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Abstract

Agents in a large population are randomly matched to play a material payoff game. They may have preferences that are different from the material payoffs. Agents learn equilibrium strategies according to their preferences before evolution changes the preference distribution in the population according to fitness. When agents know the preferences of the opponent in a match, only efficient symmetric strategy profiles of the material payoff game can be stable. When agents do not know the preferences of the opponent, only Nash equilibria of the material payoff game can be stable. For 2×2 symmetric games I characterize preferences that are stable.

Keywords: two-speed evolution; symmetric games; evolutionary stability.

JEL Codes: C72, A13

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1 Introduction

The goal of this paper is to analyze the stability of strategies and preferences in symmetric games under two-speed evolution when the set of admissible preferences is not restricted. Two-speed evolution refers to the case when the evolution of behavior for given preferences is much (infinitely) faster than the evolution of preferences. Two-speed evolution is an extension of the indirect evolution approach of Güth and Yaari (1992) to multiple equilibria that are stable under evolution of strategies.

Indirect evolution works on preferences through equilibrium strategies. A game is given by a function that for each strategy profile specifies the material payoff *(fitness)* to the players. Each player, however, is supplied with genetically programmed preferences over strategy profiles, represented by a von Neumann-Morgenstern utility function that does not necessarily coincide with the material payoff function. When matched, players play the game given these preferences. Players arrive at an equilibrium of this subjective game by a learning process and therefore only 'stable' equilibria are possible.[1] The evolutionary success of the players is determined by the fitness they receive from playing equilibrium strategies. Different preferences lead to different strategies and, generally, to different fitness. Evolution selects preferences with higher fitness. I am interested in certain stable stationary points of this process.

Some work using the indirect evolution approach has been done before for certain games. For example, evolution of trust (Güth and Kliemt, 1998), evolution of fairness (Huck and Oechssler, 1999), evolution of reciprocity (Sethi and Somanathan, 2001), and evolution of preferences for sales in duopoly (Dufwenberg and Güth, 1999) have been analyzed. However, the set of admissible preferences in those papers is usually assumed to be a one-dimensional subset of all possible preferences. This restriction may lead to results that are not robust to an enlargement of the set of admissible preferences (see Bester and Güth, 1998; Bolle, 2000; and Possajennikov, 2000). Therefore, it is important to consider as large a set of admissible preferences as possible. I consider as admissible any preferences that can be represented by a von Neumann-Morgenstern expected utility function.

I define a population state as a distribution of preferences and distribu-

[1] Güth and Yaari (1992) analyzed a game in which 'stable' equilibrium was unique. Here I do not require a 'stable' equilibrium to be unique.

tions of strategies in each subpopulation with given preferences, and I check for stability of population states against particular perturbations. Preferences that are present in a stable population state are called *indirectly stable*. A population state induces a strategy profile. Strategy profiles that are induced by a stable population state are considered *indirectly stable*. I am interested in the following questions. Which preferences are stable for a given game? Are selfish preferences, that is, preferences whose utility function coincides with the material payoff function, stable? Is a stable strategy profile an equilibrium of the material payoff game? Is it efficient from the material payoff point of view? For the first question, I give a relatively complete answer only for 2×2 games, since in larger games the set of admissible preferences is too large. Other questions can be answered quite generally.

I analyze two informational assumptions. In the complete information case agents in a match know the preferences of the opponent. With incomplete information they know only the distribution of preferences in the population. The results are strikingly different: with complete information only efficient strategy profiles can be stable, while with incomplete information only Nash equilibria of the material payoff game can be stable. Selfish preferences are not necessarily stable with complete information, while with incomplete information they are (almost) always stable.

These results are in line with the results of similar models of Ely and Yilankaya (2001), Ok and Vega-Redondo (2001), and Dekel et al. (1998). The last paper has the model closest to mine as it also considers an infinite population playing a symmetric game, discrete distribution of preferences, and a static stability concept with respect to evolution of preferences. The main difference of my model is the consideration of two-speed evolution, that is, the requirement that the equilibrium is 'stable' with respect to learning. To my knowledge, only in Sandholm (2001) two-speed evolution was explicitly considered, but the analysis was restricted to certain 2×2 games and certain preferences. Another difference is in the formulation of stability in the incomplete information case, where I allow for small changes in the strategy of incumbents in the post-entry population. Compared with the results of Dekel et al. the first difference leads to a stronger condition for stability of strategy profiles, while the second one leads to a weaker condition. These differences are illustrated on examples.

I formulate the model of two-speed evolution in symmetric games in Section 2. Section 3 analyzes the complete information case and Section 4 the incomplete information case. Section 5 concludes.

2 The Model

2.1 Games

2.1.1 General Games

The basis for the analysis is a given two-player symmetric finite game $G = (N, S, u), N = \{1, 2\}, u : S \times S \to \mathbb{R}^2, u(s_i, s_j) = u_1(s_i, s_j) = u_2(s_j, s_i)$. The payoff function $u(\cdot)$ is the *material payoff function* and the game G is the *material payoff game*. The material payoff function represents *fitness* on which evolution works. The mixed strategy extension ΔS of the set S is denoted by Σ. Let $|S| = m$, $\sigma^i = (p_1^i, \dots, p_m^i) \in \Sigma$ and let $\sigma = (\sigma^1, \sigma^2) \in \Sigma \times \Sigma$ be a strategy profile. The material payoff function $u(\cdot)$ extends to the set of mixed strategy profiles $u(\sigma) = \sum_{i=1}^m \sum_{j=1}^m p_i^1 p_j^2 u(s_i, s_j)$. A strategy profile $\sigma = (\sigma^1, \sigma^2)$ is symmetric if $\sigma^1 = \sigma^2$. A symmetric strategy profile σ is *efficient* if for any other symmetric strategy profile σ' $u(\sigma) \geq u(\sigma')$. A symmetric strategy profile σ is *strongly efficient* if for any other strategy profile $\sigma' \in \Sigma$ (not necessarily symmetric) $u(\sigma) \geq u(\sigma')$. A *correlated* strategy profile σ_c specifies the probability with which each pair of pure strategies is played, that is, $\sigma_c \in \Delta(S \times S)$, while a usual strategy profile $\sigma \in \Delta S \times \Delta S$.

For a given strategy σ^j of player j the best response $BR^i(\sigma^j)$ of player i is the set of strategies σ^i such that for any other strategy $\rho^i \in \Sigma$ $u(\sigma^i, \sigma^j) \geq u(\rho^i, \sigma^j)$. The best response correspondence BR maps each strategy profile $\sigma = (\sigma^1, \sigma^2)$ to the set $BR(\sigma) = BR^1(\sigma^2) \times BR^2(\sigma^1)$. A strategy profile σ is *Nash equilibrium* if $\sigma \in BR(\sigma)$. A Nash equilibrium σ is symmetric if σ is symmetric. A strategy σ^i is *neutrally stable strategy* if $\exists \varepsilon^* > 0$ such that $(1-\varepsilon)u(\sigma^i, \sigma^i) + \varepsilon u(\sigma^i, \sigma^j) \geq (1-\varepsilon)u(\sigma^j, \sigma^i) + \varepsilon u(\sigma^j, \sigma^j)$ $\forall \sigma^j \in \Sigma$ $\forall \varepsilon \in (0, \varepsilon^*)$.

2.1.2 2×2 Games

Some of the results in the paper are for 2×2 games. A 2×2 symmetric material payoff game is given by the symmetric bimatrix

	s_1	s_2
s_1	α, α	β, γ
s_2	γ, β	δ, δ

I focus on games where one symmetric pure strategy profile is more efficient than the other, that is, $\alpha > \delta$. The analysis is easily adapted for the case $\alpha = \delta$ but it adds an extra case without much additional insight.

By adding a constant to all payoffs and multiplying all payoffs by a positive constant, the game can be transformed into

	s_1	s_2
s_1	$1,1$	b,c
s_2	c,b	$0,0$

where $b = \frac{\beta-\delta}{\alpha-\delta}, c = \frac{\gamma-\delta}{\alpha-\delta}$. These transformations do not affect efficiency of strategy profiles, the equilibria of the game and the notion of stability I use.

The following lemma is useful. I identify a mixed strategy $\sigma^i = (p, 1-p)$ with the probability p of playing s_1.

Lemma 1 *If $b + c \leq 2$ the symmetric efficient strategy profile is $(1,1)$ with fitness 1; otherwise the symmetric efficient strategy profile is (p,p), where $p = \frac{b+c}{2(b+c-1)}$, with fitness $\frac{(b+c)^2}{4(b+c-1)}$.*

Proof. The material payoffs of both players in a symmetric strategy profile (p,p) is $p^2 + p(1-p)(b+c)$. The maximum of this expression with respect to $p \in [0,1]$ is achieved at $p^* = 1$ if $b+c \leq 2$, and at $p^* = \frac{b+c}{2(b+c-1)}$ if $b+c > 2$. Substituting the values p^* into the expression for the material payoffs leads to the fitness in the formulation of the lemma. ■

The class of symmetric 2×2 games can be divided into two subclasses according to whether the efficient symmetric strategy profile is pure or mixed. Further division can be done, according to the best reply correspondence.

1. $b + c > 2$. The efficient symmetric strategy profile is mixed.
   (a) $1 \geq c, b > 0$. The unique symmetric equilibrium is (s_1, s_1).
   (b) $1 < c, b \leq 0$. The unique symmetric equilibrium is (s_2, s_2).
   (c) $1 < c, b > 0$. The unique symmetric equilibrium is mixed, but is generally not equal to the efficient symmetric strategy profile. They are equal only when $b = c$.

2. $b + c \leq 2$. The efficient symmetric strategy profile is (s_1, s_1).
   (a) $1 \geq c, b \geq 0$, at least one inequality is strict. If $b > 0$ the unique symmetric equilibrium is (s_1, s_1), otherwise (s_2, s_2) is also an equilibrium.
   (b) $1 > c, b < 0$. Coordination Problem: two pure strategy symmetric equilibria $(s_1, s_1), (s_2, s_2)$ and one mixed.

(c) $1 \leq c, b \leq 0$, at least one inequality is strict. If $1 < c$ it is Prisoners' Dilemma: the unique equilibrium is (s_2, s_2). Otherwise (s_1, s_1) is also an equilibrium.

(d) $1 < c, b > 0$. Chicken type game. The unique symmetric equilibrium is mixed.

(e) $1 = c, b = 0$. Any symmetric strategy profile is an equilibrium.

I want to find which combinations of preferences and strategy profiles are stable for each type of games.

2.2 Preferences

2.2.1 General Games

Let a material payoff game G be given. Subjective preferences, which do not have to coincide with the material payoffs, are similarly defined on the set of strategy combinations $S \times S$. Preferences of agent i can be represented by a utility function $v_i : S \times S \to \mathbb{R}$. Preferences are assumed to satisfy the axioms of expected utility of von Neumann and Morgenstern. Then the utility function $v_i(\cdot)$ extends to the set of mixed strategy profiles $\Sigma \times \Sigma$ in the straightforward way, $v_i(\sigma^1, \sigma^2) = \sum_{j=1}^{m} \sum_{k=1}^{m} p_j^1 p_k^2 v_i(s_j, s_k)$. Preferences are determined by the values of the utility function representing these preferences on pure strategy profiles. The set of admissible preferences W_G for a given game G is equivalent to the set $\mathbb{R}^{m^2}$. In what follows I identify preferences with the utility function representing them.

Analogously with best responses with respect to the material payoffs, best responses with respect to subjective preferences are defined. For a given strategy $\sigma^k \in \Sigma$ the best response $BR_i(\sigma^k)$ of a player with preferences v_i is the set of strategies $\sigma^i \in \Sigma$ such that for any $\rho^i \in \Sigma$ $v_i(\sigma^i, \sigma^k) \geq v_i(\rho^i, \sigma^k)$.

2.2.2 2×2 Games

In 2×2 games it is convenient to divide admissible preferences into the following types:

1. (St1): $v_i(s_1, s_1) \geq v_i(s_2, s_1), v_i(s_1, s_2) \geq v_i(s_2, s_2)$, at least one inequality is strict;

2. (CO): $v_i(s_1, s_1) > v_i(s_2, s_1), v_i(s_1, s_2) < v_i(s_2, s_2)$;

3. (NC): $v_i(s_1, s_1) < v_i(s_2, s_1), v_i(s_1, s_2) > v_i(s_2, s_2)$;

4. (St2): $v_i(s_1, s_1) \leq v_i(s_2, s_1), v_i(s_1, s_2) \leq v_i(s_2, s_2)$, at least one inequality is strict;

5. (BB): $v_i(s_1, s_1) = v_i(s_2, s_1), v_i(s_1, s_2) = v_i(s_2, s_2)$.

Preferences v_i belong to type k if v_i satisfies the inequalities for type k. Players with type (St1) preferences perceive the game as having (possibly weakly) dominant strategy s_1, while players of type (St2) think that s_2 is dominant. Type (CO) players (COordinators or COnformists) perceive that s_1 is best reply to s_1 and s_2 on s_2, while type (NC) (NonConformists) players prefer to play s_1 on s_2 and s_2 on s_1. Finally, there are preferences of type (BB) ("Big Bores") for which the strategies are equivalent. The players with such preferences are indifferent between strategies for any strategy of the opponent and therefore can play any strategy in equilibrium.

An interpretation of having different preferences can be seen on the example of Prisoners' Dilemma. Some agents may have selfish preferences while others might not like to let their opponents down and therefore have a higher subjective utility from mutual cooperation than from defecting against a cooperator. Yet others can be heroic unconditional cooperators who derive a higher utility even from being defected upon, that is, they prefer to sacrifice themselves in favor of the other player.

The subjective utility functions can represent many preferences. It is clear from the definition of types that such preferences as biases towards a particular strategy (Sandholm, 2001), the desire to conform, and the desire to differ can be represented. Altruistic and spiteful preferences (Possajennikov, 2000) can be represented as well since one can compute the sums (or the differences) of the material payoffs for each strategy combination. Furthermore, preferences represented by any well behaved function of material payoffs are admissible too. Note that the preferences of a player are independent of the preferences of the opponent; thus the reciprocal preferences of Levine (1998) and Sethi and Somanathan (2001) are not directly considered in this setup. However, their main property of being able to use in equilibrium different strategies against opponents with different preferences who nevertheless play the same strategy can be imitated by (BB) preferences.

Though the agents know their preferences, they do not need to know what the material payoffs are. Evolution, described in the next subsection, will choose those preferences that have higher fitness.

2.3 Evolution

There is a large (infinite) populations of agents randomly matched each period to play the given symmetric material payoff game G. The agents are characterized by the subjective preferences they have, and by strategies they play. Both the distribution of preferences in the population and strategies used by agents evolve. The change in strategies is, however, much faster, and is referred to as learning, while the change in the distribution of preferences is truly evolutionary.

With respect to the strategies of the players I consider two models differing in informational assumptions. In a match, the individuals either know the preferences of the opponent or they do not. The models differ significantly to deserve to be described separately in the following sections, though they have some common features. The differences also influence the evolutionary process on preferences, but in this section I highlight common points of evolution of preferences that are independent of informational assumptions. Suppose for the moment that given the strategies played by the individuals one can calculate the expected fitness from a play of the material payoff game.

I focus on states that contain finite number of different preferences $\{v_1, \ldots, v_n\}$. The state of the population at a given period of evolutionary time can be described by the proportions of players with each preferences, $\mu_1, \ldots, \mu_n, \sum_{i=1}^{n} \mu_i = 1$. Due to evolution the proportions change over time. Let the average (over different strategies employed in the subpopulations) expected material payoff in the population of players with preferences v_i from an encounter with a player with preferences v_j be u_{ij}. Then the average expected fitness of players with preferences v_i is $u_i = \sum_{j=1}^{n} \mu_j u_{ij}$. The main assumption on the evolutionary process is that it is monotone in the following sense: the proportion of players with preferences v_i increases relative to the proportion of players with preferences v_j iff $u_i > u_j$.

I do not specify the process further, but focus instead on stationary states. A state is stationary if average fitness in all subpopulations is the same. I check which stationary states are robust against the appearance of an arbitrarily small proportion of mutants with some other preferences, similar to the concepts of evolutionarily and neutrally stable strategy. Before specifying this evolutionary stability concept I formalize the learning process on strategies.

3 Complete Information

3.1 General Games

In this section the individuals in a match know the preferences of the opponent. Therefore they can use different strategies against opponents with different preferences. The state of the subpopulation of players with preferences v_i is described by the proportions of players that use each strategy s_k against an opponent with preferences v_j. Denote this proportion by x_k^{ij}. The state of the population can be described by a 3-dimensional matrix

$$\begin{array}{cccc} \mu_1 & \mu_2 & \cdots & \mu_n \\ (x_1^{11}, \ldots, x_m^{11}) & (x_1^{21}, \ldots, x_m^{21}) & \cdots & (x_1^{n1}, \ldots, x_m^{n1}) \\ (x_1^{12}, \ldots, x_m^{12}) & (x_1^{22}, \ldots, x_m^{22}) & \cdots & (x_1^{n2}, \ldots, x_m^{n2}) \\ \cdots & \cdots & \cdots & \cdots \\ (x_1^{1n}, \ldots, x_m^{1n}) & (x_1^{2n}, \ldots, x_m^{2n}) & \cdots & (x_1^{nn}, \ldots, x_m^{nn}) \end{array}$$

where the first row represents the proportions of agents with different preferences in the population, the column i under μ_i represents what strategies players with preferences v_i use against players with each of the other preferences, and the row j represents what strategies are used against players with preferences v_j. Let $x^{ij} = (x_1^{ij}, \ldots, x_m^{ij})$. The vector x^{ij} induces a mixed strategy. Note that players cannot condition their strategies on the role in the game (player 1 or 2) but only on the preferences of the opponent. Therefore vectors x^{ii} induce a symmetric strategy profile in the symmetric game between players with preferences v_i, while the pair of vectors $\{x^{ij}, x^{ji}\}$ induces a strategy profile in the asymmetric game between players with preferences v_i and v_j. I refer to x^{ii} as the state of the game between players with preferences v_i and to the pair $\{x^{ij}, x^{ji}\}$ as the state of the game between players with preferences v_i and v_j. The state of the whole population is denoted as $\{(\mu_i)_{i=1}^n; (x^{ij})_{i,j=1}^n\}$, where $\mu_i \neq 0$.

In each subpopulation with given preferences there is a learning process. Due to this process the proportions of players using given strategies change over time. The learning process operates on the subjective preferences of the players. Learning in a match against a player with preferences v_i is independent of learning in a match against a player with other preferences v_j. Therefore there are n one-population learning processes for each subpopulation and $\frac{n(n-1)}{2}$ two-population learning processes, one for each pair of subpopulations. The learning processes are much faster than the evolutionary process on preferences, thus μ_i's are fixed from the point of view of learning.

The game between players with preferences v_i and v_j is a finite game $G_{i,j} = (N, S, \{v_i, v_j\})$. The best response correspondence maps each strategy profile $\sigma = (\sigma^i, \sigma^j)$ to $BR_{i,j}(\sigma) = BR_i(\sigma^j) \times BR_j(\sigma^i)$. A strategy profile σ is a Nash equilibrium in the game $G_{i,j}$ if $\sigma \in BR_{i,j}(\sigma)$. For a set X of strategy profiles $BR_{i,j}(X) = \cup_{\sigma \in X} BR_{i,j}(\sigma)$.

I use the following (weak) notion of stability with respect to the learning process. First, if the learning process in not in a Nash equilibrium, players will be tempted to change strategies. Therefore only Nash equilibria can be learning stable. Second, players may change occasionally to alternative best replies. Learning should be able to lead back to the original state. The notion of the minimal set closed under rational behavior (curb), due to Basu and Weibull (1991), fulfills these requirements.

Definition 1 *A set X of strategy profiles is **closed under rational behavior (curb)** in the game between players with preferences v_i, v_j if X is the product of nonempty compact subsets of strategy sets Σ and $BR_{i,j}(X) \subset X$. A **minimal curb** set is a curb set that does not contain any proper subset that is a curb set.*

For a given pair of preferences, the strategy profile used in the play between the subpopulations is learning stable if it is a Nash equilibrium and belongs to a minimal curb set. The idea is that if an equilibrium does not belong to a curb set, occasional use of alternative best replies will lead out of the set and the play will never come back. For a game between players with the same preferences, that is, for a one-population learning process the additional requirement is that the Nash equilibrium is symmetric.

Definition 2 *The state x^{ii} of the game between players with preferences v_i is **learning stable** if x^{ii} induces a strategy profile that is a symmetric Nash equilibrium and belongs to a minimal curb set in this game.*

Definition 3 *The state $\{x^{ij}, x^{ji}\}$ of the game between players with preferences v_i and v_j is **learning stable** if $\{x^{ij}, x^{ji}\}$ induce a strategy profile that is a Nash equilibrium and belongs to a minimal curb set in this game.*

I will sometimes call an equilibrium strategy profile that belongs to a minimal curb set 'learning stable equilibrium'.

Now consider the whole population. The population state is learning stable if all games in it reached learning stable states.

Definition 4 *A population state* $\{(\mu_i)_{i=1}^n; (x^{ij})_{i,j=1}^n\}$ *is* ***learning stable*** *if* x^{ii} *is learning stable* $\forall i$ *and pairs* $\{x^{ij}, x^{ji}\}$ *are learning stable* $\forall i, j, i \neq j$.

Given the population state, the expected fitness of each subpopulation can be calculated. The expected fitness of a player with preferences v_i against a player with preferences v_j is $u_{ij} = \sum_{k=1}^m \sum_{l=1}^m x_k^{ij} x_l^{ji} u(s_k, s_l)$. Against a randomly chosen opponent, the expected fitness of a player with preferences v_i is $u_i = \sum_{j=1}^n \mu_j u_{ij}$. This is the average expected fitness of the subpopulation of players with preferences v_i that is used for evolution.

Let the population state be $\{(\mu_i)_{i=1}^n; (x^{ij})_{i,j=1}^n\}$. The set $\{v_1, \ldots, v_n\}$ is called the *support* of the population state. A population state is stationary with respect to evolution if all subpopulations have the same average expected fitness u_i.

The evolutionary process alone does not bring new preferences to the population. Any monomorphic population, that is, a population where all agents have the same preferences is stationary. Therefore I check the robustness of a state against an invasion by mutants.

Definition 5 *A stationary population state* $\{(\mu_i)_{i=1}^n; (x^{ij})_{i,j=1}^n\}$ *is* ***indirectly evolutionarily stable*** *if*

(i) it is learning stable;

(ii) $\forall v_k \notin \{v_i\}_{i=1}^n \exists \varepsilon^* > 0$ *such that* $\forall \varepsilon \in (0, \varepsilon^*), \forall i\ \forall x^{ik}, x^{ki}, x^{kk}$ *such that* x^{kk} *is learning stable and* $\{x^{ik}, x^{ki}\}$ *are learning stable,* $u_i > u_k$ *in the population state* $\{(1-\varepsilon)(\mu_i)_{i=1}^n + \varepsilon\mu_k, (x^{ij})_{i,j=1}^n, (x^{ik})_{i=1}^n, (x^{ki})_{i=1}^n, x^{kk}\}$.

The second part of the definition requires the state to be evolutionary stable against an appearance of an arbitrarily small proportion of mutants with arbitrary preferences, whose behavior, and the behavior of other players against them is learning stable. This justifies the use of the term 'indirect' in the definition: stability with respect to evolutionary process is affected only through learning stable behavior. Note that the relative proportions and the behavior of the incumbents against each other do not change after the appearance of the mutants. With probability $(1-\varepsilon)$ an incumbent is matched against another incumbent and gets on average the same fitness as before the appearance of the mutants. The second condition in the definition can be rewritten as $(1-\varepsilon)u + \varepsilon u_{ik} > (1-\varepsilon)\sum_j \mu_j u_{kj} + \varepsilon u_{kk}\ \forall i$ for sufficiently small ε, where u is the fitness incumbents get against each other (necessarily the same for all incumbents in a stationary state), u_{ik} is the fitness incumbents i get against mutants, u_{kj} is the fitness mutants get against incumbents j and u_{kk} is the fitness mutants get among themselves. Thus, one can compare first the fitness of incumbents among themselves and the fitness of mutants against incumbents, and only if they are equal, the fitness against mutants counts.

Lemma 2 *No population state with finite n is indirectly evolutionarily stable.*

Proof. Consider mutants that are indifferent between all strategies. There are infinite number of such preferences obtained by the linear transformations from one another. Therefore there are such preferences that are not present in the population with finite number of preferences. The mutants can mimic such equilibrium strategies in the population that bring the highest fitness against particular incumbents (see the proof of Lemma 3). All these equilibria in the games between incumbents and mutants are learning stable since for mutants the unique minimal curb set is the set of all strategies. The mutants then get not lower fitness than the incumbents. ■

One needs to relax conditions on stability in the flavor of neutral stability, so that mutants can appear but they will not grow.

Definition 6 *A stationary population state $\{(\mu_i)_{i=1}^n, (x^{ij})_{i,j=1}^n\}$ is **indirectly stable** if*

(i) it is learning stable;

(ii) $\forall v_k \notin \{v_i\}_{i=1}^n, \exists \varepsilon^* > 0$ such that $\forall \varepsilon \in (0, \varepsilon^*), \forall i\ \forall x^{ik}, x^{ki}, x^{kk}$ such that x^{kk} is learning stable and $\{x^{ik}, x^{ki}\}$ are learning stable, $u_i \geq u_k$ in the population state $\{(1-\varepsilon)(\mu_i)_{i=1}^n + \varepsilon\mu_k, (x^{ij})_{i,j=1}^n, (x^{ik})_{i=1}^n, (x^{ki})_{i=1}^n, x^{kk}\}$.

The notion of indirectly stable state will be the central notion in the paper. This notion of stability is a rather weak one, since it allows mutants to appear (though not grow). Nevertheless, as will be shown, even this weak notion of stability is often too restrictive.

A population state consists of preferences and strategies the players with these preferences use. Below I formulate when given preferences are stable, and when given strategies are stable.

Definition 7 *Preferences v_k are **indirectly stable** if there exists an indirectly stable population state $\{(\mu_i)_{i=1}^n, (x_{ij})_{i,j=1}^n\}$ with v_k in the support of this state.*

A given population state induces a symmetric strategy profile that may be correlated. A pair of players with given preferences $\{v_i, v_j\}$ is matched with probability $\mu_i \mu_j$. This match induces strategy profile (x^{ij}, x^{ji}). Averaging over all matches, the induced strategy profile in the population state is $\sum_{i=1}^n \sum_{j=1}^n \mu_i \mu_j (x^{ij}, x^{ji})$. A given pure strategy profile (s_k, s_l) is played with probability $p_{kl} = \sum_{i=1}^n \sum_{j=1}^n \mu_i \mu_j x_k^{ij} x_l^{ji}$. Denote by $x = (p_{kl})_{k,l=1}^m$ the correlated strategy profile induced by a population state.

Definition 8 *A (correlated) strategy profile $x = (p_{kl})_{k,l=1}^m$ is **indirectly stable** if there exists an indirectly stable population state that induces this strategy profile.*

Compared with the definition of stable outcome in Dekel et al. (1998) there is a couple of differences. First, I require that the state is learning stable, that is, only equilibria that belong to a minimal curb set are played while Dekel et al. allow for any equilibrium. This, taken by itself, would lead to more stringent conditions for stability. But I also require that the mutants play such an equilibrium among themselves, while Dekel et al. also allow for any equilibrium. This would lead to weaker conditions for stability since some equilibria are now unattainable for mutants. However, since the mutants always can be of the type that is indifferent between strategies (see Lemma 2), for whom any equilibrium is learning stable, the latter requirement does not restrict the mutants. Therefore, my stability requirement is stronger, and the set of indirectly stable strategy profiles of this paper is a subset of stable outcomes of Dekel et al. Furthermore, I consider the state of population as the basic concept, while Dekel et al. focus on outcomes, i.e. strategy profiles.

Analogously with Dekel et al. one can show that in an indirectly stable state the incumbents get the same fitness in any match.

Lemma 3 *In an indirectly stable population state $\{(\mu_i)_{i=1}^n, (x_{ij})_{i,j=1}^n\}$ $u_i = u_j = u_{ij}$ $\forall i, j$.*

Proof. Let mutants be of the type that is indifferent among all strategies. Suppose that players with preferences v_i get the highest fitness against players with preferences v_j, i.e. $u_{ij} = \max_k u_{kj}$. Let the mutants play against the players with preferences v_j the same strategy as players with preferences v_i do, and let the players with preferences v_j play the same strategy against mutants as they did against the preferences v_i. This state of the game between mutants and players with preferences v_j is learning stable. In the same way let the mutant imitate against each incumbent the subpopulation that plays a strategy that has highest fitness against this incumbent. Such states of the games between mutants and incumbents are learning stable. Then the mutants achieve fitness composed of maximal fitness against each incumbent, and so, unless all other incumbents also achieve the same fitness, the mutants have higher fitness than the incumbents. Thus, against a given incumbent other incumbents should have the same fitness.

Suppose now that $\exists v_i, v_j$ such that $u_{ij} < u_i$. From the reasoning above $u_{jj} = u_{ij} < u_i$. Consider again the mutants that are indifferent among all strategies. Let the mutants play the same strategy against everybody as players with preferences v_j play, and let the mutants play among themselves a strategy that players with preferences v_k play among themselves, where v_k is such that $u_{kk} = u_{ik} > u_i$. The fitness of players with preferences v_j and the mutants against the incumbents is the same. Against the mutants the players with preferences v_j get u_{jj} while the mutants get fitness equal to $u_{ik} > u_i > u_{jj}$. ■

The lemma shows that in an indirectly stable state, either all incumbents play the same strategy against each other, or, if they play different strategies, they have the same fitness. Since in a population state players with given preferences play among themselves a symmetric strategy profile, the fitness in an indirectly stable state can be induced by a non-correlated symmetric strategy profile.

Still following the line of reasoning of Dekel et al. (1998) one shows that only efficient strategy profiles are indirectly stable, and that efficient strict Nash equilibria are indirectly stable.

Lemma 4 *If a symmetric strategy profile x is indirectly stable, then it is efficient.*

Proof. Suppose there exists an inefficient strategy profile that is indirectly stable. Then there exists an indirectly stable population state that induces this strategy profile. Consider again the mutants that are indifferent among all strategies. Suppose they imitate the behavior of the incumbents everywhere, and play the efficient strategy profile among themselves. Such mutants will have higher fitness than the incumbents, therefore the population state is not indirectly stable, a contradiction. ■

Preferences that do not play the efficient symmetric strategy profile among themselves cannot be stable. The questions posed in the introduction have the following answers in the complete information case: selfish preferences are not always stable (since they play a Nash equilibrium of the material payoff game and it is not necessarily efficient); stable strategy profiles are not always Nash equilibria of the material payoff game (by the same argument); stable strategy profiles are efficient.

Lemma 5 *If a symmetric strategy profile x is efficient and it is a strict Nash equilibrium of the material payoff game, it is indirectly stable.*

Proof. Consider a monomorphic population of players for whom the pure strategy of the efficient strategy profile is dominant. If mutants appear, the incumbents continue to play the same strategy. Since it is a strict Nash equilibrium, the mutants can achieve the same payoff against incumbents only by playing the same strategy. Then both mutants and incumbents play the same strategy and have the same fitness. ■

The indifferent preferences, so often used to upset an unstable population state, can always play the efficient symmetric strategy profile. However, even if they do so, they are not stable if there exists an asymmetric strategy profile with a higher fitness at least for one player.

Lemma 6 *Preferences that are indifferent among strategies are indirectly stable iff the symmetric efficient strategy profile is strongly efficient and there is no asymmetric strategy profile in which one player has the same fitness as in the efficient symmetric strategy profile while the other not.*

Proof. Consider a mutant that is also indifferent among all strategies. All strategy profiles, including asymmetric ones, are learning stable equilibria of the game between mutants and incumbents. Therefore, if the efficient strategy profile is not strongly efficient, the mutants can achieve fitness of the asymmetric strategy profile, higher that the incumbents are getting. If there is no such asymmetric strategy the mutants cannot achieve fitness higher than the incumbents. If there is asymmetric strategy with the same fitness for one player but lower for the other, the mutants can achieve the same fitness as incumbents against incumbents and play the efficient symmetric strategy profile among themselves. The incumbents then have lower fitness against mutants and thus the state is not indirectly stable. ■

3.2 2×2 Games

For 2×2 symmetric games the notion of learning stability in one population has following consequences. If one of the strategies is strictly dominant, the unique Nash equilibrium is strict and therefore learning stable. If the game has two pure strategy symmetric equilibria but one is in dominated strategies, the dominated equilibrium is not learning stable. In coordination games the two pure strict equilibria are learning stable, while the mixed equilibrium is not. In chicken type games the unique symmetric mixed equilibrium is learning stable. In games with equivalent strategies any symmetric strategy profile is a learning stable equilibrium.

In previous section I divided preferences in 2×2 games into types. Players with preferences of type (St1) always play strategy s_1 in a one-population learning stable state; analogously players of type (St2) always plays s_2. A mixed strategy can be played in a one-population learning stable state only by players with preferences of types (NC) and (BB). Preferences of type (BB), however, cannot be stable, if the symmetric efficient strategy profile is not strongly efficient. Since only efficient symmetric strategy profiles can be played in an indirectly stable state, I have to check only the states with efficient strategy profiles.

Proposition 1 *In symmetric 2×2 games following preferences are indirectly stable:*

1. *If $b + c > 2$ no preferences are indirectly stable;*
2. *If $b + c \leq 2$ then*
   (a) *if $1 > c$ then*
      i. *if $b > 1$ preferences of type (St1), and some preferences of type (CO);*
      ii. *if $b = 1$ preferences of types (St1) and (CO);*
      iii. *if $b < 1$ preferences of types (St1), (CO), and (BB);*

   (b) *if $c = 1$ then if $b = 1$ preferences of types (St1), (CO), and (BB);*

(c) otherwise no preferences are indirectly stable.

Proof. Consider first the case with $b + c > 2$. The symmetric efficient outcome is mixed, with $p = \frac{b+c}{2(b+c-1)}$ and fitness $\frac{(b+c)^2}{4(b+c-1)}$. Since only the efficient strategy profile can be stable, only preferences of types (BB) and (NC) can be stable. When $b + c > 2$ it holds that either $b > \frac{(b+c)^2}{4(b+c-1)}$ or $c > \frac{(b+c)^2}{4(b+c-1)}$, therefore preferences of type (BB) cannot be stable. Mutants of type (St1) get b against preferences of type (NC) in learning stable equilibrium, and mutants of type (St2) get c against preferences of type (NC) in learning stable equilibrium. Therefore preferences of type (NC) cannot be stable either.

Consider now the case when $b+c \leq 2$. Preferences of types (NC) and (St2) cannot be stable because they never play the efficient symmetric strategy profile (s_1, s_1). From Lemma 6 preferences of type (BB) are stable iff $b < 1, c < 1$ or $b = 1, c = 1$. In this case there is no possibility for a mutant to get fitness higher than 1, or get fitness 1 while incumbents get less, therefore preferences (St1) and (CO) that play (s_1, s_1) are indirectly stable as well.

In prisoners' dilemma and chicken type games ($c > 1$) preferences of type (St1) are not stable since mutants of type (St2) can appear and get $c > 1$ in a learning stable state. Let any preferences of type (CO) be parametrized without loss of generality by the completely mixed strategy $(\sigma_{CO}, 1 - \sigma_{CO})$ against which they are indifferent between s_1 and s_2 and analogously any preferences of type (NC) by the mixed strategy $(\sigma_{NC}, 1 - \sigma_{NC})$. The unique equilibrium $(\sigma_{NC}, \sigma_{CO})$ of the game between players with preferences of type (CO) and preferences of type (NC) is learning stable. In this equilibrium the fitness of the players with preferences of type (NC) is $\sigma_{NC}(\sigma_{CO} + (1 - \sigma_{CO})c) + (1 - \sigma_{NC})\sigma_{CO}b$. Since $c > 1$, $\sigma_{CO} + (1 - \sigma_{CO})c > 1$ $\forall \sigma_{CO}$. But then for σ_{NC} sufficiently close to 1 $\sigma_{NC}(\sigma_{CO} + (1 - \sigma_{CO})c) + (1 - \sigma_{NC})\sigma_{CO}b > 1$ and the mutants of type (NC) get the higher payoff than incumbents. If $c = 1, b < 1$ the mutants of type (BB) can appear and play against incumbents of either type (St1) or (CO) the learning stable equilibrium (s_1, σ_{CO}), and efficient (s_1, s_1) among themselves. The mutants then have fitness 1 both against incumbents and against themselves, while incumbents have $\sigma_{CO} + (1 - \sigma_{CO})b < 1$ against mutants. Thus no preferences are stable in such games.

Consider now the case when $b \geq 1, c < 1$. Players with preferences (St1) with $v(s_1, s_1) > v(s_2, s_1)$ either play s_1 in equilibrium, or the opponent play s_2

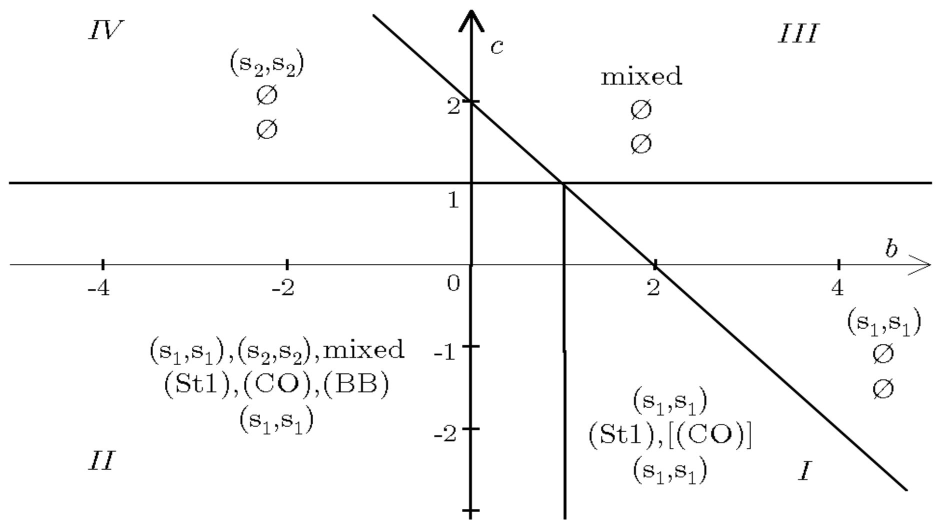

Figure 1: Indirectly stable preferences and strategy profiles

in equilibrium. In either case the mutants can get only a convex combination of $1, c$, and 0, which is not higher than 1, and if it is 1 then the incumbents also get 1 against mutants. In a game of incumbents with preferences (CO) with mutants of type (BB) (s_2, σ_{CO}) is a learning stable equilibrium. In it mutants have fitness $\sigma_{CO} b$. Therefore if $\sigma_{CO} > \frac{1}{b}$ mutants have higher fitness than the incumbents. When $\sigma_{CO} = \frac{1}{b}$ incumbents get $\sigma_{CO} c < 1$ against mutants, while mutants can get 1 among themselves, so such preferences of type (CO) are not indirectly stable. When $\sigma_{CO} < \frac{1}{b}$, preferences of type (CO) are indirectly stable, since in any equilibrium mutants have fitness not higher than 1 against them, and if it is 1 the incumbents also have fitness 1 against mutants. ■

The proposition is illustrated in Figure 1. The figure shows, for each type of the material payoff game, symmetric Nash equilibria of it, preferences that are indirectly stable, and the strategy profiles that are indirectly stable. For example, in region III $(1 > c, b > 0)$ where the game is a chicken type game, the unique symmetric equilibrium is mixed, no preferences are indirectly stable, and no strategy profiles are indirectly stable. In Prisoners' Dilemma (region IV, $1 > c, b < 0$) (s_2, s_2) is the unique equilibrium while no preferences and no strategy profiles are stable. In coordination games (region II, $1 <$

$c, b < 0$) there are three symmetric equilibria, and preferences of types (St1), (CO), and (BB) that play the efficient equilibrium (s_1, s_1) are stable. Finally, in region I ($1 > c, b > 0$) if (s_1, s_1) is efficient symmetric strategy combination some preferences playing it (types (St1) and (CO)) are stable. If there exists a mixed strategy combination that is more efficient than (s_1, s_1) no preferences and no strategy profiles are indirectly stable.

There are several differences from the analysis of Dekel et al. (1998). In distinction from Dekel et al. I do not have cooperation as a stable outcome in Prisoners' Dilemma. Dekel et al. found that cooperation can be supported by preferences of type (St2) that are indifferent between strategies when the opponent plays s_1. Then (s_1, s_1) was also an equilibrium for them. In my model, however, such equilibrium is not learning stable. Small amount of experimentation inside the population of such players will upset this equilibrium and the play will never return back. The same holds for the result of Dekel et al. that in chicken type games with $b = c$ the mixed efficient equilibrium can be supported by players of type (CO). This equilibrium again is not learning stable. I consider it more natural to have, along with the possibility of mutations, also the possibility of experimentation. Arguably, experimentation with strategies happens more often than mutations in preferences. Cooperation in Prisoners' Dilemma is upset either by mutants who defect by 'conviction', or by experimentators who try defection as alternative best reply.

It is hard for a population state to be indirectly stable. Since I consider all possible preferences, there are often mutants that upset a population state. More often than not there is no stable state or strategy profile. In this case simulations can help to see what could be the possible outcomes. I analyze this in the companion paper Possajennikov (2002).

4 Incomplete Information

4.1 General Games

If agents do not know the preferences of the opponent in a match, they cannot condition their strategy on them. The state of the subpopulation with preferences v_i is described by a distribution of strategies that are used in any match, $x_1^i, \ldots, x_m^i$. The state of the population is described by a matrix

$$\begin{array}{cccc} \mu_1 & \mu_2 & \dots & \mu_n \\ x_1^1 & x_1^2 & \dots & x_1^n \\ x_2^1 & x_2^2 & \dots & x_2^n \\ \dots & \dots & \dots & \dots \\ x_m^1 & x_m^2 & \dots & x_m^n \end{array}$$

where the first row is the proportion of preferences, column i corresponds to the distribution of strategies players with preferences v_i use, and row j shows how often strategy s_j is used. The state of the population will be denoted $\{(\mu_i)_{i=1}^n; (x^i)_{i=1}^n\}$ with $\mu_i \neq 0$.

Again, in each subpopulation there is a learning process that operates on subjective preferences v_i. However, all these processes are interconnected into one one-population learning process. Each match corresponds now to a two-player symmetric Bayesian game with incomplete information, in which players can be of n types $v_1, \dots, v_n$, a player knows own type and the distribution of opponent's types $\mu_1, \dots, \mu_n$, and the players simultaneously choose a strategy from Σ. I call the distributions of strategies $(x^i)_{i=1}^n$ the state of the Bayesian game. The population state induces a symmetric mixed strategy profile (x, x) in an obvious way, $x = \sum_{i=1}^n \mu_i x^i$. Since the players cannot condition their strategy on their role, only symmetric states are possible. Learning is much faster than evolution, thus from the point of view of learning μ_i's are fixed.

Analogously with the complete information case, I use the following weak notion of learning stability. If the play is not in a Bayesian-Nash equilibrium of the game, players in at least one of the subpopulations change strategy. If players experiment with alternative best replies, there should be a possibility for a play to come back to the original equilibrium. The notion of curb sets, used in the complete information section, can be easily extended to incomplete information games.

Let the distribution of types $(\mu_i)_{i=1}^n$ be given. Let $\{z = (z^1, \dots, z^n)\} = \Sigma^n$ be the set of strategy combinations, with the interpretation that strategy z^i is used by players of type v_i. For a given type v_i and a given z the incomplete information best response $BR_i^{in}(z)$ is the set of strategies that are best responses to the mixed strategy induced by the proportions of types in the population, i.e. the set of strategies $\sigma \in \Sigma$ such that $\sigma \in BR_i(\sum_{j=1}^n \mu_j z^j)$. The incomplete information best response correspondence maps a given z to $BR^{in}(z) = \times_{i=1}^n BR_i^{in}(z)$. A strategy combination z is a Bayesian-Nash equilibrium if $z \in BR^{in}(z)$. For a set Z of strategy combinations $BR^{in}(Z) = \cup_{z \in Z} BR^{in}(z)$. A subset $Z \subset \Sigma^n$ is *closed under rational behavior in the in-*

complete information game (curb^{in}) if Z is the product of non-empty convex subsets of Σ and $BR^{in}(Z) \subset Z$. A minimal curb^{in} set is the curb^{in} set that does not contain any proper subset that is a curb^{in} set.

For a given distribution of preferences $(\mu_i)_{i=1}^n$ a strategy combination $(x^i)_{i=1}^n$ is *learning stable* if it is a symmetric Bayesian-Nash equilibrium and belongs to a minimal curb^{in} set of the game. I sometimes will refer to learning stable strategy combination as learning stable equilibrium. The population state is learning stable if the strategy profile in it is learning stable given the distribution of preferences in the state.

Definition 9 *A population state* $\{(\mu_i)_{i=1}^n; (x^i)_{i=1}^n\}$ *is* ***learning stable****, if the state* $(x^i)_{i=1}^n$ *of the Bayesian game is symmetric Bayesian-Nash equilibrium of the game and belongs to a minimal curb*^{in} *set, given* $(\mu_i)_{i=1}^n$.

The evolutionary process uses the expected fitness of the players. Players with preferences v_i, if matched with players with preferences v_j, get in expected terms $u_{ij} = \sum_{k=1}^m \sum_{l=1}^m x_k^i x_l^j u(s_k, s_l)$. Before matching, the expected average fitness of players with preferences v_i is $u_i = \sum_{j=1}^n \mu_j u_{ij}$. The proportion μ_i of players with preferences v_i increase relative to the proportion μ_j of players with preferences v_j iff $u_i > u_j$. A population state is stationary with respect to the evolutionary process if all subpopulations have the same expected average fitness u_i.

Like in the complete information case, any monomorphic population is stationary. Therefore I again check the robustness of a state against an invasion by mutants.

Compared with the complete information case, an additional issue arises in defining the indirect stability in the incomplete information case. With complete information, the incumbents did not need to change their strategies in games between themselves, therefore for small proportion of mutants the strategy profile in a perturbed state was close to the strategy profile of the original state. With incomplete information, even with small proportion of mutants, incumbents may change their strategy considerably in a new learning stable equilibrium. The definition should take care of this possibility.

Consider two (mixed) strategies $x = (x_1, \ldots, x_m)$ and $y = (y_1, \ldots, y_m)$. Let the distance between the two strategies be defined as $d(x, y) := \sum_{j=1}^m |x_j - y_j|$.

Definition 10 *A population state* $\{(\mu_i)_{i=1}^n; (x^i)_{i=1}^n\}$ *is* ***indirectly evolutionarily stable with incomplete information*** *if* *(i) it is learning stable;*

(ii) $\forall v_k \notin \{v_i\}_{i=1}^n, \exists a < \infty, \exists \varepsilon^* > 0$ *such that* $\forall \varepsilon \in (0, \varepsilon^*)$ $\exists (y^i)_{i=1}^n, y^k$ *such that the strategy combination* $(y^i)_{i=1}^n, y^k$ *is learning stable with* $(1-\varepsilon)(\mu_i)_{i=1}^n + \varepsilon\mu_k$ *and* $d(y^i, x^i) \leq a\varepsilon$ $\forall i$. *For any such* $(y^i)_{i=1}^n, y^k$ $u_i > u_k$ $\forall i$ *in the state* $\{(1-\varepsilon)(\mu_i)_{i=1}^n + \varepsilon\mu_k; (y^i)_{i=1}^n, y^k\}$.

The idea in the second part of the definition is that after the appearance of the mutants there exists a new learning stable strategy combination where incumbents play strategies close to their original strategies. In each such new learning stable state of the game the incumbents should receive higher fitness than the mutants.

As in the complete information case no state with finite number of preferences can be evolutionarily stable.

Lemma 7 *No population state with finite* n *is evolutionarily stable.*

Proof. Consider a mutant of the type that is indifferent between all strategies. Suppose that in the new learning stable state of the game mutants play the strategy corresponding to the induced symmetric strategy profile x, and the incumbents play the same strategies as before. Such a state is learning stable since incumbents face the same distribution of strategies and therefore old strategies are still best responses, for the mutants all strategies are best responses, and the distance between new learning stable equilibrium and the old one is 0 for all incumbents. The mutants in such a state receive the same expected fitness as the incumbents. ■

Relaxing the definition in the flavor of neutral stability, I get

Definition 11 *A population state* $\{(\mu_i)_{i=1}^n, (x^i)_{i=1}^n\}$ *is* ***indirectly stable with incomplete information*** *if*

(i) it is learning stable;

(ii) $\forall v_k \notin \{v_i\}_{i=1}^n, \exists a < \infty, \exists \varepsilon^* > 0$ *such that* $\forall \varepsilon \in (0, \varepsilon^*)$ $\exists (y^i)_{i=1}^n, y^k$ *such that the strategy combination* $(y^i)_{i=1}^n, y^k$ *is learning stable with* $(1-\varepsilon)(\mu_i)_{i=1}^n + \varepsilon\mu_k$ *and* $d(y^i, x^i) \leq a\varepsilon$ $\forall i$. *For any such* $(y^i)_{i=1}^n, y^k$ $u_i \geq u_k$ $\forall i$ *in the state* $\{(1-\varepsilon)(\mu_i)_{i=1}^n + \varepsilon\mu_k; (y^i)_{i=1}^n, y^k\}$.

In an indirectly stable state the mutants may get the same fitness as the incumbents but they cannot get higher fitness.

Analogously with the complete information case, I define indirectly stable preferences and indirectly stable strategy profiles.

Definition 12 *Preferences v_k are* ***indirectly stable with incomplete information*** *if there exists an indirectly stable with incomplete information population state $\{(\mu_i)_{i=1}^n, (x^i)_{i=1}^n\}$ with v_k in the support of this state.*

Definition 13 *A symmetric strategy profile x is* ***indirectly stable with incomplete information*** *if there exists an indirectly stable with incomplete information population state $\{(\mu_i)_{i=1}^n, (x^i)_{i=1}^n\}$ that induces x.*

Compared with the definition in Dekel et al. (1998), there is the same difference as in the complete information case: I allow only for learning stable equilibria. There is also another difference. Dekel et al. do not allow in a post-entry population for equilibria in which incumbents play a strategy different from the one they played before the entry. I allow for small changes in the strategy of the incumbents, which is, in my view, an assumption that is more in line with the complete information case, when incumbents had time to learn the equilibrium strategy against the mutants. Though Dekel et al. mention this possibility, they do not elaborate on its implications. I will comment in the end of the section about the possibility of relaxing the informational assumptions further that would lead to the same results as in Dekel et al.

The definition is similar to the one of the complete information case and suffers from the same drawback as it allows mutants have the same fitness as the incumbents. Perhaps more important, since stability of the preferences is defined through stability of population states that takes into account stability of strategies, it may lead to counterintuitive results, as I discuss at the end of the section. But for the moment I stick to the current definition.

Lemma 8 *Suppose a symmetric strategy profile x is indirectly stable with incomplete information. Then it is a Nash equilibrium of the material payoff game.*

Proof. Consider any population state $\{(\mu_i)_{i=1}^n, (x^i)_{i=1}^n\}$ that induces x. Suppose x is not a Nash equilibrium of the material payoff game. Then there exists strategy y that has higher fitness against x than x against itself,

$u(y,x) > u(x,x)$. Suppose that mutants for whom strategy y is strictly dominant appear. In any new learning stable state of the game the mutants play the dominant strategy y. If there is no new learning stable strategy combination in which the incumbents play strategies that are close to x, then the population state $\{(\mu_i)_{i=1}^n, (x^i)_{i=1}^n\}$ that induced x is not indirectly stable.

Consider now any new learning stable strategy combination where the incumbents play (possibly different) strategies z^i such that $d(z^i, x^i) \leq a\varepsilon \ \forall i$. The aggregate strategy of the incumbents is $z^i = \sum_{i=1}^n \mu_i y^i$. Then $d(z,x) \leq ma\varepsilon$. By linearity of the material payoff function the mutants get fitness $u_m = (1-\varepsilon)u(y,z)+\varepsilon u(y,y) = u(y,x)+[u(y,z)-u(y,x)]+\varepsilon[u(y,y)-u(y,z)]$, and the incumbents get on average $u_i = (1-\varepsilon)u(z,z) + \varepsilon u(z,y) = u(x,x) + [u(z,z)-u(z,x)+u(z,x)-u(x,x)]+\varepsilon[u(z,y)-u(z,z)]$. Then $u_m - u_i > 0 \Leftrightarrow u(y,x)-u(x,x) > [u(z,z)-u(z,x)]+[u(z,x)-u(x,x)]+[u(y,x)-u(y,z)]+ \varepsilon[u(z,y)-u(z,z)+u(y,z)-u(y,y)]$. Since $d(z,x) \leq ma\varepsilon$, for any strategy $w \in \Sigma$ $|u(w,z)-u(w,x)| \leq u_{\max}m^2a\varepsilon$ and $|u(z,w)-u(x,w)| \leq u_{\max}m^2a\varepsilon$, where $u_{\max} = \max_{(s,t)\in S\times S}|u(s,t)|$. The right hand side of the strict inequality above can be written as $d\varepsilon$, where d is a finite constant. For sufficiently small $\varepsilon < \frac{u(y,x)-u(x,x)}{d}$ it holds that $u_m - u_i > 0$, thus there exist incumbents that have lower fitness than the mutants and so any state that induces x is not stable. ■

Thus in terms of played strategy profiles, only Nash equilibria can be observed in an indirectly stable population state. A sufficient condition for a Nash equilibrium to be indirectly stable with incomplete information is that it is in neutrally stable strategies.

Lemma 9 *Suppose a symmetric strategy profile x is a Nash equilibrium of the material payoff game in neutrally stable strategies. Then it is indirectly stable with incomplete information.*

Proof. Consider the population state $\{(\mu_i)_{i=1}^n, (x^i)_{i=1}^n\}$ with $n = 1$, and the only incumbent players have preferences of the type that is indifferent among strategies. After appearance of mutants, there are new learning stable states in which incumbents play x, thus one can take $a = 0$ in the definition of indirectly stable state. If mutants plays strategy y different from x in a new learning stable state, the expected fitness of the incumbents is $u_i = (1-\varepsilon)u(x,x)+\varepsilon u(x,y)$, while that of the mutants is $u_m = (1-\varepsilon)u(y,x)+\varepsilon u(y,y)$. Clearly, $u_i \geq u_m$ for sufficiently small ε by the definition of neutral stability. If mutants play x, then the incumbents and the mutants have the same fitness, so the population state and the induced strategy profile x are indirectly stable. ■

Not all Nash equilibria are stable, and the result cannot be strengthened to the result of Dekel et al. that only neutrally stable strategy profile can be stable, as examples below show.

Example 1 *Nash equilibria of the material payoff game that are not indirectly stable with incomplete information.*

Consider the symmetric game

	s_1	s_2	s_3
s_1	$1,1$	$0,1$	$1,0$
s_2	$1,0$	$1,1$	$0,1$
s_3	$0,1$	$1,0$	$1,1$

.

Symmetric pure strategy combinations $(s_i, s_i), i = 1, 2, 3$ are Nash equilibria but none of them is in neutrally stable strategies (Weibull, 1995, Ch.2). Consider a population state that induces equilibrium (s_1, s_1). Consider a mutant for whom strategy s_2 is strictly dominant. If there is no new learning stable state in which incumbents play a strategy close to s_1, then the population state is not stable. Consider therefore a new learning stable state where the mutants play s_2 while the incumbents play a strategy close to s_1. If the strategy of each incumbent is at the distance not more than $a\varepsilon$ from s_1, the average strategy of the incumbents is also in the $a\varepsilon$-neighborhood of s_1, and can be written as $(1-\delta_2-\delta_3)s_1+\delta_2 s_2+\delta_3 s_3$ for $\delta_2+\delta_3 \leq \frac{a\varepsilon}{2}$. Then the expected fitness of the mutants is $u_m = (1-\varepsilon)(1-\delta_3)+\varepsilon$, while the expected average fitness of the incumbents is $u_i = (1-\varepsilon)(1-\delta_2-\delta_3+\delta_2^2+\delta_2\delta_3+\delta_3^2)+\varepsilon(\delta_2+\delta_3)$. For given $\delta_3 \leq \frac{a\varepsilon}{2}$, and for $\delta_2 \in [0, \frac{a\varepsilon}{2} - \delta_3]$ u_i reaches maximum at $\delta_2 = 0$. Its maximum value is $u_i^* = (1-\varepsilon)(1-\delta_3) + (1-\varepsilon)\delta_3^2 + \varepsilon\delta_3 < u_m$ for $\varepsilon < \frac{2}{a}$ if $\frac{2}{a} \geq 1$ and for $\varepsilon < \frac{4}{a^2}$ if $\frac{2}{a} < 1$. Thus there are incumbents that have lower fitness that mutants, and so any population state that induces equilibrium (s_1, s_1) is not stable. Analogous reasoning with obvious modifications applies to (s_2, s_2) and (s_3, s_3).

Example 2 *Nash equilibrium of the material payoff game that is not in neutrally stable strategies but that is indirectly stable with incomplete information.*

Consider 2×2 coordination game with $1 > c, b < 0$. The mixed strategy equilibrium is not neutrally stable strategy in this game. Consider the population state $\{(\mu_i)_{i=1}^n, (x^i)_{i=1}^n\}$ with $n = 1$, consisting of players with preferences of type (NC) such that they play the mixed strategy equilibrium of the material payoff game, $\sigma_{NC} = \frac{b}{b+c-1}$. This population state is stable. Consider mutants that in a new learning stable state play strategy σ_m. The incumbents are indifferent between strategies if their own strategy σ_i is such that $(1-\varepsilon)\sigma_i + \varepsilon\sigma_m = \sigma_{NC}$. In the post-entry learning stable equilibrium the incumbents play $\sigma_i = \frac{\sigma_{NC}-\varepsilon\sigma_m}{1-\varepsilon}$, with $d(\sigma_i, \sigma_{NC}) = \frac{2|\sigma_{NC}-\sigma_m|}{1-\varepsilon}\varepsilon \leq 4|\sigma_{NC} - \sigma_m|\varepsilon$ for $\varepsilon \leq \frac{1}{2}$. The fitness of the incumbents in the post-entry population is $u_i = (1-\varepsilon)u(\sigma_i, \sigma_i) + \varepsilon u(\sigma_i, \sigma_m) = u(\sigma_i, \sigma_{NC})$ and the fitness of the mutants is $u_m = (1-\varepsilon)u(\sigma_m, \sigma_i) + \varepsilon u(\sigma_m, \sigma_m) = u(\sigma_m, \sigma_{NC})$. Since σ_{NC} is completely mixed equilibrium of the material payoff game, any strategy is best response to it, and any strategy has the same fitness against it. Thus the incumbents and the mutants have the same fitness, so the population state and the mixed equilibrium it induces are indirectly stable.

In the model of Dekel et al. (1998) only neutrally stable strategy profiles are stable. Example 2 shows, however, that Nash equilibria that are not neutrally stable can be indirectly stable in the current model.

The following lemma shows when monomorphic population consisting of players with indifferent preferences is indirectly stable with incomplete information.

Lemma 10 *The monomorphic population consisting of players that are indifferent among strategies is indirectly stable iff the players play neutrally stable Nash equilibrium of the material payoff game.*

Proof. By Lemma 9 if players that are indifferent among strategies play a neutrally stable equilibrium x, the monomorphic population state is stable. If they play x that is not neutrally stable, there exists y such that $(1-\varepsilon)u(y, x) + \varepsilon u(y, y) > (1-\varepsilon)u(x, x) + \varepsilon u(x, y)$. Consider mutants that play strategy y in the new learning stable state. Their fitness is higher than the fitness of the incumbents. ■

4.2 2×2 Games

In this subsection I apply the results above to 2×2 games. It is difficult to give complete characterization of indirectly stable preferences for polymorphic population states consisting of players with different preferences. I give results for monomorphic populations consisting of players all having the same preferences.

Proposition 2 *In symmetric* 2×2 *games following preferences are indirectly stable in preference monomorphic population states:*

1. *If* $1 \geq c, b \geq 0$*, at least one inequality is strict, preferences of types (St1), (CO), and (BB);*
2. *If* $1 > c, b < 0$ *preferences of any type;*
3. *If* $1 \leq c, b \leq 0$*, at least one inequality is strict, preferences of types (St2), (CO), and (BB);*
4. *If* $1 < c, b > 0$ *preferences of types (NC) and (BB);*
5. *If* $c = 1, b = 0$ *preferences of any type;*

Proof. When $1 \geq c, b \geq 0$, (s_1, s_1) is neutrally stable Nash equilibrium. For preferences of all three types that play this equilibrium (St1), (CO), and (BB) choose $a = 0$ in the definition of indirectly stable population state. Similar to the proof of Lemma 9 it follows that these preferences are indirectly stable. Strategy profile (s_2, s_2) is an equilibrium if $b = 0$, but reasoning similar to the one in Example 1 shows that it is not indirectly stable. The situation in Prisoners' Dilemma, when $1 \leq c, b \leq 0$, is similar, only mirrored with respect to pure strategies s_1 and s_2 and types (St1) and (St2).

In coordination games ($1 > c, b < 0$) preferences of types (St1), (CO), (St2), and (BB) are indirectly stable when they play one of the pure equilibria of the game because one can choose $a = 0$ in the definition of indirectly stable population state. Example 2 shows that preferences of type (NC) are indirectly stable when they play the mixed equilibrium of the coordination game. The idea of the example works also for chicken type games ($1 < c, b > 0$), thus preferences of type (NC) playing the mixed equilibrium are indirectly stable in such games. The mixed equilibrium in chicken type games is neutrally stable, so preferences of type (BB) playing this equilibrium are indirectly stable by Lemma 10.

Finally, in games with equivalent strategies ($c = 1, b = 0$) all players face the same distribution of strategies x. Since all strategies are equivalent, independently of which strategy incumbents and mutants use their expected fitness is the same, so any preferences are indirectly stable. ■

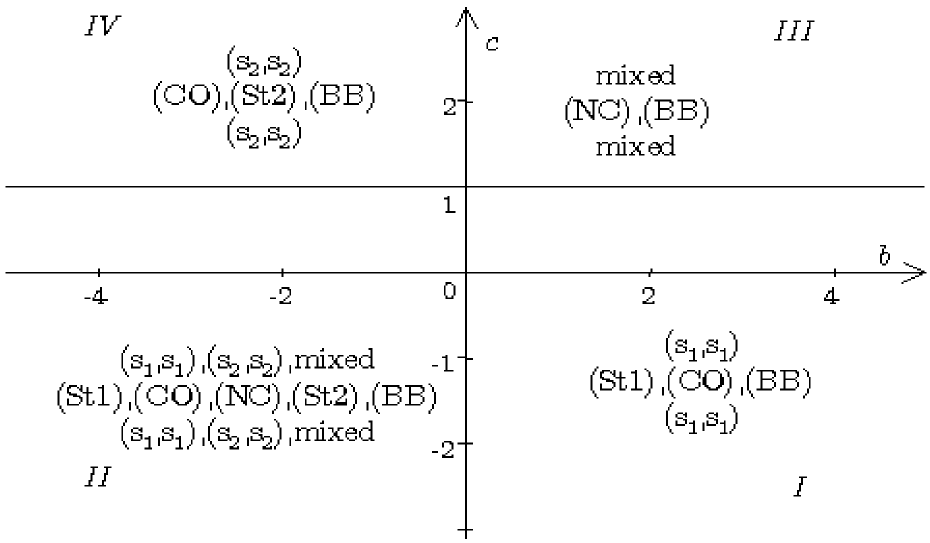

Figure 2: Indirectly stable preferences and profiles under incomplete information

The proposition is illustrated in Figure 2. The figure shows that in 2×2 games any non-dominated Nash equilibrium can be supported by stable preferences. Note that in the model of Dekel et al. (1998) the mixed equilibrium of a chicken type game is not supported by the population of players with preferences (NC) since there are mutants of type (St1), for example, after whose entry the equilibrium of the Bayesian game has incumbents play slightly different strategy. In my model such population state is stable, but then also some equilibria that are not neutrally stable, like the mixed equilibrium in coordination games, become stable.

4.3 Discussion

4.3.1 Stability of Preferences

Consider again the game of Example 1. Consider a monomorphic population of players with selfish preferences. The example showed that states that induce pure strategy symmetric equilibria $(s_i, s_i), i = 1, 2, 3$ are not stable. Consider a state that induces the only remaining equilibrium $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. After the appearance of mutants for whom strategy s_1 is strictly dominant the unique equilibrium in which the incumbents play a strategy close to $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ is not learning stable. If a curb^{in} set includes s_3 for the incumbents, it also includes s_1, and then also s_2. However, the set $\Delta(s_1, s_2)$ of mixed strategies not including strategy s_3 for the incumbents, together with the set $\{s_1\}$ for the mutants, is a curb^{in} set. Thus, the fully mixed equilibrium with incumbents' strategy close to $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ does not belong to a minimal curb^{in} set. The monomorphic population consisting of players with selfish preferences is not stable with incomplete information in this game.

In the model of this paper stability of preferences is defined through stability of population states that also takes into account stability of strategy profiles. Without the last requirement, an alternative definition could be as following. Preferences are stable if there exists a population state so that after the appearance of mutants in any learning stable equilibrium incumbents have fitness not lower than the mutants. The difference is that now there is no restriction on how far the new strategy profile is from the old one. With this new definition the monomorphic population of players with selfish preferences is always stable, since after the appearance of mutants both incumbents and mutants face the same distribution of preferences in the population, and the selfish incumbents maximize material payoffs, hence having fitness not lower than the mutants.

With the original definition, however, the analysis in this section shows that the answers to the questions from the introduction in the incomplete information case are: selfish preferences are almost always stable (see the example above when they are not; this is a non-generic case); stable outcomes are Nash equilibria of the material payoff game; stable outcomes are not always efficient.

4.3.2 Ignorance

The following informational assumption can justify the requirement of Dekel et al. (1998) that the incumbents continue to play the same strategy in the post-entry state. The incumbents had time to learn and has arrived to an equilibrium x according to their subjective preferences. Suppose now that a small proportion of the mutants appear but the incumbents are ignorant of the arrival of the mutants. Since the incumbents were playing equilibrium x of their subjective preferences game, they continue playing the same strategy x. If the equilibrium x of the subjective preferences game was not neutrally stable strategy of the material payoff game, there exists strategy $y \in \Sigma$

and $\varepsilon > 0$ such that $(1-\varepsilon)u(y,x) + \varepsilon u(y,y) > (1-\varepsilon)u(x,x) + \varepsilon u(x,y)$. Consider mutants that play y. The left hand side in the inequality above is the expected fitness of the mutants in the post-entry population, while the right hand side is the expected fitness of the incumbents. Thus the mutants have higher fitness than the incumbent and x is not stable. On the other hand, if x is neutrally stable strategy, for any strategy y of the mutants $(1-\varepsilon)u(y,x) + \varepsilon u(y,y) \leq (1-\varepsilon)u(x,x) + \varepsilon u(x,y)$, so x is stable. Therefore a strategy profile x is stable under ignorance if and only if x is neutrally stable strategy of the material payoff game, the same result as in the model of Dekel et al.

5 Conclusion

The main message of the analysis in the paper is an old one and can be found, for example, in Frank (1987): information about opponents improves efficiency if one can commit to a cooperative action. Preferences provide such commitment, therefore inefficient strategy profiles cannot be stable with complete information. The paper shows that these results are of quite a general nature in the realm of games played by randomly matched pairs. The innovative feature of the paper is the introduction of learning together with evolution that makes it more difficult to sustain efficient strategies than in Dekel et al. (1998).

A very important limitation of the analysis, in my view, is the use of static concepts of evolution. Moreover, it is further restricted to finite preference distributions and checked only against particular perturbations. It is possible to formulate explicitly a dynamic process of evolution, together with a dynamic process of learning of equilibrium, but the analysis is hard due to the infinite space of possible preferences. With the help of simulations, however, one can achieve some insight, particularly in the case when there are no stable preferences, as in prisoners' dilemma and chicken type games. This possibility is developed in Possajennikov (2002).

Another limitation is the strong assumptions on information agents have: either they observe opponent's preferences perfectly, or do not at all. Instead of such extreme cases one can consider the case with a given technology to obtain (or hide) such information. The evolutionary process can then be expanded to information acquisition. An analysis of such a model for a simple trust game is given in Güth et al. (2000); it would be interesting to extend the analysis to other games.

References

[1] Basu K., Weibull J. (1991) "Strategy Subsets Closed under Rational Behavior", *Economics Letters*, 36, 141-146.

[2] Bester H., Güth W. (1998) "Is Altruism Evolutionary Stable?", *Journal of Economic Behavior and Organization*, 34, 193-209.

[3] Bolle F. (2000) "Is Altruism Evolutionary Stable? And Envy and Malevolence?", *Journal of Economic Behavior and Organization*, 42, 131-133.

[4] Dekel E., Ely J., Yilankaya O. (1998) "Evolution of Preferences", working paper, Northwestern University, available at http://www.faculty.econ.northwestern.edu/faculty/ely.

[5] Dufwenberg M., Güth W. (1999) "Indirect Evolution versus Strategic Delegation: A Comparison of two Approaches to Explaining Economic Institutions", *European Journal of Political Economy*, 15, 281-295.

[6] Ely J., Yilankaya O. (2001) "Nash Equilibrium and the Evolution of Preferences", *Journal of Economic Theory*, 97, 255-272.

[7] Frank R. (1987) "If Homo Economicus Could Choose His Own Utility Function, Would He Want One with a Conscience?", *American Economic Review*, 77, 593-604.

[8] Güth W., Kliemt H. (1998) "The Indirect Evolutionary Approach", *Rationality and Society*, 10, 377-399.

[9] Güth W., Kliemt H., Peleg B. (2000) "Co-evolution of Preferences and Information in Simple Games of Trust", *German Economic Review*, 1, 83-110.

[10] Güth W., Yaari M. (1992) "Explaining Reciprocal Fairness in Simple Strategic Games: An Evolutionary Approach", in: Witt U. (ed.) *Explaining Process and Change - Approaches to Evolutionary Economics*, The University of Michigan Press.

[11] Levine D. (1998) "Modeling Altruism and Spitefulness in Experiments", *Review of Economic Dynamics*, 1, 593-622.

[12] Ok E., Vega-Redondo F. (2001) "On the Evolution of Individualistic Preferences: An Incomplete Information Scenario", *Journal of Economic Theory*, 97, 231-254.

[13] Huck S., Oechssler J. (1999) "The Indirect Evolutionary Approach to Explaining Fair Allocation", *Games and Economic Behavior*, 28,13-24.

[14] Possajennikov A. (2000) "On the Evolutionary Stability of Altruistic and Spiteful Preferences", *Journal of Economic Behavior and Organization*, 42, 125-129.

[15] Possajennikov A. (2002) "Cooperative Prisoners and Aggressive Chickens: Evolution of Strategies and Preferences in 2×2 Games", working paper, University of Mannheim, available at http://www.sfb504.uni-mannheim.de/~possajen.

[16] Sandholm W. (2001) "Preference Evolution, Two-Speed Dynamics, and Rapid Social Change", *Review of Economic Dynamics*, 4, 637-679.

[17] Sethi R., Somanathan E. (2001) "Preference Evolution and Reciprocity", *Journal of Economic Theory*, 97, 273-297.

[18] Weibull J. (1995) *Evolutionary Game Theory*, MIT Press.

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