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Dominance-Solvable Lattice Games

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Abstract

Dominance-solvable games possess the desirable property that all strategic solution concepts - equilibrium solution concepts as well as iterative solution concepts - determine the same unique solution. The theoretical part of this paper derives sufficient and necessary conditions for dominance-solvability of so-called *lattice games* whose strategy sets have a *lattice structure* while they simultaneously belong to some metric space. As conceptual main contribution, two - hitherto - different strands in the literature about dominance-solvability of strategic games, namely Moulin's (1984) approach for *nice games* and Milgrom and Roberts' (1990) approach for *supermodular games*, are combined and considerably extended. For example, in addition to Milgrom and Roberts (1990) my findings also apply to games where players' actions are *strategic substitutes* or *only partial strategic complementarities*. This is further elaborated in the applicational part of this paper where I establish dominance-solvability of several non-supermodular games such as n-firm Cournot oligopolies, auctions with bidders who are *optimistic* - respectively *pessimistic* - with respect to an imperfectly known allocation rule, and Two-player Bayesian models of bank runs.

Keywords: Supermodular Games, Strategic Complementarities, Strategic Substitutes, Cournot Oligopoly, Auctions with unknown Allocation Rule, Bank Runs

JEL Classification Numbers: C72, C62.

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1 Introduction

In the literature there exist several game theoretic solution concepts which, in general, determine different solutions of strategic games. Different *equilibrium concepts* arise from different proposals for refining Nash's (1950a,b) definition of equilibrium points (see, e.g., Selten, 1975; Myerson, 1978; for an overview see van Damme, 1991). Different *iterative solution concepts* - presuming successive elimination of *unreasonable* strategies - apply different definitions of an *unreasonable* strategy (Bernheim, 1984; Moulin, 1984; Pearce, 1984; Börgers, 1993). However, if a game is *dominance-solvable* - in the sense that only a unique strategy survives *iterated elimination of strictly dominated strategies* - all these different strategic solution concepts determine the same unique solution. Thus, dominance-solvable games describe the class of games for which most game theorists agree in their predictions of how *strategically sophisticated* individuals will act in a decision situation of strategic interdependency.

This paper extends existing results about dominance-solvability of strategic games by providing sufficient and necessary conditions for dominance-solvability of so-called *lattice games*. On the one hand, strategy sets of *lattice games* exhibit a *lattice structure,* that is, strategies are partially ordered and there exists for every pair of strategies an infimum and a supremum in the strategy set (Topkis, 1979; Milgrom and Roberts, 1990; Vives, 1990; Milgrom and Shannon, 1994). On the other hand, strategy sets of *lattice games* are assumed to be simultaneously subsets of some *metric space* such that the distance between the smallest and the largest element - in lattice order - of some subset is greater than the distance between the remaining elements of this subset. Due to this dual property of strategy sets of lattice games, I am able to combine and generalize two different strands in the literature about dominance-solvability of strategic games, namely Moulin's (1984) approach for *nice games* and Milgrom and Roberts' (1990) approach for *supermodular games*.

The most prominent results about strategic solutions of games - where strategy sets have a lattice structure - concern existence and structure of Nash equilibria of supermodular games (see, e.g., Topkis, 1978, Milgrom and Roberts, 1990; Vives, 1990) and of games with a *single-crossing property* (Milgrom and Shannon, 1994; Athey, 2001). Supermodular games are characterized by players whose actions are *strategic complementarities* (Bulow, Geneakoplos, and Klemperer, 1985), that is, a player's incentive of choosing a larger strategy - with respect to the lattice order - increases if her opponents also choose larger strategies. Milgrom and Roberts (1990) now derive the remarkable result that supermodular games are dominance-solvable if and only if they exhibit a unique Nash equilibrium.

This paper's findings extend Milgrom and Roberts' (1990) result along two dimensions. First, if a supermodular game is also a lattice game - which is the case for standard strategy sets - I can characterize uniqueness of the Nash equilibrium by a rather simple necessary and sufficient mathematical condition. Thus, in addition to Milgrom and Roberts (1990), this paper provides a specific technical criterion for uniqueness of the Nash equilibrium in supermodular games which might be easier to verify (or to falsify, for that matter) than standard theorems establishing existence of a unique fixed point.

Second, and more importantly, the results of this paper also establish dominance-solvability of games that are not supermodular, and that therefore do not necessarily exhibit increasing best response functions (as implied by supermodular games or by games with the single-crossing property). Many relevant economic situations can not be described as supermodular games since players actions are either *strategic substitutes* (Bulow, Geneakoplos, and Klemperer, 1985) or *only partial strategic complementarities.* In the applicational part of this paper I further elaborate on this point by applying this paper's theoretical findings to demonstrate dominance-solvability of games that are not supermodular. In particular, I consider n-firm Cournot oligopolies, auctions with bidders who are *optimistic* - respectively *pessimistic* - with respect to an imperfectly known allocation rule, and a simple Two-player Bayesian model of bank runs.

In the remainder of this introduction I explain in deeper detail this paper's technical contributions and the economic examples presented in the applicational part.

1.1 Technical Contributions

Exploiting the lattice structure of strategy sets I derive equivalence conditions - referring to players' utility functions - which imply that a game is dominance-solvable if and only if it admits a unique *point-rationalizable* strategy. Point-rationalizability (Bernheim, 1984; Moulin, 1984; Pearce, 1984) is an iterative solution concept where strategies are eliminated as unreasonable if and only if they are not a best response to some pure strategy profile (or equivalently: to some degenerated point-belief about pure strategy profiles). Since, in general, point-rationalizability is a significantly stronger solution concept than iterated elimination of dominated strategies such equivalence conditions are not trivial.

Exploiting the metric-space property of strategy sets I derive uniqueness conditions for point-rationalizable strategies that refer to properties of players' best response functions. Moreover, the lattice structure of strategy sets allows for a particularly convenient characterization of unique point-rationalizable strategies if best response functions are either increasing for all players or decreasing for all players. Roughly speaking, a combination of these equivalence- and uniqueness conditions then establishes dominance-solvability of lattice games.

1.1.1 Equivalence Results

This paper derives two different equivalence results that identify conditions so that a game, satisfying these conditions, is dominance-solvable if and only if it has a unique point-rationalizable strategy. Both equivalence results generalize similar findings appearing in Moulin (1984) and in Milgrom and Roberts (1990), respectively.

First consider strategy sets with an arbitrary lattice structure. Under the assumption that players' utility functions are *supermodular* with respect to their own strategies, proposition 1 of this paper then derives equivalence between dominance-solvability and uniqueness of point-rationalizable strategies if differences in players' utility functions either increase or decrease with an increase in the opponents' strategy choice. Increasing utility differences formally define strategic complementarities whereas decreasing utility differences define strategic complementarities. Thus, proposition 1 extends an equivalence result due to Milgrom and Roberts (1990), which is restricted to supermodular games where all players have increasing utility differences only, to games where players may have arbitrary monotonic - increasing or decreasing - utility differences. As a consequence, proposition 1 may therefore establish equivalence between dominance-solvability and uniqueness of point-rationalizable strategies for games where, e.g., one or all players have decreasing best response functions.

Now consider the degenerated case of a lattice structure where strategy sets are *totally ordered*. Proposition 2 of this paper then demonstrates equivalence between dominance-solvability and uniqueness of point-rationalizable strategies if players' utility functions satisfy a condition I call *order-quasiconcavity* (which is a straightforward generalization of the definition of quasiconcavity on convex sets to arbitrary sets that are partially ordered). As a consequence, proposition 2 generalizes an equivalence result due to Moulin (1984) - who shows equivalence under the assumptions of *strictly quasiconcave* utility functions and *real-valued convex* strategy sets - to games with general *totally ordered* strategy sets that are not necessarily convex. Moreover, in contrast to the equivalence result of proposition 1, proposition 2 establishes equivalence without entailing monotonic best response functions. However, whereas proposition 1 applies to games where strategy sets exhibit an arbitrary lattice structure, the equivalence result of proposition 2 requires the rather strong assumption of totally ordered strategy sets.

1.1.2 Dominance-solvability Results

The concept of lattice games serves two technical purposes. First, since for lattice games the *diameter* (defined as the least upper-bound of all distances between the elements of a given set) of some *order-complete* set of strategies coincides with the distance between the set's smallest and largest - in lattice order - element, a non-empty set of point-rationalizable strategies contains a unique strategy if and only if the diameter of the set of strategies, which survive iterated elimination by the point-rationalizability criterion, converges towards zero when the number of iteration steps approaches infinity. Second, by the lattice structure of the strategy sets of lattice games, the equivalence conditions, derived in proposition 1 and proposition 2, ensure that a lattice game is dominance-solvable if and only if it admits a unique point-rationalizable strategy.

Proposition 3 of this paper derives a necessary and sufficient mathematical condition establishing dominance-solvability of lattice games. In particular, it is shown that convergence properties of a k–fold application of the best response function to itself are necessary and sufficient to guaranteeing for lattice games that the set of point-rationalizable strategies is non-empty while the diameter of the set of strategies, which survive iterated elimination by the point-rationalizability criterion, converges towards zero. The proof idea parallels an approach due to Zimper (2003a) who generalizes findings of Bernheim (1984) and of Moulin (1984). However, while the proofs in Zimper (2003a) rely on the assumption of *compact*, respectively *bounded* and *complete*, subsets of some metric space, the proof of proposition 3 exclusively refers to lattice properties of strategy sets.

For lattice games with real-valued strategy sets, a corollary to proposition 3 establishes dominance-solvability if the first-order partial derivatives of functions - resulting from a k-fold application of the best response function to itself - have sufficiently small values. Moulin's (1984) sufficiency condition for dominance-solvability of nice games obtains as special case of the corollary when only 1-fold applications of the best response function to itself are considered and when only the distance, induced by the *supremum-norm*, is considered.

Under the additional assumption that players have monotonic best response functions, which move in the same direction[1], proposition 4 presents a significantly simpler characterization of dominance-solvability than proposition 3. The result of proposition 4 refers to convergence-properties of the k-fold application of the best response function to itself only evaluated at the smallest and at the largest strategy of the strategy set. On the one hand, proposition 4 therefore provides a simple characterization of unique Nash equilibria of supermodular games that are also lattice games. On the other hand, the finding of proposition 4 will prove very useful for establishing dominance-solvability in the applicational part of this paper where non-supermodular games with decreasing best response functions are considered.

[1] That is, if the best response functions are either increasing for all players or decreasing for all players.

1.2 Examples and Applications

1.2.1 n-Firm Cournot Oligopolies

Strategic solutions of *n-firm Cournot oligopolies* have been extensively studied in the literature (see, e.g., Bernheim, 1984; Moulin, 1984; Novshek, 1984 and 1985; Bamon and Fraysse, 1985; Vives, 1990; Basu, 1992; Amir, 1996). Two issues emerging from this literature are of particular relevance to this paper's topic. First, applied to classical models of n-firm Cournot oligopolies, iterative solution concepts perform rather poorly since they blow up the set of possible solutions when there are more than two firms involved (Bernheim, 1984; Basu, 1992). Second, the application of lattice-theory to characterizing strategic solutions is typically restricted to Cournot duopolies only (Milgrom and Roberts, 1990; Vives, 1990) since only Cournot duopolies can be transformed into supermodular games whereas actions in general n-firm Cournot oligopolies are strategic substitutes (but see Amir, 1996, who describes non-standard n-firm Cournot oligopolies where players have increasing best response functions).

Proposition 5 and proposition 6 of this paper apply this paper's theoretical findings to identify conditions which assure dominance-solvability of n-firm Cournot oligopolies. For the classical model of an n-firm Cournot oligopoly - exhibiting a linear inverse demand function and constant marginal costs - proposition 5 establishes dominance-solvability if the different firms' products are not perceived as perfect substitutes by the customers. If there are three firms in the oligopoly then dominance-solvability already obtains in case firms' products are arbitrarily close to being perfect substitutes, but are not actually perfect substitutes. For more than three firms a further weakening of the perfect substitute assumption is required to assure dominance-solvability. Assuming that firms' products are perfect substitutes, Bernheim (1984) shows for the classical model of an n-firm Cournot oligopoly that virtually any output decision can be justified by iterative solution concepts if the oligopoly consists of more than two firms. Thus, proposition 5 demonstrates that Bernheim's (1984) negative result about the predictive performance of iterative solution concepts strongly relies on the assumption that firms compete with products that customers perceive as perfect substitutes.

Proposition 6 of this paper derives conditions which imply dominance-solvability of n-firm Cournot oligopolies under the assumption that cost functions are quadratic and that firms compete on so-called *large markets*, as described by Börgers and Janssen (1995), where an increase in the number of firms is matched by an increase in market demand. The findings of proposition 6 suggest, maybe somewhat contrary to intuition, that an increase in the market-size rather increases than decreases the difficulties for establishing dominance-solvability.

1.2.2 Auctions with Optimistic - respectively Pessimistic - Bidders

A second application concerns auctions where bidders are uncertain about the actual allocation rule and exhibit optimistic, respectively pessimistic, attitudes with respect to this rule. Such auctions are relevant whenever the organizer of the auction pays attention to matters like bidder's "moral standing", solvency, and so on[2].

Auctions where one bidder is *handicapped*, in the sense that she has to offer a much higher bid than a *favored* bidder in order to win the auction, have been studied by Feess, Muehlheusser, and Walzl (2002). However, while in their model it is common-knowledge among the bidders who is handicapped and who is favored, I describe a - very simple - model of an auction where pessimistic bidders *believe* they are *handicapped* by the allocation rule in the sense that they only expect to win with certainty if they offer significantly higher monetary bids than their competitors. Analogously, optimistic bidders believe they are *favored* by the allocation rule so that they expect to win even if they bid less money than their competitors.

Proposition 7 of this paper then shows that in auctions with pessimistic bidders all bidders bid the highest amount allowed by their budget constraints, whereas in auctions with optimistic bidders every bidder just offers the reservation price demanded by the auction's organizer. Thus, the findings of proposition 7 imply that an auction-organizer, who wants to gain high profits while she does not know bidders' evaluations or budget-constraints, better lets the allocation rule imprecise if she expects pessimistic bidders, whereas she should be very specific about the allocation rule's details in the case of optimistic bidders.

1.2.3 A Two-Player Model of Bank-Runs

Ever since the seminal contributions of Bryant (1980) and of Diamond and Dybvig (1983) models of bank runs have been the subject of intensive study. At their core game-theoretic models of bank runs presume a coordination problem where patient investors achieve the *good* outcome when they simultaneously do NOT WITHDRAW whereas they only achieve the *bad* outcome when they simultaneously WITHDRAW. While the early models of Bryant (1980) and of Diamond and Dybvig (1983) describe this coordination

[2] Think, for example, of the Lottery Commission's zigzagging when it had to decide, on behalf of the British Goverment, whether the new seven-year operating licence for the British Lottery was granted to Sir Richard Brenson's People's Lottery or to its competitor Camelot. At some point of time the Commission said that it would neither grant the licence to the People's Lottery nor to Camelot because People's Lottery had so-called "technical problems over finances" whereas Camelot was judged as "not to be a "fit and proper" operator, largely because of its association with the American gaming software company GTech" (quoted from a DAILY-TELEGRAPH internet article).

problem as a coordination game exhibiting multiple equilibria, more recent approaches (e.g., Postlewaite and Vives, 1987; Goldstein and Pauzner, 2002), try to deduce the likelihood of bank-runs from a unique strategic solution.

As third application I present Two-Player Bayesian games of bank runs where players choose between *switching strategies* that are characterized by a *cutoff-point* so that the player chooses to WITHDRAW (NOT WITHDRAW) for signals below (above) the cutoff point. Each player's signal is independently drawn from a uniform distribution over the unit interval where signals may be interpreted, e.g., as an investment project's success (as in Goldstein and Pauzner, 2002) or as investor's preferences for intertemporal consumption (as in Postlewaite and Vives, 1987).

Proposition 8 then establishes dominance-solvability of Two-Player Bayesian games of bank runs and it derives the likelihood of bank runs from the unique strategic solution of such games. Postlewaite and Vives (1987) also describe bank runs by a Two-player Bayesian game. Their model admits only three different signals about players intertemporal consumption preferences and it exhibits, for a particular range of parameters, dominant strategies for both players. In contrast to the model of Postlewaite and Vives (1987), the dominance-solvable Two-Player Bayesian games of bank runs that I consider exhibit a more complicated strategic structure since they do not possess solutions in dominant strategies. In analogy to the *global game* approach of Carlsson and van Damme (1993), Goldstein and Pauzner (2002) show existence of a unique equilibrium for their model of bank runs. However, they do not establish dominance-solvability[3].

The remainder of this paper is organized as follows. In section 2 notation and basic definitions are introduced. Section 3 derives conditions implying that a game is dominance-solvable if and only if this game has a unique point-rationalizable strategy. Lattice games are formally defined in section 4; examples for possible strategy sets of lattice games are provided. Section 5 contains this paper's technical main results concerning dominance-solvability of lattice games. The theoretical findings of this paper are applied to establishing dominance-solvability of n-firm Cournot oligopolies (section 6), of auctions with bid- respectively optimistic - with respect to an imperfectly known allocation rule (section 7), and of Two-player Bayesian games of bank runs (section 8). All technical proofs are relegated to the appendix.

[3] The model of Goldstein and Pauzner (2002) is actually not a global game in the typical sense since it does not satisfy the supermodularity assumptions required for global games (compare Morris and Shin, 2002; Frankel, Morris, and Pauzner, 2003). Thus, in contrast to global games, a unqiue equilibrium does here not imply dominance-solvability.

2 Preliminaries: Notation, Lattice Theory

For a finite set of players I, let $G = (S_i, U_i)_{i \in I}$ denote a game in normal form where S_i denotes the individual strategy set of player $i \in I$ and where $U_i : S_i \times S_{-i} \to \mathbb{R}_+$ represents player i's preferences over strategies in S. Let $f_i : S_{-i} \to 2^{S_i}$ denote player i's individual best response correspondence such that, for all $s_{-i} \in S_{-i}$,

$$f_i(s_{-i}) = \arg\max_{s_i \in S_i} U_i(s_i, s_{-i})$$

For the sake of presentational simplicity, throughout this paper only games with *individual best response functions* are considered, that is, for all $i \in I$ and all $s_{-i} \in S_{-i}$, $f_i(s_{-i})$ is assumed to be single-valued[4]. Function $f : S \to S$, with $f(s) = \times_{i=1}^{I} f_i(s_{-i})$, is then called the game's *best response function*.

Recall the following notions of *lattice theory* (see, e.g., Topkis, 1979; Milgrom and Roberts, 1990; Vives, 1990; Fudenberg and Tirole, 1996):

Given a reflexive, transitive, and antisymmetric binary relation $\leq_L$ on a set S_i. If there exists for all elements $s_i, t_i \in S_i$ a *supremum* $s_i \vee t_i$ and an *infimum* $s_i \wedge t_i$ in S_i then $(S_i, \leq_L)$ denotes a *lattice*.

$(S_i, \leq_L)$ is a *complete lattice* if, for every non-empty subset $T \subset S_i$, $\inf T \in S_i$ and $\sup T \in S_i$. In particular, completeness of S_i implies existence of a *smallest* - the unique minimal - element $s_i \in S_i$ such that $s_i <_L s_i'$ for all $s_i' \in S_i$ with $s_i' \neq s_i$, and of a *largest* - the unique maximal - element $t_i \in S_i$ such that $s_i' <_L t_i$ for all $s_i' \in S_i$ with $s_i' \neq t_i$.

$(S_i, \leq_L)$ is *totally ordered*, i.e., a *chain*, if, for all $s_i, t_i \in S_i$, $s_i \nleq_L t_i$ implies $t_i \leq_L s_i$.

If $(S_i, \leq_L)$ is a lattice for all $i \in I$ then $(S, \leq_L)$ denotes a lattice such that $s \leq_L t$ if and only if, for all $i \in I$, $s_i \leq_L t_i$.

U_i is *supermodular* on $(S_i, \leq_L)$ if, for all $s_i, t_i \in S_i$ and all $s_{-i} \in S_{-i}$,

$$U_i(s_i, s_{-i}) + U_i(t_i, s_{-i}) \leq U_i(s_i \wedge t_i, s_{-i}) + U_i(s_i \vee t_i, s_{-i})$$

Note that *supermodularity* of U_i on $(S_i, \leq_L)$ is trivially satisfied if $(S_i, \leq_L)$ is a chain since

$$\begin{aligned} s_i \wedge t_i &= \min\{s_i, t_i\} \\ s_i \vee t_i &= \max\{s_i, t_i\} \end{aligned}$$

In particular, U_i is supermodular on $(S_i, \leq_L)$ if $S_i \subset \mathbb{R}$ and $\leq_L$ denotes the standard order $\leq$ of the real numbers.

[4] This paper's findings immediately generalize to games whose best response correspondences reduce to best response functions after some arbitrary round of eliminating *unreasonable* strategies.

U_i has *increasing differences* on $(S_{-i}, \leq_L)$ if, for all $t_i \leq_L s_i$, $U_i(s_i, s_{-i}) - U_i(t_i, s_{-i})$ is non-decreasing in s_{-i}. Conversely, U_i has *decreasing differences* on $(S_{-i}, \leq_L)$ if, for all $t_i \leq_L s_i$, $U_i(s_i, s_{-i}) - U_i(t_i, s_{-i})$ is non-increasing in s_{-i}.

Furthermore, I call U_i *order-quasiconcave* on $(S_i, \leq_L)$ if, for all chains $(C_i, \leq_L) \subset (S_i, \leq_L)$, and for all $s_i, t_i \in C_i$ and all $s_{-i} \in S_{-i}$,

$$U_i(s_i, s_{-i}) \geq U_i(t_i, s_{-i})$$

implies $U_i(s_i', s_{-i}) \geq U_i(t_i, s_{-i})$ for all $s_i' \in C_i$ such that $s_i \leq_L s_i' \leq_L t_i$ or $t_i \leq_L s_i' \leq_L s_i$.

If f is *order-continuous* then its values converge on every chain, i.e., totally ordered subset of S, in decreasing or in increasing direction. That is, for any chain $(C, \leq_L) \subset (S, \leq_L)$,

$$\begin{aligned} \lim_{s \in C, s \downarrow \inf C} f(s) &= f(\inf C) \\ \lim_{s \in C, s \uparrow \sup C} f(s) &= f(\sup C) \end{aligned}$$

f_i is *increasing* on $(S_{-i}, \leq_L)$ if $s_{-i} \leq_L t_{-i}$ implies $f_i(s_{-i}) \leq_L f_i(t_{-i})$; and f_i is *decreasing* on $(S_{-i}, \leq_L)$ if $s_{-i} \leq_L t_{-i}$ implies $f_i(t_{-i}) \leq_L f_i(s_{-i})$. Note that an individual best response function f_i is increasing, respectively decreasing, on $(S_{-i}, \leq_L)$ if U_i has increasing, respectively decreasing, differences on $(S_{-i}, \leq_L)$ whereas the converse statement is not necessarily true.

Remark. By Topkis' characterization theorem (Theorem 1.1 in Topkis, 1979), a real-valued function is submodular (supermodular) on product space $L = \times_{k=1}^{m} L_k$ with lattice structure if and only if it has decreasing (increasing) differences on all L_k, $k \in \{1, ..., m\}$, while keeping L_{-k} fixed. Notice that results of this paper may refer to utility functions U_i that are supermodular on individual strategy sets $(S_i, \leq_L)$ but not necessarily on $(S, \leq_L)$. Moreover, the considered utility functions U_i may have increasing or decreasing utility differences on $(S_{-i}, \leq_L)$ but not necessarily on $(S, \leq_L)$. Thus, presuming utility functions U_i that are supermodular on $(S_i, \leq_L)$, while they simultaneously have decreasing utility differences on $(S_{-i}, \leq_L)$, does not contradict Topkis' characterization theorem.

3 Equivalence Results

Iterative solution concepts can be justified by the assumption that players involve in an internal process of reasoning which succeedingly excludes *unreasonable* strategies (see Pearce, 1984; Tan and Werlang, 1988; Guesnerie, 2002 for an epistemic foundation of iterative solution concepts by the assumption that *it is common-knowledge among* *players that players do not choose unreasonable strategies*). *Rationalizability concepts* (Bernheim, 1984; Moulin, 1984; Pearce, 1984; Börgers, 1993) treat any strategy as unreasonable if it is not a best response to some *belief*. These beliefs are either defined as non-additive probability measures (Ghirardato and Le Breton, 1997 and 2000), or as additive probability measures which may be further restricted to independent- or even to degenerated probability measures.

Dominance solution concepts (e.g., Moulin, 1984; Milgrom and Roberts, 1990; Börgers 1993) treat a strategy as unreasonable if it is dominated - strongly versus weakly - by another - mixed versus pure - strategy. Assume, for example, that players only choose strategies such that no alternative strategy results in a strictly higher utility for all possible strategies of the player's opponents. This assumption may effectively eliminate some strategies as *strongly dominated*. In a next step presume that players only choose strategies such that no alternative strategy gives a strictly higher utility for all opponents' strategies surviving the first round of elimination. Repeating this argument gives in the limit the *dominance solution* of a game, that is, the set of all strategies that survive iterated elimination of strategies that are strongly dominated.

Definition: *The* dominance solution *of game $G = (S_i, U_i)_{i \in I}$ is defined as the set*

$$D(G) = \bigcap_{k=0}^{\infty} \theta^k(S)$$

such that $\theta^0(S) = S$ and for all $k \geq 1$: for every $i \in I$, $s_i \in \theta_i^k$ if and only if there does not exist some $t_i \in \theta_i^{k-1}$ such that, for all $s_{-i} \in \theta_{-i}^{k-1}$, $U_i(t_i, s_{-i}) > U_i(s_i, s_{-i})$.

Moreover, game $G = (S_i, U_i)_{i \in I}$ is called dominance-solvable *if and only if there exists a unique strategy $s \in S$ such that $s \in D(G)$.*

Point-rationalizability (Bernheim 1984, Moulin 1984, Pearce 1984) starts out with the assumption that players only choose best responses to some strategy choice of their opponents. This assumption may effectively eliminate some strategies and in a next step point-rationalizability requires the players to choose only best responses to the remaining strategy choices of her opponents. Iteration of this argument gives in the limit the set of point-rationalizable strategies.

Definition: *The set of* point-rationalizable *strategies of game $G = (S_i, U_i)_{i \in I}$ is defined as*

$$P(G) = \bigcap_{k=0}^{\infty} \lambda^k(S)$$

such that $\lambda^0(S) = S$ *and, for all* $k \geq 1$,

$$\lambda^k(S) = \bigcup_{s \in \lambda^{k-1}(S)} f(s)$$

Since point-rationalizability only considers best responses to strategies, i.e., to *probability one* beliefs, it is - from a decision theoretic point of view - less convincing than alternative rationalizability concepts. However, the great advantage of point-rationalizability is its technical simplicity which will be later exploited for deriving mathematical conditions guaranteeing dominance-solvability. In what follows, conditions are presented implying equivalence between dominance solvability and a unique point-rationalizable strategy.

3.1 Monotonic Utility Differences

My first equivalence result generalizes findings of Milgrom and Roberts (lemma 1, proposition 5; 1990) to games where some players' utility functions may have decreasing differences.

Lemma 1. *Suppose that game* $G = (S_i, U_i)_{i \in I}$ *satisfies the following assumptions:*
(A1) $(S_i, \leq_L)$ *is a complete lattice for all* $i \in I$.
(A2) There exists an order-continuous best response function f.
(A3) U_i *is supermodular on* $(S_i, \leq_L)$ *for all* $i \in I$.
(A4) U_i *has either increasing or decreasing utility differences on* $(S_{-i}, \leq_L)$ *for all* $i \in I$.

Then the set of point-rationalizable strategies, $P(G)$*, and the dominance-solution,* $D(G)$*, are complete lattices such that the largest (smallest) elements of both sets coincide, i.e.,*

$$\sup P(G) = \sup D(G) \tag{1}$$
$$\inf P(G) = \inf D(G) \tag{2}$$

Since every point-rationalizable strategy belongs to the dominance solution, lemma 1 immediately implies:

Proposition 1. *Suppose that game* $G = (S_i, U_i)_{i \in I}$ *satisfies assumptions (A1) - (A4) of lemma 1. Then* $G = (S_i, U_i)_{i \in I}$ *is dominance-solvable if and only if there exists* *a unique point-rationalizable strategy of* $G = (S_i, U_i)_{i \in I}$*, i.e.,* $P(G) = \{s\}$ *for some* $s \in S$*.*

Remark. Although the assumptions of the lemma imply $\sup P(G) = \sup D(G)$ and $\inf P(G) = \inf D(G)$ they are not sufficient for guaranteeing that the set of point-rationalizable strategies coincides with the dominance-solution, i.e., $P(G) = D(G)$. To see this, consider the following example of a symmetric two-player game with payoff-matrix given by

B

A		$b1$	$b2$	$b3$
	$a1$	$1 \quad \epsilon$	$1 \quad \epsilon$	$0 \quad \epsilon$
	$a2$	$0.7 \quad \epsilon$	$0.7 \quad \epsilon$	$0.7 \quad \epsilon$
	$a3$	$0 \quad \epsilon$	$1 \quad \epsilon$	$1 \quad \epsilon$

Let $a1 \leq_L a2 \leq_L a3$ and $b1 \leq_L b2 \leq_L b3$, and observe that the assumptions of the lemma are satisfied. However, while the individual strategy $a2$ is not a best response to any pure strategy it is not strictly dominated either. Thus,

$$\begin{aligned} \sup P(G) &= \sup D(G) = (a3, b3) \\ \inf P(G) &= \inf D(G) = (a1, b1) \end{aligned}$$

but, e.g., $(a2, b2)$ is not a point-rationalizable strategy although it belongs to the dominance-solution.

3.2 Order-quasiconcave Utility Functions

For the second equivalence result I utilize an idea already appearing in Moulin (lemma 2; 1984) who shows for so-called *nice games*, where individual strategy sets are compact and convex subsets of the real numbers and utility functions are continuous and strictly quasiconcave, equivalence between the iterative procedures of the dominance-solution[5] and of point-rationalizability.

Lemma 2. *Suppose that game* $G = (S_i, U_i)_{i \in I}$ *satisfies the following assumptions:*

[5] Moulin (1984) actually considers successive elimination of *weakly* dominated strategies. However, as shown in Zimper (2003b), a strategy is weakly dominated in a *nice game* if and only if it is also strongly dominated.

(B1) $(S_i, \leq_L)$ *is a complete lattice for all* $i \in I$.

(B2) $(S_i, \leq_L)$ *is totally ordered for all* $i \in I$.

(B3) There exists an order-continuous best response function f.

(B4) U_i *is order-quasiconcave on* $(S_i, \leq_L)$ *for all* $i \in I$.

Then the set of point-rationalizable strategies, $P(G)$, *and the dominance-solution,* $D(G)$, *are complete lattices such that the largest (smallest) elements of both sets coincide, i.e.,*

$$\begin{aligned} \sup P(G) &= \sup D(G) \\ \inf P(G) &= \inf D(G) \end{aligned}$$

Proposition 2. *Suppose that game* $G = (S_i, U_i)_{i \in I}$ *satisfies assumptions (B1) - (B4) of lemma 2. Then* $G = (S_i, U_i)_{i \in I}$ *is dominance-solvable if and only if there exists a unique point-rationalizable strategy of* $G = (S_i, U_i)_{i \in I}$, *i.e.,* $P(G) = \{s\}$ *for some* $s \in S$.

Remark 1. As a relevant generalization of Moulin's (1984) assumptions, proposition 2 admits totally ordered individual strategy sets that are not convex. However, Moulin's convexity-assumption is crucial for proving, by the intermediate value theorem, equivalence in nice games between the dominance solution and the set of point-rationalizable strategies regardless whether the point-rationalizable solution is unique or not. Note therefore, that, by dropping Moulin's (1984) convexity assumption, uniqueness of the point-rationalizable solution is required to assure equivalence between both iterative solution concepts in proposition 2.

Remark 2. One might wonder whether assumption (B2) of proposition 2 could be generalized from totally ordered to just partially ordered individual strategy sets. The following example shows that this is not the case. Presume players' payoffs given by

B

A		$b1$	$b2$
	$(1,1)$	$1 \quad \epsilon$	$1 \quad \epsilon$
	$(1,2)$	$2 \quad \epsilon$	$0 \quad \epsilon$
	$(2,1)$	$0 \quad \epsilon$	$2 \quad \epsilon$
	$(2,2)$	$1 \quad \epsilon$	$1 \quad \epsilon$

For $(x_1, x_2), (y_1, y_2) \in S_A$, let $(x_1, x_2) \leq_L (y_1, y_2)$ iff $x_1 \leq y_1$ and $x_2 \leq y_2$, and observe that A's utility function is order-quasiconcave. Although all assumptions of lemma 2, except for (B2), are satisfied, its conclusion is violated since

$$\begin{aligned} \sup P(G) &= (2,1) \neq (2,2) = \sup D(G) \\ \inf P(G) &= (1,2) \neq (1,1) = \inf D(G) \end{aligned}$$

4 Lattice Games

A *lattice game* is a game whose strategy set is simultaneously described as a subset of a metric space (X, d) and as a lattice $(S, \leq_L)$ such that the partial order $\leq_L$ and the distance function $d : X \times X \to \mathbb{R}_+$ satisfy a particular condition, implying that the distance between the *smallest* and the *largest* element of a complete subset $T \subset S$ is not smaller than the distance between arbitrary elements in T.

Definition. *$G = (S_i, U_i)_{i \in I}$ is a* lattice game *if and only if S is a bounded, non-empty subset of some metric space (X, d) as well as a complete lattice $(S, \leq_L)$ such that for all $s, s', t', t \in S$*

$$s \leq_L s', t' \text{ and } s', t' \leq_L t \text{ implies } d(s', t') \leq d(s, t) \tag{3}$$

Recall that a normed Riesz space is an ordered vector space that is a lattice as well as a metric space with norm-induced metric, (see, e.g., Aliprantis and Border, 1994). Consequently, whenever all S_i are normed Riesz spaces the strategy set S can be characterized as a lattice and as a subset of a metric space under the max-norm $\|s\| = \max_{i \in I} \|s_i\|$. Consider the following examples to see that typical individual strategy sets of economic interest are describable as normed Riesz spaces satisfying condition (3).

Example. Let S_i be a subset of the Riesz space $B(X)$ of all bounded real functions on X under the supremum-norm $\|s_i\|_\infty = \sup\{|s_i(x)| \mid x \in X\}$. Impose the following lattice structure on S_i: $s_i \leq_L t_i$ if and only if $s_i(x) \leq t_i(x)$ for all $x \in X$. Now suppose that $s_i \leq_L s'_i, t'_i$ and $s'_i, t'_i \leq_L t_i$, and without restricting generality assume further that $\|t'_i - s'_i\|_\infty = \sup\{t'_i(x) - s'_i(x) \mid x \in X\}$. Since $t_i(x) \geq t'_i(x)$ and $s'_i(x) \geq s_i(x)$ for all $x \in X$, it obtains $\|t_i - s_i\|_\infty \geq \|t'_i - s'_i\|_\infty$.

Example. Let S_i be a subset of the Riesz space l_∞ of all continuous real functions on $\mathbb{N}$ with compact support, i.e.,

$$l_\infty = \left\{s_i \in \mathbb{R}^{\mathbb{N}} \mid \|s_i\|_\infty < \infty\right\}$$

which is obviously a special case of the preceding example by letting $X = \mathbb{N}$. Since l_∞ is nothing else than the space of sequences with bounded entries, the individual strategy sets S_i can therefore be described for typical settings of dynamic games with infinite time-horizon as a lattice and as a subset of a metric space satisfying condition (3), (compare the "Arms race"-example of a supermodular game in Milgrom and Roberts, 1990).

Example. Let S_i be a subset of the Riesz space $B([0,1])$ of all bounded real functions on $[0,1]$ under the L_1-norm, i.e. $\|s_i\| = \int_0^1 |s_i(x)|\,dx$, such that s_i and t_i are considered as identical if $\int_0^1 |s_i(x) - t_i(x)|\,dx = 0$. Impose the following lattice structure on S_i: $s_i \leq_L t_i$ if and only if the set $\{x \mid s_i(x) > t_i(x)\}$ is of measure zero, (compare Theorem 3 in Milgrom and Roberts, 1990). Now suppose $s_i \leq_L s_i', t_i'$ and $s_i', t_i' \leq_L t_i$ and note that

$$\begin{aligned}
\int_0^1 t_i(x) - s_i'(x)\,dx + \int_0^1 t_i(x) - t_i'(x)\,dx &= d(s_i', t_i) + d(t_i, t_i') \geq d(s_i', t_i') \\
\int_0^1 t_i'(x) - s_i(x)\,dx + \int_0^1 s_i'(x) - s_i(x)\,dx &= d(t_i', s_i) + d(s_i, s_i') \geq d(t_i', s_i')
\end{aligned}$$

Summing up the l.h.s and the r.h.s of the above inequalities gives the desired result

$$\begin{aligned}
2\int_0^1 t_i(x) - s_i(x)\,dx &\geq d(s_i', t_i') + d(t_i', s_i') \\
d(s_i, t_i) &\geq d(s_i', t_i')
\end{aligned}$$

Counter-example. Consider $S_i = [0,1) \cup \{1.5\}$ with Euclidean metric. Let $s_i \leq_L t_i$ if $s_i \leq t_i$ and observe that S_i is a complete lattice satisfying condition (3). Now define $\leq_L$ by

$$\begin{aligned}
1.5 &\leq {}_L s_i \text{ for all } s_i \in S_i \\
s_i &\leq {}_L t_i \text{ if and only if } s_i \leq t_i \text{ for all } s_i, t_i \in [0,1]
\end{aligned}$$

Again, S_i is a complete lattice, however, condition (3) is not satisfied.

5 Technical Main Results: Dominance-Solvability

This section presents the main findings of this paper which refer to contraction properties of the game's best response function. Let $f^0(s) = s$ and define for a given best response function $f : S \to S$ the function $f^k : S \to S$ such that, for all $k \in \mathbb{N}$, $f^k(s) = f\left(f^{k-1}(s)\right)$.

Proposition 3: *Consider a lattice game $G = (S_i, U_i)_{i \in I}$ satisfying assumptions (A1) - (A4) of lemma 1 or assumptions (B1) - (B4) of lemma 2.*

Then the following two statements are equivalent:

(i) $G = (S_i, U_i)_{i \in I}$ is dominance-solvable.

(ii) There exists for all $s, t \in S$, with $s \neq t$, some $k \in \mathbb{N}$ dependent on s, t, such that $d\left(f^k(s), f^k(t)\right) < d(s,t)$.

Call a best response function f *T-contractive* if and only if f^T, with $T \in \mathbb{N}$, is a contractive mapping, that is, for all $s, t \in S$ with $s \neq t$, $d\left(f^T(s), f^T(t)\right) < d(s,t)$. For T-contractive f statement (ii) in proposition 2 is trivially satisfied since $d\left(f^T(s), f^T(t)\right) < d(s,t)$ for number $T \in \mathbb{N}$ being the same for all $s, t \in S$ with $s \neq t$. For real valued and continuously differentiable individual best response functions T-contractivity of f can be verified by properties of the partial derivatives of f^T whose values are easily computed, for all $s \in S$, via successive application of the chain-rule:

$$\begin{aligned} \frac{\partial f_i^1}{\partial s_j}(s) &= \frac{\partial f_i}{\partial s_j}(s) \\ \frac{\partial f_i^T}{\partial s_j}(s) &= \sum_{k \neq i} \frac{\partial f_i}{\partial s_k} \frac{\partial f_k^{T-1}}{\partial s_j}(s) \text{ for } T \geq 2 \end{aligned}$$

Corollary: *Consider a game $G = (S_i, U_i)_{i \in I}$ such that, for all $i \in I$,*

(C1) S_i is a non-empty, compact, and convex subset of $\mathbb{R}$.

(C2) f_i is continuously differentiable.

(C3a) U_i has either increasing or decreasing utility differences on $(S_{-i}, \leq_L)$ where $\leq_L$ denotes the natural order $\leq$ on $\mathbb{R}$.

or (C3b) U_i is quasiconcave on $(S_i, \leq_L)$ where $\leq_L$ denotes the natural order $\leq$ on $\mathbb{R}$.

Then $G = (S_i, U_i)_{i \in I}$ is dominance-solvable

if there exists a $T \geq 1$ such that, for all $i \in I$ and all $s \in S$,

$$\sum_{j \in I} \left| \frac{\partial f_i^T}{\partial s_j}(s) \right| < 1 \tag{4}$$

or if there exists a $T \geq 1$ such that, for all $j \in I$ and all $s \in S$,

$$\sum_{i \in I} \left| \frac{\partial f_i^T}{\partial s_j}(s) \right| < 1 \tag{5}$$

For the special case $T = 1$, Moulin (Theorem 4, 1984) already proves that an equivalent formulation of condition (4) - in terms of second-order partial derivatives of the utility function - implies dominance-solvability of nice games (which satisfy assumptions (C1), (C2), and (C3b) of the corollary).

If best response functions are monotonic and move, furthermore, for all players in the same direction, a simple characterization of dominance-solvability can be obtained which only refers to the smallest and largest strategy in strategy set S.

Proposition 4: *Consider a lattice game $G = (S_i, U_i)_{i \in I}$ satisfying assumptions (A1) - (A4) of lemma 1 or assumptions (B1) - (B4) of lemma 2. If either, for all $i \in I$, the individual best response function f_i is increasing or, for all $i \in I$, f_i is decreasing, then the following two statements are equivalent:*

(i) $G = (S_i, U_i)_{i \in I}$ is dominance-solvable.

(ii) $\lim_{k \to \infty} d\left(f^k(s), f^k(t)\right) = 0$, where s denotes the smallest and t denotes the largest element in S.

The proof of proposition 4 immediately implies the following restatement of a finding of Milgrom and Roberts who characterize dominance-solvability of supermodular games by uniqueness of the Nash equilibrium.

Observation 1 (Milgrom and Roberts, 1990)**:** *Consider a lattice game $G = (S_i, U_i)_{i \in I}$ satisfying the assumptions (A1) - (A4) of lemma 1. If, for all $i \in I$, the utility differences are increasing then there exists a smallest strategy $s \in D(G)$ and a largest strategy $t \in D(G)$ such that $f(s) = s$ and $f(t) = t$, i.e., s and t are Nash equilibria. Consequently, $G = (S_i, U_i)_{i \in I}$ is dominance-solvable if and only if $G = (S_i, U_i)_{i \in I}$ has a unique Nash equilibrium.*

Since a supermodular game exhibits increasing best response functions, observation 1 implies that statement (ii) of proposition 4 characterizes a unique Nash equilibrium for supermodular games that are also lattice games. In addition to supermodular games, as considered by Milgrom and Roberts (1990), proposition 4 also applies to games where all players have decreasing best response functions. The following observation 2, implied by the proofs of lemma 1 and proposition 4, demonstrates that the conclusion of observation 1 is not valid for lattice games that are not supermodular. Moreover, observation 2 shows why - in contrast to supermodular games - uniqueness of the Nash equilibrium does not necessarily imply dominance-solvability of games if players have decreasing utility differences (as observed by Bernheim (1984) for n-firm Cournot oligopolies).

Observation 2: *Consider a lattice game $G = (S_i, U_i)_{i \in I}$ satisfying the assumptions (A1) - (A4) of lemma 1. If, for all $i \in I$, the utility differences are decreasing then there exists a smallest strategy $s \in D(G)$ and a largest strategy $t \in D(G)$ such that $f(s) = t$, $f^2(s) = s$ and $f(t) = s$, $f^2(t) = t$, i.e., s and t are Nash equilibria of $G = (S_i, U_i)_{i \in I}$ if and only if $G = (S_i, U_i)_{i \in I}$ is dominance-solvable.*

Remark. In the light of the proof of proposition 4, statement (ii) of proposition 4 provides a sufficient and necessary condition for a unique point-rationalizable strategy in lattice games with monotonic best response functions. However, in contrast to the stronger assumption of decreasing utility differences, the assumption of monotonic best response functions is not sufficient to also guarantee dominance-solvability. To see this, consider the following game with payoff matrix

A \ B	$b1$	$b2$
$a1$	$2,0$	$2,3$
$a2$	$0,2$	$3,0$
$a3$	$3,3$	$0,2$

Let $a1 \leq_L a2 \leq_L a3$ and $b1 \leq_L b2$, and impose the discrete metric on S_A and on S_B, to obtain a lattice game, which exhibits decreasing best response functions. Strategy $(a3, b1)$ is the unique point-rationalizable strategy of this lattice game (e.g., statement (ii) of proposition 2 is satisfied for $k = 3$), however, all strategies of this game belong to the dominance solution. Note that there are neither monotonic utility differences nor order-quasiconcave utility functions.

6 Dominance-solvable n-Firm Cournot Oligopolies

For the classical n-firm Cournot oligopoly - presuming a linear inverse demand function, constant marginal costs, and products that are perfect substitutes - a unique Nash equilibrium exists for any number n of firms. However, Bernheim (1984) observes that any output-decision of a firm, ranging between zero and the monopoly-output, is a point-rationalizable strategy if there belong more than two firms to an n-firm Cournot oligopoly. In another line of research, the application of lattice theory to the analysis of strategic solutions of n-firm Cournot oligopolies is restricted (for an exception see Amir, 1996) to Cournot duopolies only, because typical n-firm Cournot oligopolies with more than two firms are not supermodular games (Milgrom and Roberts, 1990; Vives, 1990).

6.1 Relaxing the Perfect Substitute Assumption

This section introduces a simple model of an n-firm Cournot oligopoly where the perfect substitute assumption is weakened and, by an application of this paper's theoretical findings, conditions are identified which guarantee dominance-solvability. Contrary to Bernheim's (1984) observation, iterative solution concepts therefore re-gain maximal predictive power for n-firm Cournot oligopolies if the according conditions are satisfied. Moreover, the application of lattice theory proves fruitful to n-firm Cournot oligopolies that are not supermodular games.

Definition. *Call a game* $G = (S_i, U_i)_{i \in I}$ *an* n-firm Cournot oligopoly with imperfect substitutes *if, for all* $i \in I = \{1, ..n\}$, $S_i = [0, 1]$ *and*

$$U_i(s) = P_i(s) \cdot s_i - c_i s_i$$

such that, for all $i \in I$, $c_i \in (0, 1)$ *and, for all* $s \in S$,

$$P_i(s) = \max\left\{0, \left(1 - \sum_{j \neq i} \beta_{ij} s_j - s_i\right)\right\}$$

where, for all $j \neq i$, $\beta_{ij} \in [0, 1]$.

The individual strategy set S_i stands here for the possible output-decisions of firm $i \in I$. Function $P_i : S \to [0, 1]$ is interpreted as the inverse demand function for the product of firm $i \in I$ which determines, for a given market output, the maximal price firm $i \in I$ can charge pro-unit of its product. Furthermore, the number c_i denotes the constant pro-unit production costs of firm $i \in I$.

If $\beta_{ij} = 1$, for all $i \neq j$, the above definition of an n-firm Cournot oligopoly with imperfect substitutes coincides with the classical n-firm Cournot oligopoly as considered by Bernheim (1984). However, if, for some $i \neq j$, $\beta_{ij} \neq 1$ the product of firm j is not anylonger a *perfect substitute* for the product of firm i since one-unit output of firm j influences the residual demand for the product of firm i differently than one-unit output of firm i. Such an n-firm Cournot oligopoly could be interpreted as a model of oligopolistic competition on n different *home-markets* where each firm i suffers some negative externality - measured by β_{ij} - on its home-market from firm j's output.

Obviously, the smaller the externality weight β_{ij} the greater firm i's market-power on its home-market (in the extreme case $\beta_{ij} = 0$ for all $j \neq i$, implying that firm i has a monopoly on its home-market). Since firms rarely compete with products that are perfect substitutes the introduction of externality-weights has, in my opinion, realistic appeal.

Note that the n-firm Cournot oligopoly, as defined above, is a lattice game that satisfies all assumptions of proposition 2 if the lattice order $\leq_L$ is taken to be the standard order $\leq$ on $\mathbb{R}$. To see that the utility differences are decreasing verify that, for all $i \in I$ and for all $t_i \leq_L s_i$, $U_i(s_i, s_{-i}) - U_i(t_i, s_{-i})$ is non-increasing in s_{-i} if and only if

$$\left(\sum_{j \neq i} \beta_{ij} s_j\right) \cdot (t_i - s_i)$$

is non-increasing in s_{-i} - which is obviously satisfied since $(t_i - s_i)$ is non-positive. Basically an application of the corollary shows then that a sufficiently small impact of competitors on firms' home-markets assures dominance-solvability of n-firm Cournot oligopolies.

Proposition 5: *An n-firm Cournot oligopoly with imperfect substitutes is dominance-solvable if, for all $i \in I$,*

$$\sum_{j \neq i} \beta_{ij} < 2 \tag{6}$$

or if, for all $j \neq i$,

$$\sum_{i \in I} \beta_{ij} < 2 \tag{7}$$

In the case of the classical n-firm Cournot oligopoly, where products are perfect substitutes, i.e., for all $i, j \in I$, $\beta_{ij} = 1$, conditions (6) and (7) are violated for more than two firms. However, in the case of three firms condition (6) is already satisfied if, for all $i, j \in I$, $\beta_{ij} < 1$. Thus, Bernheim's (1984) observation that in an n-firm Cournot oligopoly with three firms every output-decision between zero and the monopoly output belongs to the dominance-solution, is not any longer valid if a marginal deviation from the perfect substitute assumption is considered.

6.2 Large Markets and Quadratic Cost Functions

Börgers and Janssen (1995) investigate dominance-solvability of an n-firm Cournot oligopoly under the assumption of *large markets* where an increase in the number of firms is matched by an increase in market size. In particular, let n be the number of firms then Börgers and Janssen speak of an *n-th Cournot game* if the inverse demand function is given by $P^n(s) = P(s/n)$ where P is the inverse demand function of the unreplicated game. Motivated by the fact that, with increasing n, an n-th Cournot game converges towards a perfectly competitive market whose Walrasian equilibrium may be approached by a so-called *cobweb-process*, Börgers and Janssen 1995 show that if the cobweb process is *strictly globally stable* (cf. Börgers and Janssen, 1995) an n-th Cournot game is dominance-solvable for sufficiently great n.

The following model of a *large market n-firm Cournot oligopoly* adopts Börgers and Janssen's (1995) definition of a large market to a *n-firm Cournot oligopoly* with linear inverse demand function where products are - as in the classical model - perfect substitutes. Under the assumption of quadratic cost functions, conditions on the market size, n, are identified which guarantee dominance-solvability.

Definition. *Call a game* $G = (S_i, U_i)_{i \in I}$ *a* large market n-firm Cournot oligopoly with quadratic cost function *if, for all* $i \in I = \{1, ..n\}$*,* $S_i = [0, 1]$ *and*

$$U_i(s) = P_i(s) \cdot s_i - c \cdot (s_i)^2$$

such that, for all $i \in I$*,* $c_i \in (0, 1)$ *and, for all* $s \in S$*,*

$$P_i(s) = \max\left\{0, \left(1 - \frac{1}{n}\sum_{j=1}^{n} s_j\right)\right\}$$

Presuming the standard order of the real numbers and, e.g., the metric induced by the absolute-value norm, the large market n-firm Cournot oligopoly with quadratic cost function is a lattice game satisfying assumption (A1) - supermodularity - and assumption (A2) - decreasing utility differences - of proposition 1.

Proposition 6:

If $c \geq \frac{1}{2}$ *then, for any* $n \in \mathbb{N}$*, a large market n-firm Cournot oligopoly with quadratic cost function is dominance-solvable .*

If $c < \frac{1}{2}$ *then the large market n-firm Cournot oligopoly with quadratic cost function is dominance-solvable if the market size* $n \in \mathbb{N}$ *satisfies*

$$n < \frac{1}{0.5 - c} \tag{8}$$

i.e., if $n \in \mathbb{N}$ *is sufficiently small.*

As the second statement of proposition 6 shows, an increase in market size increases the difficulty for satisfying condition (8). Therefore, rather the assumption of convenient cost functions than the large market assumption assures here for the n-firm Cournot oligopoly dominance-solvability.

7 Dominance-solvable Auctions with Optimistic - respectively Pessimistic - Bidders

Imagine that *pessimistic* bidders consider themselves as handicapped by the - unknown - allocation rule by believing that they have to make higher monetary bids than their competitors in order to win the good. Analogously, optimistic bidders think they are favored by the allocation rule so that they expect to win even if they bid less money than their competitors. It will be assumed that bidders strongly prefer to obtain the good with - subjectively believed - certainty whereas they strongly abhor to certainly not obtaining the good. Moreover, if bidders can not afford bids as high as to win the auction with certainty they still offer the highest bid possible in order to have maximal - as subjectively perceived - chances of winning. Instead of providing any axiomatic foundation, which would give rise to such decision making under uncertainty, I simply presume utility functions that are consistent with the described behavior.

In particular, each bidder $i \in I$ offers a monetary amount $s_i \in [0,1]$ - her *bid* - which she has to pay if she wins the good, and each bidder' utility - measured in monetary units - of winning the good is given by $W_i > 1$. Each bidder presumes she has no chance of winning if she bids less than the threshold amount $\underline{b} > 0$ - believed to be the auction-organizer's reservation prize. A pessimistic bidder believes with certainty that she obtains the good if her bid meets at least the maximal bid of her opponents, denoted $\max s_{-i}$, times some pessimism-factor $\gamma > 1$. Similarly, an optimistic bidder believes with certainty that she obtains the good if her bid at least equals the maximal bid of her opponents times some optimism-factor $\gamma < 1$.

Definition. *Consider a game $G = (S_i, U_i)_{i \in I}$ such that, for all $i \in I$, $S_i = [0,1]$ and*

$$
\begin{array}{ll}
U_i(s_i, s_{-i}) = W_i - s_i & \text{if } \max\{\underline{b}, \gamma \cdot \max s_{-i}\} \leq s_i \\
U_i(s_i, s_{-i}) = W_i - s_i & \text{if } s_i = 1 \text{ and } 1 < \gamma \cdot \max s_{-i} \\
U_i(s_i, s_{-i}) = 0 & \text{if } s_i < 1 \text{ and } 1 < \gamma \cdot \max s_{-i} \\
U_i(s_i, s_{-i}) = 0 & \text{if } s_i < \max\{\underline{b}, \gamma \cdot \max s_{-i}\}
\end{array}
$$

where $\underline{b} \in (0,1)$ and $\gamma \in \mathbb{R}_+$.

Call $G = (S_i, U_i)_{i \in I}$ *a* simple auction with optimistic bidders *if* $\gamma < 1$.
Call $G = (S_i, U_i)_{i \in I}$ *a* simple auction with pessimistic bidders *if* $1 < \gamma$.

For example, in a simple two-player auction a pessimistic bidder with pessimism-factor $\gamma = 1.2$ wants to offer 20 percent more than her competitor if this bid exceeds $\underline{b}$. However, if, due to budget constraints, this pessimistic bidder can not afford to offer 20 percent more than her competitor she bids the maximal amount allowed by her budget constraints to maintain a "maximal chance" of winning.

In general, the individual best response functions in simple auctions with optimistic - respectively pessimistic - bidders are given as follows, for all $i \in I$,

$$\begin{array}{ll} f_i(s_{-i}) = 1 & \text{if } 1 < \gamma \cdot \max s_{-i} \\ f_i(s_{-i}) = \gamma \cdot \max s_{-i} & \text{if } \underline{b} \leq \gamma \cdot \max s_{-i} \leq 1 \\ f_i(s_{-i}) = \underline{b} & \text{if } \gamma \cdot \max s_{-i} < \underline{b} \end{array}$$

Presuming the standard order of the real numbers, these best response functions are increasing and order-continuous. However, despite increasing best response functions, simple auctions with optimistic - respectively pessimistic - bidders are not supermodular games since utility differences are not increasing. The intuition for bids being only partial strategic complementarities is straightforward: A bidder gains utility by increasing her bid as long as this bidding is decisive for winning the good, but if the high bid is not anylonger decisive it diminishes utility since then high bidding increases the amount to pay in the case of winning. Although the assumptions of lemma 1 are therefore not satisfied, simple auctions with optimistic - respectively pessimistic - bidders can be described as lattice games satisfying the assumptions of lemma 2 since utility functions are order-quasiconcave.

For the above auction environment, it is intuitively clear that pessimistic bidders rather tend to higher bids than optimistic bidders. The contribution of proposition 7 is to show, that the strategic logic of dominance-solvability brings this tendency to the extreme: whereas pessimistic bidders go to their budgetary limit for winning the good, optimistic bidders just bid the reservation price.

Proposition 7:

A simple auction with pessimistic bidders is dominance-solvable and the dominance solution, $\{s^*\} = D(G)$, *is given by*

$$s^* = (1, ..., 1)$$

A simple auction with optimistic bidders is dominance-solvable and the dominance solution, $\{s^*\} = D(G)$*, is given by*

$$s^* = (\underline{b}, ..., \underline{b})$$

8 A Dominance-solvable Two-Player Model of Bank-Runs

This section introduces a simple Two-player Bayesian model of bank runs. Both investors privately observe a *signal* - independently drawn from a uniform distribution over $[0, 1]$ - that determines their *types* before they either decide to WITHDRAW or to NOT WITHDRAW money from the bank. Instead of offering a particular interpretation of these types - e.g., a measure of the investment project's success (as in Goldstein and Pauzner, 2002) or as the investor's preferences for intertemporal consumption (e.g., the investor's *life span* in Postlewaite and Vives, 1987) - I simply presume, in accordance with the literature, that for any given action of her opponent the utility of NOT WITHDRAWING increases with the investor's type. Another stylized fact of bank run models is captured by the assumption that the utility of NOT WITHDRAWING is higher[6] when the opponent also chooses NOT WITHDRAWING than when she chooses WITHDRAWING.

Given some number $r \in (0, 1)$, consider a symmetric Two-player Bayesian game where player A's payoffs, for realized type $\theta_A \in \Theta_A = [0, 1]$, depend on opponent B's decision as follows:

B

A

	NOT WITHDRAW	WITHDRAW
NOT WITHDRAW	$2 \cdot \theta_A$	θ_A
WITHDRAW	1	r

Thus, whenever A receives signal $\theta_A > \max\{0.5, r\}$ her strictly dominant action is to NOT WITHDRAW. Conversely, for signals $\theta_A < \min\{0.5, r\}$ to WITHDRAW strictly dominates to NOT WITHDRAW. For remaining signals, i.e., $\min\{0.5, r\} \leq \theta_A \leq \max\{0.5, r\}$, a coordination problem arises since then a player's optimal action depends on the action of her opponent. For example, if $0.5 \leq \theta_A \leq r$ then A prefers to WITHDRAW when B

[6] Here simply to be taken as *twice* as high.

WITHDRAWS whereas A prefers NOT WITHDRAW when B chooses to NOT WITHDRAW. If instead $r \leq \theta_A \leq 0.5$ then A prefers to WITHDRAW when B does NOT WITHDRAW, and she prefers to NOT WITHDRAW when B WITHDRAWS.

For $i \in \{A, B\}$, define a *switching strategy* s_i as a map

$$s_i : [0,1] \rightarrow \{\text{WITHDRAW}, \text{NOT WITHDRAW}\}$$

such that, for some *cutoff-point* $\sigma_i \in [0,1]$,

$$s_i(x) = \begin{cases} \text{WITHDRAW} & \text{if } x \leq \sigma_i \\ \text{NOT WITHDRAW} & \text{if } \sigma_i < x \end{cases}$$

Denote by S_i the set of all switching strategies of player i, and observe that every switching strategy $s_i \in S_i$ is completely characterized by its cutoff-point $\sigma_i \in [0,1]$. The assumption of a uniform-distribution implies that if player A expects B to choose switching strategy s_B with cutoff-point σ_B, then A expects B to WITHDRAW with probability σ_B.

Suppose that players are risk-neutral expected utility maximizers. Then the above payoff-specification implies for A's type-dependent utility function $U_A : \Theta_A \times S_A \times S_B \rightarrow \mathbb{R}_+$ that

$$U_A(\theta_A, s_A, s_B) = \begin{cases} 2 \cdot \theta_A \cdot (1 - \sigma_B) + \theta_A \cdot \sigma_B & \text{if } \sigma_A \leq \theta_A \\ 1 \cdot (1 - \sigma_B) + r \cdot \sigma_B & \text{if } \theta_A < \sigma_A \end{cases} \tag{9}$$

where σ_A (σ_B) denotes the cutoff-point of strategy s_A (s_B). Moreover, the ex-ante expected utility - before learning her type - of player A is given by

$$\begin{aligned} U_A(s_A, s_B) &= \int_0^1 U_A(\theta_A, s_A, s_B)\, d\theta_A \qquad (10) \\ &= \int_0^{\sigma_A} 1 \cdot (1 - \sigma_B) + r \cdot \sigma_B d\theta_A \\ &\quad + \int_{\sigma_A}^1 2 \cdot \theta_A \cdot (1 - \sigma_B) + \theta_A \cdot \sigma_B d\theta_A \\ &= 1 \cdot (1 - \sigma_B) \cdot \sigma_A + r \cdot \sigma_B \cdot \sigma_A \\ &\quad + (1 - \sigma_B) \cdot \left(1 - \sigma_A^2\right) + \frac{1}{2} \cdot \sigma_B \cdot \left(1 - \sigma_A^2\right) \end{aligned}$$

Definition. *Call a game $G = (S_i, U_i)_{i \in I}$ a* Two-Player Bayesian model of bank runs *if, for all $i \in \{A, B\}$, S_i is the set of switching strategies as defined by (9), and U_i is the ex-ante expected utility of player i as defined by (10).*

For all $i \in \{A, B\}$, let $t_i \leq_L s_i$ if their cutoff-points satisfy $\tau_i \leq \sigma_i$, and endow S_i with the metric $d(s_i, t_i) = |\sigma_i - \tau_i|$, thereby describing any *Two-Player Bayesian model of bank runs* as a lattice game with totally ordered individual strategy sets. Presume $t_A \leq_L s_A$ and calculate as utility differences

$$U_A(s_A, s_B) - U_A(t_A, s_B) = [1 - \sigma_B + r \cdot \sigma_B] \cdot (\sigma_A - \tau_A) - \left(1 + \frac{1}{2} \cdot \sigma_B\right) \cdot \left(\sigma_A^2 - \tau_A^2\right)$$

implying

$$\begin{aligned} \frac{\partial}{\partial \sigma_B} [U_A(s_A, s_B) - U_A(t_A, s_B)] &\leq 0 \Leftrightarrow \\ r + \frac{1}{2} \cdot (\sigma_A + \tau_A) &\leq 1 \end{aligned}$$

Thus, for every $r \in (0, 1)$, there exist, on the one hand, some large $t_A, s_A \in S_A$ with $t_A \leq_L s_A$ such that A's utility differences are increasing in s_B, while there also exist, on the other hand, some small $t_A, s_A \in S_A$ with $t_A \leq_L s_A$ such that A's utility differences are decreasing in s_B. Switching strategies in any *Two-Player Bayesian model of bank runs* are therefore only partial strategic complementarities; a situation that is typical for bank-run models (compare, e.g., Goldstein and Pauzner, 2002).

While *Two-Player Bayesian models of bank runs* do not admit monotonic utility differences, they imply order-quasiconcave utility functions. To see this, note that

$$\begin{aligned} \frac{\partial U_A(s_A, s_B)}{\partial \sigma_A} &= (1 - \sigma_B) + r \cdot \sigma_B - 2\sigma_A + \sigma_B \cdot \sigma_A \leq 0 \Leftrightarrow \\ \frac{(1 - \sigma_B) + r \cdot \sigma_B}{2 - \sigma_B} &\leq \sigma_A \end{aligned}$$

i.e., for given $r \in (0, 1)$ and $\sigma_B \in [0, 1]$, U_A increases in lattice-order in s_A until cutoff-point

$$\sigma_A^* = \frac{(1 - \sigma_B) + r \cdot \sigma_B}{2 - \sigma_B} \tag{11}$$

while it decreases afterwards.

Proposition 8: *For any $r \in (0, 1)$, a Two-Player Bayesian model of bank runs is dominance-solvable and the dominance solution, $\{s^*\} = D(G)$, is characterized by the cutoff-points*

$$\sigma_i^* = \frac{3}{2} - \frac{1}{2} \cdot r - \frac{1}{2} \cdot \sqrt{(5 - 6 \cdot r + r^2)}$$

for $i \in \{A, B\}$. As the likelihood of bank runs where both investors withdraw therefore obtains

$$\sigma_A^* \cdot \sigma_B^* = \frac{1}{4}\left[-3 + r + \sqrt{(5 - 6 \cdot r + r^2)}\right]^2$$

which is, of course, increasing in r.

9 Appendix

Proof of the lemma:

Step 1. I start by proving, for all $k \geq 0$,

$$\sup \lambda^k (S) = \sup \theta^k (S) \tag{12}$$

$$\inf \lambda^k (S) = \inf \theta^k (S) \tag{13}$$

At first verify that $\sup \lambda^k (S)$ and $\inf \lambda^k (S)$ exist for all $k \geq 0$ since, by completeness of $(S, \leq_L)$, there must exist a largest and a smallest element of S, i.e., $\sup \lambda^0 (S)$ and $\inf \lambda^0 (S)$. Presume that $\sup \lambda^{k-1} (S)$ and $\inf \lambda^{k-1} (S)$ exist and observe that

$$\begin{aligned}
\sup \lambda_i^k (S) &= f_i \left(\inf \lambda_{-i}^{k-1} (S)\right) \\
\inf \lambda_i^k (S) &= f_i \left(\sup \lambda_{-i}^{k-1} (S)\right)
\end{aligned}$$

if U_i has decreasing differences; and

$$\sup \lambda_i^k (S) = f_i \left(\sup \lambda_{-i}^{k-1} (S)\right) \tag{14}$$

$$\inf \lambda_i^k (S) = f_i \left(\inf \lambda_{-i}^{k-1} (S)\right) \tag{15}$$

if U_i has increasing differences. Thus, by induction, monotonic individual best response functions imply existence of $\sup \lambda^k (S)$ and $\inf \lambda^k (S)$ for all $k \geq 1$.

Now suppose player $i \in I$ has decreasing utility differences on $(S_{-i}, \leq_L)$, (for players with increasing utility differences compare the proof of lemma 1 in Milgrom and Roberts, 1990). Given an interval $[s_{-i}, t_{-i}]$ such that $s_{-i} \leq_L t_{-i}$, let $\theta_i [s_{-i}, t_{-i}]$ denote the set of undominated strategies and let $\lambda_i [s_{-i}, t_{-i}]$ denote the set of best responses to elements in $[s_{-i}, t_{-i}]$. Simply write $\hat{s}_i$ for $\sup f_i (s_{-i})$ and $\check{s}_i$ for $\inf f_i (t_{-i})$ where existence of $\sup f_i (s_{-i})$ and $\inf f_i (t_{-i})$ is assured by (14) and (15). Observe that any r_i with $r_i \nleq_L \hat{s}_i$ is strongly dominated by the strategy $\hat{s}_i \wedge r_i$ since, for all $x_{-i} \in [s_{-i}, t_{-i}]$,

$$\begin{aligned}
U_i (r_i, x_{-i}) - U_i (\hat{s}_i \wedge r_i, x_{-i}) &\leq U_i (r_i, s_{-i}) - U_i (\hat{s}_i \wedge r_i, s_{-i}) \text{ by (A2)} \\
&\leq U_i (\hat{s}_i \vee r_i, s_{-i}) - U_i (\hat{s}_i, s_{-i}) \text{ by (A1)} \\
&< 0
\end{aligned}$$

where the last inequality results from $\hat{s}_i \in f_i (s_{-i})$ and $\hat{s}_i <_L \hat{s}_i \vee r_i$, i.e., $\hat{s}_i \vee r_i \notin f_i (s_{-i})$. This proves $r_i \leq_L \hat{s}_i$ for any $r_i \in \theta_i [s_{-i}, t_{-i}]$. Accordingly, it can be shown that any strategy r_i with $\check{s}_i \nleq_L r_i$ is dominated by a strategy $\check{s}_i \vee r_i$. Consequently, $\check{s}_i \leq_L r_i$ for any $r_i \in \theta_i [s_{-i}, t_{-i}]$. The set-inclusion $\lambda_i [s_{-i}, t_{-i}] \subset \theta_i [s_{-i}, t_{-i}]$ then implies $\sup \lambda_i [s_{-i}, t_{-i}] = \sup \theta_i [s_{-i}, t_{-i}]$ and $\inf \lambda_i [s_{-i}, t_{-i}] = \inf \theta_i [s_{-i}, t_{-i}]$ for any interval $[s_{-i}, t_{-i}]$ with $s_{-i} \leq_L t_{-i}$.

Step 2. It remains to prove that the equations (12) and (13) indeed entail (1) and (2).

By set-inclusion, the sequences $\left\{\inf \theta^k (S)\right\}_{k\geq 0}$, $\left\{\sup \theta^k (S)\right\}_{k\geq 0}$ are monotonically increasing, respectively decreasing. Completeness of S implies then existence of order-limits such that

$$\lim_{k\to\infty} \inf \theta^k (S) = \lim_{k\to\infty} \inf \lambda^k (S) = \check{s}$$
$$\lim_{k\to\infty} \sup \theta^k (S) = \lim_{k\to\infty} \sup \lambda^k (S) = \hat{s}$$

by (13) and (12). Moreover, by definition of $P(G)$ and $D(G)$

$$P(G) \subset D(G) \subset [\check{s}, \hat{s}]$$

and, therefore, $\check{s}, \hat{s} \in P(G)$ would prove the claim. I proceed by showing that all $\hat{s}_i$ are either best responses to $\hat{s}_{-i}$ or to $\check{s}_{-i}$ and that all $\check{s}_i$ are either best responses to $\hat{s}_{-i}$ or to $\check{s}_{-i}$, thereby implying $\check{s}, \hat{s} \in P(G)$.

By order-continuity of f

$$\lim_{k\to\infty} f\left(\inf \lambda^k (S)\right) = f(\check{s}), \quad \lim_{k\to\infty} f\left(\sup \lambda^k (S)\right) = f(\hat{s})$$

i.e., for all $i \in I$,

$$\lim_{k\to\infty} f_i\left(\inf \lambda^k_{-i} (S)\right) = f_i(\check{s}_{-i}), \quad \lim_{k\to\infty} f_i\left(\sup \lambda^k_{-i} (S)\right) = f_i(\hat{s}_{-i})$$

Moreover, if player $i \in I$ has decreasing utility differences

$$\lim_{k\to\infty} f_i\left(\inf \lambda^k_{-i} (S)\right) = \lim_{k\to\infty} \sup \lambda^{k+1}_i (S) = \hat{s}_i$$
$$\lim_{k\to\infty} f_i\left(\sup \lambda^k_{-i} (S)\right) = \lim_{k\to\infty} \inf \lambda^{k+1}_i (S) = \check{s}_i$$

implying $f_i(\check{s}_{-i}) = \hat{s}_i$ and $f_i(\hat{s}_{-i}) = \check{s}_i$, i.e., $\hat{s}_i$ ($\check{s}_i$) is a best response to $\check{s}_{-i}$ ($\hat{s}_{-i}$).

Similarly, if player $i \in I$ has increasing utility differences

$$\lim_{k\to\infty} f_i\left(\inf \lambda^k_{-i} (S)\right) = \lim_{k\to\infty} \inf \lambda^{k+1}_i (S) = \check{s}_i$$
$$\lim_{k\to\infty} f_i\left(\sup \lambda^k_{-i} (S)\right) = \lim_{k\to\infty} \sup \lambda^{k+1}_i (S) = \hat{s}_i$$

That is, $\hat{s}_i$ ($\check{s}_i$) is a best response to $\hat{s}_{-i}$ ($\check{s}_{-i}$).$\square$

Proof of proposition 2:

Proceed as in the proof of lemma 1 to see that $\sup \lambda^k (S)$ and $\inf \lambda^k (S)$ exist for all $k \geq 0$. Moreover, since

$$\sup \lambda^0 (S) = \sup \theta^0 (S)$$
$$\inf \lambda^0 (S) = \inf \theta^0 (S)$$

presume

$$\sup \lambda^{k-1} (S) = \sup \theta^{k-1} (S)$$
$$\inf \lambda^{k-1} (S) = \inf \theta^{k-1} (S)$$

and proceed by proving, for an arbitrary $i \in I$,

$$\sup \lambda_i^k(S) = \sup \theta_i^k(S) \tag{16}$$
$$\inf \lambda_i^k(S) = \inf \theta_i^k(S) \tag{17}$$

Abbreviate $\hat{s}_i = \sup \lambda_i^k(S)$ and $\check{s}_i = \inf \lambda_i^k(S)$. Since any $r_i \in S_i$, satisfying $r_i <_L \check{s}_i$ or $\hat{s}_i <_L r_i$, is, by assumption, no best response to any strategy $s_{-i} \in \lambda_{-i}^{k-1}(S)$ there exists for every $s_{-i} \in \lambda_{-i}^{k-1}(S)$ some $s_i' \in \lambda_i^k(S)$ such that

$$U_i(s_i', s_{-i}) > U_i(r_i, s_{-i})$$

By assumption, S_i, and therefore $\lambda_i^k(S)$, is a chain, i.e., $\check{s}_i \leq_L s_i' \leq_L \hat{s}_i$ for all $s_i' \in \lambda_i^k(S)$. Order-quasiconcavity then implies, for all $s_{-i} \in \lambda_{-i}^{k-1}(S)$,

$$U_i(\hat{s}_i, s_{-i}) > U_i(r_i, s_{-i})$$

if $\hat{s}_i <_L r_i$, and

$$U_i(\check{s}_i, s_{-i}) > U_i(r_i, s_{-i})$$

if $r_i <_L \check{s}_i$. Thus, any $r_i \in S_i$, satisfying $r_i <_L \check{s}_i$ ($\hat{s}_i <_L r_i$) is strongly dominated by $\check{s}_i$ ($\hat{s}_i$), which proves (16) and (17).

By set-inclusion, the sequences $\left\{\inf \theta^k(S)\right\}_{k\geq 0}$, $\left\{\sup \theta^k(S)\right\}_{k\geq 0}$ are monotonically increasing, respectively decreasing. Completeness of S implies existence of order-limits such that

$$\lim_{k\to\infty} \inf \theta^k(S) = \lim_{k\to\infty} \inf \lambda^k(S) = \check{s}$$
$$\lim_{k\to\infty} \sup \theta^k(S) = \lim_{k\to\infty} \sup \lambda^k(S) = \hat{s}$$

Since, by assumption, $\check{s} = \hat{s}$

$$P(G) \subset D(G) \subset \{\check{s}\}$$

Finally, observe that $P(G)$ is non-empty, i.e., $P(G) = D(G) = \{\check{s}\}$, because order-continuity of f implies $f(\check{s}) = \check{s}$.$\square$

Proof of proposition 3: First I demonstrate that statement (ii) characterizes uniqueness of point-rationalizable strategies for lattice games with order-continuous best response function.

(ii) implies (i)**.** By lattice-completeness of S and order-continuity of f there exist for all $k \geq 1$ strategies $s^k, t^k \in \lambda^k(S)$ such that, for all $s' \in \lambda^k(S)$, $s^k \leq_L s' \leq_L t^k$. Thus, by condition (3), for all $k \geq 0$, $diam\left(\lambda^k(S)\right) = d\left(s^k, t^k\right)$. Observe that, by set-inclusion, $\left(t^k\right)_{k\geq 1}$ is a monotonically decreasing sequence bounded from below, and $\left(s^k\right)_{k\geq 1}$ is a monotonically increasing sequence bounded from above. Since S is order-complete the order-limits $t^* = \inf t^k$ and $s^* = \sup s^k$ exist and, by condition (3), $diam(P(G)) = d(s^*, t^*)$. However, since, for all $k \geq 1$, f^k is order-continuous it is also true that $diam(P(G)) = d\left(f^k(s^*), f^k(t^*)\right)$ for any $k \geq 1$. Consequently, $P(G)$ must be single-valued if there exists for every pair of strategies $s \neq t$ some finite k such that $d\left(f^k(s), f^k(t)\right) < d(s, t)$.

(i) implies (ii). Suppose, on the contrary, that statement (ii) is violated such that there exist some $s', t' \in S$, with $s' \neq t'$, and $\lim_{k\to\infty} d\left(s'^k, t'^k\right) > 0$. But then, by set-inclusion,

$$diam(P(G)) = d(s^*, t^*) \geq \lim_{k\to\infty} d\left(s'^k, t'^k\right) > 0$$

implying $s^* \neq t^*$. Thus, $P(G)$ is not single-valued.

Finally note, that the assumptions of proposition 3 assure, by proposition 1, that $G = (S_i, U_i)_{i\in I}$ is dominance-solvable if and only if $G = (S_i, U_i)_{i\in I}$ has a unique point-rationalizable strategy. $\square$

Proof of the corollary:

Part A. Let $g_i(\lambda) = f_i^T(\lambda(s-t)+t)$, and observe that $g_i(\lambda)$ is continuously differentiable on $[0,1]$. The mean-value inequality for real-valued functions with a real-valued domain implies

$$|g_i(1) - g_i(0)| \leq \left|\frac{\partial g_i}{\partial \lambda}(\lambda^*)\right| \cdot |1-0| \tag{18}$$

for some λ^* such that $\lambda^* = \arg\max_{[0,1]} \left|\frac{\partial g_i}{\partial \lambda}(\lambda)\right|$. By an application of the chain-rule:

$$\begin{aligned}
\frac{\partial g_i}{\partial \lambda}(\lambda^*) &= \sum_{j\in I} \frac{\partial f_i^T}{\partial s_j}(\lambda^*(s_j-t_j)+t_j)\cdot(s_j-t_j) \\
\left|\frac{\partial g_i}{\partial \lambda}(\lambda^*)\right| &\leq \left|\sum_{j\in I} \frac{\partial f_i^T}{\partial s_j}(\lambda^*(s_j-t_j)+t_j)\right| \cdot \|s-t\|_\infty
\end{aligned}$$

Substituting for the terms in inequality (18):

$$\left|f_i^T(s) - f_i^T(t)\right| \leq \left|\sum_{j\in I} \frac{\partial f_i^T}{\partial s_j}(r)\right| \cdot \|s-t\|_\infty$$

with $r = \lambda^*(s-t)+t$. Since this is true by assumption for all $i \in I$ we obtain for the supremum norm

$$\left\|f^T(s) - f^T(t)\right\|_\infty \leq \left|\sum_{j\in I} \frac{\partial f_i^T}{\partial s_j}(r)\right| \cdot \|s-t\|_\infty$$

Consequently, condition (4) of the corollary implies T-contraction of f in the supremum norm.

Note that *T-contraction*, and not just *T-contractivity*, is satisfied because $\sum_{j\in I}\left|\frac{\partial f_i^T}{\partial s_j}(s)\right|$ is a continuous function obtaining a maximum on the compact set S. Consequently, if $\sum_{j\in I}\left|\frac{\partial f_i^T}{\partial s_j}(s)\right| < 1$ for all i and all $s \in S$ then there exists some $c < 1$ such that $\sum_{j\in I}\left|\frac{\partial f_i^T}{\partial s_j}(s)\right| \leq c$ for all i.

Part B. Let again $g_i(\lambda) = f_i^T(\lambda(s-t)+t)$, and observe that the mean-value inequality implies

$$|g_i(1) - g_i(0)| \leq \left|\frac{\partial g_i}{\partial \lambda}(\lambda^i)\right| \cdot |1-0|$$

for some $\lambda^i = \arg\max_{[0,1]} \left|\frac{\partial g_i}{\partial \lambda}(\lambda)\right|$. By the chain-rule and substitution

$$\left|f_i^T(s) - f_i^T(t)\right| \leq \left|\sum_{j\in I} \frac{\partial f_i^T}{\partial s_j}(r^i)\cdot(s_j-t_j)\right|$$

with $r^i = \lambda^i s + (1-\lambda^i)t$. Summing up over all i and rearranging

$$\begin{aligned}
\sum_{i\in I}\left|f_i^T(s) - f_i^T(t)\right| &\leq \sum_{i\in I}\left|\sum_{j\in I}\left(\frac{\partial f_i^T}{\partial s_j}(r^i)\right)\cdot(s_j-t_j)\right| \\
\sum_{i\in I}\left|f_i^T(s) - f_i^T(t)\right| &\leq \max_{j\in I}\left\{\left|\sum_{i\in I}\left(\frac{\partial f_i^T}{\partial s_j}(r^i)\right)\right|\right\}\cdot\sum_{j\in I}|s_j-t_j| \\
\left\|f^T(s) - f^T(t)\right\|_1 &< \|s-t\|_1
\end{aligned}$$

Where the last step follows from the assumption, for all j and all $r^i \in \{\lambda s + (1-\lambda)t \mid \lambda \in [0,1]\}$,

$$\sum_{i \in I} \left(\frac{\partial f_i^T}{\partial s_j} \left(r^i\right) \right) < 1$$

Consequently, $d\left(f^T(s), f^T(t)\right) < d(s,t)$ with d induced by the absolute value norm. Thus, condition (5) of the corollary implies T-contraction in the absolute value norm.$\square$

Proof of proposition 4: By the proof of proposition 3 the point-rationalizable strategy of $G = (S_i, U_i)_{i \in I}$ is unique if and only if $\lim_{k \to \infty} d\left(s^k, t^k\right) = 0$ where, for all $k \geq 1$, $s^k, t^k \in \lambda^k(S)$ such that, for all $s' \in \lambda^k(S)$, $s^k \leq_L s' \leq_L t^k$.

If all players best response functions are increasing then, for all $k \geq 1$, $s^k = f\left(s^{k-1}\right)$ and $t^k = f\left(t^{k-1}\right)$. Consequently, $\lim_{k \to \infty} d\left(s^k, t^k\right) = 0$ if and only if $\lim_{k \to \infty} d\left(f^k(s), f^k(t)\right) = 0$, where s denotes the smallest and t denotes the largest element in S.

Analogously, if all players have decreasing best response functions then, for all $k \geq 1$, $s^k = f\left(t^{k-1}\right)$ and $t^k = f\left(s^{k-1}\right)$. Thus, for $k' = 1, 3, 5, ...$, $\lim_{k' \to \infty} d\left(s^{k'}, t^{k'}\right) = 0$ if and only if

$$\lim_{k' \to \infty} d\left(f^{k'}(s), f^{k'}(t)\right) = 0$$

which is, by set-inclusion, equivalent to $\lim_{k \to \infty} d\left(s^k, t^k\right) = 0$.$\square$

Proof of proposition 5: The individual best response functions of the n-firm Cournot oligopoly are given, for all $i \in I$, by

$$f_i(s_{-i}) = \max\left\{0, \frac{1 - \sum_{j \neq i} \beta_{ij} s_j - c_i}{2}\right\}$$

Since the individual best response functions f_i are not differentiable everywhere - they have a *kink* at strategies s_{-i} where the interior and the boundary solutions of the utility maximization problem coincide - the corollary is not immediately applicable to f. Consider therefore the functions $h_i : \mathbb{R}^{n-1} \to \mathbb{R}$ such that, for all $i \in I$,

$$h_i(s_{-i}) = \frac{1 - \sum_{j \neq i} \beta_{ij} s_j - c_i}{2}$$

Thus, while the values of f_i must not be smaller than zero, h_i is not restricted to non-negative values. Since T-contractivity of the maximizers $h = \times_{i \in I} h_i$ with domain $\mathbb{R}$ entails T-contractivity of the maximizers f with domain $\mathbb{R}_+$, dominance-solvability of $G = (S_i, U_i)_{i \in I}$ is proved by showing that h satisfies condition (4) or condition (5) of the corollary. Note that this is, already for $T = 1$, the case if condition (6) or condition (7) of proposition 5 are satisfied.$\square$

Proof of proposition 6: As individual best response functions obtain, for all $i \in I$,

$$f_i(s_{-i}) = \max\left\{0, \left(1 - \frac{1}{n} \sum_{j \neq i} s_j\right) \cdot \frac{n}{2(n \cdot c + 1)}\right\}$$

Proceeding as in the proof of proposition 4 yields, for all $j \neq i$,

$$\frac{\partial h_i(s_{-i})}{\partial s_j} = \left|\frac{-1}{2(n \cdot c + 1)}\right|$$

Thus, condition (4) is satisfied if

$$\left|\frac{-1}{2\left(n\cdot c+1\right)}\right|<\frac{1}{n}$$

Rearrangement gives the two statements of proposition 6.□

Proof of proposition 7:
Since the individual best response functions are increasing it suffices, by proposition 4, to show

$$\lim_{k\to\infty} f^{k}\left(0,...,0\right) \;=\lim_{k\to\infty} f^{k}\left(1,...,1\right)$$

Part A. Pessimistic bidders, i.e., $\gamma>1$
Obviously, $f\left(1,...,1\right)=\left(1,...,1\right)$ implying the desired result

$$\lim_{k\to\infty} f^{k}\left(1,...,1\right)=\left(1,...,1\right)$$

Turn to $s=\left(0,...,0\right)$ and observe that

$$\begin{aligned}\lim_{k\to\infty}\gamma^{k}\cdot\left(\underline{b},...,\underline{b}\right) &= \lim_{k\to\infty} f^{k+1}\left(0,...,0\right) \text{ if } \lim_{k\to\infty}\gamma^{k}\cdot\left(\underline{b},...,\underline{b}\right)\leq\left(1,...,1\right)\\ \lim_{k\to\infty} f^{k+1}\left(0,...,0\right) &= \left(1,...,1\right) \text{ else}\end{aligned}$$

Since, by assumption, $0<\underline{b}$ and $1<\gamma$, the desired result obtains:

$$\lim_{k\to\infty} f^{k}\left(0,..,0\right)=\left(1,...,1\right)$$

Part B. Optimistic bidders, i.e., $\gamma<1$
First consider $s=\left(0,...,0\right)$. Then $f\left(0,...,0\right)=\left(\underline{b},...,\underline{b}\right)$ and $f\left(\underline{b},...,\underline{b}\right)=\left(\underline{b},...,\underline{b}\right)$ imply

$$\lim_{k\to\infty} f^{k}\left(0,...,0\right)=\left(\underline{b},...,\underline{b}\right)$$

. Turn to $s=\left(1,...,1\right)$ and observe that, by $\gamma<1$, there exist some finite number $M\in\mathbb{N}$ such that

$$f^{M}\left(1,...,1\right)=\gamma^{M}\cdot\left(1,...,1\right)<\left(\underline{b},...,\underline{b}\right)$$

Thus,

$$f^{M+1}\left(1,...,1\right)=\left(\underline{b},...,\underline{b}\right)$$

implying

$$\lim_{k\to\infty} f^{k}\left(1,...,1\right)=\left(\underline{b},...,\underline{b}\right)$$

which proves the proposition.□

Proof of proposition 8:
By equation (11), the best response function with respect to cutoff-points is given by

$$f_{A}\left(s_{B}\right)=\frac{\left(1-\sigma_{B}\right)+r\cdot\sigma_{B}}{2-\sigma_{B}} \tag{19}$$

which could have been alternatively obtained from rearranging for the indifferent - the cutoff-point - type θ_A^*:

$$\begin{aligned} f_A\left(s_B\right) &= \theta_A^* \text{ such that} \\ \sigma_B \cdot r + (1-\sigma_B) \cdot 1 &= \sigma_B \cdot \theta_A^* + (1-\sigma_B) \cdot \theta_A^* \cdot 2 \end{aligned}$$

In the light of the corollary, uniqueness of the point-rationalizable strategies is guaranteed if the first order derivative of A's best response function with respect to B's cutoff-points, i.e.,

$$\frac{df_A}{d\sigma_B}\left(s_B\right) = \frac{2r-1}{\left(s_B-2\right)^2}$$

is smaller than one. That is, for all $\sigma_B \in [0,1]$,

$$\begin{aligned} |2r-1| &< \left(\sigma_B-2\right)^2 \\ |2r-1| &< 1 < \sigma_B^2 - 4\sigma_B + 4 \end{aligned}$$

whereby the second inequality is satisfied for all $r \in (0,1)$. Since the assumptions of proposition 2 are fulfilled, the unique dominance-solvable strategy is given by the fixed point of (19) in $[0,1]$.$\square$

10 References

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