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Contest Architecture

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Abstract

A contest architecture specifies how the prize sum is split among several prizes, and how the contestants (who are here privately informed about their abilities) are split among several sub-contests. We compare the performance of such schemes to that of grand winner-take-all contests from the point of view of designers who maximize either the expected total effort or the expected highest effort. An important explanatory variable is the form of the agents' cost functions. The analysis is based on simple but powerful results about various stochastic dominance relations among order statistics and functions thereof.

1 Introduction

Contests are situations in which agents spend resources in order to win one or more prizes. A major feature that differentiates these types of interactions from standard auctions is that, independently of success, all contestants bear the cost of their "bids". Numerous applications of such winner-take-all grand contests have been made to rent-seeking and lobbying in organizations, R&D races, political contests, promotions in labor markets, trade wars, military and biological wars of attrition. Due to the pervasive nature of such competitions (which are either designed or arise naturally), there exist large scientific and popular literatures[1] on the subject. Most of the scientific attention has focused on contests where a unique prize is awarded, and where all contestants compete against each other in one grand contest. But a casual survey of designed real-life contests reveals that: 1) Several prizes are often awarded; and 2) It is often the case that contestants do not compete in an "each against all" fashion, but are rather divided into several sub-contests.

[1] Two very entertaining popular books are Frank and Cook (1995) and Sobel (1995).

Applications of multi-prize contests can be found, for example, in the field of R&D inducement[2] where the designer's goal is either to achieve the highest possible performance[3] (or some pre-specified level of that performance), or to induce a general increase of activity in the specific field. For example, the European Information Technology Society annually awards three "grand" prizes worth 200000 euros each[4] for "novel products with high information technologies content and evident market potential"; R&D intensive firms such as Dow Chemical and IBM sponsor annual tournaments in which several substantial and renewable grants ($30000 and $50000 yearly, respectively) are awarded to workers in order to encourage the development of their ideas for commercial use.

Parallel sub-contests are often observed in the organization of internal labor markets in large firms and public agencies. The sub-contests are usually regional or divisional, and the prizes are promotions to well-defined (and usually equally-payed[5]) positions on the next rung of the hierarchy-ladder. The organization of several sub-contests is very popular in the world of sport. For example, in the first stage of international ball-game competitions (soccer, basketball, etc...) clubs or national teams compete first in groups[6], and the best competitors win a prize - the possibility of competing at the next stage.

Given the wealth of contests with multiple prizes and sub-contests, it is of interest to ask: What is the optimal contest architecture ? Galton (1902) considered a contest designer with a fixed prize sum to be split among two prizes, and asked: "What ratio should a first prize bear to that of a second one ? Does it depend on the number of competitors, and if so, why ?" Without specifying the designer's goal, Galton's answer focused on limit distribution of differences of *order statistics* based on a normal random vari-

[2] Wright (1983), Taylor (1995), Fullerton and McAfee (1999) and Che and Gale (2001) study various research tournaments models with one winning firm, and emphasize the need to restrict entry. Note though that some R&D intensive procurement contracts specify "dual-sourcing", thus have at least two prizes.Wilson (1979) and Anton and Yao (1992) analyze standard auction models where prize-splitting has an adverse effect on the designer's revenue.

[3] e.g., the 1795 French competition promising Fr 12000 for the best method of food canning for the military; the 1829 £ 500 prize for the best engine to power a train between Liverpool and Manchester; the 1992 $30 million prize awarded by US utilities for the development of an energy-efficient refrgerator.

[4] There are also 20 "participation" prizes worth 5000 euros each.

[5] Equal benefits for similar positions in the hierarchy (independent of the occupants' abilities) can be also rationalized by the designer's wish to minimize influence costs. Inderst et.al. (2001) show that a hierarchical contest consisting of several several sub-contests may be beneficial for such a designer.

[6] In some leagues this is also motivated by the the need to reduce the number of games. But this motive is not always present (e.g., in the NBA)

able representing the distribution of abilities in the population. This work pioneered the scientific literature on contests and introduced the important concept of order statistics.

In Moldovanu and Sela (2001) we revisited Galton's problem and we calculated the optimal ratio of prizes for a designer that maximizes the expected total effort in the grand contest where each agent compete against everyone else[7]. It turns out that, except for the case where contestants have an increasing marginal cost of effort, the value of the second and lower prizes should be zero. If cost functions are convex, several prizes may be optimal, but the precise optimal ratio depends in a complex way on joint properties of the cost function and the function governing the distribution of abilities in the population.

In this paper, we allow the designer to split the contestants into several parallel sub-contests (each with an equal number of contestants), and to specify the number of equal prizes in each sub-contest. Since we assume that the designer has an overall fixed prize-sum, these two operations are not independent: an increase in the number of sub-contests implies a decrease of the total prize-sum in each one of them. In order to account for these intertwined effects, our present analysis of the optimal contest architecture relies on several tools (borrowed from mathematical statistics) that can yield important insights for the study of multi-prize contests.

We consider a contest model where n contestants exert effort in order to win one of p prizes. Each contestant i exerts an observable effort. The contestant with the highest effort wins the first prize, the contestant with the second-highest effort wins the second prize, and so on until all the prizes are allocated. All contestants (including those that did not win any prize) incur a cost that is a strictly increasing function of their effort. The contestants have private information[8] about a parameter ("ability") that affects their effort cost function. Cost functions are assumed to be strictly increasing in effort and are either linear, concave or convex. The function governing the distribution of abilities in the population is common knowledge, and abilities are drawn independently of each other. Our basic model[9] for the grand

[7] Related questions have been addressed in various models by Glazer and Hassin (1988), Barut and Kovenock (1998) and Krishna and Morgan (1998).

[8] Contest models with complete information about the value of a unique prize include, among others: Tullock (1980), Varian (1980), Moulin (1986), Dasgupta (1986), Hillman and Samet (1987), Dixit (1987), Baye et. al. (1993). Baye et. al. (1996) offers a complete characterization of equilibrium behavior in the complete information all-pay auction with one prize.

[9] A different model emphasizes the use of contests in order to extract effort under "moral hazard" conditions (see Lazear and Rosen (1981), Green and Stokey (1983) , Nalebuff and Stiglitz (1983), and Rosen (1986)) In that literature agents uually have the same known

contest with linear cost functions is isomorphic to a "private values" all-pay auction with several prizes[10]

The designer chooses the contest architecture: she determines the number of prizes (while the prize sum is kept fixed) and how the prizes and the contestants should be split among several sub-contests. We consider here two designer goals[11]: 1) Maximization of the expected total effort; 2) Maximization of the expected highest effort.

In the case of a designer who wishes to maximize the expected total effort, we first show that the designer's payoff in the linear cost case increases in the number of contestants and decreases in the number of prizes. As a consequence, the optimal contest architecture for the case of a linear cost function is the one where all the contestants compete in a single grand contest for a unique large prize. We next show that this result extends to the case of concave cost functions. For the case of convex cost functions, the grand architecture may not be optimal: the designer can benefit by splitting the contestants in several sub-contests and/or by splitting the prize sum in several prizes. While the precise optimal architecture for the case of convex cost functions depends on properties of the function governing the distribution of abilities in the population (which is unlikely to be precisely known to the designer), we can show the following: if the grand contest is not optimal for a given cost function, then it continues to be dominated by split contests also for all cost functions that are more convex (i.e., for all functions with a higher Arrow-Pratt curvature index). Moreover, it can be shown that the advantage of the splits increases when we increase the degree of convexity.

In the case where the designer wishes to maximize the expected value of the highest effort, we show that the designer's payoff falls in the number of prizes, and therefore she wants to award a unique prize in each possible sub-contest (if any) when cost functions are linear. The dependence on the number of contestants is, however, more subtle. When the number of contestants is increased, the highest effort goes up, while the effort of more types goes down. In addition, more efforts are "wasted" (since only the highest effort counts here). As the number of contestants goes to infinity, we can show

ability, but output is a stochastic function of the unobservable effort. The identity of the most productive agent is determined by an external shock

[10] All-pay auction models with linear cost functions and incomplete information about the prize's value to different contestants include Weber (1985), Hillman and Riley (1989), Krishna and Morgan (1997). Contest models with several identical prizes include Clark and Riis (1998) (they compare simultaneous versus sequential designs under complete information) and Bulow and Klemperer (1999) (who study a war of attrition).

[11] Since the equilibrium effort functions only dependent on the contest architecture, the methods of this paper can be used to analyse other goals arising in various applications.

that the designer's payoff converges to just half the payoff of a designer who maximizes total effort, i.e., half of the expected total effort comes from the ablest contestant. But, the designer's payoff is not necessarily monotonically increasing in the number of contestants. This demonstrates the advantages of restricting entry[12] if the designer maximizes the expected highest performance. In order to generate such examples we do not need to rely on additional "frictions."[13]

Due to the possible non-monotonicity in the number of contestants, it is not clear whether the grand architecture is better than a parallel one. We can nevertheless show that, for linear cost functions, the grand architecture is preferred to any other where contestants are split in parallel sub-contests. The reason is that in the parallel architecture we also award a smaller prize in each sub-contest, and this has a strong negative influence on the effort made by the ablest competitors. Finally, for the case of convex cost functions, we show that if the grand contest is dominated by a parallel one for a designer who maximizes the highest effort, then it is also dominated for a designer who maximizes the total effort.

In this paper, we study the performance of contest designs that can be implemented by a designer without detailed knowledge about the underlying situation. We do not perform here a "mechanism design analysis" in the usual sense, since the basic contest structure remains fixed: we only allow for a restricted set of design variations. In particular, none of our main results relies on properties of the underlying distribution of abilities in the population (which is unlikely to be known to the designer in each instance where the contest is used). It seems intuitive that our "architectures" are relatively robust to non-dramatic changes in the environment, and that these architectures can be ex-ante specified, and implemented, without any precise knowledge about the particularities in each application. In addition, our results can be seen as the first step in a comprehensive theory of hierarchical tournament design that rationalizes the use of particular architectures at each stage of the tournament.[14]

The rest of the paper is organized as follows: In Section 2 we present the

[12]We display an example where the optimal number of contestants is two. Interestingly, many competitions for US defense procurement involved only two firms: the F-15 and F-16 engine competition (General Electric versus Pratt and Whitney), the Sparrow air-to-air missile competition (General Dynamics versus Raytheon), the SSN-688 submarine competition (Electric Boat versus Newport News).

[13]Taylor (1995), and Fullerton and McAfee (1999) assume positive fixed costs of entry. Che and Gale (2001) study a model where firms invest in quality prior to the contest. Without that feature the designer's utility increases in the number of firms

[14]In his pioneering paper on the subject, Rosen (1986) simply assumes that each sub-contest has two competitors and a unique prize.

contest model. In Section 3 we derive the symmetric equilibrium effort functions and we show that they are determined by differences among densities belonging to successive order statistics. In Section 4 we study the comparative statics obtained by varying the number of contestants or the number of equal prizes. When we increase the number of contestants, or decrease the number of prizes, the equilibrium effort of high ability types increases while the equilibrium effort of low ability types decreases. There exists exactly one type of contestant whose equilibrium effort is unaffected by the change. These *single-crossing* properties are central to the analysis in the sequel[15]. In sections 5 and 6 we analyze the optimal contest architectures when the contest designer wishes to maximize the expected total effort and the expected highest effort, respectively. Section 7 concludes, and mentions possible avenues for future research. In Appendix A we set up the necessary analytical tools on which the whole technical analysis is based. These tools involve various stochastic dominance relations among linear combinations (and other functions) of order statistics.

2 The Model

Consider a contest where p prizes are awarded. The value of the $j-th$ prize is V_j, where $V_1 \geq V_2 \geq ... \geq V_p \geq 0$. The values of the prizes are common knowledge. We assume that $\sum_{i=1}^{p} V_i = 1$ - this is just a normalization.

The set of contestants is $N = \{1, 2, ..., n\}$, where $n \geq 2$ and $n > p$. Each player i makes an effort x_i. These efforts are submitted simultaneously. An effort x_i causes a cost denoted by $c_i\gamma(x_i)$, where $\gamma : R_+ \rightarrow R_+$ is a strictly increasing function with $\gamma(0) = 0$, and where $c_i > 0$ is an ability parameter.[16] Note that a **low** c_i means that i has a **high** ability and vice-versa. We denote by g the inverse function γ^{-1}.

The ability (or *type*) of contestant i is private information to i. Abilities are drawn independently of each other from an interval $[m, 1]$ according to a distribution function F which is common knowledge. We assume that F has a continuous density $dF > 0$. In order to avoid infinite bids caused by zero costs, we assume that m, the type with the highest possible ability, is

[15] The reader may be familiar with single-crossing as an assumed property on utility functions (see for example Athey, 2000, who uses it in order to establish monotone comparative statics in the theory of one-person decision making under risk) In contrast, single-crossing is here an endogenously arising property of equilibrium effort functions.

[16] The treatment of the case in which $i's$ cost function is $\delta(c_i)\gamma(x_i)$, where δ is strictly monotonically increasing, is completely analogous. The main assumption here is the separability of ability and effort.

strictly positive.[17]

The contestant with the highest effort wins the first prize V_1. The contestant with the second highest effort wins the second prize V_2, and so on until all the prizes are allocated.[18] That is, the payoff of contestant i who has ability c_i and exerts an effort x_i is either $V_j - c_i\gamma(x_i)$ if i wins prize j, or $-c_i\gamma(x_i)$ if i does not win a prize. In the case of p equal prizes, the contestants with the highest p efforts win the available prizes.

Each contestant i chooses his effort in order to maximize expected utility (given the other competitors' actions and the values of the prizes). The contest designer can organize one grand contest or she can split the contestants into several parallel sub-contests. In addition, she can determine the number of prizes in each (sub)contest. We consider two forms of utility for the designer: 1) The designer maximizes the expected value of total effort $E(\sum_{i=1}^{k} x_i)$ and 2) The designer maximizes the expected value of the highest effort $E(x_{\max})$.

3 Equilibrium Characterization

Denote by $X_1, X_2, ...X_n$ the identical, independently distributed random variables governing the distribution of the contestants' abilities. Denote by $X_{(1,n)}, X_{(2,n)}, ..., X_{(n,n)}$ the corresponding order statistics, and by $F_{(1,n)}, F_{(2,n)}, ..., F_{(n,n)}$ their respective distribution functions. In Appendix A we list the explicit formulae for the distributions and densities of order statistics.

Proposition 1 *Consider a contest with n contestants where the designer awards $p < n$ prizes, $V_1 \geq V_2 \geq ... \geq V_p \geq 0$. In a symmetric equilibrium, each contestant makes an effort according to the strictly decreasing function*[19]

$$b(c) = g[\sum_{i=1}^{p} V_i \int_c^1 \frac{1}{s}(dF_{(i,n-1)}(s) - dF_{(i-1,n-1)}(s))] \tag{1}$$

Proof. In Moldovanu and Sela (2001) we used the first-order maximization condition in order to obtain a differential equation involving the equilibrium effort function and its derivative[20]. That condition involves the different

[17] The case where $m = 0$ can be treated as well, but requires slightly different methods. The choice of the interval $[m, 1]$ is a normalization.

[18] If $h > 1$ agents tie for a prize, each one of them gets the respectuve prize with probability $\frac{1}{h}$.

[19] We use the convention $dF_{(0,n-1)}(s) \equiv 0$.

[20] It is interesting to note that the resulting differential equation is the one with separated variables (i.e., of the form $H(y, y') = D(x)$) which can always be explicitly integrated.

probabilities with which an agent expects to win each of the p prizes. We proved that the symmetric equilibrium effort function is given by

$$b(c) = g[\sum_{i=1}^{p} V_i \int_c^1 -\frac{1}{s} dF_i^n(s)] \tag{2}$$

where $F_i^n(s), 1 \leq i \leq n$, denotes the probability that an agent with type s meets $n-1$ competitors such that $i-1$ of them have lower types and $n-i$ have higher types. The representation in the statement of the Proposition follows by relations 1,2 of Lemma 20 in Appendix A. ■

The above representation is very useful since it allows us to employ various stochastic dominance results among order statistics and functions thereof. For the case of equal prizes (on which we focus in this paper) we obtain an even more compact characterization:

Corollary 2 *Consider a contest with n contestants where the designer awards $p < n$ equal prizes, each worth $\frac{1}{p}$. The symmetric equilibrium effort function is given by*

$$b_{n,p}(c) = g[\frac{1}{p} \int_c^1 \frac{1}{s} dF_{(p,n-1)}(s)] \tag{3}$$

Proof. The result follows by the telescopic nature of the equilibrium effort function in Proposition 1. ■

4 Single-Crossing Properties

It is clear from the above results that the equilibrium for a strictly increasing cost function γ is obtained by applying the inverse function $\gamma^{-1} = g$ to the equilibrium obtained for the linear cost function $\gamma(x) = x$. The equilibrium properties in the linear cost case are therefore central to our analysis, and we next display several important structural properties for this case. The following two results make explicit the trade-offs induced by varying the number of contestants and the number of prizes: 1) For a fixed number of contestants, increasing the number of prizes has a negative effect on the equilibrium effort of high ability contestants and a positive effect on the equilibrium effort of low ability contestants[21] and 2) For a fixed number of prizes, increasing the number of contestants has a positive effect on the

[21] Another operation that induces a similar change is the imposition of a bid cap. The effects of this operation in an all-pay auction with a unique prize (whose value is private information) are studied by Gavious et.al. (2000).

equilibrium effort of high ability contestants and a negative effect on the equilibrium effort of low ability contestants.

Proposition 3 *Consider a contest with* n *contestants. For any number of prizes* p, r *such that* $n > r > p$*, the equilibrium effort functions* $b_{n,r}(c)$ *and* $b_{n,p}(c)$ *are single-crossing. That is, there exists a unique* $c^* = c^*(n, r, p) \in (m, 1)$ *such that:*

1. $b_{n,r}(c^*) = b_{n,p}(c^*)$;
2. $b_{n,p}(c) > b_{n,r}(c)$ *for all* $c \in [m, c^*)$;
3. $b_{n,p}(c) < b_{n,r}(c)$ *for all* $c \in (c^*, 1)$.

Proof. The proof consists of several steps:
1) We first show that:

$$\begin{aligned} b_{n,p}(m) &> b_{n,r}(m); & (4) \\ b_{n,p}(c) &< b_{n,r}(c) \text{ for } c \text{ in a neighborhood } [1-\varepsilon, 1). & (5) \end{aligned}$$

This proves that the continuous equilibrium effort functions $b_{n,r}(c)$ and $b_{n,p}(c)$ must cross at least once in the interval $(m, 1)$.

By equation 3, we obtain $b_{n,i}(m) = \frac{1}{i}E[\frac{1}{X_{(i,n-1)}}]$. By Proposition 19-1, we know that $X_{(p,n-1)} \leq_{st} X_{(r,n-1)}$. Since the function $h(x) = \frac{1}{x}$ is strictly decreasing, we obtain by Proposition 16-5 that $\frac{1}{X_{(p,n-1)}} \geq_{st} \frac{1}{X_{(r,n-1)}}$, and hence that $E[\frac{1}{X_{(p,n-1)}}] > E[\frac{1}{X_{(r,n-1)}}]$. Since $r > p$, we finally obtain $\frac{1}{p}E[\frac{1}{X_{(p,n-1)}}] > \frac{1}{r}E[\frac{1}{X_{(r,n-1)}}]$, which means that $b_{n,p}(m) > b_{n,r}(m)$.

In order to prove relation 5, note that $b_{n,p}(1) = b_{n,r}(1) = 0$. Moreover, we have $b^{(i)}_{n,p}(1) = 0$ for all derivatives of order i, $1 \leq i \leq n-p-1$, and $b^{(i)}_{n,r}(1) = 0$ for all derivatives of order i, $1 \leq i \leq n-r-1$. This yields

$$\lim_{c \to 1} \frac{b^{(n-r)}_{n,r}(c)}{b^{(n-r)}_{n,p}(c)} = \infty \qquad (6)$$

The result for the neighborhood of 1 follows then by L'Hospital's rule.

2) We now show that the equation $b'_{n,p}(c) = b'_{n,r}(c)$ has a unique solution in the interval $(m, 1)$. We have:

$$
\begin{aligned}
& b'_{n,p}(c) - b'_{n,r}(c) \\
= \; & \frac{1}{p}\frac{1}{c} dF_{(p,n-1)}(c)F'(c) - \frac{1}{r}\frac{1}{c} dF_{(r,n-1)}(c)F'(c) \\
= \; & \frac{1}{p}\frac{1}{c}\frac{(n-1)!}{(p-1)!(n-p-1)!}F(c)^{p-1}(1-F(c))^{n-p-1}F'(c) - \\
& \frac{1}{r}\frac{1}{c}\frac{(n-1)!}{(r-1)!(n-r-1)!}F(c)^{r-1}(1-F(c))^{n-r-1}F'(c) \\
= \; & \frac{1}{c}(n-1)!F(c)^{p-1}(1-F(c))^{n-r-1}F'(c) \times \\
& [\frac{1}{p!(n-p-1)!}(1-F(c))^{r-p} - \frac{1}{r!(n-r-1)!}F(c)^{r-p}] \qquad (7)
\end{aligned}
$$

Hence, for $c \in (m, 1)$

$$
b'_{n,p}(c) - b'_{n,r}(c) = 0 \Leftrightarrow \frac{r!(n-r-1)!}{p!(n-p-1)!} = (\frac{F(c)}{1-F(c)})^{r-p} \qquad (8)
$$

The function $H(c) = (\frac{F(c)}{1-F(c)})^{r-p}$ is strictly monotonically increasing with $H(m) = 0$ and $H(1) = \infty$. Hence,

$$
H(c) = \frac{r!(n-r-1)!}{p!(n-p-1)!} \Leftrightarrow b'_{n,p}(c) - b'_{n,r}(c) = 0 \qquad (9)
$$

has a unique solution in $(m, 1)$, as desired.

3) By step 1, we know that the equation $b_{n,p}(c) = b_{n,r}(c)$ must have at least one solution in the interval $(m, 1)$. It remains to show that the solution is unique.

Assume, by contradiction, that on $(m, 1)$ the equation $b_{n,p}(c) - b_{n,r}(c) = 0$ has two distinct solutions c_1, c_2 with $c_2 > c_1$. On the interval $[m, 1]$ there are then exactly three distinct solutions (the additional one is of course $c = 1$). Applying Rolle's Theorem, we obtain two points d_1 and d_2 such that $d_1 \in (c_1, c_2)$, $d_2 \in (c_2, 1)$ and

$$
b'_n(d_1) - b'_k(d_1) = b'_n(d_2) - b'_k(d_2) = 0. \qquad (10)
$$

Since both $d_1, d_2 \in (m, 1)$ we obtain a contradiction to step 2. ■

Proposition 4 *Consider a contest with p prizes. For any numbers of contestants n, k such that $n > k > p$, the equilibrium effort functions $b_{n,p}(c)$ and $b_{k,p}(c)$ are single-crossing. That is, there exists a unique $c^* = c^*(n, k, p) \in (m, 1)$ such that:*

1. $b_{n,p}(c^*) = b_{k,p}(c^*)$;

2. $b_{n,p}(c) > b_{k,p}(c)$ *for all* $c \in [m, c^*)$;

3. $b_{n,p}(c) < b_{k,p}(c)$ *for all* $c \in (c^*, 1)$;

Proof. The proof uses exactly the same steps and arguments analogous to those presented in the proof of Proposition 3. ■

5 Maximization of Total Effort

In this section we assume that the contest designer wishes to maximize the expected value of the total effort. The results in the previous section pointed out that high and low ability contestants are affected by changes in the number of prizes and contestants in opposite ways. The interesting questions naturally are: 1) What is the aggregate effect of variations in the number of prizes and the number contestants on the designer's payoff ? and 2) What is the optimal preferred architecture?

Proposition 5 *Assume that the designer's payoff is the expected value of total effort, and assume that the cost functions are linear. Then the following hold:*

1. *The designer's payoff increases in the number of contestants;*

2. *The designer's payoff decreases in the number of prizes.*

Proof. Let $R_{n,p}$ denote the designer's payoff in a contest with n contestants and p equal prizes. Then we have:

$$\begin{aligned}
R_{n,p} &= n\int_m^1 b_{n,p}(c)dF(c) \\
&= \frac{n}{p}\int_m^1 [\int_c^1 \frac{1}{s}dF_{(p,n-1)}(s)]dF(c) \\
&= \frac{n}{p}\left(F(c)\int_c^1 \frac{1}{s}dF_{(p,n-1)}(s)\,\Big|_m^1 + \int_m^1 F(c)\frac{1}{c}dF_{(p,n-1)}(c)\right) \\
&= \int_m^1 \frac{1}{c}\frac{n}{p}F(c)dF_{(p,n-1)}(c) \\
&= \int_m^1 \frac{1}{c}dF_{(p+1,n)}(c) = E[\frac{1}{X_{(p+1,n)}}] \qquad (11)
\end{aligned}$$

We integrated by parts in the second line, and we used Lemma 20-3 in fourth line. Note also that

$$F(c)\int_{c}^{1}\frac{1}{s}dF_{(p,n-1)}(s)\,|_{m}^{1}=0 \tag{12}$$

.

By Theorems 19-1,3 and 16-1 in Appendix A, we know that $X_{p,n} \leq_{st} X_{p+1,n}$ and that $X_{p,n} \leq_{st} X_{p,n-1}$. Since the function $\frac{1}{x}$ is decreasing, we obtain by Theorem 16-5 that $\frac{1}{X_{(p+1,n)}} \leq_{st} \frac{1}{X_{(p,n)}}$ and that $\frac{1}{X_{(p,n-1)}} \leq_{st} \frac{1}{X_{(p,n)}}$. The results follow then by Theorem 16-2 in Appendix A. ■

Consider now the following three types of contest architectures for a given group of n contestants:

1. In the *Grand Architecture* (GA) the entire group of n contestants competes for one prize worth 1.
2. In the *t-Parallel-Architecture* (t-PA) there are $t > 1$ separate subgroups, each consisting of $\frac{n}{t}$ contestants[22], competing for one prize worth $\frac{1}{t}$.
3. In the *p-Split-Prize Architecture* (p-SPA) there is one group of n contestants competing for $p > 1$ equal prizes, each worth $\frac{1}{p}$.

It is of course possible to perform a joint split into several sub-contests where, in each one of them, contestants compete for several prizes. The results for this architecture will be simple consequences from those obtained for the separate splits defined above.

Proposition 6 *Assume that cost functions are linear, and that the designer's payoff is given by the expected value of total effort. Then the designer's payoff in the Grand Architecture (GA) is larger than the respective payoffs in any Parallel (t-PA) or Split-Prize (p-SPA) Architecture.*

Proof. The designer's payoffs are given by $R_{n,1}$ in GA; by $t\cdot\frac{1}{t}R_{\frac{n}{t},1} = R_{\frac{n}{t},1}$ in t-PA; and by $R_{n,p}$ in p-SPA (see the proof of Proposition 5). The result follows by repeated applications of Proposition 5. ■

[22]We assume here that $\frac{n}{t}$ is an integer. It can be verified that a symmetric partition of the contestants (which are symmmetric ex-ante) to sub-contests is indeed optimal for the designer.

The above Proposition shows that splitting the prize sum or splitting the contestants into several sub-contests is not beneficial for a designer who maximizes the total effort, as long as the agents have linear cost functions.[23]

For the case of linear cost functions, the preference relations among contest architectures were simple consequences of the monotonicity relations established in Proposition 5. In fact, there is an equivalence relation between the two phenomena. Once we introduce non-linear cost functions, this equivalence breaks down, and the respective proofs are more delicate. Our next result shows that the preference relation established in Proposition 6 extends to the case of concave cost functions. The proof relies on role of the single-crossing properties displayed in Proposition 3, 4 in establishing variability orders among functions of order statistics.

Proposition 7 *Assume that cost functions are concave and assume that the designer's payoff is given by the expected value of total effort. Then the designer's payoff in the Grand Architecture is larger than her respective payoffs in any Parallel or Split-Prize Architecture.*

Proof. Let g denote the inverse of the cost function. Then g is increasing and convex.

1) We first show that GA dominates any t-PA architecture. Let $B_{n,1}$ denote the random variable governing the equilibrium bid with n contestants, one prize worth 1 and linear cost functions. Similarly, $B_{k,1}$ denotes the random variable for the same case with k contestants,where $k < n$.

By Proposition 4, there exists c^* such that $B_{n,1} \geq B_{k,1}$ for $c \leq c^*$ and $B_{n,1} \leq B_{k,1}$ for $c \geq c^*$. Since $\frac{k}{n} < 1$, the same property holds for the random variables $B_{n,1}$ and $\frac{k}{n}B_{k,1}$. Note that since these two functions are strictly decreasing (!), their distribution functions are, respectively, $1 - F(B_{n,1}^{-1})$ and $1 - F((\frac{k}{n}B_{k,1})^{-1})$, where F is the distribution of abilities.

By the single-crossing property of $B_{n,1}$ and $\frac{k}{n}B_{k,1}$, their distribution functions are also single-crossing in the sense of Theorem 18-1 in Appendix A. By Proposition 5-1, we know that

$$kE[B_{k,1}] \leq nE[B_{n,1}] \Leftrightarrow E[\frac{k}{n}B_{k,1}] \leq E[B_{n,1}] \quad (13)$$

Single-crossing and inequality 13 imply that $\frac{k}{n}B_{k,1} \leq_{icx} B_{n,1}$ (for the *increasing convex stochastic order*, see Definition 17 and Theorem 18 in Appendix

[23] It should be obvious from the above derivations that a joint split of both the prize sum and the set of contestants is also not beneficial. Moreover, it can be shown that the result continues to hold even if we allow for several unequal prizes

A). Hence, we obtain that for any increasing convex function g,

$$E[g(\frac{k}{n}B_{k,1})] \leq E[g(B_{n,1})] \tag{14}$$

In GA the designer has a payoff $nE[g(B_{n,1})]$. In t-PA the designer has a payoff of $t \cdot \frac{n}{t}E[g(\frac{1}{t}B_{\frac{n}{t},1})] = nE[g(\frac{1}{t}B_{\frac{n}{t},1})]$. The desired result follows by setting $k = \frac{n}{t}$ in inequality 14.

2) We now show that GA dominates architecture p-SPA. Let $B_{n,1}$ denote the random variable governing the equilibrium bid with n contestants, one prize and linear cost functions, and let $B_{n,p}$ denotes the random variable governing the equilibrium bid with n contestants, $p > 1$ prizes and linear cost functions. By Proposition 3, these functions are single-crossing. By Proposition 5-2, we know that $E[B_{n,1})] \geq E[B_{n,p}]$. The rest of the proof follows exactly as above. ■

The next example shows that splitting the contestants or splitting the prize sum can be beneficial if the cost functions are convex.[24]

Example 8 *Let $n = 6$, and let abilities be uniformly distributed on the interval $\left[\frac{1}{2}, 1\right]$. Consider the convex cost function $\gamma(x) = x^2$. The designer's payoff in GA is*

$$R_{6,1} = 6\int_{0.5}^{1} 2(\int_{c}^{1} (6-1)\frac{1}{s}(1-(2s-1))^{6-2}2)^{0.5} = 2.1014 \tag{15}$$

The designer's payoff in 2-SPA is

$$R_{6,2} = 6\int_{0.5}^{1} 2(\int_{c}^{1} \frac{1}{2}(6-1)\frac{1}{s}(1-(2s-1))^{6-3}(6-2)(2s-1)2)^{0.5} = 2.3043 \tag{16}$$

Finally, the designer's payoff in 2-PA is

$$2 \cdot R_{3,1} = 2 \cdot 3\int_{0.5}^{1} 2(\int_{c}^{1} \frac{1}{2}(3-1)\frac{1}{s}(1-(2s-1))^{3-2}2)^{0.5} = 2.4299 \tag{17}$$

When the cost functions are convex, the relations between the designer's payoffs in the various architectures depend on the precise relations between the function governing the distribution of abilities and the (convex) cost function. There is no general ranking of architectures. But we can offer a comparative statics result by varying the degree of convexity. As usual, we

[24] If the cost function is convex enough, it is possible that a joint split of contestants and prizes in each sub-contest is more beneficial than the simple splits.

say that an increasing convex [concave] function β is "more convex" [more concave] than another increasing convex [concave] function α if there exists a strictly monotonic increasing and convex [concave] function μ such that $\beta = \mu \circ \alpha$. This partial order is equivalent to the one obtained by comparing curvatures according to the Arrow-Pratt index.[25]

Proposition 9 *Assume that a designer facing contestants with a given convex cost function γ prefers either a Parallel or a Split-Prize architecture to the Grand Architecture. Then this preference extends to any situation where contestants have a more convex cost function δ.*

Proof. Let g denote the inverse of γ and d denote the inverse of δ. By assumption, there exists a strictly increasing and concave function μ such that $d = \mu \circ g$.

1) We first compare GA with p-SPA. For contestants with cost function γ, the designer's payoff is $nE[g(B_{n,1})]$ in GA and $nE[g(B_{n,p})]$ in p-SPA. By our assumption we know that

$$E[g(B_{n,p})] \geq E[g(B_{n,1})] \tag{18}$$

By Proposition 3, the random variables $B_{n,p}$ and $B_{n,1}$ are single-crossing. Since g is increasing, we obtain that $g(B_{n,p})$ and $g(B_{n,1})$ are single-crossing in the sense of Theorem 18-2 (recall that these random variables are decreasing). Together with inequality 18, this yields $g(B_{n,p}) \geq_{icv} g(B_{n,1})$ (see Definition 17 in Appendix A for the *increasing concave stochastic order*). Hence, for the concave function μ we obtain that

$$E[\mu(g(B_{n,p}))] \geq E[\mu(g(B_{n,1}))] \Leftrightarrow E[d(B_{n,p}))] \geq E[d(B_{n,1}))] \tag{19}$$

as desired.

2) The proof for GA and t-PA is analogous, and therefore it is omitted here. ■

The same methods as above generally show that an increase in the convexity of the cost function makes splits more advantageous (combine Propositions 6 and 9 to get an instance of this phenomenon). Hence, a standard design where only two contestants compete against each other (and where the winner gets the prize, which may be the ability of competing at the next stage) can be rationalized by strongly increasing marginal costs of effort.

[25] This index is used for comparing the risk-aversion of agents with increasing and concave utility functions. The same logic applies of course to increasing convex functions.

6 Maximization of Highest Effort

We now consider a principal whose payoff is given by the expected value of the highest effort. The comparative statics with respect to changes in the number of prizes are clear:

Proposition 10 *Assume that the designer's payoff is the expected value of the highest effort, and assume that the cost functions are linear. Then the designer's payoff decreases in the number of prizes. Consequently, her payoff in the Grand Architecture is higher than that in any Split-Prize Architecture.*

Proof. We showed in Proposition 3 that an increase in the number of prizes causes an increase in the equilibrium effort of low ability types and a decrease in the equilibrium effort of high ability types. Let $b_{n,p}(c)$ be the equilibrium effort, and denote $\Delta(c) \equiv b_{n,p}(c) - b_{n,p-1}(c)$. Let $c^* = c^*(n,p) \in (m,1)$ the unique type satisfying $\Delta(c) = 0$. For every $p > 1$,

$$\Delta(c) > 0 \text{ for } c \in (c^*, 1) \tag{20}$$

$$\Delta(c) < 0 \text{ for } c \in (m, c^*) \tag{21}$$

In Proposition 5-2 we showed that the designer's aggregate payoff in the case where she maximizes the expected value of the total effort decreases in the number of prizes:

$$n \int_m^1 \Delta(c) f(c) < 0 \tag{22}$$

In the present case, the designer's payoff relies on the distribution of the highest (i.e., $n-$th) order statistic, and is given by:

$$n \int_m^1 \Delta(c)(1 - F(c))^{n-1} f(c) \tag{23}$$

Since $H(c) = (1 - F(c))^{n-1}$ is decreasing, and by inequalities 20 and 21, we obtain that expression 23 is obtained from expression 22 by multiplying all negative terms $\Delta(c)$ by relatively high values of $H(c)$, and all positive terms $\Delta(c)$ by relatively lower values. Therefore,

$$\int_m^1 \Delta(c)(1 - F(c))^{n-1} f(c) < \int_m^1 \Delta(c) f(c) < 0$$

Thus, the designer's payoff decreases in the number of prizes. ■

We consider below only contests (or sub-contests) with one prize. The dependence on the number of contestants is more subtle now. By the formula obtained in the proof of Proposition 5, the payoff of a principal who maximizes the expected value of the total effort monotonically increases, and converges to $\frac{1}{m}$ when the number of contestants n tends to infinity. The monotonicity result was, a-priori, rather delicate since, using Proposition 4, there are four effects at play when we increase the number of contestants. On the positive side: 1) The effort of high ability contestants goes up; 2) We add up the effort of more contestants. On the negative side: 3) The effort of low ability contestants goes down; 4) The measure of types whose effort goes comparatively down increases.[26] Roughly speaking, we showed that effects 1 and 2 are together stronger than effects 3 and 4.

In the present situation, we completely lose effect 2, but effect 1 has more weight (since we are only interested in the highest effort, which naturally comes from high ability contestants).

Proposition 11 *Assume that the designer's payoff is given by the expected value of the highest effort. As the number of contestants tends to infinity, the designer's payoff converges to $\frac{1}{2m}$. The convergence need not be monotonic and, moreover, the designer's payoff may be maximized for a number of contestants $n^* < \infty$.*

Proof. The equilibrium effort function is given by

$$b_{n,1}(c) = \int_c^1 \frac{1}{s} dF_{(1,n-1)}(s) \tag{24}$$

The designer's payoff is given by :

$$P_{n,1} = E[b_{n,1}(X_{(1,n)})] \tag{25}$$

[26] This last effect is not formally proven, but follows easily by the proof of Proposition 4.

We have:

$$
\begin{aligned}
P_{n,1} &= \int_m^1 b_{n,1}(c)dF_{(1,n)}(c) = \int_m^1 [\int_c^1 \frac{1}{s}dF_{(1,n-1)}(s)]dF_{(1,n)}(c) \\
&= \int_m^1 F_{(1,n)}(s)\frac{1}{s}dF_{(1,n-1)}(s) \\
&= \int_m^1 \frac{1}{s}[1-(1-F(s))^n]dF_{(1,n-1)}(s) \\
&= \int_m^1 \frac{1}{s}dF_{(1,n-1)}(s) - \int_m^1 \frac{1}{s}(1-F(s))^n dF_{(1,n-1)}(s) \\
&= \int_m^1 \frac{1}{s}dF_{(1,n-1)}(s) - (n-1)\int_m^1 \frac{1}{s}(1-F(s))^{2n-2} \\
&= \int_m^1 \frac{1}{s}dF_{(1,n-1)}(s) - \frac{n-1}{2n-1}\int_m^1 \frac{1}{s}F_{(1,2n-1)}(s) \\
&= b_n(m) - \frac{n-1}{2n-1}b_{2n}(m) \qquad (26)
\end{aligned}
$$

We used integration by parts in the second line, and the formula for $F_{(1,n)}$ and the binomial expansion formula in the third line. Taking the limit, we obtain:

$$
\begin{aligned}
\lim_{n\to\infty} P_{n,1} &= \lim_{n\to\infty}[b_n(m) - \frac{n-1}{2n-1}b_{2n}(m)] \\
&= \frac{1}{m} - \frac{1}{2m} = \frac{1}{2m}
\end{aligned}
$$

For the other part of the result, see the Example below. ∎

Example 12 *Assume that the abilities are drawn from the interval* $[0.5, 1]$ *according to the distribution function* $F(s) = (2s-1)^{0.1}$. *We obtain that* $P_{2,1} = 1.2116$. *Since* $P_{2,1} > \lim_{n\to\infty} P_{n,1} = 1$, *the designer's payoff cannot be monotonically increasing. A simple numerical calculation reveals that* $n = 2$ *is in fact the optimal number of contestants. For the uniform distribution* $F(s) = 2s - 1$ *the designer's payoff is monotonically increasing, hence* $n^* = \infty$.

Recall that, for linear cost functions, a designer who maximizes the expected value of total effort preferred the Grand Architecture to any Parallel Architecture (see Proposition 6). The result was an immediate consequence of the monotonicity of the designer's payoff in the number of contestants (see Proposition 5-1).Whenever the designer's payoff is not monotonically increasing in the number of contestants (as it may happen here), a parallel design may, in principle, be advantageous. But, in a parallel design, the prize awarded in each sub-contest is only a fraction of the total prize, thus causing a decrease in the effort of high ability contestants. Roughly speaking, our next result shows that the second effect always dominates:

Proposition 13 *Assume that cost functions are linear and that the designer's payoff is given by the expected value of the highest effort. Then her payoff in the Grand Architecture is larger than her payoff in any Parallel Architecture.*

Proof. The designer's expected payoff in GA is

$$\int_m^1 b_{n,1}(c)dF_{(1,n)}(c) \tag{27}$$

In t-PA there are n contestants, each exerting an effort of $\frac{1}{t}b_{\frac{n}{t},1}(c)$. The designer is interested in the highest realization, thus her payoff is given by

$$\int_m^1 \frac{1}{t}b_{\frac{n}{t},1}(c)dF_{(1,n)}(c) \tag{28}$$

Denote $\Delta_t(c) = b_{n,1}(c) - \frac{1}{t}b_{\frac{n}{t},1}(c)$, and recall that $b_{n,1}(c)$ and $\frac{1}{t}b_{\frac{n}{t},1}(c)$ are single crossing: there exists a unique point $c^* = c(n,t)$ such that $\Delta_t(c) > 0$ for all $c \in [m, c^*)$ and $\Delta_t(c) < 0$ for all $c \in (c^*, 1]$. The difference between the designer's payoffs in GA and t-PA is

$$\Delta = \int_m^1 \Delta_t(c)dF_{(1,n)}(c) = n\int_m^1 \Delta_t(c)(1-F(c))^{n-1}dF(c) \tag{29}$$

The analog difference for the case where the designer maximizes total effort is positive by Proposition 6:

$$\widetilde{\Delta} = n\int_m^1 \Delta_t(c)dF(c) > 0 \tag{30}$$

Note that the expression for Δ is obtained by multiplying each term in expression $\widetilde{\Delta}$ by the decreasing function $H(c) = (1-F(c))^{n-1}$. Hence all positive terms in $\widetilde{\Delta}$ are multiplied by relatively high values of $H(c)$, while all negative terms are multiplied by relatively lower values. Therefore, if $\widetilde{\Delta}$ is positive, Δ must be positive too. ■

For the case of convex cost functions we have seen that a designer who maximizes expected total effort may benefit from splitting the contestants into several parallel contests. Exactly the same intuition applies here as well. Our last result connects the two designer's goals by showing that in any instance where the parallel design is beneficial to a maximizer of the expected value of highest effort, it is also beneficial to a maximizer of the expected total effort.

Proposition 14 *Assume that cost functions are convex, and assume that a Parallel Architecture dominates the Grand Architecture for a designer who maximizes the expected value of the highest effort. Then the same preference extends to a designer who maximizes the expected value of total effort.*

Proof. For a maximizer of the expected highest effort, the payoffs in GA and in t-PA are, respectively:

$$\int_m^1 g(b_{n,1}(c))dF_{(1,n)}(c) \tag{31}$$

$$\int_m^1 g(\frac{1}{t}b_{\frac{n}{t},1}(c))dF_{(1,n)}(c) \tag{32}$$

In the case of a preferred t-PA we must have

$$\int_m^1 g(\frac{1}{t}b_{\frac{n}{t},1}(c))dF_{(1,n)}(c) \geq \int_m^1 g(b_{n,1}(c))dF_{(1,n)}(c) \tag{33}$$

For the maximizer of expected total effort, the payoffs in GA and in t-PA are, respectively:

$$n\int_m^1 g(b_{n,1}(c))dF(c) \tag{34}$$

$$t\cdot\frac{n}{t}\int g(\frac{1}{t}b_{\frac{n}{t},1}(c))dF(c) = n\int_m^1 g(\frac{1}{t}b_{\frac{n}{t},1}(c))dF(c) \tag{35}$$

By the same method as in the proof[27] of Proposition 13, we obtain that inequality 33 implies that

$$n\int_m^1 g(\frac{1}{t}b_{\frac{n}{t},1}(c))dF(c) \geq n\int_m^1 g(b_{n,1}(c))dF(c) \tag{36}$$

■

[27] The reader will easily see that this method also yields that the grand architecture is preferred to any parallel architecture also for the case of concave cost functions.

7 Conclusion

We have compared the performance of robust architectures in multi-prize contests with privately informed agents. The form of the contestants' cost functions play a major role in the analysis. In particular, independently of the number of contestants and the distribution of abilities in the population, constant or decreasing cost functions justify the organization of a grand competition where every agent competes against everyone else. On the other hand, increasing marginal costs of effort justify the organization of several parallel sub-contests instead of a grand competition. The optimal number of sub-contest depends then on the number of contestants, on the distribution of abilities in the population, and on the curvature of the cost functions.

We see two main avenues for future research: 1) Embedding the present analysis in a full theory of hierarchical contest design. Such an analysis needs to take into account dynamic aspects which have been absent here; 2) Embedding the present analysis in a model of competition among contest designers. Models of competing mechanism designers are rare (either because they are notoriously difficult, or because they immediately lead to "Bertrand paradoxes"), but we think that significant progress can be made by studying the realistic and relevant scenario where only the contest architecture may be varied (but not other features). Since the contest architecture influences the expected payoffs of the participating agents, it is interesting to analyze which agents engage in which contests. The technical tools provided in this paper should also be of great use for these extended models.

8 Appendix A

We set here the framework for stochastic dominance arguments involving order statistics and functions thereof. The results given without proofs are taken from the excellent textbook by Shaked and Shanthikumar (1994).

Definition 15 *For any two random variables, Y and Z with distributions G and H respectively denote their hazard rates by $r_y = \frac{G'(s)}{1-G(s)}$ and $r_z = \frac{H'(s)}{1-H(s)}$. Y is said to be smaller than Z in the hazard rate order (denoted by $Y \leq_{hr} Z$) if $\forall s,\ r_y(s) \geq r_z(s)$. Y is said to be smaller than Z in the usual stochastic order (denoted by $Y \leq_{st} Z$) if $\forall s,\ G(s) \geq H(s)$.*

Theorem 16 *The following relations hold:*

1. *If $Y \leq_{hr} Z$ then $Y \leq_{st} Z$;*

2. *If* $Y \leq_{st} Z$ *then* $E[Y] \leq E[Z]$;

3. *If* $Y \leq_{st} Z$ *and* $E[Y] = E[Z]$ *then* $G = H$;

4. *If* $Y \leq_{hr} Z$ *and* w *is any increasing [decreasing] function then* $w(Y) \leq_{hr} [\geq_{hr}] w(Z)$;

5. *If* $Y \leq_{st} Z$ *and* w *is any increasing [decreasing] function then* $w(Y) \leq_{st} [\geq_{st}] w(Z)$.

Definition 17 *Let* Y, Z *be two random variables such that* $E[g(Y)] \leq E[g(Z)]$ *for all increasing convex [concave] functions* g. *Then* Y *is said to be smaller than* Z *in the increasing convex order, denoted by* $Y \leq_{icx} Z$ *[*Y *is said to be smaller than* Z *in the increasing concave order*[28]*, denoted by* $Y \leq_{icv} Z$ *]*

Theorem 18 *Let* Y *and* Z *be two random variables with distributions* H *and* G *respectively, such that* $E[Y] \leq E[Z]$.

1. *Assume that the distributions* H *and* G *are single-crossing such that* $G \geq H$ *for* $x \leq x^*$ *and* $G \leq H$ *and for* $x \geq x^*$. *Then* $Y \leq_{icx} Z$.

2. *Assume that the distributions* H *and* G *are single-crossing such that* $G \geq H$ *for* $x \geq x^*$ *and* $G \leq H$ *and for* $x \leq x^*$. *Then* $Y \leq_{icv} Z$.

In order to apply the above results to our framework, let X denote the random variable governing a contestant's ability, and let F be the corresponding distribution function. We denote by $X_{(i,n)}$ the random variable corresponding to the i-th order statistic out of n independent variables, each identical to X (that is, $X_{(n,n)}$ is the highest order statistic, etc...), and we denote by $F_{(i,n)}$ the respective distributions. It is well known that:

$$F_{(i,n)}(s) = \sum_{j=i}^{n} \binom{n}{j} F(s)^j (1 - F(s))^{n-j} \tag{37}$$

$$dF_{(i,n)}(s) = \frac{n!}{(i-1)!(n-i)!} F(s)^{i-1} (1 - F(s))^{n-i} F'(s) \tag{38}$$

Theorem 19 *The following relations hold*[29]*:*

[28] In the economics literature this order is sometimes called "second-order stochastic dominance." But note that some authors use this term to obtain a variability ranking of random variables with the same mean. Here we need the more general definition.

[29] For future references we reproduce here the strong results, involving the hazard rate order. In this paper we employ the weaker versions (implied by Theorem 16-1) for the usual stochastic order .

1. $X_{(i,n)} \leq_{hr} X_{(i+1,n)}$ *for* $i = 1, 2, ..., n-1$

2. $X_{(i-1,n-1)} \leq_{hr} X_{(i,n)}$ *for* $i = 2, 3, ..., n-1$

3. $X_{(i,n)} \leq_{hr} X_{(i,n-1)}$ *for* $i = 1, 2, ..., n-1$

Fix agent j, and let $F_i^n(s), 1 \leq i \leq n$ denote the probability that agent j with type s meets $n-1$ competitors such that $i-1$ of them have lower types, and $n-i$ have higher types. We then have

$$F_i^n(s) = \frac{(n-1)!}{(i-1)!(n-i)!}(F(s))^{i-1}(1-F(s))^{n-i} \tag{39}$$

Lemma 20 *The following relations hold:*

1. $F_1^n(s) = 1 - F_{(1,n-1)}(s)$

2. $F_i^n(s) = F_{(i-1,n-1)}(s) - F_{(i,n-1)}(s),$ *for all* $i = 2, ..., n-1$

3. $nF(s)dF_{(i,n-1)} = idF_{(i+1,n)},$ *for all* $i = 2, ..., n-1$

Proof. The relations follow immediately from the respective definitions given above. ■

9 References

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