

SHOPPER CITY

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Abstract: The bulk of the literature on retail location looks at the topic from the perspective of either the retail firm or the individual shopper. Another branch of the literature examines the spatial distribution of retail activities within a city or region, drawing on either central place theory or the Lowry model, neither of which incorporates either markets or agglomeration economies. This paper looks at retail location from the perspective of a general equilibrium model of location and land use, with agglomeration economies in retailing. In particular, drawing on the Fujita-Ogawa (1982) model of non-monocentric cities, it develops a model of retail location, assuming that retail firms behave competitively, subject to spatial agglomeration economies. Locations are distinguished according to the effective variety of retail goods they offer. Shoppers are willing to pay more for goods at locations with greater effective variety, and in their choice of where to shop trade off retail price, product variety, and accessibility to home. Retail prices and land rents at different locations adjust to achieve spatial equilibrium.

Keywords: retail, agglomeration, variety, land use

JEL codes: R10, R20, R30

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1. *Introduction*

One of the authors has recently been participating in the development of a large-scale, microeconomic metropolitan simulation model (tentatively called METRO-LA) that aims to forecast transportation, land use, and pollution in the Los Angeles metropolitan area. One of the many modeling questions that has arisen is: How should retail location be modeled at such a geographic scale? The literature on retail location that is most familiar to economists is strategic firm location theory/ spatial competition theory, to which Curtis Eaton has made distinguished contributions¹, many in co-authorship with Richard Lipsey. Models in this literature solve for the Nash equilibrium of a game between firms, with locations, prices, and perhaps entry as strategy variables. With space interpreted as geographic space, this literature has provided many insights into the location and pricing of firms; and with space interpreted as characteristics space, into product differentiation. But spatial competition theory is not well suited to the analysis of retail location on a broad geographical scale. For one thing, attempting to solve for the Nash equilibrium of a game between thousands of firms, with many different types of consumer goods, is intractable. For another, firm location theory is partial equilibrium in nature, taking the location of consumers as given, but a satisfactory model of land use needs to take into account the simultaneous location equilibrium of firms and households. For yet another, land rent is ignored in spatial competition theory but in an urban setting plays an essential role in location choice.

¹ These include Eaton (1976), Eaton and Lipsey (1975, 1979, 1982), and Eaton and Wooders (1985).

There is also a large literature on retail location outside economics, in marketing and geography/regional science. The bulk of the marketing literature on retail location looks at the topic from the perspective of either the individual retail firm or the individual shopper. The former examines the most profitable location of a single retail firm or shopping center, often taking into account strategic interaction with other retail firms². The latter examines the individual's decision of where to shop³. The geography/regional science literature on the subject, for which Harris (1985) provides an excellent review, examines the spatial distribution of retail activities within a city or region, drawing on central place theory (e.g. Berry, 1967), the Lowry model (Lowry, 1964,1967; and Goldner, 1971), and spatial interaction theory (Wilson, 1970,1974). This branch of the literature does not incorporate prices and markets, and does not treat agglomeration economies, at least explicitly.

This paper looks at retail location from the perspective of a general equilibrium model of location and land use, with agglomeration economies in retailing, adapting the approach developed by Fujita and Ogawa (1982). Fujita and Ogawa solves for the simultaneous location equilibrium of firms and households in a city. The model is essentially competitive. Each firm decides where to locate taking prices, rents, household location, and other firm locations as given, and produces under constant returns to scale. The

²There is a vast literature on the subject. Some well-known papers include Brown (1989, 1993), Clark, Bennisson, and Pal (1997), Craig, Ghosh, and McLafferty (1984), Drezner and Drezner (1996), Huff (1964, 1966), and Weisbrod, Parcells, and Kern (1984). Kohsaka (1984) is representative of papers that look at optimal shopping center location.

³ The literature on this topic is less extensive. Three well-known papers are Bell, Ho, and Tang (1998), Bucklin, Gupta, and Siddarth (1998), Cadwallar (1975).

economies of scale that give rise to agglomeration are external to the individual firm. Closer proximity to other firms makes a firm more productive; firms learn from other firms, incur lower transport costs in intermediate goods exchange, and have access to a broader labor pool. In deciding where to locate, a firm trades off the higher productivity of more proximate locations against the higher wages and rents there. Each individual makes two location decisions, where to live and where to work, taking prices, rents, firm location, and the location of all other households as given. In deciding where to live, she trades off the higher commuting cost of a less central location against the lower land rent; and in deciding where to work, she trades off the higher wage at a more central location against the higher commuting cost. Equilibrium obtains when, by changing location, no firm can increase its profits and no household can increase its utility. In equilibrium, land and labor markets clear at all locations, and land goes to that use which bids the most for it. Since there are economies of scale, there may be multiple equilibria, corresponding to different location patterns. One possible equilibrium is monocentric, in which all firms are located at the city center with residences surrounding them; another possible equilibrium is completely mixed, with firms and households co-locating at all urban locations; another has three centers; and so on. Four parameters are particularly important in determining which location patterns are equilibria, commuting cost per unit distance, the spatial decay rate -- the exponential rate at which benefits from proximity to other firms decays with distance, the population, and a parameter characterizing the intensity of agglomeration economies in production. When, for instance, the spatial decay rate is large and commuting costs are moderate, there are equilibria with many small employment centers. Also, as the urban population increases, subcentering occurs.

This paper adapts the Fujita-Ogawa model by having firms sell differentiated retail goods rather than produce. In order to highlight the basic economics, the model is made as simple as possible. Agglomeration economies occur via individuals' taste for product variety rather than via external economies of scale in production; in particular, stores have an incentive to co-locate in a shopping center since doing so raises the *effective variety* there, which makes shopping there more attractive. Each of the identical individuals receives an endowment of a generic good, which she spends on her lot, transportation for shopping, and the differentiated retail goods. She decides where to live, trading off rent against transport cost, and where to shop, trading off the greater variety against the higher price and likely higher transport cost of shopping at a larger center. Using only land, competitive retail firms transform the generic good into differentiated retail goods and sell them. Thus, individuals and stores compete in the land market. If population and shopping transport costs are low, if the taste for variety is high, and if the benefit one store derives from proximity to other stores falls off moderately rapidly with distance, then there exists a monocentric equilibrium with all stores at a single, central shopping center. If shopping transport costs are high and if the taste for variety is low, then an equilibrium exists in which stores and residences are intermixed. The model is called "Shopper City" since individuals do nothing but shop and enjoy consuming their differentiated retail goods on their lots.

A companion paper, Arnott and Erbil (2008), enriches the above model, developing a computable static general equilibrium model with agglomeration in both retailing and

production. The CGE model is standard (allowing for labor and capital, as well as land, in retail goods production, intermediate and wholesale goods production, structures of variable density, multiple types of retail goods, multiple household groups, commodity transportation, etc.) except that it treats residential location, household transportation for commuting and shopping, and agglomeration economies in both production and retailing. METRO-LA will have the same model structure, except that it will be dynamic. History dependence will be incorporated through durable structures and the transmission of industry- and zone-specific location potentials and zone-specific indices of effective varieties from one period to the next. The present will be linked to the future via property markets, with property values being determined under perfect foresight.

Section 2 lays out the basic model. Section 3 derives the parameter restrictions such that a monocentric equilibrium exists. Section 4 performs the same exercise for a completely mixed urban configuration. And section 5 concludes.

2. *Model Description*

The model adapts Fujita and Ogawa (1982), replacing agglomeration economies in production with agglomeration economies in retailing.

- geography, population, and transportation

N identical individuals live in the city. Each resides on a lot of size S and requires s units of retail land area⁴. Thus, the residential area is NS , the retail area Ns , and the

⁴ An earlier version of the paper employed the more realistic assumption that physical sales volume per unit area is fixed. With this assumption, the algebra was considerably more complex and little additional insight was obtained.

total urban area $N(S + s)$. The city is long and narrow, of unit width. The central location is taken to be the origin, and both x and y are used to index location. The boundaries of the city are $-N(S + s)/2$ and $N(S + s)/2$. Every day each individual makes a return journey from her home to the shopping location⁵ of her choice at a transport cost of t per unit distance.

- tastes

Each individual derives utility from differentiated retail goods and her lot. Since lot size is fixed, utility can be treated as a function only of differentiated consumer goods. The utility she receives from retail goods is a function of the quantity she purchases, Q , as well as the *effective variety*, v :

$$U = u(Q, v) = Qv. \quad (1)$$

The multiplicative form is chosen to simplify the algebra. The effective variety for a shopper who travels to location y to shop is measured as:

$$v(y) = 1 + k \int_{-b}^b a(x) \exp\{-\alpha|x - y|\} dx, \quad (2)$$

where $a(x)$ is the proportion of land at x that is used by stores, k is a parameter indexing the intensity of taste for variety or the degree of variety, and α is the exponential rate of spatial attenuation of benefits from variety. Thus, effective variety is additive in the contribution to effective variety over locations, and the contribution to effective variety of a store at location x to a shopper who travels to location y to shop decreases exponentially in the distance between x and y . Observe that the effective

⁵ Trip frequency could be endogenized by adding home inventory costs. An individual residing at a location that is less accessible to shops would travel less frequently to shop and keep a larger inventory of goods at home.

variety offered by a completely isolated store is normalized to be unity. Note also that (2) is a “reduced form” specification, only implicitly taking into account search costs. Eq. (2) is the same as the Fujita-Ogawa location potential function⁶, except for the addition of the 1.

- individual choice

Each individual decides where to reside, x , and where to shop, y , so as to maximize utility, given by (1) and (2), subject to the budget constraint

$$Y - R(x)S - p(y)Q - t|x - y| = 0, \quad (3)$$

where Y is exogenous income (endowment of the generic good), $R(x)$ is the rent function at x , $p(y)$ is the retail price function relating the retail price to shopping location, and t is transport cost per unit distance.

- land ownership and alternative land uses

All land rents accrue to absentee landlords. Land not in urban use is employed in agriculture at a rent of R_a .

- retail technology

Retailing is characterized by constant returns to scale. An atomistic store at x purchases the generic good from households, transforms it into differentiated retail goods, which it then sells at the competitively-determined retail price $p(x)$. Stores incur in addition a fixed cost per unit area K , which can be interpreted as capital costs, as well as land rent. Thus, the profit function per unit area is

⁶ $v(x)$ could be termed the retail location potential function, but this term is used in the earlier literature (e.g., Lowry, 1964) to refer to the profitability of a location to a store, whereas $v(x)$ refers to the attractiveness of a location from the perspective of a customer.

$$\pi(x) = [p(x) - 1]Q(x)/s - K - R(x); \quad (4)$$

$Q(x)$ is the equilibrium quantity of retail goods purchased by an individual who shops at x and $1/s$ is the number of individuals who shop at x , so that $Q(x)/s$ is the retail sales volume at x .

Equilibrium is defined to be a location pattern, described by $a(x)$ the proportion of land in retail use at x , $a^-(x)$ the proportion of land in residential use at x , b the city boundary, a retail price function $p(x)$, and a rent function $R(x)$, such that all markets clear, no store can increase its profits per unit area by changing location, and no individual can increase her utility by changing either her residential or her shopping location. In equilibrium, all urban land is developed so that $a(x) + a^-(x) = 1$ for $x \in [-b, b]$.

The constructive procedure to solve for equilibrium is the same as that employed in Fujita and Ogawa. For each qualitatively different location pattern, one solves for the set of parameter values consistent with the equilibrium conditions. To illustrate the procedure, the next section derives the set of parameter values consistent with a symmetric monocentric equilibrium, in which stores occupy the central area and on both sides residential lots extend from the outer boundary of the retail area to the city boundary, beyond which land is used in agriculture. Section 4 derives the set of parameter values consistent with a completely mixed equilibrium, in which each individual purchases at a backyard store.

One might reasonably object to our specification of agglomeration economies in retailing. If one were to ask store owners why one location is more attractive than other, the first thing they would mention is likely customer volume. In our model, in contrast, from the perspective of store owners locations are differentiated according to the competitive price. We defend our specification on three grounds: first, it is in keeping with our competitive assumptions⁷; second, if the model were extended to allow for variable structural density, shopping volume would be higher at locations with greater shopping variety; and third, it is our impression that retail prices do differ significantly over locations⁸.

3. *Monocentric Urban Configuration*

The city is symmetric around the origin. Letting f denote the distance of the retail-residential boundary from the city center, the retail area, which extends from $-f$ to f , is flanked by two residential areas, one extending from $-b$ to $-f$, the other from f to b . To simplify, where applicable, the right-hand side of the city shall be considered, for which the location index is positive. Thus:

⁷ One model could be adapted without difficulty so that retailers are monopolistically competitive rather than perfectly competitive, but the treatment of other industry structures (except monopoly, which is unrealistic) would result in intractability.

⁸ This is difficult to document because of sales, discounts, and product choice. Consider, for example, goods that have a suggested retail price. A store owner can lower his average markup on such goods by selling them at a deeper discount, by selling them at the discounted price a greater proportion of the time, by have deeper and more frequent store-wide sales, and by choosing to sell those goods for which the ratio of the suggested retail price to the wholesale price is lower. The higher price of groceries in ghetto locations is documented. Labor economists have used the McDonald's wage to measure intra-metropolitan spatial variation in wages. Perhaps the same could be done for product prices.

$$f = Ns/2 \quad b - f = NS/2 \quad b = N(S + s)/2. \quad (5)$$

This is a convenient point at which to record some properties of the effective variety function, (2):

For $x \in [0, f]$:

$$\begin{aligned} v(x) &= 1 + k \int_{-f}^x \exp\{-\alpha(x - y)\} dy + k \int_x^f \exp\{-\alpha(y - x)\} dy > 0 \\ v'(x) &= k(\exp\{-\alpha(x + f)\} - \exp\{-\alpha(f - x)\}) < 0 \\ v''(x) &= -k\alpha(\exp\{-\alpha(x + f)\} + \exp\{-\alpha(f - x)\}) < 0. \end{aligned} \quad (6)$$

For $x \in (f, b]$:

$$\begin{aligned} v(x) &= 1 + k \int_{-f}^f \exp\{-\alpha(x - y)\} dy = 1 + (k/\alpha)(\exp\{-\alpha(x - f)\} - \exp\{-\alpha(x + f)\}) \\ v'(x) &= -k(\exp\{-\alpha(x - f)\} - \exp\{-\alpha(x + f)\}) = -\alpha(v(x) - 1) < 0 \\ v''(x) &= -\alpha v'(x) > 0. \end{aligned} \quad (7)$$

Also,

$$\begin{aligned} v(0) &= 1 + (2k/\alpha)(1 - \exp\{-\alpha f\}), \quad v(f) = 1 + (k/\alpha)(1 - \exp\{-2\alpha f\}) \\ v(b) &= 1 + (k/\alpha) \exp\{-\alpha NS/2\} (1 - \exp\{-2\alpha f\}). \end{aligned} \quad (8)$$

Thus, on the right-hand side of the city, the effective variety function declines monotonically with distance from the city center, is positive everywhere, is concave in the retail area, and convex in the residential area.

The approach taken to solve for the monocentric equilibrium is essentially the same as that employed in Fujita and Ogawa. First, solve for the retail bid-rent function and the residential bid-rent function, taking as given two endogenous parameters, the equilibrium

level of utility and the equilibrium retail price at the retail-residential boundary. Second, apply two equilibrium conditions to determine the two endogenous parameters, first that the retail bid rent equal the residential bid rent at the retail-residential boundary, and second that the residential bid rent equal the agricultural bid rent at the urban boundary. Finally, check that the solution is consistent with the final equilibrium condition that “land goes to that use which bids the most for it”; specifically, check that the retail bid rent exceeds the residential bid rent everywhere in the retail area, and that the residential bid rent exceeds the retail bid rent everywhere in the residential area.

- The retail bid-rent function

The retail bid rent at x , $\Phi(x)$, is the maximum amount a retail firm is willing to bid in rent per unit area of land at x , which is the amount that drives its profits to zero. Thus, the retail bid rent at x equals revenue minus non-land costs, the cost of wholesale goods plus the fixed cost:

$$\Phi(x) = [p(x) - 1]Q(x) / s - K. \quad (9)$$

In equilibrium, all identical individuals receive the same level of utility, U^* .

Furthermore, $U = Qv$, so that

$$Q(x) = U^* / v(x). \quad (10)$$

Substituting (10) into (9) yields

$$\Phi(x) = [p(x) - 1]U^* / (sv(x)) - K. \quad (11)$$

- The residential bid-rent function

The residential bid rent, $\Psi(x, U)$, is the maximum amount an individual residing at x is willing to pay in rent per unit area of land, consistent with utility U . For the moment, consider the residential bid-rent function only in the residential area. Since, in

equilibrium, all individuals are indifferent as to where they shop in the retail area, without loss of generality the individual who shops at f is considered. The residential bid-rent function for $x \in (f, b)$ is

$$\Psi(x, U) = (Y - p(f)U / v(f) - t(x - f)) / S . \quad (12)$$

The individual at x spends $t(x - f)$ in shopping transport costs and when she shops at f has to spend $p(f)U / v(f)$ to achieve utility U , leaving $Y - p(f)U / v(f) - t(x - f)$ to spend on lot rent. Observe that, over the residential area, the residential bid rent varies with residential location so as to offset transport costs, that the residential bid-rent curve is linear in x , and that with fixed lot size the sum of the expenditures on transport costs and lot rent is constant across residential locations, which leaves a constant amount left over to spend on the differentiated retail goods. Thus, over the residential area, individual expenditure on differentiated retail goods is independent of both residential and shopping location. The form of the residential bid-rent function in the retail area will be considered later.

- Equal rent conditions

One of the equal rent conditions is that the residential bid-rent equal the agricultural bid rent at the city boundary:

$$R_a = \Psi(b, U) = (Y - p(f)U / v(f) - t(b - f)) / S . \quad (13)$$

Since transport costs at the urban boundary, as well as the boundary location are known, this equation can be solved for the equilibrium expenditure on differentiated retail goods:

$$p(f)U / v(f) = Y - t(b - f) - R_a S = Y - tNS / 2 - R_a S . \quad (14)$$

The other equal rent condition is that the residential bid rent equal the retail bid rent at the residential-retail boundary:

$$\Phi(f) = [p(f) - 1]U / (sv(f)) - K = (Y - p(f)U / v(f)) / S = \Psi(f, U), \quad (15)$$

which can be rewritten as

$$[p(f)U / v(f)][1/s + 1/S] = U / (sv(f)) + K + Y / S. \quad (16)$$

Substituting (14) into (16) and rearranging yields

$$U^* = v(f)\{(Y - [tNS / 2 + R_a S][1 + s / S] - Ks)\}, \quad (17)$$

which gives the equilibrium level of utility U^* as a function of exogenous parameters and $v(f)$, which can be calculated using (5) and (8). The causality underlying (17) is complicated by the endogeneity of the retail price function. Knowing the location of the city boundary and the rent there determines the equilibrium residential bid-rent curve (eqs. (12) and (13)). In the standard monocentric city model, knowing the equilibrium residential bid-rent curve permits determination of the equilibrium utility level, but here it permits determination only of the equilibrium expenditure on retail goods. Since the equilibrium residential bid rent at the residential-retail boundary is known, the equal rent condition gives the equilibrium retail bid rent there. Knowing the retail bid rent there, as well as the equilibrium expenditure on the differentiated retail goods, permits determination of the equilibrium retail price at that location. Knowledge of the equilibrium expenditure $p(f)U / v(f)$, the equilibrium retail price, and the effective variety at that location then permits determination of the equilibrium level of utility, U^* .

From (14) we obtain

$$p^*(f) = (Y - tNS / 2 - R_a S)v(f) / U^*, \quad (18)$$

which gives the equilibrium $p(f)$ as a function of exogenous parameters.

- Completing the solution

Thus far, the following have been solved for: the equilibrium residential bid-rent function in the residential area, the equilibrium level of utility, and the equilibrium retail price and retail bid rent at the residential-retail boundary. It remains to solve for the complete equilibrium residential bid-rent, retail bid-rent, and retail price functions.

At whatever retail location x an individual shops, in equilibrium she receives utility

$U^* = Q^*(x)v(x)$ from consumption of the differentiated retail goods, yielding

$Q^*(x) = U^* / v(x)$. Furthermore, in equilibrium, an individual who shops at location x in

the retail area and lives in the residential area spends $tNS / 2 + R_a S + t(f - x)$ on her lot

rent, leaving $Y - tNS / 2 - R_a S - t(f - x)$ to spend on the retail good. Thus,

$p^*(x)Q^*(x) = Y - tNS / 2 - R_a S - t(f - x)$. Combining these results yields

$$\begin{aligned} p^*(x) &= [Y - tNS / 2 - R_a S - t(f - x)]v(x) / U^* \\ &= [Y - R_a S - t(b - x)]v(x) / U^* \quad \text{for } x \in [0, f]. \end{aligned} \quad (19)$$

To derive the retail price function in the residential area, consider a store at location y in

the residential area. Only individuals who live further away from the city center than y

would patronize this store. Such an individual who lives at $x > y$ pays $R_a S + t(b - y)$ in

lot rent and transport cost, leaving $Y - R_a S - t(b - y)$ to spend on retail goods. Thus,

$p^*(y)Q^*(y) = Y - R_a S - t(b - y)$. Combining this result with $Q^*(x) = U^* / v(x)$, and with

(19), gives

$$p^*(x) = [Y - R_a S - t(b - x)]v(x) / U^* \quad \text{for } x \in [0, b]. \quad (20)$$

Notice that the retail price function is the product of two terms. The first, in square brackets, which is increasing in x , reflects the reduced transport cost associated with less accessible shopping locations; the second, which is decreasing in x , reflects the reduced product variety at less accessible locations. As a result, the retail price function may be either increasing or decreasing in x .

Having solved for the equilibrium retail price function, it is straightforward to solve for the equilibrium retail bid-rent function:

$$\Phi^*(x) = [p^*(x) - 1]U^* / (sv(x)) - K. \quad (21)$$

Notice that this function is a product of two terms. The first term, the retail markup, may be either increasing or decreasing in x ; the second term, the quantity purchased, is increasing in x . Consequently, the equilibrium retail bid-rent function may be either increasing or decreasing in x .

The equilibrium residential bid-rent function for residential locations has already been solved for. To solve for the function in the retail area, consider an individual who resides in the retail area, $x \in [0, f)$. She is indifferent between shopping at her residential location and at any location closer to the city center; assume that she shops at her residential location. She consumes a lot of size S there, receives a level of utility $U^* = Q^*(x)v(x)$ from retail goods, and spends $p^*(x)Q^*(x)$ on them. Thus, for $x \in [0, f]$:

$$\Psi^*(x, U) = (Y - p^*(x)U^* / v(x)) / S = R_a + t(b - x) / S \quad (\text{using (20)}). \quad (22)$$

Expressed in terms of exogenous parameters, from (12), (19), (20), and (22), the equilibrium residential bid-rent function is

$$\Psi^*(x, U) = R_a + t(b - x)/S \quad \text{for } x \in [0, b], \quad (23)$$

which is linearly decreasing in x . And, using (20), and (21), the equilibrium retail bid-rent function expressed in terms of exogenous parameters and $v(x)$ is

$$\Phi^*(x) = [Y - R_a S - t(b - x)]/s - U^*/(sv(x)) - K \quad \text{for } x \in [0, b]. \quad (24)$$

As noted earlier, the retail bid-rent function may be either positively or negatively sloped.

Also,

$$\text{sgn}[d^2\Phi^*(x)/dx^2] = \text{sgn}[v''v - 2(v')^2]. \quad (25)$$

From (6) it follows that $\Phi^*(x)$ is concave in the retail area. In the residential area, from (7), $v''v - 2(v')^2 = -\alpha^2(v-1)(v-2)$, so that the retail bid-rent function is concave at those residential locations for which $v > 2$ and convex where $v < 2$.

- Checking that the solution is indeed a monocentric urban configuration

For the solution to characterize a monocentric urban configuration, it must be the case that the residential bid rent exceed the retail bid rent everywhere in the retail area and that the retail bid rent exceed the residential bid rent everywhere in the residential area. Since the residential bid-rent curve is linear while the retail bid-rent curve is concave in the retail area and can change curvature only once in the residential area, from concave to convex as x increases, it follows that these two conditions are satisfied if, first, the retail bid rent exceeds the residential bid rent at the city center, and, second, if the residential bid rent exceed the retail bid rent at the urban boundary. The former condition, which we term the first inequality, is

$$\Phi(0) = [Y - R_a S - tb] / s - U^* / (sv(0)) - K > R_a + tb / S = \Psi(0). \quad (26)$$

The latter condition, which we term the second inequality, is that

$$\Phi(b) = [Y - R_a S] / s - U^* / (sv(b)) - K < R_a = \Psi(b). \quad (27)$$

These conditions can be simplified by using the equilibrium condition that the residential and retail bid rents are the same at the retail-residential boundary:

$$\Phi(f) = [Y - R_a S - t(b - f)] / s - U^* / (sv(f)) - K = R_a + t(b - f) / S = \Psi(f). \quad (28)$$

Combining (26) and (28), the first inequality reduces to

$$(U^* / s)(1 / v(f) - 1 / v(0)) - tf / s > tf / S. \quad (29)$$

Substituting out for f and for $U^* / v(f)$ using (17), this inequality reduces to

$$\begin{aligned} & \{(Y - R_a(s + S) - Ks) [1 - v(Ns/2) / v(0)] \\ & > [tN(s + S) / 2] (1 + s / S - v(Ns/2) / v(0)) \}. \end{aligned} \quad (30)$$

Note that, as intuition suggests, the first inequality is easier to satisfy the lower is unit transport cost. Now consider the second inequality. Combining (27) and (28) yields the inequality

$$(U^* / s)(1 / v(b) - 1 / v(f)) - t(b - f) / s > t(b - f) / S. \quad (31)$$

Substituting out for f and for $U^* / v(f)$ using (17), this inequality reduces to

$$\begin{aligned} & \{(Y - R_a(s + S) - Ks) [v(Ns/2) / v(N(s + S) / 2) - 1] \\ & > [tN(s + S) / 2] v(Ns/2) / v(N(s + S) / 2) \}. \end{aligned} \quad (32)$$

The second inequality, too, is easier to satisfy the lower is unit transport cost. That both inequalities are easier to satisfy the lower is unit transport cost implies that a monocentric urban configuration is “more likely” to be an equilibrium, the lower is unit transport cost,

as intuition would suggest. Note that the exogenous parameters in the above inequalities are Y , t , N , S , s , K , α , and k .

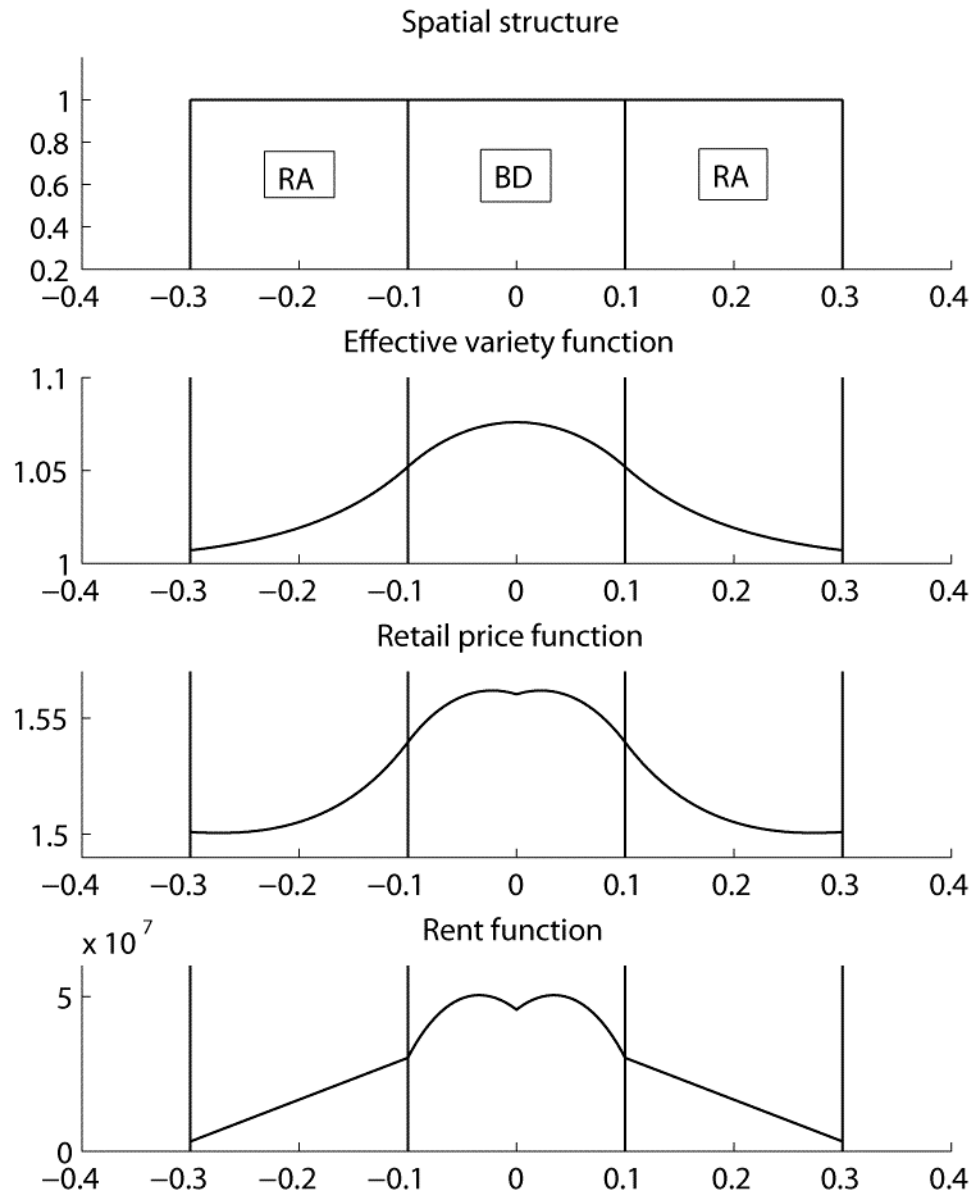
Throughout the paper, we consider a single numerical example. The following parameter values are employed: $N = 2 \times 10^4$, $S = 2 \times 10^{-5} \text{ mile}^2$, $s = 10^{-5} \text{ mile}^2$, $K = 10^9 \text{ \$}/\text{mile}^2$, $Y = 3 \times 10^4 \text{ \$}$, $t = 2700 \text{ \$}/\text{mile}$, $R_a = 3.2 \times 10^6 \text{ \$}/\text{mile}^2$, $\alpha = 10$, and $k = 0.6$. With these parameters, both a monocentric equilibrium and a completely mixed equilibrium exist.

Figure 1 plots the equilibrium spatial configuration, and effective variety, retail price, and rent, as functions of location, with these parameter values. The graphs are as expected.

The top panel shows that the retail district, marked as BD, is at the city center, and flanked by residential areas, marked as RA. The effective variety function declines monotonically from the city center with an inflection point at the retail-residential boundary, between a concave region inside the retail-residential boundary and convex regions outside it. The retail price function is concave on each side of the retail area (as was derived above) and has its maximum away from the city center. The rent

Figure 1 Monocentric urban configuration

Note: $N = 2 \times 10^4$, $S = 2 \times 10^{-5} \text{ mile}^2$, $s = 10^{-5} \text{ mile}^2$, $K = 10^9 \text{ \$}/\text{mile}^2$, $Y = 3 \times 10^4 \text{ \$}$, $t = 2700 \text{ \$}/\text{mile}$, $R_a = 3.2 \times 10^6 \text{ \$}/\text{mile}^2$, $\alpha = 10$, and $k = 0.6$.



function is the upper envelope of the equilibrium retail bid-rent curve and the equilibrium residential bid-rent curve. In the example, the retail bid-rent function is concave on each side of the city center and achieves the maximum away from the city center. The residential bid rent falls off linearly from the city center.

Figure 2, panel A plots the set of (α, t) for which a monocentric urban configuration exists. The other parameters are held fixed at their base case values. The area below the dashed line satisfies the first inequality, and that below the solid line the second inequality. The shaded area is the region in which both inequalities are satisfied, and in which therefore a monocentric urban equilibrium exists⁹. Consider first holding α fixed and raising the unit transport cost until a monocentric equilibrium fails to exist. There are two cases to consider. If the first inequality is violated, the residential bid rent at the city center exceeds the retail bid rent there, so that with the supposed rent function an individual can achieve a higher utility at the city center than in the residential area. If the second inequality is violated, the retail bid rent at the urban boundary exceeds the residential bid rent there, so that it would be profitable for a firm to relocate from the retail area to the urban boundary. Consider next holding t fixed and varying α -- the rate of decay of the benefit in terms of effective variety from proximity to other stores. At very low levels of α , with a monocentric urban configuration effective variety would decline so slowly with distance from the city center that the benefit to the individual from shopping at a more central location would be more than offset by the higher travel costs. At high levels of α , with a monocentric urban configuration effective variety declines so rapidly with distance from the city center that a store could profit by moving from the retail area to the urban boundary.

Figure 2.A Equilibrium conditions on $\alpha - t$ for a monocentric urban configuration

⁹ In Fujita-Ogawa, in contrast to this paper, one of the inequalities implies the other. The region in which both inequalities hold has the same qualitative shape as in Figure 2.A. We suspect that the difference derives primarily from a difference in the assumed form of the spatial decay function. In our paper, the effective variety at an isolated store is unity; in Fujita-Ogawa, in contrast, the productivity of an isolated firm is zero.

Note: $N = 2 \times 10^4$, $S = 2 \times 10^{-5} \text{ mile}^2$, $s = 10^{-5} \text{ mile}^2$, $K = 10^9 \text{ \$/mile}^2$, $Y = 3 \times 10^4 \text{ \$}$, $R_a = 3.2 \times 10^6 \text{ \$/mile}^2$ and $k = 0.6$. The area below the dashed line satisfies the first inequality (30) and that below the solid line satisfies the second inequality (32). The cross-hatched area satisfies both inequalities.

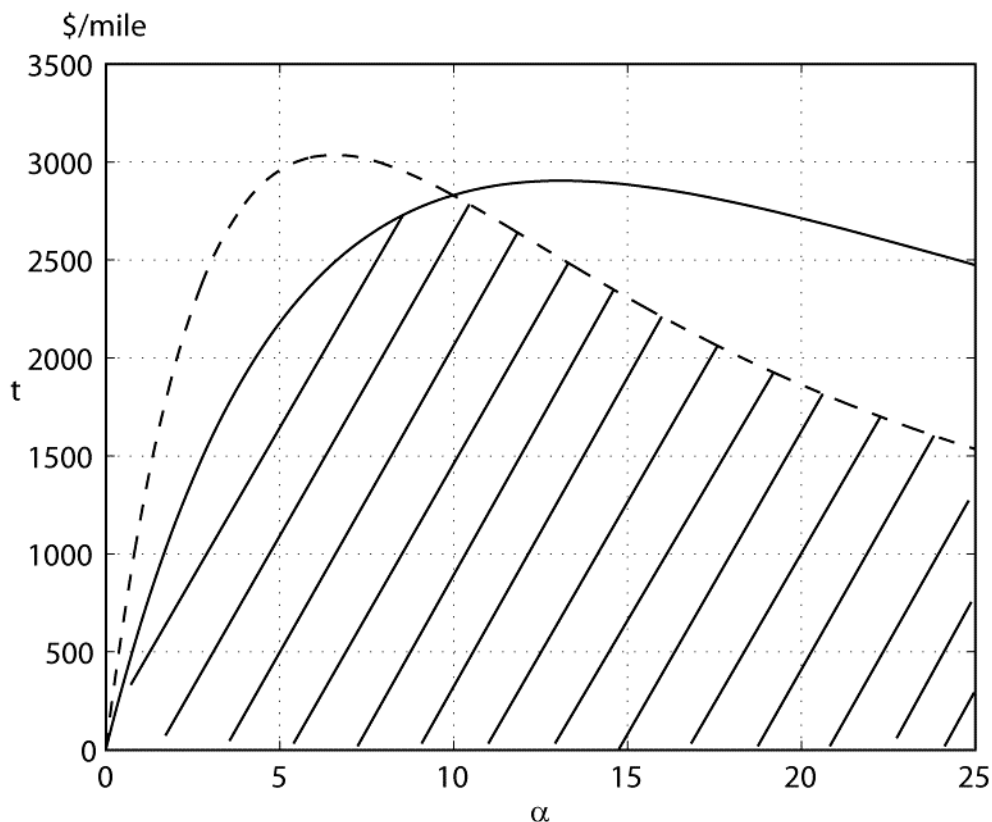
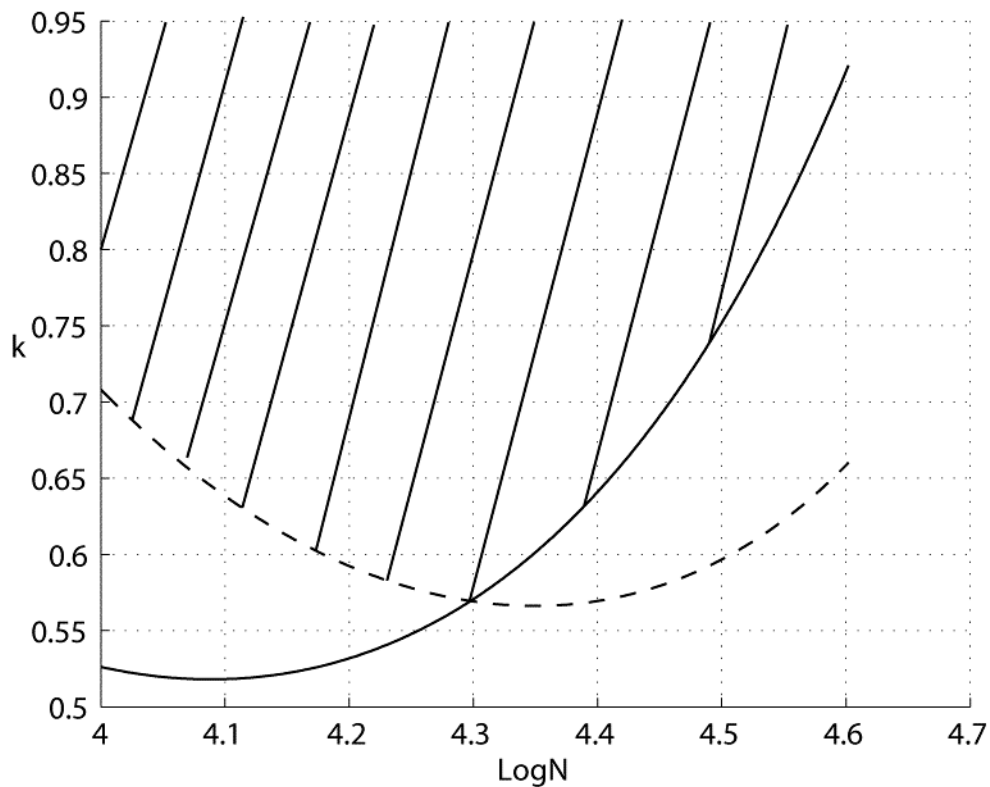


Figure 2, panel B plots the set of $(\text{Log}N, k)$ for which a monocentric urban configuration exists. The other parameter values are held at their base case levels. Holding population fixed, a monocentric equilibrium exists when the preference for variety is sufficiently strong; otherwise, an individual does not find it worthwhile to travel to a more central location to do her shopping. Now hold k fixed. For low levels of population, the rate at which effective variety increases with proximity to the center does not increase sufficiently to justify traveling to shop. At high levels of population, it is profitable for a single store to relocate to the urban boundary.

Figure 2.B Equilibrium conditions on $\text{Log}N - k$ for a monocentric urban configuration

Note: $S = 2 \times 10^{-5} \text{ mile}^2$, $s = 10^{-5} \text{ mile}^2$, $K = 10^9 \$ / \text{mile}^2$, $Y = 3 \times 10^4 \$$, $t = 2700 \$ / \text{mile}$, $R_a = 3.2 \times 10^6 \$ / \text{mile}^2$, and $\alpha = 10$. The area above the dashed line satisfies the first inequality (30) and that above the solid line satisfies the second inequality (32). The cross-hatched area satisfies both inequalities. $\text{Log}N$ is the base ten log, so that $\text{Log}N = 5.0$ corresponds to a population of 10^5 .



4. *Completely Mixed Urban Configuration*

In a completely mixed urban configuration, retail and residential land uses are interspersed so that each individual essentially has a store in his backyard. This section determines the set of parameter values for which this configuration is an equilibrium.

Note that the completely mixed urban configuration and the monocentric urban configuration are extreme cases; in one, stores are as centralized as possible, in the other as decentralized as possible. Intuition therefore suggests that the parameter set for which equilibria of both types exist may be empty, as it is in Fujita and Ogawa.

The city extends from $-b$ to b , with $b = N(S + s)/2$, with a proportion $s/(s + S) = a$ of the land at each location being allocated to retail and the rest to residential use. The effective variety function is the same as for the monocentric city configuration, except that $a(x) = a$ throughout the urban area, rather than equaling 1 in the retail area and 0 in the residential area as was the case in the monocentric urban configuration. Thus,

For $x \in [0, b]$:

$$v(x) = 1 + ka \int_{-b}^x \exp\{-\alpha(x - y)\} dy + ka \int_x^b \exp\{-\alpha(y - x)\} dy > 0$$

and

$$\begin{aligned} v'(x) &= ka(\exp\{-\alpha(x + f)\} - \exp\{-\alpha(f - x)\}) < 0 \\ v''(x) &= -ka\alpha(\exp\{-\alpha(x + f)\} + \exp\{-\alpha(f - x)\}) < 0. \end{aligned} \quad (33)$$

Also,

$$v(0) = 1 + (2ka/\alpha)(1 - \exp\{-\alpha b\}), \quad v(b) = 1 + (ka/\alpha)(1 - \exp\{-2\alpha b\}) \quad (34)$$

Thus, the effective variety function declines monotonically from the city center and is concave throughout the city.

- The retail bid-rent function

Eqs. (9) through (11) continue to apply.

- The residential bid-rent function

Everyone shops at her backyard store and therefore incurs no transport costs. Thus,

$$\Psi(x, U) = (Y - p(x)U / v(x)) / S . \quad (35)$$

- Equal rent conditions

Since retail and residential land co-exist at all urban locations, in equilibrium

$$\Phi(x) = \Psi(x) = R(x) \quad \text{for } x \in [0, b],$$

where $R(x)$ is the rent function, or

$$[p(x) - 1]U / (sv(x)) - K = (Y - p(x)U / v(x)) / S . \quad (36)$$

Rewrite (36) as

$$p(x) = [1 + (Ks + Ys / S)v(x) / U] / (1 + s / S) . \quad (36')$$

Also, in equilibrium at the city boundary both the retail and residential bid rents equal the agricultural bid rent:

$$Y - p(b)U / v(b) = R_a S . \quad (37)$$

Substituting (37) into (36') evaluated at $x = b$ yields

$$U^* = v(b)[Y - R_a(s + S) - Ks], \quad (38)$$

which gives the equilibrium level of utility in terms of exogenous parameters.

Substituting (38) into (36') would give an expression for $p(x)$ in terms of only $v(x)$ and exogenous parameters. Now, insert (36') into (35) to give the rent function:

$$R(x) = \{Y - [U^* / v(x) + (Ks + Ys / S)]\} / (s + S). \quad (39)$$

The condition for the existence of a completely mixed configuration is that the rent function have a slope with absolute value less than t/S , since otherwise it would be worthwhile for some individuals to commute inwards to shop. Now,

$$R'(x) = U^* v'(x) / [v^2(x)(s + S)]. \quad (40)$$

Since $d(v'(x)/v^2(x))/dx = (1/v^2)(vv'' - 2(v')^2) < 0$ from (33), $|R'(x)|$ takes on its highest value at $x = b$. Thus, the condition for the existence of a completely mixed urban configuration is that $-U^* v'(b)/v^2(b) < t(s + S)/S$ or, using (33), (34), and (38), that

$$\begin{aligned} [Y - R_a(s + S) - Ks][ks(1 - \exp\{-2\alpha N(s + S)/2\})/(s + S)] \\ > t(1 + s/S)[1 + (ks/\alpha)(1 - \exp\{-2\alpha N(s + S)/2\})/(s + S)]. \end{aligned} \quad (41)$$

Two features of the inequality bear note. Most importantly, there exists some critical level of t , $\hat{t}(Y, R_a, S, s, k, N)$, above which a completely mixed urban configuration exists and below which it does not. Also, the conditions for such a configuration are more stringent the higher is per capita “net endowment”, $Y - R_a(s + S) - Ks$.

We now return to the numerical example. Figure 3 plots the spatial structure, and effective variety, retail price, and rent, as functions of location. The graphs are as expected. The top panel shows that the spatial structure, marked ID, is uniform over the city. The effective variety, retail price, and rent functions decline monotonically with distance from the city center and are concave.

Figure 3 Completely mixed urban configuration

Note: $N = 2 \times 10^4$, $S = 2 \times 10^{-5} \text{ mile}^2$, $s = 10^{-5} \text{ mile}^2$, $K = 10^9 \text{ \$/mile}^2$, $Y = 3 \times 10^4 \text{ \$}$,
 $t = 2700 \text{ \$/mile}$, $R_a = 3.2 \times 10^6 \text{ \$/mile}^2$, $\alpha = 10$, and $k = 0.6$.

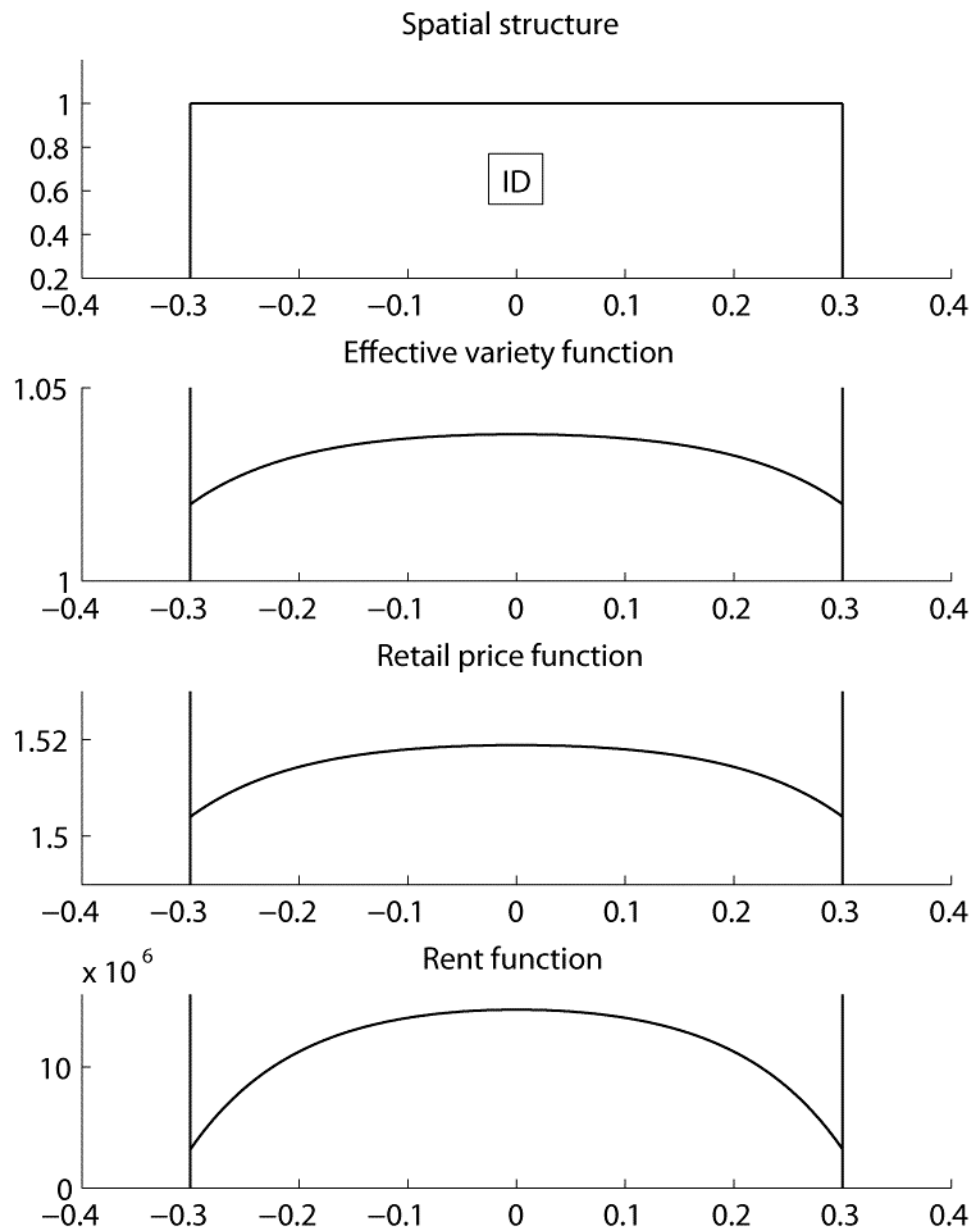
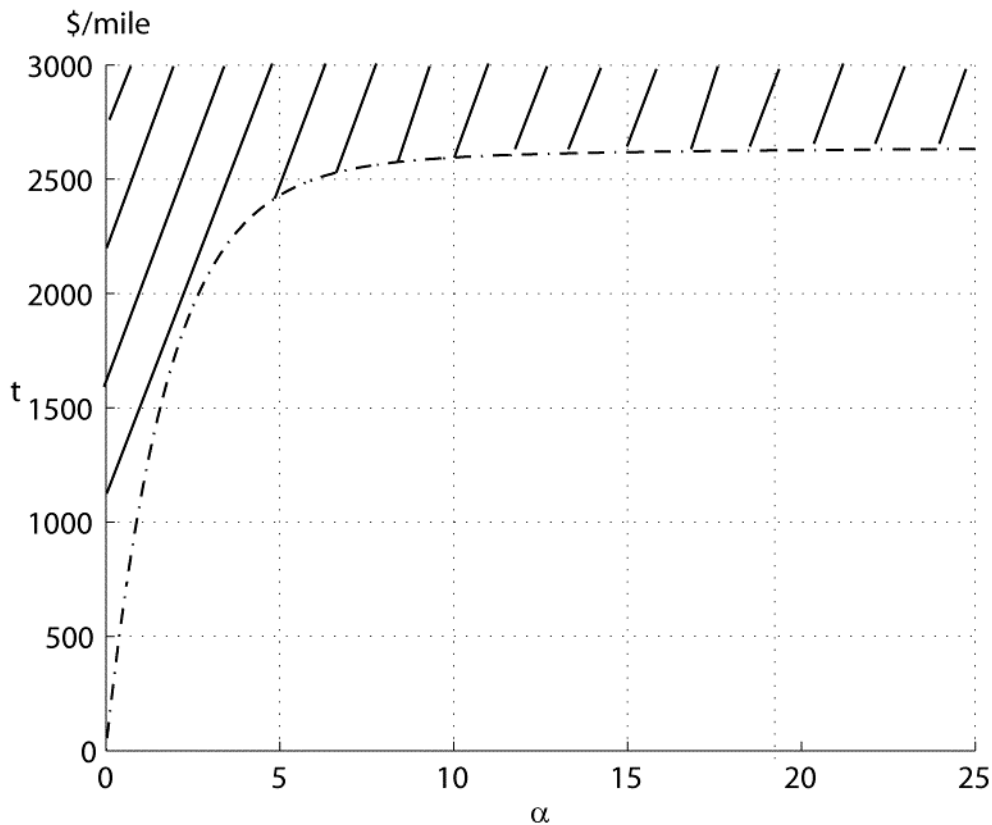


Figure 4, panel A plots the set of (α, t) for which a completely mixed urban equilibrium exists. The other parameters are held at their base case levels.

Figure 4.A Equilibrium condition on α - t for a completely mixed urban configuration

Note: $N = 2 \times 10^4$, $S = 2 \times 10^{-5} \text{ mile}^2$, $s = 10^{-5} \text{ mile}^2$, $K = 10^9 \$ / \text{mile}^2$, $Y = 3 \times 10^4 \$$, $R_a = 3.2 \times 10^6 \$ / \text{mile}^2$ and $k = 0.6$. The cross-hatched area above the dashdot line satisfies third inequality (42).



With this spatial configuration there is only one inequality that needs to be satisfied. The set of parameter pairs satisfying the inequality, and for which therefore a completely mixed urban equilibrium exists, is indicated by the cross-hatched area. Consider first holding α fixed and lowering the unit transport cost. Below a critical level of the unit transport cost, a completely mixed urban equilibrium does not exist since, at some locations at least, an individual can improve her utility by shopping at a more central location, which offers greater product variety. Consider next holding t fixed and varying α . For low levels of α , effective variety does not fall off sufficiently rapidly with

distance from the city center to make travel to shop at more central locations, with greater retail variety, worthwhile, while for higher levels of α effective variety does fall off sufficiently rapidly to make it worthwhile.

Figure 4.B Equilibrium condition on $\text{Log}N - k$ for a completely mixed urban configuration

Note: $S = 2 \times 10^{-5} \text{ mile}^2$, $s = 10^{-5} \text{ mile}^2$, $K = 10^9 \$ / \text{mile}^2$, $Y = 3 \times 10^4 \$$, $t = 2700 \$ / \text{mile}$, $R_a = 3.2 \times 10^6 \$ / \text{mile}^2$, and $\alpha = 10$. The cross-hatched area below the dashdot line satisfies third inequality (41). $\text{Log}N$ is the base 10 log, so that 5.0 corresponds to a population of 100,000.

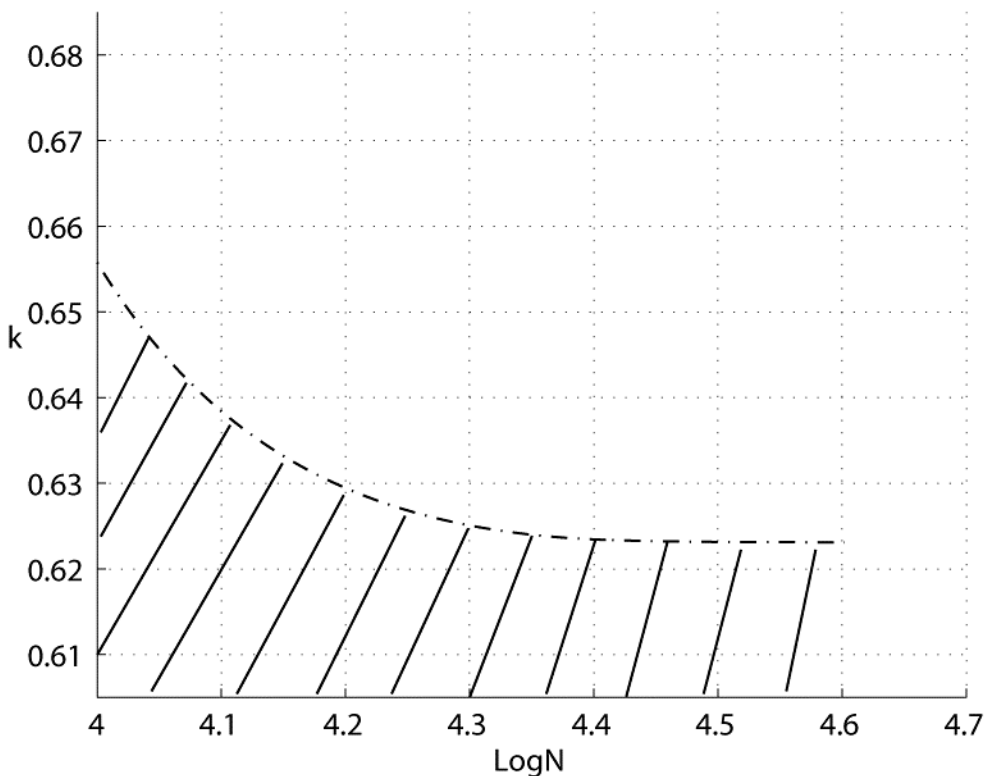


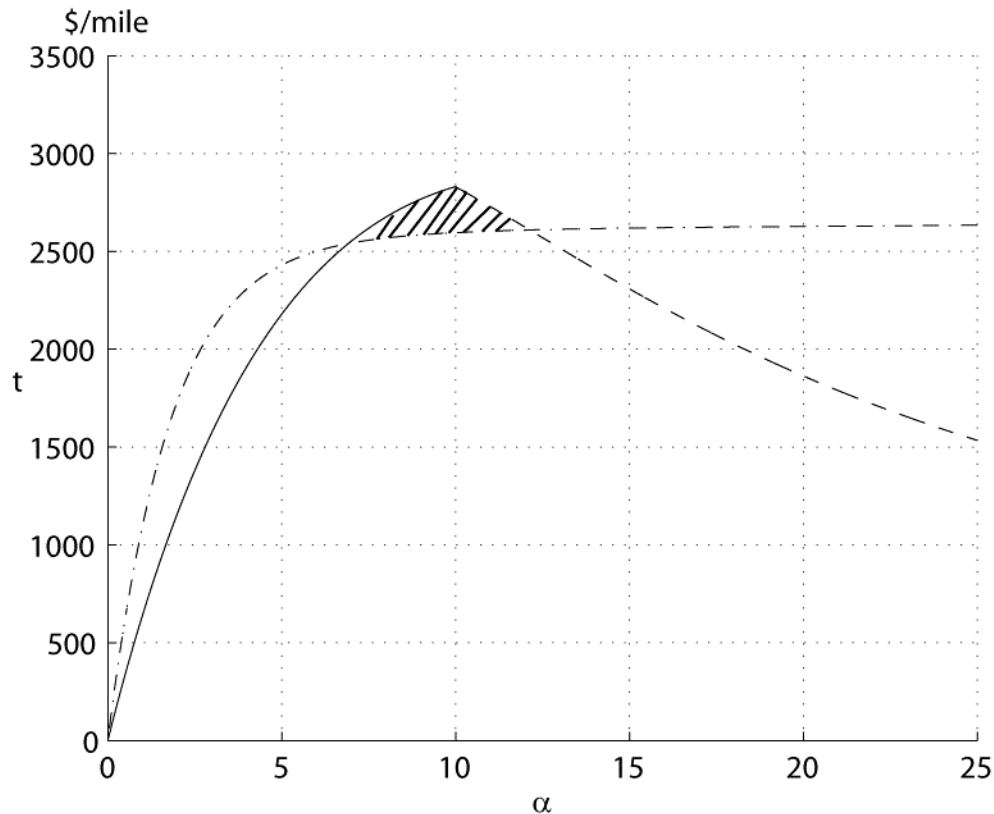
Figure 4, panel B shows the region in $(\text{Log}N, k)$ space for which a completely mixed urban configuration exists. When the taste for variety is sufficiently high, there are some locations at which effective variety/the taste for variety increases sufficiently rapidly with

proximity to the city center to justify travel to shop, and for which therefore a completely mixed equilibrium does not exist.

Figure 5 displays the region of (α, t) space for which both the monocentric and completely mixed equilibria co-exist. A monocentric equilibrium exists in the region below the dashed and solid line and a completely mixed urban equilibrium in the region above the dashdot line. In the small, cross-hatched region, which is centered on the parameter values of the numerical example, both types of equilibria co-exist. We have already provided some intuition for the shapes of the regions in this space for which each of the two equilibria exist. Start at the point in the cross-hatched area corresponding to the values of t and α in the numerical example, and move SE to the area where neither type of equilibrium exists. The monocentric equilibrium ceases to exist since the first inequality is violated -- with a monocentric urban configuration, an individual can achieve a higher level of utility at the center than in the residential area.

Figure 5 Equilibrium conditions on $\alpha - t$ for monocentric/ completely mixed urban configurations

Note: $N = 2 \times 10^4$, $S = 2 \times 10^{-5} \text{ mile}^2$, $s = 10^{-5} \text{ mile}^2$, $K = 10^9 \$ / \text{mile}^2$, $Y = 3 \times 10^4 \$$, $R_a = 3.2 \times 10^6 \$ / \text{mile}^2$, $k = 0.6$. The area below the dashed and solid line satisfies the monocentric equilibrium conditions, and that above dashdot line satisfies the completely mixed equilibrium condition. The cross-hatched area satisfies both sets of equilibrium conditions.



The completely mixed equilibrium ceases to exist since in the completely mixed configuration it becomes worthwhile for the individual at the urban boundary to travel a small distance towards the city center to shop; the cost of travel falls and the benefit increases. Now, instead, move SW to the area where neither type of equilibrium exists. The monocentric equilibrium ceases to exist since the second inequality ceases to be satisfied -- with a monocentric urban configuration, it becomes profitable for a single store to move from the retail area to the urban boundary; the decrease in the spatial decay rate of proximity benefits causes the difference in the effective variety at the retail location compared to the urban boundary to fall, and this effect more than offsets the increased attractiveness of shopping in the retail area due to the decline in transport costs. The completely mixed equilibrium ceases to exist because in a completely mixed urban configuration it becomes worthwhile for the individual at the urban boundary to travel a

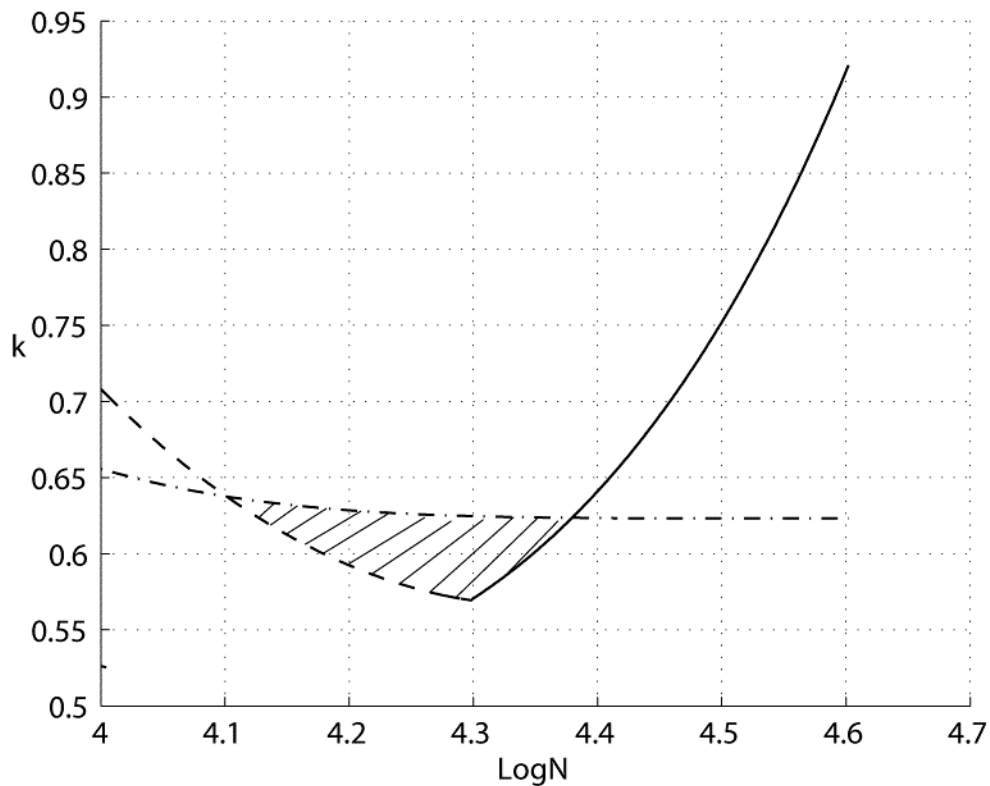
small distance towards the city center; the cost of travel falls by more than the benefit does.

Figure 6 displays the region of $(\text{Log}N, k)$ space in which the two types of equilibria exist.

A monocentric equilibrium exists in the region above the dashed and solid line and a completely mixed equilibrium in the region below the dashdot line. In the small, cross-hatched region, which is centered on the alternative set of parameter values, both types of equilibria co-exist. Start at the point in the cross-hatched area with the values of k and N in the numerical example. Holding population fixed: in the completely mixed urban configuration, with an increase in the taste for variety the benefit from shopping closer to the city center increases, resulting in the completely mixed equilibrium failing to exist; and with a decrease in the taste for variety, in the monocentric urban configuration, either an individual can increase his utility by living at the city center or it becomes profitable for a store to relocate to the urban boundary. The explanation for how the existence of the two types of equilibria depends on population, holding fixed the taste for variety, is left to the reader.

Figure 6 Equilibrium conditions on $\text{Log}N - k$ for monocentric/ completely mixed urban configurations

Note: $S = 2 \times 10^{-5} \text{ mile}^2$, $s = 10^{-5} \text{ mile}^2$, $K = 10^9 \$ / \text{mile}^2$, $Y = 3 \times 10^4 \$$, $t = 2700 \$ / \text{mile}$, $R_a = 3.2 \times 10^6 \$ / \text{mile}^2$, and $\alpha = 10$. The area above the dashed and solid line satisfies the monocentric equilibrium conditions. The area below the dashdot line satisfies completely the mixed equilibrium condition. The cross-hatched area satisfies both sets of equilibrium conditions.



We have considered two qualitative spatial configurations, the monocentric and the completely mixed. There are many other possibilities: incompletely mixed, duocentric, --. The procedure for determining the parameter set for each of these other configurations are variations of the procedures developed for the monocentric and completely mixed configurations.

5. *Concluding Comments*

In this paper, we have developed a competitive model of retail agglomeration. While the model is very simple, the simplifying assumptions are not substantive in the sense that the same approach can – and will be – applied in the dynamic, competitive general equilibrium simulation model of the LA Metropolitan Area, tentatively named METRO-

LA, which builds on Alex Anas' RELU-TRAN model for Chicago (Anas and Liu, 2007), and in the development of which Anas and Arnott are participating. In these concluding comments, we describe briefly how this paper's model could be extended for this application.

One module of the LA model solves for the temporary (static) equilibrium for each period. Each period inherits a stock of properties (vacant land and land with structures on it) indexed by type and zone, as well as location potentials indexed by industry type and zone, and the effective varieties indexed by zone. The temporary equilibrium is like a static Arrow-Debreu competitive equilibrium with (commodity) transport costs¹⁰, except that: first, each household consumes property at only one location and incurs transport costs, for commuting from its chosen residential location to its chosen work location, for shopping, recreational activities, and so on¹¹; second, retail goods are distinguished from wholesale goods; third, the economy is endowed with structures, as well as conventional factors of production; and fourth, agglomeration economies in production and retailing are treated via industry-specific location potential functions and an effective variety function¹². A temporary equilibrium is characterized by a market-clearing set of prices

¹⁰ Each household maximizes its utility and each firm maximizes its profits, taking prices as fixed. There is potentially an arbitrarily large but finite number of types of individuals and of retail and intermediate goods.

¹¹ In the Arrow-Debreu model, the assumption of convex preferences would result in households diversifying their housing consumption over locations, and people transport costs are not treated.

¹² An industry-specific location potential function can depend on proximity to firms in other industries as well to firms in the same industry. The effective variety function can be differentiated by type of retail good.

It is possible to introduce agglomeration economies in retailing on the production as well as on the consumption side.

and the corresponding allocation. The location potential functions and effective variety function can be solved for from the location pattern of the temporary equilibrium of one period and then treated as exogenous in determining the next period's temporary equilibrium.

The second module links the time periods. The present is linked to the past via the inheritance of real properties, location potential functions, and the effective variety function¹³. The present is linked to the future via real property markets. Economic agents have perfect foresight, and the market value of a property equals the expected present discounted value of future net rents¹⁴. Between periods developers make profit-maximizing conversion decisions, building on vacant land, demolishing structures, allowing some existing properties to deteriorate and rehabilitating others, etc., which moves the property stock forward from one period to the next¹⁵. Currently, the demography of the model is exogenous, but the aim is to make it responsive to economic

¹³ In the model of the paper, the effective variety function is determined as part of the static equilibrium, taking as given the qualitative spatial configuration. In a corresponding dynamic model, effective variety as a function of location could be calculated as part of that period's temporary equilibrium, taking as the starting point in the computation the equilibrium effective variety function from the previous period, which would generate some history dependence in the location pattern. METRO-LA assumes instead that the locational potential functions and effective variety functions calculated from one period's temporary equilibrium are taken as exogenous in the next period's temporary equilibrium. This (substantive) simplifying assumption is made to reduce computational costs.

¹⁴ A non-stationary, infinite horizon model can obviously not be solved exactly with a computer, which has finite computational ability. The model must be truncated somehow. In the LA model, the truncation is done by assuming that at some terminal time the economy's property values correspond to those of a stationary equilibrium.

¹⁵ Individual utilities and developer conversion costs are treated as idiosyncratic (in particular, the logit algebra is employed) so as to smooth adjustment, which facilitates computation.

conditions. The base industries grow according to the time path of export prices, which are taken as exogenous.

Despite the higher complexity of the LA model, the economics of retail agglomeration are essentially the same as those described in this paper. The essential component is the effective variety function, which relates the attractiveness to individuals of shopping at different locations to the spatial distribution of stores. Taking the spatial pattern of retail location, as reflected in the effective variety function, and of retail prices as given, individuals choose where to shop, trading off the greater variety of retail goods at larger shopping centers against the higher prices there and (for most individuals) the higher transport costs of traveling to a larger center. Stores choose where to locate trading off the higher price they can charge at a larger center against the higher rent they must pay there, and in equilibrium make zero profits. The spatial pattern of retail prices, as well as the retail and residential bid-rent functions and the effective variety function, adjust simultaneously to clear the location-specific markets for retail goods as well as the location-specific land markets.

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