Centre for Energy Policy and Economics
Swiss Federal Institutes of Technology

What if?
Policy Analysis with
Calibrated Equilibrium Models

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Contents

1 Problem description

1.1 The model in a nutshell

The goal of this paper is to build up and apply a simple static model of world oil markets. For this purpose two market levels are defined: a regional (regions *r*) and an international (*world*). In each region, crude oil may be produced or may be not. There is one representative refinery in each region. It buys crude oil – either imported from the world market or produced in the region – and refines it into the fuels *f*. This process needs labor and capital, too, combined in a value added to the value of the crude oil (from the point of view of the refinery this is just an additional input cost for its production process). The produced fuels are sold in the region to cover the regional fuel demand or exported to the world market.

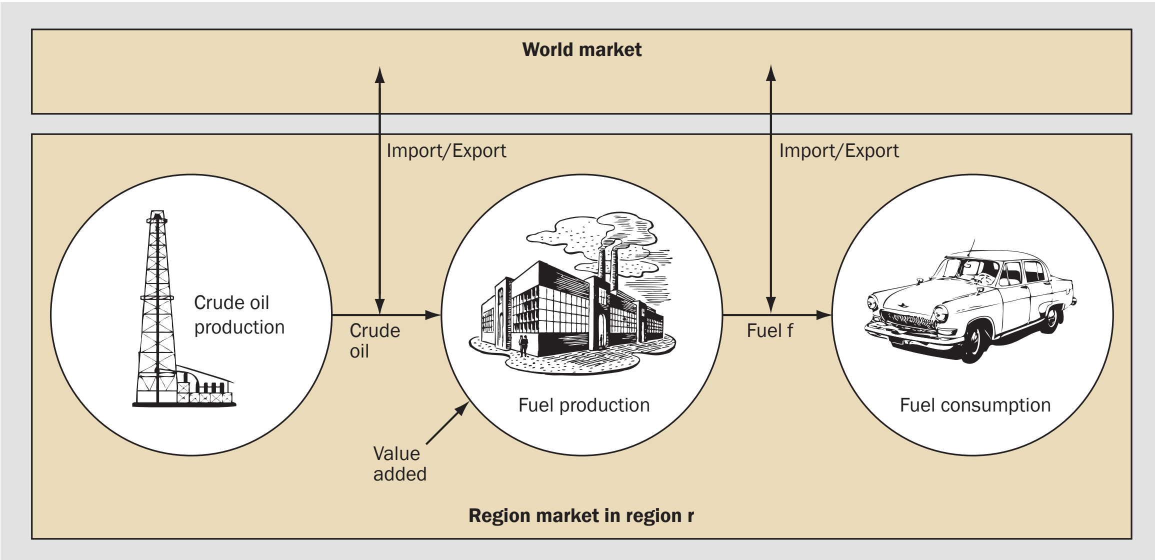


Figure 1: Elements of the oil market model.

1.2 Assumptions

- Basic elements: Regions *r*, goods *g* (*crude* and fuels *f*).
- Regional fuel consumption: The consumption of fuel *f* in region *r* ($d_{r,f}$) is characterized by a linear demand-function, with $p_{r,f}$ being the price and $A_{r,f}$, $B_{r,f}$ two region- and fuel-specific coefficients.

$$d_{r,f} = A_{r,f} - B_{r,f} \cdot p_{r,f}$$

- Regional fuel production: The production of fuels in region *r* ($s_{r,f}$) is characterized by a CET-function (see commentary box, page 4).

$$d_{r,crude} = \left[\sum_f a_{r,f} \cdot \left(s_{r,f}\right)^{\frac{1+\eta_r}{\eta_r}} \right]^{\frac{\eta_r}{1+\eta_r}}$$

($d_{r,crude}$ is the crude oil input to the refinery, η_r is the elasticity of transformation and $a_{r,f}$ are region- and fuel-specific coeffients.)

- The price (costs) of the value added to the refining process ($p_{r,va}$) is a linear (increasing) function of the crude oil input to the refinery ($d_{r,crude}$), with $p_{r,va}$ being the price of the value added and E_r, F_r two region-specific coefficients.

$$p_{r,va} = E_r + F_r \cdot d_{r,crude}$$

- Regional crude oil production: The production of crude oil in region r ($s_{r,crude}$) is characterized by a linear supply-function, with $p_{r,crude}$ being the price and G_r, H_r two region-specific coefficients.

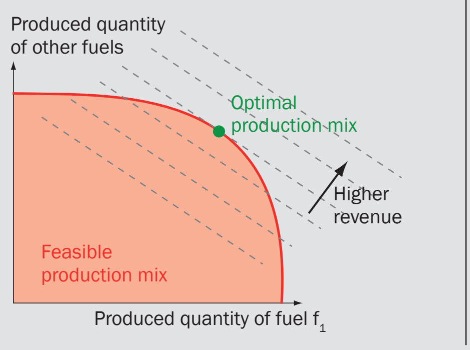
$$s_{r,crude} = G_r + H_r \cdot p_{r,crude}$$

- Imports and exports: Every unit of crude oil or fuel exported to the world market induces an export trade cost margin ($exc_{r,g}$). Analogously every unit of crude oil or fuel imported from the world market induces an import trade cost margin ($imc_{r,g}$).

CET-function: Which output-mix to choose?

Regarding the refining process with one input (crude oil) and several outputs (fuels) the CET-production-function describes the input as a function of the outputs. The basic idea of the CET-function is the possibility to transform one output into other outputs, characterized by a constant elasticity of transformation (CET). To illustrate this, the relationship of one refinery-output to the others is illustrated in figure 2 (left) where one product – let's say gasoline – is compared to the other products. The CET-function defines the frontier of the feasible production mix – i.e. the trade-off between producing the fuel f_1 or producing other fuels: By lowering the production of f_1 (i.e. by accepting an «opportunity cost»), the refinery can produce more of the other fuels (and vice versa).
The shape of the CET-function changes with different elasticities of transformation (see figure 3, right). The higher the elasticity is, the flatter becomes the production-frontier – i.e. the opportunity cost of producing one more unit of f_1 does not change much with the production-level of f_1.
Given the CET-production-function and a fixed level of input, the refinery has to find the optimal output-mix – i.e. the one generating the highest revenues (see figure 2). This is the optimization problem the refinery in region r has to solve. The analytical solution is presented in detail in appendix A1 (page 16).

Optimization problem of the refineries producing according to a CET-function:



Influence of the elasticity of transformation:

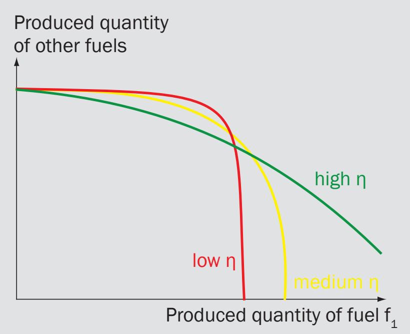


Figure 2 (left): The refinery produces according to a CET-function and has to find the optimal output mix.

Figure 3 (right): Influence of the elasticity of transformation.

2 Model formulation

2.1 COP- and MCP-approach

Basically there are two approaches to solve the described problem.

- An intuitive way to find an equilibrium is to formulate a constrained optimization problem (COP). The total surplus (consumer and producer surplus) is an appropriate objective to maximize – assuming cleared markets ($g + r{\cdot}g$ constraints) and CET-production in the regional refineries (r constraints). Since the COP can be solved in Excel which is available to all students, this paper focuses mainly on the COP-approach.
- An alternative approach is to formulate the problem as a mixed complementary problem (MCP). This involves the conversion of the problem into a system of equations solved simultaneously. The MCP-approach is discussed in appendix A2 (page 18).

COP-approach	MCP-approach
Maximize the total surplus subject to the following constraints: ▪ All markets are cleared. ▪ Refineries produce according to a CET-function. (Remark: The crude oil supply-function and the fuel demand-function are implicitly considered – namely in the total surplus. The detailed calculation is presented in the following chapter.)	Solve a system of equations: ▪ CET-refineries maximize profits which have to non-positive (perfect competition). ▪ Crude oil is produced according to the given supply-function. ▪ Fuels are consumed according to the given demand-function. ▪ All markets are cleared. ▪ No «import-export-short-cuts» allowed: The regional price has to lie between the import-price (world price plus import cost) and the export-price (world price minus export cost).

Table 1: Comparison of th COP- and the MCP-approach.

2.2 Building up the COP

Step 1: Specification of the model dimension

The COP-model has $4{\cdot}r{\cdot}g$ variables:

- $s_{r,crude}$ Supply (i.e. production) of crude oil in region r
- $s_{r,f}$ Supply (i.e. production, output of the refinery) of fuel f in region r
- $d_{r,crude}$ Demand (i.e. refinery input) for crude oil in region r
- $d_{r,f}$ Demand (i.e. consumption) of fuel f in region r
- $ex_{r,g}$ Exports of good g from region r to the world market
- $im_{r,g}$ Imports of good g from the world market to region r

Step 2: Calibration of the demand-, supply- and production-functions

«Calibrated» models base upon economic benchmark data, i.e. demand and supply statistics at prices in a reference year ($s0_{r,g}$, $d0_{r,g}$, $p0_{r,g}$). They have two main advantages: Since the benchmark data represent a solution to the model by definition, it is possible to conduct a replication check. And secondly, most of the coefficients of the model are «observable» values such as price elasticities or benchmark value shares.

- Calibration of the fuel demand function:

The price elasticity of demand is defined as $$\alpha_{r,f} = -\frac{\delta d_{r,f}}{\delta p_{r,f}} \cdot \frac{p0_{r,f}}{d0_{r,f}}$$

Solve for $\frac{\delta d_{r,f}}{\delta p_{r,f}}$ and integrate to get $$d_{r,f}(p_{r,f}) = \text{Const} + \frac{p_{r,f}}{p0_{r,f}} \cdot d0_{r,f} \cdot \alpha_{r,f}$$

One can easily check that this calibrated demand-function is equal to uncalibrated function with $A_{r,f}$ and $B_{r,f}$ being

$$A_{r,f} = d0_{r,f} \cdot \left(1+\alpha_{r,f}\right) \quad and \quad B_{r,f} = \frac{d0_{r,f}}{p0_{r,f}} \cdot \alpha_{r,f} \qquad with\ A_{r,f}, B_{r,f}, \alpha_{r,f} > 0$$

For the counterfactual analysis (chapter 4, page 10), a region-specific scaling factor GDP_r is introduced into the fuel demand-function:

$$d_{r,f} = d0_{r,f} \cdot GDP_r \cdot \left[1 - \alpha_{r,f} \cdot \left(\frac{p_{r,f}}{p0_{r,f}} - 1\right)\right]$$

- The linear crude oil demand- and supply-functions are calibrated analogously.

$$d_{r,crude} = d0_{r,crude} \cdot \left[1 - \beta_{r,va} \cdot \left(\frac{p_{r,va}}{p0_{r,va}} - 1\right)\right] \qquad \text{and}$$

$$s_{r,crude} = s0_{r,crude} \cdot \left[1 + \gamma_{r,crude} \cdot \left(\frac{p_{r,crude}}{p0_{r,crude}} - 1\right)\right]$$

- And finally, the calibrated CET-function is:

$$d_{r,crude} = d0_{r,crude} \cdot \left[\sum_f \theta0_{r,f} \cdot \left(\frac{s_{r,f}}{s0_{r,f}}\right)^{\frac{1+\eta_r}{\eta_r}}\right]^{\frac{\eta_r}{1+\eta_r}}$$

(The calibration of the CET-function is presented in detail in appendix A1, page 16.)

Step 3: Objective – maximize total surplus

The total surplus is the consumer's willingness to pay for all the consumed goods minus all costs to provide them with these goods.

The total surplus (TS) is the sum of the consumer and the producer surplus. In a typical school book example it is the triangle spanned by the demand and the supply curve. In our model, the total surplus can be calculated as the consumer's willingness to pay for all the consumed goods minus the sum of all costs to provide the consumers with these goods.

- The willingness to pay for all the consumed goods (TV) is the cumulated area under the fuel demand curves in all the regions r.

$$TV = \sum_r \sum_f \int_0^{d_{r,f}} p_{r,f}(d^*_{r,f})\,\partial d^*_{r,f}$$

Now, the fuel demand-function is solved for $p_{r,f}$ and inserted – this leads to

$$TV = \sum_r \sum_f \int_0^{d_{r,f}} p0_{r,f} \cdot \left(1 - \frac{d^*_{r,f}/(d0_{r,f} \cdot GDP_r) - 1}{\alpha_{r,f}}\right) \partial d^*_{r,f}$$

And finally, integrate to get

$$TV = \sum_r \sum_f p0_{r,f} \cdot d_{r,f} \cdot \left(1 - \frac{d_{r,f}/(d0_{r,f} \cdot GDP_r) - 2}{2 \cdot \alpha_{r,f}}\right)$$

- The total costs consists of

1. The cost for the crude oil production (C^{crude})which is the cumulated area under the crude oil supply (production) function in all the regions r. (Calculation as done for TV)

$$C^{crude} = \sum_r \int_0^{s_{r,crude}} p_{r,crude}(s^*_{r,crude})\,\partial s^*_{r,crude} = \sum_r p0_{r,crude} \cdot s_{r,crude} \cdot \left(1 + \frac{s_{r,crude} / s0_{r,crude} - 2}{2 \cdot \gamma_{r,crude}}\right)$$

2. The cost for the value added to the refining process (C^{va})which is the cumulated area under the supply curve for the value added in all the regions *r*. (Dito)

$$C^{va} = \sum_r \int_0^{d_{r,crude}} p_{r,va}(d^*_{r,crude})\,\partial d^*_{r,crude} = \sum_r p0_{r,va} \cdot d_{r,crude} \cdot \left(1 + \frac{d_{r,crude} / d0_{r,crude} - 2}{2 \cdot \beta_{r,crude}}\right)$$

3. The cumulated import and export cost (C^{trade}) due to the trade between regional and world markets.

$$C^{trade} = \sum_r \sum_g \left(ex_{r,g} \cdot exc_{r,g} + im_{r,g} \cdot imc_{r,g}\right)$$

So, the total surplus – the objective function of the COP – simply becomes

$$TS = TV - \left(C^{crude} + C^{va} + C^{trade}\right)$$

Step 4: Constraints

As mentioned in the beginning of this chapter, there are two kinds of constraints to be complied with:

- All markets have to be cleared.

Regional markets ($r \cdot g$ constraints): $s_{r,g} + im_{r,g} = d_{r,g} + ex_{r,g}$

World market (g constraints): $\sum_r ex_{r,g} = \sum_r im_{r,g}$

- The refineries produce according to a CET-function (as mentioned above, the calibration of the CET-function is presented in appendix A1, page 16).

$$d_{r,crude} = d0_{r,crude} \cdot \left[\sum_f \theta 0_{r,f} \cdot \left(\frac{s_{r,f}}{s0_{r,f}}\right)^{\frac{1+\eta_r}{\eta_r}}\right]^{\frac{\eta_r}{1+\eta_r}}$$

Step 5: COP – Summary

To sum up, the COP becomes

Maximize	$TS = TV - \left(C^{crude} + C^{va} + C^{trade}\right)$	Total surplus
Subject to	$s_{r,g} + im_{r,g} = d_{r,g} + ex_{r,g}$ $\sum_r ex_{r,g} = \sum_r im_{r,g}$	Market clearing
	$d_{r,crude} = d0_{r,crude} \cdot \left[\sum_f \theta 0_{r,f} \cdot \left(\frac{s_{r,f}}{s0_{r,f}}\right)^{\frac{1+\eta_r}{\eta_r}}\right]^{\frac{\eta_r}{1+\eta_r}}$	CET-production

3 Data preparation

It would go beyond the scope of this paper to explain all the data preparation steps in detail. Nevertheless it is worth to give a short overview.

Step 1: Get the data

We used benchmark data from the year 2005, published in the Intertiol Energy Annual 2006 (Energy Information Administration, EIA) containing demand and supply statistics (including exports and imports) for more than 200 regions (mostly coutries) and 8 good categories (crude oil, motor gasoline, jet fuel, kerosene, distillate fuel oil, residual fuel oil, liquefied petroleum gases, other products). In addition to the demand and supply statistics some rough price statistics are used (www.eia.doe.gov/oiaf /aeo/pdf/aeohptab_12.pdf).

Data source: Intertiol Energy Annual 2006 (Energy Information Administration, EIA)

Step 2: Balance the data

After the elimination of outliers, a least squares model is defined to precisely reconcile supply and demand statistics. This makes it easier to verify consistency of the resulting equilibrium model, as all flows will be balanced precisely at the reference point. Now, a complete balanced benchmark data set is available for the modeller ($d0_{r,g}$, $s0_{r,g}$).

Step 3: Aggregate the data

Especially for the implementation in Excel but also to simplify the analysis of counterfactual scenarios, it is convenient to aggregate the benchmark data to a smaller – in this example actually to a very small – set of regions and goods. The original regions are aggregated to

- United States (USA)
- Japan (JAP)
- Europe (EUR)
- Other OECD countries (OEC)
- Saudi Arabia (SAU)
- Other OPEC countries (OOP)
- China (CHN)
- Russia (RUS)
- Rest of the World (ROW)

The eight original good categories are aggregated to

- Crude oil
- Motor gasoline
- Diesel (distillate fuel)
- Other fuels

(Remark regarding the aggregation: The demand and supply statistics are summed up to get the data for the aggregated regions and goods. The price of aggregated goods is calculated by a demand weighted average over the original goods.)

Step 4: Define other benchmark values

There are some more benchmark values which can be calculated given the data:

1. Assuming that the world and regional markets are balanced, the benchmark region market prices are calculated by adding the import cost margin to the world price in regions which are net importers ($p0_{r,g} = p0_{world,g} + imc0_{r,g}$), respectively by subtracting the export cost margin from the world price in regions which are net exporters ($p0_{r,g} = p0_{world,g} - exc0_{r,g}$).

2. The benchmark fuel value shares ($\theta 0_{r,f}$) are calculated, for example for gasoline:

$$\theta 0_{r,f} = \frac{p0_{r,gasoline} \cdot s0_{r,gasoline}}{\sum_{f} p0_{r,f} \cdot s0_{r,f}}$$

3. The price (cost) of the value added to the refinery in region r can be calculated under the assumption of zero profits (in this case the value of the refinery output has to match the value of the inputs):

$$p0_{r,va} = \frac{\sum_{f} p0_{r,f} \cdot s0_{r,f}}{d0_{r,crude}} - p0_{r,crude}$$

Step 5: Define elasticities

In a more serious model, the used elasticities could be the result of extensive (econometric) studies about how a certain variable – let's say a demanded quantity – reacts on changes of another variable (e.g. the price). For the sake of this simple model some reasonable values are assigned to the price elasticities ($\alpha_{r,f}$ = 0.25, $\beta_{r,va}$ = 0.75, $\gamma_{r,crude}$ = 0.5) and the constant elasticity of transformation (η_r = 0.5 over the output mix of the refinery) without justification.

For the analysis the defined elasticities and parameter can be changed to simulate counterfactual scenarios.

Step 6: Define parameter for counterfactual scenarios

- The region-specific fuel demand scaling factor is set to 1 ($GDP0_r$ = 1) in the benchmark case and can be used to simulate growth of a certain region.
- The following trade cost margins are set for the benchmark case: $imc0_{r,f}$ = 2, $imc0_{r,crude}$ = 0.5, $exc0_{r,f}$ = 1, $exc0_{r,crude}$ = 0.25. The trade cost margins can be used to simulate trade sanctions on certain regions.

4 Analysis

In this chapter, first the benchmark data and secondly the modelling results of two different counterfactual simulations are illustrated and commented. For this purpose, the COP-model presented in chapter 2 has been implemented in Excel. The implementation is straight forward and does not need special Excel-skills. You can use the prepared «OilMarketModel.xls»-file to reproduce the presented simulations.

4.1 Benchmark case: «Business as usual»

Crude oil: Production and demand

Figure 4 shows the crude oil production and demand (input to the refinery). Two interesting points are mentioned here:

Every day, more than 80 million barrels of crude oil are produced, transported and processed.

- Production and demand of crude oil are not balanced within the regions – the world is divided into big importers (USA, Europe, Japan and China) and exporters (Saudi Arabia, other OPEC-members and Russia). The model assumption of a perfect competition oil production has to be questioned since there are only two parties – namely OPEC and Russia – being able to export large amounts of crude oil.
- The total amount of crude oil produced, transported and processed every day is unimaginably high – more than 80 million barrels! Already after 300 days one could fill up the lake of Zurich with crude oil. Note that more than half of this amount is consumed in the US, Europe and Japan.

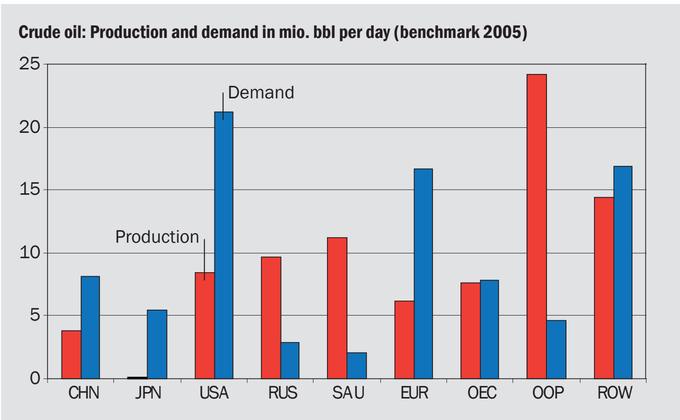


Figure 4: Crude oil production and demand in the reference year 2005.

Fuel supply and demand

In Figure 5 to 8 (page 11) the data for refined products are illustrated (production and consumption, net imports, the calculated value shares and the refinery cost shares). Basically, every region has two possibilities to cover its demand for refined products:

- It buys enough crude oil and covers the demand with its own refineries.
- It buys the fuels directly on the world market.

The trade flows given by the benchmark data can not be interpreted without further information regarding trade and refining costs. But – considering the low fuel trade flows – it seems to be advantageous to keep the refining within the region: While the crude oil production is «centralized by nature», the refining is preferred to be done «locally».

Figure 5 to 8: Benchmark data (2005) for refined products.

4.2 Counterfactual case 1: «China's growth»

In the first counterfactual scenario China's fuel demand is assumed to be three times higher than in the benchmark case. The region specific scaling factor for China is switched to 3 (GDP_{CHN} = 3) and the COP is solved again. The results are shown in figure 9 to 16 (page 12) and interpreted in table 2.

«What if... China's demand for fuels is growing to a much higher level?»

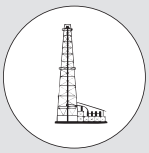
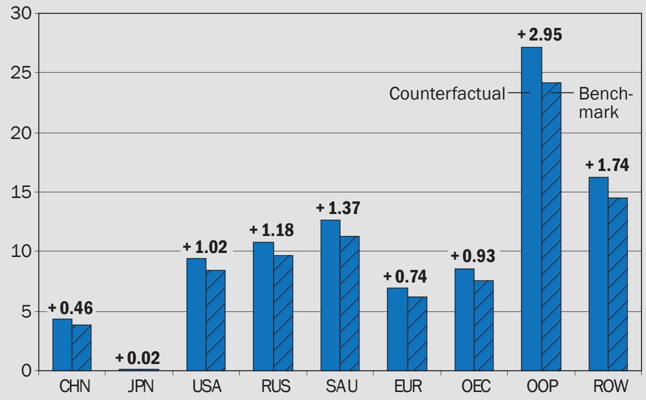
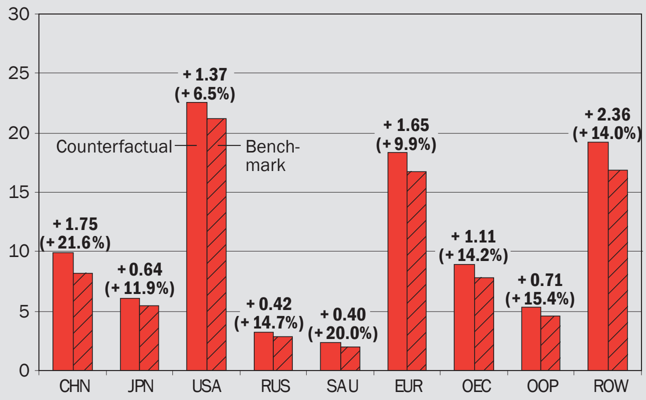
«China's growth» – Interpretation of the model results	
	**Fuel consumption** The world demand for fuels increases, driven by the positive demand shock in China. The interesting question is by how much: The consumption-increase in China induces also an increase in prices, which has a negative effect on the consumption in the regions. This effect is characterized by the price elasticity of fuel demand.
	**Fuel production (refineries)** The world demand for crude oil and the production of fuels increases, driven by the higher fuel demand. The increase is dispersed over the refineries in all the regions to minimize the total cost of the value added to the refining process. China covers only a small part of its fuel demand with its own regional refineries, the rest is imported from the other regions. The reason lies in the assumed cost characteristic of the refineries. As indicated in figure 17 the value added to the refining process is an increasing function of the crude oil input, i.e. processing more crude oil (e.g. + 30 %) means needing even more labour and capital (e.g. + 50 %).
	**Crude oil production** All the regions increase the production of crude oil according to their crude oil supply functions. This is driven by the increase of the crude oil demand in the refineries. Remark: Since the elasticity of supply is the same for all regions, the relative increase is the same, too – namely 12 % of the benchmark production. An alternative simulation with different elasticities is done in the following subchapter («Impact of the elasticity of crude oil supply», page 13).

Table 2: Counterfactual case 1, «China's growth». Interpretation of the simulation results illustrated on the following page 12.

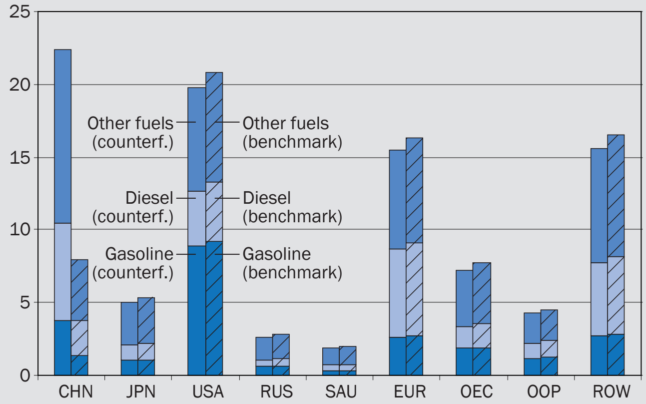
Crude oil production in mio. bbl per day, benchmark (hatched) and counterfactual case



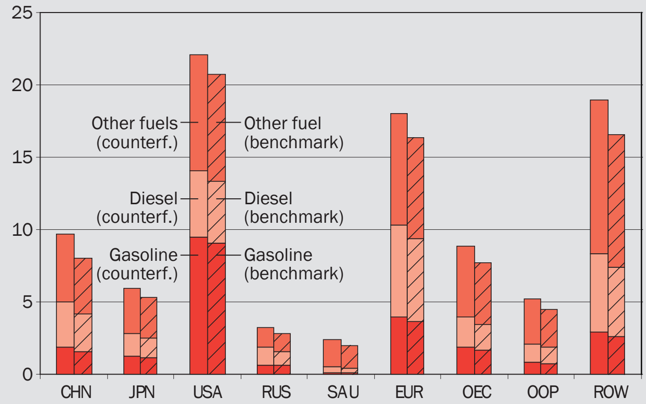
Processed crude oil in mio. bbl per day, benchmark (hatched) and counterfactual case



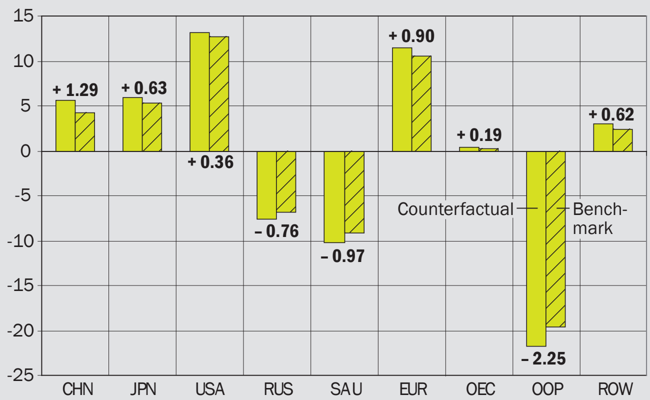
Fuel consumption in mio. bbl per day, benchmark (hatched) and counterfactual case



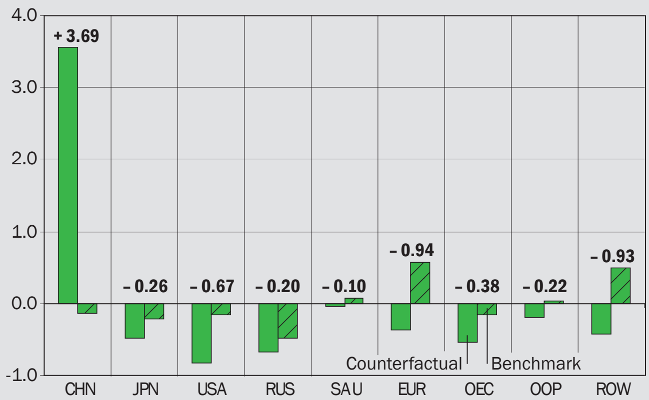
Produced fuels in mio. bbl per day, benchmark (hatched) and counterfactual case



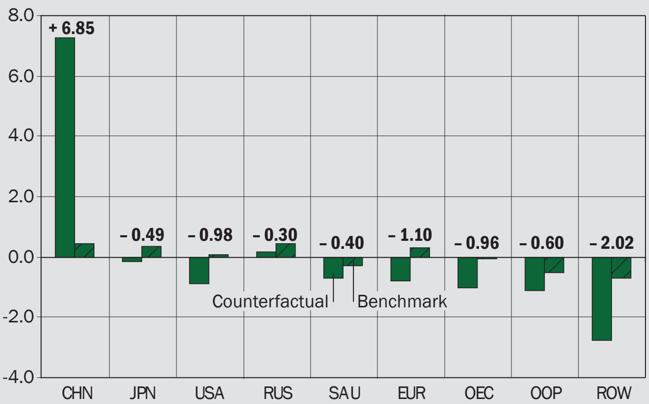
Crude oil: Net imports in mio. bbl per day, benchmark (hatched) and counterfactual case



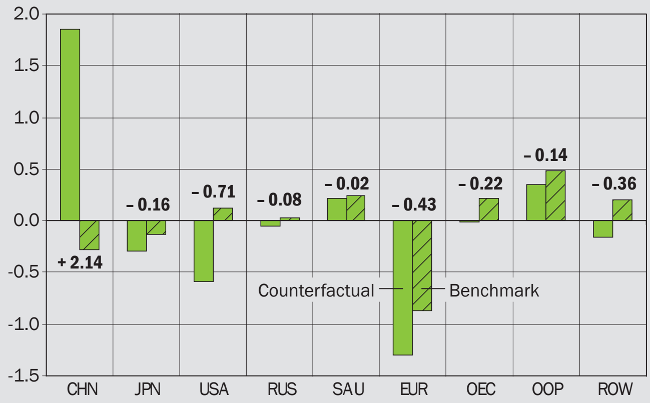
Diesel: Net imports in mio. bbl per day, benchmark (hatched) and counterfactual case



Other fuels: Net imports in mio. bbl per day, benchmark (hatched) and counterfactual case



Gasoline: Net imports in mio. bbl per day, benchmark (hatched) and counterfactual case



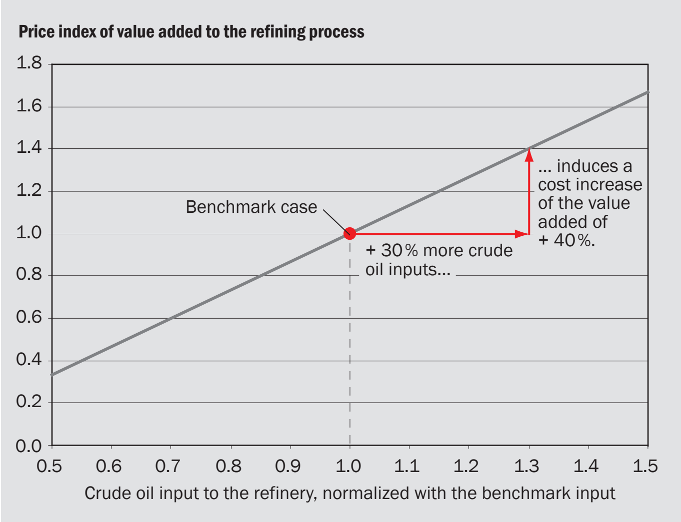


Figure 17: As defined in the model assumptions (page 4), the price (i.e. cost) of the value added increases with the crude oil input.

Impact of the elasticity of crude oil supply

To point out the influence of a price elasticity, the elasticity of crude oil supply (production) is varied as an alternative to counterfactual case 1. While the scaling factor of China is set to 3 (GDP_{CHN} = 3) the elasticity of crude oil supply in China (CHN), Japan (JAP), the USA, Europe (EUR), the other OECD countries (OEC) and the rest of the world (ROW) is set to a much lower value (0.1 instead of 0.5). The idea of this measure is the following: Russia and the OPEC-countries – sitting on comfortable resources of crude oil – can react much better, i.e. with lower costs, to changes in demand.

The resulting crude oil production is illustrated in figure 18. With regard to the benchmark case the crude oil production is higher in all the regions both for the counterfactual case 1 («China's growth») and its alternative (very inelastic crude oil supply in countries not belonging to OPEC or Russia). But one can notice that Saudi Arabia, the other OPEC-countries and Russia increase their production in the alternative even more, while the other regions have a lower production. This is caused by the lower elasticity of supply: If an oil producer has very high marginal cost to increase its production it can not react on a rise in demand and will only get a small piece of the additional cake.

«What if… the costs to increase the crude oil production is much higher in regions not belonging to the OPEC or Russia?»

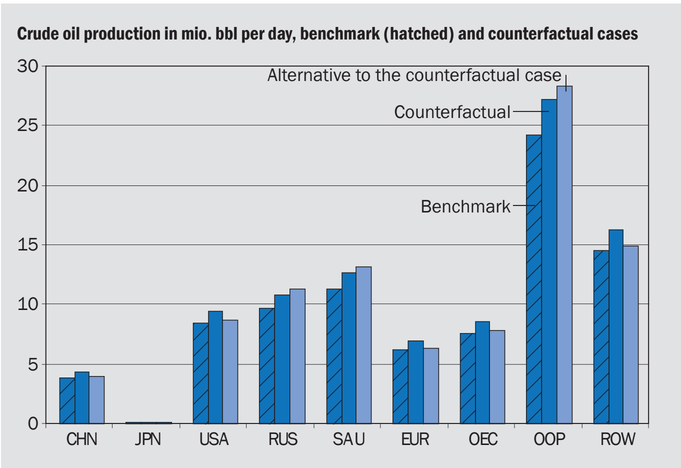


Figure 18: Comparison of the counterfactual case 1 («China's growth») with an alternative simulation in which China (CHN), Japan (JAP), the USA, Europe (EUR), the other OECD countries (OEC) and the rest of the world (ROW) have a much lower crude oil price elasticity of supply (0.1 instead of 0.5).

Figure 9 to 16 (page 12): Simulation results for the counterfactual case 1, «China's growth».

4.3 Counterfactual case 2: «Sanction on OPEC»

In the second counterfactual scenario, the OPEC-countries have to pay a much higher marginal cost to export their crude oil to the world market. To point out the difference to the benchmark case, very high marginal costs are chosen ($ex_{SAU,crude} = ex_{OOP,crude} = 10$ instead of $ex0_{SAU,crude} = ex0_{OOP,crude} = 0.25$ in the benchmark case). The simulation results are illustrated in figure 19 to 26 (page 15) and interpreted in table 3.

«What if… a sanction is imposed on crude oil exports from the OPEC?»

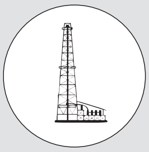
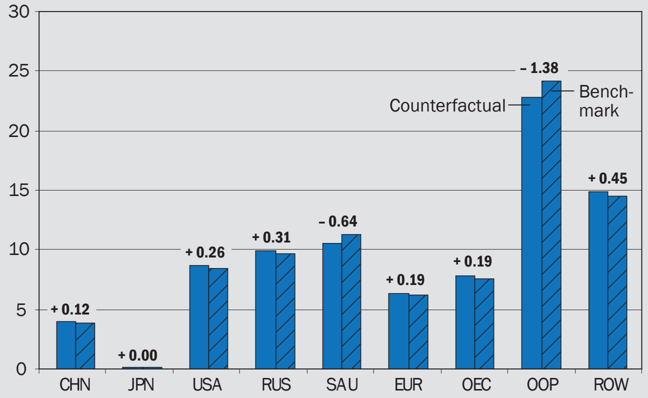
«Sanction on OPEC» – Interpretation of the model results	
	**Crude oil production** The world production of crude oil decreases by a little amount (0.5 million barrels per day) due to the higher total trade costs. The non-OPEC-countries can increase their crude oil production at the expense of the OPEC-countries. But this «redistribution-effect» is not that strong: The OPEC-countries make use of the possibility to refine more of their crude oil and export the produced fuels.
	**Fuel production (refineries)** The world demand for crude oil decreases by a little amount (0.5 mio. barrels per day), according to the lower crude oil production. As mentioned above, there is a «leakage» in the trade barrier against the OPEC-countries. Despite the higher costs for the value added (see figure 17), their refineries demand now more crude oil than in the benchmark case and increase their fuel exports according to the shift in the relative costs: Relatively to crude oil, fuels become cheaper to export. On the other side, the refineries in the non-OPEC countries lower their production level and more fuels are imported (or less fuels exported respectively).
	**Fuel consumption** The world demand for fuels decreases by a little amount. Finally, the higher total trade costs cause prices to rise which again has a negative effect on the fuel demand.

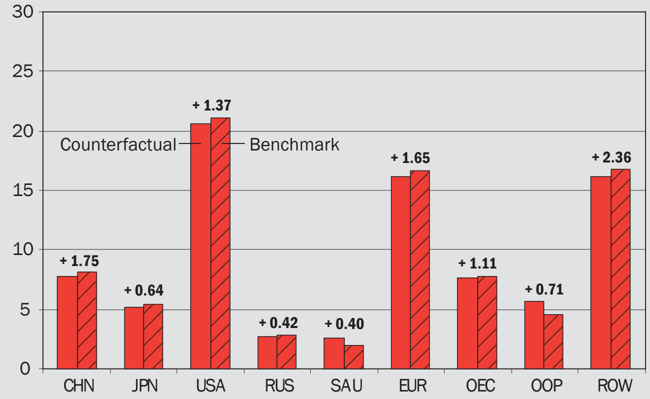
Table 3: Counterfactual case 2, «Sanction on OPEC». Interpretation of the simulation results illustrated on the following page 15.

Figure 19 to 26 (page 15): Simulation results for the counterfactual case 2, «Sanction on OPEC»

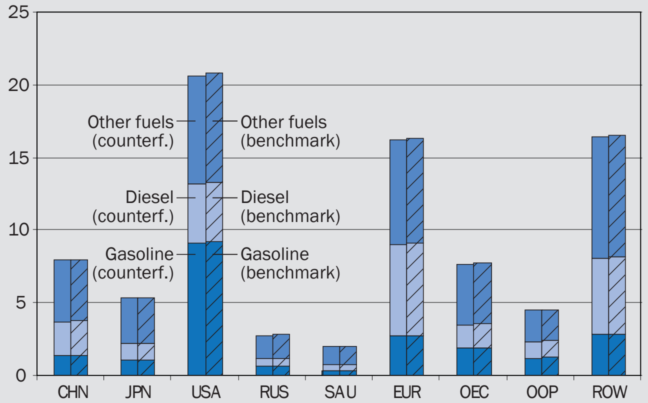
Crude oil production in mio. bbl per day, benchmark (hatched) and counterfactual case



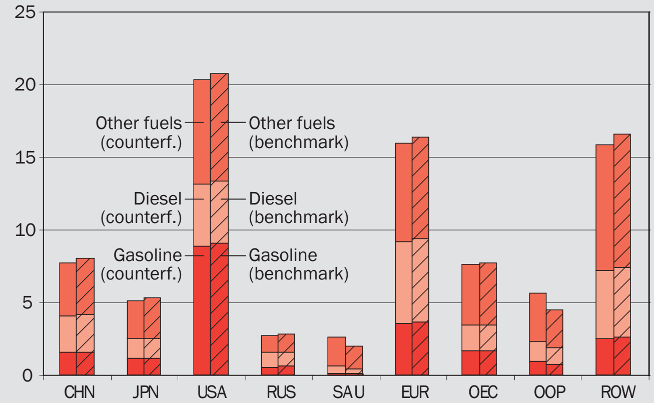
Processed crude oil in mio. bbl per day, benchmark (hatched) and counterfactual case



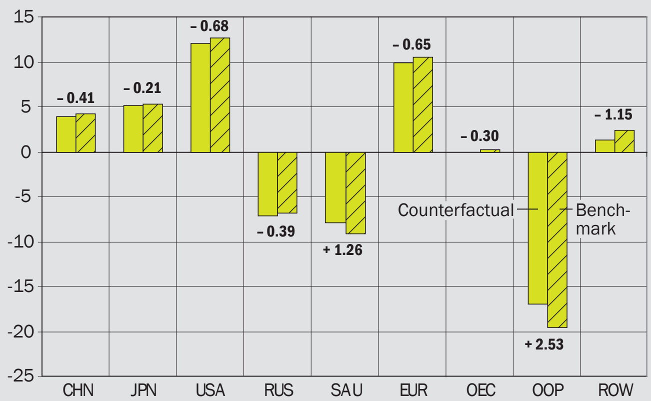
Fuel consumption in mio. bbl per day, benchmark (hatched) and counterfactual case



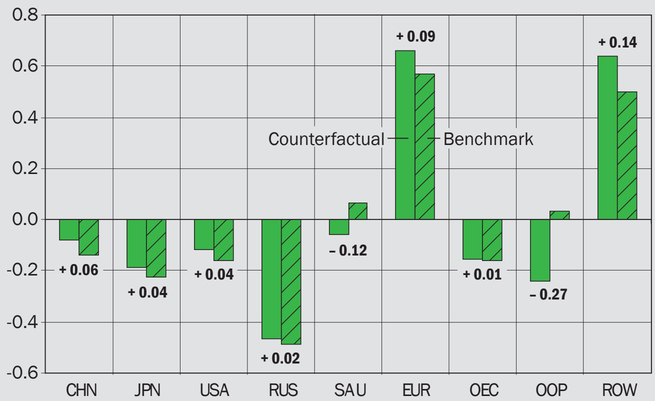
Produced fuels in mio. bbl per day, benchmark (hatched) and counterfactual case



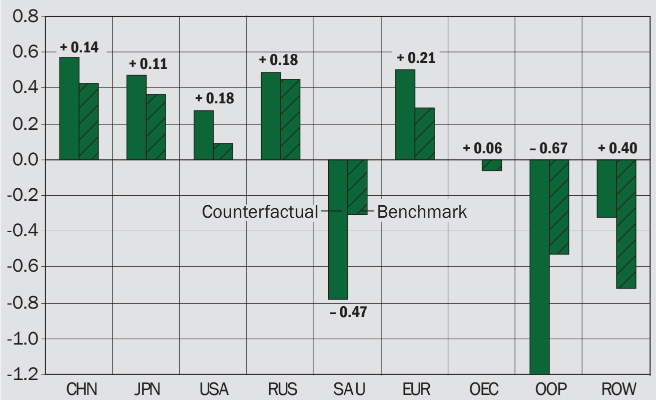
Crude oil: Net imports in mio. bbl per day, benchmark (hatched) and counterfactual case



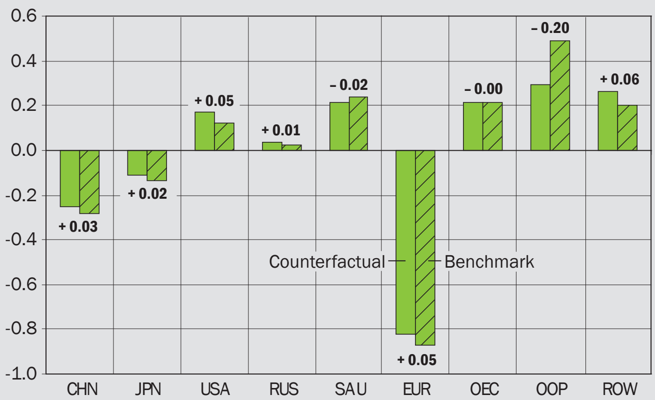
Diesel: Net imports in mio. bbl per day, benchmark (hatched) and counterfactual case



Other fuels: Net imports in mio. bbl per day, benchmark (hatched) and counterfactual case



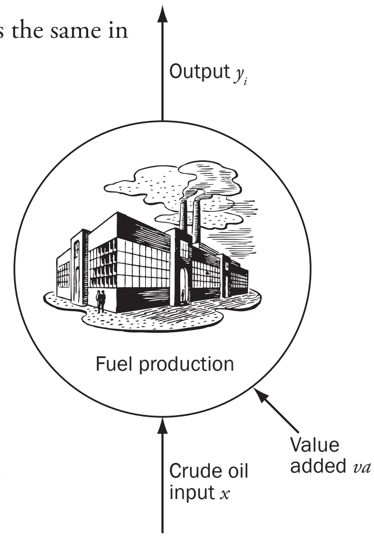
Gasoline: Net imports in mio. bbl per day, benchmark (hatched) and counterfactual case



A Appendix

A1 CET-function – calculus

Using a CET-production-function for the presented calibrated benchmark equilibrium model involves two «preparation-steps»:
1. Solving the profit maximization problem
2. Calibration of the production- (2a.) and the output-function (2b.)



For the sake of readability, the region index is omitted here – the problem is the same in each region anyway – and the following more general notation is used:

y_i, p_i	amount and price of output i of the refinery (fuels)
x, p_x	amount and price of crude oil input to the refinery
v_a, p_{va}	amount and price of value added to the refining process
$f(\cdot)$	CET-function
a_i	output-specific coefficient
η	elasticity of transformation
£	Lagrangian function
λ	Lagrangian multiplier
r	marginal revenue of input
θ_i	output value share of output i
$\cdot 0$	adding a 0 to a symbol indicates the value at the benchmark point (i.e. $y0_i$, $x0$, $r0$ and $\theta 0_i$)

1. Solving the profit maximization problem

Every refinery maximizes profits, i.e. revenues in our case. The input level x can be set to an arbitrary level (e.g. to 1) since this does not change the optimal output mix. In this case, the value added to the refining process is fixed, too, and does not influence the optimal output mix either. So the problem reduces to a «revenue-maximization problem».

$$\max \underbrace{\sum_i p_i \cdot y_i - p_x \cdot x - p_{va} \cdot va}_{\text{profits of the refinery}} \Rightarrow \max \sum_i p_i \cdot y_i \quad \Bigg\} \text{ Objective: the refinery maximizes profits}$$

$$\text{subject to} \quad x = f(y) = \left[\sum_i a_i \cdot (y_i)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}} = 1 \quad \Bigg\} \text{ Constraint: the refinery produces according to a CET-function at an arbitrary input level of 1}$$

This classical optimization problem is solved using the Lagrangian:

$$£ = \sum p_i \cdot y_i - \lambda \cdot (f(y) - 1)$$

The Lagrange multiplier, λ, equals to the marginal revenue of input, r. Hence, the first order condition for y_i reduces to:

$$\frac{\partial £}{\partial y_i} = p_i - r\frac{\partial f}{\partial y_i} = 0 \quad \Rightarrow \quad p_i = r \cdot a_i \cdot (y_i)^{\frac{1+\eta}{\eta}-1} \left[\overbrace{\sum_i a_i \cdot (y_i)^{\frac{1+\eta}{\eta}}}^{=1}\right]^{\frac{\eta}{1+\eta}-1} = r \cdot a_i \cdot (y_i)^{\frac{1+\eta}{\eta}-1}$$

(Note that the simplification is possible since the input level is fixed to 1.)

Solving for y_i to get the output-function: $\quad y_i = \left(\dfrac{p_i}{a_i \cdot r}\right)^{\eta}$

The marginal revenue of input can be found by substituting y_i into the objective-function:

$$r = \sum_i p_i \cdot y_i = \sum_i a_i^{-\eta} \cdot p_i^{1+\eta} \cdot r^{-\eta} = r^{-\eta} \cdot \sum_i a_i^{-\eta} \cdot p_i^{1+\eta} \quad \Rightarrow \quad r = \left[\sum_i a_i^{-\eta} \cdot p_i^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

2a. Calibration of the production function

The unspecified coefficients a_i can be calibrated by inverting the output-function. The output-function derived above is that which maximizes the revenues by using one unit of input x (crude oil). With constant returns to scale (CRTS) – and the CET-function actually exhibits CRTS as shown below – the revenue maximizing output mix can be scaled up with the crude oil input x without loosing optimality.

$$\left.f(\lambda \cdot y) = \left[\sum_i a_i \cdot (\lambda \cdot y_i)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}} = \lambda \cdot \left[\sum_i a_i \cdot (y_i)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}} = \lambda \cdot f(y) \quad \right\}$$

Proof that the CET-function exhibits CRTS.

Back to the calculus... Insert the benchmark data ($y0_i$, $x0$, $r0$, $p0_i$) into the output-function and solve for a_i:

$$y0_i = x0 \cdot \left(\frac{p0_i}{a_i \cdot r0}\right)^{\eta} \quad \Rightarrow \quad a_i = \frac{p0_i}{r0 \cdot \left(\dfrac{y0_i}{x0}\right)^{\frac{1}{\eta}}} \qquad with \quad r0 = \frac{\sum_i p0_i \cdot y0_i}{x0}$$

Scale up with the input:
correct because of CRTS

Inserting the calibrated a_i into the production-function leads to the calibrated form of the CET-production-function:

$$x = \left[\sum_i a_i \cdot (y_i)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}} = \left[\sum_i \frac{p0_i}{r0 \cdot \left(\dfrac{y0_i}{x0}\right)^{\frac{1}{\eta}}} \cdot (y_i)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}$$

$$= \left[\sum_i \overbrace{\frac{p0_i \cdot y0_i}{r0 \cdot x0}}^{=\theta 0_i} \cdot \frac{(y0_i)^{-\frac{1}{\eta}-1}}{(x0)^{-\frac{1}{\eta}-1}} (y_i)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}} = x0 \cdot \left[\sum_i \theta 0_i \cdot \left(\frac{y_i}{y0_i}\right)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}$$

2b. Calibration of the output-function

Inserting the calibrated a_i into the output-function leads to the calibrated form of the CET-production function:

$$y_i = x \cdot \left(\frac{p_i}{a_i \cdot r}\right)^{\eta} = x \cdot \left(\frac{p_i}{\dfrac{p0_i}{r0 \cdot \left(\dfrac{y0_i}{x0}\right)^{\frac{1}{\eta}}} \cdot r}\right)^{\eta} = y0_i \cdot \frac{x}{x0} \cdot \left[\frac{p_i}{p0_i} \cdot \left(\frac{r}{r0}\right)^{-1}\right]^{\eta}$$

Dito: correct
because of CRTS

And the marginal revenue of input, r, is calibrated analogously:

$$r = \left[\sum_i a_i^{-\eta} \cdot p_i^{1+\eta}\right]^{\frac{1}{1+\eta}} = \left[\sum_i \left(\frac{p0_i}{r0 \cdot \left(\frac{y0_i}{x0}\right)^{\frac{1}{\eta}}}\right)^{-\eta} \cdot p_i^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

$$= r0 \cdot \left[\sum_i \overbrace{\frac{p0_i \cdot y0_i}{r0 \cdot x0}}^{=\theta 0_i} \cdot \left(\frac{p_i}{p0_i}\right)^{1+\eta}\right]^{\frac{1}{1+\eta}} = r0 \cdot \left[\sum_i \theta 0_i \cdot \left(\frac{p_i}{p0_i}\right)^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

Summary

For the COP-model the calibrated form of the CET-production-function is needed – that is:

$$x = x0 \cdot \left[\sum_i \theta 0_i \cdot \left(\frac{y_i}{y0_i}\right)^{\frac{1+\eta}{\eta}}\right]^{\frac{\eta}{1+\eta}}$$

For the MCP-model the calibrated form of the output-function (including the marginal revenue of output) is needed:

$$y_i = y0_i \cdot \frac{x}{x0} \cdot \left[\frac{p_i}{p0_i} \cdot \left(\frac{r}{r0}\right)^{-1}\right]^{\eta} \qquad \text{with} \quad r = r0 \cdot \left[\sum_i \theta 0_i \cdot \left(\frac{p_i}{p0_i}\right)^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

A2 MCP-approach

For the sake of completeness the MCP-approach is shortly explained as an alternative to the COP. The MCP-approach is a standard approach to solve economic equilibrium problems. It bases upon the idea of complementarity between equilibrium variables and equilibrium conditions. Positive market prices imply market clearance – otherwise commodities are in excess supply and the respective prices fall to zero. Activities like the refining in our example will be operated only as long as they break even – negative revenues would imply the shutdown of the refinery. In this context, the term «mixed complementarity problem» (MCP) is straightforward: «mixed» indicates that the mathematical program includes equalities as well as inequalities; «complementarity» refers to complementary slackness between system variables and system conditions.

Oil market model as MCP

As mentioned in chapter 2, solving the MCP means solving a system of equations:

- CET-refineries maximize profits which have to be zero (perfect competition).
- Crude oil is produced according to the given supply-function.
- Fuels are consumed according to the given demand-function.
- All markets are cleared.
- No «import-export-short-cuts» allowed: The regional price has to lie between the import-price (world price plus import cost) and the export-price (world price minus export cost).

In mathematical terms solving the MCP means solving the ten equations, illustrated in figure 27, simultaneously. Please note that the solution of the refinery optimization problem – the output-function defining the optimal fuel output mix (see appendix 1, page 16) – is embedded directly into the model.

A

Fuels are consumed according to the given demand-function:

$$d_{r,f} = d0_{r,f} \cdot GDP_r \cdot \left[1 - \alpha_{r,f} \cdot \left(\frac{p_{r,f}}{p0_{r,f}} - 1\right)\right]$$

B

The refinery produces according to a CET-function and maximizes profits. The optimal output mix is (solved in appendix 1, page 00)

$$s_{r,f} = s0_{r,f} \cdot \frac{d_{r,crude}}{d0_{r,crude}} \cdot \left[\frac{p_{r,f}}{p0_{r,f}} \cdot \left(\frac{r_r}{r0_r}\right)^{-1}\right]^{\eta}$$

with the revenue index being

$$r_r = r0_r \cdot \left[\sum_f \theta0_{r,f} \cdot \left(\frac{p_{r,f}}{p0_{r,f}}\right)^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

The price (cost) of the value added is an increasing function of the crude oil input:

$$d_{r,crude} = d0_{r,crude} \cdot \left[1 - \beta_{r,va} \cdot \left(\frac{p_{r,va}}{p0_{r,va}} - 1\right)\right]$$

And finally, the refinery has zero profits (costs equal revenues):

$$\left(1 - \theta0_{r,crude}\right) \cdot \frac{p_{r,va}}{p0_{r,va}} + \theta0_{r,crude} \cdot \frac{p_{r,crude}}{p0_{r,crude}} = \frac{r_r}{r0_r}$$

C

Crude oil is produces according to the given supply-function:

$$s_{r,crude} = s0_{r,crude} \cdot \left[1 + \gamma_{r,crude} \cdot \left(\frac{p_{r,crude}}{p0_{r,crude}} - 1\right)\right]$$

D

All the markets are cleared:

$$s_{r,g} + im_{r,g} = d_{r,g} + ex_{r,g}$$

$$\sum_r ex_{r,g} = \sum_r im_{r,g}$$

E

No «trade short-cuts» allowed:

$$p_{r,g} + exc_{r,g} \geq p_{world,g}$$

$$p_{world,g} + imc_{r,g} \geq p_{r,g}$$

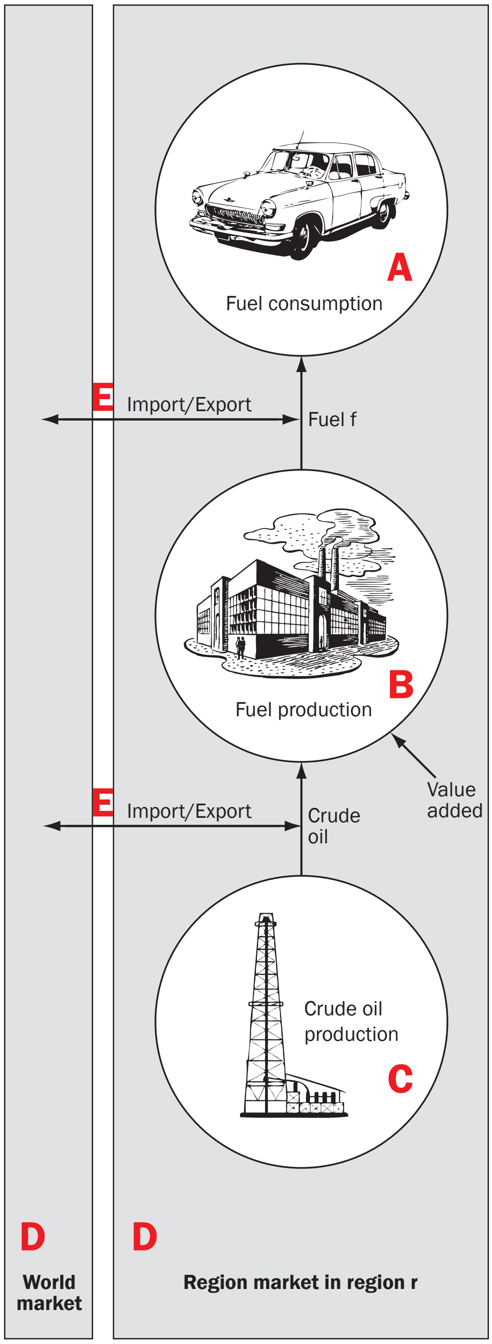


Figure 27: Solving the MCP means solving the illustrated model equations simultaneously.

This paper can be downloaded from the CEPE-homepage, including a xls- (Excel) and a gms-file (GAMS) with the implemented model.

http://www.files.ethz.ch/cepe/CEPE_WP72.zip

References

Christoph Böhringer, Thomas F. Rutherford and Wolfgang Wiegard, «Computable General Equilibrium Analysis: Opening a Black Box», Discussion Paper No. 03-56, Centre for European Economic Research. Download: ftp://ftp.zew.de/pub/zew-docs/dp/dp0356.pdf

Thomas F. Rutherford, ETH Zurich, handouts to the lecture «Intermediate Microeconomics» 2008, lecture 3: «Inequality constrained optimization» and lecture 8: «Calibrated CES Cost Functions». Download: www.cepe.ethz.ch/education/IntermediateMicro/lecture3.pdf and .../lecture8.pdf)