Simple Non-Exclusive Contracting under Adverse Selection

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Abstract

This paper studies how implicit collusion may take place in non-exclusive contracting under adverse selection when multiple agents (e.g., entrepreneurs with risky projects) non-exclusively trade with multiple firms (e.g., banks). It shows that any price schedule can be supported as equilibrium terms of trade in the market if each firm's expected profit is no less than its reservation profit. Firms sustain collusive outcomes through the triggering trading mechanism in which they change their terms of trade contingent only on agents' reports on the lowest average price that the deviating firm's trading mechanism would induce.

1 Introduction

Trading in decentralized markets is frequently non-exclusive by nature and involves asymmetric information between contracting parties. For example, a bank may lend money to many entrepreneurs who have private information on their risky projects and vice versa an entrepreneur may borrow money from many banks to finance his risky project. Various financial assets including derivatives are also non-exclusively traded among sellers and buyers.

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Some traders tend to have more information on the underlying values of structured assets such as derivatives than others do.

Non-exclusive contracting with multiple agents (e.g., entrepreneurs in loan contracting) is generally a complex process for firms (e.g., banks in loan contracting) because agents can also contract with competing firms. In this contracting environment, agents may well communicate with firms at the contracting stage because firms can ask agents about competing firms' terms of trade (e.g. loan amount and interest pairs in loan contracting). Importantly, when multiple agents communicate with firms, firms can compare what agents are telling. This may make it easier for firms to acquire the true information on competing firms' terms of trade from multiple agents. Subsequently, they may want to offer trading mechanisms in which their terms of trade depend on agents' reports on competing firms' terms of trade. In this way, firms can actively punish a deviating firm by changing their terms of trade upon agents' reports on the deviating firm's terms of trade and hence they may sustain many collusive outcomes that are not possible when there is only one agent.

The idea of collusion through complex communication mechanisms is in fact the central theme that motivates the literature on competing mechanism design. Epstein and Peters (1999) construct a very rich language that agents can use in describing the market information when they communicate with firms. Peters and Szentes (2012) characterize equilibrium allocations and equilibrium contracts when a firm has unlimited commitment in the sense that it can make its contract directly contingent on the other firms' contracts.[1]

Yamashita (2010) considers the competing mechanism game in Epstein and Peters (1999) where the firm has limited commitment so that it can make its contract contingent on agents' messages only. He then shows that firms can sustain various collusive outcomes if each firm offers the recommendation mechanism that asks each agent to report his type and the direct mechanism the firm should choose. When all agents report the same direct mechanism, the firm chooses that direct mechanism, which then determines the firm's decision according to agents' type reports. His approach tells us how one can view firms' implicit collusion via their commitment to the recommen-

[1]There are no distinction between principals and agents in Peters and Szentes (2012) and Peters and Troncoso Valverde (2010) because (i) all players can offer mechanisms to other players and hence (ii) all players communicate with one another.

dation mechanisms but it does not identify equilibrium allocations because, in his approach, equilibrium allocations are specified by the firm's minmax value relative to the set of all complex mechanisms but it is not feasible to specify the exact set of all complex mechanisms. For the characterization of equilibrium allocations, Peters and Troncoso Valverde (2010) incorporate two rounds of communication into Yamashita's recommendation mechanism.

The recommendation mechanism needs at least three or more agents for their truthful reports but it provides a perfectly nice way of understanding implicit collusion in general. Each agent's message in the recommendation mechanism is simpler than the message in the universal language (Epstein and Peters 1999). However, the message in the recommendation mechanism is still complex and in particular it becomes increasingly complicated as the number of agents increases. The reason is that each agent must report the entire mapping of a direct mechanism that specifies an action for every possible profile of all agents' types and hence each agent's burden of communication exponentially increases in the number of agents.

The simplicity of an agent's message however seems important to understand implicit collusion in some applications such as non-exclusive trading problems mentioned earlier. For example, it is hard to imagine that a bank asks each entrepreneur to report the bank's entire lending plan that specify loan contracts for all entrepreneurs contingent on every feasible profile of their project types. The purpose of this paper is to develop a simple equilibrium mechanism that can minimize the agent's communication burden, for a better understanding of implicit collusion in non-exclusive contracting under adverse selection such as investment financing, insurance, and various other trading problems. The key to such a simple mechanism is that each agent's message should not depend on the number of agents nor does it take a complex form.

Consider a market for a good where each privately-informed agent can trade with any number of firms and each firm can also trade with any number of privately-informed agents. Firms can freely offer any arbitrary trading mechanism that make quantity and monetary payment pairs across agents contingent on their messages. The market terms of trade can be characterized by a price schedule that specify monetary payment from the agent as a function of the quantity that the agent trades. The key result of the paper is to show how to construct an equilibrium trading mechanism for firms, given their implicit agreement on a price schedule, in a way that no firm gains by deviating to any arbitrary complex trading mechanism. Then, we show that any price schedule can be supported as equilibrium terms of trade in the market as long as it ensures that each firm receives no less profit than its reservation profit.

This paper proposes the triggering weakly incentive compatible extended (WICE) direct mechanism with which firms can maintain their implicit agreement on a price schedule, say $\tilde{\mathbf{y}}$. A triggering WICE direct mechanism asks each agent to report, along with the quantity that he wants to trade with the firm, whether there is a deviating firm and, if so, what would be the deviating firm's lowest average price that he believes he would face if he was the only one who traded with the deviating firm. When agents are anonymous so that the trading mechanism is anonymous, each agent has the same belief on the lowest average price that the deviating firm's trading mechanism would induce when he would be the only one who participated in the deviating firm's trading mechanism. As shown later, this approach is easily extended to the case in which agents are ex-ante heterogeneous.

The triggering WICE direct mechanism has the following structure. When two or more agents participate in a firm's triggering WICE direct mechanism, and more than half of their reports on the deviating firm's lowest average price are all p, then the firm offers a linear price schedule such that its unit price matches the minimum between p and the lowest average price of $\tilde{\mathbf{y}}$, which is a price schedule firms implicitly agree on. In all other cases, the firm continues to offer $\tilde{\mathbf{y}}$.

Suppose that some firm indeed deviates to an arbitrary mechanism and each agent reports his true belief p to non-deviating firms. Then, each non-deviating firm's price schedule is the linear price schedule in which the unit price matches the minimum between p and the lowest average price of $\tilde{\mathbf{y}}$. When there are three or more agents, one agent cannot unilaterally change the non-deviating firm's price schedule given the other agents' truthful reports, p. When there are only two agents, one agent can unilaterally change the non-deviating firm's price schedule by reporting p' $(\neq p)$ given the other agent's truthful report, p. In this case, the triggering WICE direct mechanism shoot them both by continuing to offer $\tilde{\mathbf{y}}$ to them. It not only makes each agent truthfully report p to each non-deviating firm given that the other agents do the same: It also makes it optimal for each agent to trade only with non-deviating firms. Consequently, a deviating firm ends up with its reservation profit upon any deviation to any arbitrary mechanism because no agents trade with the deviating firm in truthful continuation equilibrium.

When no firm deviates, each agent truthfully reports each firm, along with the quantity that he wants to trade with each firm, that no firm has deviated and then each firm continues to offer $\tilde{\mathbf{y}}$. Because all agents report that no firm has deviated, each firm also continues to offer $\tilde{\mathbf{y}}$ upon any agent's unilateral deviation to an alternative message and hence no agent has an incentive to tell a lie. As long as a price schedule ensure that each firm receives no less profit than its reservation profit, no firm has an incentive to deviate to any arbitrary trading mechanism because it only receives its reservation payoff upon deviation to any trading mechanism.

The triggering WICE direct mechanism features convenience in a large class of applications. Because each agent's message is simply two numbers (the deviating principal's lowest average price and the quantity that the agent wants to buy), it is simple and independent of the number of agents. Finally, it also works for any multiple number of agents, including the case of two agents, and the set of equilibrium payoffs is defined in terms of each firm's reservation profit which is independent of trading mechanisms.

2 Literature Review

In common agency (multiple firms and a single agent), Pavan and Calzolari (2009, 2010) propose a tractable class of extended direct mechanisms that can be used in deriving an equilibrium relative to any complex mechanisms or equivalently menus (Peters 2001 and Martimort and Stole 2002). They show that a firm can ask the agent about his choice of payoff-relevant alternatives from all the other firms, along with his type. The agent's communication is simpler than the communication with the universal language (Epstein and Peters 1999) or the communication in the recommendation mechanism (Yamashita 2010). However, it is not obvious how to extend Pavan and Calzolari's approach to multiple agency (i.e., multiple firms and multiple agents). Our paper shows that a single number, i.e., the deviating firm's lowest average price, becomes a sufficient statistic for the market information in a large class of applications for multiple agency. This enables us to view firms' implicit collusion under adverse selection through the triggering WICE direct mechanism in which the agent's communication is even simpler than what is required in Pavan and Calzolari's extended direct mechanism.

In terms of applications, our paper allows for the common-value case (the agent's type affects the principal's payoff) as well as the private value case (the agent's payoff does not affect the principal's payoff). An adverse selection problem, known as the lemon's problem in the common-value case, was identified as early as Akerlof's seminal paper (1970) but he identified the lemon's problem with as hoc restrictions such as price-taking behavior and exclusive trading. Rothchild and Stiglitz (1976) considered strategic contracting for the common-value case with multiple firms and a single agent but trading is exclusive in the sense that an agent trades with only one firm. They showed that the lemon's problem may be less severe in their screening model. However, equilibrium may not exist.

Jaynes (1978) and Hellwig (1988) show that when insurance firms directly disclose and share information on who accepts the insurance contract in non-exclusive contracting, the non-existence problem of equilibrium in the common-value case under exclusive contracting can be resolved. Pouyet, Salanié, and Salanié (2008) show that the adverse selection problem does not occur and efficiency is achieved in the private-value case even with the restriction of exclusive trading.

Recently Attar, Mariotti, and Salanié (2011) relax the restriction of exclusive trading to examine whether the lemon's problem is still present in the strategic model of non-exclusive trading for the common-value case where a single seller who is privately-informed about the quality of her product can sell it to multiple buyers.[2] They extend the results to bilateral contracting in which each buyer offers a menu of quantity and price pairs to each seller.[3] They show that the lemon's problem is the necessary equilibrium feature even in bilateral contracting in the sense that equilibrium aggregate allocation in bilateral contracting is unique, and the equilibrium price of the good is always equal to the expected quality of the good traded in the market, and a seller with a good of quality higher than the equilibrium price stays out of the market.

Our paper studies equilibrium allocation and trading mechanism in the fully generalized contracting environment where each firm's terms of trade

[2] A buyer is the contracting party who offers a trading mechanism so he is equivalent to a firm in our paper or a principal in general. A seller is the contracting party who sends a report to buyers given trading mechanisms so she is equivalent to an agent in our paper or an agent in general.

[3] Prat and Rustichini (2003) extend non-exclusive trading to bilateral contracting in which multiple principals negotiate terms of trades with multiple agents independently. However, agents have no private information in Prat and Rustichini's model. Han (2006) shows why principals can rely on menus instead of complex mechanisms in bilateral contracting.

for an agent can be determined based on communication with all agents. The results in our paper imply that, in the general contracting environment, any price schedule and the associated allocation can be supported in equilibrium as long as they provide each firm with expected profit no less than its reservation profit. Therefore, the lemon's problem in the unique equilibrium allocation of bilateral contracting is no longer the necessary equilibrium feature in the general contracting environment. Combining the results in Attar, Mariotti, and Salanié (2011), it suggests that the lemon's problem can be the necessary equilibrium feature with contractual restrictions such as bilateral contracting but it arises as a coordination failure in the general contracting environment without contractual restrictions.

Ales and Maziero (2009) derive a similar result for the common-value case to the one in Attar, Mariotti, and Salanié (2011). Finally, Biais, Martimort, and Rochet (2000) study non-exclusive financial asset trading for the common-value case in a common agency framework. In their model, multiple market makers compete in price schedules to supply liquidity to a single agent who is privately informed about the value of the asset and his hedging needs. With a continuum of the agent's types, they show that there exists a unique equilibrium in convex price schedules, which leads to Cournot-type equilibrium outcomes in the sense that each market maker makes positive expected profits but these profits go away as the number of market makers increases.

3 Model

Suppose that I ex-ante anonymous agents ($I \geq 2$) trade with J firms ($J \geq 2$) in a market for a good. Each agent can trade a good with any number of firms and each firm can also trade with any number of agents. Let x_i^j denote the quantity of the good that agent i buys from firm j. If $x_i^j > 0$, then agent i is the buyer and firm j is the seller between the two; If $x_i^j < 0$, then firm j is the buyer and agent i is the seller. Let $X \subset \mathbb{R}$ be the set of feasible quantities that each agent i buys from each firm j. Let m_i^j be the monetary payment from agent i to firm j. We assume that $m_i^j \times x_i^j > 0$ if $x_i^j \neq 0$. It means that the buyer who buys the good pays a positive amount of money to the seller and hence a unit price is positive. Let $(x_i, m_i) = (\sum_{k=1}^{J} x_i^k, \sum_{k=1}^{J} m_i^k) \in X \times \mathbb{R}$ be the pair of the total quantity that agent i trades with firms and the total monetary payment that he makes to firms. Let ω_i denote agent i's payoff type, which is assumed to be agent i's own private information. Let Ω be the set of all feasible payoff types for each agent. When agent i of type ω_i trades the total quantity x_i at the total payment m_i, his utility is

$$u(x_i, m_i, \omega_i).$$

We assume that $u(x_i, m_i, \omega_i)$ is decreasing in m_i at each (x_i, ω_i). Each firm j's profit associated with $\mathbf{x}^j = [x_1^j, \ldots, x_I^j]$ and $\mathbf{m}^j = [m_1^j, \ldots, m_I^j]$ is denoted by

$$v^j(\mathbf{x}^j, \mathbf{m}^j, \omega)$$

at each $\omega = [\omega_1, \ldots, \omega_I]$. Note that the formulation of each firm j's profit function allows for the common value of each agent i's type as well as the private value.

A seller trades with multiple buyers and a buyer also trades with multiple sellers in a variety of settings. The examples for the common-value case includes investment financing, insurance, and trading goods or services:

Investment Financing: Entrepreneur i has a risky investment project. It generates profit $f(x_i)$ when the amount of money invested in the project is x_i. Let x_i^j be the amount of money borrowed from lender j and m_i^j the amount of money that the entrepreneur agrees to pay back when the project turns out to be successful. Let ω_i be the probability of success. Let ρ be the risk-free (gross) interest rate. Entrepreneur i's (expected) payoff is $u(x_i, m_i, \omega_i) = \omega_i[f(x_i) - m_i]$, where $(x_i, m_i) = (\sum_{k=1}^J x_i^k, \sum_{k=1}^J m_i^k)$. Lender j's (expected) profit associated with $\mathbf{x}^j = [x_1^j, \ldots, x_I^j]$ and $\mathbf{m}^j = [m_1^j, \ldots, m_I^j]$ is $v^j(\mathbf{x}^j, \mathbf{m}^j, \omega) = \sum_{k=1}^I \omega_k m_k^j - \rho \sum_{k=1}^I x_k^j$ at $\omega = [\omega_1, \ldots, \omega_I]$.

Insurance: Risk-averse individual i has total wealth W. Let $U(\cdot)$ be his Bernoulli utility function for money. An accident occurs with probability $1 - \omega_i$. The accident entails a monetary loss L. Individual i pays insurance premium m_i^j to insurance company j and is reimbursed x_i^j in the case of the accident. The individual's expected utility is $\omega_i U(W - m_i) + (1 - \omega_i) U(W - L - m_i + x_i)$, where $(x_i, m_i) = (\sum_{k=1}^J x_i^k, \sum_{k=1}^J m_i^k)$. The profit for insurance company j associated with $\mathbf{x}^j = [x_1^j, \ldots, x_I^j]$ and $\mathbf{m}^j = [m_1^j, \ldots, m_I^j]$ is $v^j(\mathbf{x}^j, \mathbf{m}^j, \omega) = \sum_{k=1}^I m_k^j - \sum_{k=1}^I (1 - \omega_k) x_k^j$ at $\omega = [\omega_1, \ldots, \omega_I]$.

Trading: Each seller i produces a good. Let x_i^j be the quantity of the good sold to buyer j (firm) and $-m_i^j$ be the monetary payment made by buyer j. The quality of the good produced by seller i is his own private information and is denoted by ω_i. The cost of producing x_i units of the good to seller i is $c(x_i, \omega_i)$ so that seller i's payoff is $u(x_i, m_i, \omega_i) = -m_i - c(x_i, \omega_i)$. Buyer j's payoff associated with $\mathbf{x}^j = [x_1^j, \ldots, x_I^j]$ and $\mathbf{m}^j = [m_1^j, \ldots, m_I^j]$ is $v^j(\mathbf{x}^j, \mathbf{m}^j, \omega)$ at $\omega = [\omega_1, \ldots, \omega_I]$.

We consider a market for non-exclusive trading in which firms may freely offer agents any trading mechanisms that they want. Firms do not observe trading mechanisms offered by competing firms. An alternative interpretation is that firms do observe competing firms' trading mechanisms but they cannot write binding contracts directly contingent on competing firms' offers that they observe. However, firms can make their terms of trade for an agent contingent on all agents' reports in their trading mechanism. Messages are private in the sense that the message that agent i sends to firm j are observable only between them. This is consistent with the formulation in Epstein and Peters (1999) and Yamashita (2010).

A firm's trading mechanism determines the quantity and payment pair for each agent contingent on all agents' messages. For each firm j, let C be the set of messages available for each agent i. Because agents are ex ante anonymous, the firm offers an anonymous trading mechanism. Given firm j's trading mechanism $\gamma^j : C^I \to X \times \mathbb{R}$, $\gamma^j(c_i^j, c_{-i}^j) \in X \times \mathbb{R}$ denotes the quantity and payment pair for each agent i when his message is c_i^j and the other agents' messages are c_{-i}^j. For notational simplicity, let C include the null message $\varnothing$. We assume that if an agent decides not to participate in firm j's trading mechanism, it is equivalent to sending the null message $\varnothing$ to firm j. Let $\gamma^j(C, c_{-i}^j)$ denote the set of all quantity and monetary payment pairs that each agent i can induce by sending messages in C when the other agents' messages are c_{-i}^j.

Let Γ^j be the set of all feasible trading mechanisms for each firm j. Let $\Gamma \equiv \times_{k=1}^J \Gamma^k$. A competing mechanism game relative to Γ starts when each firm j simultaneously offers a trading mechanism from Γ^j. After observing a profile of trading mechanisms, each agent sends messages, one to each firm. Each firm j decides quantity and monetary payment pairs, one for each agent, contingent on the messages that it receives from agents. A trading mechanism can be very complex because the set of messages in a trading mechanism can be quite general in the degree and nature of the communication that it permits regarding what the other firms are doing: It could ask the agent to report not only about his type but also about the whole set of trading mechanisms offered by the other firms, the terms of trade that the agent chooses from the other firms, and so on. We adopt the notion of perfect Bayesian equilibrium for the solution concept of the competing mechanism game relative to Γ.

4 Collusion through Trading Mechanisms

Now we examine how firms can maintain their implicit collusion on terms of trade. The market terms of trade can be characterized by a price schedule $\mathbf{y}\colon X \to \mathbb{R}$ that specifies the agent's payment to a firm as a function of the quantity that the agent trades with it. Let $\mathbf{Y}$ be the set of all feasible price schedules such that for all $\mathbf{y} \in \mathbf{Y}$, (i) $\mathbf{y}(x) \times x > 0$ if $x \neq 0$ and (ii) $\mathbf{y}(x) = 0$ if $x = 0$.

Suppose that firms implicitly agree that they will trade with agents according to a price schedule $\tilde{\mathbf{y}}$. If the agent can trade with each firm according to the price schedule $\tilde{\mathbf{y}}$, the agent's payoff maximization problem can be stated as follows: For each $\omega_i \in \Omega$,

$$\max_{(x^1,\ldots,x^J)\in X} u\left(\sum_{k=1}^{J} x^k, \sum_{k=1}^{J} \tilde{\mathbf{y}}(x^k), \omega_i\right). \tag{1}$$

Let $(\tilde{x}^1(\omega_i), \ldots, \tilde{x}^J(\omega_i))$ be a solution to problem (1). Then, the maximum payoff for agent i of type ω_i becomes

$$\tilde{U}(\omega_i) \equiv u\left(\sum_{k=1}^{J} \tilde{x}^k(\omega_i), \sum_{k=1}^{J} \tilde{\mathbf{y}}(\tilde{x}^k(\omega_i)), \omega_i\right).$$

Let $u_\circ(\omega_i) \equiv u(0, 0, \omega_i)$ be the reservation payoff for the agent of type ω_i. Because $\tilde{\mathbf{y}}(x) = 0$ for $x = 0$, we can assure that $\tilde{U}(\omega_i) \geq u_\circ(\omega_i)$ for all $\omega_i \in \Omega$.

Given a price schedule for each agent, the expected payoff for firm j can be accordingly expressed as

$$V^j(\tilde{\mathbf{y}}) \equiv \mathbb{E}\left[v^j(\tilde{x}^j(\omega_1), \ldots, \tilde{x}^j(\omega_I), \tilde{\mathbf{y}}(\tilde{x}^j(\omega_1)), \ldots, \tilde{\mathbf{y}}(\tilde{x}^j(\omega_I)), \omega)\right],$$

where $\mathbb{E}[\cdot]$ is the expectation operator over $\omega = [\omega_1, \ldots, \omega_I]$. Let $v^j_\circ \equiv \mathbb{E}[v^j(0, \ldots, 0, 0, \ldots, 0, \omega)]$ be the reservation profit for firm j when it does not trade at all. The level of the firm's reservation profit depends on the application we consider. If the firm is a (potential) trader who owns a good such as a car owner or an asset holder, then its reservation profit is its payoff associated with keeping the good. If the firm is a producer that can make a production decision contingent on contracting with buyers, then its reservation profit is simply the zero profit associated with producing nothing.

We now examine how firms can implicitly support any price schedule $\tilde{\mathbf{y}}$ with $V^j(\tilde{\mathbf{y}}) \geq v_\circ^j$ for all j, as their equilibrium terms of trade. To this end, we first construct each firm's equilibrium trading mechanism that prevents any firm's deviation to any complex trading mechanism. We call it a triggering WICE direct mechanism.

For an arbitrary price schedule $\tilde{\mathbf{y}}$ with $V^j(\tilde{\mathbf{y}}) \geq v_\circ^j$ for all j, each firm j's triggering WICE direct mechanism is denoted by $\gamma_E^j\colon E^I \to X \times \mathbb{R}$. The set of messages available for each agent i is $E \equiv P \times X$, where $P = \mathbb{R}_{++} \cup \{\eta\}$. Each agent i reports $(p, x) \in E$.[4] The message $x \in X$ is the quantity that the agent wants to trade with the firm. The message p has the following meaning. If $p = \eta$, then it means either (i) no other firms deviate from the triggering WICE direct mechanisms or (ii) a deviating firm's price schedule for each agent is $\tilde{\mathbf{y}}$ and it is independent of the other agent's messages to the deviating firm. If $p \in \mathbb{R}_{++}$, then it means (a) there exists a deviating firm whose trading mechanism does not induce (ii) and (b) p is the the deviating firms' lowest average price for the agent if he was the only agent who participated in the deviating firm's mechanism.

Suppose that firm k deviates to a mechanism $\gamma^k\colon C^I \to X \times \mathbb{R}$. When each agent i is the only agent who participates in the deviating firm's mechanism, the deviating principal's lowest average price for the agent is defined as

$$\inf\left\{p' \in \mathbb{R}_{++} : p' = \frac{m}{x} \text{ for } (x, m) \in \gamma^k(C, \varnothing^{I-1}) \text{ and } x \neq 0\right\}.$$

For an arbitrary price schedule $\tilde{\mathbf{y}}$ with $V^j(\tilde{\mathbf{y}}) \geq v_\circ^j$ for all j, the triggering WICE direct mechanism $\tilde{\gamma}_E^j\colon E^I \to X \times \mathbb{R}$ has the following properties:

D1. If the number of participating agents is two or more and more than half of participating agents report $p \in \mathbb{R}_{++}$, the firm offers a linear price schedule $\tau(p)$ such that $\tau(p)(x) = ax$ for all $x \in X$. Each participating agent i then pays $\tau(\mathbf{y})(x) = ax$ for the quantity x that he submits along with his report on some other firm's lowest average price.

[4] If an agent decides not to trade with a firm, it is assumed to be equivalent to sending $x = 0$ to the firm. Accordingly the mechanism assigns zero monetary payment for the agent.

D2. In all other cases, the price schedule is $\tilde{\mathbf{y}}$. Each agent i then pays $\tilde{\mathbf{y}}(x)$ for the quantity x that he submits along with his report on some other firm's price schedule.

The key to the triggering WICE direct mechanism is to set up $\tau(p)$ for all $p \in \mathbb{R}_{++}$ in a way that it induces agents not to trade with a deviating firm in truth-telling continuation equilibrium. As shown later, non-deviating firms' triggering WICE direct mechanisms in fact lead to truth-telling continuation equilibrium in which each agent reports, to each non-deviating firm, the lowest average price p that he believes he would face from the deviating firm if he was the only one who participated in the deviating firm's trading mechanism.

Suppose that a deviating firm's price schedule is $p \in \mathbb{R}_{++}$ for each agent if he was the only one who traded with the deviating firm. When two or more agents participate in the non-deviating firm's triggering WICE direct mechanism and more than half of participating agents report $p \in \mathbb{R}_{++}$, then the triggering WICE direct mechanism assigns the linear price schedule $\tau(p)(x) = ax$ that satisfies

$$a = \min\left[p, \inf_{x \in X \setminus \{0\}} \left(\frac{\tilde{\mathbf{y}}(x)}{x}\right)\right]. \tag{2}$$

Note that $\inf_{x \in X \setminus \{0\}} \left(\frac{\tilde{\mathbf{y}}(x)}{x}\right)$ is the lowest average price based on the price schedule $\tilde{\mathbf{y}}$.

Consider an arbitrary $\tilde{\mathbf{y}}$ that induces $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for all j. Our main result shows that when every firm offers the triggering WICE direct mechanism with $\tau(\cdot)$ that satisfies (2) for any $p \in \mathbb{R}_{++}$, there exists the truth-telling continuation equilibrium in which no firm j can make more profit than $V^j(\tilde{\mathbf{y}})$ by deviating to any complex trading mechanism. Therefore, any price schedule $\tilde{\mathbf{y}}$ with $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for all j can be supported as equilibrium terms of trade in the market.

Theorem 1 *Suppose that each firm offers the triggering WICE direct mechanism associated with a price schedule $\tilde{\mathbf{y}}$ with $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for all j. It is the equilibrium mechanism for each firm in perfect Bayesian equilibrium in which the truth-telling continuation equilibrium is characterized as follows:*

1. *When no firm deviates or firm k deviates to a mechanism that induces $\tilde{\mathbf{y}}$ to each agent regardless of the other agents' reports to firm k, each*

agent i of type ω_i sends the message $(\eta, \tilde{x}^j(\omega_i))$ to each non-deviating firm j and a message, to firm k, which leads him to trade $\tilde{x}^k(\omega_i)$ at $\tilde{\mathbf{y}}(\tilde{x}^k(\omega_i))$ with firm k.

2. *When firm k deviates to any other mechanism, each agent i of type ω_i trades $\hat{x}(\omega_i)$ only with every non-deviating firm by reporting $(p, \hat{x}(\omega_i))$, where p is each agent's belief on the lowest average price that the deviating firm's mechanism would induce if only one agent participated in its mechanism and $\hat{x}(\omega_i)$ satisfies*

$$\hat{x}(\omega_i) \in \arg\max_x u((J-1)x, (J-1)\tau(p)(x), \omega_i).$$

Proof. Choose an arbitrary price schedule $\tilde{\mathbf{y}}$ that induces $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for each firm j based on the solution $(\tilde{x}^1(\omega_i), \ldots, \tilde{x}^j(\omega_i))$ to problem (1). Each firm offers the triggering WICE direct mechanism associated with the price schedule $\tilde{\mathbf{y}}$. We will show that the triggering WICE direct mechanism is the equilibrium trading mechanism for each firm in perfect Bayesian equilibrium in which agents truthfully communicate with non-deviating firms on their beliefs on the lowest average price that a deviating firm's trading mechanism would induce no matter how complex the deviating firm's trading mechanism is. First of all, consider the truth-telling continuation equilibrium on the equilibrium path

(a) On the equilibrium path: When no firm deviates from its triggering WICE direct mechanism, each agent i participates in all firms' triggering WICE direct mechanisms by sending the message $(\eta, \tilde{x}^j(\omega_i))$ to each firm j. Suppose that an agent considers a deviation from the report η when he communicates with a firm. Because of condition (D2), an agent cannot unilaterally change a firm's price schedule away from $\tilde{\mathbf{y}}$ with any other report in P given that all the other agents send η to the firm. Therefore, it is incentive compatible for each agent to send η to each firm when the other agents also send η to each firm. Because each firm's price schedule becomes $\tilde{\mathbf{y}}$, it is in fact optimal for each buyer i of type ω_i to participate in each firm j's triggering WICE direct mechanisms by sending $\tilde{x}^j(\omega_i)$ along with η.

(b) Off the equilibrium path: Now we consider firm k's deviation to any complex trading mechanism. There are two types of deviation.

(b-1) Suppose that firm k deviates to a trading mechanism $\gamma^k \colon C^I \to X \times \mathbb{R}$ such that (i) for all $c^k_i \in C$ and all $c^k_{-i}, \acute{c}^k_{-i} \in C^{I-1}$,

$$\gamma^k(c^k_i, c^k_{-i}) = \gamma^k(c^k_i, \acute{c}^k_{-i}) \tag{3}$$

and (ii) for each $x \in X$,

$$\min\{m \in \mathbb{R}\colon (x,m) \in \gamma^k(C, c_{-i}^k)\} = \tilde{\mathbf{y}}(x). \tag{4}$$

(3) implies that the quantity and payment pair for each agent i depends only on his message but not on the other agents' messages. For any $c_{-i}^k \in C^{I-1}$, recall that $\gamma^k(C, c_{-i}^k)$ denotes the set of all quantity and payment pairs that each agent i can induce from firm k.

When agent i chooses to trade x with firm k, there may be many messages that can induce the same quantity x along with different amounts of payment. If agent i ever chooses to trade x with firm k, it is always optimal for him to trade x at the minimum payment. Therefore, the left-hand side of (4) is the minimum payment that the agent will pay if he trades x with firm k. Note that (4) already presumes that firm k deviates to a mechanism in which the minimum on the left-hand side of (4) exists. In fact, when firm k deviates to a mechanism satisfying (3) and (4), it is equivalent to offering the price schedule $\tilde{\mathbf{y}}$.

Assume that, given firm k's deviation to a mechanism satisfying (3) and (4), each agent i trades with all firms including the deviating firm. Each agent i of type ω_i sends the message $(\eta, \tilde{x}^j(\omega_i))$ to each non-deviating firm j and sends a message to firm k in a way that it induces him to trade $\tilde{x}^k(\omega_i)$ with firm k at $\tilde{\mathbf{y}}(\tilde{x}^k(\omega_i))$. As proved in part (a), each agent finds it optimal to send η to each non-deviating firm when all the other agents send the message η to each non-deviating firm. This leads each non-deviating firm to assign the price schedule $\tilde{\mathbf{y}}$ given its triggering WICE direct mechanism. Because all firms' price schedules, including the deviating firm's, are $\tilde{\mathbf{y}}$, it is again optimal for each agent i of type ω_i to trade $\tilde{x}^\ell(\omega_i)$ with firm ℓ at $\tilde{\mathbf{y}}(\tilde{x}^\ell(\omega_i))$ for all $\ell = 1, \ldots, J$. Parts (a) and (b-1) complete the proof of the first part of Theorem 1.

(b-2) Suppose that firm k deviates to any other trading mechanism $\gamma^k\colon C^I \to X \times \mathbb{R}$ that does not belong to (b-1). Suppose that agent i is the only one agent who participates in firm k's trading mechanism. Then, $\gamma^k(C, \varnothing^{I-1})$ is the set of all quantity and payment pairs that agent i can choose from firm k and hence the lowest average price for the agent becomes

$$p = \inf\left\{p' \in \mathbb{R}_{++}\colon p' = \frac{m}{x} \text{ for } (x,m) \in \gamma^k(C, \varnothing^{I-1}) \text{ and } x \neq 0\right\}. \tag{5}$$

We will show that, upon firm k's deviation to a trading mechanism $\gamma^k : C^I \to X \times \mathbb{R}$, each agent i of type ω_i trades with only non-deviating firms by sending the message $(p, \hat{x}(\omega_i))$ to each non-deviating firm, where p satisfies (5) and $\hat{x}(\omega_i) \in \arg\max_x u((J-1)x, (J-1)\tau(\mathbf{y})(x), \omega_i)$.

When every agent reports $p \in \mathbb{R}_{++}$ to each non-deviating firm, the non-deviating firm's price schedule becomes $\tau(p)$ according to (D1) so that the agent pays $\tau(p)(x) = ax$ for any x that the agent trades with the non-deviating firm. We first show that it is optimal for each agent to truthfully report p defined in (5) to each non-deviating firm when the other agents do the same.

Assume that all agents truthfully report p defined in (5) to each non-deviating firm upon firm k's deviation to $\gamma^k \colon C^I \to X \times \mathbb{R}$. Suppose that an agent reports p'' ($p'' \neq p$) to any non-deviating firm given that all other agents report p. If $I \geq 3$, then the non-deviating firm's price schedule is still $\tau(p)$ according to (D1) because still more than half of participating agents report p. Therefore, the agent has no incentive to deviate away from p. If $I = 2$, then the non-deviating firm's price schedule becomes $\tilde{\mathbf{y}}$ according to (D2) because one agent reports p and the other agent reports p''. Subsequently, the agent pays $\tilde{\mathbf{y}}(x)$ for any x that the agent trades with the non-deviating firm. Because of (2), $\tau(p)$ satisfies $\tau(p)(x) = ax \leq \tilde{\mathbf{y}}(x)$ for any x. Hence even when $I = 2$, it is optimal for an agent to truthfully report p to each non-deviating firm given that the other agent does the same.

Finally we will show that it is optimal for each agent to trade $\hat{x}(\omega_i)$ only with each non-deviating firm. Suppose that agent i currently trades x with a non-deviating firm given that all agents report p to the non-deviating firm and that he is the only agent who trades with the deviating firm. Let x' be the quantity that agent i trades with the deviating firm. Then, the total payment associated with trading x with the non-deviating seller and x' with the deviating seller is no less than $ax + px'$ because of the definition of $\tau(p)$ in (2) and the definition of p in (5). However, if the agent trades $x + x'$ only with the non-deviating seller, the monetary payment is $a(x + x')$, which is no more than $ax + px'$ because of (2). It implies that the agent can trade $x + x'$ with the same or less amount of monetary payment when he trades only with the non-deviating firm. Therefore, it is optimal for each agent not to trade with the deviating firm when all the other agents do not trade with the deviating firm. Because each non-deviating firm's price schedule is the linear price schedule $\tau(p)$, each agent i of type ω_i optimally trades the equal quantity with each non-deviating firm by sending $(p, \hat{x}(\omega_i))$ to it. This completes the proof of the second part of Theorem 1.

When firm k deviates to a trading mechanism that belongs to (b-1), it receives the same expected profit $V^k(\tilde{\mathbf{y}})$ that it would receive with the triggering WICE direct mechanism. When firm k deviates to any other mechanism, i.e., one that belongs to (b-2), it receives its reservation profit $v^j_\circ$ because no agents trade with firm k in truthful continuation equilibrium. Because the expected profit $V^k(\tilde{\mathbf{y}})$ associated with the triggering WICE direct mechanism is no less than $v^j_\circ$, firm k cannot gain by deviating to any alternative mechanism. ■

When all firms maintain their triggering WICE direct mechanisms, their price schedules are $\tilde{\mathbf{y}}$ in truth-telling continuation equilibrium. When a firm deviates to an arbitrary mechanism that is essentially equivalent to offering $\tilde{\mathbf{y}}$ to each agent independent of the other agents' messages, non-deviating firms do not punish the deviating firm and their price schedules continue to be $\tilde{\mathbf{y}}$ in truth-telling continuation equilibrium. If a firm deviates to any other mechanism, then each agent reports, to each non-deviating firm, the lowest average price p that the deviating firm's mechanism could induce if he participated in the deviating firm's trading mechanism alone in truth-telling continuation equilibrium. Subsequently, each non-deviating firm offers a linear price schedule that has the unit price equal to the minimum between the average unit price of $\tilde{\mathbf{y}}$ and p. This makes it optimal for agents not to trade with the deviating firm. Therefore, no firm j can find a profitable deviation to any trading mechanism as long as the firm's expected profit $V^j(\tilde{\mathbf{y}})$ associated with a price schedule $\tilde{\mathbf{y}}$ is no less than $v^j_\circ$.

When there are three or more agents, an agent's unilateral deviation in his report to a non-deviating firm does not chnage the non-deviating firm's price schedule because still more than half of agents report the true lowest average price that would be induced by a deviating firm's mechanism. When there are only two agents, the WICE triggering mechanism shoots both agents upon their different reports by continuing to assign $\tilde{\mathbf{y}}$ for both agents. In this way, the WICE triggering mechanism can induce truth-telling continuation equilibrium as long as there are multiple agents.[5]

[5] As in the single principal case, an equilibrium in a competing mechanism game is derived by the truth-telling continuation equilibrium in which agents truthfully reports on what principals ask. To examine robustness of equilibrium, Han (2007) and Peters (2001) study the notion of the strongly robust equilibrium. An equilibrium is said to be strongly robust if it survives in all continuation equilibria upon any firm's deviation. Attar, Mariotti, and Salanié (2011) pointed out that strongly robustness is too demanding and especially it is inconsistent with equilibrium in the market for lemons (i.e., the common-value case).

The triggering WICE direct mechanism features convenience in a large class of applications for non-exclusive trading problems under adverse selection. Each agent's message is two numbers (the deviating principal's lowest average price and the quantity that the agent wants to buy) and hence it is simple and independent of the number of agents. Therefore, each agent's communication with a firm is very simple. The triggering WICE direct mechanism also works for any multiple number of agents, including the case of two agents, and the set of equilibrium payoffs is defined in terms of each firm's reservation profit which is independent of trading mechanisms.

5 Conclusion

In terms of applications, we can show how our results on multiple equilibria differ from the multiplicity of equilibria in Akerlof (1970). According to Akerlof's result, the Walrasian market price reflects only the average quality of the good and we may have multiple fixed-point Walrasian prices that lead to different average quality of the good traded in the market. Because the Walrasian market price correctly reflects the average quality of the good, equilibrium profits are zero in any equilibrium. Our results differ from those in Akerlof for the following reasons. First of all, we showed that firms can maintain a wide range of collusive outcomes through the sophisticated trading mechanisms that make firms' terms of trade responsive to agents' report on competing firms' lowest average price. Subsequently, positive equilibrium profits and corresponding prices may arise given the same probabilistic beliefs on the quality of a good. Secondly, our multiplicity of equilibrium profits and prices is based on the full game-theoretical approach without contractual restrictions.

Theorem 1 can be easily extended to ex-ante heterogeneous agents. Let $u_i(\cdot, \cdot, \omega_i)$ is the payoff function for agent i of type ω_i. Assume that firms agree to offer an array of price schedules $[\tilde{\mathbf{y}}_1, \ldots, \tilde{\mathbf{y}}_I]$ for agents, where $\mathbf{y}_i$ is for agent i. Given the price schedule $\tilde{\mathbf{y}}_i$, we can find a profile of quantities that agent i of type ω_i will trade with each firm j. Given an array of price schedules $[\tilde{\mathbf{y}}_1, \ldots, \tilde{\mathbf{y}}_I]$, one can construct the triggering WICE direct mechanism for each firm j that asks each agent to report a quantity that he wants to trade and an array of the lowest average prices $[p_1, \ldots, p_I]$, where p_i is the lowest average price that agent i would face if he was the only one who participated in a deviating firm's trading mechanism.

Consider the case in which the number of participating agents is two or more, and more than half of their reports on some other firm's lowest average prices, one for each agent i, are all $[p_1, \ldots, p_I]$ and $p_i \neq \eta$ for some i. The triggering WICE direct mechanism then assigns the price schedule $\tau_i(p_i)$ for agent i such that $\tau_i(p_i)(x) = a_i x$, where $a_i = \min\left[p_i, \inf_{x \in X \setminus \{0\}} \left(\frac{\tilde{\mathbf{y}}_i(x)}{x}\right)\right]$. In all other cases, the triggering WICE direct mechanism continues to assign $\tilde{\mathbf{y}}_i$ for each agent i. Given this triggering WICE direct mechanism, we can show that Theorem 1 is extended for ex-ante heterogeneous agents.

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