SONDERFORSCHUNGSBEREICH 504

Rationalitätskonzepte,
Entscheidungsverhalten und
ökonomische Modellierung

No. 02-07

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January 2002

The authors are grateful to Tilman Börgers, Holger Müller and seminar participants in Gerzensee for useful comments. We also thank Mathias Meisel, portfolio manager at Zurich Invest, for insightful discussions about the telecommunications industry. Financial support by the Sonderforschungsbereich 504 is gratefully acknowledged.

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A Stylized Model of the German UMTS Auction[1]

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Abstract

This paper discusses some economic aspects of the recent German and Austrian UMTS license auctions. We consider a stylized model of the open ascending auction with incomplete information and market externalities. It is shown that, if the dominant incumbent is not successful in pushing the weakest entrant out of the market, he will face ex-post spurious price increments. We argue that this feature of the German auction design caused a significant risk for the bidding firms. In particular, being aware of these risks, an incumbent may be willing to accommodate the entrant earlier than what one would expect from the valuations alone. We compare our predictions with the observed outcomes.

JEL classification: D44, H21

[1]- incomplete and preliminary - This version: November 2001. First version: October 2000. The authors are grateful to Tilman Börgers, Holger Müller and seminar participants in Gerzensee for useful comments. We also thank Mathias Meisel, portfolio manager at Zurich Invest, for insightful discussions about the telecommunications industry. Financial support by the Sonderforschungsbereich 504 is gratefully acknowledged.

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1. Introduction

This paper illustrates the possibility that rational bidders in prominent open ascending auctions may suffer from regret subsequent to an auction because it turns out that they have paid significantly more than necessary. A very illustrative example is the recent German UMTS spectrum auction. In this particular case, the two dominant incumbents Mannesmann Mobilfunk and Deutsche Telekom, in an attempt to push one of the potential entrants out of the market, did initially not want to give up bidding on a third frequency block. While the seventh bidder dropped out at a level of approximately 6 bn Euro, the final price was near to 8 bn Euro, and the allocation of licenses was the same that would have been obtained if the winning bidders had reduced their demands at the lower level. In this sense, the incumbents' strategy remained unsuccessful. Telekom officials later said: "The levels reached were insane." (Financial Times, 18.8.2000).

The formal analysis is based on a stylized model of the German UMTS auction incorporating elements of incomplete information, bidder asymmetry, and market externalities. Our central result says that, in the flexible design that was chosen by German regulators,[4] there exists an equilibrium where one of the incumbents tries to fight the weakest potential entrant out of the market, but where he remains unsuccessful in doing so with positive probability. In the latter case, i.e., when preemption remains unsuccessful, then an allocation arises that could have resulted at lower prices if the winning bidders had reduced their demand earlier. As a consequence, prices generated by the German design, conditional on the event that a six-player market would occur, can be higher than those obtained in a less flexible design in which the number of licenses is determined exogeneously.

[4] In fact, Germany and Austria were the only European countries in which the number of licenses was determined endogeneously.

There are at least two reasons why the policy maker has an interest to avoid such outcomes. Firstly, to the extent that bidders are aware of the risks involved, negative incentive effects may reduce the number of bidders, and thereby reduce competition. This was a very realistic feature of the German UMTS auction, prior to which six eligible bidders withdraw from participation, so that the auction started ultimately with only seven bidders for a maximum of six licenses. Even worse, in the subsequent Austrian auction in autumn 2000, where the outcome of the German mechanism was still fresh in minds, only six bidders entered the auction that was equivalent to the German design. A second reason to avoid regretful outcomes in auction design is that, as it was the case for Germany, the outcome of the auction may generate the impression that the government tried to unfairly exploit the bidding firms, and lead to attempts to renegotiate by those asked to pay the license fees. Finally, the exposure may lead to shareholder value destruction, and may therefor have the potential to affect the financial stability of the telecommunications industry.

In a related paper, Cramton (1997) has argued with a simple example that not allowing package bids may lead to inefficient participation, even under complete information. The example says that there is one bidder with a car and a trailer, who values two parking spots together at \$100, and another bidder with just one car, who values only one spot at \$75. The only equilibrium here is that the first bidder does not participate, and the second bidder places a minimum bid. Cramton mentions that in the presence of incomplete information, the first bidder will bid for the pair only if there is a sufficiently high probability that the second bidder has a low valuation. While this example illustrates the exposure problem very well, there is a significant difference to the present study in that there need not be a priori complementarities between the units to be auctioned. Rather, in our setting the demand reduction is a result of a expected competition in the UMTS market which generates endogeneous externalities from any additional entrant in that market. We show that even with decreasing marginal valuations, the exposure problem may arise in auctions with externalities. As a consequence, it may happen that a bidder ends up making losses with positive probability, when compared to using a more defensive bidding strategy.

Regretful outcomes may occur for other reasons than from a flexible auction design. In a study for the Dutch Ministry of Finance, the CPB Netherlands Bureau of Economic Policy Analysis identified a number of sources of overbidding in spectrum auctions (cf. Bennett and Canoy, 2000). These sources refer essentially to misaligned management incentives such as fear of reputational loss, but also to profitability misperceptions. While we do not deny that these factors might have played a role especially in the European UMTS auctions, it seems nevertheless appropriate to clarify that the specific auction format used in Germany and Austria had an additional problem that would have to be faced even by perfectly rational bidders.

There are at least four related lines of research in the literature. Firstly, the auction formats used in the European UMTS auctions is currently being studied extensively, e.g., by Börgers and Dustmann (2001), Grimm, Riedel, and Wolfstetter (2001), Jehiel and Moldovanu (2000), Klemperer (2001), and van Damme (2001). Secondly, there are theoretical contributions by Engelbrecht-Wiggans and Kahn (1998) and by Noussair (1995) that analyze more general uniform-price auctions, a variant of which is used in this paper. Thirdly, our analysis is also related to Gilbert and Newbery (1982) who stressed the natural asymmetry between incumbents and entrants in patent auctions. Finally, Jehiel and

Moldovanu (1996) consider the question of strategic non-participation in markets with externalities.

The rest of the paper is structured as follows. In Section 2, we briefly describe the German design, and the outcomes that resulted in Austria and Germany. Section 3 describes the model. In Section 4, we derive an equilibrium, and discuss the outcome. Section 5 concludes. Appendix A provides somewhat technical details of the auction model, while appendix B contains proofs.

2. Brief description of the German and Austrian UMTS auctions

This section briefly describes some features of the design and the outcome of the German and the Austrian UMTS auction in the year 2000.

The UMTS auction[5] had two stages. The first stage followed an open upward simultaneous multiple round format in two stages. In this stage, 12 frequency packages of approximatly 2x5 MHz (Megahertz) each in the so-called paired band, were put up for auction. A bidder may obtain in this stage between two and three frequency packages. As a result, there may be between four and six license holders.

The bidders who have obtained a license in the first stage are entitled to participate in the second stage. In the second stage five frequency packages of approx. 5 MHz each in the unpaired band were put up for auction, as well as those packages in the paired band which may have not been auctioned off in the first stage. The maximum number of frequency packages that can be obtained in this second

[5]For a complete description, see the official document by the Regulierungsbehörde für Telekommunikation und Post (2000).

stage is two frequency packages in the unpaired band and one frequency package in the paired band.

The German auction was conducted by the German Regulierungsbehörde für Telekommunikation und Post, represented by Klaus-Dieter Scheurle. Initially, there were 7 bidders, after 6 other potential bidders ultimately withdrew from the auction. The first stage of the German auction ended August 17, 2000, after 3 weeks or 173 rounds of bidding, and resulted in 6 licenses being awarded. The licensed firms were E-Plus Hutchison, Group 3G, Mannesmann Mobilfunk, MobilCom Multimedia, T-Mobil, and VIAG Interkom. The total of the bids was approximatly Euro 50 bn. Each licensed firm acquired 2 blocks of paired spectrum, paying approximately Euro 8.4 Bn. One prominent feature of the auction was that after one of the potential entrants, Debitel, left the auction after 125 rounds and after the price level reached Euro 2.5 Bn per block. Since 6 firms were left bidding for a maximum of 6 licenses, the auction could have stopped immediately. Instead, the remaining firms and in particular the two large incumbents continued bidding in order to acquire more capacity. But no other firm was willing to quit, and bidding stopped in round 173. Compared to round 125, there was no change in the physical allocation, but collectively firms lost Euro 20 Bn.

The Austrian auction was opened November 2, 2000, at Vienna. The following companies have been participating in the auction: Connect, Hutchison 3G, Mannesmann 3G, max.mobil., Mobilkom, and 3G Mobile (Telefonica). For a frequency package in the paired band the minimum bid was Euro 50 m. In Austria there were exactly 6 bidders for a maximum of 6 licenses. Hence, in principle, the license auction could have ended immediately at the reserve price. Nevertheless, the bidding continued for another 16 rounds, before stopping with 6 licensed firms, each paying on average about Euro 118 m. per license.

For more thorough discussion of the design and the possible outcomes, see Jehiel and Moldovanu (2001) or Grimm, Riedel, and Wolfstetter (2001).

3. The model

There are 12 frequency blocks to be auctioned off, and $n > 0$ bidders. Each bidder i possesses valuations $v^i_m(k)$, where m is the number of frequency blocks obtained, and k is the number of players in the market.

The model developed below will focus on the first stage of the German design. The valuations therefore capture the valuations that the firms attribute to specific outcomes of the first stage. We will discuss later why the second stage does not affect the arguments.

We will also abstract from the fact that the German auction must be properly considered as a part of a more global process, in which international telecom firms have fought about the position in the European market. E.g., it has been suggested by van Damme (2001) that the high prices in Germany resulted from a struggle mainly between KPN, represented by E-Plus, and Telefonica, represented by Group 3G.

We consider a specific setting with $n = 6$ bidders. In Germany, the auction started with debitel as a seventh bidder. The abstraction from additional participants is for simplicity only. It will become clear that the outcome described below can also be rationalized when there are additional bidders with sufficiently weak valuations. In Austria, the auction indeed started with six bidders.

We assume that bidders can be ordered according to their valuations, i.e., that

$$v_m^1(k) > v_m^2(k) > ... > v_m^6(k) \tag{1}$$

for all $m \in \{2,3\}$ and all $k \in \{4,5,6\}$. These valuations are assumed to be increasing in the number of frequency blocks m and decreasing in the number of license holders k.

We assume that bidder $i = 6$ has an ex-ante unknown valuation

$$v := v_2^6(6) \in [\underline{v}, \overline{v}] \tag{2}$$

which is private information to him. The distribution of v is assumed to have full support on $[\underline{v}, \overline{v}]$. The corresponding cumulative distribution function is denoted by $F(v)$, and assumed to be differentiable. So in particular, there is not valuation v that arises with strictly positive probability.

To focus the analysis on the case where the dominant incumbent fights the weakest entrant, we assume that it is ex-ante not clear whether the dominant incumbent's per-unit valuation for a large license in a five player market is below or above the weakest entrant's per-unit valuation of a small license in a six-player market, i.e., we assume

$$\frac{\underline{v}}{2} < \frac{v_3^1(5)}{3} < \frac{\overline{v}}{2}. \tag{3}$$

We also assume that for all bidders but bidder 1, the value of the third frequency block is not too large, i.e., that

$$\frac{v_3^2(4)}{3} < \frac{\underline{v}}{2}. \tag{4}$$

Some important consequences of the above conditions on the valuations are illustrated in Figure 1. This diagramm will be especially useful in determining - Figure 1 here -

the price and allocation that will result from unilateral deviations of profit-maximizing firms.

The auction proceeds as follows. Bidders may bid for either two or three blocks, yet under the restriction that they may not increase the number of requested units during the auction ("activity rule"). To capture these strategic possibilities, we assume that bidder i bids up to b_3^i for three blocks, and up to b_2^i for two blocks, where

$$0 \leq b_3^i \leq b_2^i. \tag{5}$$

We may then summarize i's strategy by a bid $b^i = (b_3^i, b_2^i)$. While our choice of the strategy spaces is simplistic in comparison to the actual possibilities, it will become clear later that it is not overly restrictive because no firm will observe new information before the end of the auction.

The frequency blocks are assigned to the 12 highest unit bids under the provision that no bidder obtains just one block, and subject to a uniformly randomizing tie-breaking rule. The *final price* p^* is then the price of the highest losing bid. The precise mechanics that determine price and allocation is explained in Appendix A. For most of what follows, however, it suffices to work with the intuitive notion that the highest bids win the auction, and that the final price is the highest losing bid.

4. Analysis

Proposition 1 (equilibrium). *Under the assumptions made above, the following strategy profile constitutes an equilibrium in the stylized UMTS auction*

$$b^{*,i} = \begin{cases} (\beta^*, \frac{v_2^1(6)}{2}) & i = 1 \\ (\frac{v_3^i(5)}{3}, \frac{v_2^i(6)}{2}) & i = 2, ..., 6 \end{cases} \tag{6}$$

where the dominant incumbent either chooses either $\beta^* = v_3^2(5)/3$ *("accommodate"), or* $\beta^* \in [\underline{v}/2, v_3^1(5)/3]$ *("fight").*

Proof. See the appendix. □

The strategy profile described above allows for two basic ways in which the dominant incumbent (bidder 1) may behave in equilibrium. Either, he reduces demand to two units in accordance with bidder 2. Or, he tries to push the weakest bidder 6 out of the market.

Note that the equilibrium is not an artifact of our sealed-bid specification. Indeed, because there is uncertainty about $v_2^6(6)$ only, no useful information is revealed until the "end" of the auction. More precisely, there is no value of conditioning one's bid on the observed equilibrium behavior of the weakest bidder 2 because his only information-bearing action is his exiting the auction. However, this action either does not occur or it ends the auction in equilibrium.

The introduction of a second stage, as in the actual auction format in Germany and Austria, is not likely to affect the structure of the above equilibrium outcome. The second stage is strategically most relevant in the case where one unit is not frequency block is not sold in the first period. In our equilibrium, this is a result of a successful preemption by the dominant incumbent. The frequency block left over from the first stage is the auctioned among all winners of the first stage, i.e. of all 3G license holders. Applying the logic of backward induction, rational bidders will have formed beliefs about the expected value that they could obtain in such a setting. While we have not tried to provide conditions under which the second stage does not affect bidding behavior, we believe that deriving these conditions would probably not add much to the current analysis.[6]

[6]See, however, Grimm et al. (2001) for a reduced-form model in which the second stage is

The payoff consequences of accommodating vs. fighting are as follows. If the dominant incumbent reduces his demand early, then all six bidders obtain two frequency blocks for a unit price of

$$p_0 := \frac{v_3^2(5)}{3}, \tag{7}$$

and bidder 1's expected payoff is correspondingly

$$\underline{U}_1 = v_2^1(6) - 2p_0. \tag{8}$$

If, however, he fights the weakest entrant by bidding for three blocks up to $\beta \in [\underline{v}/2, v_3^1(5)/3]$, then his expected payoff is

$$U_1(\beta) = \int_{\underline{v}}^{2\beta} \{v_3^1(5) - \frac{3v}{2}\} dF(v) + \int_{2\beta}^{\overline{v}} \{v_2^1(6) - 2\beta\} dF(v). \tag{9}$$

Indeed, if $v/2 < \beta$, then the incumbent wins a large license in a five-player market, and pays the price level $v/2$ per unit, at which the entrant gives up. On the other hand, if $v/2 > \beta$, then the incumbent wins only a small license in a six-player market, and pays a per-unit price at which he gives up bidding for three blocks.

Write $\Delta U_1 = U_1(\beta) - \underline{U}_1$.

Proposition 2 (exposure). *In terms of expected utility, the difference between accomodating and fighting the entrant is*

$$\begin{aligned} \Delta U_1 \quad &= \quad pr(\frac{v}{2} < \beta)\left\{v_3^1(5) - v_2^1(6) - p_0 - 3\Delta^w\right\} \\ + \quad & \quad pr(\frac{v}{2} \geq \beta)\left\{-2\Delta^l\right\}, \end{aligned} \tag{10}$$

where

$$\Delta^w \ : \ = E[\frac{v}{2} | \frac{v}{2} < \beta] - p_0 \tag{11}$$

$$\Delta^l \ : \ = \beta - p_0 \tag{12}$$

modeled explicitly.

is the expected increment in the price per unit in case the incumbent wins and loses the battle, respectively.

Proof. Immediate from (8) and (9). □

Equation (10) captures the dilemma in which a dominant incumbent might find himself. The bidding contest would generate an uncertainty about the outcome as follows. If the incumbent wins, i.e., if $v < \beta$, then he realizes scale effects and ensures oligopoly gains. He has to pay p_0 for the additional unit, and a price of exemption Δ^w for each frequency block. This will be a desirable outcome for the incumbent. However, if the entrant wins the auction, the incumbent pays an additional increment Δ^l for each of the two frequency blocks he would have obtained anyway. This illustrates the exposure problem that resulted from the German auction design.

We will now derive the optimal bidding strategy for bidder 1.

Proposition 3 (bid shading). *If the dominant incumbent fights the weakest entrant, then the optimal bid is given by*

$$v_3^1(5) - v_2^1(6) = \beta^* + \frac{1 - F(2\beta^*)}{f(2\beta^*)}. \tag{13}$$

Moreover, to mitigate the exposure problem, the dominant incumbent reduces his bid for the third block, i.e., $\beta^* < v_3^1(5)/3$.

Proof. See the appendix. □

The above bid shading is an instance of a more general phenomenon of bid shading in uniform price auctions (cf. Ausubel and Cramton, 1998). The incumbent lowers his demand for the third unit because a higher bid for the third frequency block increases the price for the first two blocks, as described by Proposition 2, which lowers the incentives for bidding up to the true valuation.

Proposition 3 suggests the following comparative statics result. Consider a setting in which the dominant incumbent fights the weakest entrant. Consider an alternative setting in which the weakest entrant is more aggressive in the sense that the distribution function of his valuation for a small license is $G(v)$, and $G(v)$ dominates $F(v)$ in the hazard-rate order.[7] Then, from (13), the equilibrium bid of the dominant incumbent is lower, and he may even choose to accommodate the entrant without fighting. As a consequence, the entrant should like to appear strong, in order to demotivate the incumbent, which would lead to lower prices and a higher likelihood of winning for the entrant. On the other hand, the revenue-raising government would have incentives to make entrants look weak, in order to encourage a battle.

For completeness, we note that the incumbent's bid function can be interpreted as a reserve price, for which he would be willing to sell the third frequency block to the entrant in the presence of imperfect information.

The above expression for the optimal bid shows that the equilibrium bid depends only on the marginal valuation that a third block and the advantage of operating in a five-player market. As higher prices must be paid for all three blocks, this opens up the possibility of regret. We say that regret occurs whenever the bidding strategies do not form an ex-post equilibrium, i.e., the dominant incumbent could have reached the same allocation at a lower payment (by not fighting the entrant),

[7] I.e., for all v, we have
$$\frac{F'(v)}{1-F(v)} \leq \frac{G'(v)}{1-G(v)}.$$

under the assumption that he knows the outcome of the auction.

Proposition 4 (regret). *There exist parameter values of the model under which the dominant incumbent fights the weakest entrant in equilibrium, but does so in vain with positive probability.*

Proof. Consider the distribution function

$$F(v) = \begin{cases} 0 & v < \underline{v} \\ 1 - (\frac{\overline{v} - v}{\overline{v} - \underline{v}})^4 & \underline{v} \leq v < \overline{v} \\ 1 & v \geq \overline{v}. \end{cases}$$

for the entrant's valuation. We determine equilibrium strategies under the condition that the incumbent participates with a demand of three. From Propositions 1, we know that the weakest entrant bids up to $v/2$. Proposition 3 predicts that the fighting incumbent will bid up to

$$\beta^* = 2\{v_3^1(5) - v_2^1(6)\} - \frac{\overline{v}}{2}. \tag{14}$$

The dominant incumbent's utility can be higher from fighting, as the following argument shows. Choose $p_0 = v_3^2(5)$ close to $\overline{v}$, and note that

$$\frac{\partial U_1}{\partial \beta}(\frac{v}{2}) = \frac{8}{\overline{v} - v}\{v_3^1(5) - v_2^1(6) - \beta\} - 2 \tag{15}$$

$$= 4\frac{\overline{v}/2 - \beta^*}{\overline{v} - v} > 0. \tag{16}$$

Thus, accommodating is always suboptimal. Note now that whenever $v/2 > \beta^*$, the entrant obtains a license, and the incumbent regrets his bidding strategy. □

Intuitively, Example 1 extends immediately to the more general case where there is a sufficiently high probability for a valuation v close to, but still above $\underline{v}$. This is because in such a case, demand reduction effects are not very strong, so bidder 1 bids actively for the third frequency block, at least not when the entrants minimum per-unit valuation $\underline{v}/2$ is not very much above the accomodating level p_0. However, the incumbent will in general obtain the third frequency block only with probability strictly smaller than one.

It is useful to compare the design with an alternative design in which six small licenses are auctioned off. It is clear that then the weakest entrant, i.e., bidder $i = 6$, obtains a license in any case, even when his valuation is comparatively low. In the actual setting, if the weakest entrant had possessed a sufficiently low valuation, then it would have been possible for dominant incumbents to induce a five-player market. So this additional flexibility in fact can work in a beneficial manner. However, for the firms, this additional flexibility comes at the cost of having to bear a significant risk. If the dominant players remain unsuccessful in their battle for the small market, all bidders have to bear significantly raised prices.

With our assumptions, we have focused the analysis on the outcome actually observed. The four-player outcome is not unlikely from an ex-ante perspective, if bidders are very keen to obtain a third block. If the additional value of the third frequency block, and a four-player market is very high, as from an ex-ante perspective it was plausible, then demand reduction effects could have been of minor importance, and a four-license outcome could have occurred.

We believe that it is feasible to modify the assumptions in a way that the regret is sufficiently strong to engender non-participation. More precisely, for lower values of $v_2^1(6)$, any participation of bidder 1 may create value for this bidder only if a large license can be obtained. However, this can only be achieved with a probability strictly below 1. If $v_2^1(6)$ is sufficiently low, then the losses incurred from having lost the battle are so large, that the incumbent may prefer not to participate.

Grimm et al. (2001) consider a three-player four-stage game of incomplete information, in which the weakest entrant be one of two types. They show that under certain conditions, there is an equilibrium in this game in which one dominant incumbent tries predation, and another resigns. The authors point out that there is a free-rider problem between the two incumbents when a remaining frequency block can be purchased cheaply in the second stage of the auction. While some aspects of their analysis are related, it is not clear with discrete types why the incumbent can in fact end up in regret. After all, if the model is a proxy for an open ascending auction, then the incumbent will be able to observe whether the entrant has a high valuation already if the level of bids exceeds the entrant's low valuation.

5. Conclusion

This paper offered a stylized model of the German UMTS auction, which allows to study the interplay of incomplete information and market externalities. We describe an equilibrium in which bidders reduce demand from three to two one by one, until with positive probability, the six-bidder outcome is reached. The central result says that there the auction outcome may be ex-post inefficient, and in fact the dominant incumbent may make losses from having tried to push the weakest entrant out of the market.

The predictions of the model can be interpreted in a way that the auction design in German and Austrian UMTS auctions incurred significant risks to the involved bidding consortia. Because of the spurious price increments in the later rounds, the observed outcome in Germany could correspond to the regret outcome in our model.

Appendix A. Details on the auction model

This appendix serves the purpose to describe in more formal terms the determination of final price and allocation in our auction model.

Individual demand of bidder i at price p is given by

$$D^i(p) = \begin{cases} 3 & \text{if } p \leq b_3^i \\ 2 & \text{if } b_3^i < p \leq b_2^i \\ 0 & \text{if } p > b_2^i \end{cases} . \tag{17}$$

Aggregate demand is then

$$D(p) = \sum_{i=1}^{n} D^i(p). \tag{18}$$

Define individual and aggregate *robust* demand by

$$D_+^i(p) = \lim_{\substack{p' \to p \\ p' > p}} D^i(p) \tag{19}$$

and

$$D_+(p) = \sum_{i=1}^{n} D_+^i(p), \tag{20}$$

respectively. The ascending auction format determines the smallest price which yields a robust demand of at most 12, i.e.,

$$p^* := \min\{p | p \geq 0 \text{ and } D_+(p) \leq 12\}. \tag{21}$$

Lemma 1. *The unit price p^* realized in the auction is well-defined. Moreover, if $n \geq 5$, then $p^* \geq 0$ is uniquely characterized by the property that the demand is strictly larger than 12, but robust demand is at most 12, i.e., by*

$$D(p^*) > 12 \geq D_+(p^*). \tag{22}$$

Proof. The set $M := \{b | b \geq 0 \text{ and } D_+(b) \leq 12\}$ is nonempty since demand goes to zero for high prices. As $D_+(b)$ is semicontinuous from the right, a minimum always exists, so that b^* is well-defined. Next we show that b^* satisfies property (22). By definition, $12 \geq D_+(b^*)$. To provoke a contradiction, assume $D(b^*) \leq 12$. Then, because $n \geq 5$, we have $D(0) > 12$, so that $b^* > 0$. Since $D(b)$ is piecewise constant and semicontinuous from the left, there is an $\varepsilon > 0$ such that $b^* - \varepsilon > 0$ and still $D_+(b^* - \varepsilon) \leq 12$, which is a contradiction to the definition of b^*. Assume now that $b' \geq 0$ satisfies $D(b') > 12 \geq D_+(b')$. Then, $b' \in M$. If $b' = 0$, then clearly $b^* = 0$. If $b' > 0$, then $D(b' - \varepsilon) = D(b') > 12$ for all sufficiently small $\varepsilon > 0$. In particular, $D_+(b' - \varepsilon) > 12$ for small ε, so that $b' = \min M$. □

The allocation assigns $D^i_+(p^*)$ frequency blocks to bidder i, when $D_+(p^*) = 12$. If $D_+(p^*) < 12$, then the allocation is uniformly random on assignments D^i satisfying

$$D^i_+(p) \leq D^i \leq D^i(p^*), \tag{23}$$

and

$$\sum_{i=1}^{n} D^i = 12. \tag{24}$$

Appendix B. Technical proofs.

Proof of Proposition 1. The proof has the following structure. We will first show that it is suboptimal for bidder 1, i.e., the dominant incumbent, to choose $b^1_3 \notin [v^1_3(5)/3, v^2_3(5)/3]$. We then specify β^* and show that bidder 1 has no incentive to deviate. We will show then that no bidder $i = 2, ..., 6$ has an incentive to deviate. To simplify the wording in the sequel, we will follow the tradition and argue as if in case of indifference, bidders prefer to use the proposed strategies. Of course, this does not affect the formal argument. We will also make continous use of our assumptions, which are graphically summarized in Figure 1. Use Figures 2 and 3 to keep oversight.

Write $p_0 := v_3^2(5)/3$, and $D^{-i}(p)$ for the aggregate demand of the bidders $j \neq i$ at price p.

- Figure 2 here -

Claim 1. *The dominant incumbent chooses* $b_2^1 \geq p_0$. Consider Figure 2, which exhibits the set of feasible strategies for bidder 1. Assume that $b_2^1 < p_0$. Then, from Figure 1, we have $D^{-1}(b_2^1) \geq 11$, so that it is impossible for bidder 1 to obtain a license. So his utility will be zero in this case. However, accommodating by bidding $b^1 = (p_0, v_2^1(6)/2)$ wins a small license in a six-player market with certainty at a price $p_0 < v_2^1(6)/2$, and gives a strictly positive utility. Hence, the dominant incumbent will set $b_2^1 \geq p_0$.

Claim 2. *The dominant incumbent chooses* $b_3^1 \geq p_0$. Assume that $b_3^1 < p_0$. Then, using claim 1, and with a view on Figure 1, we have $D_+(p_0) = 12$. Thus, this amounts to accommodating the entrant in terms of expected payoffs. So bidder 1 will not lose by setting $b_3^1 = p_0$.

Claim 3. *The dominant incumbent does not choose* $b_3^1 \in (p_0, \underline{v}/2)$. Assume $b_3^1 \in (p_0, \underline{v}/2)$. Then bidder 1 would obtain a small license at the per-unit price $b_3^1 > p_0$. Hence, he could decrease the final price by lowering b_3^1 to p_0.

Claim 4. *The dominant incumbent chooses* $b_2^1 \geq \overline{v}/2$. Assume that $b_2^1 < \overline{v}/2$. Then, using claim 2, increasing b_2^1 to $\overline{v}/2$ will leave expected payoffs unchanged.

Claim 5. *The dominant incumbent chooses* $b_3^1 \leq v_3^1(5)/3$. Assume that $b_3^1 > v_3^1(5)/3$. From claim 3, we may assume without loss of generality that $b_2^1 > \overline{v}/2$. Then reducing b_3^1 to $v_3^1(5)/3$ may affect payoffs only in those cases where $v/2 \geq$

$v_3^1(5)/3$. However, in these cases, payoffs with a negative sign are converted into payoffs with a positive sign, and therefore it is never optimal to choose $b_3^1 > v_3^1(5)/3$.

Claim 6. *The dominant incumbent chooses $b_2^1 = v_2^1(6)/2$.* There are two cases. Either $b_3^1 = p_0$ or $b_3^1 \in [\underline{v}/2, v_3^1(5)/3]$. In both cases, we may assume that $b_2^1 = v_2^1(6)/2$.

We specify β^* as a payoff-maximizing choice of b_3^1 given that $b_2^1 = v_2^1(6)/2$. Such an optimal choice does always exist because b_3^1 is chosen from the compact interval $[v_3^1(5)/3, p_0]$, and the expected payoff-function is continuous on this interval (recall that we assumed that the distribution of v has no atoms). It is clear now that the dominant incumbent has no incentive to deviate. We proceed by checking that deviations are also not profitable for the remaining bidders. - Figure 3 here -

Claim 7. *Bidders $i = 2, ..., 6$ choose $b_3^i \leq \overline{v}/2$.* If $b_3^i > \overline{v}/2$, then, from Figure 1, bidder i will obtain a large license with certainty. The resulting market will have either four or five players, depending on whether $v/2 < b_3^1$ or not. Payoffs will be $v_3^i(4) - \frac{3}{2}v$ in the former case, and $v_3^i(5) - v_3^1(5)$ in the latter. Both expressions are negative because of assumption (4). So bidder i would make a loss, which can be easily avoided by sticking to the candidate equilibrium strategy.

Claim 8. *If bidders $i = 2, ..., 5$ choose (b_3^i, b_2^i) such that $b_3^i < \underline{v}/2$, then they set $b_2^i = v_2^i(6)/2$.* Assume that $b_3^i < \underline{v}/2$. Then, since $D_+^{-i}(\underline{v}) \geq 10$, bidder i cannot hope for a large license. Hence, it is a best reply to bid any number $b_2^i > \overline{v}/2$. This proves the claim.

Claim 9. *Bidders $i = 2, ..., 5$ do not choose (b_3^i, b_2^i) such that $b_3^i \geq \underline{v}/2$ and $b_2^i > \overline{v}/2$.* Assume $b_3^i \geq \underline{v}/2$ and $b_2^i > \overline{v}/2$. By claim 7, we know that then

$b_3^i \in [\underline{v}/2, \overline{v}/2]$. Without loss of generality, we may assume $b_2^i = v_2^i(6)/2$. But then lowering b_3^i to the candidate equilibrium level $v_3^i(5)/3$ reduces the probability of winning a large license and making losses, and can only reduce the price that has to be paid for a small license.

Claim 10. *Bidders $i = 2, ..., 5$ do not choose (b_3^i, b_2^i) such that $b_3^i \geq \underline{v}/2$ and $b_2^i > \underline{v}/2$.* It is always preferable to switch to the candidate equilibrium strategy because this avoids losses from winning an expensive large license, and increases gains from winning a small license more often.

Claim 11. *Bidders $i = 6$ chooses $(b_3^6, b_2^6) = (v_3^6(5)/3, v/2)$.* Note that the payoff is zero for bidder 6 if $b_2^6 < v_3^1(5)/3$. Moreover, any $b_3^6 > v_3^1(5)/3$ is dominated. Hence $b_3^6 \leq v_3^1(5)/3 \leq b_2^6$. Thus, bidder 6 wins a small license at $v_3^1(5)/3$ if $v \geq v_3^1(5)/3$, and does not win otherwise.

The above sequence of claims shows that a deviation is not profitable for any bidder. This proves the assertion. □

Proof of Proposition 3. By Proposition 1, if the dominant incumbent fights, then $\beta^* \in [\underline{v}/2, v_3^1(5)/3]$. Note first that bidder 1 never sets $b_3^1 = \underline{v}/2$ because the distribution of v has no atom at $\underline{v}/2$, and he may lower his bid for the third block without loss, thereby lowering the final price. Thus, $b_3^1 > \underline{v}/2$. Differentiation of (9) gives

$$\frac{\partial U_1}{\partial \beta} = 2\{v_3^1(5) - v_2^1(6) - \beta\} f(2\beta) - 2\{1 - F(2\beta)\}. \tag{25}$$

In particular,

$$\frac{\partial U_1}{\partial \beta}(\frac{v_3^1(5)}{3}) = 2\{\frac{2v_3^1(5)}{3} - v_2^1(6)\} f(\frac{2v_3^1(5)}{3}) - 2\{1 - F(\frac{2v_3^1(5)}{3})\} < 0 \tag{26}$$

because

$$\frac{2v_3^1(5)}{3} < \overline{v} < v_2^1(6) \tag{27}$$

by assumptions (3)and (1). Thus, the first-order boundary condition is not satisfied, and $\beta^* < v_3^1(5)/3$, which proves that a fighting bidder 1 shades his bid in equilibrium. The necessary first-order condition (13) follows from (25). $\square$

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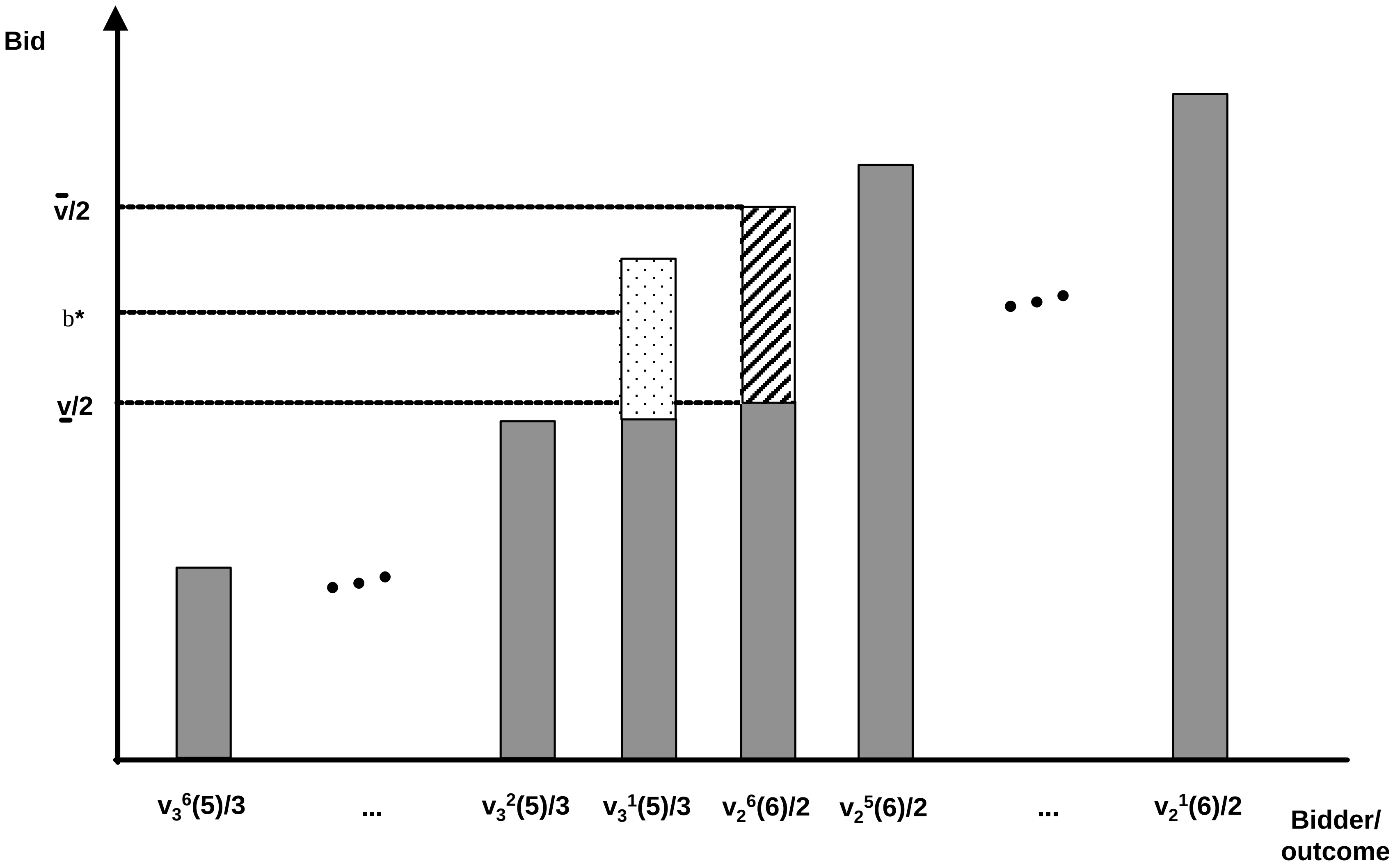

Figure 1: Bidder valuations in the stylized UMTS auction

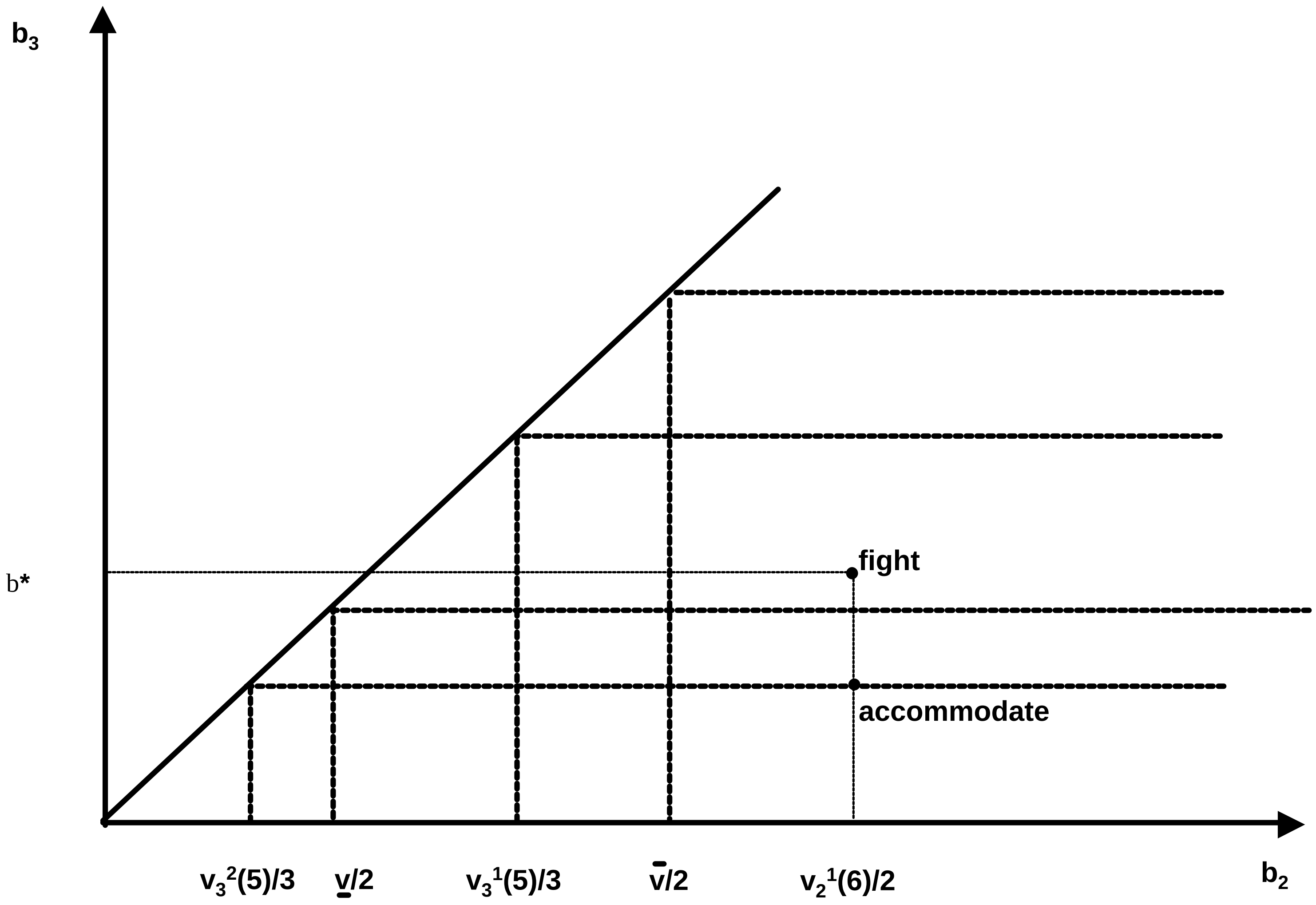

Figure 2: Equilibrium analysis for the dominant incumbent

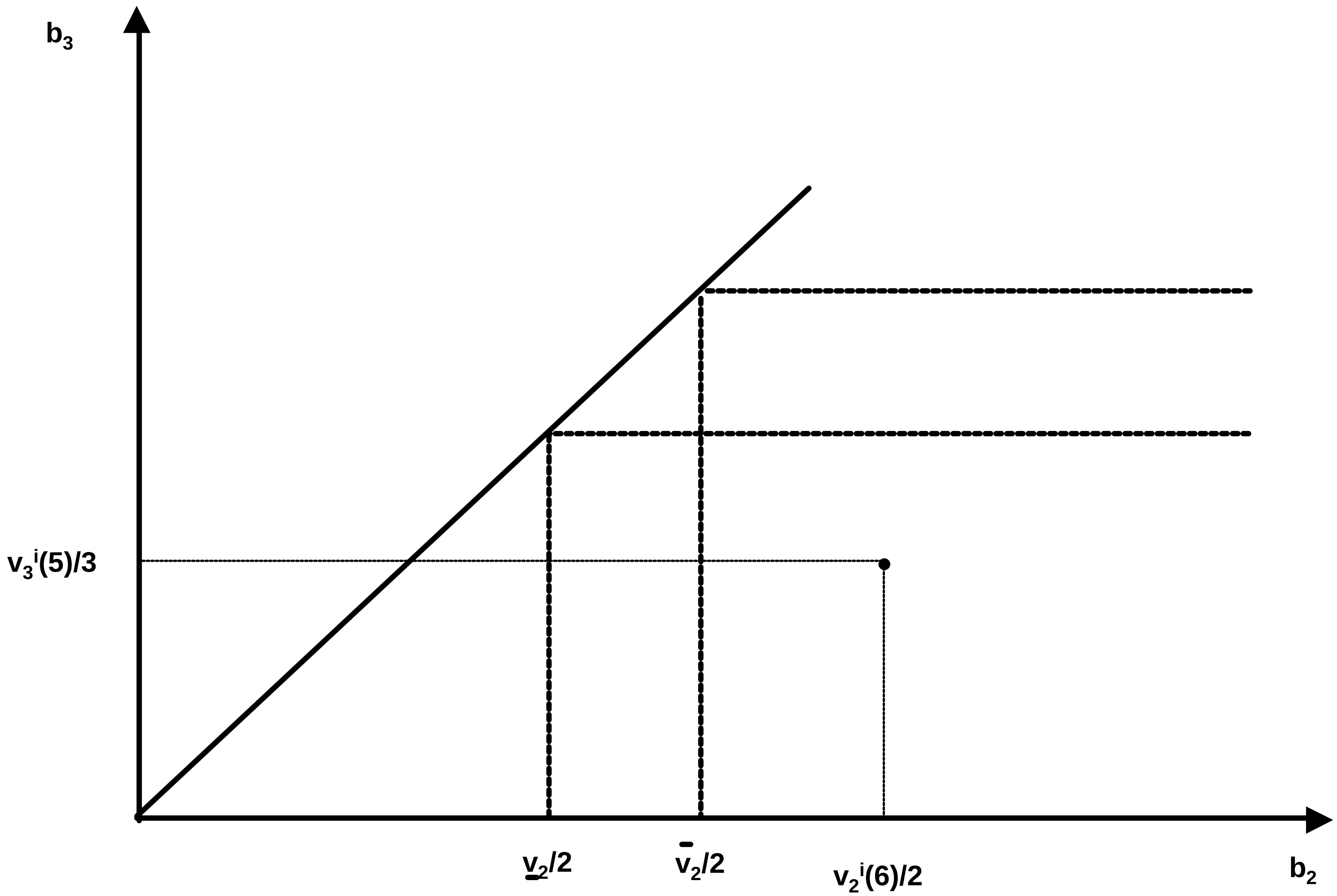

Figure 3: Equilibrium analysis for the non-dominant bidders

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