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Pairwise Interaction on Random Graphs

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Abstract

We analyze dynamic local interaction in population games where the local interaction structure (modeled as a graph) can change over time: A stochastic process generates a random sequence of graphs. This contrasts with models where the initial interaction structure (represented by a deterministic graph or the realization of a random graph) cannot change over time.

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1 Introduction

Frequently, interaction in an economic, social, political or computational context is local in the sense that it consists of pairwise interactions between neighbors. In widely used pairwise matching models, if an agent interacts at all in a particular period, he interacts with only one (temporary or permanent) partner. Since the early 90s, a sizeable literature on pairwise interactions between neighbors on a graph has emerged.[1] The novel feature is that direct interaction of an agent is confined to his neighhbors, frequently but not necessarily a small group, while indirect interaction via a chain of neighbors may occur between any pair of agents. As a rule, it has been assumed that the underlying interaction structure (network, graph) does not change over time. This assumption captures the case of rather rigid social ties.

Very valuable insights have been gained from studying pairwise interaction under the assumption of a fixed interaction structure. However, social ties are not always rigid. First, they may change because of stochastic shocks. Examples of the latter are random encounters or noisy communication. Second, social ties may be formed or severed as the consequence of deliberate actions taken by individuals.[2] Third, network design and network utilization may go hand in hand. The specific contributions by Droste, Gilles and Johnson (2000), Jackson and Watts (2002), Goyal and Vega-Redondo (2005), Hojman and Szeidl (2006), and Ehrhardt, Marsili and Vega-Redondo (2006) will be discussed in subsection 5.3.

Here we explore the first possibility. The interaction structure is exogenous and random. It changes over time, following an i.i.d. process. This assumption captures the case of rather loose social ties. It allows us to directly compare the effects of two polar cases — fixed versus random interaction structure, rigid versus loose social ties — without altering the local interaction model in other respects. The assumption of an exogenous, randomly changing interaction structure also offers an interesting contrast to models where both network formation and network utilization are endogenous.

[1]For pioneering contributions to this literature, see Anderlini and Ianni (1996), Berninghaus and Schwalbe (1996a,b), Blume (1993, 1995), Ellison (1993), Goyal and Janssen (1997), among others.

[2]See the literature on strategic network formation in the tradition of Jackson and Wolinsky (1996) and Bala and Goyal (2000).

The bulk of the literature on local interaction with a fixed interaction structure studies spatial games. Such games are characterized by pairwise strategic interactions between neighbors on a graph. They constitute a substantial part of the economically motivated literature on evolutionary games. The class of spatial games is especially useful in explaining and investigating the emergence, coexistence, persistence as well as disappearance of conventions and social norms. It also lends itself to the study of the formation of industry standards and norms. In this paper, we consider a generalization of the traditional concept of spatial games. We develop and analyze an evolutionary game that constitutes a dynamic spatial game with randomly changing interaction structure, evolves in discrete time, and has the following particular features:

■ **Base or constituent game.** In every period, the players play a local interaction or spatial game. Every time, each player has to make a binary choice between two actions, X and Y, and receives the sum of his payoffs from interacting once with each of his neighbors. Payoffs from each pairwise interaction are based on the same symmetric 2×2 coordination game, called base or constituent game in the literature. Consequently, players are conformists: If all his neighbors play X, a player prefers to play X; if all the neighbors play Y, he prefers Y.

■ **Interaction structure.** An interaction structure describes who interacts with whom. It specifies for each agent a set of neighbors, the set of agents with whom the agent interacts. In the sequel, an "interaction structure" is modeled as an undirected finite graph whose vertices or nodes are the members of the player population. Two players are neighbors, if they form an edge of the graph. Global interaction prevails if any two players are neighbors. Otherwise, interaction is local. In principle, one can generalize and replace the graph by a weighted graph. But we refrain from doing so in the present paper and rather generalize in a different direction, allowing for random graphs.

■ **Dynamics.** The model has two dynamic elements. On the one hand, the interaction structure (modeled as a graph) can change over time: A stochastic process generates a **random sequence of graphs**. This contrasts with models where the initial interaction structure (represented by a deterministic graph or the realization of a random graph) cannot change over time.[3]

On the other hand, we consider **best response dynamics** where at each time, one or every player chooses a (static) best response against the empirical distribution of the last strategies played by his neighbors. This constitutes rational behavior impaired by myopia. Myopia in the temporal sense means that the player is not forward looking, does not take into account that other players might be changing their strategies.[4] Myopia in the spatial sense means that the player is influenced only by his local environment, the previous choices of his neighbors. Deterministic best response dynamics of local interaction games has been pioneered by Blume (1995) and Berninghaus and Schwalbe (1996a,b). Blume (1995) studies and compares local and global interaction for specific interaction structures (infinite and finite two-dimensional lattices). In an otherwise deterministic model, Blume assumes asynchronous updating where each period, a player is selected at random and plays a myopic best response against his neighbors' previous actions while the other players repeat their previous actions. Berninghaus and Schwalbe (1996a,b), in a model with simultaneous updating, analyze deterministic best response dynamics with global or local interaction. Unless specified otherwise, we shall proceed under the assumption of simultaneous updating.

■ **Contagion.** Contagion is said to occur if one action can spread by a contact effect from a particular group of agents, typically a small group, to the entire population. Suppose that originally, only a small group of people chooses action X. We say that contagion (with respect to X) occurs, if with probability 1, the entire population ends up playing X. Under the myopic best response dynamics assumed in the present paper, an agent is playing X in the current period if and only if a sufficiently high proportion of his current contacts (neighbors) has played X in the last period. Analogous definitions

[3]In the setting of Blume (1995) and Berninghaus and Schwalbe (1996a,b), the same exogenously given interaction structure (graph) is present in all periods. In the model of López-Pintado (2006), an interaction structure (network, graph) is chosen at random prior to the game and does not change throughout a particular play of the game. For a more detailed comparison, see subsection 5.2.

[4]Hence, unlike the standard treatment of repeated games, our analysis is not concerned with perfectly rational outcomes such as the perfect equilibria of the repeated game.

and observations apply to the alternative action, Y.

Our definition of contagion, which requires that an action, trait, defect, virus, etc. spreads to the entire population, has been used in computer science [Flocchini *et al.* (2001, 2004), Peleg (1998, 2002)], economics [Morris (2000), Lee and Valentinyi (2000)], and game theory [Berninghaus *et al.* (2006), Durieu *et al.* (2006)]. Different definitions of contagion have been proposed and are appealing in certain contexts. In a social context, our definition rules out the long-run coexistence of conventions. A less demanding definition of contagion would require that an action spreads to a significant proportion of the player population and remains there forever. This definition — which allows for the long-run coexistence of conventions — is akin to that adopted by López-Pintado (2006) in a social interaction setting and by Pastor-Satorrs and Vespignani (2001) in an epidemiological model.

■ **Main results.** Our analysis is focused on models with two kinds of random graphs. In the first type of models, the support of the random graph consists of regular graphs, where all players have the same number of neighbors. In the second type of models, the underlying random graph is binomial.

Among regular graphs, **circular graphs** are of special interest. Presumably, circular graphs are the local interaction structures most frequently studied in game theory and economics. In such a graph, the players can be arranged in a circle so that each player has one neighbor to the left and one neighbor to the right. We find that if the support of the underlying random graph consists of all circular graphs, at least one player chose the risk dominant action initially, and updating is simultaneous, then contagion with respect to the risk dominant action occurs. In contrast, with simultaneous updating, an even number of players, and a fixed circular graph, convergence to a two-cycle can occur. We extend the analysis to so-called **OR networks** à la Goles and Hernández (2000). We obtain less conclusive results for arbitrary regular graphs where each player has more than two neighbors.

In a **binomial random graph**, also known as an Erdős-Rényi random graph, each possible edge is included independently of others with a given probability, which is the same for all edges. This is the random graph model most commonly studied in mathematics and statistics, sometimes even referred to as "the random graph." In a dynamic context it reflects best the idea of loose social ties. We find that when the evolution of the interaction structure is based on a binomial random graph, then with probability one, contagion (either with respect to action X or with respect to action Y) occurs. We further show that if at least one player chooses the risk dominant action initially, then with positive probability, all players choose the risk dominant action in period 2. However, given a limited number of players choosing an action initially, the probability that eventually all players choose the alternative action approaches one as the population size goes to infinity. Thus, if just a few players choose the risk dominant action initially, then with positive probability, contagion with respect to the risk dominant action occurs very rapidly; but in a large population, it is much more likely that contagion with respect to the alternative action occurs.

In the next section, we present the general model. In Section 3, we consider interaction on regular graphs, with a special emphasis on circular graphs and OR networks. In Section 4, we study interaction on binomial graphs. In Section 5, we offer qualifying remarks. Specifically, subsection 5.1 deals with asynchronous updating, subsection 5.2 puts our results into context, and subsection 5.3 elaborates on the endogenous co-evolution of local interaction and the local interaction structure.

2 Preliminaries

We consider a dynamic game that evolves in discrete time, with periods $t = 0, 1, 2, \ldots$. The game is played by a finite population of $N \geq 3$ players $i \in I = \{1, \ldots, N\}$. In each period t each player i is matched with every player in his neighborhood $V_i(t)$ to play a 2-person coordination game.

The primitive data of the model are the player set I, a binary individual action set $\{X, Y\}$, payoffs for the 2-player coordination game, a stochastic process $\widetilde{V}(0), \widetilde{V}(1), \ldots$ generating sequences of interaction structures (graphs, networks) $V(0), V(1), \ldots$, and an updating rule.

2.1 Base or constituent game

In each period, each player i chooses an action $s_i \in S_i = \{X, Y\}$. The binary action set is the same for all players at all times. $S = \{X, Y\}^N$ is the set of all action profiles. If players i and j interact once, i chooses action s_i and j chooses action s_j, then player receives payoff $\pi(s_i, s_j)$ and player j receives payoff $\pi(s_j, s_i)$ from their pairwise interaction. The pairwise interaction payoff function $\pi : \{X, Y\}^2 \to \mathbb{R}$ applies to all players at all times, where the first argument is always the action taken by the player, say i, whose payoff is to be determined while the second argument is the action of the particular player j with whom i interacts. If i and j, $i \neq j$, is an arbitrary pair of players, then their one time pairwise interaction is described by a symmetric 2×2 game, the **base or constituent game**. In case i is treated as row player and j as column player, the base game assumes the matrix form

j

i		X	Y
	X	a,a	b,c
	Y	c,b	d,d

where $a = \pi(X, X), b = \pi(X, Y), c = (Y, X), d = (Y, Y)$. We assume that the base game is a **coordination game**, that is $a > c$ and $d > b$.[5] In that case, the Nash equilibrium (X, X) is payoff dominant if $a > d$ whereas (Y, Y) is payoff dominant if $a < d$. The Nash equilibrium (X, X) or the action X is risk dominant if $a - c > d - b$. (Y, Y) or Y is risk dominant if $a - c < d - b$.

2.2 Interaction structures

An interaction structure describes who interacts with whom. This can simply be achieved by listing, for each player $i \in I$, the group $V_i \subseteq I \setminus \{i\}$ of players with whom i interacts. Now a binary relation $V \subseteq I \times I$ can be equivalently represented by its sections $V_i \equiv \{j \in I | (i, j) \in V\}, i \in I$. The pair (I, V) is also called a graph or network with I as its set of nodes and V as the set of edges. Thus listing who interacts with whom amounts to fixing a graph. It often proves convenient to refer to V rather than (I, V) as the graph.

We only consider graphs which have no loops [V is irreflexive: $(i, i) \notin V$ for $i \in I$] and are undirected [V is symmetric: $(i, j) \in V \Rightarrow (j, i) \in V$ or, equivalently, $j \in V_i \Leftrightarrow i \in V_j$]. Let $\mathbb{V}$ denote the set of all such "graphs" V. The elements $V \in \mathbb{V}$ are interpreted as **interaction structures**, describing who interacts with whom: If V is the prevailing interaction structure and $i, j \in I$, then i can interact with j and vice versa if and only if $j \in V_i$ and

[5]The base game is an anti-coordination game if $a < c$ and $d < b$.

$i \in V_j$. In particular, $V(t)$ denotes the interaction structure at time t and the section $V_i(t)$ is the set of neighbors or partners with whom player i is matched and interacts in period t.

Instead of representing an undirected graph as a symmetric binary relation, it is often advantageous to resort to an alternative representation: Edges are unordered pairs $\{i, j\}$. $j \in V_i$ and $i \in V_j$ iff $\{i, j\} \in V$. We shall resort to the latter representation from now on.

2.3 Dynamics

Let $s_i(t) \in S_i$ denote the action chosen by player i in period t. Further let $x(t) = |\{i \in I : s_i(t) = X\}|$ denote the number of players choosing action X and $s(t) = (s_1(t), \ldots, s_N(t))$ denote the action profile in period t.

We are interested in properties of the sequences $s(t)$, $t = 0, 1, \ldots$, and $x(t)$, $t = 0, 1, \ldots$, which of course depend on how the sequences are generated. In contrast to most models of biological evolution, models of economic and social evolution have explored the assumption that a player can only interact with a subset of the population, his neighborhood or reference group.[6] Moreover, it has been typically assumed that the exogenously given interaction structure is totally rigid over time: $V_i(t) = V_i(0)$ for all i and t. While many socio-economic relations are fairly stable, others are not or emerge only over time, and few are totally stable. Here we keep the concept of an interaction structure, but allow for the possibility that the structure changes over time. In principle, there could be inertia in the system so that only a few edges are added or deleted from one period to the next. The interaction structure of the next period could depend on the current interaction structure as well as the current action profile. Here we ignore these possibilities and focus on the direct opposite of a totally rigid interaction structure. We assume *a stochastic process of i.i.d. random variables* $\widetilde{V}(t) : \Omega \to \mathcal{V}$, $t = 0, 1, \ldots$, defined on an underlying probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and assuming values in a subset $\mathcal{V}$ of $\mathbb{V}$. The realizations of the process $\widetilde{V}(0), \widetilde{V}(1), \ldots$ constitute sequences of interaction structures (graphs, networks) $V(0), V(1), \ldots$. The independence assumption means that past connections do not influence current or future connections and, thus, captures the idea that social connections

[6]See Berninghaus and Schwalbe (1996a,b), Blume (1993, 1995), and references therein and thereafter.

may not be rigid at all.

Given an action profile $s = (s_1, \dots, s_N) \in S$ and an interaction structure V in a particular period, player i receives an aggregate payoff

$$u_i[s\,; V] = \sum_{j \in V_i} \pi(s_i, s_j) \tag{1}$$

from pairwise interactions in that period. The dynamic game starts with an initial state $s(0) \in S$ and an initial interaction structure, $V(0)$. In every period $t \geq 1$, given the realization $V(t)$ of $\widetilde{V}(t)$, players update their strategies, following a "myopic best response rule": Player i chooses a best response against the previous play of his current neighbors,

$$s_i(t) \in \arg\max \{ s_i \in S_i \mid u_i[(s_i, s_{-i}(t-1)); V(t)] \} . \tag{2}$$

Ties are broken in favor of X unless a player has no neighbors. In some realizations of some random graph, a player may end up neighborless, $V_i(t) = \emptyset$, in which case we assume that the player exhibits inertia: $s_i(t) = s_i(t-1)$. Let $n_i(t) = |\{ j \in V_i(t) : s_j(t-1) = X \}|$ be the number of i's neighbors who chose X last period. Then i with $V_i(t) \neq \emptyset$ will choose X this period if

$$n_i(t) \geq |V_i(t)| \cdot \frac{d-b}{a-c+d-b}. \tag{3}$$

Player i with $V_i(t) \neq \emptyset$ will choose Y in period t if

$$n_i(t) < |V_i(t)| \cdot \frac{d-b}{a-c+d-b}. \tag{4}$$

On purely descriptive grounds, one can envisage that an updating player maximizes against previous play of previous neighbors, that is $s_i(t)$ maximizes $u_i[(s_i, s_{-i}(t-1)); V(t-1)]$. Yet given the i.i.d. assumption on the process $\widetilde{V}(0), \widetilde{V}(1), \dots$, this descriptive difference merely translates into a notational difference and not a material one.

2.4 Long-run behavior and contagion

We examine the long-run behavior of the path $s(0), s(1), \dots$. **Contagion** (with respect to action X) occurs from an initial subset I_0 of I, if $s_i(0) = X$ for $i \in I_0$, $s_i(0) = Y$ for $i \notin I_0$, and there exists $T \in \mathbb{N}$ such that $s_i(t) = X$ for all $i \in I$ and $t \geq T$ or, equivalently, $x(t) = N$ for all $t \geq T$. Contagion with respect to action Y is defined in an analogous way. Contagion is especially forceful, if an action spreads to the entire population starting from a small group of players.

In the case of a fixed deterministic interaction structure, that is $V \in \mathbb{V}$ such that $V(t) = V$ for $t = 1, 2, \ldots$, one obtains that $s(t)$ converges either to a fixed point (steady state) or to a two-cycle; see Berninghaus and Schwalbe (1996b, Theorem 2), Goles (1987), Goles and Olivos (1980), Goles and Martinez (1990). Notice that convergence to a steady state does not necessarily imply occurrence of contagion. Berninghaus and Schwalbe (1996b) give an example of a discrete 6×6 torus (lattice with von Neumann neighborhoods) and a steady state where both conventions or actions, X and Y, coexist.

In general, the path $s(0), s(1), \ldots$ generated by simultaneous myopic best response updating is random when the sequence of interaction structures is random.[7] The fact that the interaction structure is newly formed each period can drastically affect contagion, even if the same network architecture is realized every period. The difference between a fixed deterministic interaction structure and a random sequence of interaction structures becomes evident in the case of circular graphs (2-regular connected graphs) studied in the next section.

3 Interaction on Regular Graphs

For a player or node $i \in I$ and a graph $V \in \mathbb{V}$, the degree of i in V, $d_i(V)$, is defined as the number of i's neighbors in V: $d_i(V) \equiv |V_i|$. Let n be a nonnegative integer. The graph V is n-**regular**, if each player has n neighbors in V: $d_i(V) = n$ for all $i \in I$. Let $\mathcal{C}(n)$ denote the class of all connected and n-regular graphs on I.

[7] In models of adaptive play, there can be other sources of randomness: With asynchronous updating, there may be a random selection of the player who updates. Players may play stochastically perturbed best responses caused by errors, trembles, or mutations. A player may respond to a random sample of his neighbors.

3.1 Interaction on circular graphs

Circular graphs very likely are the local interaction structures most frequently studied in game theory and economics. In such a graph, the players can be arranged in a circle so that each player has one neighbor to the left and one neighbor to the right. Formally, such a graph assumes the form V^β with sections $V_i^\beta = \{j \in I|\, \beta(j) = \beta(i) \pm 1\,\text{modulo}\ N\}$ where β is a permutation of I. In V^β, we say that j is the left neighbor of i with respect to β if $\beta(j) = \beta(i) - 1 \mod N$ and j is the right neighbor of i with respect to β if $\beta(j) = \beta(i) + 1 \mod N$. Notice that there exist permutations $\alpha \neq \beta$ such that $V^\alpha = V^\beta$, but j is a left neighbor of i with respect to α and a right neighbor of i with respect to β. Let $\mathcal{R}(2)$ denote the class of circular graphs on I. It is easy to see that $\mathcal{R}(2) = \mathcal{C}(2)$.

For the sake of comparison, let us briefly reconsider the adjustment dynamics in the case of a fixed deterministic interaction structure of the form V^β, with β a permutation of I. Let $k \in I$, $s_k(0) = X$ and $s_i(0) = Y$ for all $i \neq k$. If action Y is risk dominant then in one step contagion with respect to action Y occurs. If action X is risk dominant and N is even then convergence to a two-cycle occurs. If action X is risk dominant and N is odd, then contagion with respect to action X occurs.

Let us next consider i.i.d. random variables $\widetilde{V}(t) : \Omega \to \mathcal{R}(2)$, $t = 0, 1, \ldots$, so that $\mathcal{V} = \mathcal{R}(2) = \{V^\beta | \beta \text{ a permuation of} I\}$ and each $\widetilde{V}(t)$ is uniformly distributed on $\mathcal{R}(2)$. Now again, if $k \in I$, $s_k(0) = X$, $s_i(0) = Y$ for all $i \neq k$, and action Y is risk dominant, then in one step contagion with respect to action Y occurs. However, if action X is risk dominant, contagion occurs regardless of N:

Proposition 1 *Suppose each $\widetilde{V}(t)$ is uniformly distributed on the set of circular graphs. If X is risk dominant and $x(0) > 0$ then with probability one, the entire population will end up choosing X in finite time.*

PROOF. Let X be risk dominant. Notice that then on a circular graph, a player chooses X if at least one of his neighbors has chosen X. First we show

Claim 1: *$x(t)$ is nondecreasing in t.*

Namely, let $V(t) = V^{\beta}$ for some permutation β. If $s_i(t-1) = X$ and $s_i(t) = Y$, then $s_j(t-1) = Y$ and $s_j(t) = X$ where j is i's right-hand neighbor with respect to β. Hence there are at least as many switches from Y to X as there are from X to Y, which shows the claim. Next we show

Claim 2: *If* $0 < x(t-1) < N$ *then* $x(t) > x(t-1)$
with probability $\geq 1/|\mathcal{R}(2)|$.

For if $x(t-1) = 1$, $s_i(t-1) = X$, and $V(t) = V^{\beta}$ then i's left and right neighbors with respect to β choose X in period t. If $1 < x(t-1) < N$ then with probability at least $1/|\mathcal{R}(2)|$, $V(t)$ assumes the form $V^{\beta^{-1}}$ such that
$s_{\beta(1)}(t-1), s_{\beta(2)}(t-1), \ldots, s_{\beta(N)}(t-1)$ equals
$XX \ldots XY$ or $YX \ldots XY$ or $YX \ldots XYY$ or
$YX \ldots XYY \ldots Y$, respectively,
and, therefore,
$s_{\beta(1)}(t), s_{\beta(2)}(t), \ldots, s_{\beta(N)}(t)$ equals
$XX \ldots XX$ or $XX \ldots XX$ or $XX \ldots XXY$ or
$XX \ldots XXY \ldots Y$, respectively.
This shows the claim. The assertion of the proposition follows from the two claims. Q.E.D.

In the deterministic case (with a fixed interaction structure of the form $V^{\beta^{-1}}$) when action X is risk dominant and N is even, convergence to a two-cycle occurs. Given the prevailing interaction structure $V^{\beta^{-1}}$, the two elements s', s'' of the two-cycle exhibit an alternating pattern of X's and Y's, say $s'_{\beta(1)}, \ldots, s'_{\beta(N)} = XYXY \ldots XY$ and $s''_{\beta(1)}, \ldots, s''_{\beta(N)} = YXYX \ldots YX$. In the stochastic case, even if at some time t, $s(t)$ exhibits an alternating pattern with respect to $V(t)$, this does not lead to a permanent two-cycle. For the probability that $s(\tau)$ constitutes an alternating pattern with respect to $V(\tau)$ for all $\tau > t$ is zero. With probability one, the cycle will be interrupted and contagion occurs.

The proposition can be generalized in two ways. First, it suffices to assume that each of the i.i.d. random variables $\widetilde{V}(t)$ has as support the set of circular graphs. In the proof, the probability $1/|\mathcal{R}(2)|$ has to be replaced by $\min\{\mathsf{Prob}(V(1) = V) : V \in \mathcal{R}(2)\} > 0$. Second, the set of all circular graphs can be replaced by the set of all 2-regular graphs, that is graphs where each player has exactly two neighbors. The reason is that the connected components of a 2-regular graph are circular graphs.

3.2 Interaction with more than two neighbors

Proposition 1 shows the relevance of risk dominance for contagion in societies with small neighborhood groups. The following two examples and Proposition 2 demonstrate that only weaker conclusions can be drawn when neighborhood size increases.

Suppose $n \in \mathbb{N}, 2 \leq n < N$ and the support of each $\widetilde{V}(t)$ is contained in the set of $n-$*regular* graphs on I, the graphs where each player has exactly n neighbors. If $x(0)/n < (d-b)/(a-c+d-b)$, then (4) holds for $t = 1$ and all i and, therefore, contagion with respect to action Y occurs in one step.

Example 1. Let $a = 3, d = 2, b = c = 0$. Then action X is both payoff dominant and risk dominant. Further let $N \geq 4$ be even — to guarantee the existence of 3-regular graphs — and $x(0) = 1$. If $n = 2$, then a variant of Proposition 1 applies: With probability one, contagion with respect to action X occurs. If $n = 3$, then $x(0)/3 = 1/3 < 2/5 = (d-b)/(a-c+d-b)$ and contagion with respect to Y occurs in a single step. While risk dominance of an action favors contagion with respect to that action, nevertheless the risk dominant action gets extinct when only a relatively small fraction of players has chosen it and the random graph generates rather large neighborhoods. □□

Now let $n \geq 2$ be even and in analogy to $\mathcal{R}(2)$, define the set $\mathcal{R}(n)$ of n-regular graphs where each player has $n/2$ left neighbors and $n/2$ right neighbors (with respect to a permutation β of N that defines the graph).

Proposition 2 *Suppose each $\widetilde{V}(t)$ has support $\mathcal{R}(n)$. If X is risk dominant and $N > x(0) > n/2$ then with positive probability, the entire population will end up choosing X in finite time.*

PROOF. Let $p = \min\{\mathsf{Prob}(V(1) = V) : V \in \mathcal{R}(n)\} > 0$. If $t \geq 1$ and $N > x(t-1) > n/2$, then with probability at least p, there exists a permutation β of I such that $V(t)$ is defined by β^{-1},

$s_{\beta(1)}(t-1) = \ldots = s_{\beta(x(t-1))}(t-1) = X$ and

$s_{\beta(1)}(t) = \ldots \ \ldots = s_{\beta(x(t-1)+1)}(t) = X$.

Hence for some $k \in \{1, \ldots, N - x(0)\}$:

$x(0) < x(1) < \ldots < x(k) = N$ with probability at least p^k,

since the random variables $\widetilde{V}(1), \widetilde{V}(2), \ldots$ are i.i.d. Q.E.D.

Proposition 2 draws a weaker conclusion than Proposition 1. Indeed, in Proposition 2, "positive probability" cannot be replaced by "probability one". To see this, consider

Example 2. Suppose each $\widetilde{V}(t)$ has support $\mathcal{R}(n)$. Let $a = 3, d = 2, b = c = 0$. Then action X is both payoff dominant and risk dominant. Further let $N = 18$ and $n = 4$ so that $n/2 = 2$. Suppose that $s_i(0) = X$ for $i \in \{1, 7, 13\}$ and $s_i(0) = Y$ for $i \notin \{1, 7, 13\}$. Then $x(0) = 3 > n/2$. Now let $V' \in \mathcal{R}(4)$ be defined by the permutation $\beta = \mathrm{id}_N$, that is $\{i, j\} \in V'$ iff $j = i \pm 1$ or $i \pm 2 \mod N$. If $V(1) = V'$, then $n_i(1) \in \{0, 1\}$ and $|V_i(1)| \cdot (d - b)/(a - c + d - b) = n(d - b)/(a - c + d - b) = 8/5$ for all $i \in I$; hence (4) holds for $t = 1$ and all i and contagion with respect to action Y occurs in one step. With positive probability, $V(1) = V'$. Thus, the hypothesis of Proposition 2 is satisfied whereas the event "the entire population will end up choosing X in finite time" has probability less than one. □□

3.3 OR networks

A successful approach to the contagion question in the computer science literature is known under the rubric of AND and OR networks; see Goles and Hernández (2000). Let us consider **OR networks** in more detail. These are defined by the property

$$s_i(t) = X \iff n_i(t) \geq 1. \tag{5}$$

By (3) and (4), in our context condition (5) is equivalent to $1 \geq |V_i(t)| \cdot (d - b)/(a - c + d - b)$. In case $V(t)$ is n-regular with $n \geq 2$, the latter is equivalent to $1 \geq n(d - b)/(a - c + d - b)$ or $a - c \geq (n - 1)(d - b)$. This condition obviously requires that X be *sufficiently risk dominant* in order to obtain an OR network. In case $n = 2$, the risk dominance condition $a - c > d - b$ suffices. In case $n = 4$, the stronger condition $a - c \geq 3(d - b)$ is required.

Concerning contagion it is easy to see that similar arguments as in the proof of Proposition 1 yield the following result:

Proposition 3 *Let $n \geq 2$ be even and $a - c \geq (n-1)(d-b)$. Suppose each $\widetilde{V}(t)$ has support $\mathcal{R}(n)$ — so that each realization $V(t)$ constitutes an OR network. If $x(0) > 0$ then with probability one, the entire population will end up choosing X in finite time.*

Proposition 3 can be extended to random graph processes where the support of each $\widetilde{V}(t)$ contains $\mathcal{R}(n)$ and possibly includes other members of $\mathcal{C}(n)$ with the property that each node can be assigned a "select right-hand neighbor". In the proof of Proposition 1, the argument for Claim 1 relies on the fact that for $V \in \mathcal{R}(2)$, $V = V^\beta$ and $i \in I$, we can assign to i its right-hand neighbor j with respect to β, given by $\beta(j) = \beta(i) + 1 \mod N$. Similarly, for any even $n > 2$, $V \in \mathcal{R}(n)$ defined by a permutation β of N and $i \in I$, i's neighborhood is $V_i = \{j \in I : \beta(j) = \beta(i) \pm 1 \text{ or } \ldots \text{ or } \beta(i) \pm n/2 \mod N\}$ and i can be assigned a "select right neighbor j with respect to β", again given by $\beta(j) = \beta(i) + 1 \mod N$. Therefore, the argument for Claim 1 can be made again. It can be generalized to graphs $V \in \mathcal{C}(n) \backslash \mathcal{R}(n)$ in which every node i can be assigned a "select right neighbor".

For instance, let $N = 60$ and $n = 4$. Then each $i \in I$ has a unique representation $i = 20i_1 + i_2$ with $i_1 \in \{0, 1, 2\}$ and $i_2 \in \{1, \ldots, 20\}$. Define a von Neumann neighborhood for i via $V_i \equiv$

$$\left\{ j \in I \;\middle|\; \begin{array}{l} j = 20j_1 + j_2 \text{ with } j_1 \in \{0,1,2\}, j_1 \neq i_1, j_2 = i_2 \text{ or} \\ j = 20j_1 + j_2 \text{ with } j_1 = i_1, j_2 \in \{1, \ldots, 20\}, j_2 = i_2 \pm 1 \mod 20 \end{array} \right\}.$$

Then the 3×20 lattice or torus V given by the neighborhoods V_i, $i \in I$, belongs to $\mathcal{C}(4) \backslash \mathcal{R}(4)$. To $i = 20i_1 + i_2$, one can assign the right-hand neighbor $j = 20j_1 + i_2$ with $j_1 \in \{0, 1, 2\}, j_1 = i_1 + 1 \mod 3$. This is not the only possibility: The assignment $j = 20i_1 + j_2$ with $j_2 \in \{1, \ldots, 20\}, j_1 = i_1 + 1 \mod 20$ will also do. The construction can be applied to any $V \in \mathcal{C}(n) \backslash \mathcal{R}(n)$ which is representable as a finite lattice without boundary (discrete torus). Still for $N = 60, n = 4$, one can also consider a 5×2 lattice or torus or a 3-dimensional $2 \times 2 \times 15$ lattice or cube, etc.

Goles and Hernádez (2000) observe that deterministic OR networks form a class of neural networks. Any deterministic OR network dynamics with simultaneous updating exhibits contagion (with respect to action X or with respect action Y) or converges to a unique 2-cycle. In contrast to $n = 2$, deterministic OR network dynamics based on a fixed network $V \in \mathcal{R}(n)$

with n even, $n > 2$, and $x(0) > 0$ always exhibits contagion with respect to action X. Thus, in the long-run, interaction based on a fixed interact structure and interaction on a sequence of random interaction structures results in the same outcome. However, as elaborated in the previous paragraph, the assumed support $\mathcal{R}(n)$ in Proposition 3 can be replaced by a superset of $\mathcal{R}(n)$ which contains other elements V' of $\mathcal{C}(n)$ where each node can be assigned a "select right neighbor". For example, in case $N = 16$, $n = 4$, a 4×4 torus (lattice without boundary) V' will qualify. Deterministic OR network dynamics based on V' with $x(0) = 1$ converges to a 2-cycle. In that case, interaction on a sequence of random interaction structures makes a difference: Replace the support $\mathcal{R}(n)$ in Proposition 3 by $\mathcal{R}(n) \cup \{V'\}$ and set $\epsilon \equiv \mathsf{Prob}(V(1) \neq V') > 0$. For arbitrarily small ϵ, contagion with respect to action X occurs although V' has a much greater chance to be realized than not.

The OR network property, that is sufficiently strong risk dominance of action X, proves very conducive to contagion with respect to action X. If, on the other hand, the OR network property fails to hold, then it is perhaps not surprising that contagion with respect to action X may fail in some instances as well. However, *ceteris paribus*, the impact of the lack of the OR network property turns out to be quite drastic: Suppose $a - c < (n-1)(d-b)$ while the other assumptions of Proposition 3 remain unchanged. Then in large populations, with positive probability contagion with respect to action Y occurs even when a vast majority of the players initially chooses action X.

Proposition 4 *Let $n \geq 2$ be even and $a - c < (n-1)(d-b)$. Suppose each $\widetilde{V}(t)$ has support $\mathcal{R}(n)$ — so that none of the realizations $V(t)$ constitutes an OR network. If $N - x(0) \geq 5n^2$ then with positive probability, the entire population will end up choosing Y in finite time.*

PROOF. Let again $p = \min\{\mathsf{Prob}(V(1) = V) : V \in \mathcal{R}(n)\} > 0$. By assumption, there are at least $5n^2$ players who initially choose action Y. In case $x(0) \geq 2n+1$ there exists a permutation β of I such that

$$s_{\beta(i)}(0) = \begin{cases} Y & \text{for} \quad i = 1, \ldots, 2n; \\ X & \text{for} \quad i = (1+k)2n+1, \; k = 0, 1, \ldots, 2n; \\ Y & \text{for} \quad i = (1+k)2n+\ell, \; k = 0, 1, \ldots, 2n, \; \ell = 0, 2, \ldots, 2n; \\ Y & \text{for} \quad i = (2n+3)2n, \ldots, (2n+3)2n \\ & \qquad\qquad + N - x(0) - (4n^2 - 1 + 2n) = 4n + 1 + N - x(0); \\ X & \text{for} \quad i = 4n + 1 + N - x(0), \ldots, N. \end{cases}$$

If $V(1)$ is the element of $\mathcal{R}(n)$ defined by β^{-1}, then the $2n+1$ players $\beta(i)$ with $i = (1+k)2n+1$, $k = 0, 1, \ldots, 2n$ have at most one neighbor in $V(1)$ who previously played X. Because of $a - c < (n-1)(d-b)$, they switch from X in period 0 to Y in period 1. Each player $\beta(i)$ with $i \in \{n+1, \ldots, 4n+1+N-x(0)-n\}$ and $s_{\beta(i)}(0) = Y$ has at most one neighbor in $V(1)$ who previously played X and, therefore, $\beta(i)$ keeps playing Y in period 1. Therefore, at most $2n$ players switch from Y in period 0 to X in period 1. Hence the number of players switching from Y to X (at most $2n$) is less then the number of players switching form X to Y (at least $2n+1$). With probability at least p, this realization of $V(1)$ occurs and $x(1) < x(0)$.

In case $x(0) < 2n+1$, there exists a permutation β of I such that

$$s_{\beta(i)}(0) = \begin{cases} X & \text{for} \quad i = 2kn+1, \; k = 0, 1, \ldots, 2n-1; \\ Y & \text{for} \quad i = 2kn+\ell, \; k = 0, 1, \ldots, 2n, \; \ell = 0, 2, \ldots, 2n; \\ Y & \text{for} \quad i = 4n^2 + 2n, \ldots, N; \end{cases}$$

If $V(1)$ is the element of $\mathcal{R}(n)$ defined by β^{-1}, then each player has at most one neighbor in $V(1)$ who previously played X. Because of $a - c < (n-1)(d-b)$, every player chooses action Y in period 1. With probability at least p, this realization of $V(1)$ occurs and $x(1) = 0$.

We conclude that with probability at least p, $x(1) \leq \max\{0, x(0) - 1\}$ and $N - x(1) \geq N - x(0) \geq 5n^2$. Since the random variables $\widetilde{V}(1), \widetilde{V}(2), \ldots$ are i.i.d., induction with respect to t yields existence of $k \in \{1, \ldots, x(0)\}$ such that $x(k) = 0$ with probability at least p^k. Q.E.D.

In the first part of the proof, the case $x(0) \geq 2n+1$, consider the players arranged on a circle, each with $n/2$ neighbors on either side. The circle is partitioned into four segments A, B, C, and D which are arranged clockwise, say. Segment A consists of $2n$ players choosing action Y initially. Segment B consists of $2n+1$ initial X-players plus $2n(2n-1)$ initial Y-players where any two X-players are separated by $2n-1$ initial Y-players. Segment C consists of the remaining at least $2n$ players choosing action Y initially. Segment D consists of the rest of the initial X-players. The initial X-players in segment B will switch from X to Y. All initial Y-players in segment B keep playing Y. At most n players (close enough to segment D) in segment A and at most n players (close enough to segment D) in segment C will switch from Y to X.

Returning to our earlier assertion, let us assume that $N = Kn^2$ and $N - x(0) = 5n^2$. Then $x(0)/N = 1 - 5/K \to 1$ as $K \to \infty$. Hence for sufficiently large population size, with positive probability contagion with respect to action Y occurs even when a vast majority of the players initially chooses action X.

4 Interaction on Binomial Random Graphs

In the random graph model most commonly studied in mathematics and statistics, each possible edge $\{i, j\}$ is included independently of others with probability $p \in (0, 1)$. The model is denoted by $G(N, p)$ and known as the Erdős-Rényi random graph or the binomial random graph. Sometimes, the term "random graph" is used for this particular random graph model.

Throughout this section, we assume an i.i.d. process $\widetilde{V}(t) : \Omega \to \mathbb{V}$, $t = 0, 1, \ldots$, where each $\widetilde{V}(t)$ is distributed according to a binomial random graph $G(N, p)$. Hence $\mathsf{Prob}\,(V(1) = V) = p^{|V|}(1-p)^{N(N-1)/2-|V|}$ for $V \in \mathbb{V}$. In the literature, frequently $p = p(N)$ is assumed and asymptotic properties of the random graph $G(N, p(N))$ as $N \to \infty$ are investigated. We are going to encounter two special cases: (a) fixed p and N; (b) fixed p while $N \to \infty$.

Proposition 5 *Let $N > 2$ and $p \in (0, 1)$ be given. If $x(0) > 0$ and X is risk dominant, then with positive probability, the entire population will choose the risk dominant action X in period 2.*

PROOF. Suppose $s_i(0) = X$. With probability $p^{N-1} \cdot (1-p)^{(N-1)(N-2)/2}$ the star with center i is formed in period 1, in which case $s_j(1) = X$ for all $j \neq i$. With probability $p^{N(N-1)/2}$, the complete network is formed in period

2. In that case $s_j(1) = X$ for $j \neq i$ implies $s_j(2) = X$ for all j. Hence with probability at least $p^{(N+2)(N-1)/2} \cdot (1-p)^{(N-1)(N-2)/2}$, the population will choose X in period 2. Q.E.D.

In general, the probability $p^{(N+2)(N-1)/2} \cdot (1-p)^{(N-1)(N-2)/2}$ given in the proof is merely a lower bound, since other sequences of networks can also have the population choose X in period 2. Nonetheless, the probability that the entire population chooses X in finite time may be very small, as the following result shows. Indeed, the probability that the population ends up choosing Y in period 1 (and in all subsequent periods) can become extremely high.

Proposition 6 *Let $p \in (0,1)$ be fixed. Suppose that exactly one player chooses the risk dominant strategy X in period 0. Then as N tends to infinity, the probability that the population chooses Y in period 1 goes to one.*

Notice that Y is player i's best reply if (4) holds. In case exactly one player has chosen X, $n(i)$ equals 0 or 1 for every player. Then

$$1 < |V(i)| \cdot \frac{d-b}{a-c+d-b} \tag{6}$$

for all i is a sufficient condition for the population choosing Y. We are going to show that with high probability, all neighborhoods are "too" large in the sense of inequality (6) as N becomes large. To be precise, let us define the event $E \equiv \langle\, (6)$ holds for all i in period $1\rangle$. We shall show $\mathsf{Prob}(E) \to 1$ as $N \to \infty$.

PROOF. We want to show $\mathsf{Prob}(E) \to 1$ as $N \to \infty$. Let m denote the smallest integer greater than or equal to $(a-c+d-b)/(d-b)$. Then $|V(i)| > m$ is sufficient for (6).

Now consider $N > 4m$. Define $N' = (N-2)/2$ for N even and $N' = (N-1)/2$ for N odd. For $i \in I$, consider the set of pairs (links, edges) $L_i = \{\{i,j\} : j = i + \ell \bmod N \text{ for some } \ell \in \{1, \ldots, N'\}\}$. Then $|L_i| > m$ for all i and $L_i \cap L_{i'} = \emptyset$ for $i \neq i'$. Further consider, for $i \in I$, the events $E'_i \equiv \langle\,$More than m of the links in L_i exist$\rangle$ and $F'_i \equiv \langle\,$At most m of the links in

L_i exist⟩. Now

$$\begin{aligned} \mathsf{Prob}\,(F_i') &= \sum_{k=0}^{m} \binom{N'}{k} p^k (1-p)^{N'-k} \\ &\leq (m+1)(N')^m/(m!)q \cdot (1-p)^{N'} = c \cdot (N')^m \cdot (1-p)^{N'} \end{aligned}$$

where $q = \max_{k=0}^{m}(p/(1-p))^k$ and $c = q(m+1)/(m!)$. Since $N' \to \infty$ for $N \to \infty$, it follows $c \cdot (N')^m \cdot (1-p)^{N'} \to 0$ for $N \to \infty$.

Next observe that $E_1', E_2', \ldots, E_N'$ are independent events. Hence for sufficiently large N, $c \cdot (N')^m \cdot (1-p)^{N'} < 1$ and

$$\mathsf{Prob}\,(\bigcap_{i\in I} E_i') = \prod_{i\in I} \mathsf{Prob}\,(E_i') = \prod_{i\in I}[1 - \mathsf{Prob}\,(F_i')]$$

$$\geq \left[1 - c\cdot (N')^m \cdot (1-p)^{N'}\right]^N \geq 1 - Nc \cdot (N')^m \cdot (1-p)^{N'}$$

$$\geq 1 - 4c \cdot (N')^{m+1} \cdot (1-p)^{N'}.$$

Finally, consider the events $E_i^* \equiv \langle\, |V(i)| > m$ holds in period 1⟩ and $E_i \equiv \langle$ (6) holds in period 1⟩ for $i \in I$. Then $\bigcap_i E_i' \subseteq \bigcap_i E_i^* \subseteq \bigcap_i E_i = E$ and, consequently, $\mathsf{Prob}(E) \geq \mathsf{Prob}\,(\bigcap_i E_i') \geq 1 - 4c \cdot (N')^{m+1} \cdot (1-p)^{N'}$. Hence $\mathsf{Prob}(E) \to 1$ for $N \to \infty$ as asserted. Q.E.D.

The basic insight behind Proposition 6 is that large neighborhoods are very likely in a large population. In the event that all neighborhoods are sufficiently large, a single X cannot survive. Obviously, the result can be generalized to the case where $s_i(0) = X$ for at most h players, for a fixed positive integer h. In that case replace the left-hand side of (6) by h and define m as the smallest integer greater than or equal to $h\cdot(a-c+d-b)/(d-b)$.

In view of Propositions 5 and 6, several questions arise:

1. Can one obtain good approximations of the probabilities that in finite time, the population ends up choosing X and that in finite time, the population ends up choosing Y?

2. Is it the case that with probability one, the population in finite time either will end up choosing X or will end up choosing Y? Note that

in addition to coordination on X or coordination on Y, two more possibilities might exist when the graph or interaction structure is deterministic: 2-cycles and stationary points where some players choose X while others choose Y.

3. How does the dynamics evolve if X is prevalent in the initial state?

Whereas the first question appears to be hard, the other two have straightforward answers. To begin with, we obtain almost analogous results after switching the roles of X and Y in Propositions 5 and 6 — which answers the third question. Namely, the analog of Proposition 5 holds under the assumption $1/(N-1) < (d-b)/(a-c+d-b)$. The analog of Result 6 holds without further stipulations.

To address the second question, we consider the evolution of the variable $x(t) = |\{i \in I : s_i(t) = X\}|$. For given N, let $\mathfrak{X} = \{0, 1, \ldots, N\}$. Note that the random graph evolves independently of players' choices. Moreover, the random graph process $\widetilde{V}(t) : \Omega \to \mathcal{V}$, $t = 0, 1, \ldots$, based on the binomial random graph $G(N, p)$ is *symmetric* in the sense that if $V, V' \in \mathbb{V}$ and β a permutation of I such that $\{i, j\} \in V \iff \{\beta(i), \beta(j)\} \in V'$, then $\mathsf{Prob}\,(V(t) = V) = \mathsf{Prob}\,(V(t) = V')$. Hence for $x, x' \in \mathfrak{X}$ and $t \geq 1$, $q(x'|x) = \mathsf{Prob}\,(x(t) = x' \text{ if } x(t-1) = x)$ is well defined (and independent of t) and $x(t)$ follows a Markov process with state space $\mathfrak{X}$ and transition probabilities $q(x'|x)$. Under the assumption $1 < (N-1)(d-b)/(a-c+d-b)$, the transition probabilities satisfy

$q(0|0) = q(N|N) = 1$;

$q(x-1|\,x) > 0$ for $x \neq 0, N$.

Under the assumption $N-2 \geq (N-1)(d-b)/(a-c+d-b)$, the transition probabilities satisfy

$q(0|0) = q(N|N) = 1$;

$q(x+1|\,x) > 0$ for $x \neq 0, N$.

But either $1 < (N-1)(d-b)/(a-c+d-b)$ or $(N-1)(d-b)/(a-c+d-b) \leq 1 \leq N-2$. Therefore, $\{0\}$ and $\{N\}$ are the only ergodic sets of the process. Consequently, $\lim_{t\to\infty} \mathsf{Prob}\,(x(t) \in \{0, N\}) = 1$. This provides the answer to our second question:

Proposition 7 *With probability one, the population will be coordinated in finite time (on action X or on action Y).*

5 Remarks and Ramifications

Here we review some of our findings in a broader context. We briefly elaborate on some alternatives, like asynchronous updating and the endogenous co-evolution of local interaction structures and local interaction.

5.1 Asynchronous updating on circular graphs

Following in the footsteps of Berninghaus and Schwalbe (1996a,b), we have assumed simultaneous or synchronous updating so far. The main alternative, **asynchronous updating** means that only one player is updating his action at any time. Formally, this is modeled by means of a sequence $K(t)$, $t = 0, 1, 2, \ldots$, of I-valued random variables. $K(t)$ determines which of the players will have a chance to alter his action at time t. The selected player $i = K(t)$ will update his action according to the rule (2), amended by the tie breaking and inertia conventions. Players $j \neq K(t)$ repeat their previous action, $s_j(t) = s_j(t-1)$. Similar to Blume (1995), we assume for the current purposes that the stochastic process $K(t)$, $t = 0, 1, \ldots$ is independent of the process $\widetilde{V}(t)$, $t = 0, 1, \ldots$ and consists of a sequence of i.i.d. random variables with full support I.[8]

Huberman and Glance (1993) suggest that the order of updating, synchronous versus asynchronous, can matter. This can be easily demonstrated in the context of circular interaction structures. First, with a fixed deterministic interaction structure of the form V^{β}, each $K(t)$ uniformly distributed on I, risk dominance of action X, and $x(0) = 1$, contagion with respect to action X occurs with probability 2/3 and contagion with respect to action Y occurs with probability 1/3. This is significantly different from the corresponding case with simultaneous updating, where either contagion with respect to action X (for N odd) or convergence to a 2-cycle (for N even) occurs. Second, we have seen that with simultaneous updating, a random change of the interaction structure can interrupt cycles and, therefore, is conducive for contagion with respect to the risk dominant action. In stark contrast, with asynchronous updating, a random change of the interaction structure can reverse the contagion process and proves detrimental to conta-

[8] The assumption is frequently made in models with logit perturbed best responses. See Blume (1997), Young (1996, Ch. 6), Baron *et al.* (2002a, b).

gion with respect to the risk dominant action.

To see this, assume $N \geq 4$, that each $\widetilde{V}(t)$ is uniformly distributed on the set of circular graphs, and as before, each $K(t)$ is uniformly distributed on I, action X is risk dominant, and $x(0) = 1$. Let P be the probability that contagion with respect to action X occurs. Further let $P' \equiv \mathsf{Prob}\,(\text{contagion occurs}|\, x(1)=2)$. Recall that in the deterministic case, $P = 2/3$. Also notice that in the deterministic case, $P' = 1$, since the two players choosing X at time $t = 1$ will be neighbors forever.

We find that randomness of the interaction structure proves detrimental to contagion with respect to the risk dominant action, indeed:

Claim: *With the random interaction structure, $P' < 1$ and $P < 2/3$.*

Namely, without loss of generality, suppose that $s_1(1) = s_2(1) = X$ and $s_j(1) = Y$ for $j \neq 1, 2$. Since $N \geq 4$, there exists $V^\beta \in \mathcal{R}(2)$ such that 1 and 2 are not neighbors. Now if $V(2)$ is such that 1 and 2 are not neighbors and $K(2) \in \{1, 2\}$, then the selected player $i \in \{1, 2\}$ will change his action from $s_i(1) = X$ to $s_i(2) = Y$. The specific realizations will occur with probability $\frac{1}{|\mathcal{R}(2)|} \cdot \frac{2}{N}$. Hence $\mathsf{Prob}\,(x(2) = 1|\, x(1) = 2) \geq \frac{1}{|\mathcal{R}(2)|} \cdot \frac{2}{N}$. Further $\mathsf{Prob}\,(x(3) = 0|\, x(2) = 1) = 1/N$. Therefore, $\mathsf{Prob}\,(x(3) = 0|\, x(1) = 2) > 0$ and $P' < 1$. Finally,

$$\begin{aligned} P &= \mathsf{Prob}\,(\text{contagion occurs}|\, x(0)=1) \\ &= \mathsf{Prob}\,(x(1) = 1|\, x(0) = 1)\, P + \mathsf{Prob}\,(x(1) = 2|\, x(0) = 1)\, P' \\ &= \frac{N-3}{N}\, P + \frac{2}{N}\, P' \end{aligned}$$

or $P = (2/3)\, P' < 2/3$ which proves the claim.

5.2 Related literature and concepts

The parameter $\vartheta = (d-b)/(a-c+d-b)$ is what López-Pintado (2006) calls the "degree of risk dominance" of action X. X is risk dominant if $\vartheta < 1/2$. X becomes more risk dominant when ϑ becomes smaller. Condition (3) can be rewritten as $n_i(t)/|V_i(t)| \geq \vartheta$. That means, *ceteris paribus*, the more risk dominant is action X, the more likely occurs contagion with respect to action X. Following Morris (2000), let us define the **contagion threshold** ξ as the largest ϑ for which contagion with respect to action X is possible.

Let us first consider circular graphs. In case a single player chooses X initially, we observe the following. With a fixed circular graph and N odd, $\xi = 1/2$. With a fixed circular graph and N even, $\xi = 0$, since limit cycles do not qualify as contagion according to our definition. With a random circular graph, $\xi = 1/2$. Further, with asynchronous updating and (fixed or random) circular graph, $\xi = 1/2$, since contagion with respect to action X occurs with positive probability if and only if X is risk dominant. Now $d_i(V) = 2$ for all $i \in I$, $V \in \mathcal{R}(2)$. Hence a player's degree distribution is always concentrated on the value 2, regardless whether the circular graph is fixed or random. Notice that even though the degree distribution is not affected by a change from a fixed circular graph to a random graph, there is an effect on the probability of contagion with respect to the risk dominant action X. With N even and simultaneous updating, the probability shifts from zero to one. In the instance of asynchronous updating investigated in 5.1, the probability shifts from $P = 2/3$ to $P < 2/3$. Therefore, it makes a difference, in terms of the probability of contagion with respect to the risk dominant action X, whether a new graph is drawn at random every period or only once before the play begins. We find that in our context, it matters both qualitatively and quantitatively whether the random graph is generated every period or only once. However, López-Pintado (2006, p. 374) states "... the model that we analyze with the mean-field equations is analogous to one where the random network is generated every period, although the connectivity of each individual remains constant. We believe that the qualitative results of this alternative model coincide with the results of the original model where the network is fixed throughout the dynamics but has been generated by a random process."

Further notice that while risk dominance, $\vartheta \leq 1/2$, is necessary for contagion with respect to X, stronger risk dominance does not make a difference: If $x(0) = 1$ and $\vartheta' < \vartheta \leq 1/2$, the dynamics corresponding to ϑ' and ϑ are the same. Finally notice that for the n-regular OR networks studied in 3.3, $\xi = 1/n$.

Let us consider binomial graphs next. In case a single player chooses X initially, we observe the following. If $\vartheta \leq 1/2$, then with positive probability contagion with respect to action X occurs. If $\vartheta > 1/2$, then with probability one contagion with respect to action Y occurs. Hence $\xi = 1/2$. The probability of contagion with respect to action X is weakly decreasing in ϑ. It jumps at the points $\vartheta = r/s$, $r, s \in \mathbb{N}, r \leq s/2$. *Ceteris paribus*, the probability goes to zero as N goes to infinity. It also converges to zero, *ceteris paribus*, when $p \to 1$ — which is easy to prove. To understand these asymptotic properties, let us look at the distribution of individual degrees. For a binomial random graph $G(N, p)$ and a player (node) $i \in I$, the degree $d_i(\cdot)$ is a random variable. For $k \in \{0, 1, \ldots, N-1\}$, let $p_{ik}(N, p) = \mathsf{Prob}\,(d_i = k)$ denote the probability that i has degree k in the random graph $G(N, p)$. Obviously,

$$p_{ik}(N, p) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$

It follows $\frac{\partial}{\partial p} p_{ik}(N, p) > 0 \Leftrightarrow k > (N-1)p$ and $p_{ik}(N+1, p) > p_{ik}(N, p) \Leftrightarrow k > Np$. Hence an increase of p or N leads to a new distribution of individual degrees that first-order stochastically dominates the old one. But such a shift towards higher degrees, that is larger neighborhoods, can be detrimental to the occurrence of contagion. Namely, for contagion to have a chance at all when j is the player who chooses X initially, players i adjacent to j must have few neighbors.

Instead of the distribution of individual degrees, the literature considers the degree or connectivity distribution $\mathbb{P}^V$ across nodes for a graph $V \in \mathbb{V}$. The probability vector $\mathbb{P}^V = (\mathbb{P}^V_0, \mathbb{P}^V_1, \ldots, \mathbb{P}^V_{N-1} \in \mathbb{R}^N_+$ is defined by $\mathbb{P}^V_k = \frac{1}{N}|\{i \in I : d_i(V) = k\}|$ for $k = 0, \ldots, N-1$. $\mathbb{P}^V_k$ is the fraction of nodes in V with degree k. For a probability vector $\mathbb{P} = (\mathbb{P}_0, \mathbb{P}_1, \ldots, \mathbb{P}_{N-1}) \in \mathbb{R}^N_+$, let $\Pi(\mathbb{P}) = \{V \in \mathbb{V} : \mathbb{P}^V = \mathbb{P}\}$, denote the set of graphs with connectivity distribution $\mathbb{P}$. López-Pintado (2006) and others consider random graphs whose support is contained in a set $\Pi(\mathbb{P})$. Binomial random graphs do not belong to that category, since a binomial random graph has full support $\mathbb{V}$. For a given binomial random graph $G(N, p)$, the empirical distributions $\mathbb{P}^V$ are random. However, we can compute the expectations $\widehat{\mathbb{P}} \equiv E[\mathbb{P}^V]$. Because of the homogeneity of $G(N, p)$, the average or expected connectivity degree distribution coincides with each of the individual degree distributions. To see this, define the indicator functions $1_{ik}(V)$ for $i \in I, k = 0, 1, \ldots, N-1$, with value 1 if $d_i(V) = k$ and value 0 if $d_i(V) \neq k$. Then for $k = 0, 1, \ldots, N-1$,

$$\widehat{\mathbb{P}}_k = E[\mathbb{P}^V_k] = E[\frac{1}{N}\sum_i 1_{ik}(V)] = \frac{1}{N}\sum_i E[1_{ik}(V)] = \frac{1}{N}\sum_i p_{ik}(N, p).$$

Hence $\widehat{\mathbb{P}}_k = p_{ik}(N, p)$ for all k and i. $\widehat{\mathbb{P}}$ responds to changes in p or N the same way as the individual degree distributions.

5.3 Endogenous co-evolution of local interaction structures and local interaction

In principle, players may take actions which determine payoffs from local interaction as well as the local interaction structure. Such a scenario gives rise to the endogenous co-evolution of local interaction structures and local interaction.

Jackson and Watts (2002) consider a model where maintaining a link $\{i, j\}$ is costly for players i and j and requires the consent of both players, corresponding to pairwise stability à la Jackson and Wolinsky (1996). The authors allow for the possibility that over time, i and j get an opportunity to add or delete the link. Also, there is asynchronous updating of the strategies in the 2×2 coordination game. Jackson and Watts determine the stochastically stable states (*sss*) based on uniform trembles. They find, among other things, that for some parameter configurations, there exist a *sss* where players coordinate on a strategy which is neither efficient nor risk dominant and the network is fully connected (complete).

Droste, Gilles and Johnson (2000) consider a model where the players are located on a circle, in the order $1, \ldots, N$ for N players and with equal distances between adjacent players. The cost of a link between two players is proportional to their shortest distance on the circle. In the *sss* based on uniform trembles, the players coordinate on the risk dominant action. The *sss* network V belongs to some $\mathcal{R}(n)$ where n is depends on the model parameters and V is given by the identity permutation of I, $\beta(i) = i$, that is the canonical order $1, \ldots, N$. The unperturbed dynamics exhibits additional absorbing states: Coordination on the action which is not risk dominant as well as "coexistence of conventions" are possible.

Hojman and Szeidl (2006) adopt a version of Bala and Goyal's (2000) one-way flow model of network formation, where each player unilaterally can form links and bears the cost of these links. A player receives benefits from playing a finite symmetric game Γ with each (direct and indirect) neighbor.

In the unperturbed dynamics, contagion always occurs. When Γ is a 2×2 coordination game, then in contrast to Jackson and Watts (2002), a strategy which is both efficient and risk dominant will always be selected in the *sss* if it exists; otherwise, there is a tradeoff between payoff dominance and risk dominance. The limit networks are wheels.

Goyal and Vega-Redondo (2005) adopt a version of Bala and Goyal's (2000) two-way flow model of network formation, where each player unilaterally can form links and bears the cost of these links. A player receives benefits from playing a symmetric 2×2 coordination game with each (direct) neighbor where the player or the neighbor may have formed the link. It is assumed that one action is risk dominant while the other one is payoff dominant. In the *sss*, a complete and essential network obtains, except when link formation costs are extremely high, in which case the empty network obtains. In the *sss* with a complete and essential network, players coordinate on the risk dominant action if costs per link are below a certain threshold and they coordinate on the payoff dominant action if costs per link are above the threshold.

Ehrhardt, Marsili, and Fernando Vega-Redondo (2006) primarily focus on the process of network formation and how it is affected by efforts to coordinate on one of several action. Over time, new links are created if they are profitable and existing links may disappear due to decay. Efforts to coordinate and the network co-evolve in continuous time. The authors find that in the long-run, the finite set of nodes (players) is partitioned into the set G_0 of isolated nodes plus, for each action r, the (possibly empty) set G_r consisting of the non-isolated nodes (players) choosing action r. Hence coexistence of actions is a likely outcome. Concerning the long-run network architectures, the degree distribution is determined.

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