Capitalizing R&D Expenditures

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Abstract

The next international version of the System of National Accounts will recommend that R&D (Research and Development) expenditures be capitalized instead of being immediately expensed as in the present *System of National Accounts 1993*. An R&D project creates a new technology, which in principle does not depreciate like a reproducible asset. A new technology is however subject to obsolescence, which acts in a manner that is somewhat similar to depreciation. The paper looks at the net benefits of an R&D project in the context of a very simple intertemporal general equilibrium model and suggests that R&D expenditures be amortized using the matching principle that has been developed in the accounting literature to match the fixed costs of a project to a stream of future benefits. Of particular interest is the evaluation of the net benefits of a publicly funded project where the results are made freely available to the public.

Key Words

Cost benefit analysis, R&D project, intertemporal general equilibrium theory, money metric utility scaling, matching principle, amortization, depreciation, growth accounting, equivalent variations, duality theory, welfare economics, Allen quantity index, obsolescence, foreign direct investment, monopolistic markups, endogenous growth theory.

JEL Classification Numbers

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1. Introduction

The present paper looks at some of the accounting problems associated with capitalizing R&D expenditures. A highly simplified general equilibrium approach is taken as opposed to the usual partial equilibrium treatments of this topic. One reason for taking a general equilibrium approach is that we can deal with the case of a publicly funded R&D project where the results of the project are made freely available to the public.

The basic problem associated with investments in R&D projects is that the expenditures required to develop a new technology are made now but the benefits of the new technology occur in subsequent periods. But what are the benefits of a new technology? From the viewpoint of a firm developing a new technology, the benefits would appear to be the discounted stream of (monopoly) profits that the new technology is expected to generate.[2] However, it does not seem to be "fair" to charge all of the costs of the project to the present period and allow all of the benefits to occur in future periods since the income of the firm will be unduly depressed in the present period and unduly exaggerated in future periods. Thus it seems appropriate to *amortize* the present period expenditures on R&D and spread these costs out to future periods so that costs can be better matched to benefits period by period. This is the point of view taken by Diewert (2005b) (2005c).

Note that the amortization of R&D expenditures over future periods is conceptually different from *wear and tear depreciation* of a reproducible asset: the R&D expenditures are a *sunk cost* whereas wear and tear depreciation is a result of *use* of the reproducible asset during each period that the asset can deliver services. Thus wear and tear depreciation within a period is a definite phenomenon that depends on use of the asset and can in principle be measured, whereas R&D amortization is largely arbitrary and depends on whatever accounting principle seems "reasonable" under the circumstances that will match costs to benefits in each period.

There are some problems with the above view on how to amortize R&D expenditures:

- The above approach to measuring the benefits of an R&D project sets the net benefits of the project equal to discounted monopoly profits that can be attributed to the project less the current period cumulated R&D expenditures. This view of the net benefits of the project is firm oriented and may not capture the social benefits of the project.
- The above approach totally fails if the R&D project is a government financed project where the new technology that results from the project is made freely available to the public.

[2] This is the point of view taken by Pitzer (2004), Diewert (2005b) (2005c) and Copeland, Medeiros and Robbins (2007).

Thus there is a need to develop a *welfare oriented perspective* to evaluate the net benefits of an R&D project and this is what is done in the present paper. However, it must be noted that our model is rather crude and only represents a start on modeling the benefits of an R&D project.

In section 2, we lay out a simple intertemporal model and consider how the traditional growth accounting approach to R&D works in this highly simplified framework. In section 3, we briefly explain the *knowledge production function* that creates new technologies. Sections 4 and 5 assume that a new *process innovation* has been created and we look at the problems associated with measuring the welfare gains associated with the innovation. Section 4 looks at a publicly funded R&D project while section 5 looks at a privately funded R&D project and this is where the interesting accounting problems emerge. Section 6 extends our analysis to a *product innovation* (rather than a process innovation). Section 7 notes how our initial general equilibrium model can be generalized to account for *obsolescence* and section 8 concludes.

2. Disembodied Technical Change and Growth Accounting

In order to highlight the differences between the traditional Solow (1957), Jorgenson and Griliches (1967) (1972) growth accounting methodology and the R&D accounting methodology that will be developed in this paper, it is useful to review the traditional methodology in the case where the economy is producing only one output and using only one input.[3]

Thus let $y^t > 0$ and $x^t > 0$ denote the output produced and input used in period t for t = 0,1,... and let $p^t > 0$ and $w^t > 0$ denote the corresponding output and input prices. We assume that production is subject to constant returns to scale and there is competitive pricing in each period so that the value of outputs equals the value of inputs; i.e., we have:

(1) $p^t y^t = w^t x^t$; $\qquad t = 0,1,2, \ldots$.

Assume that the output and input data for periods 0 and 1 can be observed. Then the period 0 and 1 *productivity levels*, a^0 and a^1, can be defined as the output input ratio in each period; i.e., we have[4]

(2) $a^0 \equiv y^0/x^0$; $a^1 \equiv y^1/x^1$.

The *Total Factor Productivity Growth* of the economy going from period 0 to 1, $\tau^{0,1}$, is defined as (one plus) the rate of growth of productivity levels; i.e., we have:

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[3] Index number complications are not present in this model. This one output and one input methodology is developed in Diewert (1992a), Balk (2003) and Diewert and Nakamura (2003). For extensions to many outputs and inputs, see Jorgenson and Griliches (1967) (1972), Caves, Christensen and Diewert (1982), Diewert and Morrison (1986), Kohli (1990) and Balk (1998).

[4] Thus period 0 production function is $y = a^0x$ and the period 1 production function is $y = a^1x$ where y is the output that can be produced by the amount of input x.

(3) $\tau^{0,1} \equiv a^1/a^0 = [y^1/y^0]/[x^1/x^0] = [w^1/w^0]/[p^1/p^0]$

where the last two equalities in (3) follow using (1) and (2). If a^1 is greater than a^0 (the usual case), then $\tau^{0,1}$ is greater than one and we say that there has been a (total factor) *productivity improvement* going from period 0 to 1.

The above algebra captures the main aspects of TFP measurement and growth accounting. Note that the productivity improvement is not "earned"; it just happens! Thus we say that there is *disembodied technical progress* if $\tau^{0,1}$ is greater than one.

In the following sections of this paper, we are going to relax the assumption of disembodied technical change and we will assume that productivity improvements require some *effort* in order to create new technologies. However, once we recognize that technological improvements generally require an investment of effort in a prior period before the benefits can be realized in subsequent periods, then a simple two period framework is no longer adequate in order to develop the welfare effects of an effort driven innovation. In order to prepare for this many period modeling effort, it will be useful to conclude this section with a multiple period disembodied technical change model.

Thus let $y \equiv [y^0,y^1,y^2,...]$ be the sequence of expected outputs of the economy for periods 0, 1, 2, ... We assume that all output is consumed in each period (so there are no durable outputs for simplicity). We need a Social Welfare Function or intertemporal utility function, W(y), that will enable us to evaluate the relative worth of different sequences of consumption. We choose the following very simple additively separable SWF to make these welfare evaluations:

(4) $W(y) \equiv y^0 + (1+r)^{-1}y^1 + (1+r)^{-2}y^2 + ...$

where $r > 0$ can be interpreted as a *reference real interest rate*.[5]

Let $x^t \geq 0$ be the amount of primary input that is expected to be available for use by the production sector of the economy in period t for $t = 0,1,2,...,\infty$. If the economy has only the period 0 technology available for all future periods, then y^t equal to a^0x^t is the output that can be expected to be produced in period t and the economy's *expected welfare using the period 0 technology* will be W^0 defined as follows:

$$
\begin{aligned}
(5)\ W^0 &\equiv a^0x^0 + a^0x^1/(1+r) + a^0x^2/(1+r)^2 + a^0x^3/(1+r)^3 + ... \\
&= a^0x^0 + a^0X^1
\end{aligned}
$$

where the *future period discounted input aggregate* X^1 is defined as follows:

[5] Alternatively, $1/(1+r)$ can be interpreted as a social discount factor. We assume that the x^t are bounded from above as are the one period technical coefficients a^t that define the period t production functions and so when we evaluate W(y) using feasible input vectors and feasible technologies, W(y) is finite using the assumption that $r > 0$.

(6) $X^1 \equiv x^1/(1+r) + x^2/(1+r)^2 + x^3/(1+r)^3 + \ldots$.

Now suppose that starting in period 1, the economy has a new constant returns to scale technology that is defined by the production function $y = a^1x$ where the new output input coefficient a^1 is strictly greater than a^0. Then if we use this new technology in period 1 and subsequent periods, the new level of expected social welfare that can be attained using the new technology is W^1 defined as follows:

$$
\begin{aligned}
(7)\ W^1 &\equiv a^0x^0 + a^1x^1/(1+r) + a^1x^2/(1+r)^2 + a^1x^3/(1+r)^3 + \ldots \\
&= a^0x^0 + a^1X^1 \qquad \text{using definition (6).}
\end{aligned}
$$

Using the above definitions, we can calculate a *benefit measure* B which reflects the expected increase in discounted real consumption due to the disembodied productivity improvement going from the period 0 technology (represented by a^0) to the period 1 technology (represented by a^1 which is assumed to be greater than a^0):

$$
\begin{aligned}
(8)\ B &\equiv W^1 - W^0 \\
&= a^0x^0 + a^1X^1 - [a^0x^0 + a^0X^1] \qquad \text{using (5) and (7)} \\
&= [a^1 - a^0]X^1 .
\end{aligned}
$$

Recall that $\tau^{0,1}$ equal to a^1/a^0 is a traditional *ratio type measure* of a productivity improvement whereas $a^1 - a^0$ is a *difference type measure* of a productivity improvement.[6] We shall find that when we study the welfare effects of an R&D project, the difference approach is much more convenient.

The above disembodied model of technical progress assumed that the new technology dropped from heaven without requiring any sacrifices on the part of households and firms to create the new technology. In the following section, we will relax this assumption.

3. The New Technologies Production Function

We now recognize that in many cases, the creation of a new process technology cannot be done without the expenditure of some effort. Thus from the perspective of creating a new technology in period 0, we could think of a period 0 *process innovation production function* f^0 such that

(9) $a^1 = f^0(z)$

where $z \geq 0$ is the expenditure in period 0 that is required to produce a new technology characterized by the output input coefficient a^1. We assume that $f^0(0) = a^0$ so if we expend no effort, we end up getting the same old period 0 technology, a^0, that we already

[6] The difference approach to productivity measurement is pursued by Balk (2007) and Diewert and Mizobuchi (2007) while the difference approach to the measurement of welfare change is pursued by Chambers (2001), Balk, Färe and Grosskopf (2004), Diewert (2005), Diewert and Fox (2005) and Fox (2006).

have access to. We also assume that f^0 is *nondecreasing* (so that increased effort inputs cannot create worse technologies than we already have in period 0) and *concave* (so that there are constant or diminishing returns to scale in the creation of new technologies).

From the viewpoint of a central planner in period 0 who needs to decide how much of society's period 0 input x^0 should be allocated to the creation of new technologies, the following social welfare maximization problem seems to be relevant:

(10) $\max_z \{a^0(x^0 - z) + f^0(z)X^1 : 0 \le z \le x^0\}$.

Thus increased investments in R&D (i.e., bigger levels of z) lead to lower period 0 consumption, $a^0(x^0 - z)$, but the efficiency of the economy in subsequent periods is increased: $a^1 = f^0(z)$ is generally bigger than a^0 so that discounted future consumption, $f^0(z)X^1$, is generally bigger than a^0X^1, which is discounted future consumption using the old technology. Thus the diminished period 0 consumption is offset by increased consumption in subsequent periods and the z^0 which solves (10) balances these two effects. If z^0 is close to 0, there is no problem with this setup but if z^0 is equal to x^0 or is close to x^0 (so that it is enormously productive to invest in the creation of new technologies), then the maximization problem (10) is not reasonable from a practical point of view since the solution leads to starvation of the populace in period 0! Thus in this case, the upper bound to z in (10) will have to be reduced.

In the sections which follow, we will not assume that investment in new technologies is necessarily "optimal" in the sense that z^0 solves (10). We will simply assume that the government or private sector investors allocate the amount z^0 of input in period 0 to create a new technology with output input coefficient $a^1 \equiv f^0(z^0) > a^0$ where $0 < z^0 < x^0$.

It would be of some interest to investigate the "exogenous" determinants of the period 0 technology creation function $f^0(z)$; i.e., this function will in general depend on the level of education in the economy under consideration, on the stocks of domestic "knowledge" that exist in period 0, on the access to the stocks of foreign "knowledge" and many other factors. However, this is not the focus of the present paper which has much narrower accounting goals in mind.[7]

In the following section, we will look at the effects of a government R&D project and in section 5, we will turn our attention to private sector R&D projects.

4. R&D and Process Innovation: The Case of a Government Funded Project

It turns out that the analysis of a government funded R&D project cannot be analyzed as a single case: we need to consider various alternative ways of *financing* the government R&D project. We will consider four different cases in this section. Privately funded projects will be considered in the following section.

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[7] There is a huge literature on endogenous growth models that looks at these questions; see Aghion and Howitt (1998) and Aghion and Durlauf (2005) for a sample of this literature.

Case 1: *The Centrally Planned Economy*

We first consider the case of a centrally planned economy since this is the easiest (and least relevant) case to consider. We suppose that the central planner allocates z^0 units of input to the creation of new technologies in period 0 where $0 < z^0 < x^0$. This investment creates a new production function with output input coefficient $a^1 > a^0$. The resulting value of expected discounted consumption is W^{1*} defined as follows:

$$(11)\ W^{1*} \equiv a^0[x^0 - z^0] + a^1x^1/(1+r) + a^1x^2/(1+r)^2 + a^1x^3/(1+r)^3 + ...$$
$$= a^0[x^0 - z^0] + a^1X^1 \qquad \text{using definition (6)}$$
$$< W^1$$

where W^1 was the value of discounted consumption defined by (7) which assumed that the creation of the new technology was costless. Thus the present model is more realistic in assuming that technology creation is *not* generally costless.

Of course, we can now define a new *net benefit measure* B^* associated with the costs and benefits of the creation of the new technology:

$$(12)\ B^* \equiv W^{1*} - W^0$$
$$= a^0[x^0 - z^0] + a^1X^1 - [a^0x^0 + a^0X^1] \qquad \text{using (5) and (11)}$$
$$= [a^1 - a^0]X^1 - a^0z^0$$
$$< [a^1 - a^0]X^1$$
$$= B$$

where the costless measure of benefit B was defined by (8). Thus not surprisingly, our new measure of the net benefits of a government investment in the creation of a new technology, B^*, is less than our section 2 estimates of a disembodied (or costless) creation of a new technology.[8]

Even if a^1 is greater than a^0, it may be the case that B^* is negative if the fixed costs of creating the new technology, a^0z^0, are sufficiently large.[9] In what follows, we will neglect the case of such an impoverishing investment and we will assume that a^1 is greater than a^0 and

$$(13)\ B^* \equiv W^{1*} - W^0 = [a^1 - a^0]X^1 - a^0z^0 > 0$$

so that the R&D project generates an increase in discounted real consumption.[10]

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[8] If our present (costly) model of technical progress is correct, then assuming incorrectly that the disembodied technical change model presented in section 2 is correct will not surprisingly generate errors. Thus using the framework in section 2 would lead to an estimated period 0 output input coefficient equal to $a^{0*} \equiv a^0(x^0-z^0)/x^0 = a^0 - (z^0/x^0)a^0 < a^0$. Thus the disembodied model would incorrectly assume that the productivity growth rate going from period 0 to 1 was a^1/a^{0*} which is greater than the actual rate a^1/a^0.

[9] See Romer (1994) for a good discussion of this point in a slightly different context.

[10] We will assume that (13) holds also for the case of a private R&D project.

We now turn our attention to the case of a market economy where the government has to raise the revenue required to fund the R&D project by taxing households.

Case 2: *An Income Tax for Period 0.*

We now have to specify what is happening to prices in the economy in each period. We will assume that there is a central bank in the background that acts to stabilize the producer price of output in each period so that p^t is expected to equal 1 in each period. In period 0, primary inputs x^t are paid the value of what they produce (before any income taxes) so that w^0 is equal to a^0 in period 0 and w^t is equal to a^1 for periods $t \geq 1$. As usual, y^0 equals a^0x^0 in period 0 and y^t equals a^1x^t in subsequent periods. It is convenient to list these assumptions since we will draw on them in the cases to be developed later:

(14) Period 0: $p^0 = 1$; $w^0 = a^0$; $y^0 = a^0x^0$; Period t: $p^t = 1$; $w^t = a^1$; $y^t = a^1x^t$ for $t \geq 1$.

The primary input prices w^t defined above are producer prices and are before any income taxes. We now assume that the government imposes an income tax in period 0 in order to finance the R&D investment. The size of the income tax, t^0, that is required to balance the government's budget in period 0 is:

(15) $t^0 \equiv z^0/x^0$.

Thus primary input suppliers in period 0 face the after tax wage rate of $w^0(1-t^0)$ so that their total period 0 after tax income is:

(16) $w^0x^0(1-t^0) = a^0x^0(1-t^0) = a^0x^0[1-(z^0/x^0)] = a^0[x^0-z^0] = p^0c^0$

where $c^0 \equiv a^0[x^0-z^0]$ is period 0 consumption (recall that $p^0 = 1$). The value of expected discounted consumption is still W^{1*} defined by (11) and the net benefit associated with the R&D project is still B^* defined by (12). Thus there is no problem in attaining the planned economy level of welfare using an income tax in period 0 to finance the project in a decentralized market economy.

Case 3: *A Consumption Tax for Period 0.*

Again, we assume that expected producer output and input prices are defined by (14). However, in this case, instead of assuming that the government imposes an income tax in period 0 in order to finance the R&D investment, we now assume that it imposes a consumption tax. The size of the consumption tax, t^{0*}, that is required to balance the government's budget in period 0 is defined by the following equation:

(17) $1+t^{0*} \equiv [1-(z^0/x^0)]^{-1} = 1/(1-t^0)$

where t^0 was the income tax defined by (15). In this case, households in period 0 have primary factor income equal to w^0x^0 to spend on consumption which is priced at $p^0(1+t^{0*})$. Thus the period 0 household expenditure equals income equation is:

$$(18)\quad p^0(1+t^{0*})c^0 = w^0x^0 \text{ or}$$
$$(1+t^{0*})a^0[x^0-z^0] = w^0x^0 \text{ or}$$
$$a^0[x^0-z^0] = (1-t^0)\, w^0x^0 \qquad \text{using (17)}$$
.

But the last equation in (18) is equivalent to (16). Again, the value of expected discounted consumption is W^{1*} defined by (11) and the net benefit associated with the R&D project is still B^* defined by (12). Thus there is no problem in attaining the planned economy level of welfare using a consumption tax in period 0 to finance the project in a decentralized market economy.[11]

Case 4: *Financing the Project by Foreign Borrowing*

In the previous 3 cases, households who are alive in period 0 bear all of the burden of financing the government R&D project whereas subsequent generations get all of the benefits of the project without suffering any of the costs. Since we did not allow for durables in our model (or any other form of saving) and since we assumed that primary inputs were supplied inelastically in each period, we were unable to avoid this asymmetric bearing of the burden problem. We now relax our previous assumptions and assume that the government can borrow from abroad if it wishes to do so (at the same real interest rate r that appeared in our intertemporal utility function W(y) defined by (4) above). Thus we now assume that the government faces the following *intertemporal balance of payments constraint*:

$$(19)\ b^0 + b^1(1+r)^{-1} + b^2(1+r)^{-2} + ... = 0$$

where b^t is the period t export (if b^t is positive) or import (if b^t is negative) of output for the economy in period t. Thus suppose the government imports the output commodity in period 0 so that b^0 is negative (and we can regard $-b^0$ as a capital import) and then repays the loan in the following period. Since capital made available in period 0 is more valuable in period 0 than in period 1, if the government repays the loan in period 1, then the government must export b^1 equal to $(1+r)(-b^0)$ in order to repay the loan.

We will assume that the government borrows an amount in period 0 that is just sufficient to maintain the consumption level that would have resulted if the government R&D project were not implemented. Thus b^0 is defined as follows:

$$(20)\ b^0 = -a^0z^0.$$

When we substitute (20) into the intertemporal balance of payments constraint (19), we could pick any pattern of future period b^t which satisfy the constraint; i.e., the repayment plan has many degrees of freedom. But fairness and ability to pay considerations might

[11] Our model is too simple to record any difference in social welfare due to the imposition of a consumption tax versus an income tax. Of course, in more complex real life economies, there would be differences between the two methods for financing the government R&D project.

suggest that the burdens imposed by the loan repayment be proportional to the future period consumption levels. Consumption in period t before the imposition of any taxes will be equal to production y^t which is equal to a^1x^t for $t \geq 1$. Thus we let period t exports b^t be proportional to period t production; i.e., we want to choose a fraction f such that[12]

(21) $b^t = fy^t = fa^1x^t$; $t \geq 1$.

Now substitute (20) and (21) into (19) and we obtain the following equation:

(22) $a^0z^0 = \sum_{t=1}^{\infty} fa^1x^t/(1+r)^t$
$= fa^1X^1$ using definition (6).

Solving (22) for f gives us:

(23) $f = a^0z^0/a^1X^1$;
(24) $b^t = a^0z^0[x^t/X^1]$ for $t \geq 1$;
(25) $c^t = y^t - b^t$ for $t \geq 1$;
$= a^1x^t - a^0z^0[x^t/X^1]$ using (24)
$= [a^1X^1 - a^0z^0][x^t/X^1]$.

Now define *social welfare* under this method of financing the project in the usual way as discounted consumption:

(26) $W^{1**} \equiv c^0 + c^1(1+r)^{-1} + c^2(1+r)^{-2} + ...$
$= a^0x^0 + \sum_{t=1}^{\infty} [a^1X^1 - a^0z^0][x^t/X^1](1+r)^{-t}$ using (25)
$= a^0x^0 + [a^1X^1 - a^0z^0]$ using (6)
$= a^0[x^0 - z^0] + a^1X^1$
$= W^{1*}$ using (11).

Thus under this borrowing from abroad financing scheme for the government funded R&D project, we attain the same level of net benefit, B^* defined by (13), as was obtained in cases 1-3. However, the present scheme will be intertemporally much more equitable than the previous methods of financing the project, assuming that the inequality (13) is satisfied.[13]

In order to implement the above equilibrium in a decentralized fashion, the government need only impose an income tax at the rate f defined by (23) for periods $t \geq 1$. Alternatively, the government could implement the above equilibrium by imposing a consumption tax for periods $t \geq 1$ at the rate t^* defined as follows:

[12] We will obtain the same c^t solution as is given by (25) if we make period t loan repayments proportional to the *incremental consumption* that is made possible by the innovation; i.e., we obtain the same pattern of consumption and social welfare if we set $b^t = f[a^1 - a^0]x^t$ for $t \geq 1$.
[13] In order to ensure that future consumption with the loan repayment is greater than preproject consumption, we require that $a^1/a^0 > 1 + z^0/X^1$ and this inequality is equivalent to (13).

(27) $t^* \equiv (1-f)^{-1} - 1$

where f is defined by (23).

We now turn our attention to privately funded projects.

5. R&D and Process Innovation: The Case of a Privately Funded Project

Case 5: *A Domestically Funded Project with No Outsourcing of Production*

In this case, we assume that domestic investors forego consumption in period 0 in order to finance the R&D project and in return, these investors will get a stream of monopoly profits which they can spend on consumption in subsequent periods. The characteristics of the R&D project are the same as was the case in the previous section: the project uses up z^0 units of primary input in period 0 and it produces a new technology which is characterized by the output input coefficient, a^1, where a^1 is strictly larger than the preproject technical coefficient, a^0. We will assume that this project is successful so that the inequality (13) is satisfied. In period 0, we assume that prices and quantities satisfy (14). In subsequent periods, we assume that the new technology immediately displaces the old technology and the monopolist produces output using the new technology. We assume that outputs and inputs, y^t and x^t, and the corresponding producer prices, p^t and w^t, satisfy assumptions (14). Producer prices are equal to final demand prices in period 0 but in subsequent periods, final demand output prices differ from the producer output prices described in (14) because the owners of the new technology can charge a monopolistic markup, m say, on the sales of the output.[14] Thus for periods $t \geq 1$, the owners charge households the final demand price, $p^t(1+m) = (1+m)$, instead of the producer price, $p^t = 1$.[15] We now have to determine the size of the markup.

After the introduction of the new technology, real wages will increase from w^0 equal to a^0 to w^t equal to a^1 for $t \geq 1$. In order to produce one unit of output in period $t \geq 1$ using the old technology, an amount of input equal to $1/a^0$ would have to be used since the equation:

(28) $1 = a^0x^*$

has the solution x^* equals $1/a^0$. Thus potential competitors (using the old technology) to the monopolist would sell units of output at the period t price p^{t*} defined as follows in order to cover costs:

(29) $p^{t*} = w^tx^* = a^1(1/a^0) = a^1/a^0 = (1+m)p^t = (1+m)$; $\qquad t \geq 1.$

Thus the owners of the new technology can charge (in the limit) a monopolistic markup m defined as

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[14] Thus we are assuming that the new technology is proprietary and is not made available to other producers.

[15] Recall that we are assuming that the central bank stabilizes the producer price of output in each period.

(30) $m \equiv (a^1/a^0) - 1$.

Thus if the owners of the project fully exploit their monopoly position, households will have to pay the final demand prices p^{t*} defined by (29) for $t \geq 1$.

With the above preliminaries out of the way, we can now look at the streams of factor incomes and expenditures at final demand prices and see if income equals expenditure in each period. The period by period sequence of consumption quantities, c^t, is the following sequence:

(31) $c^0 = a^0(x^0-z^0)$; $c^t = a^1x^t$, $t \geq 1$.

The corresponding sequence of consumption expenditures at final demand prices is:

(32) $p^0c^0 = c^0$; $p^t(1+m)c^t = (1+m)c^t = (1+m)a^1x^t$, $t \geq 1$.

Turning our attention to the quantities and values of inputs, the sequence of input quantities is the usual x^t for $t \geq 0$. The corresponding primary input income sequence is given by:

(33) $w^0x^0 = a^0x^0$; $w^tx^t = a^1x^t$, $t \geq 1$.

However, primary input income does not exhaust income because for periods later than 0, there will be some monopoly profits that will be distributed back to the investors in the R&D project. This stream of monopoly profits is given by:

(34) 0 ; $mp^tc^t = mc^t = ma^1x^t$, $t \geq 1$.

Comparing the stream of household expenditures given by (32) with the sum of the two income streams defined by (33) and (34), it can be seen that total income from all sources will be equal to household consumption expenditures at final demand prices for all periods except for period 0. Thus for the periods beyond period 0, this monopoly model is perfectly consistent. However, in period 0, it can be seen that the value of consumption is p^0c^0 equal to $a^0(x^0-z^0)$, which is *less* than the corresponding factor income, w^0x^0 equal to a^0x^0. The problem is that we have not accounted for the period 0 *investment* in developing the new technology. This investment is equal to z^0 units of input. We could revalue this input measure of investment into units of output that are foregone and thus we define the period 0 investment I^0 as follows:

(35) $I^0 \equiv a^0z^0$.

It can be seen that if we add period 0 investment I^0 to period 0 consumption c^0, then the period 0 value of outputs will equal the value of period 0 primary inputs.

Now we are ready to define *social welfare* under this private sector financing of the R&D project:

$$\begin{aligned}(36)\ W^{1***} &\equiv c^0 + c^1(1+r)^{-1} + c^2(1+r)^{-2} + \ldots \\ &= a^0(x^0-z^0) + \textstyle\sum_{t=1}^{\infty} a^1x^t/(1+r)^{-t} && \text{using (31)} \\ &= a^0(x^0-z^0) + a^1X^1 && \text{using (6)} \\ &= W^{1*} && \text{using (11).}\end{aligned}$$

Thus somewhat surprisingly, the level of social welfare that is attained by this private sector model of R&D investment, W^{1***}, is exactly equal to the level of social welfare W^{1*} that was attained in the previous section with the various government financing schemes. Thus for this private sector financing model, *we attain the same level of net benefit*, B^* defined by (13), as was obtained in Cases 1-4. However, at this point, we again must point to the inadequacies of our social welfare function, which does not distinguish between households who invest in the project and those who do not invest. A more adequate social welfare function would take into account the increased income inequality between households that results from the monopoly profit stream and the fact that some households invested in the R&D project and some did not.

Our measurement problems are not quite over yet. We need to discuss the *accounting problems* that are associated with this privately funded R&D project. For accounting purposes, it is useful to break up the activities of the R&D project into two *divisions*:

- A *production division* which oversees the production of the output using the new technology and
- An *R&D management division* which finances the expenditures for the project and collects the monopoly revenues from consumers.

The accounting for the production division is straightforward and need not be analyzed in detail here. The accounting problems associated with management division are much more interesting and will be discussed in more detail. The sequence of costs (in period 0) and revenues (in subsequent periods) associated with the R&D management division is the following one:

$$(37)\ \ -w^0z^0 = -a^0z^0\ ;\ p^tmc^t = ma^1x^t\ ,\ t \geq 1.$$

The problem with the above sequence of net revenues earned by the management division is that *all* of the costs occur in the first period and *all* of the benefits occur in subsequent periods. Thus if we look at the net income earned by the division in any given period, it will not be "representative" for the project as a whole: the period 0 costs are not matched up with the period t revenues. Thus in order to construct more representative estimates of income for the management division, accountants have invented the *matching principle* for allocating costs that occur in period 0 but whose benefits occur in later periods.[16] The basic idea can be explained as follows. Let $\{d^t: t = 0,1,....\}$ be a sequence of net revenue allocations that has the following property:

(38) $d^0 + d^1(1+r)^{-1} + d^2(1+r)^{-2} + d^3(1+r)^{-3} + ... = 0.$

If we add the period t imputation d^t to the period t net revenues of the division for each t, and then take the present value of the above imputed net revenues to the actual net revenues of the firm, then the present value of the total revenue stream (including actual and imputed net revenues) will of course be equal to the present value of the actual net revenue stream. Thus we will attempt to choose a sequence of imputations d^t that will result in more *representative* period by period net revenues; i.e., the resulting stream of period by period net revenues will better *match* the fixed costs of period 0 to the monopoly revenues that occur in subsequent periods.

We now proceed to an explicit application of the above "matching" model. In period 0, we create an imputed investment output d^0 defined to be equal to the period 0 cost of the R&D project:

(39) $d^0 = w^0z^0 = a^0z^0.$

We need to choose a sequence of cost allocations d^t for $t \geq 1$ which will satisfy equation (38) when we substitute (39) into (38). We will choose to make the period t cost allocation, d^t, proportional to the period t monopoly revenue, mc^t. Letting f be the factor of proportionality so that d^t equals fmc^t for $t \geq 1$, we want to solve the following equation for f:

(40) $$\begin{aligned} -a^0z^0 &= f\,mc^1(1+r)^{-1} + fmc^2(1+r)^{-2} + fmc^3(1+r)^{-3} + ... \\ &= f\,m\{c^1(1+r)^{-1} + c^2(1+r)^{-2} + c^3(1+r)^{-3} + ...\} \\ &= f\,ma^1X^1 \qquad \text{using (6) and (31).} \end{aligned}$$

Thus f is equal to:

(41) $f \equiv -a^0z^0/ma^1X^1$

and the sequence of period t cost allocations d^t is given by

(42) $$\begin{aligned} d^t &\equiv f\,mc^t & t \geq 1 \\ &= -[a^0z^0/ma^1X^1][ma^1x^t] & \text{using (31) and (41)} \\ &= -a^0z^0[x^t/X^1]. \end{aligned}$$

Thus period t net income n^t for the management division is equal to actual net income plus imputed net income in period t, d^t, which is actually an imputed cost for period $t \geq 1$; i.e., we have the following sequence of net incomes for the management division of the monopolist:

[16] For references to the accounting literature on the matching principle, see Diewert (2005c).

$$(43)\ n^0 \equiv 0;$$

$$(44)\ n^t \equiv mc^t + d^t \qquad t = 1,2,\ldots$$
$$= ma^1x^t - a^0z^0[x^t/X^1] \qquad \text{using (31) and (42)}$$
$$= [ma^1 - a^0(z^0/X^1)]x^t$$
$$= [\{(a^1/a^0) - 1\}a^1 - a^0(z^0/X^1)]x^t \qquad \text{using (30)}$$
$$> 0$$

where the above inequality follows from $[(a^1/a^0) - 1] > (z^0/X^1)$, which is equivalent to (13), and $a^1 > a^0$.

The point of all of these imputations is that the cost of the R&D research made in period 0 is now spread out over future periods, reducing the gross monopoly profits in period t of mc^t by the amount $-d^t$ equal to $a^0z^0[x^t/X^1]$ and the period 0 original negative income for the R&D management division of $-a^0z^0$ is increased to the zero level; i.e., the original fixed costs of the R&D project are intertemporally reallocated to subsequent periods in a way which matches costs to revenues in a "reasonable" manner. It is evident that d^t could be interpreted as a period t *depreciation allowance* but it is more properly interpreted as an *amortization amount*; it is simply an imputation that somewhat arbitrarily allocates the period 0 fixed cost to future periods. Note that the absolute value of d^t is equal to the product of a^0z^0 (the fixed cost that is to be amortized over future periods) times x^t divided by the future input aggregate X^1. Note also that

$$(45)\ \sum_{t=1}^{\infty} x^t/X^1(1+r)^t = 1.$$

Equations (42) and (45) mean that the sum of the amortization amounts, $\sum_{t=1}^{\infty} (-d^t)$, will exceed the original period 0 R&D costs, a^0z^0, due to the discounting by the interest rate.[17]

It can be seen that accounting for R&D leads to an accounting framework which is, unfortunately, much more complex than the usual Solow-Jorgenson-Griliches growth accounting framework!

Case 6: A Domestically Funded Project with No Outsourcing of Production and with Foreign Financing of the Project

Domestic investors could decide to fund the R&D project by borrowing from abroad. Thus in this case, we will assume that domestic investors finance the period 0 costs of the R&D project by borrowing an amount of the consumption good that is equal to the period 0 consumption that is foregone by investing in the R&D project; this amount is a^0z^0, which we set equal to $-b^0$; i.e., we define b^0 by (20) as in Case 4 where we considered the case of a government R&D project which was financed by foreign borrowing. As in our analysis of Case 4, we assume that the private investors in the R&D project face the intertemporal balance of payments constraint (19), where b^t for $t \geq 1$ is the amount of the

[17] See section 11 in Diewert (2005b) and Diewert (2005c) for examples of how the matching methodology works.

consumption good which must be exported in period t in order to repay the loan. As in Case 4, the R&D investors could pick any pattern of future period b^t which satisfy the constraint (19); i.e., the repayment plan has many degrees of freedom. But following the logic of the matching approach to the problem of amortizing the loan, it seems appropriate to make future period loan repayments proportional to the expected net benefit that the R&D project generates in period t, which is the amount of monopoly profits equal to ma^1x^t for $t \geq 1$. Thus b^t is defined as:

$$(46)\ b^t \equiv f\,[(a^1/a^0) - 1]a^1x^t = f\,[a^1 - a^0](a^1/a^0)x^t\ ; \qquad t \geq 1$$

where f is a parameter to be determined. Substituting (46) into the intertemporal balance of payments equation (19) with b^0 defined by (20) leads to the following equation:

$$\begin{aligned}(47)\ a^0z^0 &= \textstyle\sum_{t=1}^{\infty} b^t/(1+r)^t \\ &= \textstyle\sum_{t=1}^{\infty} f\,[a^1 - a^0](a^1/a^0)x^t/(1+r)^t && \text{using (46)} \\ &= f\,[a^1 - a^0](a^1/a^0)X^1 && \text{using definition (6).}\end{aligned}$$

Solving (47) for f gives us:

$$(48)\ f = a^0z^0/[a^1 - a^0](a^1/a^0)X^1.$$

Substituting (48) into (46) gives us:[18]

$$(49)\ b^t \equiv a^0z^0\, x^t/X^1 \qquad \text{for } t \geq 1.$$

Equations (25) and (26) can be used in order to define period t consumption for the home economy, $c^t = y^t - b^t$ for $t \geq 1$, and then (26) can be used to define the economy's social welfare under this method for financing the project. Not surprisingly, we obtain the same level of social welfare that we have obtained in all of our previous cases.

We now calculate the *period t monopoly revenue*, ma^1x^t, less the *period t loan repayment*, b^t:

$$\begin{aligned}(50)\ ma^1x^t - b^t &= ma^1x^t - a^0z^0\, x^t/X^1 && \text{using (49) for } t \geq 1 \\ &= \{(a^1/a^0) - 1\}a^1x^t - a^0z^0\, x^t/X^1 && \text{using definition (30)} \\ &= n^t && \text{using (44)}\end{aligned}$$

where n^t was previously defined to be equal to the imputed net income of the management division of the monopolist. Thus in the present case where the R&D project is financed by a loan from abroad, our old imputed net income in period t is equal to

[18] We get the same answer for b^t if we note that the net social benefit of the project in period t is the extra production that the project makes possible which is $[a^1 - a^0]x^t$ and so we could set b^t to be proportional to this net social benefit so set $b^t = f[a^1 - a^0]x^t$ and then solve equation (47) for f, etc. However, there is no reason for a monopolist to make cost allocations or loan repayments based on a consideration of social benefits.

actual monopoly revenue less actual loan repayments in period t. *Thus the net income that accrues to the management division is no longer an imputed accounting income; in the present case, it is an actual income*. This equality helps to justify the case for the imputed income concept that we developed in the previous case. However, note that for both the present case and the previous case, there are ambiguities in our estimates of period by period income; i.e., in Case 5, we had to decide how the fixed cost should be matched to future revenues and in Case 6, we had to decide exactly how the loan from abroad should be repaid.

Cases 7 and 8: *A Domestically Funded Project with Licensing of Production (with domestic funding of the project or foreign borrowing to fund the project)*

The setup here is exactly the same as in the previous two cases, except that now we assume that the developers of the new process *license* the technology to other producers. If we assume that the license fees are proportional to the quantity of output produced by the new technology, then no new algebra needs to be developed: simply reinterpret the monopoly markup m in the previous case as the *per unit output royalty* that must be paid by independent producers for the right to use the new technology. Thus the welfare effects for these cases are exactly the same as in Cases 5 and 6. However, note that the accounting is slightly different in these licensing cases: we require a slightly augmented set of production accounts so that the flows generated by these royalty payments can be accommodated.

Case 9: *A Foreign Funded Project*

In this case, foreign investors fund the period 0 expenditures on primary inputs, w^0z^0, in return for the stream of monopoly profits or licensing fees generated by the R&D project. In this case, the welfare effects of the R&D project are (finally!) different: *the foreign investors will get all of the benefits of the R&D project*, leaving domestic residents no better off than they would be if the project never took place. We leave the details to the reader.

This completes our discussion of a publicly or privately funded R&D project that develops a new process. In the following section, we turn our attention to R&D projects that develop a *new product* as opposed to a *new process*.

6. R&D and Product Innovation

Fortunately, it is not necessary to develop any new algebra for the case of a product innovation. We can adapt the analysis in the previous two sections very easily under two alternative sets of assumptions concerning the new product:

- The new product is simply a new mixture of characteristics that purchasers value and a hedonic regression methodology can be used to quality adjust a unit of the new product into an equivalent number of units of the existing product that is displaced. Once this has been done, then we look at the input requirements for producing one constant quality unit of the new and old products and this will give us the output input coefficients, a^0 and a^1, for producing units of the old and new products. The rest of the analysis proceeds as in sections 5 and 6.

- The new product has one or more really new characteristics so hedonic regression techniques may be problematic in this case, we may be forced to rely on the methodology developed by Hicks (1940) to value the contribution of new goods, except that we make the further restriction that the preferences of households over combinations of new and old products are linear. Thus we assume that one unit of primary input produces an amount of the "old" commodity which consumers value at v^0 and one unit of primary input produces an amount of the "new" commodity which consumers value at $v^1 > v^0$. Now let a^0 equal v^0 and a^1 equal v^1 and we can use the analysis developed in the previous sections.

It should be noted that our analysis is not entirely satisfactory; i.e., in general, we have not modeled *substitution effects* in an adequate manner. On the production side of our model, we have essentially assumed Leontief no substitution type technologies in each period and on the consumer side of our model, we have assumed linear intertemporal preferences, which imply perfect substitutability between consumption at different points in time. Obviously, it would be desirable to relax these rather restrictive assumptions. However, the reader will have noted that even with our simplifying assumptions, accounting for R&D investments is rather complex.

7. Towards a More Realistic Model of R&D Investment

Another problem with our modeling of an R&D investment is that we have aggregated outputs into a single commodity and the R&D investment leads to a new technology which entirely displaces the old technology. This is obviously unrealistic. Thus in this section, we will develop a more realistic model of an R&D investment. In this new model, we will still have only one aggregate input but now we distinguish two outputs:

- Output 1 is general consumption and the technology that produces this output is unaffected by the R&D investment and
- Output 2 is a specific commodity where the R&D investment generates a new technology that can produce this specific commodity more efficiently. This second commodity will typically make up only a small fraction of the entire economy.

Since we now have two outputs, our old single output social welfare function, W(y) defined by (4), has to be modified. Our new welfare function will essentially be a discounted sum of period t (cardinal) utility levels, $U^t(y_1^t,y_2^t)$, $t = 0,1,\ldots$, where y_1^t is the period t household consumption of the general commodity and y_2^t is the period t consumption of the specific commodity whose production is affected by the R&D investment in period 0. We will assume that the *period utility functions* U^t are of the no substitution Leontief type; i.e., we define U^t as follows:

(51) $U^t(y_1,y_2) \equiv \min_{y\text{'s}} \{y_1, y_2/\beta_t\}$

where β_t is a period t nonnegative[19] *taste parameter*; i.e., the bigger β_t is, the more important will be the specific commodity to the consumer in period t.[20] Generally, we expect the sequence of β_t parameters to decline over time as household tastes change and demand shifts away from the specific product until finally, for some period $T > 1$, the β_t are all equal to 0 for $t > T$ and for these distant future periods, the specific commodity is no longer demanded by any household.

Turning now to the production side of the model, we assume that each commodity is produced by a single common input. Let $x_1^t \geq 0$ and $x_2^t \geq 0$ denote the amounts of input used to produce the period t outputs $y_1^t \geq 0$ and $y_2^t \geq 0$ respectively. If there is no R&D investment, then we assume that period t outputs and inputs are related by the following *constant returns to scale production functions*:

$$(52)\ y_1^t = x_1^t\ ;\ \ y_2^t = a^0 x_2^t\ ; \qquad\qquad t \geq 0$$

where $a^0 > 0$ is the output input coefficient for sector 2. Note that we have chosen units so that output equals input for the production of the general consumption commodity; i.e., in sector 1, output in period t, y_1^t, is equal to the amount of input used during period t, x_1^t.

The total amount of input available to the economy in period t is $x^t > 0$ as in the previous sections. Thus we have the following constraint in each period t on the allocation of input between the two sectors:

$$(53)\ x_1^t + x_2^t = x^t\ ; \qquad\qquad t \geq 0.$$

We now work out the (anticipated) competitive allocation of resources between each of the two sectors for each period under the assumption that there is no R&D project. Using definition (51) of the period t utility function U^t, it can be seen that if β_t is positive, then we have the following relationships between the period t level of utility, u^t, and the period t consumption levels of the two commodities, y_1^t and y_2^t:

$$(54)\ u^t = y_1^t = y_2^t/\beta_t\ ; \qquad\qquad t \geq 0,\ \beta_t > 0.$$

The above equations imply that the following equations hold:

$$(55)\ y_2^t = \beta_t\, y_1^t\ ; \qquad\qquad t \geq 0$$

and it can be verified that equations (55) hold even if β_t equals 0. Substituting (52) and (55) into (53) leads to the following anticipated allocation of inputs for period t:

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[19] If $\beta_t = 0$, then we define $U^t(y_1,y_2) \equiv y_1$. Thus if β_t equals zero, then the specific commodity is no longer desired by the consumer in period t.

[20] The *expenditure function* $E^t(u^t,p_1^t,p_2^t)$ that is dual to U^t is $E^t(u^t,p_1^t,p_2^t) \equiv \min_{y\text{'s}} \{p_1^t y_1 + p_2^t y_2 : U^t(y_1,y_2) \geq u^t\} = [p_1^t + \beta_t p_2^t]u^t$ for $t = 0,1,2, \ldots$.

(56) $x_1{}^t = [1 + (a^0)^{-1}\beta_t]^{-1}x^t$; $t \geq 0$;
(57) $x_2{}^t = (a^0)^{-1}\beta_t[1 + (a^0)^{-1}\beta_t]^{-1}x^t$; $t \geq 0$.

Substituting (56) and (57) into (52) gives us the anticipated production of each commodity in period t:

(58) $y_1{}^t = x_1{}^t = [1 + (a^0)^{-1}\beta_t]^{-1}x^t = u^t$; $t \geq 0$;
(59) $y_2{}^t = a^0x_2{}^t = \beta_t[1 + (a^0)^{-1}\beta_t]^{-1}x^t$; $t \geq 0$.

We turn our attention to the prices that are anticipated to prevail in each period t. We now assume that the central bank stabilizes the price of input w^t in each period and so we set w^t equal to unity for each t. Given assumptions (52) on the technology of the sectors, the price of output 1 in period t, $p_1{}^t$, will be equal to the period t price of input so we will have $p_1{}^t$ equal to one in each period as well. For sector 2, we must have the period t value of outputs, $p_2{}^ty_2{}^t$, equal to the corresponding value of input, $w^tx_2{}^t = x_2{}^t$, and this equation along with the second equation in (52) will imply that $p_2{}^t$ will equal the reciprocal of a^0. Putting this altogether, anticipated period t prices in the economy will be equal to the following specific values:

(60) $w^t = 1$; $p_1{}^t = 1$; $p_2{}^t = 1/a^0$; $t \geq 0$.

We could use the above information in order to calculate the discounted stream of period t utilities and this would lead to a measure of (anticipated) social welfare for the economy; i.e., we could use the same social welfare function as was defined by (4) above except that period t utility u^t would replace our old period t consumption y^t. However, we will find it more convenient to measure period t utility by period t household expenditure at constant prices; i.e., we will use *money metric utility* as our measure of period t utility.[21] Let $E^t(u,p_1,p_2)$ be the expenditure function that is dual to the utility function $U^t(y_1,y_2)$ that is defined by (51). Then period t money metric utility e^t is defined as follows:

(61) $e^t \equiv E^t(u^t,p_1{}^t,p_2{}^t)$ for t = 0,1,2,...
$= p_1{}^ty_1{}^t + p_2{}^ty_2{}^t$ where $y_1{}^t$ and $y_2{}^t$ are the period t household quantities
$= y_1{}^t + (a^0)^{-1}y_2{}^t$ using (60)
$= x_1{}^t + x_2{}^t$ using (58) and (59)
$= x^t$ using (53).

Thus period t money metric utility (or expenditure) e^t is equal to period t aggregate input x^t, which is reasonable, given our assumptions of no technical progress in the economy and our pricing conventions, (60).

Define the (anticipated) *money metric level of social welfare*, W^0, as discounted period by period money metric utility e^t:

[21] The term money metric utility scaling is due to Samuelson (1974) but the idea of using an expenditure function with prices fixed to cardinalize utility can be traced back to Hicks (1942) and Allen (1949).

$$(62)\ W^0 \equiv \sum_{t=0}^{\infty} (1+r)^{-t}e^t$$
$$= \sum_{t=0}^{\infty} (1+r)^{-t}x^t \qquad \text{using (61).}$$

Thus for our base case where there is no R&D investment in developing a new technology for sector 2, social welfare is equal to the discounted sum of period by period aggregate input availability for the economy.

We now consider the case of a government R&D project that develops a new technology for sector 2 in period 0 and is made freely available to the economy in subsequent periods. We suppose that the period 0 government R&D expenditures on primary input are equal to the quantity of input z^0, which is assumed to be less than the available total primary input for period 0, x^0. This R&D investment produces a new technology for sector 2 which has output input coefficient a^1, where as usual, we assume that a^1 is greater than a^0:

$$(63)\ a^1 > a^0.$$

We again assume (somewhat unrealistically) that the new technology immediately displaces the old technology used in sector 2 for all periods t greater than 0. The algebra associated with equations (52)-(59) can now be repeated for all periods $t \geq 1$, except that a^1 replaces a^0. Letting x_i^{t*} and y_i^{t*} denote the quantity of input used by sector i and output produced by sector i in period t for i = 1,2, it can be seen that we obtain the following equations:

$$(64)\ y_1^{t*} = x_1^{t*}\ ;\ \ y_2^{t*} = a^1x_2^{t*}\ ; \qquad t \geq 1;$$
$$(65)\ x_1^{t*} + x_2^{t*} = x^t\ ; \qquad t \geq 1;$$
$$(66)\ y_2^{t*} = \beta_t\, y_1^{t*}\ ; \qquad t \geq 1;$$
$$(67)\ x_1^{t*} = [1 + (a^1)^{-1}\beta_t]^{-1}x^t\ ; \qquad t \geq 1;$$
$$(68)\ x_2^{t*} = (a^1)^{-1}\beta_t[1 + (a^1)^{-1}\beta_t]^{-1}x^t\ ; \qquad t \geq 1;$$
$$(69)\ y_1^{t*} = x_1^{t*} = [1 + (a^1)^{-1}\beta_t]^{-1}x^t = u^{t*}\ ; \qquad t \geq 1;$$
$$(70)\ y_2^{t*} = a^1x_2^{t*} = \beta_t[1 + (a^1)^{-1}\beta_t]^{-1}x^t\ ; \qquad t \geq 1.$$

In a similar manner, we obtain the following counterparts to equations (60); i.e., anticipated period t prices in the economy after the R&D investment will be equal to the following specific values:

$$(71)\ w^{t*} = 1\ ;\ p_1^{t*} = 1\ ;\ p_2^{t*} = 1/a^1\ ; \qquad t \geq 1.$$

Using assumption (63) and our assumption that the period t taste parameter β_t is nonnegative, we can establish the following inequalities:[22]

$$(72)\ [1 + (a^1)^{-1}\beta_t]^{-1} \geq [1 + (a^0)^{-1}\beta_t]^{-1}\ ; \qquad t \geq 1.$$

[22] If $\beta_t > 0$, then inequality t in (72) holds strictly.

Using (58), (59), (69), (70) and (72), it can be shown that the following relationships hold between the period t anticipated outputs with no R&D investment, y_i^t, and the period t anticipated outputs with the period 0 R&D investment, y_i^{t*}, for i = 1,2:

$$(73)\ y_i^{t*} \geq y_i^t\ ; \qquad\qquad i = 1,2\ ;\ t \geq 1.$$

It can also be shown that the two inequalities in (73) will hold strictly for any period t where the taste parameter β_t is positive, so that the specific consumption commodity is demanded in that period. The inequalities (73) make good intuitive sense, since the increased productivity in the production of the specific commodity affected by the process innovation allows more of both commodities to be produced for any period where the second commodity is actually demanded by households.

We still need to calculate the period 0 allocation of resources between the two sectors after the R&D effort z^0 is subtracted off from total available primary input x^0 in period 0. If the required revenue to finance the government R&D project in period 0 is financed by an income tax in period 0, then the size of the required income tax t^0 is still equal to z^0/x^0, the same tax rate that was defined by (15) above. The period 0 producer price for input will be $w^{0*} \equiv 1$ and the corresponding consumer price will be $w^{0*}(1-t^0)$. The producer and consumer prices for the two outputs in period 0 will be defined as follows:

$$(74)\ p_1^{0*} \equiv 1\ ;\ p_2^{0*} \equiv 1/a^0\ .$$

The amounts of the two consumption goods that will be produced in the R&D equilibrium in period 0 turn out to be the following amounts:

$$(75)\ y_1^{0*} = [1 + (a^0)^{-1}\beta_0]^{-1}[x^0 - z^0]\ ;$$
$$(76)\ y_2^{t*} = \beta_0[1 + (a^0)^{-1}\beta_0]^{-1}[x^0 - z^0]\ .$$

We will find it convenient to define the monopolistic markup m that the new technology would allow if the technology were closely held by private investors (even though in the present case, the government makes the new technology freely available):

$$(77)\ m \equiv [a^1/a^0] - 1 > 0$$

where the inequality follows from assumption (63).

We now calculate the value of consumption in each period for the new equilibrium, e^{t*}, but at the prices that prevailed in the equilibrium with no R&D investment. For period 0, this *money metric utility level* is given by the following expression:

$$\begin{aligned}(78)\ e^{0*} &\equiv E^0(u^{0*},p_1^0,p_2^0) \\ &= p_1^0 y_1^{0*} + p_2^0 y_2^{0*} \\ &= y_1^{0*} + (a^0)^{-1} y_2^{0*} && \text{using (74)} \\ &= [1 + (a^0)^{-1}\beta_0]^{-1}[x^0 - z^0] + (a^0)^{-1}\beta_0[1 + (a^0)^{-1}\beta_0]^{-1}[x^0 - z^0] && \text{using (75) and (76)} \\ &= x^0 - z^0.\end{aligned}$$

Thus the value of consumption in period 0 (valued at the period 0 consumer prices corresponding to the no R&D equilibrium prices p_i^0 which turn out to equal the period 0 consumer prices of the R&D equilibrium p_i^{0*}), e^{0*}, is equal to the value of primary input that is allocated to the production of consumer goods and services, $w^{0*}[x^0 - z^0] = [x^0 - z^0]$, where the equality follows since w^{0*} equals unity. In a similar fashion, we calculate the *period t money metric utility level* e^{t*}, which is equal to the value of consumption in period t (valued at the period t consumer prices corresponding to the no R&D equilibrium consumer prices p_i^t):

$$
\begin{aligned}
(79)\quad e^{t*} &\equiv E^t(u^{t*},p_1^t,p_2^t) && t \geq 1 \\
&= p_1^t y_1^{t*} + p_2^t y_2^{t*} \\
&= y_1^{t*} + (a^0)^{-1} y_2^{t*} && \text{using (60)} \\
&= x_1^{t*} + (a^0)^{-1} a^1 x_2^{t*} && \text{using (69) and (70)} \\
&= x_1^{t*} + [1+m] x_2^{t*} && \text{using (77)} \\
&= x^t + m x_2^{t*} && \text{using (65).}
\end{aligned}
$$

Thus the value of consumption in period t (valued at the period t consumer prices corresponding to the no R&D equilibrium prices p_i^t), e^{t*}, is equal to the aggregate value of primary input that is available in period t, x^t, plus the imputed monopoly markup over the old technology that the new technology generates, m, times the amount of primary input that is allocated to the new technology in period t, x_2^{t*}. This is a rather nice result.

We can use the above information in order to calculate *an intertemporal money metric estimate of social welfare* W^1 that the investment in the R&D project makes possible; i.e., define W^1 as the following discounted stream of period t money metric utility levels e^{t*} defined above by (78) and (79):

$$
\begin{aligned}
(80)\quad W^1 &\equiv \textstyle\sum_{t=0}^{\infty} (1+r)^{-t} e^{t*} \\
&= x^0 - z^0 + \textstyle\sum_{t=1}^{\infty} (1+r)^{-t}[x^t + m x_2^{t*}] && \text{using (78) and (79).}
\end{aligned}
$$

Finally, the intertemporal money metric estimates of social welfare defined by (62) and (80) can be differenced in order to give us an estimate of the *expected net benefits* B of the government R&D project:[23]

$$
\begin{aligned}
(81)\quad B &\equiv W^1 - W^0 \\
&= x^0 - z^0 + \textstyle\sum_{t=1}^{\infty} (1+r)^{-t}[x^t + m x_2^{t*}] - \textstyle\sum_{t=0}^{\infty} (1+r)^{-1} x^t && \text{using (62) and (80)} \\
(82)\quad &= - z^0 + m\textstyle\sum_{t=1}^{\infty} (1+r)^{-t} x_2^{t*} \\
(83)\quad &= - z^0 + m\textstyle\sum_{t=1}^{\infty} (1+r)^{-t} p_2^{t*} y_2^{t*} && \text{using (70) and (71).}
\end{aligned}
$$

Expression (82) says that the net benefits of the R&D project are equal to the imputed monopolistic markup m times the discounted value of the input that is used by the new

[23] It can be seen that the net benefit measure B is *an intertemporal Hicksian equivalent variation*; see Hicks (1945-46) and Diewert (1992b) for a discussion of the Hicksian variation measures.

process, $\sum_{t=1}^{\infty} (1+r)^{-t}x_2^{t*}$,[24] less the amount of input that is allocated to the development of the new process in period 0, z^0. Expression (83) provides an interpretation of the net benefits of the project in terms of outputs rather than inputs and is a counterpart to our earlier measure of net benefits B^* defined by (12). Expression (83) says that the net benefits of the R&D project are equal to the imputed monopolistic markup m times the discounted value of the anticipated output that will be produced by the new process, $\sum_{t=1}^{\infty} (1+r)^{-t}p_2^{t*}y_2^{t*}$, less the amount of input that is allocated to the development of the new process in period 0, z^0, which in turn is equal to the value of the output of commodity 1 that is foregone due to the R&D investment. The measure of the net benefits of the project defined by (83) is very close to the partial equilibrium measure of the net benefits of a privately funded R&D project that was suggested by Diewert (2005b) (2005c).[25]

Obviously, we could go through various cases where the government funds the R&D project in various ways or we could look at the various cases where the private sector funds the R&D investment in our present more realistic model. However, the results of analyzing all of these cases will be similar to the cases that we analyzed in sections 4 and 5 above for our initial highly simplified model.

There are considerable measurement problems associated with implementing the R&D measurement model defined by the net benefit measure (82). It will generally be possible to obtain estimates of R&D effort in the current period, z^0, and it will be generally possible to make somewhat realistic estimates of the appropriate real interest rate r or the discount factor 1/(1+r) but it will be very difficult to form estimates of the monopolistic markup factor m defined by (77)[26] or to form estimates of the amounts of input that the new technology will use in future periods.

At this point, it is necessary to discuss some of the limitations of our present model. We have allowed for obsolescence of the technology due to changing tastes. However, there are other sources of obsolescence. There are a number of factors that determine the rate of obsolescence for a newly developed technology:

- Competitors develop new processes or products that erode the comparative advantage of the original new process or product.
- Tastes change, leading to changes in demand for the products that the new technology produces over time.
- Economic growth and nonunitary income elasticities change the demand for products over time; e.g., horses give way to bicycles which give way to motor

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[24] This summation of terms will be a finite one if $\beta_t = 0$ for all $t \geq T$.

[25] The main difference between our present measure defined by (83) and Diewert's suggested measure is that Diewert allowed for an erosion of the monopoly markups over time. However, it should be noted that Diewert's partial equilibrium measure of the benefits of a privately funded project will usually understate the social benefits; see the discussion at the end of this section.

[26] More realistically, we need to form estimates of the future expected sequence of monopolistic markup factors; i.e., m is unlikely to be constant over the life of the project technology. For a publicly funded R&D project where the technology is made freely available, it will be particularly difficult to form estimates of the monopolistic markup m or equivalently, of the output input coefficient a^0 that corresponds to the displaced old technology.

bikes which in turn give way to automobiles which in turn, may give way to public transit!

Of the three factors listed above, we have modeled only the second factor; i.e., we have introduced exogenous changes in tastes that cause households to shift their demands away from the products that the new technology produces over time. The third factor could be accommodated but as soon as nonhomothetic period preferences are introduced, it becomes necessary to distinguish rich and poor household groups and our model would become very complex indeed.[27]

In order to model the first factor listed above in the context of a privately developed innovation, we could imagine that in period t, competitors have developed a competing technology that is characterized by the output input coefficient a^{t*} which satisfies the following inequalities:

$$(84)\ a^0 \leq a^{t*} < a^1 ; \qquad t = 1,2,...,T^*;$$
$$(85)\ a^{t*} \geq a^1; \qquad t > T^*.$$

Thus for periods between 1 and T^*, the new technology that corresponds to the output input coefficient a^1 will enjoy a monopolistic advantage with the *imputed markup* in period t, m^t, defined as follows:

$$(86)\ m^t \equiv [a^1/a^{t*}] - 1 > 0 ; \qquad t = 1,2,...,T^*.$$

Thus instead of assuming a constant markup m which lasts forever, we could assume that there will be a sequence of expected monopoly markups m^t which will eventually become 0 after enough time has passed. However, note that the *social benefit* of the process innovation is still given by (82) or (83); i.e., in this case, *the private benefit derived by the monopolist will be less than the social benefit*. The difference between the social benefit of the innovation and the monopolist's private benefit will show up as disembodied TPF growth!

In addition to the above difficulties with the model presented in this section, we note some of the other important limitations of our analysis:

- Our model can give only a first order approximation to the effects of an innovation; i.e., *substitution effects*, both on the consumer and producer sides of our model are absent.[28]
- Our social welfare function is very simple and highly aggregated. In particular, we did not distinguish how various segments of society would be affected by the innovation. This limitation is particularly important when studying the effects of

[27] If each household has identical homothetic period preferences, our assumption of a single representative consumer is justified.

[28] Consumer substitution effects are eliminated because of our assumption of Leontief within period preferences and the assumption of linear intertemporal preferences between periods. Producer substitution effects are eliminated due to the high degree of aggregation in our producer production functions.

a privately funded R&D project where the distribution of the gains generated by the project need not be spread evenly over all households in the economy.

- Our model assumed that the new technology could be developed in a single period. In fact, many innovations (such as the development of a new drug) require many years to develop.[29]
- Our model assumed that the new technology immediately displaced the incumbent technology when in fact, the old technology will not be displaced immediately; i.e., the new technology will only gradually *diffuse* into the economy.
- Our model assumed stable prices over time; i.e., we did not model how *general inflation* would affect accounting for the project.
- Due to the high degree of aggregation in our model, many *index number problems* were suppressed; e.g., how exactly should the deflator for R&D effort be constructed and how exactly should the monopolistic markup factor be defined when there are many inputs in the economy so that the new technology cannot be summarized by a single output input coefficient?

8. Conclusion

Many details remain to be worked out when we account for R&D investments. We believe that the models presented in this paper may be particularly helpful in evaluating the benefits of publicly funded R&D projects, since this topic has proved to be very resistant to analysis.

Some of the more important implications that emerge from our analysis are the following ones:

- The benefits of both privately and publicly funded R&D projects can only be evaluated in the context of an *intertemporal general equilibrium model*.
- In the case of a *publicly funded project* where the results of the project are made freely available, there are a large number of alternative ways that the government can fund the project, giving rise to a large number of alternative accounting treatments of the R&D investment. In our highly simplified models, these alternative methods for funding the project did not affect social welfare but in more realistic disaggregated models, the alternative methods of funding will affect social welfare.
- In the case of a *privately funded R&D project*, the discounted stream of monopoly profits generated as a result of the project is a *lower bound* to the social benefits of the project. In general, the social benefits will be considerably larger than the private benefits.
- We suggested that the costs of an R&D project should be amortized over time according to the *matching principle* but this matching will inevitably be somewhat arbitrary, both for private and publicly funded projects.

[29] This limitation of our model is not too serious: we need only to cumulate the R&D expenditures over the development periods until the new technology is being used in production. Note that each period's expenditures need to be carried forward at the rate given by one plus the interest rate.

- A "correct" accounting treatment of an R&D project will lead to significant changes to the traditional Solow-Jorgenson-Griliches *growth accounting paradigm*. The production accounts in the current System of National Accounts will also require significant modifications. In particular, monopoly profits will have to be accommodated in a revised system of production accounts: an R&D "asset" is not at all like a depreciable capital stock asset.

As we indicated at the end of the previous section, there are some significant shortcomings in our modeling of the effects of an R&D project. However, we have provided a start on the development of a reasonable methodology for the treatment of R&D investments.

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