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Evolution of Wealth and Asset Prices in Markets with Case-Based Investors

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I am indebted to my advisor Juergen Eichberger for his helpful guidance. I would like to thank Hans Haller, Emanuela Sciubba, Carlos Allos-Ferrer, Itzhak Gilboa, Hans Gersbach, Larry Blume, David Easley, Thorsten Hens, Johannes Becker, as well as the participants of the workshop in Honour of David Schmeidler in Yale, of the workshop on Evolutionary and Behavioral Finance in Zurich and of seminars at the University of Heidelberg and Cornell University for helpful discussions and comments.

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Evolution of Wealth and Asset Prices in Markets with Case-Based Investors[1]

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I analyze whether case-based decision makers (CBDM) can survive in an asset market in the presence of expected utility maximizers. Conditions are identified, under which the CBDM retain a positive mass with probability one. CBDM can cause predictability of asset returns, high volatility and bubbles. It is found that the expected utility maximizers can disappear from the market for a finite period of time, if the mispricing of the risky asset caused by the case-based decision-makers aggravates too much. Only in the case of logarithmic expected utility maximizers do the case-based decision makers disappear from the market for all parameter values.

Keywords: asset pricing, bubbles, volatility, case-based decision theory, evolution.
JEL classification: D81, G 11, G 12

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1 Introduction

The empirical literature in financial markets has identified multiple paradoxes of asset pricing. Bubbles, high volatility and predictability of returns seem to be common phenomena in real markets, see e.g. Kindelberger (1978), Sunder (1995) on bubbles in financial markets, Jegadesh (1990) on predictability of asset returns and Shiller (1981) on excessive volatility of asset prices. The standard literature on asset pricing does not explain these phenomena, which contradict the joint hypothesis of expected utility maximization and rational expectations. In this paper, I provide a model in which traders causing asset prices to deviate from prices under rational expectations can survive in a market populated by expected utility maximizers. Moreover, they are able to influence equilibrium prices and cause excess volatility, bubbles and predictability of returns. The model further shows that under certain conditions, the expected utility maximizers can be driven out of the market for a finite period of time, during which an asset with a positive fundamental value sells at a 0-price. Hence, it can provide an explanation for price crashes occurring on assets with positive fundamentals.

I use the case-based decision theory proposed by Gilboa and Schmeidler (1995) to model investors whose behavior significantly deviates from expected utility maximization with correct expectations. Case-based decision-makers are not assumed to have knowledge of possible states of nature or of the distribution of state-dependent outcomes. Instead they learn from experience and evaluate an alternative by the past performance of similar alternatives, taking into account whether the past results have been satisfactory or not compared to a bench-mark called an aspiration level.

In Guerdjikova (2003), it is shown that an asset market populated only by case-based investors can exhibit 0-price equilibria and a price dynamic featuring excess volatility, bubbles and predictability of asset returns.

The current model applies these results to study the evolutionary dynamic of wealth in a market populated by both case-based decision-makers and expected utility maximizers. The proportion of the two types of investors and, therefore, their wealth share, evolve according to the relative success of both groups. The higher the returns achieved by one type of investors, the higher the share of the initial endowment, they receive in the subsequent period. This endogenizes the initial endowment of the investors and allows to address the issue of the relative performance of these two strategies.

Two questions are of main interest:

First, whether and under which conditions case-based decision makers are able to retain a positive share in the market;

Second, whether the effects observed in a market populated solely by case-based decision makers also transfer to a market with expected utility maximizers.

To analyze these issues, the paper will be organized as follows: section two surveys shortly the literature on evolutionary finance. Section three gives a description of the economy. and introduces the evolutionary dynamic of investor types. In section four the evolutionary dynamic is analyzed in order to ask the question, whether case-based decision makers can survive in a financial market. Section five discusses how empirically observed phenomena, such as bubbles or price crashes can emerge in the presence of case-based decision makers in the market. Section six concludes. The proofs of all propositions are stated in the appendix.

2 Survey of the Literature

In the last years, the problems of evolution in financial markets have been gaining attention in the economic literature. As a starting point serves the common view, formulated by Friedman (1953), that markets select for rational traders with correct beliefs.

Thorough analysis of this hypothesis, however, leads to ambiguous results. De Long, Shleifer, Summers and Waldmann (1990, 1991) demonstrate that if the misperceptions of the noise traders force them to choose a riskier portfolio than the one chosen by rational traders, then noise traders dominate the market by achieving higher expected returns than traders with correct beliefs. In a similar setting, but assuming a non-competitive market for assets, Palomino (1996) shows that noise traders can dominate the market even if the evolutionary selection accounts for the disutilities of risk-bearing.

These results opened a discussion on the criteria according to which investment strategies are selected by the market. Several studies on this issue, see Blume and Easley (1992), Hens and Schenk-Hoppé (2001), Evstigneev, Hens and Schenk-Hoppé (2002, 2003), show that the most successful strategy consists in maximizing a logarithmic expected utility function with correct beliefs. Since the logarithmic function has the property to maximize the expected growth of wealth, investors with such utility functions accumulate the whole market wealth over time and drive other types of investors to extinction.

Should a logarithmic utility maximizer be absent from the market, Blume and Easley (1992) show that the market selects for patient investors if relative risk-aversion is controlled for and for investors with relative risk-aversion close to 1 if the discount factors are controlled for. The influence of risk-aversion for survival is hence not unequivocal.

Correct beliefs are shown to be the only robust selection criterium in complete markets, see Blume and Easley (2001) and in markets with perfect foresight, Sandroni (2000).

Although criteria for survival and dominance have been identified in the literature, I cannot aply these results directly to the model at hand. The reason is that the assumptions used to derive the results in the models cited above (such as short-lived assets, simple strategies, strategies independent of the current price, perfect foresight and Pareto-optimality of the market equilibrium) are not satisfied in the economy described in section 3.

Whereas the major part of the literature searches for the best strategy, there is still little research into how different investment rules perform relative to each other. Sciubba (2001) analyzes the relative performance of the CAPM rule as compared to logarithmic utility maximization with correct beliefs and mean-variance utility maximization. She shows that CAPM-traders vanish, whereas those maximizing a mean-variance utility imitate the logarithmic utility maximizers and, therefore, survive.

Similarly, the current paper does not look for the most successful strategy in the market, but addresses the question of relative performance of the two strategies: expected utility maximization versus case-based decision-making.

3 The Economy

I consider an economy, consisting of a continuum of investors, uniformly distributed on the interval $[0;1]$. Time is discrete: $t = 0, 1....$ In period t a proportion e_t of the investors are expected utility maximizers, whereas the rest, $c_t = 1 - e_t$ are case-based decision makers. No population growth is considered.

The model has an overlapping generations structure. Each investor lives for two periods. Investors consume only in the second period of their life. Preferences are represented by a CRRA utility function

$$u_{\beta}(x) = x^{\beta}, \beta \in (0;1]$$
$$u_{\beta}(x) = \ln x, \beta = 0,$$

which is identical for all investors. $(1-\beta)$, therefore, denotes the coefficient of relative risk aversion. There is one consumption good in the economy with price normalized to 1. The initial endowment of the investors consists of one unit of the consumption good in the first period and is 0 in the second period.

There are two possible ways to transfer consumption between two periods: either using a riskless storage technology b, or investing in a risky asset a. The storage technology b delivers $(1+r)$ units of consumption good in period t for each unit of consumption good, stored in period $(t-1)$. It is available in a perfectly elastic supply at a price of 1.

The supply of the risky asset a is fixed at $A = 1$. The payoff of one unit of the asset in period t is:

$$\delta_t = \left\{ \begin{array}{ll} \delta & \text{with probability } q \\ 0 & \text{with probability } 1-q \end{array} \right\},$$

and is identically and independently distributed in each period. Its price is p_t. New emissions are not considered, since I am interested in the behavior of prices on the secondary asset market. I assume, that the payoffs satisfy $1 > \delta > r > 0$.

Short sales are not permitted. Therefore, the set of available acts reduces to:

$$\gamma_t^i \in [0;1],$$

where γ denote the share of initial endowment invested into the risky asset and $i \in \{eu; cb\}$, where eu and cb identify the expected utility maximizers and the case-based decision makers, respectively. Since diversification is possible, I will assume that all investors of a given type choose identical portfolios. This amounts to replacing each of the types by a representative investor.

Given the act chosen by an investor of type i at time $(t-1)$, his indirect utility from consumption at time t can be written as:

$$v_t\left(\gamma_{t-1}^i\right) = u_\beta\left(\gamma_{t-1}^i\left(\frac{p_t}{p_{t-1}} + \frac{\delta_t}{p_{t-1}}\right) + \left(1-\gamma_{t-1}^i\right)(1+r)\right). \tag{1}$$

Note, that the utility derived, when choosing act a depends not only on the dividend of the risky asset, but also on the price of a in t, therefore on the decisions of the young investors at time t.

3.1 Information and Individual Decisions

The individual decision-making process will predetermine the evolution of asset prices, as well as of the shares of different investor types in the economy.

3.1.1 Case-Based Decision Makers

First consider the case-based decision makers, as introduced in Gilboa and Schmeidler (2001). Their description of the situation contains the statement of the problem, they have to solve: "Invest your initial endowment in one of the two assets, a or b, so as to be able to consume tomorrow", as well as the acts, which are available to them:

$$\gamma_t^{cb} \in [0;1].$$

Unlike expected utility maximizers, case-based decision makers do not use information about possible states of nature, state-contingent outcomes and their probability distribution. Therefore, they can only base their decisions on the experience of previous generations, they know about. This information is called memory. I assume that the memory consists only of the act chosen and utility realized by the case-based investors in the previous generation. The experience of the expected utility maximizers is not taken into account. Hence, the memory at time t can be written as

$$M_t = \left(\left(\gamma_{t-1}^{cb}; v_t\left(\gamma_{t-1}^{cb}\right)\right)\right)$$

The utility obtained from γ_{t-1}^{cb} is then compared to an aspiration level $\bar{u}$ assumed to be identical for all case-based investors and constant over the time. If an act is considered satisfactory, it is chosen again, else, it is abandoned and a different act is chosen next.

The perceived similarity among acts allows the decision-maker to evaluate acts that weren't chosen before. Since the available acts are situated on the one-dimensional simplex, I assume that similarity between two acts γ and γ', $s(\gamma;\gamma')$, is a strictly decreasing function in the Euclidean distance $\|\gamma-\gamma'\|$.

Given these assumptions, each act is evaluated according to its cumulative utility:

$$U_t(\gamma) = \left[v_t\left(\gamma_{t-1}^{cb}\right) - \bar{u}\right] s\left(\gamma; \gamma_{t-1}^{cb}\right)$$

and

$$\gamma_t^{cb} = \arg\max_{\gamma\in[0;1]} U_t(\gamma).$$

Hence, the decision of a single case-based investor takes the form:

$$\gamma_t^{cb} = \left\{ \begin{array}{ll} \gamma_{t-1}^{cb} & \text{if } v_t\left(\gamma_{t-1}^{cb}\right) \geq \bar{u} \\ \arg\max_{\gamma\in[0;1]} & \text{if } v_t\left(\gamma_{t-1}^{cb}\right) \leq \bar{u} \end{array} \right\}. \tag{2}$$

The following lemma obtains directly from (2).

Lemma 1 *Let $\gamma_0^{cb} \in (0;1)$. If $\bar{u}$ satisfies*

$$u_\beta\left(\gamma_0^{cb} + \left(1-\gamma_0^{cb}\right)(1+r)\right) \geq \bar{u},$$

then $\gamma_t^{cb} = \gamma_{t-1}^{cb} = \gamma_0^{cb}$ for all t. If

$$u_\beta\left(\gamma_0^{cb} + \left(1-\gamma_0^{cb}\right)(1+r)\right) < \bar{u},$$

then $\gamma_t^{cb} \in \{0;1\}$ almost surely holds for all $t \geq \bar{t}$ for some finite $\bar{t}$.

To derive the share of the endowment of case-based investors invested into a, denote by $\tilde{p}_t$ the price of a, for which the case-based investors are indifferent among all portfolios (if such a price exists):

$$\tilde{p}_t : u_\beta\left(\gamma_{t-1}^{cb}\left(\frac{\tilde{p}_t}{p_{t-1}} + \frac{\delta_t}{p_{t-1}}\right) + \left(1-\gamma_{t-1}^{cb}\right)(1+r)\right) - \bar{u} = 0$$

As long, as $\tilde{p}_t > 0$ and $\gamma_{t-1}^{cb} \geq \frac{1}{2}$

$$\gamma_t^{cb} = \left\{ \begin{array}{ll} \gamma_{t-1}^{cb} & \text{if } p_t > \tilde{p}_t \\ \left[0;\gamma_{t-1}^{cb}\right] & \text{if } p_t = \tilde{p}_t \\ 0 & \text{if } p_t < \tilde{p}_t \end{array} \right\}.$$

For $\gamma_{t-1}^{cb} \in \left(0;\frac{1}{2}\right)$,

$$\gamma_t^{cb} = \left\{ \begin{array}{ll} \gamma_{t-1}^{cb} & \text{if } p_t > \tilde{p}_t \\ \left[0;\gamma_{t-1}^{cb}\right] & \text{if } p_t = \tilde{p}_t \\ 1 & \text{if } p_t < \tilde{p}_t \end{array} \right\}.$$

For $\gamma_{t-1}^{cb} = 0$, $\tilde{p}_t$ exists only if $1+r=\bar{u}$:

$$\gamma_t^{cb} = \left\{ \begin{array}{ll} 0 & \text{if } 1+r > \bar{u} \\ [0;1] & \text{if } 1+r = \bar{u} \\ 1 & \text{if } 1+r < \bar{u} \end{array} \right\}.$$

Figure 1 gives an illustration of $\gamma_t^{cb}(p_t)$ for these three cases.

Note that $\gamma_t^{cb}(p_t)$ is a non-empty, convex-valued, upper hemicontinuous correspondence.

The three cases demonstrate that the share of the endowment invested into the risky asset by case-based investors is monotonically increasing in the price p_t, except if $\gamma_{t-1}^{cb} \in \left(0;\frac{1}{2}\right)$. Since the

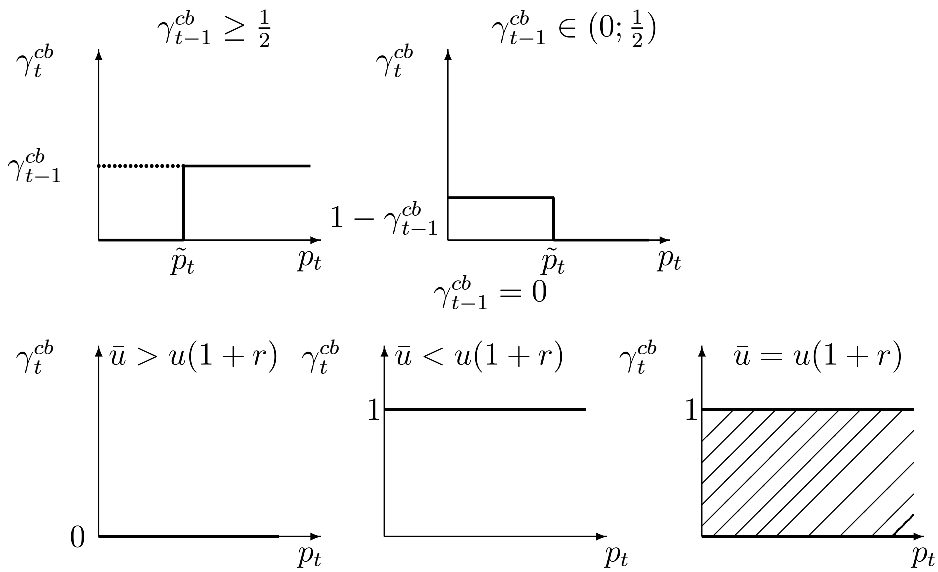

Figure 1

main interest will be on the case of relatively high aspiration levels, which imply that diversified portfolios are held only for a finite number of periods, cases 1 and 3 will describe the demand of case-based investors for assets. Hence, the relationship between p_t and $\gamma_t^{cb}(p_t)$ will be positive. This will be the main difference between the case-based decision makers and the expected utility maximizers, whose share of endowment invested into the risky asset will decrease in p_t.

3.1.2 Expected Utility Maximizers

Now I turn to the description of the expected utility maximizers. I will assume, as usual, that expected utility maximizers have expectations about the state-contingent payments of each of the assets. Still, it will not be assumed, that these expectations are necessarily rational. Even if an expected utility maximizer is informed about the correct distribution of the dividends of the risky asset and of the returns of the safe technology, it is not clear, that he will be able to predict the influence of the case-based decision makers on the asset prices. To do so, he would have to take into account the constitution of the population, the case-based decision making process, as well as the influence the evolution of types will have on prices and returns in the economy. I will assume that expected utility maximizers neglect these issues. They act boundedly rational, taking into account the information about the correct distribution of dividends and the correct interest rate, but building their expectations about the price as if the economy consisted only of expected utility maximizers, identical to themselves.

Let p_β^{eu} denote the price which would emerge in a stationary equilibrium if only expected utility maximizers were present in the market. To compute p_β^{eu}, write the maximization problem of an investor given that the price today is equal to the price tomorrow[3], p

$$\max_{\tilde{\gamma}_t^{eu}\in[0;1]} q\left[\left(1+\frac{\delta}{p}\right)\tilde{\gamma}_t^{eu}+(1+r)\left(1-\tilde{\gamma}_t^{eu}\right)\right]^{\beta}+(1-q)\left[\tilde{\gamma}_t^{eu}+(1+r)\left(1-\tilde{\gamma}_{t^{eu}}\right)\right]^{\beta}$$

where $\tilde{\gamma}_t^{eu}$ denotes the share of the initial endowment invested into a given these expectations and observe that the first order condition simplifies to:

$$\tilde{\gamma}_t^{eu}(p)=\frac{(1+r)\left[\left(\frac{(1-q)r}{q\left(\frac{\delta}{p}-r\right)}\right)^{\frac{1}{\beta-1}}-1\right]}{\frac{\delta}{p}+r\left[\left(\frac{(1-q)r}{q\left(\frac{\delta}{p}-r\right)}\right)^{\frac{1}{\beta-1}}-1\right]}, \tag{3}$$

which describes the optimal $\tilde{\gamma}_t^{eu}$, as long as the term on the r.h.s. of (3) is between $[0;1]$. If the r.h.s. of (3) exceeds 1 or lies below 0, $\tilde{\gamma}_t^{eu}$ takes the values 0 and 1, respectively[4]:

$$\tilde{\gamma}_t^{eu}(p)=\left\{\begin{array}{ll} 0, & \text{if } \frac{(1+r)\left[\left(\frac{(1-q)r}{q\left(\frac{\delta}{p}-r\right)}\right)^{\frac{1}{\beta-1}}-1\right]}{\frac{\delta}{p}+r\left[\left(\frac{(1-q)r}{q\left(\frac{\delta}{p}-r\right)}\right)^{\frac{1}{\beta-1}}-1\right]}<0 \\ \frac{(1+r)\left[\left(\frac{(1-q)r}{q\left(\frac{\delta}{p}-r\right)}\right)^{\frac{1}{\beta-1}}-1\right]}{\frac{\delta}{p}+r\left[\left(\frac{(1-q)r}{q\left(\frac{\delta}{p}-r\right)}\right)^{\frac{1}{\beta-1}}-1\right]}, & \text{if } \frac{(1+r)\left[\left(\frac{(1-q)r}{q\left(\frac{\delta}{p}-r\right)}\right)^{\frac{1}{\beta-1}}-1\right]}{\frac{\delta}{p}+r\left[\left(\frac{(1-q)r}{q\left(\frac{\delta}{p}-r\right)}\right)^{\frac{1}{\beta-1}}-1\right]}\in[0;1] \\ 1, & \text{if } \frac{(1+r)\left[\left(\frac{(1-q)r}{q\left(\frac{\delta}{p}-r\right)}\right)^{\frac{1}{\beta-1}}-1\right]}{\frac{\delta}{p}+r\left[\left(\frac{(1-q)r}{q\left(\frac{\delta}{p}-r\right)}\right)^{\frac{1}{\beta-1}}-1\right]}>0 \end{array}\right\}.$$

Obviously, p_β^{eu} is the fixed point of $\tilde{\gamma}_t^{eu}(p)$:

$$\tilde{\gamma}_t^{eu}\left(p_\beta^{eu}\right)=p_\beta^{eu}.$$

Two special cases are of interest. If $u(x)=x$ then

$$p_1^{eu}=\min\left\{\frac{q\delta}{r};1\right\},$$

hence the price equals the fundamental value of the asset. If $u(x)=\ln x$, then

$$p_0^{eu} \;=\; \min\left\{p_{\log}^{eu};1\right\} \text{ with}$$

[3] The expressions are given for values of $\beta\in(0;1]$. The respective equations for the logarithmic utility function are derived analogously.

[4] For the interior solutions, the no-arbitrage conditions are satisfied. For the corner solutions, the short sale constraints prevent arbitrage even if the no-arbitrage conditions fail.

$$p_{\log}^{eu} = \frac{1+r+\delta-\sqrt{(1+r+\delta)^2-4q\delta(1+r)}}{2r}.$$

Since the expected utility maximizers perceive p^{eu} to be the "true" price of the risky asset[5], the share of their endowment invested into the risky asset γ_t^{eu} is determined as a solution to

$$\max_{\gamma_t^{eu}\in[0;1]} q\left[\left(\frac{p_\beta^{eu}+\delta}{p_t}\right)\gamma_t^{eu}+(1+r)(1-\gamma_t^{eu})\right]^\beta+(1-q)\left[\frac{p_\beta^{eu}}{p_t}\gamma_t^{eu}+(1+r)(1-\gamma_{t^{eu}})\right]^\beta$$

and is decreasing in the price p_t. Differently from the case-based decision makers, expected utility maximizers, therefore, hold the risky asset only if its price is relatively low. Note that if $p_\beta^{eu}=1$, then the expected utility maximizers will invest their whole endowment into the risky asset, irrespectively of the price p_t.

$\gamma_t^{eu}(p_t)$ is a continuous function for $\beta\in[0;1)$, whereas for $\beta=1$ it is a non-empty, convex-valued and upper hemicontinuous correspondence. The value of demand for a of the whole population is obtained as:

$$d_t(p_t)=e_t\gamma_t^{eu}(p_t)+(1-e_t)\gamma_t^{cb}(p_t).$$

It is a correspondence, which also has the characteristics stated above and maps the interval $[0;1]$ into $[0;1]$.

3.2 The Evolution of Investor Types

After describing the decision process of the investors in the economy, I now introduce the selection dynamic. I measure the fitness of a given type of investors by the actual average returns they achieve relative to the average returns of the society as a whole. This gives rise to a replicator dynamic, in which the share of the type of investors who perform better grows. A higher wealth share for a particular type of investors in the economy then implies greater influence on market processes[6].

An overlapping generations structure does not allow for a natural wealth dynamic to arise as in the works cited in section 2. Note, however that since each investor is born with the same initial endowment of 1 unit of the consumption good, the share of a type of investors can be

[5] Although expected utility maximizers do not need to have rational expectations in general, they do have rational expectations in the limit, when $e\to 1$.

[6] This property, which follows from the market clearing condition in this model, need not hold in general. See for instance Kogan, Ross, Wang and Westerfield (2003) for a model, in which noise traders can influence the price process, even though their share converges to 0 in the limit.

identified with the total income of the investors of this type. Hence, the replicator dynamic can be interpreted as a wealth dynamic in the model at hand.

3.2.1 Replicator Dynamic

Differently from the usual approach in evolutionary game theory, the replicator dynamic in this model is applied not to the portfolio strategy chosen by an individual, but to the ”meta”-strategies used by the two types of investors, hence to the performance of case-based decision-making versus expected utility maximization[7].

The following replicator dynamic is introduced, following Weibull (1995, pp. 124-125)[8].

Denote by

$$\begin{aligned} \tilde{v}_t^i &= \gamma_{t-1}^i \frac{p_t+\delta_t}{p_{t-1}} + \left[1-\gamma_{t-1}^i\right](1+r) \\ \tilde{v}_t &= e_{t-1}\left[\gamma_{t-1}^{eu}\frac{p_t+\delta_t}{p_{t-1}} + \left[1-\gamma_{t-1}^{eu}\right](1+r)\right] \\ &\quad + (1-e_{t-1})\left[\gamma_{t-1}^{cb}\frac{p_t+\delta_t}{p_{t-1}} + \left[1-\gamma_{t-1}^{cb}\right](1+r)\right] \end{aligned} \tag{4}$$

the average returns achieved by an investor of type $i \in \{eu; cb\}$ and by the society as a whole. The replicator dynamic is written as:

$$e_t = \frac{\tilde{v}_t^{eu}}{\tilde{v}_t} e_{t-1} \tag{5}$$

Hence, the equilibrium share of expected utility maximizers becomes:

$$e_t^* = \frac{\left[\frac{p_t^*(e_t^*)+\delta_t}{p_{t-1}}\gamma_{t-1}^{eu} + (1+r)\left(1-\gamma_{t-1}^{eu}\right)\right]e_{t-1}}{\frac{p_t^*(e_t^*)+\delta_t}{p_{t-1}}\left(\gamma_{t-1}^{eu}e_{t-1}+\gamma_{t-1}^{cb}\left(1-e_{t-1}\right)\right) + (1+r)\left(1-\left(\gamma_{t-1}^{eu}e_{t-1}+\gamma_{t-1}^{cb}\left(1-e_{t-1}\right)\right)\right)}. \tag{6}$$

Note, that the numerator represents the wealth of the old expected utility maximizers at time t, the denominator is the wealth of the old investors in the economy at t. Hence, the proportion of young investors following a decision rule at time t is equal to the relative share of wealth held by the old investors following the strategy. I therefore claim, that the replicator dynamic can be interpreted as a relative wealth dynamic in the sense of Blume and Easley (1992, 2001) and

[7] This approach is therefore similar to the indirect evolutionary approach initiated by Güth and Yaari (1992). In their setup the genetic phenotype describes a decision rule for choosing a strategy in a game. The solution of the game, computed in accordance with the proportions in which these phenotypes are present, determines the payoffs and hence the evolution of decision rules (not of strategies) in the population. The present model differs however from the work of Yaari and Güth by the fact that instead by a game the payoffs are determined by a market.

[8] In this model the length of period is assumed to be 1 and the growth rate of the population is 0. This correspondes to $\tau = 1$ and $\beta = 0$ in the overlapping-generations model presented by Weibull.

Hens and Schenk-Hoppé (2001), Evstigneev, Hens and Schenk-Hoppé (2002, 2003).

The analysis of $\gamma^{cb}(p)$ in section 3 shows that if $e_{t-1} = 0$, $p_{t-1} = 0$ might obtain, see figure 1. To insure that e_t^* is always well defined, compute the limit of (6) as e_{t-1} and p_{t-1} converge to 0. Obviously, $p_{t-1} = \gamma_{t-1}^{eu} e_{t-1}$ and $\gamma_{t-1}^{eu} = 1$ will hold for prices near 0. Substituting in (6) gives:

$$\lim_{e_{t-1}\to 0} e_t^* = \lim_{e_{t-1}\to 0} \frac{\frac{p_t^*(e_t^*)+\delta_t}{e_{t-1}} e_{t-1}}{\frac{p_t^*(e_t^*)+\delta_t}{e_{t-1}} e_{t-1} + (1+r)(1-e_{t-1})} = \frac{p_t^*(e_t^*)+\delta_t}{p_t^*(e_t^*)+\delta_t+1+r},$$

which is well defined. This means, especially, that starting with $e_t = 0$ the mass of expected utility maximizers may become strictly positive if expected utility maximizers hold an asset with positive fundamental value, the price of which is 0. On the other hand, if the initial mass of the case-based decision makers is 0, then it remains 0 in all subsequent periods.

3.2.2 Temporary Equilibrium With Replicator Dynamic

Given $\left(e_{t-1}; \gamma_{t-1}^{eu}; \gamma_{t-1}^{cb}; p_{t-1}\right)$, a temporary equilibrium with replicator dynamic at time t is defined as a vector: $\left(e_t^*; \gamma_t^{*eu}; \gamma_t^{*cb}; p_t^*\right)$, such that:

(i) $\gamma_t^{*eu} = \gamma_t^{eu}(p_t^*)$

(ii) $\gamma_t^{*cb} = \gamma_t^{cb}(p_t^*)$

(iii) $p_t^*(e_t^*)$ clears the market for the risky asset given e_t^*;

(iv) e_t^* is determined by the replicator dynamic:

$$e_t^* = \frac{\left[\frac{p_t^*(e_t^*)+\delta_t}{p_{t-1}} \gamma_{t-1}^{eu} + (1+r)\left(1-\gamma_{t-1}^{eu}\right)\right] e_{t-1}}{\frac{p_t^*(e_t^*)+\delta_t}{p_{t-1}} \left(\gamma_{t-1}^{eu} e_{t-1} + \gamma_{t-1}^{cb}(1-e_{t-1})\right) + (1+r)\left(1-\left(\gamma_{t-1}^{eu} e_{t-1} + \gamma_{t-1}^{cb}(1-e_{t-1})\right)\right)}.$$

It is possible to show that such an equilibrium exists in each period as long as the initial state $\left(e_{t-1}; \gamma_{t-1}^{eu}; \gamma_{t-1}^{cb}; p_{t-1}\right)$ is an equilibrium. The evolution of the system is therefore well defined.

4 The Evolution of Wealth

The definition of a temporary equilibrium with evolutionary dynamic, combined with the dividend process determine the evolution of the system. I first discuss the stationary states.

4.1 Stationary states

Proposition 2 (i) *$e_t^* = 1$, $p_t^* = \gamma_t^{eu} = p_\beta^{eu}$ is a stationary state.*

(ii) $e_t^* = 0$, $p_t^* = \gamma_t^{cb} = \gamma_0^{cb}$ *is a stationary state, if* $\gamma_0^{cb} > 0$ *and*

$$\bar{u} < u_\beta \left(\gamma_0^{cb} + (1+r)\left(1 - \gamma_0^{cb}\right)\right)$$

hold.

If only expected utility-maximizers are present in the market, a rational expectations equilibrium will emerge.

For the case $e = 0$, the price $p_t^* = \gamma_0^{cb}$ is constant over time, but need not coincide with p_β^{eu}. In this case, arbitrage opportunities might remain unused in the market. However, only a relatively low aspiration level that insures that $\gamma_t^{cb} > 0$ holds for each t allows the case-based decision makers to keep their mass at 1 in the market. As explained above, if $p_t^* = 0$ holds in some period of time, $e_T^* > 0$ would almost surely obtain for some finite $T > t$. Hence, $e = 0$ would not be stationary.

Proposition 3 *Let*

$$\bar{u} < u_\beta \left(p_\beta^{eu} + (1+r)\left(1 - p_\beta^{eu}\right)\right).$$

Then each $e \in [0;1]$ *is a stationary state, if the portfolios held and the price of* a *fullfil:*

$$\gamma_t^{eu} = \gamma_t^{cb} = p_\beta^{eu} = p_t^*$$

for all $t \geq 0$*.*

Proposition 3 identifies stationary states, in which both types of traders coexist. The price coincides with the price under rational expectations and both types of investors hold the same optimal portfolio, given the market price. By imitating the expected utility maximizers, case-based decision makers with relatively low aspiration levels are thus able to survive in a financial market. However, they will not influence prices and it would not be possible to empirically reject the hypothesis of rational expectations and expected utility maximization in such a market. It is, therefore, interesting, whether a positive share of case-based decision-makers can survive if the portfolio strategies of expected utility maximizers and case-based decision makers differ.

4.2 Can the Case-Based Investors Survive?

It has been shown, that case-based decision makers with a relatively low aspiration level can survive in a market, without influencing prices. Now I shall look at the dynamics of the system for case-based investors satisfying:

$$\bar{u} > u_\beta \left(\gamma_0^{cb} + (1+r)\left(1 - \gamma_0^{cb}\right)\right)$$

If the aspiration level of the case-based decision makers is relatively high, their behavior might influence prices. The price dynamic in a market populated only by case-based decision makers is discussed in Guerdjikova (2003).

If

$$u_\beta (1+r) > \bar{u} > u_\beta \left(\gamma_0^{cb} + (1+r)\left(1-\gamma_0^{cb}\right)\right),$$

$p_t^* = 0$ in each period holds and all investors hold b in every period.

For relatively high aspiration levels,

$$u_\beta (1+\delta) > \bar{u} > u_\beta (1+r),$$

the price process is a stochastic cycle with two states: $p_h = 1$ and $p_l = 0$. The Markov transition matrix, describing this process is given by:

	$p_{t+1} = p_h$	$p_{t+1} = p_l$
$p_t = p_h$	q	$1-q$
$p_t = p_l$	1	0

.

Of course, for $e > 0$ the price in state $p_l > 0$, and (as long as $p_\beta^{eu} < 1$) $p_h < 1$ will obtain in equilibrium. Nevertheless, it is possible that similar cycles occur even in the presence of expected utility maximizers in the market. In the current model, the magnitude of these cycles depends positive on the mass of case-based decision makers in the economy. Therefore, such cycles will persist, only if a positive mass of case-based decision makers survives.

In the following I examine the stability of the stationary state $e = 1$ to determine whether the case-based investors survive and influence the prices in the market.

The further discussion will concentrate on the case, in which the stationary states in which case-based investors are present in the market, as described in propositions 2 and 3 do not occur, i.e. on

$$\bar{u} > u_\beta \left(\gamma_0^{cb} + (1+r)\left(1-\gamma_0^{cb}\right)\right)$$

for $\gamma_0^{cb} > 0$, since then the case-based investors will change their portfolio holdings over time generating a non-trivial dynamic. The discussion of the results for asset markets without expected utility maximizers shows, that the dynamic of the system crucially depends on the aspiration level of the case-based decision makers. Two cases will be of importance: $\bar{u} \in \left(u_\beta (1+r); u_\beta (1+\delta)\right)$, referred to as high aspiration level and

$$\bar{u} \in \left(u_\beta \left(\gamma_0^{cb} + (1+r)\left(1-\gamma_0^{cb}\right)\right); u_\beta (1+r)\right),$$

the case of low aspiration level.

For

$$\bar{u} \in \left(u_\beta\left(\gamma_0^{cb} + (1+r)\left(1-\gamma_0^{cb}\right)\right); u_\beta(1+r)\right),$$

it is easy to see, that the return of b is satisfactory for the case-based decision makers, whereas the return of a, given that the dividend is 0 and the price of a remains unchanged or falls is not satisfactory. Hence, $\gamma^{cb} = 0$ obtains and holds forever.

For

$$\bar{u} \in \left(u_\beta(1+r); u_\beta(1+\delta)\right),$$

the return of a is considered satisfactory, when the dividend is high and the price of a weakly increases, whereas the return of b and the return of a if its dividend is low, are regarded as unsatisfactory. Hence, the case-based decision makers will switch infinitely often between $\gamma^{cb} = 1$ and $\gamma^{cb} = 0$.

4.2.1 The case of high aspiration levels

Consider first the case of high aspiration level:

$$\bar{u} \in \left(u_\beta(1+r); u_\beta(1+\delta)\right).$$

Proposition 4 *Let*

$$\bar{u} \in \left(u_\beta(1+r); u_\beta(1+\delta)\right).$$

1. If
$$q \in \left[\frac{r}{r+(\delta-r)(1+\delta)^{\beta-1}}; \frac{(1+\delta)r}{(1+r)\delta}\right),$$
then for each $\beta \in (0;1]$, there exists an $\tilde{e} \in (0;1)$, such that e_t^* is a submartingale for $e_{t-1}^* < \tilde{e}$ and a supermartingale for $e_{t-1}^* \geq \tilde{e}$.

2. For all values of $\beta \in [0;1]$ if
$$q \in \left[\frac{(1+\delta)r}{(1+r)\delta}; 1\right],$$
e_t^* is a submartingale for all $e_{t-1} \in [0;1]$. The case-based decision makers disappear with probability 1.

Proposition 4 establishes conditions under which the stationary state $e = 1$ is not stable, in the sense that the replicator dynamics does not converge to it with probability 1.

To see the intuition behind the result, consider the case of linear utility functions. The condition on q in the first part of proposition 4 insures that $p_1^{eu} = 1$, hence, the expected utility maximizaers hold a in each period of time, independently of p_t. The high aspiration level implies that the case-based investors are constantly switching between $\gamma^{cb} = 1$ and $\gamma^{cb} = 0$. The replicator dynamic of e_t^* is concave in the returns of the expected utility maximizers. Therefore, it selects for the less risky strategy, given that the expected returns of two strategies are identical. At times, when the case-based decision makers choose $\gamma^{cb} = 0$, their portfolio is less risky, than the portfolio of the expected utility maximizers, who choose $\gamma^{eu} = 1$. Moreover, as $c \to 0$ in case $p_1^{eu} = \frac{q\delta}{r} = 1$, the expected returns of both portfolios are the same and the replicator dynamic selects for the less risky one — those of the case-based investors. By continuity, the same result holds in some surrounding of $e = 1$ and in some surrounding of $q = \frac{r}{\delta}$. Hence, as long as q is not very large, a positive share of case-based decision makers survives.

If, however, q exceeds $\frac{(1+\delta)r}{(1+r)\delta}$, the excess return of the expected utility maximizers becomes sufficiently high to compensate for the higher risk of their portfolio. In this case, they accumulate the whole market wealth with probability 1. Note that in the case of a logarithmic utility function, only this case is relevant and the case-based investors almost surely disappear. This result is consistent with the findings in the literature cited in section 2.

Higher values of p_β^{eu} coincide with higher values of q ceteris paribus. The probability of high dividends has two effects on the evolutionary dynamic. On the one hand, higher q implies higher expected returns of the risky asset and therefore higher profits for the investors holding a, i.e. for the expected utility maximizers. On the other hand, higher values of q cause the case-based decision makers to switch less frequently between the two undiversified portfolios and to hold the risky asset during a larger share of time, hence to behave in a less risk-averse manner[9]. These two effects work in the same direction, making the strategy of the expected utility maximizers more successful.

A result similar to the one of proposition 4 can be derived for lower fundamental values of the risky asset, when the utility function is linear.

Proposition 5 *Suppose that $\beta = 1$. Let the aspiration level satisfy*

$$\bar{u} \in (1 + r; 1 + \delta).$$

[9] See Guerdjikova (2003) for a derivation of the limit frequencies with which the case-based decision makers choose $\gamma^{cb} = 0$ and $\gamma^{cb} = 1$.

Then there is a critical value $\tilde{p}_1^{eu} \in \left(\frac{1}{2};1\right)$ *such that* $E\left[e_{t+2}^* \mid e_t^*\right] < e_t^*$ *holds for*

$$e_t^* \in \left[\max\left\{p_1^{eu}; 1 - p_1^{eu} + \frac{r p_1^{eu^2}}{1+r}\right\}; 1\right)$$

if $p_1^{eu} \in (\tilde{p}_1^{eu}; 1)$.

Although the result is stated for the case of a linear utility function, the argument can be extended to all coefficients of relative risk-aversion strictly smaller than 1. Indeed, because of proposition 4, we know that $e = 1$ is unstable at

$$q = \frac{r}{r + (\delta - r)(1+\delta)^{\beta - 1}}.$$

Since the dynamic of the system is continuous with respect to the parameter q, it follows that this property still holds in some surrounding of $p_\beta^{eu} = 1$ and especially for

$$q \in \left(\tilde{q}; \frac{r}{r + (\delta - r)(1+\delta)^{\beta - 1}}\right)$$

for some $\tilde{q} \in \left(0; \frac{r}{r+(\delta-r)(1+\delta)^{\beta-1}}\right)$.

For lower values of p_1^{eu} (especially lower than $\frac{1}{2}$) the results are not clear. Whereas the expected share of expected utility maximizers decreases, when the case-based decision makers hold a in period t, ($E\left[e_{t+2}^* \mid e_t^*, \gamma_t^{*cb} = 1\right] < 0$ always holds near 1), their share increases in expectation, when case-based decision makers hold b: $E\left[e_{t+2}^* \mid e_t^*, \gamma_t^{*cb} = 0\right] > 0$, as long as e_t is sufficiently close to 1. It is, à priori, not clear which of these two effects will dominate. Nevertheless,the intuition suggests that for sufficiently low fundamental values of the risky asset the case-based decision makers will disappear. Indeed, imagine that e.g. $\delta = 0$, so that $p_1^{eu} = 0$ holds, hence the risky asset never pays a positive dividend. In this case, the case-based decision makers, who hold a strictly dominated asset with positive frequency disappear in the limit. By continuity this result holds in some surrounding of $p_1^{eu} = 0$ ($\delta = 0$) and, therefore, case-based decision makers with high aspiration level cannot survive for low fundamental values of the risky asset.

To summarize, if the fundamental value of the risky asset is neither too high, nor too low, there is a positive probability that the case-based decision makers will not disappear from the market. This result can be made even stronger:

Proposition 6 *Suppose that* e_t^* *is a supermartingale on some interval* $[\tilde{e}; 1]$. *Then*

$$\Pr\{e_t^* \to 1\} = 0.$$

The share of case-based decision makers thus remains almost surely positive, as long as it can be shown, that e_t^* is a supermartingale near 1. This result can be interpreted in terms of the definition of survival and dominance introduced by Blume and Easley (1992). In their terminology survival requires that the share of an investor type, say of case-based decision makers, fulfills:

$$\Pr\left\{\limsup_{t\to\infty} c_t > 0\right\} = 1, \tag{7}$$

whereas the case-based decision makers dominate the market, if

$$\Pr\left\{\lim\inf_{t\to\infty} c_t > 0\right\} = 1 \tag{8}$$

is satisfied. Proposition 6 implies that both (7) and (8) are fulfilled, as long as e_t^* is a supermartingale on some interval $[\tilde{e}; 1]$.

4.2.2 The case of low aspiration level

Now suppose that the case-based decision makers have an aspiration level which satisfies

$$u_\beta\left(\gamma_0^{cb} + (1+r)\left(1-\gamma_0^{cb}\right)\right) < \bar{u} < u_\beta\left(1+r\right),$$

implying that the case-based decision makers hold b in each period of time. Again, it is possible to identify values of the parameters, for which the state $e = 1$ is not stable and the case-based investors almost surely survive in positive proportion.

Proposition 7 *If*

$$\bar{u} \in \left(u_\beta\left(\gamma_0^{cb} + (1+r)\left(1-\gamma_0^{cb}\right)\right); u_\beta\left(1+r\right)\right)$$

and

1. $q \in \left[\frac{r}{r+(\delta-r)(1+\delta)^{\beta-1}}; \frac{(1+\delta)r}{(1+r)\delta}\right)$, then for each $\beta \in (0;1]$, there exists a cut-off point $\hat{e} \in (0;1)$, such that e_t is a supermartingale above $\hat{e}$ and a submartingale below $\hat{e}$.

2. If $q \in \left[\frac{(1+\delta)r}{(1+r)\delta}; 1\right]$, for any $\beta \in [0;1]$, the share of the case-based decision makers converges to 0 almost surely.

Note that with low aspiration levels the case-based decision makers survive for exactly the same values of q, which were found in proposition 4. Although in the case of low aspiration level, q influences the selection only by increasing the average return of the expected utility maximizers and not through the less risk-averse behavior on the side of the case-based decision makers, in the limit when $c \to 0$, the conditions for survival of the case-based investors are identical in both cases.

However, the cut-off values $\tilde{e}$ (as defined in proposition 4) and $\hat{e}$ from proposition 7 reflect the fact that the strategy of the case-based decision makers is riskier in the case of high aspiration level. Hence, case-based investors with low aspiration levels are likely to survive in a higher proportion than case-based investors with high aspiration levels, as the following proposition demonstrates:

Proposition 8 *$\tilde{e}$, as defined in proposition 4 and $\hat{e}$ from proposition 7 satisfy:*

$$\tilde{e} > \hat{e}.$$

Proposition 9 *Suppose that*

$$\bar{u} \in \left(u_\beta\left(\gamma_0^{cb} + (1+r)\left(1-\gamma_0^{cb}\right)\right); u_\beta(1+r)\right)$$

Let

$$q < \frac{r}{r + (\delta - r)(1+\delta)^{\beta-1}}.$$

Then for all $\beta \in (0;1]$, e_t^* is a supermartingale on an interval $[\breve{e}(\beta);1]$.

The result of proposition 6 applies in this case as well, implying that the share of case-based decision makers remains positive with probability 1, as long as e_t^* is a supermartingale in some interval $[\hat{e};1]$. Note that as in the case of high aspiration levels, case-based investors cannot survive in the presence of logarithmic expected utility maximizers.

5 Asset Prices in the Presence of Case-Based Decision Makers

The results of section 4 show that case-based investors can survive in strictly positive proportion in the presence of expected utility maximizers. This section analyzes the effect of their behavior on asset prices.

Consider first the case of high aspiration level. If $p_\beta^{eu} < 1$, the case-based decision makers can influence prices and cause bubbles, excessive volatility and predictability of returns, as long as their share is sufficiently high. Denote by

$$\hat{p}_\beta^{eu} = \min\{p \mid \gamma^{eu}(p) = 0\}$$

the minimal price at which the expected utility maximizers hold only bonds. Suppose now that $c_t > \hat{p}_\beta^{eu}$ holds. Since the case-based decision makers switch between a and b infinitely often, the price of a will fluctuate depending on the share of case-based decision makers and on their behavior, exhibiting excessive volatility[10]. Moreover, the returns of a are predictable.

[10] In an overlapping generations model with constant initial endowments and no population growth the price of the risky asset should remain constant over time, given rational expectations and expected utility maximization. This is the case for $e = 1$.

Especially, if in a certain period only expected utility maximizers hold a, an external observer could predict, that the price of a in the next period will (weakly) rise, since the young case-based decision makers will buy a in t, independently of the dividend paid by the risky asset.

Case-based investors can cause a bubble to emerge and to persist in the market for several periods. Suppose for instance, that the share of expected utility maximizers is lower than $\left(1-\hat{p}_{\beta}^{eu}\right)$ at some time t and that case-based decision makers choose $\gamma_t^{cb}=1$. Then the equilibrium price of a will satisfy

$$p_t^*=(1-e_t^*)>\hat{p}_{\beta}^{eu}\geq p_{\beta}^{eu}.$$

Moreover, if $\delta_{t+1}=\delta$, then the returns of the case-based investors will exceed those of the expected utility maximizers and $e_{t+1}^*<e_t^*$ will hold. Since the young case-based decision makers choose $\gamma_{t+1}^{cb}=1$,

$$p_{t+1}^*=\left(1-e_{t+1}^*\right)>p_t^*\hat{p}_{\beta}^{eu}\geq p_{\beta}^{eu}$$

holds in equilibrium. Hence, the price increases above the price under rational expectations, as long as the dividend of the risky asset remains positive. In the first period t', such that $\delta_{t'}=0$, the bubble will burst, since the case-based decision makers will switch to $\gamma^{cb}=0$ and their share will decrease. Moreover, the price of the risky asset might even fall below the price under rational expectations p_{β}^{eu}. For instance, in the case of a linear utility function this would happen, if

$$(1-e_{t'}^*)>\frac{\left(p_1^{eu}-1\right)^2}{p_1^{eu}\left(1+r\right)},$$

hence if the bubble has lasted long enough to decrease substantially the share of the expected utility maximizers.

Now consider the case of low aspiration levels, hence, $\gamma_t^{*cb}=0$ holds for each t. It turns out that case-based investors with low aspirations can drive the expected utility maximizersd out of the market for a finite number of periods.

Denote by

$$\check{p}_{\beta}^{eu}=\max\left\{p\mid\gamma^{eu}\left(p\right)=1\right\}.$$

Let $e_t\leq\check{p}_{\beta}^{eu}$ so that $\gamma_t^{eu}=1$, $p_t^*=e_t^*$ and let the next period dividend be low, $\delta_{t+1}=0$. The equilibrium share of the expected utility maximizers is given by the solution of the equation:

$$e_{t+1}^*=\frac{\frac{e_{t+1}^*}{e_t^*}e_t^*}{\frac{e_{t+1}^*}{e_t^*}e_t+\left(1+r\right)\left(1-e_t^*\right)}=\frac{e_{t+1}^*}{e_{t+1}^*+\left(1+r\right)\left(1-e_t^*\right)}. \tag{9}$$

(9) has two solutions: $e^*_{t+1} = 0$ and

$$e^{**}_{t+1} = e^*_t (1 + r) - r < e^*_t$$

Hence, if the initial share of the expected utility maximizers is relatively small:

$$e^*_t < \min\left\{\frac{r}{1+r}; 1 - \check{p}^{eu}_\beta\right\},$$

$e^{**}_{t+1} < 0$ and $e^*_{t+1} = 0$ obtains in equilibrium.

The expected utility maximizers can vanish, if they hold the risky asset, hoping that it is valuable, but if there are not enough of their type to prevent prices from falling, when the dividend of the asset is low. This effect is similar to the noise trader risk identified by De Long, Shleifer, Summers and Waldmann (1990). Although the expected utility maximizers don't have rational expectations in this model, they suffer from an undervaluation of the risky asset, caused by the case-based decision makers. If further the returns of the expected utility maximizers are relatively low compared to those of the population as a whole, then the share of the case-based decision makers will increase causing the undervaluation of the risky asset to become even more severe.

The effect arises, because of the dependence of the replicator dynamic on the price of the risky asset and therefore indirectly on e_t itself. It shows, that even in markets, in which expected utility maximizers are à priori present, the price of an asset with positive fundamental value may fall to 0 and remain so for few periods.

The expected utility maximizers will, however, not disappear forever. Since they will hold an asset with positive fundamental value, their share in the population will become positive in the first period, in which the dividend of the risky asset becomes strictly positive. Hence, the price of the asset becomes positive in finite time.

The results of this section imply that some of the phenomena empirically observed in financial markets could be attributed to the presence of case-based decision makers in the economy. However, the emergence of bubbles or price crashes requires a relatively high proportion of case-based decision makers in the market. Although the probability of such events is positive, it is not clear, whether its analytical computation is possible. Future work will therefore have to deal with simulations of the model, from which the frequency of such phenomena could be estimated.

6 Conclusion

The analysis of the model answers the two questions stated in the introduction by identifying conditions under which case-based decision makers survive in the presence of expected utility maximizers and discussing their influence on prices. It is shown that case-based investors can survive for certain ranges of the parameters if the coefficient of relative risk aversion is less than 1. Case-based investors are shown to cause predictability of asset prices, high volatility and bubbles, as well as price crashes when the share of expected utility maximizers in the market is relatively low.

A final note has to be made on the issue of introducing expected utility maximizers with rational expectations. It is straightforward to see, that in the case of a linear utility function, as long, as the short-sale constraints are not binding, the expected returns of the two assets will be identical at each period of time. Therefore, the replicator dynamic will select for the less risky strategy in each period and the case-based investors would retain a positive proportion in the market. Since the replicator dynamic and the demand of the investors are continuous with respect to β, it can be expected that similar results will obtain in a surrounding of $\beta = 1$. Hence, the results about the instability of the stationary state $e = 1$ would remain valid at least for a range of coefficients of relative risk-aversion close to 1.

Appendix

Proof of Proposition 2

(i) Let first $e_{t-1} = 1$. It follows that $(1 - e_{t-1}) = 0$. Since

$$e_t^* = \frac{\left[\frac{p_t^*(e_t^*)+\delta_t}{p_{t-1}}\gamma_{t-1}^{eu} + (1+r)\left(1-\gamma_{t-1}^{eu}\right)\right] e_{t-1}}{\frac{p_t^*(e_t^*)+\delta_t}{p_{t-1}}\gamma_{t-1}^{eu} e_{t-1} + (1+r)\left(1-\gamma_{t-1}^{eu} e_{t-1}\right)} = 1,$$

the claim of the proposition obtains.

(ii) If $p_{t-1} > 0$ and $e_{t-1} = 0$, then $e_t^* = \frac{\tilde{v}_t^{eu} \cdot 0}{\tilde{v}_t} = 0$. Therefore, if it can be insured, that the demand for a of the case-based decision makers is strictly positive over the time, the mass of the expected utility maximizers will stay 0. Hence, a condition is needed, that insures that $\gamma_t^{*cb} > 0$ for each t. Assume, as in the proposition, that $\gamma_0^{cb} > 0$ and note that all case-based decision makers will have the case $\left(\gamma_0^{cb}; v_t\left(\gamma_0^{cb}\right)\right)$ in their memory, will observe a return of at least

$$u_\beta\left(\gamma_0^{cb} + (1+r)\left(1-\gamma_0^{cb}\right)\right) > \bar{u},$$

given that the price of the asset remains unchanged. Hence, if $\gamma_t^{cb} = \gamma_0^{cb}$, then $\gamma_{t+1}^{cb} = \gamma_0^{cb}$ and

$$\gamma_t^{cb} = p_t^* = \gamma_0^{cb}$$

obtains in each period t.■

Proof of Proposition 3:

The assumption

$$\bar{u} < u\left(p_\beta^{eu} + (1+r)\left(1-p_\beta^{eu}\right)\right)$$

guarantees, that as long as the price of a remains constant over the time, none of the case-based decision makers will ever switch away from the initially chosen portfolio. Indeed, if a case-based decision maker remembers $\left(\gamma^{cb} = p_\beta^{eu}; v_t\left(p_\beta^{eu}\right)\right)$,

$$v_t\left(p_\beta^{eu}\right) \geq u_\beta\left(p_\beta^{eu} + (1+r)\left(1-p_\beta^{eu}\right)\right) > \bar{u}$$

holds and $\gamma_t^{cb} = p_\beta^{eu}$. Moreover, at p_β^{eu}, $\gamma^{eu}\left(p_\beta^{eu}\right) = p_\beta^{eu}$ is the optimal choice of the expected utility maximizers. Since both types of investors hold the same portfolio, their returns in each period of time are equal and hence their shares in the population remain constant over time.■

Proof of proposition 4

Lemma 10 *If* $q \geq \frac{r(1+\delta)}{\delta(1+r)}$, $p_\beta^{eu} = 1$ *holds.*

Proof of lemma 10

Let first $\beta = 0$. Observe that p_0^{eu} is strictly decreasing in q and

$$p_0^{eu}(q) = 1$$

obtains at $q = \frac{r(1+\delta)}{\delta(1+r)}$. It follows that $p_0^{eu} \geq 1$ is equivalent to $q \geq \frac{r(1+\delta)}{\delta(1+r)}$. Hence, $p_0^{eu} \geq 1$ is inconsistent with the condition necessary for c_t^* to be a submartingale.

Now assume $\beta \in (0;1]$. Consider first the case of $p_\beta^{eu} = 1$ and note, that the value of q, for which $\gamma_t^{eu} = p_\beta^{eu} = 1$ will still be an interior solution of (3) is given by:

$$\frac{(1+r)\left[\left(\frac{(1-q)r}{q(\delta-r)}\right)^{\frac{1}{\beta-1}} - 1\right]}{\delta + r\left[\left(\frac{(1-q)r}{q(\delta-r)}\right)^{\frac{1}{\beta-1}} - 1\right]} = 1,$$

which simplifies to

$$q = \frac{r}{r + (\delta - r)(1+\delta)^{\beta-1}}. \tag{10}$$

Denote

$$\left(\frac{(1-q)\,r}{q\left(\frac{\delta}{p} - r\right)}\right)^{\frac{1}{\beta-1}} =: z.$$

From $\frac{\partial \tilde{\gamma}_t^{eu}}{\partial z} > 0$ and

$$\frac{\partial z}{\partial q} = \frac{1}{\beta-1}\left(\frac{(1-q)\,r}{q\left(\frac{\delta}{p} - r\right)}\right)^{\frac{2-\beta}{\beta-1}} \frac{(-r(\delta-r))}{q^2(\delta-r)^2} > 0$$

it follows that $\frac{\partial \tilde{\gamma}_t^{eu}}{\partial q} > 0$ and since $\tilde{\gamma}_t^{eu}$ falls in price, it follows that the equilibrium price is increasing in the probability of high dividend q. Therefore, for values of q, higher than (10), the price under rational expectations (for a given β) is equal to 1, see figure 16.■

Hence, for all current values of $p_t \leq 1$ the expected utility maximizers (who believe that $p_{t+1} = 1$) will invest their whole initial endowment into the risky asset. Therefore, the interval of values

$$q \in \left[\frac{r}{r + (\delta - r)(1+\delta)^{\beta-1}}; 1\right]$$

corresponds to the case, in which the expected utility maximizers invest their whole initial endowment into a, independently of the price.

Given the assumptions on q and $\bar{u}$, the dynamics of prices and asset holdings can be described as follows:

$$\gamma_t^{*eu} = 1 \text{ for each } t;$$

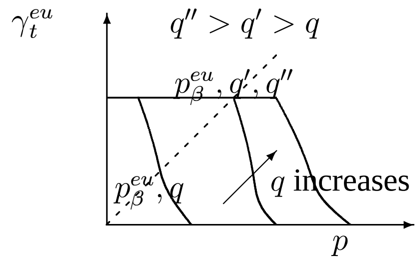

Figure 2

$$\gamma_t^{*cb} = \left\{ \begin{array}{ll} 1, \text{if} & \gamma_{t-1}^{cb} = 1 \text{ and } \delta_t = \delta \text{ or} \\ & \gamma_{t-1}^{cb} = 0 \\ 0, \text{if} & \gamma_{t-1}^{cb} = 1 \text{ and } \delta_t = 0 \end{array} \right\};$$

$$p_t^* = \left\{ \begin{array}{ll} 1, \text{if} & \gamma_{t-1}^{cb} = 1 \text{ and } \delta_t = \delta \text{ or} \\ & \gamma_{t-1}^{cb} = 0 \\ e_t^*, \text{if} & \gamma_{t-1}^{cb} = 1 \text{ and } \delta_t = 0 \end{array} \right\}.$$

Note that the average returns of the two types are identical, as long as both types hold a. Therefore the population shares remain unchanged in such periods:

$$e_t^* = e_{t-1}^*, \text{ if } \gamma_{t-1}^{*cb} = 1.$$

Hence, these periods do not influence the dynamic of population shares. It is, therefore, sufficient to analyze how e_t changes in periods, in which the holdings of both types of investors differ. In such periods the expected (since it depends on a random dividend payment) equilibrium share of case-based decision makers is given by:

$$E\left[c_t^* \mid c_{t-1}^*\right] = q\frac{(1+r)\,c_{t-1}}{(1+r)\,c_{t-1}^* + \frac{(1+\delta)}{p_{t-1}^*}e_{t-1}^*} + (1-q)\,\frac{(1+r)\,c_{t-1}}{(1+r)\left(1-e_{t-1}^*\right) + \frac{1}{p_{t-1}^*}e_{t-1}^*}.$$

Now note, that since in $(t-1)$ only the expected utility maximizers hold a, $p_{t-1}^* = e_{t-1}^*$. Therefore

$$E\left[c_t^* \mid c_{t-1}^*\right] = q\frac{(1+r)\,c_{t-1}^*}{(1+r)\,c_{t-1}^* + (1+\delta)} + (1-q)\,\frac{(1+r)\,c_{t-1}^*}{(1+r)\,c_{t-1}^* + 1} \gtreqless c_{t-1}^{**}$$

$$\Leftrightarrow$$

$$r(1+\delta)+r(1+r)c_{t-1}^{*}-q\delta(1+r)\gtreqless(1+\delta)(1+r)c_{t-1}^{*}+(1+r)^{2}c_{t-1}^{*2}$$

$$r\left(1+\delta+(1+r)c_{t-1}^{*}\right)-(1+r)c_{t-1}^{*}\left(1+\delta+(1+r)c_{t-1}^{*}\right)-q\delta(1+r)\gtreqless 0$$

$$\left(1+\delta+(1+r)c_{t-1}^{*}\right)\left[r-(1+r)c_{t-1}^{*}\right]-q\delta(1+r)\gtreqless 0. \tag{11}$$

It is clear, that for $c_{t-1}^{*}\geq\frac{r}{1+r}$, $E\left[c_{t}^{*}\mid c_{t-1}^{*}\right]<c_{t-1}^{*}$, therefore c_{t}^{*} is a supermartingale and since $e_{t}^{*}+c_{t}^{*}=1$ in each period, it follows that e_{t}^{*} is a submartingale, if $e_{t-1}^{*}\leq\frac{1}{1+r}$. On the other hand, if $c_{t-1}^{*}\rightarrow 0$, the l.h.s. of (11) becomes:

$$(1+\delta)r-q\delta(1+r)>0\text{, if }q<\frac{(1+\delta)r}{(1+r)\delta}.$$

If $q<\frac{(1+\delta)r}{(1+r)\delta}$, the continuity of the l.h.s. of (11) guarantees, that $E\left[c_{t}^{*}\mid c_{t-1}^{*}\right]>c_{t-1}^{*}$ (and hence a submartingale) for c_{t-1}^{*} close to 0. It follows, that for some $\tilde{c}\in\left(0;\frac{r}{1+r}\right)$ $E\left[c_{t}^{*}\mid c_{t-1}^{*}=\tilde{c}\right]=0$. The assertion of the first part of the proposition now follows by defining $\tilde{e}=1-\tilde{c}$.

If $q>\frac{(1+\delta)r}{(1+r)\delta}$, the l.h.s. of (11) is negative for all $c_{t-1}^{*}\in[0;1]$ and therefore $E\left[e_{t}^{*}\mid e_{t-1}^{*}\right]>e_{t-1}^{*}$ for all $e_{t-1}\in[0;1]$. It follows, that e_{t}^{*} is a submartingale on $[0;1]$. Hence, the convergence theorem for martingales applies, i.e. e_{t}^{*} converges almost surely. It follows that on almost each dividend path

$$\lim_{t\rightarrow\infty}\frac{e_{t}^{*}}{e_{t-1}^{*}}=\lim_{t\rightarrow\infty}\frac{(1+\delta_{t})}{1+(1-e_{t}^{*})r+\delta_{t}}=1$$

must hold, which is only possible, if $e_{t}^{*}\rightarrow 1$ with probability 1.

It remains to show that the condition $q<\frac{(1+\delta)r}{(1+r)\delta}$ is consistent with the assumption that $p_{\beta}^{eu}=1$ only for $\beta\in(0;1]$, but not for $\beta=0$.

Lemma 11 *$q<\frac{(1+\delta)r}{(1+r)\delta}$ and*

$$q\geq\frac{r}{r+(\delta-r)(1+\delta)^{\beta-1}}$$

can hold simultaneously only for $\beta\in(0;1]$, but not for $\beta=0$.

Indeed,

$$\frac{r}{r+(\delta-r)(1+\delta)^{\beta-1}}<\frac{(1+\delta)r}{(1+r)\delta}$$

is equivalent to

$$(\delta-r)\left[(1+\delta)^{\beta}-1\right]>0,$$

which is always satisfied for $\beta>0$. Note, that the logarithmic utility function represents the limit case, $\beta=0$, in which the equality holds.■

Proof of proposition 5:

Assume that $e_t \in \left[\max\left\{p^{eu}; 1-p^{eu}+\frac{p^{eu}r^2}{1-p^{eu}}\right\}; 1\right)$.

The proposition will be proved separately for those periods in which the case-based decision-makers hold a and those periods in which they hold b. First note that if $\gamma_t^{cb} = 1$ holds, then the case-based decision-makers continue to hold a at time $t+1$, iff $\delta_{t+1} = \delta$ so that in this case

$$p_{t+1} = p_t = p^{eu}.$$

By assumption, the share of case-based decision-makers satisfies

$$c_t^* = 1 - e_t^* \leq 1 - p^{eu} < p^{eu},$$

since $p^{eu} < \frac{1}{2}$. If the price cannot rise higher than p^{eu}, the average return of the case-based decision-makers is, therefore:

$$\tilde{v}_{t+1}^{cb} = 1 + \frac{\delta}{p^{eu}},$$

whereas the average return of the population is given by

$$\tilde{v}_{t+1} = p^{eu} + \delta + (1+r)(1-p^{eu}) = 1 + r - rp^{eu} + \delta,$$

as long as $e_{t+1}^* < p^{eu}$ holds. Furthermore, since $1 + \frac{\delta}{p^{eu}} > 1 + \delta > \bar{u}$, the young case-based decision-makers invest in a as well so that $\gamma_{t+1}^{cb} = 1$.

Alternatively, if $\delta_{t+1} = 0$, then the highest return that the case-based decision-makers can achieve from a is $1 < \bar{u}$, therefore the young case-based decision-makers will choose b, achieving an average return of at most 1. Since this average return is smaller than the average return of the population, given by:

$$\tilde{v}_{t+1} = p^{eu} + (1+r)(1-p^{eu}) = 1 + r - rp^{eu},$$

the mass of the case-based decision-makers decreases, making it possible for the expected utility maximizers to sustain the price of a at p^{eu} at time $(t+1)$.

If $\delta_{t+1} = \delta$, then the returns and the behavior of the investors in $(t+2)$ is described exactly as in $(t+1)$, except in the case, in which the share of the case-based decision-makers has risen above p^{eu} and does not allow the expected utility maximizers to reduce the price of the risky asset to its fundamental value. This can happen, if the initial c_t^* is relatively high, so that:

$$c_{t+1}^* = \frac{\left(1 + \frac{\delta}{p^{eu}}\right)}{1 + r - rp^{eu} + \delta} c_t^* > p^{eu}. \tag{12}$$

It is, therefore, shown that if p^{eu} is sufficiently large, $c_{t+1}^* < p^{eu}$ holds for all values of $c_t^* \in$

$(0; 1 - p^{eu})$. Indeed, rewrite (12) as

$$c_t^* > \frac{p^{eu}\left(1 + r - rp^{eu} + \delta\right)}{1 + \frac{\delta}{p^{eu}}}.$$

To exclude the case, in which the inequality in (12) holds, it is necessary that:

$$\frac{p^{eu}\left(1 + r - rp^{eu} + \delta\right)}{1 + \frac{\delta}{p^{eu}}} > 1 - p^{eu},$$

or that

$$-rp^{eu^3} + p^{eu^2}\left(2 + r + \delta\right) - \left(1 - \delta\right)p^{eu} - \delta > 0. \tag{13}$$

Note first that for $\delta = \frac{r}{2q}$ ($p^{eu} = \frac{1}{2}$), the l.h.s. is negative and that for $\delta = \frac{r}{q}$ ($p^{eu} = 1$), the l.h.s. is positive. Using now the fact that $p^{eu} = \frac{q\delta}{r}$, rewrite (13) as:

$$q^2\delta^2\left(1 - q\right) + \delta q\left(2q + qr + r\right) - r > 0$$

and since the l.h.s. of this expression is a convex quadratic function, there exists a $\hat{\delta}$, such that for every $\delta > \hat{\delta}$ (13) is satisfied.

The expected value of the share of the case-based decision-makers at time $(t+2)$, given their share at time t, can then be written as[11]:

$$\begin{aligned} E\left[c_{t+2}^* \mid c_t^*, \gamma_t^{cb} = 1\right] &= c_t^* q \frac{\left(1 + \frac{\delta}{p^{eu}}\right)}{R + \delta}\left[q\frac{1 + \frac{\delta}{p^{eu}}}{R + \delta} + \left(1 - q\right)\frac{1}{R}\right] + \\ &\quad + c_t^*\left(1 - q\right)\frac{1}{R}\left[q\frac{\left(1 + r\right)}{R + \delta} + \left(1 - q\right)\frac{\left(1 + r\right)}{R}\right], \end{aligned}$$

where $R = 1 + r - rp^{eu}$. Using simple algebra and the fact that $p^{eu} = \frac{q\delta}{r}$ shows that

$$E\left[c_{t+2}^* \mid c_t^*, \gamma_t^{cb} = 1\right] > c_t^*,$$

if and only if

$$\left(q + r\right)R\left(R\left(1 + r\right) + \delta\left(1 - q\right)\right) + \left(1 - q\right)\left(1 + r\right)\left(R + \delta\left(1 - q\right)\right)\left(R + \delta\right) > \left(R + \delta\right)^2 R^2 \tag{14}$$

holds. If

$$p^{eu} = \frac{q\delta}{r} = 1,$$

meaning that $R = 1$ and $q\delta = r$, condition (9) simplifies to:

$$\left(1 + \delta\right)r\left(\delta - r + qr\right) > 0,$$

[11] In fact, as above, it should be taken into account that the share of the case-based decision-makers might exceed p^{eu} in $(t+2)$ if the risky asset pays a high dividend. However, this will only increase the expected value of c_{t+2}^*. Since the argument relies on showing that the expected value of c_{t+2}^* exceeds c_t^*, neglecting this effect has no influence on the results.

which is always satisfied, since $\delta > r$ holds by assumption. On the other hand, for

$$p^{eu} = \frac{q\delta}{r} = \frac{1}{2}$$

and, hence, $R = 1 + \frac{r}{2}$ and $q\delta = \frac{r}{2}$, (14) is equivalent to

$$\frac{qr}{2} + \frac{qr^2}{4} + \frac{3r^3}{16} - \frac{1}{2} - r - \delta r - \delta^2 r - \delta r^2 > 0,$$

which is never satisfied, since

$$\begin{aligned} \frac{qr}{2} &< \frac{r}{2} < \frac{1}{2} \\ \frac{qr^2}{4} &< r^2 < r \\ \frac{3r^3}{16} &< r^3 < r^2 < \delta r \end{aligned}$$

hold according to the assumption that $\delta > r$, $r \in (0;1)$ and $q \in (0;1)$. Therefore,

$$E\left[c^*_{t+2} \mid c^*_t, \gamma^{cb}_t = 1\right] > c^*_t$$

holds for $\delta = \frac{r}{q}$ and since the expected value of c^*_{t+2} is continuous in δ, it follows that the process c^*_t, c^*_{t+2}, c^*_{t+4}... is a submartingale in some surrounding of $\delta = \frac{r}{q}$. At the same time,

$$E\left[c^*_{t+2} \mid c^*_t, \gamma^{cb}_t = 1\right] < c^*_t$$

holds for $\delta = \frac{r}{2q}$. By continuity of the expected value of c^*_{t+2}, there is, therefore, a value for δ, $\bar{\delta} \in \left(\frac{r}{2q}; \frac{r}{q}\right)$ such that the expected value of c^*_{t+2} exceeds c^*_t for $\delta > \bar{\delta}$.

Now suppose that $\gamma^{cb}_t = 0$. Similar arguments as those stated above allow to write the expected value of c^*_{t+2} as:

$$\begin{aligned} E\left[c^*_{t+2} \mid c^*_t, \gamma^{cb}_t = 0\right] &= c^*_t q \frac{(1+r)}{R+\delta}\left[q\frac{\left(1+\frac{\delta}{p^{eu}}\right)}{R+\delta} + (1-q)\frac{1}{R}\right] + \\ &\quad + c^*_t (1-q)\frac{(1+r)}{R}\left[q\frac{\left(1+\frac{\delta}{p^{eu}}\right)}{R+\delta} + (1-q)\frac{1}{R}\right]. \end{aligned}$$

Again, one should take into account that the mass of the case-based decision-makers could increase above p^{eu} in period $(t+1)$, when the dividend of the risky asset is low. However, this would require that:

$$\frac{(1+r)}{1+r-rp^{eu}} c^*_t > p^{eu},$$

or, equivalently

$$e^*_t < 1 - p^{eu} + \frac{rp^{eu^2}}{1+r},$$

which is excluded by the assumptions made.

Using simple algebra and the fact that $p^{eu} = \frac{q\delta}{r}$ shows that

$$E\left[c_{t+2}^* \mid c_t^*, \gamma_t^{cb} = 0\right] > c_t^*$$

holds if

$$(1+r)\left[R(1+r) + \delta(1-q)\right]\left[R + \delta(1-q)\right] > (R+\delta)^2 R^2 \tag{15}$$

is satisfied. Note that for $p^{eu} = 1$, condition (15) is equivalent to

$$(1+r)(1+\delta-r) > 0,$$

which is always satisfied. For $p^{eu} = \frac{1}{2}$, (15) becomes

$$\frac{3r^2}{8} + \frac{r}{4} + \frac{r^2\delta}{2} + \frac{r\delta}{2} + \frac{\delta}{2} > 0,$$

which is obviously satisfied for all positive values of r and δ. Since the expected value of c_{t+2}^* is continuous in δ, it follows that there is a $\check{\delta} \in \left[\frac{r}{2q}; \frac{r}{q}\right)$ such that

$$E\left[c_{t+2}^* \mid c_t^*, \gamma_t^{cb} = 0\right] > c_t^*$$

for all $\delta > \check{\delta}$. Now choose the maximal of the three values $\hat{\delta}$, $\check{\delta}$, $\bar{\delta}$ and denote it by $\tilde{\delta}$. Let $\tilde{p}^{eu} = \frac{q\tilde{\delta}}{r}$. It follows that $\tilde{p}^{eu} \in \left(\frac{1}{2}; 1\right)$ and that

$$E\left[c_{t+2}^* \mid c_t^*\right] > c_t^*,$$

for $p^{eu} > \tilde{p}^{eu}$ and

$$e_t \in \left[\max\left\{p^{eu}; 1 - p^{eu} + \frac{p^{eu}r^2}{1-p^{eu}}\right\}; 1\right).$$

Since c_{t+2}^* and e_{t+2}^* sum to 1, it follows that

$$\begin{aligned} E\left[e_{t+2}^* \mid e_t^*, \gamma_t^{cb} = 0\right] &< e_t^* \\ E\left[e_{t+2} \mid e_t^*, \gamma_t^{cb} = 1\right] &< e_t^*, \end{aligned}$$

if $p^{eu} > \tilde{p}^{eu}$ and

$$e_t \in \left[\max\left\{p^{eu}; 1 - p^{eu} + \frac{p^{eu}r^2}{1-p^{eu}}\right\}; 1\right)$$

are fulfilled simultaneously.■

Proof of proposition 6:

In Lemma 19 in Sciubba (1999, p. 40) it is demonstrated, that a supermartingale bounded between $[0;1]$ and starting below 1 cannot converge to its upper boundary with probability 1. The following argument follows closely the proof of Proposition 17 in Sciubba (1999, pp. 40-41). Suppose that e_t^* converges to 1 with strictly positive probability and denote the event on which this happens by Θ. Now consider e_t^* on the event Θ and suppose that on Θ $\Pr\{e_t^* \to 1\} = 1$. Denote by $\Theta_0 \subseteq \Theta_1 \subseteq ...\Theta_t \subseteq ...\Theta$ the natural filtration of Θ. Since $\Pr\{\Theta\} > 0$, and since the process of the dividends is i.i.d., the law of large numbers applies and the distribution of dividends on Θ coincides with the distribution of the dividends on Ω, the set of all possible dividend paths. Especially, $\Pr\{\delta_t = \delta \mid \Theta_{t-1}\} = \Pr\{\delta_t = \delta\} = q$. Therefore the process e_t^* on Θ can be described in exactly the same way, as the process e_t^* on Ω and therefore e_t^* is a supermartingale on Θ. But, according to Lemma 19 in Sciubba (1999, p. 40) $\Pr\{e_t^* \to 1 \mid \Theta\} \neq 1$, since e_t^* is a supermartingale bounded above by 1. Therefore, there is no event with positive probability, on which $e_t^* \to 1$ occurs almost surely. Hence, $\Pr\left\{e_t^* \to 1\right\} = 0$ and the case-based decision makers survive with probability 1.■

Proof of proposition 7:

As was demonstrated in lemma 10, the condition

$$q \geq \frac{r}{r + (\delta - r)(1 + \delta)^{\beta - 1}}$$

implies that $p_\beta^{eu} = 1$ and the expected utility maximizers will choose $\gamma^{eu} = 1$ in each period. The case-based decision makers will always hold b, since their aspiration level is between 1 and $(1 + r)$. Therefore the price of a will be $p_t^* = e_t^* = 1 - c_t^*$ for each t. The return of the case-based decision makers is $(1 + r)$ in each period, whereas the average return of the population is given by

$$\tilde{v}_t = e_t^* + \delta_t + (1 - e_t^*)(1 + r) = 1 + \delta_t + c_t^* r.$$

Hence $E\left[c_{t+1}^* \mid c_t^*\right]$ can be written as:

$$E\left[c_{t+1}^* \mid c_t^*\right] = \left[q \frac{(1 + r)}{1 + c_t^* r + \delta} + (1 - q)\frac{(1 + r)}{1 + c_t^* r}\right] c_t^* \gtreqless c_t^*$$

This simplifies to:

$$(1 - c_t^*) r (1 + c_t^* r + \delta) - q (1 + r) \delta \gtreqless 0. \tag{16}$$

For $c_t \to 0$ the left hand side becomes:

$$r(1 + \delta) - q(1 + r)\delta > 0, \text{ iff } q < \frac{(1 + \delta) r}{(1 + r)\delta}.$$

For $c_t^* = 1$, the left hand side is negative. Since the left-hand side of (16) is a quadratic function with a negative coefficient in front of c_t^{*2}, it follows that for $q < \frac{(1+\delta)r}{(1+r)\delta}$, there exists a unique $\hat{c} \in (0; 1)$, for which the left hand side of (16) is 0. For $c_t^* > \hat{c}$, c_t^* is a supermartingale and vice versa. Now denote by $\hat{e} = 1 - \hat{c}$ the share of expected utility maximizers corresponding to the share $\hat{c}$ of case-based decision makers. It follows that e_t^* is a supermartingale for $e_t^* > \hat{e}$ and a submartingale for $e_t^* < \hat{e}$.

If $q \geq \frac{(1+\delta)r}{(1+r)\delta}$, then c_t^* is a supermartingale on the whole interval $[0;1]$. Therefore e_t^* is a submartingale on $[0;1]$. Hence, the convergence theorem for martingales applies, i.e. e_t^* converges almost surely. It follows that on almost each dividend path

$$\lim_{t\to\infty} \frac{e_t^*}{e_{t-1}^*} = \lim_{t\to\infty} \frac{\frac{(e_t^*+\delta_t)}{e_t^*+\delta_t+(1+r)\left(1-e_{t-1}^*\right)}}{\frac{\left(e_{t-1}^*+\delta_{t-1}\right)}{e_{t-1}^*+\delta_{t-1}+(1+r)\left(1-e_{t-2}^*\right)}} = 1.$$

must hold. Let $e_t^* = e_{t-1}^* = e_{t-2}^*$, then it follows that:

$$\lim_{t\to\infty} \frac{(e_t^*+\delta_t)}{e_t^*+\delta_t+(1+r)(1-e_t^*)} = \lim_{t\to\infty} \frac{(e_t^*+\delta_{t-1})}{e_t^*+\delta_{t-1}+(1+r)(1-e_t^*)}.$$

Note, however, that since δ_t is a stochastic process, this equality can only hold, if $e_t^* \to 1$ with probability 1, hence if the average return of the expected utility maximizers coincide with the average return of the society in each period of time.

The fact that $q < \frac{(1+\delta)r}{(1+r)\delta}$ is consistent with $p_\beta^{eu} = 1$ only for $\beta\,(0;1]$, but not for $\beta = 0$ is analogous to the one given in lemma 11.■

Proof of proposition 8:

Rewrite conditions (11) and (16) as:

$$-(1+r)^2 c_t^{*^2} - (1+r)\,c_t^*\,(1+\delta-r) + (1+\delta)\,r - q\delta\,(1+r)$$

and

$$-r^2 c_t^{*^2} - (1+r)\,c_t^*\,(1+\delta-r) + (1+\delta)\,r - q\delta\,(1+r)\,,$$

respectively. One easily sees that

$$\begin{aligned} & -(1+r)^2 c_t^{*^2} - (1+r)\,c_t^*\,(1+\delta-r) + (1+\delta)\,r - q\delta\,(1+r) \\ < \quad & -r^2 c_t^{*^2} - (1+r)\,c_t^*\,(1+\delta-r) + (1+\delta)\,r - q\delta\,(1+r) \end{aligned}$$

always holds. Hence, the sole positive root of (11) $\tilde{e}$ is always greater than the sole positive root of (16), $\hat{e}$.■

Proof of proposition 9:

Consider first the case of $\beta = 1$. Note that for each p_1^{eu} it is possible to choose the initial mass of expected utility maximizers e_t^* to be sufficiently high so as to support the price of a at p_1^{eu} in the next period. The restrictions on the aspiration level of the case-based implies that they will hold b in each period. Their average return is therefore $\tilde{v}_t^{cb} = (1+r)$, whereas the average return of the population is $\tilde{v}_t = 1 + \delta_t + r - rp_1^{eu}$. Hence $E\left[c_{t+1}^* \mid c_t^*\right]$ can be written as:

$$E\left[c_{t+1}^* \mid c_t^*\right] = \left[q\frac{(1+r)}{1+r-rp_1^{eu}+\delta} + (1-q)\frac{(1+r)}{1+r-rp_1^{eu}}\right] c_t^* \gtreqless c_t^*$$

This easily simplifies to:

$$-q(1+r)\delta \gtreqless -rp_1^{eu}\left(1+r-rp_1^{eu}+\delta\right) \tag{17}$$

and by using the fact that $p_1^{eu} = \frac{q\delta}{r}$ one obtains that c_t^* is a submartingale, if

$$q\delta^2(1-q) > 0,$$

which is always satisfied for q and $\delta \in (0;1)$[12]. Since c_t^* is a submartingale and since $e_t^* = 1 - c_t^*$, it follows, that e_t^* is a supermartingale on $\left[\max\left\{p_1^{eu}; \frac{p_1^{eu}\left(1+r-rp_1^{eu}\right)}{(1+r)}\right\}; 1\right]$.

Now suppose that $\beta \in (0;1]$. Now it is no longer possible to choose the mass of the case-based investors in such a way that the equilibrium price equals p_β^{eu}. Indeed, to support the price at p_β^{eu} in presence of a positive mass of case-based investors, the expected utility maximizers would have to choose a higher γ^{eu}. But they would not be ready to do so, unless they could pay a lower price for the risky asset than p_β^{eu}. However, it is easy to see that if the share of case-based decision-makers in the market converges to 0, the equilibrium price will converge to p_β^{eu}. In the limit, the condition for c_t^* to be a submartingale is determined by equation (17) and is equivalent to:

$$p_\beta^{eu} \in \left((1+r+\delta) - \sqrt{(1+r+\delta) - 4q\delta(1+r)}; (1+r+\delta) + \sqrt{(1+r+\delta) - 4q\delta(1+r)}\right).$$

Since p_β^{eu} is increasing in β and since p_1^{eu} satisfies this condition, it follows that

$$p_\beta^{eu} < (1+r+\delta) + \sqrt{(1+r+\delta) - 4q\delta(1+r)}$$

for all $\beta \in [0;1]$. Moreover, for $\beta = 0$,

$$p_0^{eu} = (1+r+\delta) - \sqrt{(1+r+\delta) - 4q\delta(1+r)},$$

hence, for all $\beta \in (0;1]$, the condition is satisfied and c_t^* is a submartingale in some surrounding of $c_t = 0$, implying that there exists an $\check{e}(\beta) \in (0;1)$ such that e_t^* is a supermartingale on $[\check{e}(\beta); 1]$.■

[12] For $q\delta = 0$ both the case-based decision makers and the expected utility maximizers hold only asset b and achieve therefore identical returns in each period of time. The mass of the case-based decision makers thus remains constant.

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