Financial Contracting, R&D and Growth*

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Abstract

This paper investigates the role of financial constraints in R&D races of the type used in Schumpeterian growth theory. In a world of perfect capital markets these models predict that all innovations come from industry outsiders. In reality, however, we observe a pronounced persistence of some dominant firms. We show that this persistence can be explained by constraints on financial contracting. The paper highlights an indirect channel through which agency costs reduce growth: due to agency costs incumbents face little competition from outsiders. Therefore they can afford to innovate less often and to "rest longer on their laurels", thereby retarding growth.

1 Introduction

The present paper discusses the impact of financial contracting under moral hazard on industry dynamics when technical change is endogenous. Much of the literature on endogenous technical change, e.g. Aghion and Howitt (1992, 1998) or Grossman and Helpman (1991), has industry dynamics involving leapfrogging of incumbents by innovating firms. Such leapfrogging is inconsistent with the observation that in many industries there are a few dominant firms whose dominance persists through decades with significant technical change.

We show that such persistence can be explained by constraints on financial contracting. These constraints provide an absolute advantage to the incumbent firm which can rely on its retained earnings to finance its innovations.

The corporate finance literature suggests that the ability to selffinance is an important determinant of firm investment (Fazzari et.al (1988)). Some authors suggest that this is because retained earnings themselves are the most important

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source of finance (Mayer (1988)).[1] It may also be the case that retained earnings increase the firms' collateral and reduce the agency cost of loan finance (see Fazzari et.al (1988)) In either case, incumbency provides an absolute advantage in obtaining finance.[2] The paper shows that this advantage may overcome the usual leapfrogging effect and explain the persistence of incumbency.

The basic point is first developped in the context of a static model of a patent race. The model follows Reinganum (1982, 1984) except that players have different initial wealth, whereby the initial wealth position is correlated with the players' starting positions. One player holds a monopoly in the market whereas the other player starts from scratch. The incumbent has a deep pocket, the other player needs outside funds.

Incumbency then has two countervailing effects: since incremental profits are lower for the incumbent, he has less of an incentive to invest in research. This effect - originally due to Arrow - has been termed "replacement" effect. On the other hand the initial monopoly position creates a deep pocket for future research, which is important in a world of imperfect capital markets. Challengers have to contract with outside sources to finance their research expenditures. This contracting is affected by problems of moral hazard familiar from the finance literature (see Jensen and Meckling (1976)): we assume that financial resources as well as the efforts of entrepreneurs are essential inputs in the research process. The latter, however, are unobservable to third parties. In this setup a challenger financed by outside funds has insufficient incentives to take effort to make the venture go. Because outside investors foresee this behavior, they will supply less funds to the firm. Hence the less inside finance a firm has the less it can invest. This moral hazard effect may outweigh the replacement effect so that the incumbent will actually devote more resources to R&D than the challenger.

Subsequently the paper extends the argument to a dynamic setup. If moral hazard effects are strong enough all innovations will come from incumbent firms. This contrasts with the leapfrogging patterns in, e.g., Aghion and Howitt (1992). An interesting finding is that there may well be a negative impact of moral hazard on growth although actual innovators themselves face no financial problems. The growth retarding effect works through the market: because potential innovators are financially constrained, the competitive threat for the incumbent firms is weak and incumbents can innovate less often without fear of being replaced, i.e. they "can rest on their laurels".

Retardation of growth through agency problems has also been discussed by Aghion, Dewatripont and Rey (1996). In their model managers take insufficient effort for research and development. In the present model insufficiency of incentives for effort taken is also important but this insufficiency arises endogenously from the need of entrepreneurs to obtain outside finance. Here, agency problems not only affect the incentives of potential competitors but also the incentives of incumbents. Given that incumbency persists, the latter is what matters for

[1] For a contrary view, see Hackethal and Schmidt (1999).

[2] See also Myers and Majluf (1984).

equilibrium growth.
The persistence of monopolies has also been studied by Gilbert and Newberry (1982,1984). In their analysis persistence is supported by preemptive patenting. Preemption of patents plays no role in the present paper. The point of the analysis here is that even in the absence of preemptive patenting the problems associated with financial contracting can eliminate leapfrogging.

The rest of the paper is structured as follows: The next paragraph presents the model. In section 3, the Nash Equilibrium of the game without any financial restrictions is characterized. Section 4 introduces moral hazard and financial restrictions into the model. This can change the conclusions on the characteristics of the Nash equilibrium of the game (section 5). Section 6 extends this result to a dynamic growth model in the spirit of Aghion and Howitt (1992). The final section concludes.

2 The model

The model is a simple static equivalent of Aghion and Howitt (1992).There are two firms and two periods. One of the firms, the incumbent, is already in the market making a monopoly profit π_1. The other firm, called the challenger henceforth, initially produces nothing. During period one they both race for some cost reducing innovation which would bring down the constant marginal cost of production from the initial high level $\bar{c}$ to a lower level $\underline{c}$. The innovation is drastic, meaning that $p^m(\underline{c}) < \bar{c}$ or in words: if the successful innovator sets its monopoly price, the previous producer is no more able to compete[3]. Let π_2 denote the profit of the producer with marginal costs $\underline{c}$ setting his monopoly price $p^m(\underline{c})$ in period two. Let $\pi_2 > \pi_1 > 0$.
Research is taking place during the first period. Production with the superior technology cannot begin before the second period. If one of the firms is successful alone, then it receives a patent for the rest of all (model-) time. If nobody is successful in the research lab the incumbent can still produce with his high-cost technology. If both competitors are successful it shall be assumed that the innovation is treated as commonable and hence not patentable. Should this happen, the players engage in Bertrand competition and make zero profits.[4]

Remark 1 *Patent races as well as the new growth theories are usually modelled*

[3]In case the incumbent is the winner, there are two possible interpretations: One can think of the marginal production costs of the challenger of being either infinity or $\bar{c}$, with both assumptions yielding the same result. In the second case, there is a spillover assumption present, meaning that past innovations are immediately common knowledge.

[4]The purpose of the assumption is explained in the remark. However, one could equally well assume that the patent agency assigns the patent randomly - by flipping a fair coin - should more than one player be successful at the same time. In this case both players would assign expected value $p_c p_I \frac{\pi_2}{2}$ to this event. The key insight is that this influences their incentives symmetrically. All conclusions of the paper hinge on forces that influence the players' incentives asymmetrically. Thus, the alternative formulation would add complexity without increasing the generality of the arguments.

in continuous time. This has the advantage that the event of both players "winning" the race at the same time is of measure zero. Therefore it is accounted for in the payoff functions only with probability zero. The Bertrand assumption ensures, that the event "both win" receives the same "weight" in the present static formulation as it does in continuous time.

The gross payoffs of the two competitors are thus given by the following matrix: All variables with subscript c denote the challengers choices while subscript I stands for Incumbent.

Event	**probability**	**Chall's Payoff**	**Inc's Payoff**
both fail	$(1-p_c)(1-p_I)$	0	π_1
Inc. wins	$(1-p_c)p_I$	0	π_2
Chall. wins	$p_c(1-p_I)$	π_2	0
both "win"	$p_c p_I$	0	0

Research technologies: Decisions about research are once and for all. In order to be successful in the research lab innovators have to spend money I (invest) and effort e. Success probabilities and inputs are linked by the following Cobb-Douglas technology:

$$p_j = b e_j^{\alpha} I_j^{1-\alpha}; j = c, I; \alpha \in [0,1]; e, I \geq 0. \quad (1)$$

The interest rate (opportunity cost of funds) is zero. Effort is privately costly. Spending effort e generates nonmonetary costs $\frac{e^2}{2}$. b is a scale factor. (see below) The given situation corresponds to a noncooperative game in which the challenger and the incumbent as strategic players choose research success probabilities (and associated inputs). As a first benchmark, the game will be analyzed under the assumption that both players have deep pockets and moral hazard is not an issue. The payoff functions in this game are then given by 2 for the challenger and 3 for the incumbent:

$$b e_c^{\alpha} I_c^{1-\alpha}(1-p_I)\pi_2 - I_c - \frac{e_c^2}{2} \quad (2)$$

$$(1-p_c)\pi_1 + b e_I^{\alpha} I_I^{1-\alpha}(1-p_c)(\pi_2 - \pi_1) - I_I - \frac{e_I^2}{2} \quad (3)$$

Proposition 1 *The strategic game between the challenger and the incumbent, summarized by the payoff functions 2 and 3 has a Nash equilibrium in pure strategies.*

Proof: A success *probability* must satisfy the restriction $p_j \leq 1; j = c, I$. Let $\lambda_j; j = c, I$ be the multiplier on this constraint and let δ_I^e and δ_I^I be the multipliers on the constraints $e_I, I_I \geq 0$. The solution to the optimization problem of the incumbent must satisfy the Kuhn Tucker necessary conditions:

$$Foc_e : (1-\lambda_I)b\alpha e_I^{\alpha-1}I_I^{1-\alpha}(1-p_c)(\pi_2-\pi_1) - e_I + \delta_I^e \overset{!}{=} 0 \tag{4}$$

$$Foc_I : (1-\lambda_I)b(1-\alpha)e_I^{\alpha}I_I^{-\alpha}(1-p_c)(\pi_2-\pi_1) - 1 + \delta_I^I \overset{!}{=} 0 \tag{5}$$

$$\delta_I^I I_I = \delta_I^e e_I = \lambda_I(be_c^{\alpha}I_c^{1-\alpha} - 1) \overset{!}{=} 0 \tag{6}$$

Solving these equations (and the analogous ones for the challenger), gives the best response function of the incumbent and the challenger, respectively

$$p_I = \min\left\{\alpha(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} b^{\frac{2}{\alpha}} (\pi_2-\pi_1)^{\frac{2-\alpha}{\alpha}} (1-p_c)^{\frac{2-\alpha}{\alpha}}, 1\right\} \tag{7}$$

$$p_c = \min\left\{\alpha(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} b^{\frac{2}{\alpha}} \pi_2^{\frac{2-\alpha}{\alpha}} (1-p_I)^{\frac{2-\alpha}{\alpha}}, 1\right\} \tag{8}$$

which define a continuous map $[0,1]^2 \to [0,1]^2$. This map must have a fixed point: strategy sets are nonempty compact convex subsets of R^2, the payoff functions are continuous in the opponents actions and concave in the own actions. Hence theorem 1.2 Fudenberg and Tirole (1995)applies.[5]■

3 Properties of the Nash-equilibrium

- Assumption *A1*: $b < \frac{1}{\pi_2}; \pi_2 - \pi_1 > 2; \pi_1 > 0$.
- *Notation*: To save on space, let henceforth $\theta := \alpha(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} b^{\frac{2}{\alpha}}$; $\epsilon := \frac{2-\alpha}{\alpha}$; $c_c := \theta\pi_2^{\epsilon}$; $c_I := \theta(\pi_2-\pi_1)^{\epsilon}$; $\sigma := \frac{\pi_2-\pi_1}{\pi_2}$; $\mu := \sigma^{\epsilon}$.

  Note that $\pi_1 > 0 \Rightarrow \sigma < 1 \Rightarrow \mu < 1 \Rightarrow c_I = \mu c_c < c_c$.

Remark 2 *A1 serves two purposes: First, it excludes boundary solutions in 7 and 8, hence $0 \le p_j^*(p_i) < 1; i,j = I,c; i \neq j$. Second, it ensures (see lemma 1 below) uniqueness of the Nash equilibrium. It is important to note, that A1 is a sufficient not a necessary condition to reach these ends. Therefore the following results can reasonably be expected to hold for a broader set of parameters. Heuristically, restricting b to be small, just means that however hard you work, you can never be sure to reach success.*

In view of the simplified notation, the best response functions can now be expressed as

$$p_I = c_I(1-p_c)^{\epsilon} \tag{9}$$

[5]The theorem is due to Debreu (1952), Glicksberg (1952) and Fan (1952). See Fudenberg and Tirole (1995) for the exact references.

$$p_c = c_c(1 - p_I)^\epsilon \quad (10)$$

Appendix A establishes, that $A1 \Rightarrow 1 > c_c, c_I$. Note also that $c_c > c_I$! But then, the solutions to the system 9 and 10 define the fixed point(s) of a continuous map of $[0, c_c]^2$ into $[0, c_c]^2$.

Lemma 1 *Under Assumption A1 the Nash equilibrium of the simultaneous move game is unique.*

Proof: According to system 9 and 10 Nash-equilibria of the game are solutions to[6]

$$p_I = c_I(1 - c_c(1 - p_I)^\epsilon)^\epsilon \quad (11)$$

Uniqueness requires that the function $f = c_I(1 - c_c(1 - p_I)^\epsilon)^\epsilon$ has only one fixed point. Consider the slope of f :

$$\frac{\partial f}{\partial p_I} = (1 - c_c(1 - p_I)^\epsilon)^{\epsilon-1}(1 - p_I)^{\epsilon-1}\epsilon c_c \epsilon c_I$$

A sufficient condition for the uniqueness conclusion is that the right hand side of this equation is smaller than one everywhere. This is what we now show: Proceeding term by term, observe first that $p_I \in [0, c_c] \Rightarrow \{1 - c_c(1 - p_I)^\epsilon\} \in [1 - c_c, 1 - c_c(1 - c_c)^\epsilon]$. Hence $\{1 - c_c(1 - p_I)^\epsilon\} < 1, \forall p_I \in [0, c_c]$. Together with $\epsilon - 1 > 0$ this implies that $(1 - c_c(1 - p_I)^\epsilon)^{\epsilon-1} < 1$.
Next, observe that $p_I \in [0, c_c] \Rightarrow (1 - p_I)^{\epsilon-1} < 1$. Appendix B finally provides a proof that $A1 \Rightarrow \epsilon c_c, \epsilon c_I < 1; \forall \alpha \in [0, 1]$. □

An analysis of the game without a restriction on a unique equilibrium would be interesting in its own light. The restriction on uniqueness will be discussed right after Proposition 2 below. Throughout the analysis, we make repeated use of a result on monotone comparative statics, which shall therefore be restated here for completeness:

Lemma 2 *Let $f(p, \mu) : [0, c_c] \times M \to R$, where M is any partially ordered set and where $f(0, \mu) \geq 0$ and $f(c_c, \mu) \leq 0$. Suppose that for all $\mu \in M$, f is continuous in p. (Then there exists a solution to the equation $f(p, \mu) = 0$.) $p_L(\mu) \equiv \inf \{p \,|\, f(p, \mu) \leq 0\}$ is the lowest solution of $f(p, \mu) = 0$ and $p_H(\mu) \equiv \sup \{p \,|\, f(p, \mu) \geq 0\}$ is the highest solution. Suppose further that for all $p \in [0, c_c]$, f is monotone nondecreasing in μ. Then $p_L(\mu)$ and $p_H(\mu)$ are monotone nondecreasing for all $\mu \in M$. If f is strictly increasing in μ, then $p_L(\mu)$ and $p_H(\mu)$ are strictly increasing.*

Proof: Milgrom and Roberts (1994) Theorem 1.

[6] Of course, it does not matter, whether we search for a fixed point in p_I or p_c since both formulations contain the same information: once the solution in p_I is found, the solution in p_c can be read directly from the reaction function 10.

With these two lemmas in hands, we are finally ready to state the central result on the structure of the Nash equilibrium when both players have deep pockets.[7]

Proposition 2 *In the Nash Equilibrium of the simultaneous move game, the challenger invests more and exerts more effort and thus has a higher success probability than the incumbent.*

Proof: The proof will proceed in three steps: Step (i) verifies that all conditions in lemma 2 are satisfied. In step (ii) we will characterize the equilibrium for the case $\pi_1 = 0$ ($\mu = 1$). Finally in step (iii) we will characterize the way the equilibrium changes if $\pi_1 > 0$ ($\mu < 1$).
Step (i): Consider the function

$$g(p_I, \mu) = \mu c_c(1 - c_c(1 - p_I)^{\epsilon})^{\epsilon} - p_I \tag{12}$$

We have

$$g(0, \mu) = \mu c_c(1 - c_c)^{\epsilon} \geq 0$$

and

$$g(c_c, \mu) = \mu c_c(1 - c_c(1 - c_c)^{\epsilon})^{\epsilon} - c_c \leq 0$$

For all $\mu \in M$ g is continuous in p. We already know, that a solution must exist. Note that uniqueness implies that

$$\sup \{p \,|\, f(p, \mu) \geq 0\} \equiv p_H(\mu) = p_L(\mu) \equiv \inf \{p \,|\, f(p, \mu) \leq 0\}$$

This verifies, that g has the desired properties.$\square$
Step (ii): Suppose, that $\pi_1 = 0$ and hence($\mu(\pi_1 = 0) = 1$). Then the success probability of the incumbent in the unique equilibrium of the game must satisfy

$$g(p_I, 1) = 1c_c(1 - c_c(1 - p_I)^{\epsilon})^{\epsilon} - p_I = 0$$

Clearly, one could find the equilibrium equivalently by looking for a fixed point in p_c. This fixed point must satisfy

$$h(p_c, 1) = c_c(1 - 1c_c(1 - p_c)^{\epsilon})^{\epsilon} - p_c = 0$$

Let $\hat{p}_c$ be a solution to $h(p_c, 1) = 0$ and let $\hat{p}_I$ be a solution to $g(p_I, 1) = 0$. Suppose then that $\hat{p}_c \neq \hat{p}_I$. Since $h(p_c, 1) \equiv g(p_I, 1)$ this contradicts lemma 1. Hence $\hat{p}_c = \hat{p}_I$.$\square$
Consider finally the generalized fixed point map, parameterized by $\mu(\pi_1)$:

$$g(p_I, \mu) = \mu(\pi_1)c_c(1 - c_c(1 - p_I)^{\epsilon})^{\epsilon} - p_I$$

Since $\pi_1 > 0 \Rightarrow \mu < 1$ we know that $g(p_I, \mu(\pi_1)) < g(p_I, 1)$ for all $\pi_1 > 0$. By lemma 1 and 2, we know then that $\hat{p}_I(\mu) < \hat{p}_I(1)$ for all $\pi_1 > 0$. Since step (ii)

[7] This result is known very well from Reinganum (1983). As the method of the proof will be used repeatedly, the result is restated at this point.

established symmetry of the Nash equilibrium for $\pi_1 = 0$ and by the fact that success probabilities are strategic substitutes it follows that $\hat{p}_c(\mu) > \hat{p}_I(\mu)$ for all $\pi_1 > 0$.■

Because the incumbent would like to rest on his laurels, he ends up investing less, spending less effort and he therefore also produces a lower success probability than his rival does. This is because the only way to get in the position to make the higher profit π_2 is to destroy his own monopoly and hence loose π_1. In view of this simple logic, it might seem odd, to impose such strong conditions as Assumption 1 to ensure uniqueness. However, without uniqueness one can go no further than the general statement in lemma 2: the lowest and the highest fixed point in p_I will both be lower in a game with $\pi_1 > 0$ compared to a game with $\pi_1 = 0$. Thus, the conclusion that the initial condition $\pi_1 > 0$ puts the incumbent at a strategic disadvantage in the R&D race for a drastic product innovation is quite robust and general. However, from this one *cannot* conclude, that *all* Nash equilibria are asymmetric in favor of the challenger. Since it is this asymmetry prediction of the growth and patent race literature that the paper targets, I have chosen to sacrifice generality in favor of clarity of the results.
The asymmetry result has a close parallel in the Schumpeter growth theory. At the heart of what has been termed the creative destruction mechanism lies the replacement or Arrow effect: because incumbents already make monopoly profits, their incentives to conduct research are always lower than the outsiders' incentives. To make matters simple, models like Aghion and Howitt (1992) or Grossman and Helpman (1991) and almost every other paper in the field, assume a linear (or constant returns to scale) research technology. Together with a free entry condition into the research business this gives a nice Arbitrage condition, which has to be fulfilled at each instant in time. The price of the only resource of the economy, labor, is bid up in the general equilibrium such that incumbents would make losses, were they to do research. All in all this produces the well known bang bang result that only the challengers invest money in research and that it is only a question of when and not of if the incumbent loses his business. While in Aghion and Howitt it is sure that some challenger will win the race, in the present model this holds only true "on average". This difference in results has mainly two reasons: the partial equilibrium perspective of the present model and the strictly convex effort costs. Although the research technology here is essentially the same as in Aghion and Howitt (1992) - a constant returns to scale function - its implications are strikingly different in the present partial equilibrium model: as long as the rival does not win with probability one the optimizing choices of effort and investment will always be strictly positive, however small they turn out to be. Therefore the incumbent's success probability is bounded away from zero as long as the challenger does not choose to operate at success probability one. Section 6 relaxes both assumptions. There is a third difference between the models: the number of entrants. The free entry assumption in the growth literature of course assumes, that there is an infinite amount of challengers lurking around while the present paper has so far been working with only one of them. However, it is immediate to generalize proposition 2 to

a game with many entrants and one incumbent firm:

Proposition 3 *(i)The extended simultaneous move game with many entrants has a unique Nash equilibrium in pure strategies in which each challenger chooses the same probability. (ii)The success probability of each challenger is bigger than the success probability of the Incumbent. (iii) As the number of challengers goes out of bounds, p_I goes to zero.*

Proof:
(i) Payoff functions are still as required in the proof of proposition 1. Thus a Nash Existence in pure strategies exists. Consider the best response functions of any two representative (the h_{th} and the i_{th}) out of n Challengers:

$$p_{ch} = c_c(1-p_{ci})^\epsilon \prod_{\substack{j=1 \\ j\neq i,h}}^{n} (1-p_{cj})^\epsilon (1-p_I)^\epsilon \tag{13}$$

$$p_{ci} = c_c(1-p_{ch})^\epsilon \prod_{\substack{j=1 \\ j\neq i,h}}^{n} (1-p_{cj})^\epsilon (1-p_I)^\epsilon \tag{14}$$

Let $F := \prod_{\substack{j=1 \\ j\neq i,h}}^{n} (1-p_{cj})^\epsilon (1-p_I)^\epsilon$ and let $p_{cj}, j = 1, ..., n; j \neq i, h$ and p_I be fixed and exogenous for the moment. Under this restriction p_{ci} must satisfy

$$p_{ci} = c_c(1 - c_c(1-p_{ci})^\epsilon F)^\epsilon F$$

while p_{ch} must satisfy

$$p_{ch} = c_c(1 - c_c(1-p_{ch})^\epsilon F)^\epsilon F$$

Let $\hat{p}_{ci}$ and $\hat{p}_{ch}$ be solutions of these equations. But then, since $F < 1$ $\hat{p}_{ci} = \hat{p}_{ch}$ again by lemma 1. Since i, h where picked arbitrarily, this must be true for any two of the challengers, hence for all of them. Furthermore since this is true for any $F < 1$ it must also be true for $F = \prod_{\substack{j=1 \\ j\neq i,h}}^{n} (1-\hat{p}_{cj})^\epsilon (1-\hat{p}_I)^\epsilon$, those values that are chosen in equilibrium. This proves $p_{ci} = p_{ch} = \bar{p}_c; i, h = 1, ..., n.\square$
(ii) Consider now the best response functions of any challenger i and the Incumbent, again holding fixed the choices of all other challengers at the endogenously determined equilibrium values:

$$p_I = c_I(1-\bar{p}_c)^{\epsilon(n-1)}(1-p_{ci})^\epsilon \tag{15}$$

$$p_{ci} = c_c(1-\bar{p}_c)^{\epsilon(n-1)}(1-p_I)^\epsilon \tag{16}$$

The fact that $\hat{p}_I < \hat{p}_{ci} = \bar{p}_c$ then follows from the proof of Proposition 2.$\square$
(iii) Take limits in 15 and 16 as $n \to \infty$, we see, that p_I hits the "zero line" sooner than p_c, because $c_I < c_c$ and $(1-p_c)^\epsilon < (1-p_I)^\epsilon$ (because $p_I < p_c$). For large enough n the incumbent's success probability is then approximately zero while the challengers' success probabilities are still strictly positive.■

4 Moral Hazard: The Impact of Outside Financing Needs of the Challenger

The last section treated both the incumbent and the challenger(s) on equal footing: Both were assumed to have deep pockets or - equivalently - capital markets were assumed to be perfect. However, if a world of imperfect capital markets is considered, it turns out that the model has a natural asymmetry built in: while the incumbent can finance his investments out of retained earnings, the challenger cannot. Assuming further on that the challenger has no wealth at all he has to contract with outside financiers for financial resources. The following set of assumptions is imposed in the sequel:

- Assumption *A2*: (deep pocket) $\pi_1 > \alpha(1-\alpha)^{\frac{2-\alpha}{\alpha}} \{b(\pi_2 - \pi_1)\}^{\frac{2-\alpha}{\alpha}}$

A2 says that the endogenously chosen height of investment is in all cases, i.e. even if the challenger should abstain from doing research altogether, smaller than the amount of retained earnings the incumbent has.

- Assumption *A3*: (Information) Effort choice is not observable and hence not contractible. Apart from this everything else is observable and contractible.

Thus the source of moral hazard is the challenger's effort choice. Note however, that there is no other source of moral hazard. In particular, it can be verified costlessly ex post, wether there was success or not.

- Assumption *A4:* (degree of competition) Financial markets are perfectly competitive: Financiers make zero profits in equilibrium.

- **The timing of events:**

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$t = 0$	$t = \frac{1}{2}$	$t = 1$	$t = 2$
Incumbent produces	Financiers offer contracts	contracts chosen, Research phase (simultaneous)	the winner(s) produce(s)

Remark 3 *Financiers, when they offer contracts, have to anticipate the outcome in $t = 1$. Thanks to the uniqueness of the equilibrium this poses no problems.*

4.1 First best

The first best can be achieved if the financier is able to observe the effort choice of the challenger. Optimal contracts can and will then be contingent on this effort choice. Of course the contract must maximize the joint surplus. Optimal effort choice and investment levels will therefore be the same as if the challenger owned his business and would therefore be the only residual claimant.

Remark 4 *Having a third party, i.e. the financier, brings an element of sequentiality into the game and hence requires a new solution concept for the game. Throughout the analysis we will use weak perfect Bayesian equilibrium as our solution concept. For expositional reasons the precise meaning of the concept in the given game will be discussed in the section on second best contracts. Since we are currently in a world of perfectly enforceable contracts, we can treat w.l.o.g. the financier and the challenger as effectively one agent.*[8]*. Then, in the simultaneous move game between the Incumbent and this "super-agent", Nash equilibrium has enough bite.*

We already know from the last section what the optimum will look like. It therefore suffices here to state the contracts that will sustain this equilibrium. These contracts consist of:

1. an initial amount of money I, the financier gives to the challenger.[9]
2. an amount of effort e, the challenger must exert.
3. a repayment rule, contingent on the challenger's effort choice.

Proposition 4 *An optimal contract under complete information between challenger and financier specifies:* (1)$I^* = \alpha(1-\alpha)^{\frac{2-\alpha}{\alpha}} \{b\pi_2\}^{\frac{2}{\alpha}} (1-p_I)^{\frac{2}{\alpha}}$, (2)$e^* = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \{b\pi_2\}^{\frac{1}{\alpha}} (1-p_I)^{\frac{1}{\alpha}}$
(3)

$$R_{e^*} = \begin{cases} (1-\alpha)\pi_2 & \textit{in case of success} \\ 0 & \textit{in case of failure} \end{cases} \; if \quad e = e^*$$

$$R_e = \begin{cases} \pi_2 & \textit{in case of success} \\ 0 & \textit{in case of failure} \end{cases} \; if \quad e \neq e^*$$

Proof: Under complete information, an optimal contract must maximize social surplus. This determines e^* and I^*. The rule $R(.)$ is chosen such that the challenger optimally chooses to exert e^* : Since he has no wealth, the harshest punishment to impose on him is to give him nothing in case of $e \neq e^*$. Otherwise, if $e = e^*$, financier and challenger get shares of profits $\left\{\beta^{fb}, 1-\beta^{fb}\right\}$ in case of success. β^{fb} is pinned down by the zero profit condition of financiers:

$$p_c^*(p_I)\{1-p_I\}\beta^{fb}\pi_2 = I^*(p_I)$$

where $p_c^*(p_I)$ is given by 10, while $I^*(p_I)$ is the investment level agreed upon in the contract. Solve this for β^{fb} to conclude that $\beta^{fb} = 1-\alpha$.■[10]

[8] Although a schizophrenic one, because he contracts with himself.

[9] Upon assumption, the challenger will not walk away with the money but will invest it.

[10] The repayment is expressed as a fraction of realized profits. However, it should be noted that in the simple setting of the present paper - two realizations of the return, either 0 or π_2 - debt and equity cannot be distinguished.

Part (1) and (2) are easily understood. The contract just says that the two parties agree to behave as if they were one profit maximizing agent, whose behavior was already derived in the last section. Due to the risk neutrality of the parties all sharing rules with the same expected payoffs for the parties are equivalent for them. The one stated in the proposition is the one with the harshest punishment in case of failure s.t. the limited wealth of the challenger. Because in equilibrium the sharing rule R_e is not directly payoff relevant the financier's individual rationality constraint is exactly binding. Finally, the result on ex post payments, $\beta^{fb} = 1 - \alpha$ is well known for example from macro theory: with a Cobb-Douglas technology, the capitalist will get a share of total output, which corresponds to the elasticity of total output with respect to his input. This in turn is exactly what the exponent $1 - \alpha$ measures.[11] Note however, that the result $\beta^{fb} = 1 - \alpha$ does not mean, that the parties share the profits according to a completely inflexible rule, i.e. independently of what the Incumbent does. Ex ante, the investment of the financier varies inversely with the success probability of the Incumbent. Therefore the price of money is the higher, the tougher the Incumbents behavior in the research lab[12].

4.2 Unobservable effort: Second best contracts

Matters get really interesting, when the effort choice of the challenger is not observable. The new informational assumptions require a new concept of equilibrium of the game:

Definition *A profile of strategies and system of beliefs (s, m) is a perfect Bayesian equilibrium in extensive form game Γ if it has the following properties: (i) The strategy profile s is sequentially rational given belief system m. (ii) The system of beliefs is derived from strategy profile s through Bayes' rule whenever possible. That is, if for information set h $prob(h\,|s) > 0$, then $m(x) = \frac{prob(x|s)}{prob(h|s)}$ for all $x \in h$.*

Remark 5 *The order of play is given by the time line. However, the game can be analyzed as if the Incumbent was given the first move, as long as nobody observes this move. Then, the financier(s) and the challenger get the move and act sequentially.*

Perfect Bayesian equilibrium requires for the given game, that:

[11]Note that individual rationality of the agent has to be fulfilled too. This is assumed implicitly. It can always be guaranteed, it the reservation utility of the challenger is small enough.

[12]This mechanism has been used by Fudenberg and Tirole (1985) to explain "Predation without Reputation". In the CSV-setup a la Gale and Hellwig (1985) they use, aggressive behaviour of a rich rival reduces the amount of equity of his opponent. To finance a necessary investment of fixed size he therefore has to borrow a bigger amount. C.p. this increases the likelyhood of bankruptcy and consequently the interest rate charged. This can go so far that the financially constrained firm abstains from the business altogether.

1. The Incumbents choice of success probability must be optimal, given the financier‘s and the challenger‘s subsequent strategy.

2. For each belief the financier has as to which success probability the Incumbent has chosen, his offer of contract must be optimal for him, given his anticipation of the challenger‘s subsequent strategy.

3. For each belief the challenger has which success probability the Incumbent has chosen, his choice of contract must be optimal given his anticipation of his own subsequent strategy (i.e. choice of effort).

The incumbents problem is exactly the same as before. Hence his best response to each choice of success probability of his rival is still given by 9.
Contracts between the financiers can no more be contingent on effort choice, since misbehavior on the part of the challenger cannot be distinguished from bad luck ex post. Therefore, under this new informational assumption, contracts specify:

1. An amount of money, D, the financier hands over to the challenger.

2. A repayment rule, which is *not* contingent on effort choice.

While in the last section, there was only one contract with a unique investment level, there is no reason to believe a priori, that this will also be true in the present context. Therefore, the repayment rule has to be generalized to:

$$R(D) = \begin{cases} \beta(D)\pi_2 & \text{in case of success} \\ 0 & \text{in case of failure} \end{cases}$$

where the ex post shares are now allowed to vary with the ex ante chosen investment level. Any contract can then be summarized by a tuple $\{D, \beta(D)\}$.[13]

Remark 6 *Financiers effectively are in a Bertrand competition with each other. As usual this forces them to offer a contract that maximizes the challengers expected payoff s.t. an Individual Rationality constraint for themselves. So the analysis has to focus on what contract is optimal for the challenger given that weak perfect Bayesian equilibrium strategies are played subsequently. We assume that the financiers make take-it-or-leave-it offers and the challenger either accepts or rejects them.*

Lemma 3 *The contracts $\{D, \beta(D)\}$ financiers offer satisfy:*

$$\varphi D^{\frac{-\alpha}{2-\alpha}} \{1 - \beta(D)\}^{\frac{\alpha}{2-\alpha}} \beta(D) - 1 = 0 \tag{17}$$

[13] Since the underlying distribution of events is a two point distribution, it is easy to see, that the repayment scheme in an *optimal* contract must look like this. Moreover, it is well known that we cannot distinguish between debt and equity in this simple world. Expressing the repayment as a share of profits made is therefore without loss of generality.

with

$$\varphi := (1 - p_I) b \left\{ \alpha b (1 - p_I) \pi_2 \right\}^{\frac{\alpha}{2-\alpha}} \pi_2$$

Proof: When offering a contract, financiers must have a belief about p_I. Since players are restricted to pure strategies, this belief must put probability mass 1 on some p_I. For any belief financiers might have when they offer a contract, they must anticipate that once a contract has been chosen, the challenger's only remaining degree of freedom is the choice of effort. This must maximize his expected payoff, given the contract, or more formally:

$$\hat{e} = \arg\max \left\{ (1 - p_I) b e^{\alpha} D^{1-\alpha} (1 - \beta(D)) \pi_2 - \frac{e^2}{2} \right\} \tag{18}$$

$$\hat{e}(\beta(D), D) = \left\{ \alpha b D^{1-\alpha} (1 - p_I)(1 - \beta(D)) \pi_2 \right\}^{\frac{1}{2-\alpha}} \tag{19}$$

Moreover, the contracts they offer allow them exactly to break even on expectation if and only if the challenger chooses the incentive compatible effort level, i.e. "stays on the equilibrium path". Technically this means:

$$(1 - p_I) b \hat{e}^{\alpha} D^{1-\alpha} \beta(D) \pi_2 - D = 0 \tag{20}$$

To offer contracts, which are more favorable to them, is meaningless due to the Bertrand assumption. The combination of 18 and 20 gives the expression in the lemma.■

Thus, Incentive Compatibility of effort choice works as an effective constraint on the set of feasible contracts, financiers should offer, if they want to break even on average.
Although 17 summarizes all relevant information on contracts, it has the disadvantage of being only an implicit relation $\beta(D)$. To calculate closed form solutions[14] it is more convenient to work with the function $D(\cdot)$ *s.t.*

$$D(\beta) = \varphi^{\frac{2-\alpha}{\alpha}} \left\{ 1 - \beta \right\} \beta^{\frac{2-\alpha}{\alpha}} \tag{21}$$

This is an explicit function $D(\beta)$, with the following properties: $D(\beta) \geqslant 0; \forall \beta \in [0,1]; D(0) = D(1) = 0; \frac{\partial D}{\partial \beta}(\frac{2-\alpha}{2}) = \frac{\partial^2 D}{\partial \beta^2}(1-\alpha) = 0$; as shown in figure 1:

The nonlinearity present in $D(\beta)$ deserves some economic explanation. Why isn't it the case that financiers offer ever higher amounts of money in return for an ever higher ex post share in the venture? Initially, i.e. for low values of β, this is the case. In this range, a higher β makes them willing to offer a higher amount of outside finance (and investment), since this still has the unambiguous effect of raising the probability of success. However, to reach success, it takes investment of financial resources as well as the efforts of the challenger. Inspection of $\hat{e}(\beta)$ shows that $\hat{e} = 0$ for $\beta = 1$. Why should the challenger work hard if he ends

[14]This causes no problems. See ftn. 16.

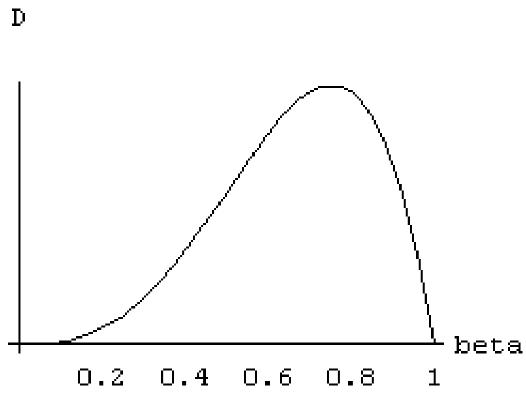

Figure 1:

up with nothing in his hands anyway? This logic tells us that $\frac{\partial e^{Ic}(\beta)}{\partial \beta}$ changes it's sign somewhere.[15] It turns out, that the outside finance capacity is reached at $\beta = \frac{2-\alpha}{2}$: $D\left(\frac{2-\alpha}{2}\right)$ is the maximum amount of finance investors are willing to provide: there is no contract with a $D > D\left(\frac{2-\alpha}{2}\right)$ which is both incentive compatible for the challenger and allows financiers to break even.
Not all contracts on the locus in the figure are relevant: consider any contract on the locus with $\beta > \frac{2-\alpha}{2}$: For each tuple $\left\{\acute{\beta}, D(\acute{\beta})\right\}$, $\acute{\beta} \in (\frac{2-\alpha}{2}, 1]$, $\exists$ a tuple $\left\{D(\tilde{\beta}), \tilde{\beta}\right\}$, such that $D(\tilde{\beta}) = D(\acute{\beta})$ but $\tilde{\beta} < \acute{\beta}$. A financier offering contracts of the form $\left\{\acute{\beta}, D(\acute{\beta})\right\}$ will thus never be able to attract any clients[16]. Having identified the locus all feasible contracts must lie on, one can finally solve for the contract which is chosen in equilibrium.

Proposition 5 *The second best optimal contract for the challenger is given by* $\left\{\beta^{sb}, D(\beta^{sb})\right\}$ *with* $\beta^{sb} = 1 - \alpha$.

Proof: Financiers, when offering contracts, take into account all subsequent effects on the challengers effort choice, knowing that effort will be chosen in an incentive compatible way. From 18 and 21 we know, that $\hat{e}$ and D are ultimately functions of β alone. Therefore, the optimal contract for the challenger is the solution to[17]:

$$\max_{\beta}(1 - p_I)\pi_2\{p_c(e(\beta), D(\beta))(1 - \beta)\} - C(e(\beta))$$

[15]see below.

[16]The function $D(\beta)$ is one-to-one only in the range $\beta \in [0, \frac{2-\alpha}{2}]$. Working with $D(\beta)$ or $\beta(D)$ is thus equivalent iff the optimal contract has a $\beta \leq \frac{2-\alpha}{2}$. See Proposition 5.

[17]This problem is not globally concave for most parameter values. However, it can be verified, that it is quasi-concave and that at the optimum the second order condition for a maximum is fulfilled.

From the envelope theorem - because effort choice must be optimal - it follows that

$$\left\{(1-p_I)\pi_2(1-\beta)\frac{\partial p_c}{\partial e} - \frac{\partial C}{\partial e}\right\}\frac{\partial e}{\partial \beta} = 0$$

Therefore the FOC to the problem is

$$(1-p_I)\pi_2\left\{(1-\beta)\frac{\partial p_c}{\partial D}\frac{\partial D}{\partial \beta} - p_c(e(\beta), D(\beta))\right\} \stackrel{!}{=} 0$$

A few algebraic transformations then show that

$$(1-\beta)(1-\alpha)D`(\beta) = D(\beta) \tag{22}$$

$D`(\beta)$ is straightforward to calculate from 21. Simplification then delivers the stated result that $\beta^{sb} = 1-\alpha$.■

We can finally characterize the weak perfect Bayesian equilibrium of the game (apart from the exact value of p_I^*, which will be derived in the next section):

Proposition 6 *The incumbent chooses* p_I^*. *The financiers belief is given by* $m_f(p_I^*) = 1$ *and he offers the challenger the contract* $\{\beta = 1-\alpha,\ D(\beta = 1-\alpha) = (1-p_I^*)^{\frac{2}{\alpha}}(b\pi_2)^{\frac{2}{\alpha}}\alpha^2(1-\alpha)^{\frac{2-\alpha}{\alpha}}\}$. *The challengers belief is given by* $m_c(p_I^*) = 1$. *He accepts and exerts effort* $e(1-\alpha) = (1-p_I)^{\frac{1}{\alpha}}(b\pi_2)^{\frac{1}{\alpha}}\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}}$. *Off the equilibrium path, the challenger is allowed to have* any *belief and subsequent sequentially rational strategy.*

Proof: Since players are restricted to pure strategies and the challenger as well as the financier can calculate the unique outcome of the game, we must have $m_c(p_I^*) = m_f(p_I^*) = 1$. The optimality of the contract and the effort level has already been proved. The fact that the challenger is allowed to have any belief and associated sequential rational strategy is due to the Bertrand assumption: financiers cannot make more than zero profits. But then they weakly prefer to stick to their equilibrium strategy no matter what the challenger should infer from a deviation on their part.■

There are two things remarkable about the result: First, the Bertrand competition between financiers destroys the usual vulnerability of the perfect Bayesian equilibria to crazy beliefs and strategies off the equilibrium path. This makes the stated results quite robust. Second, the result $\beta^{sb} = \beta^{fb}$ is remarkable. Ex post shares in the final pie are not distorted by asymmetric information; capital's share in output still corresponds to the elasticity of output with respect to financial investment. However, it is somehow clear and it will be shown formally in the next lemma that asymmetric information will not be without its costs.

5 Leapfrogging reconsidered

Intuitively, we associate the ability to finance out of retained earnings with some kind of strength. On the other hand, some notion of weakness is associated with the need to take on outside finance. The next lemma formally states, what exactly is to be understood by "weakness":

Lemma 4 *Fix any* p_I : *The best response of the challenger in the second best is always smaller than in the first best.*

Proof: From 18 and 21 one can easily derive the optimal values for $D(\cdot)$ and $e(\cdot)$ at $\beta = 1-\alpha : D(\beta = 1-\alpha) = (1-p_I)^{\frac{2}{\alpha}}(b\pi_2)^{\frac{2}{\alpha}}\alpha^2(1-\alpha)^{\frac{2-\alpha}{\alpha}}$ and $e(1-\alpha) = (1-p_I)^{\frac{1}{\alpha}}(b\pi_2)^{\frac{1}{\alpha}}\alpha^2(1-\alpha)^{\frac{1-\alpha}{\alpha}}$. Plug these values into 1 and simplify to get

$$p_c = \alpha\theta\pi_2^{\epsilon}(1-p_I)^{\epsilon}$$

Since $\alpha \in [0,1]$ this proves the claim.□

Remark 7 *It is not obvious how to derive this reaction function: While it is clear that both the financier and the challenger can calculate the outcome of the game, it is not clear that they can coordinate their beliefs for any level of* p_I*. This is however, the implicit assumption in the above derivation.*

Financial contracting thus results (unsurprisingly) in a reduction of investment. This result emerges because the investor foresees, that an agent, who has to share the fruits of his efforts with someone else, is reluctant to exert as much effort as in the first best situation. Therefore the per Dollar price charged for each unit of investment has to be higher. But this raises a question: A priori there would be two possible routes to achieve this end. One could either change, for any given amount of investment, the share of the investor in the final profit-pie. Or, one could leave the division of profits ex post unchanged and change the amount of finance ex ante. We already know from proposition 5, that the latter route is taken. An economic explanation for this observation is in order: The incentive compatible effort choice, given by 18 reaches it's maximum at $\beta^{sb} = 1-\alpha$.[18] Note that β^{sb} is located in the range, where 21 is still increasing in β. The challenger nevertheless chooses not to contract for a higher amount of finance in exchange for a higher β. Why? If he applied for more finance, the financier would recognize very well, that the challenger effectively commits himself to a *lower* level of effort thereby. Because the financier foresees any such opportunistic behavior on the part of the challenger, he has to demand a disproportionately higher share β in return for investments in excess of $D(1-\alpha)$, or in terms of 21, he offers an ever smaller amount of finance D in return for a higher β in the range $\beta \in [1-\alpha, 1]$. Technically this means, that 21 is convex in the range $[0, 1-\alpha]$ and concave thereafter. Eventually this goes so far, that

[18]The proof of this statement is obvious and therefore omitted.

the investor is reluctant to offer a higher amount of finance in exchange for a higher share: the outside finance capacity is reached at $\beta = \frac{2-\alpha}{2}$.
The reduction in all endogenous values is the more pronounced the smaller α is. What is the intuition for this result? A small α means, that effort has to be raised a lot if the success probability is to be affected significantly. Still, effort is one of the essential factors in the technology, as long as $\alpha > 0$. As explained above, the need to take on outside finance weakens the agents incentives to spend effort. If for example α is very close to one, only a tiny little bit of finance has to be taken on and the reduction in effort choice is only very small. If on the other hand α is very close to zero, then the optimal choice of investment relative to the optimal choice of effort given the cost structure will be rather big. Therefore the reduction the endogenous values will be more pronounced.

In short: Due to asymmetric information all choice variables of the challenger are reduced by the factor α. While in section 3, the incumbent was at a strategic disadvantage due to his starting position, the strategic position of the challenger is now weakened by his "shallow" pocket. There are thus now two countervailing forces at work in the model, making it worthwhile to reconsider the statement in Proposition 2. Recall that $\sigma = \frac{\pi_2 - \pi_1}{\pi_2}$ and $\mu := \sigma^\epsilon$.

Proposition 7 *If $\alpha > \mu$ then $p_{*c}(p_{*I}) > p_{*I}(p_{*c})$; if $\alpha < \mu$ then $p_{*c}(p_{*I}) < p_{*I}(p_{*c})$; finally if $\alpha = \mu$, then the Nash equilibrium is symmetric.*

Proof:
The system of equations that determines the Nash equilibrium of the game is given by:

$$p_c = \alpha\theta\pi_2^\epsilon(1 - p_I)^\epsilon$$

$$p_I = \mu\theta\pi_2^\epsilon(1 - p_c)^\epsilon$$

The rest of the proof is obvious since the same logic as in the proof of Proposition 2 applies.■

In a sense both players are a bit handicapped. The incumbent, because he wants to rest on his laurels, the challenger because he has to use outside finance with its induced agency costs. The relative magnitude of α and μ determines which of the two reaction functions is shifted more heavily inwards. In words the following mechanism is at work: holding fixed a value for α and π_2, the higher the profit the incumbent already has, the lazier he gets. The other way round: holding fixed a value for π_2 and μ, the heavier the impact of the financial restrictions on the behavior of the challenger, the likelier it gets that he ends up being beaten on average. There is thus a nontrivial interaction between financial and real factors present in the model. Again, it is of interest how the results change, if a game with many entrants is considered:

Proposition 8 *(i)The extended simultaneous move game with many entrants has a unique Nash equilibrium in pure strategies in which each challenger chooses the same probability. (ii)If $\alpha < \mu$ the success probability of each challenger is smaller than the success probability of the Incumbent. (iii) As the number of challengers goes out of bounds, p_c goes to zero.*

Proof: analogous to Proposition 3.

6 Implications for growth

6.1 The model

A complete treatment of a fully dynamic model is outside the scope of the present paper. The main results, however, have straight forward extensions in the dynamic formulation. Consider the basic version of the Aghion and Howitt (1992) growth model[19]: The economy is populated by a continuous mass L of workers with linear intertemporal preferences: $u(y) = \int_0^\infty y_\tau e^{-\rho\tau} d\tau$. Each of the workers is endowed with one unit flow of labor, so that L also equals the labor supply. In addition, every worker is endowed with the ability to exert effort, e, in potentially unbounded quantities. ρ is the discount rate. Output of the final good is produced with technology $y = Ax^\delta$, where $0 < \delta < 1$. One of the individuals, the entrepreneur, is initially the owner of the intermediate goods firm. All other individuals start without wealth. Innovations raise the efficiency parameter A by a constant factor $\gamma > 1$. The technology used in the production of the intermediate good is linear so that x also equals labor demand by that firm. The research technology is still given by 1. As the economy has only one resource, labor, *investing means hiring workers. The wage bill, however, has to be paid upfront.* Research costs C_t are now assumed to be linear in effort and labor input: investing an amount D_t generates costs $w_t D_t$, while spending effort e_t generates a cost $c_t e_t$.

Innovations raise the productivity of the intermediate goods sector. As in the previous part, these innovations are drastic. Once an innovation has been successful, the innovator faces the following static problem: the final goods sector will buy the intermediate good until its price equals its marginal value in production or $P_t = A_t \delta x^{\delta-1}$. The monopolist will then determine π_t and x_t such that

$$\pi_t = \max_x P_t(x)x - w_t x = \left(\frac{1-\delta}{\delta}\right) w_t x_t$$

[19]The present model uses only a slight modification of Aghion and Howitt (1992), therefore the description of the model is very brief. For a more extensive description of the model see Aghion and Howitt (1998) chapter 2.

and

$$x_t = \arg\max_x \left\{A_t \delta x^\delta - w_t x_t\right\} = \left\{\frac{\delta^2}{\frac{w_t}{A_t}}\right\}^{\frac{1}{1-\delta}}$$

The focus of the present section lies entirely on steady state behavior. In such a steady state all variables will grow at the same rate and the productivity adjusted wage rate, $\frac{w_t}{A_t}$, will be a constant, ϖ say. It is easy to see that in this case we will also have $\pi_t = A_t \pi(\varpi)$ and $x_t = x$. Assume, w.l.o.g. that $c_t = w_t$. Everything that is needed is that all variables grow with the same rate otherwise one of the inputs would shrink to zero over time. The index t denotes "model time": i.e. in model time, the length of the time interval between two subsequent innovations is 1. In real time, denoted by τ, of course this length is stochastic. Observe finally that there is only one investment opportunity in the economy. There is thus no possibility to save apart from the research project. Research is the only way to become wealthy!

6.2 Optimal research policies

Assume for the moment that the entrepreneur did not have to bother about competitors doing research as well. What would his preferred research policy look like? The owner manager controls the arrival rate of a Poisson process, p_τ. As is shown in Appendix C, linearity of costs $w_t e + w_t D_t$ and constant returns to scale together imply that the arrival rate is linear in D ($p_\tau = b\left(\frac{\alpha}{1-\alpha}\right)^\alpha D_\tau := \lambda D_\tau$) and that costs of research can be expressed as a linear function of D ($C_\tau = \frac{w_\tau}{1-\alpha} D_\tau$) alone. Therefore it is convenient to optimize over D directly rather than over p.
In the steady state all costs and values grow at the same rate. There is no other influence of time other than through the arrival of innovations. Therefore the research intensity is constant during time intervals between innovations. Also because costs and values grow at the same rate, the research intensity will turn out to be independent of time across time intervals.

Following these arguments the owner of the firm faces the following stochastic dynamic programming problem:

$$V(A_t) = \max_{D_t} \left\{ \begin{array}{c} \pi(A_t)\Delta\tau - C(A_t, D_t)\Delta\tau + \\ (1+\rho\Delta\tau)^{-1}\left[\lambda D_t \Delta\tau V(A_{t+1}) + (1-\lambda D_t \Delta\tau) V(A_t)\right] \end{array} \right\}$$

Take limits as $\Delta\tau \to 0$ and recognize that in a steady state $V(A_{t+1}) = \gamma V(A_t)$. Taking into account as well that the research intensity will be constant through time and that every term in the equation can be normalized by A_t we can write (with $\frac{V(A_t)}{A_t} = V(D)$)

$$V(D) = \frac{\pi - C(D)}{\rho - \lambda D(\gamma - 1)}$$

The value of a firm is simply given by operating profits net of research costs, discounted at a rate that lies somewhat below the discount rate. This reflects the fact that innovations bring the possibility of future innovations with them. Let D^* be the solution to the maximization program of the firm and let V^* the value of the objective function under policy D^*. Then, for any firm that is not yet in the position to make monopoly profits but assumes to stay forever in the industry, once it has entered, V^* represents the (maximal) incentive to do research. All these arguments are conditional on the assumption that the incumbent does not have to care about competitors doing research too. Since the incumbent only cares for the increase in value through the next innovation, which is given by $(\gamma-1)V^*$, we know that without agency costs this replacement effect will generate again the well known result that industry newcomers have the highest incentives to do research. However, there is a direct extension to proposition 7:

Proposition 9 *Assume that* $\frac{\alpha^{\frac{\alpha}{1-\alpha}}}{1-\alpha+\alpha^{\frac{2-\alpha}{1-\alpha}}} < \frac{\gamma-1}{\gamma}$. *Then, in the steady state only the incumbent does research.*

Proof: See appendix C.

In this paper we will not treat the case when the condition in the Proposition does not hold. Therefore we impose for the rest of the paper:

- Assumption *A5*: let $\frac{\alpha^{\frac{\alpha}{1-\alpha}}}{1-\alpha+\alpha^{\frac{2-\alpha}{1-\alpha}}} < \frac{\gamma-1}{\gamma}$.

Observe that the right hand side of the inequality is increasing in γ, while the left hand side is decreasing in α. Therefore this says again that for α sufficiently small relative to γ challengers will stay out. The proof uses exactly the same arguments as the previous sections: The agency cost of finance increases the costs of producing a given probability of success[20] or decreases the probability of success for any given investment. Financial resources are needed to pay the wage bill upfront. Since labor has to be compensated according to its marginal product, the total wage bill will be a fraction $(1-\alpha)$ of total value created in the research lab. Perfect competition between financiers then ensures that they exactly recover their costs: again $\beta = (1-\alpha)$. The partial equilibrium analysis of the last section had to use convex effort costs. This assumption can be dropped in the general equilibrium, because there is an additional free variable, the wage rate, which adjusts to a level such that the individual producer is indifferent between all levels of production. The research intensity is then determined by equilibrium in all markets. Workers are perfectly mobile and therefore paid the same wage everywhere. At the wage level that makes the incumbent indifferent

[20]As in Barro and Sala-i-Martin (1994), the argument is, that cost differences can make innovations profitabel for the incumbent. However, in contrast to their paper, cost differences are endogenous here.

between researching or not, the challenger will make losses if the assumption in the proposition holds: there is no contract that is individually rational for both financier and challenger since total costs exceed total expected gains.

There are several things to note about the condition in proposition 9: it was derived under the assumption that the challenger, once in the monopoly position will choose an optimal investment level over time such that V indeed reaches its maximum. This is clearly an overstatement of the matter since his effort choices are distorted downwards by the fact, that the initial contract gives the financiers the right to a share of profits of $1-\alpha$ until the indefinite future. The condition is therefore sufficient but not necessary. Note furthermore that for the argument given above it does not matter, what exact form the contract between the challenger and the incumbent takes. One can imagine many repayment schedules that all differ only in their intertemporal allocation of payments. However, ex ante they all share one feature: the expected total amount of repayment. And this is the only argument needed above.

6.3 The steady state

Consider now the maximization problem of the incumbent who faces no competition in research. The first order necessary condition for the optimality of his research policy is

$$-C'(D) + (\gamma - 1)\lambda V(D) \stackrel{!}{=} 0$$

(since $V' = 0$). At the optimum this implies that

$$wD = \frac{(1-\alpha)(\gamma-1)\lambda D\pi}{\rho} \qquad \text{(Arbitrage)}$$

which is again the familiar condition that labor, which is paid according to its marginal product, should receive a share of total final output corresponding to the elasticity of output with respect to its input. Compensations in production and research are such that a worker is indifferent between his two opportunities. The value of the ongoing firm under this policy will then simply be $V = \frac{\pi}{\rho}$. The incumbent just values the profit stream until the indefinite future, knowing that he will do research for every new good until the costs of doing so equal the gains. The remaining share α of the expected increase in value exactly compensates the entrepreneur for his effort costs. To characterize the steady state, combine the Arbitrage condition with the resource constraint of the economy:

$$L = D + x(\varpi) \qquad \text{(Labor)}$$

In the steady state the labor input into research will then satisfy:

$$1 = \frac{(1-\alpha)b(\frac{\alpha}{1-\alpha})^{\alpha}(\gamma-1)(\frac{1-\delta}{\delta})(L-D^*)}{\rho}$$

Compare this with the condition in the original Schumpeterian model: $b(\frac{\alpha}{1-\alpha})^{\alpha}$ is just the usual arrival rate λ and the factor $(1-\alpha)$ reflects the fact that labor receives only a fraction of total value added in research. Note that the condition above only indirectly reflects the impact of agency problems on growth: agency costs on the part of the newcomers generate the freedom of action for the incumbent to choose his most preferred research policy. For completeness the economy will grow (in real time) with rate

$$g^* = b(\frac{\alpha}{1-\alpha})^{\alpha} D^* \ln \gamma$$

6.4 Welfare

Assume that the social planner's objective was to maximize the expected value of consumption y_τ. Assume also that the planner can control the inputs of labor and effort directly but that the entrepreneur has to be compensated for his cost of exerting effort. Given the Poisson nature of the research process and taking into account the resource constraint on labor, the social planners problem is then to maximize

$$U(D) = \frac{(1-\alpha)A_0(L-D)^{\delta}}{\rho - (\gamma - 1)b(\frac{\alpha}{1-\alpha})^{\alpha} D}$$

The downward adjustment of the value of innovations by the factor $(1-\alpha)$ follows from the fact that the social planner calculates with a shadow cost of effort which corresponds to the marginal product of entrepreneurial effort in the research process. The solution can be written as

$$1 = \frac{(1-\alpha)b(\frac{\alpha}{1-\alpha})^{\alpha}(\gamma-1)\frac{1}{\delta}(L-D)}{\rho - (\gamma-1)b(\frac{\alpha}{1-\alpha})^{\alpha} D}$$

There are two differences relative to private optimization: (i) The entrepreneur only cares for the part of additional consumer surplus that he can really appropriate: $(1-\delta)$ instead of 1 in case of the planner. (ii) The social planner has a lower discount rate. He values the fact, that each innovation will "stand on the shoulders of the previous innovations", the intertemporal spillover effect. The entrepreneur values this fact exactly the same way. However, he knows that all private gains from future innovations will be exactly sufficient to cover the private costs, because competition for the scarce resource drives up its price.
To sum up, these two forces tend to make the intensity of research lower than the socially optimal one. The original model of Aghion and Howitt has the feature that growth can either be excessive or too slow. In the subset of parameter values that has been analyzed in this subsection growth can never be excessive. This is just because a persistent monopolist would never cannibalize his own rents: the business stealing effect is absent. No formal treatment of the complementary parameter set has been given here in the dynamic setup. But one can conclude that the tendency for growth to be excessive is reduced there too, because the

research intensity will always be smaller as the one in a world without agency costs.
If the social planner wishes to implement an optimal research intensity he faces two options. The first is to subsidize the incumbent's research. The second possibility is to finance newcomers - in *à fonds perdu* fashion - such that the economy is continuously "refreshed" by new firms entering the market. Susidizing monopolists is certainly hard to defend from a political economic perspective - not least because it undermines the possibilities of entrants even more. But so far we cannot compare these two policy options on pure efficiency grounds. Doing this would afford an explicit characterization of the long run behavior of the economy, i.e. the distribution of wealth of the economy. This is outside the scope of the present paper and left for future work.

7 Conclusions

The present paper focussed on the interplay between competition in the research labs and the financial sphere of firms. The central arguments can be repeated in a nutshell: even in a situation, where all "real" incentives are in favor of a destruction of monopolies, we may end up observing a persistence of monopolies. The advantage of being able to finance all investments out of retained earnings can give the incumbent firm a strong strategic advantage vis a vis its opponents. However, the accumulation of financial funds is not the only way how incumbents can improve upon their strategic position. The accumulation of knowledge is probably at least as important in this respect. In this sense learning and accumulating funds should be seen as complementary explanations for the observed persistence of firms. Empirically the presented model predicts - with the qualifications given in the text - that we should not observe too many vertical innovations from financially restricted industry outsiders in product lines where incumbent firms are already active. However, in industries where the elasticity of research success with respect to noncontractible inputs is high relative to the size of innovations, the real forces, i.e. the Arrow effect, will dominate the moral hazard effects and innovations should come from industry outsiders rather than from incumbents. It should be stressed that both financial restrictions and the vertical nature of innovations are crucial for the results: the model is perfectly consistent with observations like wealthy industry outsiders winning patent races for vertical innovations or financially restricted firms doing horizontal innovations.

8 Appendix A

Assumption 1 states, that $b < \frac{1}{\pi_2}; \pi_2 - \pi_1 > 2; \pi_1 > 0$. Note that this also implies that $b < \frac{1}{\pi_2 - \pi_1}$. Consider the best response functions as in the text:

$$p_I = \alpha(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} b^{\frac{2}{\alpha}} (\pi_2 - \pi_1)^{\frac{2-\alpha}{\alpha}} (1-p_c)^{\frac{2-\alpha}{\alpha}}$$

$$p_c = \alpha(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} b^{\frac{2}{\alpha}} \pi_2^{\frac{2-\alpha}{\alpha}} (1-p_I)^{\frac{2-\alpha}{\alpha}}$$

They can be rewritten as

$$p_I = \underbrace{\alpha(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}}}_{w} \underbrace{\{b(\pi_2-\pi_1)\}^{\frac{2}{\alpha}}}_{x} \underbrace{\frac{1}{\pi_2-\pi_1}}_{y} \underbrace{(1-p_c)^{\frac{2-\alpha}{\alpha}}}_{z}$$

Now $\alpha \in [0,1] \Rightarrow w < 1; y < 1$ per assumption; $b < \frac{1}{\pi_2} \Rightarrow b < \frac{1}{\pi_2-\pi_1}; hence \quad x < 1; z < 1$ because exactly the same argument can be given for the other reaction function.

Note that $b < \frac{1}{\pi_2} \Rightarrow \lim_{\alpha\to 0} \{b(\pi_2-\pi_1)\}^{\frac{2}{\alpha}} = 0$. Since the limit when $\alpha \to 1$ is no problem anyway, this establishes, that $0 \leq p_c^*(p_I), p_I^*(p_c) < 1; \forall\alpha.\square$

9 Appendix B

Consider the slope of the mapping in the text. It is given by: $\epsilon c_I(1-c_c(1-p_I)^\epsilon)^{\epsilon-1}\epsilon c_c(1-p_I)^{\epsilon-1}$. It is to be shown, that under Assumption 1, this expression can never exceed 1, no matter what value α takes on. Written out explicitly, the slope looks like

$$\begin{aligned}
&\frac{2-\alpha}{\alpha}\alpha(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} b^{\frac{2}{\alpha}} (\pi_2-\pi_1)^{\frac{2-\alpha}{\alpha}} \cdot \\
&\left\{1-\alpha(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} b^{\frac{2}{\alpha}} \pi_2^{\frac{2-\alpha}{\alpha}} (1-p_I)^{\frac{2-\alpha}{\alpha}}\right\}^{\frac{2(1-\alpha)}{\alpha}} \cdot \\
&\frac{2-\alpha}{\alpha}\alpha(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} b^{\frac{2}{\alpha}} \pi_2^{\frac{2-\alpha}{\alpha}} (1-p_I)^{\frac{2(1-\alpha)}{\alpha}}
\end{aligned}$$

Written as condensed as possible:

$$\begin{aligned}
&(2-\alpha)^2(1-\alpha)^{\frac{4(1-\alpha)}{\alpha}} \frac{1}{\pi_2-\pi_1}\frac{1}{\pi_2} \{b(\pi_2-\pi_1)\}^{\frac{2}{\alpha}} (b\pi_2)^{\frac{2}{\alpha}} \cdot \\
&(1-p_I)^{\frac{2-\alpha}{\alpha}} \left\{1-\alpha(1-\alpha)^{\frac{2(1-\alpha)}{\alpha}} (b\pi_2)^{\frac{2}{\alpha}} \frac{1}{\pi_2}(1-p_I)^{\frac{2-\alpha}{\alpha}}\right\}^{\frac{2(1-\alpha)}{\alpha}}
\end{aligned}$$

Under Assumption 1, we have $(2-\alpha)^2(1-\alpha)^{\frac{4(1-\alpha)}{\alpha}} \frac{1}{\pi_2-\pi_1}\frac{1}{\pi_2} < 1$, since even in the limit when $\alpha \to 0$ $(2-\alpha)^2(1-\alpha)^{\frac{4(1-\alpha)}{\alpha}}$ is bounded above by 4 and $\pi_2-\pi_1 > 2$ according to Assumption 1. Furthermore $\{b(\pi_2-\pi_1)\}^{\frac{2}{\alpha}}$ and $(b\pi_2)^{\frac{2}{\alpha}}$ are each smaller than one due to Assumption 1. The second line was already dealt with in Appendix A. This establishes the claim, that the slope of the function in the text cannot exceed 1.$\square$

Note again, that the real difficulties are present at the boundaries of the support of α. Were we not to impose the restriction on b, taking limits as $\alpha \to 0$ would involve products of the type $0 \cdot \infty$, which are not well defined. Assumption 1 therefore serves the purpose of putting everything on the very safest side.

10 Appendix C

Proof of Proposition 9:
We have to show that - if the condition in Proposition holds - the challenger makes losses if he does research.
Consider therefore first the incumbents research costs: he combines inputs according to the cost minimizing rule $e = \frac{\alpha}{1-\alpha}D$. His total costs are therefore proportional to $D : C(D) = \frac{w}{1-\alpha}D$. Taking cost minimization into account, we can write $p_I = be^{\alpha}D^{1-\alpha} = b\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} D$. Let $\lambda := b\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}$. The arrival rate, λD, is thus linear in D. (For completeness: the minimal costs of producing a probability p is given by $C_I(p) = \frac{w}{1-\alpha}b^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} p$.)

Consider now the challenger. To characterize his costs, we must first characterize the optimal contract. The financiers problem is:

$$\max_{e,D,\beta}(1-\beta)be^{\alpha}D^{1-\alpha}V - we$$

s.t.

$$(Comp) \quad wD = \beta be^{\alpha}D^{1-\alpha}V$$

$$(IC) \quad e = \arg\max(1-\beta)be^{\alpha}D^{1-\alpha}V - we$$

where $(Comp)$ follows from perfect competition in the financial market and (IC) from the unobservability of effort. Let $\hat{e}$ be the solution to (IC). Clearly the optimal D^* is the solution to $\max_D b\hat{e}^{\alpha}D^{1-\alpha}V - wD - w\hat{e}$. Consistency with $(Comp)$ requires then that $\beta = 1-\alpha$ as claimed in the text.
One easily infers from (IC) that for any contract (D, β) the challenger's provision of effort will be distorted downwards by a factor $(1-\beta)^{\frac{1}{1-\alpha}} < 1$ compared to the first best level . Finally, for $\beta = 1-\alpha$ his total costs and success probability for a given level of D will be $C(D) = w\left(\frac{1-\alpha+\alpha^{\frac{2-\alpha}{1-\alpha}}}{1-\alpha}\right)D$ and $p_c = b\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\alpha^{\frac{\alpha}{1-\alpha}}D$, respectively. (Again for completeness, his minimal costs of producing a probability p will be $C_c(p) = w\left(\frac{1-\alpha+\alpha^{\frac{2-\alpha}{1-\alpha}}}{1-\alpha}\right)b^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\alpha^{\frac{-\alpha}{1-\alpha}}p > C_I(p); \forall\alpha < 1.$)
We are finally ready to compare the incentives of incumbent and challenger, respectively: the Incumbent will do research as long as the marginal gain from investing is higher than the marginal costs or

$$b\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}(\gamma-1)VD > \frac{w}{1-\alpha}D$$

Such a constellation can never be part of a general equilibrium because it would generate infinite expected profits. To reestablish general equilibrium the wage rate has to increase until the value of the right hand side equals the value of the left hand side. Let this value be $\bar{w}$. Consider now the challengers problem. In general equilibrium all employers of labor pay the same wage. Hence, at $\bar{w}$

$$b\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\alpha^{\frac{\alpha}{1-\alpha}}\gamma VD < \bar{w}\frac{\left(1-\alpha+\alpha^{\frac{2-\alpha}{1-\alpha}}\right)}{1-\alpha}D$$

if

$$\frac{\alpha^{\frac{\alpha}{1-\alpha}}}{1-\alpha+\alpha^{\frac{2-\alpha}{1-\alpha}}} < \frac{\gamma-1}{\gamma}$$

and the total costs of investment will exceed the total gains.■

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