Mechanism Design with Weaker Incentive Compatibility Constraints[1]

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Abstract

We study an adverse selection problem, where an agent is able to understate his productivity, but not allowed to overstate it. The solution to this problem is generally different than the solution to the standard problem, where no restriction is made on the statements of the agent. We identify a sufficient condition, that does not depend on the distribution of types, under which these two solutions coincide.

Key Words: Mechanism design, incentive compatibility

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1 Introduction

The adverse selection problem is often studied in the context of an environment where a "principal" (she) is the residual claimant of a commodity that is produced by an "agent" (he). The productivity parameter (type of the agent) is observed by the agent, but unknown to the principal. The principal's task is designing an "incentive compatible" mechanism that would provide the incentive for the agent not to imitate another type. In most adverse selection papers, the agent is assumed to be capable of imitating any type of his choice. Yet, there is another strand of the literature, which is based on the partial verifiability of the agent's type.[1] Under partial verifiability, the agent is able to imitate only a subset of the other types. In this paper, we will study a specific form of partial verifiability, where the agent is able to understate his type but not able to overstate it.

In order to demonstrate how such a restriction to the agent's imitation capacity could arise, we will invoke the frequently used example of regulation of a monopolist. Suppose the productivity level of the monopolist is determined by its access to certain pieces of equipment. Suppose further that concealing some equipment is sufficient for an understatement of productivity, whereas an overstatement requires disclosing an equipment that does not actually exist. Under the standard paradigm, the monopolist is assumed to be capable of carrying out either one of these activities costlessly. In contrast, our partial verifiability model depicts an environment, where the concealment of an existing equipment is costless but disclosure of a non-existing equipment is prohibitively costly.[2]

Elimination of the agent's ability to overstate his type enlarges the set of incentive compatible mechanisms for the principal. Therefore it introduces the potential of improving the principal's rent extraction from the agent. In this paper, we present a sufficient condition for the elimination of overstatements not to change the principal's maximized payoff. This condition depends on how the value and the cost of production change with productivity, but unlike the widely used "regularity" conditions, does not depend on the distribution of productivity.

One result that is directly related to the current analysis is provided by Moore (1984). In an auction environment, where the principal is a seller and the agent is a buyer whose valuation for the object depends on his type, Moore shows that removing the agent's ability to imitate higher valuation types does not change the optimal solution to the principal's expected revenue maximization problem (his Theorem 1).[3] We diverge from Moore's setup by allowing for the principal's payoff to depend on the agent's type as well as the output level. The sufficient condition we identify is weaker than assuming that the principal's payoff is not responsive to the agent's type.

2 The Model

The principal is the residual claimant of the production, and the agent incurs the production costs. The principal can commit to a contract that assigns output and transfer levels to messages sent by the agent. Both players have utility functions quasilinear in money.

We assume a discrete type space for the agent, $\{1, 2, ..., N\}$. Let n be the generic element of this set. f_n is the prior probability that the agent's type is n. $F_n = \sum_{i \leq n} f_i$ is the cumulative distribution function associated with $\{f_n\}$. The cost of producing x units of output when the type is n is $c(x, n)$. This cost function is strictly increasing, convex, differentiable in x and strictly decreasing in n. We also make the following "sorting" assumption:

$$c(x, n) - c(x, n+1) \text{ is increasing in } x \text{ for all } n. \tag{1}$$

Note that the sorting condition can also be written as $c_1(x, n)$ is declining in n for all x, where the subscript 1 after a function indicates the derivative with respect to its first argument.

The value of production of x units of output for the principal is $v(x, n)$, provided that the type of the agent is n. This function is strictly increasing, strictly concave, and differentiable in x. Note that we are not stating any specification regarding the dependence of the function

$v(x,n)$ on the agent's type n. Finally, to guarantee that the optimization problems we will introduce have solutions, we assume that for all n, $v_1(x,n) - c_1(x,n)$ approaches to a negative number (or to $-\infty$) as x tends to infinity.

If the type of the agent were known by the principal, the optimal mechanism would require maximizing the value of production net of the production cost. Accordingly, x_n^{fb}, the "first best" output level for type n is defined as $x_n^{fb} \in \arg\max_x \{v(x,n) - c(x,n)\}$.[4]

We will start with stating the standard problem that an uninformed principal faces, when the agent is capable of imitating any type of his choice. This problem can be formulated as choosing the utility and output levels for each type such that there exists no type willing to imitate another one, and all types are given a non-negative utility level. Let u_n and x_n represent the utility and output levels for agent type n. Here is the principal's optimization program:

Program A1:

$$\max_{\{u_n,x_n\}} \sum_n f_n \left(v(x_n,n) - c(x_n,n) - u_n\right) \text{ s.t.}$$
$$IC(m|n) \quad : \quad u_n \geq u_m + c(x_m,m) - c(x_m,n) \text{ for all } m, \text{all } n$$
$$IR(n) \quad : \quad u_n \geq 0 \text{ for all } n$$

where IC stands for "incentive compatibility" and IR stands for "individual rationality."

One immediate implication of the incentive compatibility constraints is the monotonicity of the output levels in the productivity of the agents, i.e., $x_{n+1} \geq x_n$ for all $n < N$. By using this monotonicity requirement and the sorting condition on the cost function, one can show that the only relevant constraints of Program A1 are the "downward adjacent" IC constraints, the IR constraint of the least productive type, and the monotonicity constraints. Accordingly, Program A1 has the same solution as the "reduced" program below.

Program A2:

$$\max_{\{u_n,x_n\}} \sum_n f_n \left(v\left(x_n,n\right)-c\left(x_n,n\right)-u_n\right) \text{ s.t.}$$

$$\begin{aligned} IC\left(n|n+1\right) &: \quad u_{n+1} \geq u_n + c\left(x_n,n\right) - c\left(x_n,n+1\right) \text{ for } n<N \\ IR\left(1\right) &: \quad u_1 \geq 0 \\ x_{n+1} &\geq \quad x_n \text{ for } n<N \end{aligned}$$

At the solution to Program A2, constraints $IC\left(n|n+1\right)$ for $n<N$ and $IR\left(1\right)$ are always binding. If we ignore the monotonicity constraints, the output levels to maximize the objective function are identified as $x_n^* \in \arg\max_x P\left(x,n\right)$ where

$$P\left(x,n\right)=\begin{cases} v\left(x,n\right)-c\left(x,n\right)-\frac{1-F_n}{f_n}\left[c\left(x,n\right)-c\left(x,n+1\right)\right] \text{ for } n<N \\ v\left(x,n\right)-c\left(x,n\right) \text{ for } n=N. \end{cases}$$

Most researchers assume "regularity" conditions which secure that $\{x_n^*\}$ satisfies the monotonicity constraints and therefore constitutes an optimal output profile.[5] If the regularity conditions hold and $\{x_n^*\}$ is indeed a solution to Programs A1 and A2, removing the upward IC constraints (or removing all the IC constraints other than the downward adjacent ones) would not change the optimal solution to the problem. Notice that, since $P\left(x,n\right)$ involves the terms F_n and f_n, any such regularity conditions require assumptions on the distribution function.

On the other hand, if $\{x_n^*\}$ is not monotonic and therefore is not an optimal output profile, then the monotonicity constraints are relevant for Program A2.[6] For the purpose of this paper, what is significant about this case is that the removal of the upward IC constraints might improve the principal's rent extraction. Monotonicity is a joint implication of all the IC constraints, both the upward and the downward ones. Hence the principal is able to implement non-monotonic output profiles if upward IC constraints are eliminated. In what follows, we will not introduce any restrictions on the distribution of types. Therefore we will not be able rule out the case where monotonicity constraints are relevant for Program A2. However, we will present an alternative condition, which does not depend on the type distribution, but which guarantees that removing the upward IC constraints would not change the optimal solution to the mechanism design problem.

3 The Modified Problem

In this section, we consider the modification of Program A1, where the agent is able to understate his productivity but is unable to overstate it.

Program B1:

$$\begin{aligned}
& \max_{\{u_n, x_n\}} \sum_n f_n \left(v\left(x_n, n\right) - c\left(x_n, n\right) - u_n \right) \text{ s.t.} \\
IC\left(m|n\right) \quad &: \quad u_n \geq u_m + c\left(x_m, m\right) - c\left(x_m, n\right) \text{ for } m < n, \text{ for all } n \\
IR\left(n\right) \quad &: \quad u_n \geq 0 \text{ for all } n
\end{aligned}$$

The only IC constraints for Program B1 are the downward IC constraints. In order to induce the truthful revelation of the type, it is sufficient for the principal to make sure that there exists no type who is willing to imitate a less productive one.[7] As a first step to the simplification of the set of constraints for Program B1, we will define the function $\tilde{n}\left(\cdot\right)$ as

$$\tilde{n}\left(n\right) = \max\left\{ \arg\max_{i \leq n} \left\{x_i\right\} \right\}$$

for a given output profile $\{x_n\}$. That is, $\tilde{n}\left(n\right)$ is the type which is associated with the highest production level among the types weakly smaller than n. If there are more than one such type, $\tilde{n}\left(n\right)$ represents the highest one among them. Note that $\tilde{n}\left(\cdot\right)$ is a weakly increasing function. $\tilde{n}\left(n\right)$ takes the value of n if and only if $x_n \geq x_{\tilde{n}(n-1)}$, otherwise $\tilde{n}\left(n\right) = \tilde{n}\left(n-1\right)$. Also note that $\{x_n\}$ is monotonic if and only if $\tilde{n}\left(n\right) = n$ for all n.

Now we will introduce a simpler variant of Program B1 by removing all the constraints other than $IC\left(\tilde{n}\left(n\right)|n+1\right)$ for $n < N$ and $IR\left(1\right)$.

Program B2:

$$\max_{\{x_n,u_n\}} \sum_n f_n \left(v\left(x_n,n\right) - c\left(x_n,n\right) - u_n\right) \text{ s.t.}$$

$$IC\left(\tilde{n}\left(n\right)|n+1\right) \;:\; u_{n+1} \geq u_{\tilde{n}(n)} + c\left(x_{\tilde{n}(n)}, \tilde{n}\left(n\right)\right) - c\left(x_{\tilde{n}(n)}, n+1\right) \text{ for } n < N$$

$$IR\left(1\right) \;:\; u_1 \geq 0$$

Proposition 1 *The solutions to programs B1 and B2 are equivalent.*

Proof. We need to establish that the solution to B2 satisfies the constraints of B1. First, we will show that the solution to B2 satisfies certain equations. Then we will show that those equations imply the constraints of B1.

Step 1: The solution to B2 satisfies

$$C\left(n+1\right) : u_{n+1} = u_n + c\left(x_{\tilde{n}(n)}, n\right) - c\left(x_{\tilde{n}(n)}, n+1\right) \textbf{ for all } n < N.$$

To see this, first note that at the solution to B2, all constraints of B2 must be binding. Otherwise by reducing the value of some u_n, we could increase the value of the objective function without violating any of the constraints. To show that $C\left(n+1\right)$ is satisfied, there are two cases two consider. If $\tilde{n}\left(n\right) = n$, then $C\left(n+1\right)$ is implied by the binding $IC\left(\tilde{n}\left(n\right)|n+1\right)$ constraint. If $\tilde{n}\left(n\right) = \tilde{n}\left(n-1\right)$, then with a change of variables, we can rewrite the binding $IC\left(\tilde{n}\left(n-1\right)|n\right)$ constraint as

$$u_n = u_{\tilde{n}(n)} + c\left(x_{\tilde{n}(n)}, \tilde{n}\left(n\right)\right) - c\left(x_{\tilde{n}(n)}, n\right). \tag{2}$$

When we add $c\left(x_{\tilde{n}(n)}, n\right) - c\left(x_{\tilde{n}(n)}, n+1\right)$ to both sides, we get

$$u_n + c\left(x_{\tilde{n}(n)}, n\right) - c\left(x_{\tilde{n}(n)}, n+1\right) = u_{\tilde{n}(n)} + c\left(x_{\tilde{n}(n)}, \tilde{n}\left(n\right)\right) - c\left(x_{\tilde{n}(n)}, n+1\right). \tag{3}$$

It follows from the binding $IC\left(\tilde{n}\left(n\right)|n+1\right)$ constraint again that the right hand side of this last equation is equal to u_{n+1}. Therefore, $C\left(n+1\right)$ holds for this case as well.

Step 2: $IR\left(1\right)$ **and** $C\left(n+1\right)$ **for all** $n < N$ **imply the constraints of B1.**

$C(n+1)$ for all $n < N$ imply u_n is increasing in n. Therefore $IR(1)$ is sufficient for the other IR constraints. To see that downward IC constraints are satisfied, let n and m be two types such that $n > m$. Sum up equations $C(n)$ to $C(m+1)$ to get

$$\begin{aligned} u_n &= u_m + \left[c\left(x_{\tilde{n}(n-1)}, n-1\right) - c\left(x_{\tilde{n}(n-1)}, n\right)\right] + \left[c\left(x_{\tilde{n}(n-2)}, n-2\right) - c\left(x_{\tilde{n}(n-2)}, n-1\right)\right] + \\ &\quad ..... + \left[c\left(x_{\tilde{n}(m)}, m\right) - c\left(x_{\tilde{n}(m)}, m+1\right)\right]. \end{aligned} \tag{4}$$

By definition of $\tilde{n}(\cdot)$, we know that $x_{\tilde{n}(i)}$ is larger than x_m for all i larger than m. $IC(m|n)$ follows from the above equation and the sorting condition (1). Since n and m are arbitrarily chosen, any downward IC constraint is an implication of $C(n+1)$ for all $n < N$. ■

Unlike in Program A2, where the focus is on the downward adjacent type, in Program B2 the principal has to provide the incentive not to imitate the type that produces the highest output level among the less productive types. This points to an important distinction of our modified problem. The relevant IC constraints of Program B1 are endogenously determined by the output profile $\{x_n\}$. The exogenous ordering of the types does not reveal the relevant constraints in the absence of the upward IC constraints. In essence, the modified problem here is similar to the design problems, where the *global* incentive constraints are relevant as well as the local ones. (See Moore (1984, 1988), and Matthews and Moore (1987).)

We will close this section by presenting two properties of the solution to Program B1.

Lemma 1 *If $\{x_n\}$ is the output profile that solves Program B1 (and B2), then*

$$\begin{aligned} &i)\ x_n \leq x_n^{fb} \text{ for all } n. \\ &ii)\ x_n = x_n^{fb} \text{ for } n \text{ such that } \tilde{n}(n) < n. \end{aligned}$$

Proof. i) Consider the solution to B1. From the sorting condition, the right hand sides of the IC constraints are increasing in output levels. Suppose there exists n such that $x_n > x_n^{fb}$. Replace x_n with x_n^{fb}. The constraints of the problem are still satisfied. The objective function is higher. Contradiction.

ii) Consider the solution to B2. Suppose there exists n such that $\tilde{n}(n) < n$ and $x_n < x_n^{fb}$. Increase x_n in such a way that it is still smaller than $x_{\tilde{n}(n)}$ and x_n^{fb}. There is no change in the right hand sides of the IC constraints. The objective function is higher. Contradiction. ■

4 A Sufficient Condition

Consider the following example, where there are 3 possible types of the agent with $f_1 = \frac{3}{4}$ and $f_2 = f_3 = \frac{1}{8}$. The principal's value and the agent's cost functions are respectively $v(x,n) = v_n \ln x$ and $c(x,n) = c_n x$, where v_n and c_n are given below:

$$c_1 = 3 \quad c_2 = 2 \quad c_3 = 1$$

$$v_1 = 9 \quad v_2 = 7 \quad v_3 = 4$$

Since $\frac{\partial v(x,n)}{\partial x} = \frac{v_n}{x}$ is decreasing in n and $\frac{1-F_n}{f_n}$ is non-monotonic, this example does not satisfy the regularity conditions stated in footnote 5. The solution to Program A1 requires "bunching" types 1 and 2, with the optimal output levels $x_1 = x_2 = 2.65$, and $x_3 = 4$. It is worth noting that constraint $IC(2|1)$, an upward adjacent incentive compatibility constraint, is binding at this solution. Therefore we cannot immediately conclude that removal of the upward IC constraints would not change the optimal output levels. However, due to the small number of types in this example, we can provide a solution to Program B1 without much difficulty and observe that it is the same as the solution to Program A1. Even though $IC(2|1)$ is removed from the set of constraints, $IC(1|3)$, a downward "non-adjacent" constraint comes into effect and the principal still chooses the same outcome in the less constrained environment.

This example introduces the possibility that solutions to Programs A1 and B1 may be equivalent even when the monotonicity constraints are relevant for Program A2. With the following proposition we present a sufficient condition for equivalence.

Proposition 2 *If $\left\{x_n^{fb}\right\}$ is weakly increasing, then the solution to Program B1 (B2) is equivalent to the solution to Program A1 (A2).*

Proof. To prove the proposition, it suffices to show that the solution to B2 satisfies the constraints of A2, i.e., $\tilde{n}(n) = n$ for all n. Suppose there exists m, such that $\tilde{n}(m) < m$. This implies $x_m = x_m^{fb}$. It follows from $x_{\tilde{n}(m)} \leq x_{\tilde{n}(m)}^{fb}$ and weakly increasing $\left\{x_n^{fb}\right\}$ that $x_{\tilde{n}(m)} \leq x_m$, which is a contradiction to the definition of function $\tilde{n}(\cdot)$. ■

In the example above, the first best output levels ($x_1^{fb} = 3$, $x_2^{fb} = 3.5$, $x_3^{fb} = 4$) satisfy the sufficient condition stated in Proposition 2. This confirms our initial finding that Programs A1 and B1 yield the same solutions for this example. Moreover, since this sufficient condition does not depend on the type distribution, the equivalence of the solutions would persist under different values for f_1, f_2, and f_3.

On the other hand, if the first best output levels are not weakly increasing, solutions to Programs A1 and B1 could differ. To see this, consider a variant of the above example, where v_2 equals 5, instead of 7. Now, x_2^{fb} equals 2.5 and the first best output levels are not weakly increasing any more. The solution to Program A1 would still involve bunching with the optimal output levels $x_1 = x_2 = 2.56$, and $x_3 = 4$. But the solution to Program B1 is different and yields a higher payoff for the principal with the output levels $x_1 = 2.57$, $x_2 = 2.5$, and $x_3 = 4$.[8]

Finally, we turn our attention to a condition that would guarantee that the first best output levels are weakly increasing. If the marginal value of production, $v_1(\cdot, n)$, is weakly increasing in n then the first best output levels are weakly increasing as well.[9] A special case of this condition would be the value of production not being responsive to the type of the agent at all. The corollary below follows from this observation.

Corollary 1 *(Moore (1984)) If $v(\cdot, n) = v(\cdot, m)$ for all n and m, then the solution to Program B1 (B2) is equivalent to the solution to Program A1 (A2).*

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Notes

[1]See Green and Laffont (1986), Deneckere and Severinov (2001). Also see Maggi and Rodriguez-Clare (1995) for a model of costly misreporting.

[2]Alternatively, the regulator could be inspecting the profit generated by the monopolist rather than its access to equipment. In that case, our model would be describing a situation, where the monopolist can hide a portion of the profits but cannot inflate them. For similar assumptions, see Hurwicz et al. (1995), Hong and Page (1995) on partial verifiability of endowments; and Beaudry and Blackorby (2000) on partial verifiability of market productivity.

[3]Moore (1984) employs this result in his characterization of the optimal auction with risk aversion. Matthews and Moore (1987), and Moore (1988) apply a similar methodology to study (i) a monopolist's optimal menu of quality - warranty pairs, and (ii) the second best contract between a buyer and a seller respectively.

[4]Strict concavity of $v(\cdot, n)$ and convexity of $c(\cdot, n)$ imply that x_n^{fb} is unique.

[5]The following set of assumptions would do the trick: (i) $v_1(x, n)$ is increasing in n, (ii) $c_1(x, n) - c_1(x, n+1)$ is decreasing in n, (iii) the hazard rate of the distribution is monotonic, i.e., $\frac{1-F_n}{f_n}$ is decreasing in n.

[6]In that case, providing a solution to Programs A1 and A2 is a slightly more involved process. Fudenberg and Tirole (1991) outline this process for a model with continuous type space in the appendix to Chapter 7.

[7]One note is in order to justify our implicit reliance on the revelation principle. Since our assumptions on the agent's capacity to imitate other types satisfy the "nested range condition," i.e., since an agent who can imitate type n can also imitate all the other types that can be imitated by type n, it follows from Green and Laffont (1986) that any implementable outcome

is implementable through truthful equilibria of direct mechanisms.

[8]To see how the removal of the upward IC constraints helps in this variant of the example, note that the solution to Program A1 exhibits an "upward distortion" in the output levels. That is, the optimal level of x_2 is higher than its first best level. If the principal could reduce x_2 without changing the other output levels, she would have increased her expected payoff. However, such a reduction in x_2 violates the constraints of Program A1, since it would make imitating type 2 optimal for type 1. In contrast, since the upward IC constraints are not present in Program B1, the principal could reduce the output level for type 2 without fearing that type 1 could imitate type 2. When the first best output levels are weakly increasing, such upward distortions are ruled out in the solution to Program A1.

[9]Note that this condition is *not* a necessary condition. The example we provide above have weakly increasing first best output levels even though $v_1(\cdot, n)$ is decreasing in n.