}_T)}{p(\boldsymbol{Y}_T)},$$
(23)

where  $p(\boldsymbol{\theta}; \boldsymbol{Y}_T)$  is the joint density of the parameters and the data and  $p(\boldsymbol{Y}_T)$  is the marginal density of the data. Similarly, we can write for the density of the data conditional on the parameters (the likelihood function):

$$p(\mathbf{Y}_{T}|\boldsymbol{\theta}) = \frac{p(\boldsymbol{\theta}; \mathbf{Y}_{T})}{p(\boldsymbol{\theta})}$$

$$\Leftrightarrow p(\boldsymbol{\theta}; \mathbf{Y}_{T}) = p(\mathbf{Y}_{T}|\boldsymbol{\theta}) p(\boldsymbol{\theta}).$$
(24)

The combination of the last two equations gives the posterior density as

$$p(\boldsymbol{\theta}|\boldsymbol{Y}_T) = \frac{p(\boldsymbol{Y}_T|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\boldsymbol{Y}_T)}.$$
 (25)

Since the marginal density  $p(Y_T)$  is independent of the parameter vector  $\boldsymbol{\theta}$ , it can be treated as a constant. The posterior kernel  $\mathcal{K}(\boldsymbol{\theta}|Y_T)$  or the unnormalized posterior density corresponds to the numerator of the last expression:

$$p(\boldsymbol{\theta}|\boldsymbol{Y}_T) \propto p(\boldsymbol{Y}_T|\boldsymbol{\theta})p(\boldsymbol{\theta}) \equiv \mathcal{K}(\boldsymbol{\theta}|\boldsymbol{Y}_T).$$
 (26)

Taking logs of the kernel expression delivers:

$$\ln \mathcal{K}(\boldsymbol{\theta}|\boldsymbol{Y}_T) = \ln p(\boldsymbol{Y}_T|\boldsymbol{\theta}) + \ln p(\boldsymbol{\theta}) = \ln \mathcal{L}(\boldsymbol{Y}_T|\boldsymbol{\theta}) + \ln p(\boldsymbol{\theta}). \tag{27}$$

The next step is to maximize the log kernel with respect to  $\boldsymbol{\theta}$  in order to find estimates for the posterior mode  $\boldsymbol{\theta}^m$ . However, the analytical intractability of the log kernel (as a nonlinear and complicated function of the deep parameters  $\boldsymbol{\theta}$ ) requires the use of a numerical optimization algorithm. We use Marco Ratto's Matlab routine newrat for the purpose of obtaining the mode  $\boldsymbol{\theta}^m$  and the Hessian matrix  $\boldsymbol{H}(\boldsymbol{\theta}^m)$  evaluated at the mode<sup>8</sup>. Finally, the random-walk Metropolis-Hastings algorithm is called in order to generate the posterior distribution (mean and variance) of our parameters around the mode<sup>9</sup>. Having estimated our three model versions in the way described so far, it is possible to undertake a model comparison using posterior distributions. See Schorfheide (2000) and Rabanal and Rubio-Ramírez (2005) for a detailed overview. First, define the prior distribution over two competing models  $\boldsymbol{A}$  and  $\boldsymbol{B}$  as  $p(\boldsymbol{A})$  and  $p(\boldsymbol{B})$ . Using the Bayes rule, one can compute the posterior probability of each model as

$$p(\mathbf{A}|\mathbf{Y}_T) = \frac{p(\mathbf{A})p(\mathbf{Y}_T|\mathbf{A})}{p(\mathbf{A})p(\mathbf{Y}_T|\mathbf{A}) + p(\mathbf{B})p(\mathbf{Y}_T|\mathbf{B})}$$
(28)

and

$$p(\boldsymbol{B}|\boldsymbol{Y}_T) = \frac{p(\boldsymbol{B})p(\boldsymbol{Y}_T|\boldsymbol{B})}{p(\boldsymbol{A})p(\boldsymbol{Y}_T|\boldsymbol{A}) + p(\boldsymbol{B})p(\boldsymbol{Y}_T|\boldsymbol{B})}.$$
 (29)

The expressions above describe the probability of a model being true after observing the data. Therefore, a natural way to compare the empirical fit of two models is given by the posterior odds ratio:

$$PO_{\mathbf{A},\mathbf{B}} = \frac{p(\mathbf{A}|\mathbf{Y}_T)}{p(\mathbf{B}|\mathbf{Y}_T)} = \frac{p(\mathbf{A})}{p(\mathbf{B})} \frac{p(\mathbf{Y}_T|\mathbf{A})}{p(\mathbf{Y}_T|\mathbf{B})},$$
(30)

<sup>&</sup>lt;sup>8</sup> We also used Chris Sims's routine csminwel. However, Ratto's routine improves our results significantly. See Adjemian et al. (2011) for an overview of the Matlab routines available in Dynare.

<sup>&</sup>lt;sup>9</sup> We opt for 1,000,000 draws from each model's posterior distribution with 4 distinct chains. For details on the Metropolis-Hastings algorithm, see An and Schorfheide (2007) and Fernández-Villaverde (2009).

where  $\frac{p(Y_T|A)}{p(Y_T|B)}$  is the Bayes factor describing the evidence in the data favoring model A over model B and  $\frac{p(A)}{p(B)}$  is the prior odds ratio giving the relative probability subjectively assigned to the models. The terms entering the numerator and denominator of the Bayes factor are marginal densities of the data conditional on the respective model (the marginal likelihood of a model). They can be obtained, at least theoretically, by integrating out the deep parameters from the posterior kernel:

$$p(\mathbf{Y}_{T}|\mathbf{A}) = \int_{\boldsymbol{\theta}_{A}} p(\mathbf{Y}_{T}, \boldsymbol{\theta}_{A}|\mathbf{A}) d\boldsymbol{\theta}_{A}$$

$$= \int_{\boldsymbol{\theta}_{A}} p(\mathbf{Y}_{T}|\boldsymbol{\theta}_{A}, \mathbf{A}) p(\boldsymbol{\theta}_{A}|\mathbf{A}) d\boldsymbol{\theta}_{A}$$

$$= \int_{\boldsymbol{\theta}_{A}} \mathcal{K}(\boldsymbol{\theta}_{A}|\mathbf{Y}_{T}, \mathbf{A}) d\boldsymbol{\theta}_{A}$$
(31)

and

$$p(\mathbf{Y}_{T}|\mathbf{B}) = \int_{\boldsymbol{\theta}_{B}} p(\mathbf{Y}_{T}, \boldsymbol{\theta}_{B}|\mathbf{B}) d\boldsymbol{\theta}_{B}$$

$$= \int_{\boldsymbol{\theta}_{B}} p(\mathbf{Y}_{T}|\boldsymbol{\theta}_{B}, \mathbf{B}) p(\boldsymbol{\theta}_{B}|\mathbf{B}) d\boldsymbol{\theta}_{B}$$

$$= \int_{\boldsymbol{\theta}_{B}} \mathcal{K}(\boldsymbol{\theta}_{B}|\mathbf{Y}_{T}, B) d\boldsymbol{\theta}_{B}.$$
(32)

However, this function is analytically intractable and has to be substituted by the Laplace or the Harmonic Mean approximation. Our estimation results give the (positive) logarithmic value of the approximated marginal likelihood of each model. By assuming an uniform distribution across two models  $(p(\mathbf{A}) = p(\mathbf{B}))$ , we compare them by simply taking the logarithmic difference of the posterior odds ratio:  $\ln p(\mathbf{Y}_T|\mathbf{A}) - \ln p(\mathbf{Y}_T|\mathbf{B})$ . A positive value of this difference is interpreted as an outperformance of model  $\mathbf{A}$  over model  $\mathbf{B}$  in the description of the dataset  $\mathbf{Y}_T$  (see Schorfheide (2000) and Rabanal and Rubio-Ramírez (2005)).

### 3.3 Parameters and Priors

This section presents the subset of fixed deep parameter values and the priors<sup>10</sup>. The preliminary definition of some parameters is due to identification problems and resulting difficulties in estimating them. As pointed out by Rabanal and Rubio-Ramírez (2005), the absence of capital services in our model hinders the estimation of the household discount factor  $\beta$  and the capital share of output  $\alpha$ . Therefore, we set  $\beta = 0.99$  and  $\alpha = 0.3$ , which are standardly used values. Moreover, identification problems arise between the steady state elasticity  $\epsilon_p$  and the corresponding rigidity parameter  $\psi_p$  (BSAC and HYAC) or  $\theta_p$  (HYCC). For this reason, we assume  $\epsilon_p = 11$ , which is a commonly used value<sup>11</sup>.

Concerning the parameter priors, we opt to choose prior mean values that are mostly found in the standard literature and microeconometric studies. Their tightness (prior standard deviation) is set as loosely as possible in order to let the data drive our posterior results as much as possible. However, the looseness degree is restricted by the success of the numerical optimization of the posterior kernel across the three model versions<sup>12</sup>. This leads to prior densities for some parameters in our models that are somewhat tighter than in the prevailing literature. An overview of the common parameter priors is given by the third column of table 1. Across both adjustment cost models (BSAC and HYAC), the prior price rigidity parameter  $\psi_p$ is assumed to be normally distributed with a mean of 550 and a standard deviation (henceforth in brackets) of 100. This corresponds approximatedly to a prior for the Calvo parameter  $\theta_p$  in the HYCC model given by 0.75 (0.010). The BSAC model is characterized by  $v_p = 0$ . In contrast, we estimate this parameter in the HYAC model. We choose a loose prior for  $v_p$  and arbitrarily set 5000 (1500). Since we don't have any prior knowledge about this parameter, the imposition of a loose prior density distribution is very important. In contrast, the existing literature en-

<sup>&</sup>lt;sup>10</sup> The total numbers of deep parameters are given by: 12 (BSAC), 13 (HYAC), and 13 (HYCC). We fix a subset of 3 parameters contained in each model version. Additionally, we have 3 shocks whose priors also have to be prespecified.

 $<sup>^{11}</sup>$  This implies a steady state gross markup of  $\mu_p$  = 1.10 (10 percent).

<sup>&</sup>lt;sup>12</sup> In fact, we checked by using the Dynare command mode\_check that the maximum of the posterior log kernel is obtained for all parameters across the models.

ables us to assume standard values for the beta-distributed prior of  $\omega_p$ . It is given by 0.75 (0.015). Concerning the interest rate rule coefficients, we assume priors  $\delta_{\pi} = 1.5$  and  $\delta_y = 0.5$ , which are close to the original estimates by Taylor (1993) with annualized price inflation. We restrict those parameters to have a positive support by imposing a gamma distribution with standard deviations being equal to 0.05 and 0.01, respectively. The smoothing parameter  $\phi$  follows a beta distribution with a mean equal to 0.5 and a standard deviation of 0.05. The household parameters  $\sigma$  and  $\eta$  follow a normal distribution with mean 2.0. While these prior mean values are commonly assumed in the literature (See Smets and Wouters (2003)), we impose a somewhat tighter density for  $\eta$  of 0.05. The prior of the habit formation parameter h is assumed to follow a beta distribution with 0.8 (0.01).

Table 1 also gives the priors for the shock processes. Concerning the prior standard deviation of the shocks, we use the inverse gamma distribution with a relatively low prior mean of 0.08 for the price shock and of 0.01 for the remaining impulses. The priors for the autoregressive shock parameters  $\rho_p$  and  $\rho_A$  are assumed to follow a beta distribution with a mean and a standard deviation of 0.7 (0.01), respectively.

# 4 Empirical Findings

### 4.1 Parameter Estimates

Apart from reporting posterior parameter distributions, we also present graphical estimation results for the HYAC model. The emphasis on the model by Sienknecht (2010b) is because of two reasons. First, analogous figures were obtained across all models but the outcomes for the HYAC model are used in order to exemplify the underlying interpretations. Second, these graphical results are not very well known because estimated adjustment cost models (BSAC and HYAC) are somewhat rare in the literature. In contrast, most theoretical and empirical studies have concentrated on the (hybrid) Calvo environment.

Figure 1 displays results in the HYAC model for the posterior kernel maximization that confirm the unique and robust maximum for all model parameters. Figures 2, 3, and 4 allow for a statement on the overall and the parametrical convergence of the Metropolis-Hastings sampling algorithm. Following for example Almeida (2009), the overall convergence information is summarized in the first three graphs of figure

2 and each one of them represents a specific global convergence measure. The two distinct lines display the results within and between the chains of the Metropolis-Hastings algorithm. These measures are related to the analysis of the parameters' mean ("Interval"), of the variances ("m2") and of the third moments ("m3"). Ideally, the two lines converge to each other and become relatively stable for each one of the three measures. The figures 2, 3, and 4 show that overall and parametrical convergence was obtained<sup>13</sup>.

The last three columns of table 1 contain the posterior mean and the posterior standard deviation of estimated parameters in the BSAC, HYAC, and HYCC models. These columns also show posterior standard deviations for the shocks and the posterior distributions of the shock persistence parameters. Estimation results for the BSAC model are presented in the fourth column of table 1. Accordingly, the degree of price rigidity increases to  $\psi_p = 1000.35$ , with a lower measure of uncertainty (76.61). Most of the remaining parameter estimates in the BSAC model remain close to their priors and show a lower degree of uncertainty. The coefficients of the Taylor rule are very close to the values obtained by Clarida et al. (2000) and Rabanal and Rubio-Ramírez (2005). We obtain a posterior for the inverse of the intertemporal elasticity of substitution which is lower than in the standard literature, namely  $\sigma = 0.10^{14}$ . Another glance at the fourth column of table 1 reveals a high posterior volatility of the price markup ( $\sigma_p = 1.62$ ) and confirms the important role of price markup shocks in DSGE models. At the same time, the importance of the remaining shocks for the explanation of fluctuations is marginally increased ( $\sigma_A = 0.03$ ) or reduced ( $\sigma_R = 0.005$ ). The posterior autoregressive parameters show little deviation from our prior distribution beliefs.

Posterior parameter estimates for the HYAC model can be inspected in the fifth column of table 1. They are complemented by figure 5. Again, the posterior degree of uncertainty is lower for all parameters than in our prior subjective beliefs. We obtain a higher degree of price rigidity, namely  $\psi_p = 607.83$ . The most impor-

 $<sup>^{13}</sup>$  The overall and parametrical convergence was also obtained in the BSAC and the HYCC model.

All posterior parameter estimates in the BSAC model fulfill the Blanchard-Kahn stability condition (see Blanchard and Kahn (1980)). The same is true for the HYAC and the HYCC model. See the figures 6, 7, and 8.

tant posterior results are those for the novel parameter in the model by Sienknecht (2010b). We obtain the posterior estimate for  $v_p$  as 7547.16 (1152.06). This result clearly indicates the relevance of this parameter for the description of the underlying data. In order to reject the hybrid specification by Sienknecht (2010b), one would need to obtain a value of this parameter in the neighborhood of zero or a highly increased degree of uncertainty. Of course, the posterior positive value of  $v_p$  could be the result of the lagged inflation component alone. However, the theoretical result is an additional two-period-ahead expectation term that cannot be disentagled from the lagged term. Our posterior parameter value clearly points at the relevance of two-period-ahead expectations. The remaining posterior estimates in the HYAC model are similar to those obtained in the BSAC environment. The inverse of the intertemporal substitution in consumption is somewhat higer ( $\sigma = 0.29$  (0.04)). The fifth column of table 1 also shows a higher posterior volatility in the price markup shock, whereas the relevance of the remaining shocks is rather limited. Again, the posterior autoregressive parameters show little deviation from our prior beliefs. The estimation outcomes for the hybrid Calvo-type model (HYCC) are reported in the sixth column of table 1. All posterior parameter estimates are rather close to their priors due to the high degree of prior tightness. Since the parameters of the HYCC model have been subject to many empirical studies, our choice of priors with a low degree of uncertainty appears to be justifiable. The drop in the price markup volatility relative to the BSAC model has also been obtained by Rabanal and Rubio-Ramírez (2005). Nonetheless, other results concerning the posterior variability of shocks are mixed.

# 4.2 Model Comparison

The empirical performance of a model is assessed by means of its marginal likelihood. As a starting point, we compare the hybrid adjustment cost model (HYAC) against the baseline adjustment cost environment (BSAC) and determine which of the two dominates in terms of data description performance. The resulting outperformer is checked against the hybrid Calvo-type model (HYCC). In each comparison, we assume a uniform prior probability across the two models involved such that the prior odds ratio is equal to one. The posterior odds ratio is in logs, which implies

differences in log-marginal likelihoods as our instrument for ranking models<sup>15</sup>.

The last two rows of table 1 give the (Laplace- and Metropolis-Hastings-) approximated marginal likelihoods in logs. Note first that the HYAC model clearly outperforms the BSAC model since the differences in log-marginal likelihoods, namely 90.83 (Laplace) and 90.80 (Harmonic mean), are positive and large. This result is not surprising, as the BSAC model fails to describe the high degree of price inflation persistence present in the data. In contrast, the HYAC model performs better since its hybrid inflation curve displays a lagged term. Note again, the lagged price inflation term in the HYAC model is linked directly to the two period-ahead expectation through the parameter  $v_p$ . As stated in Sienknecht (2010b), this is relationship is unavoidable when trying to induce a lagged inflation term in the adjustment cost environment. Therefore, we cannot assess the empirical contribution of this additional forward-looking term alone.

Having established the superiority of the HYAC model against the baseline adjustment cost framework, we now ask if there is also an outperformance against the HYCC model<sup>16</sup>. From the viewpoint of the HYAC model, the differences in log-marginal likelihood are 10.38 (Laplace) and 10.98 (Metropolis-Hastings). Therefore, we find a marginal likelihood in the HYAC model which is higher than in the HYCC model. However, the marginal likehood difference is not high enough (at least a value of 30) to assess the absolute dominance of the HYAC model.

We conclude that the HYAC model should be preferred over the BSAC model. At the same time, the HYAC model seems to be able to compete the HYCC model<sup>17</sup>. As a first careful observation, the appearance of the additional expectations term in the HYAC model seems to improve the description of the underlying data. This empirical finding would support the thoretical results by Sienknecht (2010b).

Finally, we generate impulse responses in the BSAC, HYAC, and HYCC models based on our parameter estimates and with orthogonal estimated standard devia-

<sup>&</sup>lt;sup>15</sup> This practice of ranking models is the same as in Rabanal and Rubio-Ramírez (2005).

Needless to say that the HYCC model is also superior against the BSAC model since the former implies a lagged inflation term that captures the inflation persistence present in the data.

<sup>&</sup>lt;sup>17</sup> Note that the introduction of adjustment cost models (BSAC and HYAC) allows for an easier handling of the algebra than the Calvo assumption (HYCC).

tion shocks. As can be confirmed in the figures 6, 7, and 8, the posterior parameter values are such that the Blanchard-Kahn stability condition is met (see Blanchard and Kahn (1980)). The response of inflation in the HYAC model displays a high degree of persistence across all shocks. The shock impulses generate very similar responses as those in the standard literature with other rigidity mechanisms, such as the Calvo setting with rule-of-thumb setters (HYCC model).

# 5 Conclusions

The scope of this paper was to complement previous theoretical work by Sienknecht (2010b) with empirical results. Since Bayesian estimation methods are becoming increasingly popular in the DSGE literature, we opted for them to carry theoretical results towards European data. We provided evidence that the hybrid price inflation schedule by Sienknecht (2010b) performs better than purely forward-looking specifications. This is not a surprising result as it is well known that the standard New Keynesian Phillips curve is not able to capture the high degree of inflation persistence found in the data. The appearance of a lagged inflation term does improve the empirical performance of any purely forward-looking inflation relationship. However, such a lagged term in the adjustment cost environment of Hairault and Portier (1993) and Rotemberg (1982) is always connected with a two-period-ahead additional expectation term (Sienknecht (2010b)). We showed that the parameter linking this two terms adopts meaningful empirical values unequal to zero, which highlights the importance of expectations that look deeper into the future, namely two periods ahead. This is by no means an unreasonable time horizon, taking a quarterly timing into consideration. In addition, the log-marginal likelihood value of the schedule by Sienknecht (2010b) seems to be as high as the marginal density of the Calvo model with indexation. We conclude from our results that the model by Sienknecht (2010b) is a reasonable alternative to the hybrid Calvo environment.

# A Agent Optimization Problems

## A.1 Households

$$\max_{C_t(j), N_t(j), B_t(j)} E_t \sum_{k=0}^{\infty} \beta^k \left[ \frac{\left( C_{t+k}(j) - h \ C_{t+k-1}(j) \right)^{1-\sigma}}{1-\sigma} - \frac{\left( N_{t+k}(j) \right)^{1+\eta}}{1+\eta} \right]$$
(A.1.1)

subject to

$$E_{t} \sum_{k=0}^{\infty} \beta^{k} \left[ C_{t+k}(j) + \frac{B_{t+k}(j)}{P_{t+k}} \right] = E_{t} \sum_{k=0}^{\infty} \beta^{k} \left[ \frac{W_{t+k}}{P_{t+k}} N_{t+k}(j) + R_{t+k-1} \frac{B_{t+k-1}(j)}{P_{t+k}} + Div_{t+k}^{r}(j) \right].$$
(A.1.2)

### A.2 Final Good Producers

$$\max_{Y_t(i)} \left( \left( \int_0^1 Y_t(i)^{\frac{\epsilon_{p,t}-1}{\epsilon_{p,t}}} di \right)^{\frac{\epsilon_{p,t}}{\epsilon_{p,t}-1}} - \int_0^1 \frac{P_t(i) Y_t(i)}{P_t} di \right). \tag{A.2.1}$$

### A.3 Intermediate Good Producers

$$\max_{P_{t}(i)} E_{t} \sum_{k=0}^{\infty} \Delta_{t,t+k} \left[ \left( \frac{P_{t+k}(i)}{P_{t+k}} \right)^{1-\epsilon_{p,t+k}} Y_{t+k} - MC_{t+k}(i) \left( \frac{P_{t+k}(i)}{P_{t+k}} \right)^{-\epsilon_{p,t+k}} Y_{t+k} - \frac{\psi_{p}}{2} \left( \frac{P_{t+k}(i)}{P_{t+k-1}(i)} - 1 \right)^{2} - \frac{\psi_{p}}{2} \left( \frac{P_{t+k}(i)}{P_{t+k-1}(i)} - \frac{P_{t+k-1}(i)}{P_{t+k-2}(i)} \right)^{2} \right].$$
(A.3.1)

### Structural Parameters B

### Composite Parameters in Equation (1) B.1

$$\Theta_1 = \frac{h}{1 + h(1 + \beta h)}, \qquad (B.1.1) \qquad \Theta_2 = \frac{1 + \beta h(1 + h)}{1 + h(1 + \beta h)}, \qquad (B.1.2)$$

$$\Theta_3 = \frac{\beta h}{1 + h(1 + \beta h)}, \qquad (B.1.3) \qquad \Theta_4 = \frac{(1 - h)(1 - \beta h)}{\sigma(1 + h(1 + \beta h))}. \qquad (B.1.4)$$

### Composite Parameters in Equation (2) **B.2**

$$\iota_1 = \frac{\sigma (1 + \beta h^2)}{(1 - h) (1 - \beta h)}, \qquad (B.2.1) \qquad \iota_2 = \frac{h\sigma}{(1 - h) (1 - \beta h)}, \qquad (B.2.2)$$

$$\iota_3 = \frac{\sigma \beta h}{(1-h)(1-\beta h)}.$$
 (B.2.3)

### Composite Parameters in Equation (7) B.3

$$\gamma = \frac{Y(\epsilon_p - 1)}{\psi_p}, \qquad (B.3.1) \quad Y = \left(\frac{(1 - \alpha)(1 - \beta h)}{\mu_p (1 - h)^{\sigma}}\right)^{\frac{1 - \alpha}{\alpha + \eta + \sigma(1 - \alpha)}}. \quad (B.3.2)$$

### Composite Parameters in Equation (10) **B.4**

$$\gamma_{1} = \frac{\upsilon_{p}}{\upsilon_{p} (1 + 2\beta) + \psi_{p}}, \qquad (B.4.1) \qquad \gamma_{2} = \frac{\upsilon_{p} (2 + \beta) \beta + \psi_{p} \beta}{\upsilon_{p}, (1 + 2\beta) + \psi_{p}}, \qquad (B.4.2)$$

$$\gamma_{3} = \frac{\upsilon_{p} \beta^{2}}{\upsilon_{p} (1 + 2\beta) + \psi_{p}}, \qquad (B.4.3) \qquad \gamma_{4} = \frac{(\epsilon_{p} - 1) Y}{\upsilon_{p} (1 + 2\beta) + \psi_{p}}. \qquad (B.4.4)$$

$$\gamma_3 = \frac{v_p \, \beta^2}{v_p \, (1 + 2\beta) + \psi_p}, \qquad (B.4.3) \qquad \gamma_4 = \frac{(\epsilon_p - 1) \, Y}{v_p \, (1 + 2\beta) + \psi_p}. \qquad (B.4.4)$$

### Composite Parameters in Equation (11) B.5

$$\gamma_1^c = \frac{\left[1 + \alpha \left(\epsilon_p \left(1 - \beta \theta_p\right) - 1\right)\right] \omega_p}{\left[1 + \alpha \left(\epsilon_p \left(1 - \beta \theta_p\right) - 1\right)\right] \left(\theta_p + \omega_p \left(1 - \theta_p\right)\right) + \left(1 - \alpha\right) \beta \theta_p \omega_p},\tag{B.5.1}$$

$$\gamma_2^c = \frac{(1-\alpha)\beta \theta_p}{[1+\alpha(\epsilon_p(1-\beta \theta_p)-1)](\theta_p + \omega_p(1-\theta_p)) + (1-\alpha)\beta \theta_p\omega_p},$$
 (B.5.2)

$$\gamma_3^c = 0, \tag{B.5.3}$$

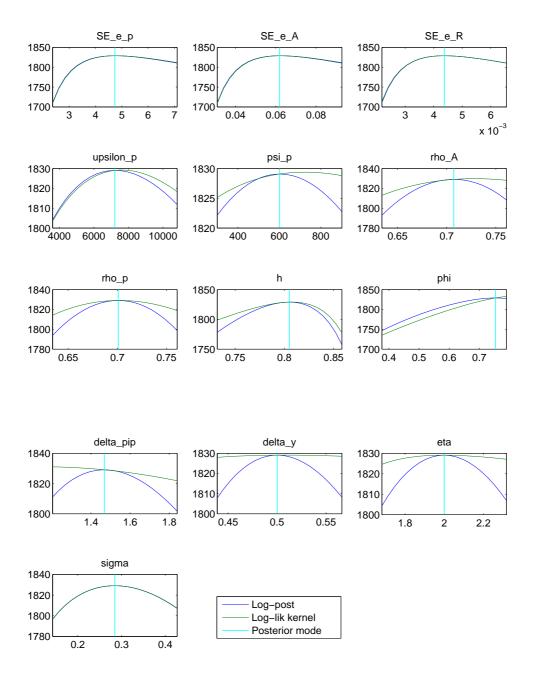
$$\gamma_4^c = \frac{(1-\alpha)(1-\beta\theta_p)(1-\theta_p)(1-\omega_p)}{[1+\alpha(\epsilon_p(1-\beta\theta_p)-1)](\theta_p+\omega_p(1-\theta_p))+(1-\alpha)\beta\theta_p\omega_p}.$$
 (B.5.4)

# C Tables

Table 1: Prior and posterior distribution of model parameters (1970Q2-2005Q4).

	Prior distribution	ı	Posterior distribution		
			BSAC	HYAC	HYCC
		Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
$\psi_p$ Rotemberg rigidity	normal	550 (100)	1000.35 (76.61)	607.83 (95.94)	()
$v_p$ Rot. infl. endogen.	normal	5000 (1500)	( <del></del> )	7547.16 (1152.06)	( <del></del> )
$ heta_p$ Calvo rigidity	normal	0.75 $(0.010)$	( <del></del> )	( <del></del> )	$0.79 \\ (0.009)$
$\omega_p$ Calvo infl. endogen.	normal	$0.75 \\ (0.015)$	( <del></del> )	( <del></del> )	0.80 $(0.012)$
$\sigma$ Consumption utility	normal	$\frac{2.0}{(1.0)}$	0.10 (0.016)	0.29 $(0.04)$	0.09 (0.01)
$\eta$ Labor utility	normal	2.0 (0.050)	1.98 (0.049)	1.99 (0.049)	2.02 (0.048)
h Consumption habit	beta	0.80 (0.010)	0.80 (0.009)	0.80 (0.009)	0.80 (0.009)
$\delta_{\pi}$ Taylor inflation	normal	1.50 (0.050)	1.44 (0.047)	1.47 (0.048)	1.49 (0.044)
$\delta_y$ Taylor output	normal	0.50 (0.010)	0.50 (0.010)	0.50 (0.010)	0.50 (0.010)
$\phi$ Taylor smoothing	normal	0.50 (0.050)	0.62 (0.034)	0.74 (0.028)	0.60 (0.036)
$\sigma_p$ Price markup	inv. gamma	0.080 (0.080)	1.62 (1.62)	4.94 (4.94)	1.04 (1.04)
$\sigma_A$ Productivity	inv. gamma	0.010 (0.010)	0.03 (0.03)	0.063 (0.063)	0.006 (0.006)
$\sigma_R$ Monetary policy	inv. gamma	0.010 (0.010)	0.005 (0.005)	0.004 (0.004)	0.006 (0.006)
$\rho_p$ Shock persistence	beta	0.70 (0.010)	0.72 (0.009)	0.71 (0.010)	0.69 (0.010)
$ ho_A$ Shock persistence	beta	0.70 (0.010)	0.70 (0.009)	0.70 (0.010)	0.70 (0.009)
$\log(L)$ : Laplace		. ,	1715.12	1805.95	1726.10
$\log(L)$ : Harmonic Mean			1715.15	1805.95	1726.13

# D Figures



**Figure 1:** Results from the posterior likelihood function maximization in the hybrid adjustment cost (HYAC) model. SE\_e\_p:  $\sigma_p$ , SE\_e\_A:  $\sigma_A$ , SE\_e\_R:  $\sigma_R$ , upsilon\_p:  $\upsilon_p$ , psi\_p:  $\upsilon_p$ , rho\_A:  $\rho_A$ , rho\_p:  $\rho_p$ , h: h, phi:  $\phi$ , delta\_pip:  $\delta_\pi$ , delta\_y:  $\delta_y$ , eta:  $\eta$ , sigma:  $\sigma$ .

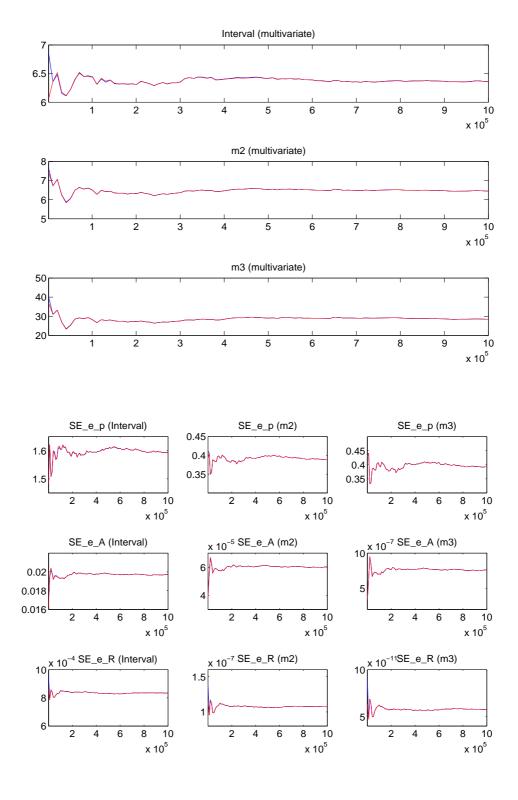


Figure 2: Metropolis-Hastings convergence diagnosis in the hybrid adjustment cost (HYAC) model. Interval: mean, m2: second moment, m3: third moment, SE\_e\_p:  $\sigma_p$ , SE\_e\_A:  $\sigma_A$ , SE\_e\_R:  $\sigma_R$ .

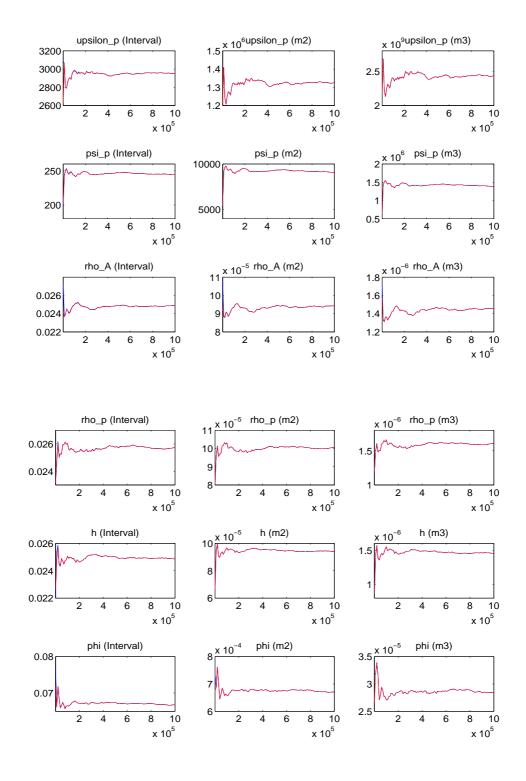


Figure 3: Metropolis-Hastings convergence diagnosis in the hybrid adjustment cost (HYAC) model. Interval: mean, m2: second moment, m3: third moment, upsilon\_p:  $\upsilon_p$ , psi\_p:  $\psi_p$ , rho\_A:  $\rho_A$ , rho\_p:  $\rho_p$ , h: h, phi:  $\phi$ .

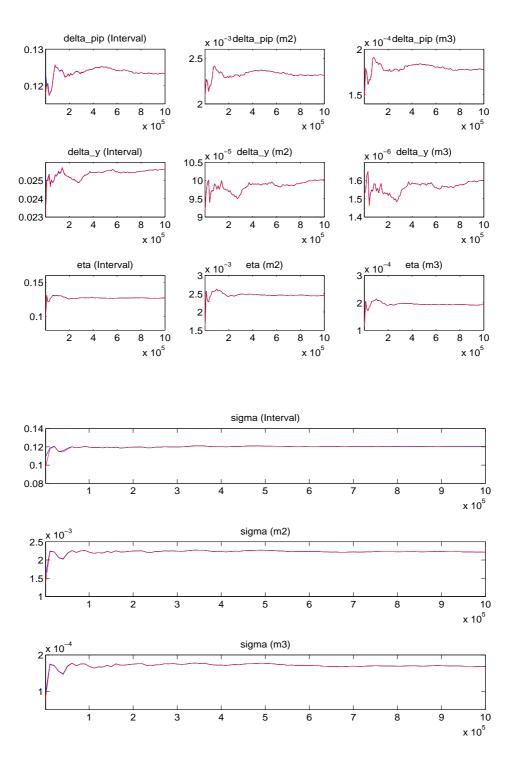
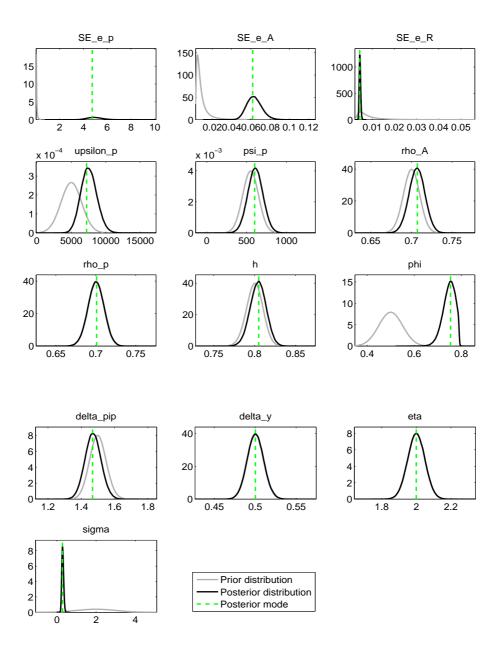
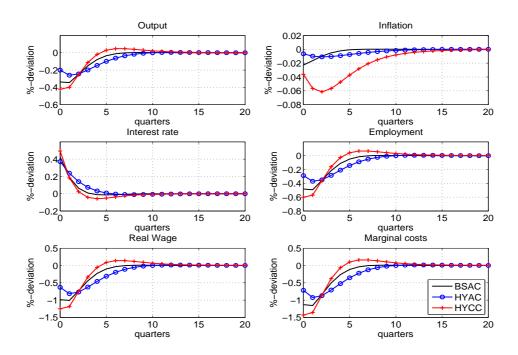


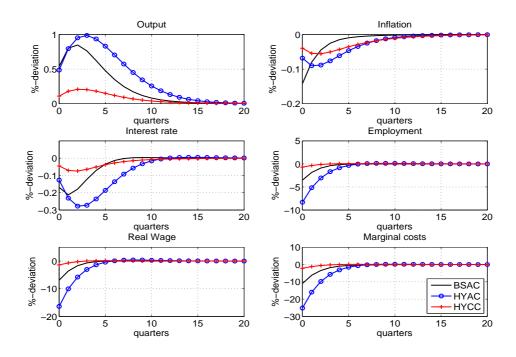
Figure 4: Metropolis-Hastings convergence diagnosis in the hybrid adjustment cost (HYAC) model. Interval: mean, m2: second moment, m3: third moment, delta\_pip:  $\delta_{\pi}$ , delta\_y:  $\delta_{y}$ , eta:  $\eta$ , sigma:  $\sigma$ .



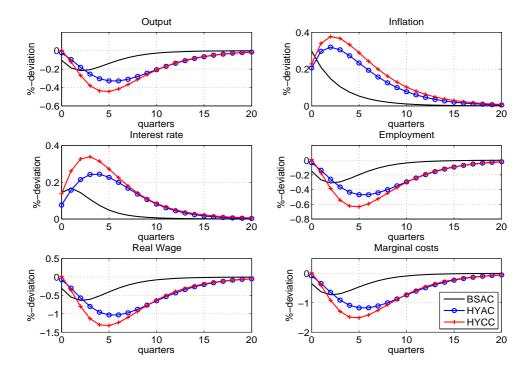
**Figure 5:** Prior and posterior distributions in the hybrid adjustment cost (HYAC) model. SE\_e\_p:  $\sigma_p$ , SE\_e\_A:  $\sigma_A$ , SE\_e\_R:  $\sigma_R$ , upsilon\_p:  $\upsilon_p$ , psi\_p:  $\psi_p$ , rho\_A:  $\rho_A$ , rho\_p:  $\rho_p$ , h: h, phi:  $\phi$ , delta\_pip:  $\delta_{\pi}$ , delta\_y:  $\delta_{y}$ , eta:  $\eta$ , sigma:  $\sigma$ .



**Figure 6:** Impulse responses based on estimation results in the BSAC, HYAC, and HYCC models after an increase of the intrument interest rate in period t = 0.



**Figure 7:** Impulse responses based on estimation results in the BSAC, HYAC, and HYCC models after a positive technology shock in period t = 0.



**Figure 8:** Impulse responses based on estimation results in the BSAC, HYAC, and HYCC models after a cost-push shock in period t = 0.

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