Implementing Behavioral Concepts into Banking Theory: The Impact of Loss Aversion on Collateralization

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In standard bank theoretic models agents are assumed to be fully rational expected utility maximizers. This fact ignores the huge amount of evidence for anomalies in human behavior found by psychologists. In this paper we argue that the implementation of behavioral concepts into banking theory might increase the predictive power of the models. As an example we consider a loan market and discuss the impact of loss aversion on the degree of collateralization in equilibrium. The very well established concept loss aversion predicts entrepreneurs to pay much more attention to the potential loss of some of their initial wealth due to a collateralized loan than they would do as expected utility maximizers. This results in a higher effort choice which in turn increases the success probability of the loan financed project. Optimal levels of collateralization are derived for different degrees of loss aversion and the problem of private information about the degree of loss aversion is addressed. It is shown that in specific situations banks can offer self selecting pairs of contracts that costlessly eliminate the private information problem.

1. Introduction and Motivation

Human nature is complex in its motivations, beliefs and behavior. While psychologic research explores this richness, economists have a quite simplified view of humans in their models. Following expected utility theory (EUT), even in descriptive models they generally assume a rational behaving individual which maximizes his stable, well-defined preferences. This is done in contrast to the variety of anomalies in human behavior identified by psychologists. Examples include framing effects, fairness considerations, representativeness heuristics, as well as

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aversion to losses and to ambiguity.[2] These deviations are too extreme and too widespread to be reconciled within EUT or another normative model of rational choice. Descriptive theories, such as Prospect Theory[3], were developed to account much better for these anomalies.

Economists justify the use of a normative analysis to explain and to predict actual behavior by different arguments.[4] First, departures from rationality and self interest are said to be eliminated by competitive markets as other market participants will take advantage of arbitrage possibilities resulting from these deviations. Even without an arbitrage mechanism, many economists believe that individual irrationalities will disappear in the aggregate.

We raise objections against this argument. Beside the fact that much economic activity is not mediated by fully competitive markets it is to doubt whether markets will always drive irrationalities to disappear. Russell and Thaler (1985) showed in a simple model that the market-reduces-irrationalities hypothesis only holds when specific conditions are satisfied. In fact, there must be a possibility to shortsell goods or to trade the goods' characteristics independent from the goods themselves. They draw the conclusion that "*these conditions are quite restrictive and are unlikely to occur in any but the most efficient financial markets*".[5] Similar results were found in a theoretical study by Haltiwanger and Waldman (1985). Markets may well exist in which both rational and irrational individuals interact and both their behaviors have an influence on equilibrium prices. Arrow (1982) surveys empirical evidence for the proposition that "*an important class of intertemporal markets show systematic deviations from individual rational behavior...*".[6] Even in financial markets evidence suggests that the forces of competition are far from perfect. Ausubel (1991) presents a collection of data from the credit card market in the 1980's that shows the presumption of a competitive spot market equilibrium to be empirically unjustified. Over- and underreaction[7] identified in stock and option markets provide other

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[2] Rabin (1996) gives a comprehensive overview.

[3] Cf. Kahneman/Tversky (1979), Tversky/Kahneman (1992).

[4] Cf. Thaler (1986) for a more comprehensive list of arguments.

[5] Cf. Thaler (1986).

[6] Arrow (1982)

[7] Cf. DeBondt/Thaler (1985,1987,1990), Schiereck/Weber (1995), Stein (1989)

striking examples.[8]

Often it is also objected that learning from past experience will eliminate biases.[9] The advocates of this thesis believe that the anomalies identified in experimental research do not translate to real world decisions. They argue that specialists and experts as well as individuals which perform repeatedly the same task account for a large portion of real world economic activity and that they learn to be less prone to departures from rationality. Therefore the observed deviations from rational behavior should not matter as this behavior does not persist in the long run.

Beside the fact that there are many important decision making tasks which are not repeated for the agents involved[10], evidence is mixed whether experience and repetition will substantially reduce biases at all. Tversky and Kahneman (1982) found sophisticated knowledge of statistics not to eliminate some observed judgement anomalies. In particular situations experience even exacerbates biases. So experts are often more prone to overconfidence than laypersons[11]. In general, research does not support the strong hypotheses that biases disappear with increased experience.

A third common objection is more fundamental. It is argued that there surely exist anomalies in individual behavior and the deviations from EUT might even be systematic. But there is just no alternative theory available, which can take into account the behavioral findings without giving up too much of the analytical power and elegance of EUT at the same time.

It is one of the main intentions of this paper to demonstrate this last argument to be false. Introducing behavioral findings into banking theory does not necessarily imply the departure from a formal analysis. There exist behavioral concepts that are easy includable into existing banking models and result in interesting and more realistic results.

It is the completely different nature of the interacting parties, which makes banking theory

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8 Cf. Thaler (1993) for a more comprehensive collection of examples .

9 Cf. Tversky / Kahneman (1988), p. 167.

10 Banking is a specially rich field of one (or few) time decisions for the nonbank-agent. Examples include saving decisions for retirement or bank loans to buy a home or to start a business.

11 Cf. Paese/Sniezek (1991).

especially attractive for the incorporation of psychological findings.[12] Banks are in contrast to their clients hardly susceptible to departures from rationality. Actually delegation may create principal-agent problems in banks but well-defined procedures limit the scope of an employee as well as preceding and following committees prepare and review the individual's decision. Due to the larger number of undertaken or examined projects the bank has moreover excellent possibilities to gain experience and to optimize its decision behavior. In our understanding of "Behavioral Banking Theory" the banks are therefore assumed to be fully rational decision makers.

The client on the other hand is usually an individual.[13] In many cases his behavior does not satisfy the requirements of full rationality. He might rely on heuristics; emotions might influence his decisions, so his behavior might be characterized by several anomalies. Due to the lack of a large number of similar projects he cannot rely on past experience. He usually is limited in his analytical capacity and has no or few controlling instances.

Anticipating its client's departures from rationality banks could benefit by offering products which incorporate particular anomalies such as time preferences or loss aversion, by considering these phenomena in setting prices and conditions, or by addressing these deviations in its marketing activities. Thus behavioral banking theory is not just a descriptive theory. From the bank's decision making point of view it's a normative analysis. By correctly anticipating their clients departures of rationality and incorporating it into their decisions the banks in our model even show a higher level of rationality.

In this paper we exemplarily introduce a simple model of a loan market with loss averse entrepreneurs. We study how the very robust phenomenon of loss aversion alters the classical, well known results about the incentive and compensation effects of collateralization. While loss aversion has similar influence on second best equilibrium contracts (given a moral hazard problem due to the unobservability of effort choice) as liquidation costs of collateral, it provides an additional private information problem. Since loss neutral entrepreneurs can gain from

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12 It is quite surprising that banking theory was mostly ignored by behavioral economists so far. There hardly exists any "Behavioral Banking Theory", though some neoclassical papers allow behavioral interpretations.E.g. the analysis on divergent opinions in Chan/Kanatas (1985) might be interpreted as addressing the phenomenon overconfidence.

13 As the decision maker in small and medium companies is in general the owner or manager relying at best on a few collaborators, the decision processes are quite similar to those of an individual.

claiming to be loss averse, the existence of self selection mechanisms has to be examined. We find that there do exist situations where the second best contracts separate entrepreneurs of different degrees of loss aversion. Hence banks can use the second best contracts even in the private information case.

In the next chapters we will give a short overview about the two streams of literature we combine in our analysis. First we survey the most important results concerning the role of collateral in loan markets with asymmetric information. Then we state the basic ideas of Prospect Theory with special emphasis on loss aversion, one of its central concepts. In section 4 our model is presented and discussed. Section 5 concludes.

2. The Role of Collateral in Loan Contracts

In the last years much theoretical research has analyzed the role of collateral in loan contracts. It should be noted that there exist two quite distinct understandings of the term collateral. Some authors regarded collateral to be an asset that belongs to the borrowing firm and hence would be available to the lenders in case of default anyway.[14] Here the point of interest is the seniority of debt for different lenders, while the borrower's loss in case of default is not influenced by additional collateralization. In the more common second point of view collateral is understood to be a private asset that is usually not attachable, but becomes available to the lender by collateralization. In this case the provision of additional collateral increases the potential loss for the borrower and might have incentive effects that influence the borrower's behavior. Our analysis deals with the behavior of individual borrowers, thus we will adopt this point of view.

The incentive effects of collateral in the light of asymmetric information were analyzed before from different perspectives. Bester (1987) and Besanko/Thakor (1987) show that collateral can serve as a sorting device, when banks are confronted with borrowers of unobservable quality. If there is a sufficient amount of collateral available credit rationing as introduced by Stiglitz/Weiss (1981) cannot be persistent[15]. Chan/Kanatas (1985) discuss how private information and differing beliefs about the project payoffs influence the degree of collateralization in equilibrium. Bester (1994) considers the role of collateral with regard to a

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[14] Cf. Rudolph (1984), Stulz/Johnson (1985)

[15] An overview on this topic can be found in Bester/Hellwig (1989)

renegotiation of the loan. The impact of collateralization on the problem of combined Moral Hazard and Private Information about the borrower quality is analyzed by Chan/Thakor (1987) under different equilibrium concepts.

Generally speaking collateral has two effects: an incentive effect and a compensation effect. While the compensation effect (transfer of assets to the lender in case of default) alone cannot explain the use of collateral as already shown by Modigliani/Miller (1958), the use of collateral has positive effects on social welfare from the incentive point of view (risk choice, effort, signalling). Assuming the compensation effect to be 'welfare neutral' it is easy to derive[16] that loan contracts should be collateralized to the highest possible degree. The analysis gets more interesting and the results more realistic if the compensation effect is assumed to have a negative impact on welfare. That might be due to insufficient risk sharing as in Bester (1987) or to transaction costs[17] associated with the transfer and the liquidation of collateral.

In this paper we claim that there is another reason for a negative compensation effect of collateralization, which is formally quite similar to the transaction cost approach, but has different implications: individual borrowers might be more or less susceptible to loss aversion. When choosing between different loan contracts they perceive collateralization as a potential loss, which has a higher impact on their evaluation of a given loan contract than the actual value of the collateral. While under symmetric information collateralized contracts are thus inferior to uncollateralized contracts, under asymmetric information regarding the entrepreneur's effort choices collateral can weaken the moral hazard problem and lead to partially collateralized contracts in equilibrium. In this respect our work is related to Boot/Thakor/Udell(1991), who also consider the impact of collateral on unobservable effort choice. But while they restrict their analysis to a discrete low/high-effort range and add private information about borrower quality, we consider a continuous effort range and derive optimal degrees of collateralization dependent on the strength of loss aversion. Further we address the very crucial problem that the degree of loss aversion is a personal property and usually unobservable by the bank.[18] Since banks will offer more attractive contracts to loss averse borrowers, which are supposed to spend more

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[16] Cf. Chan/Thakor (1987).

[17] Liquidation cost are already mentioned and their importance is discussed in the early work of Barro (1976).

[18] Especially in this respect our explanation for the asymmetric evaluation of collateral differs from the liquidation cost approach.

effort, all borrowers have the incentive to claim to be extremely loss averse. We will show that in some situations banks can costlessly overcome this private information problem by offering a set of self selecting contracts.

3. Prospect Theory and Loss Aversion

Nobel laureate Paul Samuelson once proposed the following problem to a colleague: the equal chance of winning $200 or losing $100. The colleague refused and stated: "I won't bet because I would feel the $100 loss more than the $200 gain."[19] The phenomenon that losses loom larger than gains is called loss aversion and can be observed in various situations. Examples are the increase in the perceived value of goods as soon as an individual is endowed with (endowment effect) and the different evaluation of opportunity costs and expenses.[20] In general, gains or improvements have to be more than twice as great in order to balance equal probable losses or deteriorations. Ratios between 2:1 and 2,5:1 have been confirmed in various studies.[21] These ratios, however, heavily depend on the type of goods to gain or lose. While the loss aversion ratio is rather small for easy restorable daily use goods and goods purchased for resale[22] much higher ratios have been found if health or personal wellbeing is affected.

The different impact of gains and losses on preferences is far too extreme to be explained by income effects or risk aversion in the standard framework.[23] A theory accounting for this and several other phenomena is Prospect Theory which was established by Kahneman and Tversky.[24] It differs in three important manners from expected utility theory:

1. It has been widely observed that people pay more attention to changes than to absolute levels

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[19] Cf. Samuelson (1963), pp. 108.

[20] Cf. Knetsch (1989).

[21] Cf. Tversky / Kahneman (1991), pp. 1053.

[22] Cf. Kahneman/Knetsch/Thaler (1990).

[23] The aversion to small losses would imply extreme concavity of the utility function. To explain individuals' tendency to turn down 50/50 lotteries of losing $10 and winning $11 a concavity of the utility function has to be assumed that makes individuals also turn down a 50/50 gamble of losing $100 and winning $10,000. (Rabin, 1996). It can be concluded from this thought experiment, that there must be a kink in the value function separating gains from losses.

[24] Cf. Kahneman / Tversky (1979.

which implies an evaluation of outcomes relative to a reference level. That is the reason why the value function V is defined on changes in wealth and not on final asset positions.[25]

2. Losses loom larger than gains. This fact is implemented by a value function that is steeper in the loss domain than in the gain domain. In addition the value function reflects diminishing sensitivity for losses and gains. This results in risk aversion in the gain domain but risk seeking in the loss domain.

3. Individuals misjudge probabilities in a systematic way. That's the reason why Prospect Theory uses a probability weighting function measuring the weight an individual assigns to the perceived probabilities of the consequences.

Figure 1 illustrates a typical value function of Prospect Theory. Note that the function has a kink at the origin due to loss aversion. The curvature (concavity in the gain domain, convexity in the loss domain) displays different risk attitudes for gains and losses. In our analysis we will neither incorporate the diminishing sensitivity of the value function nor the concept of probability weighting. We exclusively focus on loss aversion, thus we can clearly attribute our findings to this phenomenon.

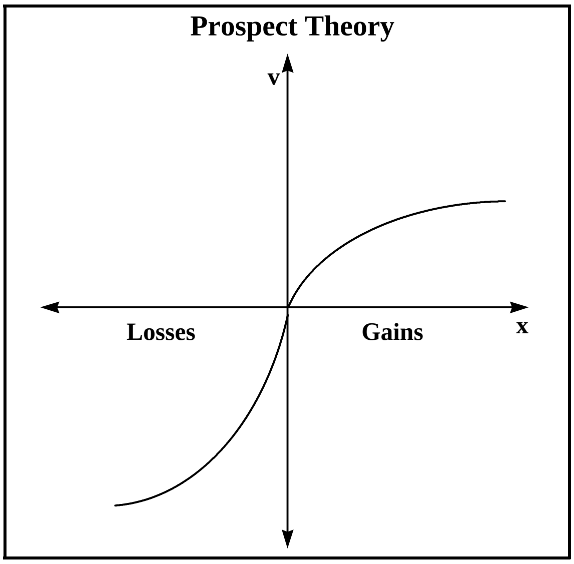

Figure 1: The value function of Prospect Theory

25 This concept is not really new, cf. e.g. Markowitz (1952).

In our model the borrowers have to post either collateral or pay higher interest rates on the loan to offset for the default risk of their project.[26] Since the status quo (ex ante, i.e. when evaluating different loan offers) usually serves as the reference point, posting more collateral is perceived as a potential higher loss. On the other hand higher interests are evaluated as a reduced gain and not as a loss. The asymmetry in the evaluation of collateral and higher interest rates resembles the different treatment of opportunity costs and expenses. Thus, loss-averse entrepreneurs will to a certain degree have a preference for higher interest rates.

4. The Model

We consider an economy with many entrepreneurs, each one endowed with an identical project requiring an input of 1$ in t=0 and yielding a stochastic return in t=1. For reasons of simplicity we restrict the return to just two possible outcomes. With probability φ the project yields X$, with probability $(1-\varphi)$ the return is 0$. The entrepreneurs do not have any capital to invest into their projects (but do have illiquid assets which can be used as external collateral) and hence need to apply for a loan from a bank. The banks in our model have unlimited access to funds at a riskless interest rate i. They are risk-neutral and compete for entrepreneurs. Hence they will make 1$-loans to entrepreneurs, where the expected return from the investment is exactly I $ $(I:=1+i)$. Given the competitive situation from now on we will focus on a single bank.

The probability of success φ is influenced by the entrepreneur's choice of effort $a\in[0,1)$. If the investor does not spend any effort at all, φ corresponds to the basic success probability $p<1$, which reflects external factors. More effort by the entrepreneur increases the success probability φ. We assume a linear relation $\varphi(a) = (1-p)a + p$.[27] Hence by spending an effort close to 1 the entrepreneur could make the project return almost riskless.

Spending effort causes some disutility $d(a)\in[0,-\infty)$ to the entrepreneur. Since the shape of the disutility function is a crucial point of our analysis, the basic properties of an appropriate

[26] Here we understand by collateral funds external to the project, e.g. private assets of the entrepreneur.

[27] This does not mean a loss of generality. Effort is *defined* by its influence on the success probability of the project. An effort value has no meaning in terms of disutility per se. Disutility enters the model by an additional transformation of effort.

disutility function should be highlighted[28]:

(1) $d'(a) \leq 0$, more effort causes more disutility.

(2) $d''(a) < 0$, the marginal disutility of effort is decreasing, i.e. an additional increase in effort is more painful at a higher effort (i.e. higher success probability) level

(3) $d(a) \to -\infty$ for $a \to 1$, trying to make a project almost riskless causes very much disutility and will never be favorable

(4) $d'(0) = 0$, a choice of a=0 (no effort at all) will never be favorable[29], since in each project there are a few possibilities to increase success probability with hardly any disutility.

Considering these basic properties, we assume a disutility function: $d(a) := z \ln(1-a)$, for some fixed $z>0$. It is easy to verify that this disutility function has the properties (1) to (3), but does not fulfill (4). This could result in the unrealistic desirability of effort choices a=0. Since $d'(0)=-z$, we can control this problem and approximate property (4) by assuming a sufficiently small z.[30]

Throughout the paper we assume the influence of effort on the success probability, the possible project outcomes and the ex post project realization to be common knowledge. We will alter the assumptions about the observability of the effort choice and the preference structure of the entrepreneurs. As a benchmark we start by analyzing the symmetric information case.

Symmetric Information

It is assumed for now, that the entrepreneurs' effort choice is observable and that the bank knows each entrepreneur's preference structure. A standard loan contract is a tripel (R,C,a), where

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28 We assume d to be twice differentiable.

29 Except in the irrelevant cases, when the entrepreneur does not profit from a higher success probability at all.

30 To be more explicit, we have to claim $z<(1-p)^2 X$ and $z<(X^{0,5} - I^{0,5})^2$ to avoid technical problems of a corner solution a=0 in the following formal analysis.

$R \in [I,X]$ is the face value of the debt, $C \in [0,R]$ defines the amount of collateral and $a \in [0,1)$ is the degree of effort to be invested by the entrepreneur. The face value R incorporates both the repayment of the loan and the payment of interest r to the bank. As the bank refinances its loans at a rate i, this rate is the minimum level for r. The term collateral refers to external illiquid assets. These assets can neither be used to finance the project initially nor can they be seized by the bank in case of default unless they are explicitly defined as collateral. There are no limitations on the amount of collateral available on the side of the entrepreneurs, i.e. in case of default they are able to liquidate assets up to I$.[31]

A contract (R,C,a) is called ***acceptable*** by the bank, if the bank's expected return equals I, i.e.:

$$J_B\ (R,C,a) := \varphi(a)(R) + (1-\varphi(a))C = I \qquad (1$$

Note that a fully collateralized (I,I,a)-contract is acceptable by the bank for all effort levels a. By R(a,C) we define the face value necessary to let the contract (R(a,C),C,a) be acceptable by the bank. We have:

$$R(C,a) := \frac{I-C}{\varphi(a)} + C \qquad (2$$

All entrepreneurs are endowed with exactly the same projects and have the same disutility function. They differ, however, in their evaluation of the project outcomes. As suggested by Prospect Theory (Kahneman/Tversky, 1979, 1992) and introduced in section 3 we assume the entrepreneurs to have different degrees of loss aversion. Thus losses might have more impact on an individual's evaluation than gains of the same size. We implement this fact into our model by defining an entrepreneur`s value function by

$$v_k(y) := \begin{cases} y\ , & \text{if } y \geq 0, \\ ky, & \text{if } y < 0, \end{cases}$$

where y is the change of wealth relative to the status quo and $k \geq 1$ measures the individual's degree of loss aversion. We ignore other ideas of Prospect Theory such as diminishing sensitivity and probability weighting and exclusively focus on the asymmetric evaluation of gains and losses. An entrepreneur with degree of loss aversion k will be called a

31 We will see that C=I is the highest possible degree of collateralization, used in a fully collateralized (I,I,a)-contract.

k-type. A k-type's evaluation of a risky alternative is given by the expected value of the v_k-transformed wealth changes. Note that a 1-type entrepreneur is a standard risk-neutral (expected value maximizing) decision maker. Hence our analysis includes the well known case of an overall risk neutral economy.

The evaluation of a (R,C,a)-contract by a k-type-entrepreneur is given by:

$$V_k(R,C,a) := \varphi(a)v_k(X-R)+(1-\varphi(a))v_k(-C)+d(a) = \varphi(a)(X-R)+(1-\varphi(a))(-kC)+d(a) \quad (3)$$

In the symmetric information case the degree of loss aversion of each individual entrepreneur is common knowledge. In equilibrium we then will have first best contracts that under all acceptable contracts maximize each k-type's evaluation V_k.

Prop.1: a.) The first best contract to offer an entrepreneur of type k>1 is of the form (R(0,a*),0,a*), where : $$a^* = 1 - \frac{z}{(1-p)\ X} \quad (4)$$

b.) For a 1-type entrepreneur all contracts of the form (R(C,a*),C,a*) with $C \in [0,I]$ are first best contracts and may exist in equilibrium.

This result is intuitively clear and we skip the straightforward formal proof. The effort a* maximizes the total welfare of the project and is found by equating marginal disutiliy and marginal project return. For a loss-neutral (1-type) entrepreneur there are several acceptable (R,C,a*) contracts which differ by the return distribution between the bank and the entrepreneur in case of success and failure without changing expected values. For loss-averse (type k>1) entrepreneurs the use of collateral decreases welfare. In case of default the entrepreneur experiences a loss of kC while the bank just gains C. Hence the avoidance of collateral and the incorporation of project risk into higher interest rates r leads to the only first best contract (R(0,a*),0,a*).[32]

[32] The proof of the positivity of a* can be found in Corollary A3 in the Appendix.

Moral Hazard

Next we consider the more realistic case of asymmetric information about the effort level chosen by the entrepreneurs. If banks cannot observe an entrepreneur's effort, contracts cannot be defined contingent on the effort level a. Then a loan contract is simply of the form (R,C).

Given an (R,C)-contract a k-type will choose an effort level $a_k(R,C)$ to maximize his welfare, i.e.

$$a_k(R,C) = \operatorname{argmax}_{a \in [0,1)} \{\varphi(a)(X-R) + (1-\varphi(a))(-kC) + d(a)\}.$$

Solving this maximization problem we get[33]:

$$a_k(R,C) = 1 - \frac{z}{(1-p)(X-R+kC)} \quad (5)$$

Two easy observations about the endogenous effort choice $a_k(R,C)$ follow in order.

1. Monotonicity: $a_k(R,C)$ is increasing in C and decreasing in R. Thus while higher collateralization leads to additional effort, a shrinking excess return X-R causes effort to decrease. For C>0 the effort $a_k(R,C)$ is increasing in k, so in case of a collateralized loan a more loss averse entrepreneur will spend more effort.

2. Suboptimality: Except in the case of a fully collateralized contract with C=R the effort level $a_k(R,C)$ chosen by an entrepreneur is always lower than the socially optimal effort level:[34]

$$a_k^{opt}(R,C) = 1 - \frac{z}{(1-p)(X-C+kC)} \quad (6)$$

   This is due to the fact, that the entrepreneur profits only partially from a high effort and thus a higher project success probability. Unless the contract is fully collateralized (and the bank gets a riskless return of I) part of the higher ecpected project return increases the bank's expected profit. Hence the entrepreneur has less incentive to spend effort.

The bank has to take into account the optimizing behavior of the entrepreneur when offering a contract (R,C) as the contract induces a certain effort level. As we have seen above this effort level depends on the entrepreneur's degree of loss aversion reflected by the coefficient k. A

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[33] In Corollary A3 in the Appendix it is shown that for all relevant contracts (R,C) the effort $a_k(R,C)$ will always be positive.

[34] The socially optimal effort for a k-type-entrepreneur given an (R,C)-contract can be derived from the fact that in case of default there is a welfare loss of (k-1)C due to the asymmetric evaluation of the transferred collateral.

contract (R,C) is called ***k-stable***, if the contract $(R,C,a_k(R,C))$ is acceptable by the bank. A k-stable contract yields an expected return I for the bank, if the entrepreneur has chosen his optimal effort level given the contract (R,C).

Obviously the fully collateralized contract (I,I) is a k-stable contract for all $k \geq 1$. The other extreme, an uncollateralized k-stable contract, is presented in the following lemma.

Lemma 2: For each $k \geq 1$ the contract $(R°,0)$ with $R° := \frac{X+I-z}{2} - \sqrt{\frac{(X+I-z)^2}{4} - IX}$ (7) is k-stable.

Proof: For C=0 the effort $a_k(R,0)$ and hence the expected return to the bank does not depend on k. Solving $U_B(R,0,a_k(R,0))=I$ for R gives R°. It remains to show:

a.) the discriminant is positive, b.) $R° \geq I$ and c.) $a_k(R°,0) > 0$.

The proofs of these properties can be found in lemma A1 and A2 in the appendix. ❑

In addition to these two extremes there are further k-stable contracts with intermediate degrees of collateralization. The most important properties of k-stable contracts are summarized in the following proposition.

Prop. 3: a.) For all $R \in [I,R°]$ and all $k \geq 1$ there exists a unique $C_k(R) \in [0,I]$ s.t. $(R,C_k(R))$ is a k-stable contract.

b.) The function $C_k(R)$: $[1,\infty) \times [I,R°] \to [0,I]$, $(k,R) \mapsto \frac{R^2-(X+I-z)R+IX}{k(R-I)+z}$ (8) is continuous in k and R, strictly decreasing in R and on $[1,\infty) \times (I,R°)$ strictly decreasing in k .

Proof: The functional form of $C_k(R)$ is derived by solving $U_B(R,C_k(R),a_k(R,C_k(R)))=I$ for $C_k(R)$. The continuity of $C_k(R)$ is obvious. The numerator R^2-(X+I-z) +IX is nonnegative and strictly decreasing in R on $[I,½(X+I-z)] \supset [I,R°]$ The denominator $k(R-I)+z$ is positive and strictly increasing in R. Hence $C_k(R)$ is strictly decreasing in R on $[I,R°]$ for all $k \geq 1$. With $C_k(I)=I$ and $C_k(R°)=0$ we get $C_k(R) \in [0,I]$ for all $R \in [I,R°]$. The monotonicity of $C_k(R)$ in k on $[1,\infty)\times(I,R°)$ is obvious. ❑

What is the intuition behind this result? The more lossaverse the entrepreneur the higher is his incentive to avoid default of the project and the loss of collateral. Hence he will choose a higher level of effort resulting in a higher success probability of the project. Anticipating this higher effort the bank can decrease the amount of collateral which is necessary to break even. The result concerning the increase of R is less obvious. Charging a higher face value R results directly in a higher return for the bank in case of project success. On the other hand a higher face value R decreases the incentive for the entrepreneur to spend effort which results in a lower success probability of the project. Prop. 3 above states that the former effect (return increase) dominates the latter one (success probability decrease). A higher R increases the bank's expected return which will be adjusted by a lower amount of collateral C[35].

The two extreme contracts (R°,0) and (I,I) are k-stable for all $k \geq 1$ as can be seen in figure 2, where curves of k-stables contracts are displayed for k=1,2,3,4. The reason for the overall k-stability is quite different in the two cases. The (R°,0) contract provides the same expected return for the bank independent of the entrepreneur`s loss aversion, because loss aversion does not play any role in contracts without collateral. Hence all k-types choose the same amount of effort which results in the same expected return for the bank. Given a (I,I) contract as the other extreme, the more lossaverse entrepreneurs choose a higher effort level which leads to a higher success probability of the project. But since in a fully collateralized contract the banks return is riskless the higher success probability does not have any impact on the bank's expected return.

35 A lower amount of collateral always decreases the expected return for the bank. Here both effects work in the same direction. Less collateral means less return in case of default and less incentive for the entrepreneur to spend effort.

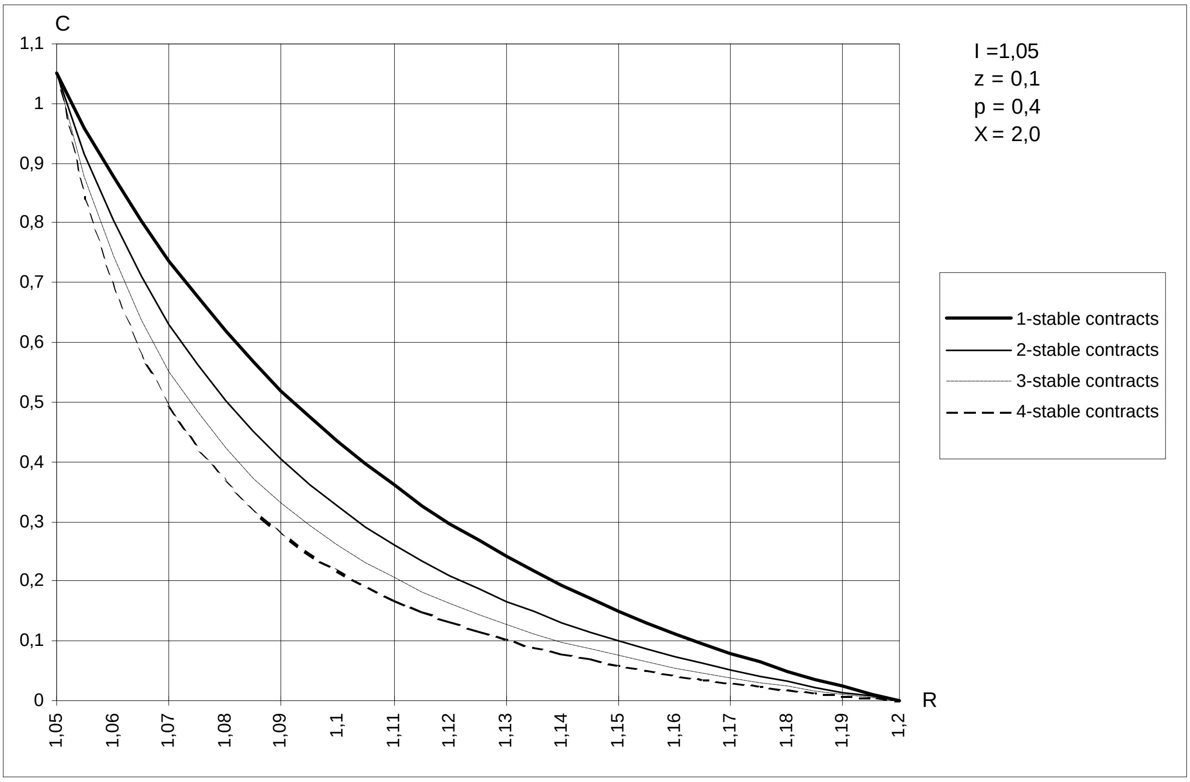

figure 2: k-stable contracts for k=1,2,3,4.

Now let us turn to the question which of the k-stable contracts the bank will offer to a k-type in equilibrium, i.e. which k-stable contract (R_k^{opt},C_k^{opt}) provides the second best solution for a k-type entrepreneur given the moral hazard problem. For k=1 this question has a well known answer and is easy to derive.

Prop. 4:	Offering the fully collateralized 1-stable (I,I)-contract to an 1-type entrepreneur leads to the unique second best solution given the moral hazard problem. It equals the first best solution even though effort choice is not observable and therefore not contractable.

Proof: From prop. 1 we know that an acceptable contract (R,C,a) is a first best solution, iff $a = a^* := 1 - \frac{z}{(1-p)\ X}$. All 1-stable contracts ($R,C_1(R),a_1(R,C_1(R))$) are acceptable and we have: $a_1(R,C_1(R)) = 1 - \frac{z}{(1-p)(X-R+C_1(R))}$. $a_1(R,C_1(R)$ equals a* iff $R=C_1(R)=I$.

❑

For a risk- and loss-neutral entrepreneur full collateralization results in optimal contracts. This is due to the fact that collateral provides a positive incentive effect without causing any disadvantages at the same time. For lossaverse entrepreneurs, however, the role of collateral is ambivalent. On the one hand it provides an incentive for the entrepreneur to choose an effort level closer to the social optimum. On the other hand the possible transfer of collateral results in a reduction of total welfare, due to the fact that the entrepreneur's evaluation of the collateral loss is not fully compensated by the evaluation of the gain on the bank's side. In the following paragraphs we analyse the trade off between the two effects.

We start by closer examining the effort $a_k(R,C) = 1 - \frac{z}{(1-p)(X-R+kC)}$ spent by k-entrepreneurs as a response to a k-stable contract. For each R $\in$ [I,R°] the effort resulting from the k-stable contract (R,C_k(R)) is called the **k-stable effort given R** and denoted by $\mathbf{a_k(R)}$. By substitution of $C_k(R) = \frac{R^2-(X+I-z)R+IX}{k(R-I)+z}$ into $a_k(R,C)$ we get:

$$a_k(R) = 1 - \frac{1}{(1-p)} \frac{K(R-I)+z}{(k-1)R+X} \tag{9}$$

The marginal disutility $\frac{\partial d(a_k(R))}{\partial R}$ following from this k-stable effort $a_k(R)$ plays an important role in the optimization process. In corollary A5 in the appendix the following properties of $\frac{\partial d(a_k(R))}{\partial R}$ are shown:

Lemma 5:	$\frac{\partial d(a_k(R))}{\partial R}$ is positive, strictly increasing in k and strictly decreasing in R on [I,R°]x[1,∞).

Now we address the central question: how does the degree of loss aversion k influence the amount of collateral used in the second best contract (R_k^{opt},C_k^{opt})? We will show that full collateralization will never be optimal except for a lossneutral 1-type entrepreneur. The other extreme, an uncollateralized contract (R°,0), can exist as the second best solution for high k-types. But its appearance as an equilibrium contract requires the project outcome X to be relatively high compared to the refinancing rate I. Otherwise second best contracts include some collateralization for all degrees of loss aversion. We will first discuss the latter case.

> Prop. 6a: If $X \le I+z+\sqrt{I^2+4Iz}$ then it holds for the second best contract (R_k^{opt},C_k^{opt}):
>
> (1) $R^{opt}:[1,\infty) \to [I,R°]$, $k \mapsto R_k^{opt}$ is continuous, strictly increasing and $R^{opt}(k) \in (I,R°)$ for all $k \in (1,\infty)$.
>
> (2) $C^{opt}:[1,\infty) \to [0,I]$, $k \mapsto C_k^{opt}$ is continuous, strictly decreasing and $C^{opt}(k) \in (0,I)$ for all $k \in (1,\infty)$.

Proof: (1) A k-type entrepreneur will respond to a k-stable contract $(R,C_k(R))$ by choosing an effort $a_k(R)$ and thus experiencing a disutility $d(a_k(R))$. His welfare out of the contract is given by:

$$U_k(R,C_k(R)) = \varphi(a_k(R))(X-R)+(1-\varphi(a_k(R)))(-kC_k(R))+d(a_k(R)) \tag{10}$$

By substituting $a_k(R)$ and $C_k(R)$ into the two left terms and doing some easy transformations the term simplifies to:

$$U_k(R,C_k(R)) = X-R-z + d(a_k(R)) \tag{11}$$

For each $k \ge 1$ this expression has to be maximized in R on [I,R°]. The first order condition is given by: $\frac{\partial d(a_k(R))}{\partial R} = 1$ and by substitution of

$$a_k(R) = 1 - \frac{1}{(1-p)}\frac{K(R-I)+z}{(k-1)R+X} \quad \text{and} \quad \frac{\partial a_k(R)}{\partial R} = \frac{-1}{(1-p)}\frac{kX+(k-1)(kI-z)}{[(k-1)R+X]^2} \qquad (12)$$

into $\frac{\partial d(a_k(R))}{\partial R} = \frac{-z\frac{\partial a_k(R)}{\partial R}}{1-a_k(R)}$ we get the equation:

$$(k-1)kR^2 + [(k-1)(z-Ik)+kX]R - z(X+Ik-z)(k-1) - XIk = 0 \qquad (13)$$

For k=1 this equation is linear and has the unique solution R=I, which we already know from prop. 4. For k>1 it is easy to see that only

$$R_k^{FOC} = \frac{-kX-(k-1)(z-Ik)}{2(k-1)k} + \sqrt{\frac{(kX+(k-1)(z-Ik))^2}{4(k-1)^2k^2} + \frac{z(X+Ik-z)}{k} + \frac{XI}{k-1}} \qquad (14)$$

the larger of the two solutions of the quadratic FOC, is greater than I.

In lemma A8 in the appendix $R_k^{FOC}<R°$ is shown for all $k\geq 1$. The concavity of $d(a_k(R))$ in R which we know from lemma 5 directly yields the concavity of $U_k(R,C_k(R))$ in R on [I,R°]. This implies $R_k^{opt} = R_k^{FOC}$. From lemma A6 in the appendix it follows that R_k^{FOC} and thus R_k^{opt} strictly increase in k. From the monotonicity and $R_1^{opt} = I$ it can be concluded that: $R^{opt}(k) \in (I,R°)$ for all $k\in(1,\infty)$.

(2) $C_k^{opt} = C_k(R_k^{opt})$ is obviously continuous in k. Since $C_k(R)$ is decreasing in k and strictly decreasing in R, we get from (1) that $C_k(R_k^{opt})$ strictly decreases in k. For all $k \in (1,\infty)$ we can conclude from R_k^{opt} in (I,R°) and prop. 3 b) that C_k^{opt} in (0,I).

❑

Next we consider the case where X is high relative to I and z. Here we find that for high degrees of loss aversion contracts without any collateral can indeed provide the second best solution.

Prop. 6b:	If $X > I+z+\sqrt{I^2+4Iz}$ then there exists a unique $k_0 \in (1,\infty)$ s.t. C^{opt} and R^{opt} are continuous functions on $[1,\infty)$ with:	
	(1)	$R^{opt}(k) = R°$ for all $k \geq k_0$ and $R^{opt}(k)$ is strictly increasing on $[1,k_0]$.
	(2)	$C^{opt}(k) = 0$ for all $k \geq k_0$ and $C^{opt}(k)$ is strictly decreasing on $[1,k_0]$.

Proof: The proof is an easy extension of the proof of prop. 6 a). It is shown in lemma A8 in the appendix that for $X > I+z+\sqrt{I^2+4Iz}$ there exists a k_0 with $R_k^{FOC} > R° \;\Leftrightarrow\; k > k_0$. From lemma 5 we know the concavity of $d(a_k(R))$ and can conclude the concavity of $U_k(R, C_k(R)) = X - R - z + d(a_k(R))$ in R on $[I,R°]$. This implies $R_k^{opt} = R_k^{FOC}$ for all $k \in [1,k_0]$. With lemma A6 in the appendix it follows that R_k^{FOC} and thus R_k^{opt} strictly increase in k on $[1,k_0]$. The continuity of R_k^{opt} on $[1,k_0]$ is derived from the obvious continuity of R_k^{FOC} and we further have: $R_{k_0}^{opt} = R_{k_0}^{FOC} = R°$.

For $k>k_0$ we have $R_k^{FOC} > R°$ and the k-type's welfare $U_k(R,C_k(R))$ strictly increases on $[I,R°]$. Hence it follows $R_k^{opt} = R°$. The overall continuity of R_k^{opt} is obvious.

The proof of (2) is trivial. The properties of $C_k^{opt} = C_k(R_k^{opt})$ are directly derived from lemma 2 and prop. 3. ❑

The propositions 6a and 6b state that fully collateralized contracts are optimal just in the special case of a loss neutral entrepreneur. The more loss averse the entrepreneur the less collateral should be provided and partial collateralization might be optimal even when there is no limit on the availability of illiquid assets. Figure 3 shows the equilibrium contracts for different k-types and parameters as considered in proposition 6a.

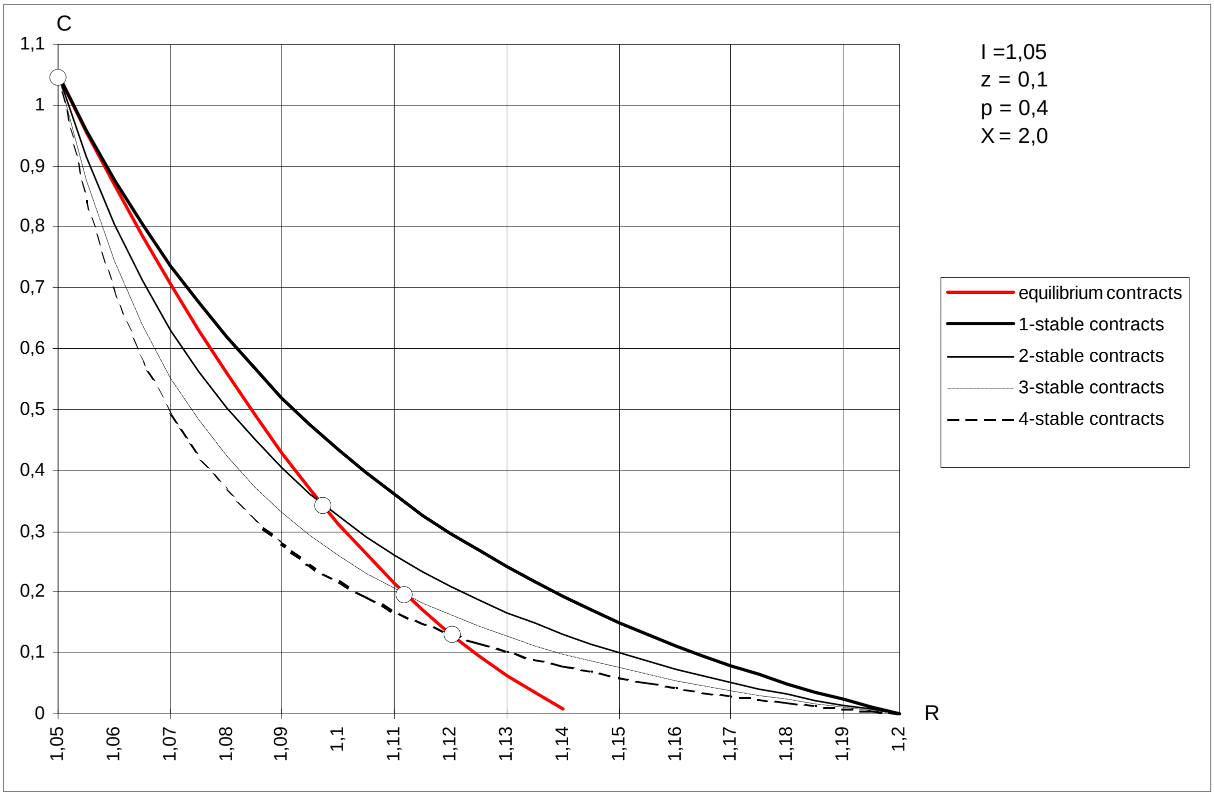

figure 3: equilibrium contracts

In section 2 it was already mentioned that our analysis can also be interpreted from a different point of view. If we assume liquidation costs attached to the transfer of collateral an asymmetric valuation of collateral is given even for risk neutral entrepreneurs. Then $\frac{1}{k}$ represents a liquidation cost factor, i.e. collateral worth C to the entrepreneur means a compensation of $\frac{C}{k}$ for the $R_{k_0}^{opt}, C_{k_0}^{opt}$ bank.

While it was shown by other authors before[36], that liquidation costs of collateral might outweigh the positive incentive effects of collateralization, our analysis can provide more detailed results about the strength of the counteracting effects. If the success return X is not too high (relative to I) we can conclude from the reinterpreted proposition 6a that contracts should always be partially collateralized even when collateral transfer causes very high liquidation costs (i.e. k is very high).

36 Cf. Chan/Kanatas (1985), Boot/Thakor/Udell (1991)

Though the formal analysis might be similar to transaction cost models we explicitly want to stress the differences of the two approaches. We argue that in addition to the obvious transaction costs of collateral, which we could easily add to our model, there might be some even more important, but less obvious costs due to loss aversion. While liquidations cost are objective and observable, loss aversion is subjective and could remain unconsidered by banks. But as we have seen in prop. 6a and 6b, social welfare could be increased by taking loss aversion into account.

So far we ignored a very severe problem concerning the observability of loss aversion for the bank. Unless banks have a long time relationship to a borrower and thus are able to learn about his preferences, the individual degree of loss aversion is private information. Every entrepreneur has an incentive to propose a very high degree of loss aversion, since he knows that the banks will then offer better contracts in expectation of a higher effort choice. If the degree of loss aversion is unobservable, separating contracts do not seem to be viable. But in the following paragraph we will show that private information about the degree of loss aversion does not necessarily imply the pooling of all types of entrepreneurs at one contract.

Moral Hazard and Private Information about the degree of loss aversion.

Our prevailing analysis relied on the rather unrealistic assumption that each individual entrepreneur's degree of loss aversion would be known to the bank. However in general a bank can not observe an entrepreneur's degree of loss aversion ex ante (and can not even conclude it ex post as the entrepreneur's choice of effort is unobservable.) Hence the degree of loss aversion is private information and contracts can not be written contingent on a degree of loss aversion. This would make it impossible for the bank to offer k-stable contracts to k-type entrepreneurs. Entrepreneurs would have an incentive to state very high degrees of loss aversion in order to get offered a contract with a very low amount of collateral C for a fixed R.

However this argument only holds, if the bank offers the full range of k-stable contracts. The low-k-type does not necessarily want to switch to the high-k-type contract, if the bank restrict itselfs to a discrete set of contracts each one designed for a special k-type. In the following we assume that there exist just two types of entrepreneurs. Some entrepreneurs are rational expected value maximizers, i.e. 1-types, the others are more emotional, loss averse entrepreneurs, i.e. k_0-

types with $k_0>1$. The bank can not distinguish between the different types.

What will happen, if the bank offers the two second best contracts (R_1^{opt},C_1^{opt}) and $(R_{k_0}^{opt},C_{k_0}^{opt})$ to all entrepreneurs applying for a loan? The k_0-types will choose their second best contract $(R_{k_0}^{opt},C_{k_0}^{opt})$, since $(R_1^{opt},C_1^{opt}) = (I,I)$ is also a k_0-stable contract. The problem arises for 1-types. They maximize their utility on the 1-stable contract curve by choosing $(R_1^{opt},C_1^{opt}) = (I,I)$. But whenever $C_{k_0}^{opt} > 0$ the contract $(R_{k_0}^{opt},C_{k_0}^{opt})$ is not a 1-stable contract. From prop.6a and prop.6b we know that in this case $C_{k_0}^{opt} < C_1(R_{k_0}^{opt})$ and thus for the 1-type entrepreneur the contract $(R_{k_0}^{opt},C_{k_0}^{opt})$ is more attractive than the 1-stable contract $(R_{k_0}^{opt},C_1(R_{k_0}^{opt}))$. Hence though we know that (R_1^{opt},C_1^{opt}) is preferred to $(R_{k_0}^{opt},C_1(R_{k_0}^{opt}))$, we do not know whether it is also preferred to the more attractive contract $(R_{k_0}^{opt},C_{k_0}^{opt})$. The figures 4 and 5 demonstrate that there is no general answer to this question. In the figures the 1-stable and 2.5-stable curves are plotted and the second best contracts are marked.

By adding the indifference curve of the 1-type which is given by $C_{ind\,1}(R) = R - X + X e^{\overline{z}}$ to the figure 4. we can see that the 1-type would prefer the contract $(R_{k_0}^{opt}, C_{k_0}^{opt})$, since $C_{k_0}^{opt} < C_{ind\,1}(R_{k_0}^{opt})$. Thus contracts are not self separating but pool all entrepreneurs at the contract $(R_{k_0}^{opt}, C_{k_0}^{opt})$. In figure 5. the value for X is increased from 1.8 to 2.2. Now the 1-types prefer the contract $(R_1^{opt}, C_1^{opt}) = (I, I)$. Here the two contracts separate the entrepreneurs. k_0-types choose their second best contract $(R_{k_0}^{opt}, C_{k_0}^{opt})$ and 1-types choose their second best contract (R_1^{opt}, C_1^{opt}).

We can state a more general result:

Prop. 7:	If X is sufficiently high, then the second best contracts (R_1^{opt}, C_1^{opt}) and $(R_{k_0}^{opt}, C_{k_0}^{opt})$ separate the entrepreneurs, i.e. 1-type entrepreneurs choose the contract (R_1^{opt}, C_1^{opt}) and k_0-type entrepreneurs choose $(R_{k_0}^{opt}, C_{k_0}^{opt})$. In this case the private information about the degree of loss aversion does not cause any additional contractual costs.

Proof: The formal proof is technical and therefore moved to lemma A9 in the appendix. The central observation is that $\lim_{k\to\infty} R_k^{opt} = I + z\frac{k-1}{k}$ while $\lim_{k\to\infty} R° =$. Therefore for an increasing X the contract $(R_{k_0}^{opt}, C_{k_0}^{opt})$ approaches the 1-stable contract (R°,0). Since the 1-type strictly prefers the (I,I) contract to the (R°,0) contract it can be shown by continuity arguments that for high X the 1-types choose (I,I) (even if $C_{k_0}^{opt}$ is still greater than zero.) ❑

5. Conclusion

Neoclassic banktheoretic modelling assumes all decision makers to be rational expected utility maximizers. This ignores the huge amount of evidence for anomalies in individual decision making found by psychologists. Many of these behavioral concepts are easily includable into the analysis of bank theoretic problems without giving up power and elegance of formal argument.

As an example we analyzed the impact of loss aversion on the degree of collateralization in a loan market equilibrium. Optimal contracts under asymmetric information are derived for different degrees of loss aversion. It is interesting to note that only in the classical case of a risk neutral entrepreneur fully collateralized contracts appear as second best solutions. For loss averse entrepreneurs the trade off between the positive incentive effect and the negative compensation effect of collateral leads to partially collateralized or uncollateralized contracts. We derive the intuitive result that the more loss averse the entrepreneur, the less collateral is used in equilibrium. Finally the problem of private information regarding the degree of loss aversion is analyzed. It turns out that for specific parameter settings a pair of second best contracts might serve as a self selection device.

There is a wide area of further research questions on this topic. On the one hand other or additional behavioral findings should be applied to the question of optimal collateralization. Two features of Prospect Theory, which were ignored in our analysis, promise interesting results. Diminishing sensitivity as well as probability weighting are intuitively supposed to weaken the effects of loss aversion demonstrated in our model. The robust phenomenon of entrepreneur overconfidence is easily includable into our analysis, too.

On the other hand there remain many open questions even in the pure loss aversion case. As one interesting example the contribution of our analysis to the discussion about the relation between collateralization and project risk[37] should be examined. It can be shown that loss averse entrepreneurs spend less effort in equilibrium than loss neutral entrepreneurs.[38] In combination with our results about collateralization this implies that the less collateralized project has the lower success probability. It is unclear though, whether this result can be extended to all different degrees of loss aversion.

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[37] Cf. Berger/Udell (1990).

[38] It is by no means a trivial fact, that $a_1(I) > a_k(R_k^{opt})$.

Finally there are many other interesting bank theoretic questions (e.g. renegotiation of loan contracts, usage of credit lines, bank run analysis), where the departure from "homo economicus" will lead to more realistic descriptions of observed human behavior.

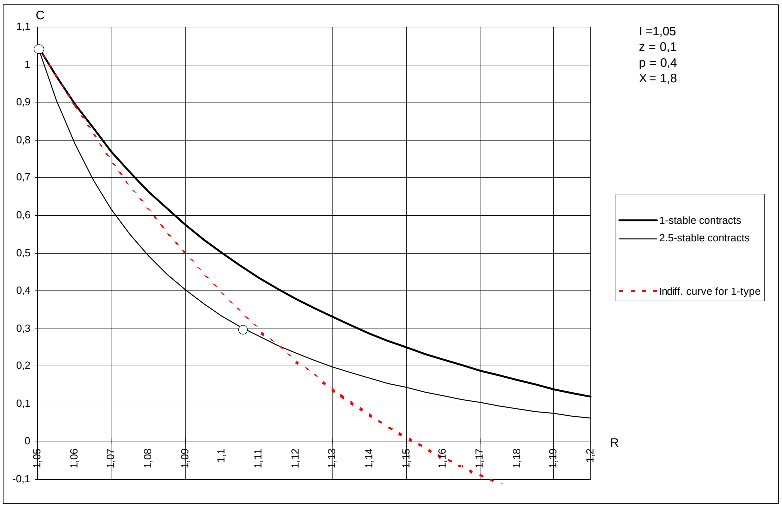

figure 4: pooling contracts

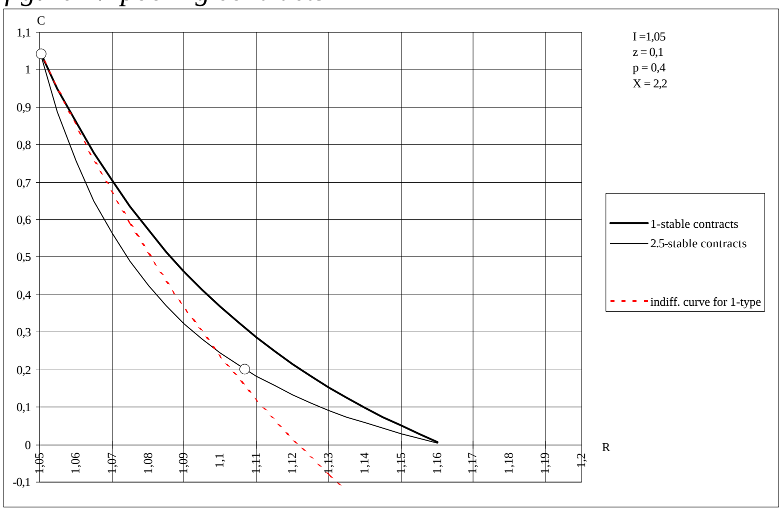

figure 5: separating contracts

Appendix

> **Lemma A1:** $\sqrt{\dfrac{(X+I-z)^2}{4}-IX}$ is positive and $R^\circ := \dfrac{X+I-z}{2}-\sqrt{\dfrac{(X+I-z)^2}{4}-IX} > I$.

Proof: The positivity of the discriminant follows directly from our assumption on the size of z: $\sqrt{z}<\sqrt{X}-\sqrt{I}$. It can be derived in order:

$$z<(\sqrt{X}-\sqrt{I})^2 \quad , \quad z<X+I-2\sqrt{IX} \quad , \quad X+I-z>2\sqrt{IX} \quad \text{and} \quad \frac{(X+I-z)^2}{4}-IX>0 \ .$$

To see the positivity of R° note that: $\dfrac{(X+I-z)^2}{4}-IX = \dfrac{(X-I-z)^2}{4}-Iz$.

Using this equality we can conclude:

$$R^\circ = \frac{X+I-z}{2}-\sqrt{\frac{(X+I-z)^2}{4}-IX} = I + \frac{X-I-z}{2}-\sqrt{\frac{(X-I-z)^2}{4}-Iz}$$

$$= I + \sqrt{\frac{(X-I-z)^2}{4}} - \sqrt{\frac{(X-I-z)^2}{4}-Iz} > I \quad .$$

❑

> **Lemma A2:** $a_k(R^\circ,0) = 1-\dfrac{z}{(1-p)(X-R^\circ)}$ is positive.

Proof: First we show $R^\circ := \dfrac{X+I-z}{2}-\sqrt{\dfrac{(X+I-z)^2}{4}-IX} < \sqrt{IX}$.

From $2\sqrt{IX}\sqrt{\dfrac{(X+I-z)^2}{4}-IX}>0$ we can conlude, that

$$\left(\sqrt{IX} + \sqrt{\frac{(X+I-z)^2}{4}-IX}\right)^2$$

$$= IX + 2\sqrt{IX}\sqrt{\frac{(X+I-z)^2}{4}-IX} + \left(\frac{(X+I-z)^2}{4}-IX\right)$$

$$> \frac{(X+I-z)^2}{4} \quad \text{holds.}$$

Taking roots on both sides yields: $R°:=\frac{X+I-z}{2}-\sqrt{\frac{(X+I-z)^2}{4}-IX} < \sqrt{IX}$.

Next we derive from our assumptions: $\sqrt{z}<\sqrt{X}-\sqrt{I}$ and $\sqrt{z}<(1-p)\sqrt{X}$ that

$\Rightarrow \sqrt{z}\sqrt{z}<(\sqrt{X}-\sqrt{I})(1-p)\sqrt{X}=(1-p)(X-\sqrt{IX})$ and thus $\frac{z}{(1-p)(X-\sqrt{IX})} < 1$.

That gives us: $a_k(R°,0)=1-\frac{z}{(1-p)(X-R°)} > 1-\frac{z}{(1-p)(X-\sqrt{IX})} > 0$. ❑

Corollary A3: For all relevant (R,C)-contracts and all $k\geq 1$ the effort levels

$$a_k^*(R,C)=1-\frac{z}{(1-p)\ (X-R+kC)} \quad , \quad a_k^{opt}(R,C)=1-\frac{z}{(1-p)\ (X-C+kC)} \quad \text{and}$$

$$a^{*}=1-\frac{z}{(1-p)\ X}$$ are positive.

Proof: It is obvious that for all relevant (R,C)-contracts the above mentioned effort levels are greater than $a_k(R°,0)$ and the proposition follows from lemma A2. ❑

Lemma A4:

(1) $a_k(R)$ is positive on $[1,\infty)x[I,R°]$.

(2) $a_k(R)$ is strictly decreasing in R and (not necessarily strictly) increasing in k on $[1\infty)x[I,R°]$.

(3) $\frac{\partial a_k(R)}{\partial R}$ is (not necessarily strictly) increasing in R and strictly decreasing in k on $[1,\infty)x[I,R°]$.

Proof: (2) We know from prop. 3 that an increase in R results in a decrease of $C_k(R)$. Both changes have a negative impact on the effort chosen. This proves the strict monotonicity in R.

For R=I the formula for $a_k(R)$ simplifies to: $a_k(R) = 1 - \frac{1}{(1-p)}\ \frac{z}{(k-1)I+X}$
and the monotonicity in k is obvious. For $R\neq I$ the k-stability condition reads:

$\varphi(a_k(R)) = \frac{I - C_k(R)}{R - C_k(R)} = 1 - \frac{R - I}{R - C_k(R)}$. From prop. 3 we know that $C_k(R)$ strictly decreases in k for $R<R°$ and is constant in k for $R=R°$. Hence $\varphi(a_k(R))$ and thus $a_k(R)$ increases in k for $R \neq I$.

(1) Using (2) it suffices to show $a_k(R°)>0$. This fact is known from lemma A2.

(3) $a_k(R)$ is continuously differentiable in R and k on $[1,\infty)x[I,R°]$ and we have: $\frac{\partial a_k(R)}{\partial R} = \frac{-1}{(1-p)} \frac{kX+(k-1)(kI-z)}{[(k-1)R+X]^2}$. The monotonicity of $\frac{\partial a_k(R)}{\partial R}$ in R is obvious (for k>1 we even have strict monotonicity).

Next consider: $\frac{\partial a_k(R)}{\partial k} = \frac{1}{(1-p)} \frac{R^2 - R(X+I-z)+IX}{[(k-1)R+X]^2}$. By definition of R° the numerator of this term is nonnegative and strictly decreasing in R on $[1,\infty)x[I,R°]$. The positive denominator is increasing in R on $[1,\infty)x[I,R°]$ (strictly increasing for k>1). Thus $\frac{\partial a_k(R)}{\partial R}$ strictly decreases in R on $[1,\infty)x[I,R°]$. Since we know $\frac{\partial^2 a_k(R)}{\partial k \; \partial R} = \frac{\partial^2 a_k(R)}{\partial R \; \partial k}$ for the totally differentiable function $a_k(R)$ this proves the strict monotonicity of $\frac{\partial a_k(R)}{\partial R}$ in k on $[1,\infty)x[I,R°]$. ❏

Corollary A5: (1) $d(a_k(R))$ is negative on $[1,\infty)x[I,R°]$.

(2) $d(a_k(R))$ is strictly increasing in R and (not necessarily strictly) decreasing in k on $[1,\infty)x[I,R°]$.

(3) $\frac{\partial d(a_k(R))}{\partial R}$ is strictly decreasing in R and strictly increasing in k on $[1,\infty)x[I,R°]$.

Proof: (1) and (2) directly follow from the properties of d(a) and lemma A4.

(3) It holds: $\dfrac{\partial d(a_k(R))}{\partial R} = \dfrac{-z\,\dfrac{\partial a_k(R)}{\partial R}}{1-a_k(R)}$. From lemma A4 we know that the positive numerator increases and the positive denominator strictly decreases in R on $[1,\infty)x[I,R°]$. This implies the strict monotonicity of $\dfrac{\partial d(a_k(R))}{\partial R}$ in R on $[1,\infty)x[I,R°]$.

The strict monotonicity in k on $[1,\infty)x[I,R°]$ follows in the same way from lemma A4. The positive numerator strictly decreases and the positive denominator increases in R on $[1,\infty)x[I,R°]$. ❑

Lemma A6: For all $\bar{k}$ with $R_{\bar{k}}^{FOC} \le R°$ it holds: $\dfrac{\partial R_{\bar{k}}^{FOC}}{\partial k} > 0$.

Proof: Consider a $\bar{k}$ in $[1,\infty)$ with $R_{\bar{k}}^{FOC} \le R$. By definition of R_k^{FOC} we have: $\dfrac{\partial d(a_{\bar{k}}(R_{\bar{k}}^{FOC}))}{\partial R} = 1$ and for a sufficiently small ε>0 we can conclude $\dfrac{\partial d(a_{\bar{k}+\varepsilon}(R_{\bar{k}}^{FOC}))}{\partial R} < 1$ from the monotonicity of $\dfrac{\partial d(a_k(R))}{\partial R}$ in k shown in corollary A5. Using the monotonicity of $\dfrac{\partial d(a_k(R))}{\partial R}$ in R $\dfrac{\partial d(a_{\bar{k}+\varepsilon}(R_{\bar{k}+\varepsilon}^{FOC}))}{\partial R} = 1$ implies $R_{\bar{k}+\varepsilon}^{FOC} > R_{\bar{k}}^{FOC}$. ❑

Lemma A7: $\qquad \lim R_k^{FOC} \gtreqless R° \quad \Leftrightarrow \quad X \gtreqless I+z+\sqrt{I^2+4Iz}$

Proof: The rewritten term: $R_k^{FOC} = -\frac{X}{2(k-1)} - \frac{z}{2k} + \frac{I}{2} + \sqrt{\left(\frac{X}{2(k-1)} + \frac{z}{2k} - \frac{I}{2}\right)^2 + \frac{z(X-z)}{k} + Iz + \frac{XI}{k-1}}$

is of the the form $\quad R_k^{FOC} = \frac{I}{2} + f(k) + \sqrt{\frac{I^2}{4} + Iz + g(k)}$, where f(k), g(k) $\rightarrow$ 0 for k$\rightarrow\infty$. The continuity of the root-function implies: $\quad \lim_{k\rightarrow\infty} R_k^{FOC} = \frac{I}{2} + \sqrt{\frac{I^2}{4} + Iz}$.

Defining $\quad \Delta := \left(\frac{X-I-z}{2}\right)^2 - \left(\frac{I}{2}\right)^2 - Iz \quad$ the following inequalities are equivalent: $\quad X > I+z+\sqrt{I^2+4Iz} \quad , \quad \frac{X-I-z}{2} > \sqrt{\left(\frac{I}{2}\right)^2 + Iz}$,

$$\left(\frac{X-I-z}{2}\right)^2 > \left(\frac{I}{2}\right)^2 + Iz \qquad \text{und} \qquad \Delta > 0.$$

By the concavity of the root-function there are further equivalent:

$$\sqrt{\left(\frac{I}{2}\right)^2 + \Delta} - \sqrt{\left(\frac{I}{2}\right)^2} \quad > \quad \sqrt{\left(\frac{I}{2}\right)^2 + Iz + \Delta} - \sqrt{\left(\frac{I}{2}\right)^2 + Iz} \qquad ,$$

$$\sqrt{\left(\frac{I}{2}\right)^2 + \Delta} + \sqrt{\left(\frac{I}{2}\right)^2 + Iz} \quad > \quad \sqrt{\left(\frac{I}{2}\right)^2 + Iz + \Delta} + \sqrt{\left(\frac{I}{2}\right)^2} \qquad ,$$

$$\sqrt{\left(\frac{X-I-z}{2}\right)^2 - Iz} + \sqrt{\left(\frac{I}{2}\right)^2 + Iz} \quad > \quad \frac{X-I-z}{2} + \frac{I}{2} \qquad ,$$

$$-\frac{I}{2} + \sqrt{\left(\frac{I}{2}\right)^2 + Iz} \quad > \quad \frac{X-I-z}{2} - \sqrt{\left(\frac{X-I-z}{2}\right)^2 - Iz} \qquad \text{and}$$

$$\lim_{k\to\infty} R_k^{opt} > R° \quad .$$

The proof for $<$ and $=$ is analogous. ❑

Lemma A8: (1) If $X \le I+z+\sqrt{I^2+4Iz}$, then $R_k^{FOC} < R°$ for all $k \ge 1$.

(2) If $X > I+z+\sqrt{I^2+4Iz}$, then there exists a k_0, such that

$$R_k^{FOC} \le R° \quad \Leftrightarrow \quad k \in [1,k_0] \ .$$

Proof: First we show: (*) $R_{k_1}^{FOC} \ge R°$ for $k_1 \ge 1$ implies $R_{k_2}^{FOC} > R°$ for all $k_2 > k_1$.

For that assume there exist k_1, k_2 with $k_2 > k_1$, $R_{k_1}^{FOC} \ge R°$ and $R_{k_2}^{FOC} \le R°$. Without loss of generality we can assume $R_{k_2}^{FOC} < R°$, since with the strict monotonicity of R_k^{FOC} in k shown in lemma A6 we could otherwise replace k_2 by k_2-ε. Now choose the largest k_3 in $[k_1,k_2]$ mit $R_{k_3}^{FOC} = R°$. The continuous function R_k^{FOC} maps the interval $[k_3,k_2]$ in $[I,R°]$ and thus by lemma A6 strictly increases on $[k_3,k_2]$. This contradicts $R_{k_3}^{FOC} = R° > R_{k_2}^{FOC}$.

(1) For $X < I+z+\sqrt{I^2+4Iz}$ we know from lemma A7 that $\lim_{k\to\infty} R_k^{FOC} < R°$. If there exists a $k_1 \ge 1$ with $R_{k_1}^{FOC} \ge R°$ we know from (*) that $R_k^{FOC} > R°$ for all $k>k_1$ in contradiction to $\lim_{k\to\infty} R_k^{FOC} < R°$. Hence for $X < I+z+\sqrt{I^2+4Iz}$ we have $R_k^{FOC} < R°$ for all $k \ge 1$.

By the obvious continuity of R_k^{FOC} in X we conclude for $X = I+z+\sqrt{I^2+4Iz}$ the property: $R_k^{FOC} \le R°$ for all $k \ge 1$. Using (*) again we can not have any $k \ge 1$ with $R_k^{FOC} = R°$. So $X = I+z+\sqrt{I^2+4Iz}$ also implies $R_k^{FOC} < R°$ for all $k \ge 1$.

(2) For $X > I+z+\sqrt{I^2+4Iz}$ we know from lemma A7 that $\lim_{k\to\infty} R_k^{FOC} > R^\circ$. Since furthermore $R_1^{FOC} = I$ the continuity of R_k^{FOC} implies the existence of a k_0 with $R_{k_0}^{FOC} = R^\circ$. For the smallest such k_0 we know $R_k^{FOC} < R^\circ$ on $[1,k_0)$ and (*) implies $R_k^{FOC} > R^\circ$ on (k_0,∞). ❑

Lemma A9: If X is sufficiently high, then the second best contracts (R_1^{opt},C_1^{opt}) and $(R_{k_0}^{opt},C_{k_0}^{opt})$ separate the entrepreneurs, i.e.. 1-type entrepreneurs choose the contract (R_1^{opt},C_1^{opt}) and k_0-type entrepreneurs choose $(R_{k_0}^{opt},C_{k_0}^{opt})$. In this case the private information about the degree of loss aversion does not cause any additional contractual costs.

Proof: The proof is in two steps:

(1) First we show that $\lim_{X\to\infty} R^\circ = I$ and $\lim_{X\to\infty} R_k^{opt} = I + z\frac{k-1}{k}$

(2) Then we argue that this implies the proposition

ad (1) From the fact that $\sqrt{A^2+B}$ is in $[A,A+\frac{B}{2A}]$ ($[A+\frac{B}{2A},A]$ resp.) for all A>0 and B≥-A², we can conclude: $\left|(\sqrt{A^2+B}-A)\right| < \frac{B}{2A}$ and thus:

(*) $\lim_{A\to\infty}(\sqrt{A^2+B}-A) = 0$.

This gives for A: = $\frac{X-I-z}{2}$ and B: = - Iz :

$$\lim_{X\to\infty}\left(\sqrt{\left(\frac{X-I-z}{2}\right)^2-Iz}-\frac{X-I-z}{2}\right) = \lim_{A\to\infty}\left(\sqrt{A^2+B}-A\right) = 0 .$$

It follows: $$\lim_{X\to\infty} R° = \lim_{X\to\infty}\left(\frac{X+I-z}{2} - \sqrt{\left(\frac{X+I-z}{2}\right)^2 - IX}\right)$$

$$= \lim_{X\to\infty}\left(I + \frac{X-I-z}{2} - \sqrt{\left(\frac{X-I-z}{2}\right)^2 - Iz}\right)$$

$$= \lim_{X\to\infty} I \; - \; \lim_{X\to\infty}\left(\sqrt{\left(\frac{X-I-z}{2}\right)^2 - Iz} \; - \; \frac{X-I-z}{2}\right) \; = \; I - 0 \; =$$

We can further write

$$R_{k_0}^{FOC} = \frac{-k_0X-(k_0-1)(z-Ik_0)}{2(k_0-1)k_0} + \sqrt{\left(\frac{k_0X+(k_0-1)(z-Ik_0)}{2(k_0-1)\,k_0}\right)^2 + \frac{z(X+Ik_0-z)}{k_0} + \frac{XI}{k_0-1}}$$

as $$R_{k_0}^{FOC} = -\frac{X}{2(k_0-1)} - \frac{z}{2k_0} + \frac{I}{2} + \sqrt{\left(\frac{X}{2(k_0-1)} + \frac{z}{2k_0} - \frac{I}{2}\right)^2 + \frac{zX}{k_0} + I - \frac{z^2}{k_0} + \frac{XI}{k_0-1}}$$

and by rearranging the discriminant we get:

$$R_{k_0}^{FOC} = I + z\left(1-\frac{1}{k_0}\right) - \left(\frac{X}{2(k_0-1)} + \frac{I}{2} + z\left(1-\frac{1}{2k_0}\right)\right) + \sqrt{\left(\frac{X}{2(k_0-1)} + \frac{I}{2} + z\left(1-\frac{1}{2k_0}\right)\right)^2 + B},$$

where B does not depend on X.

Defining $A := \frac{X}{2(k_0-1)} + \frac{I}{2} + z\left(1-\frac{1}{2k_0}\right)$ we conclude with (*):

$$\lim_{X\to\infty} R_{k_0}^{FOC} = \lim_{X\to\infty}\left(I + z\left(1-\frac{1}{k_0}\right)\right) - \lim_{A\to\infty}\left(\sqrt{A^2-B} - A\right) = I + z\left(1-\frac{1}{k_0}\right)$$

ad (2): With $\lim_{X\to\infty}\left(R_{k_0}^{FOC} - R°\right) > 0$ we get $R_{k_0}^{opt} = R°$ for sufficiently high X. Since (I,I) is optimal for the 1-type on the 1-stable curve we must have $C_{ind\ 1}(R°) < 0$. Combining theses facts we have for sufficiently high X: $C_{ind\ 1}(R_{k_0}^{opt}) = C_{ind\ 1}(R°) < 0 \leq C_{k_0}^{opt}$. This implies that the 1-type prefers the (I,I) contract to the $(R_{k_0}^{opt}, C_{k_0}^{opt})$ contract.

By continuity arguments it can be shown that there must exist some X (high, but not too high) such that the contracts are separating (i.e. $C_{ind\ 1}(R_{k_0}^{opt}) < C_{k_0}^{opt}$), without requiring $C_{k_0}^{opt}$ to be zero. ❑

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