Pure Strategy and No-Externalities with Multiple Agents

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Abstract

This note considers two properties of common agency models - pure strategy equilibria with simple competition are robust and equilibria in mechanisms can be reproduced as equilibria with simple competition provided an appropriate no-externalities assumption holds. This note provides counter examples to both these theorems when there are multiple agents.

In a recent paper (Peters 2001b) shows two properties of common agency models in which multiple principals compete by interacting through a single agent. First, if the principals compete in simple incentive schemes (contracts that specify the way that the principal's action will depend on the contractible parts of the agent's effort), then every pure strategy equilibrium in incentive schemes will be robust to the possibility that principals might use more complex mechanisms (the simple incentive contract the principal uses can depend on messages sent by the agent). This 'pure strategy theorem' is a helpful result since it is common to focus attention on pure strategy equilibria in applications. The theorem suggests that modelling competition in the naive way will reveal 'legitimate' equilibrium allocations, though not necessarily all of them.

The second result is that if the environment has the property that there are 'no externalities' in the sense that once the agent has chosen his effort, his ranking of the various actions available to any principal is independent of the actions taken by other principals, then every pure strategy equilibrium allocation that can be supported by having principals compete in mechanisms, can also be supported by having principals compete in simple incentive schemes. Thus, in environments without externalities (most well known papers on common agency analyze environments that have this property), there is nothing new to be learned by studying competition in mechanisms (at least if one is interested in pure strategy equilibria).

This is somewhat surprising since the agent typically has market information that principals would like to exploit. The would learn this information by communicating with the agent before selecting an incentive scheme.[1] The pure strategy theorem could be interpreted to mean that in a pure strategy equilibrium the principal already has all relevant market information through his knowledge of the equilibrium strategies. The 'no-externalities' result hinges on the idea that even though the agent has information that the principal would like to extract, there is no way for the principal to do this in an incentive compatible way.

The point of this note is to show that neither of these theorems extend to the case where there are multiple agents.

The pure strategy theorem for common agency starts with a group of principals offering *incentive schemes* that choose actions that potentially depend on the efforts taken by all of the agents.[2] In a pure strategy equilibrium no principal finds it profitable to deviate to any alternative incentive scheme given the schemes that are being offered by his competitors. When the principal tries to deviate to a more complex mechanism, he effectively offers the agent a menu of alternative incentive schemes. The agent then faces a simple maximization problem - choose the item from the menu of incentive schemes which, along with the schemes offered by the other principals and some appropriately chosen action, yields the agent the highest expected payoff. Select any one of the incentive schemes in the principal's menu that solve this problem for the agent. Whatever effort the agent was supposed to choose in the original game when the principal offered this incentive scheme in isolation, must also be optimal for the agent when he chooses this scheme from a menu. So there is a continuation equilibrium in which the principal's payoff when he offers the menu is the same as the payoff he would have re-

[1] Examples of equilibria supported by this sort of communication are given in (Martimort and Stole 1999),(Epstein and Peters 1999),(Peters 2001a) among others.

[2] The particular application determines the extent to which agents' efforts are contractible. None of the statements made here depend on this.

ceived by offering some simple incentive scheme by itself in the original game. In the original game this deviation would be unprofitable by the definition of equilibrium. This proves the pure strategy theorem.

This argument fails with multiple agents because the principal can offer mechanisms that change the set of continuation equilibrium efforts associated with any particular incentive scheme. In the example given below, there are multiple equilibrium efforts associated with a particular incentive scheme, and the agents play one that the principal does not like. By offering a more complex communication scheme, the principal can eliminate the undesirable continuation equilibrium outcome. The example suggests that the intuition that principals have all the market information they need in a pure strategy equilibrium is correct, but incomplete. Communications mechanisms play a strategic role that goes beyond this simple argument.

The essence of the no-externalities argument is that the way the agent ranks the different actions of the principal does not depend on the actions chosen by other principals. With symmetric information, the actions of the other principals define the agents type in some weak sense. So the no externalities condition ensures that the agents ranking of the actions of any principal are independent of his type. It is very difficult to extract this market information from agents in an incentive compatible way under these conditions. This is exactly analogous to the argument that it will be impossible to extract information on the agents willingness to pay if the only action the principal controls is price.

This fails with multiple agents essentially because market information is not fully private - each agent knows what the other does. It is possible for a principal in this case to create a mechanism in which each agent honestly reports market information because he or she expects the other agent to, and fears the consequences of disagreeing with the other agent about this.

1 Basics

Competing mechanism problems have the following general structure: there are n principals dealing with m agents. Each principal $j \in \{1, \dots n\}$ controls a simple action in the set Y_j, while each agent $i \in \{1, \dots m\}$ takes some effort from a set E_i. The principal can write *contracts* contingent on all or part of the effort levels $e \in \prod_i E_i$ taken by each of the agents. The set of feasible contracts $\mathcal{A}_j$ for seller j is a subset of the set of mappings $\alpha : \prod_i E_i \rightarrow Y_j$

These contracts are referred to henceforth as *pay for effort contracts* even though the actions taken by the principals could be more general than simple monetary transfers. To simplify it will be assumed that the sets Y_j and E_i are subsets of finite dimensional linear vector spaces (essentially probability distributions over finite sets) and that these sets, along with the set of feasible pay for effort contracts $\mathcal{A}_j$, are the same for all sellers.

Each agent's preferences are private information and are parameterized by elements in some set Ω. We refer to elements of the set Ω as *valuations* instead of types though the valuations themselves may be more complex than simple willingness to pay. Principals and agents commonly believe that the agents' valuations are jointly distributed according to some distribution F on Ω^m.

Agents and principals have expected utility preferences. The payoff to principal $j \in \{1, \ldots, n\}$ is represented by $v_j : \prod_{k=1}^{n} Y_k \times \prod_{i=1}^{m} E_i \times \Omega^m \longrightarrow [0, 1]$, while for each agent i, payoffs are represented by the function $u_i : \prod_{k=1}^{n} Y_k \times \prod_{i=1}^{m} E \times \Omega^m \longrightarrow [0, 1]$.

Principals and agents choose a pay for effort contract by engaging in a communication process or mechanism designed by the principal. Formally, a mechanism for principal j is a measurable message space C_j and a measurable mapping $\gamma_j : C_j^m \to \triangle(\mathcal{A}_j)$ that associates a distribution over pay for effort contracts with each array of messages that the agents send. To simplify the notation a bit, the message space C_j will be assumed to be fixed and common to all sellers. Also it will be assumed that the set of feasible mechanisms Γ is the same for each principal. With a slight abuse of notation, γ_j will be referred to as the *mechanism* offered by principal j.

Mechanisms are endogenous and allocations are determined by a two step process: first each principal simultaneously selects and publicly offers a mechanism from Γ; next each agent simultaneously sends a message to each principal and chooses an effort level.[3]

Agent behavior in each mechanism depends on the agent's valuation and on the mechanisms that he or she observes being offered by the other principals. A *communications strategy* for agent i is a mapping $\tilde{c}_i : \Omega \times \Gamma^n \to \triangle(C^n \times E)$ that describes the (joint probability distribution over) messages

[3] An alternative formulation would allow the principal to communicate with the agent before the agent takes his effort as in (Peters 2001a). This additional bit of communication from the principal to the agent would be useful if the principal uses a random device to hide his true mechanism from other principals, yet wishes the agents to take actions based on the outcomes of this randomizing device.

and actions that the agent will use as a function of the his valuation and the array of mechanisms that he is offered by the principals. We will sometimes refer to the *decision strategy* for agent i, given by $\tilde{\pi}_i : \Omega \times \Gamma^n \times C^n \to \triangle(E)$ which probability distribution over effort that the agent uses conditional on the messages he sends to the principals. The array $\tilde{c} = \{\tilde{c}_1, \tilde{c}_2 \ldots \tilde{c}_m\}$ of *continuation strategies* for the agents constitute a *continuation equilibrium* if for every array of mechanisms $\gamma \in \Gamma^n$ offered by the principals, the continuation strategies constitute a Bayesian equilibrium for the continuation game played by the agents. An *equilibrium* relative to the set of feasible mechanisms Γ is an array of randomizations $\{\delta_1, \ldots \delta_n\}$ and a continuation equilibrium $(\tilde{c})$ such that $\{\delta_1, \ldots \delta_n\}$ is a Nash Equilibrium for the normal form game defined by the continuation equilibrium $(\tilde{c})$.

Let Γ^D be the set of 'direct mechanisms' defined here to be the set of mechanisms for which $C = \Omega$. Formally $\Gamma^D = \{(\gamma, C) : C = \Omega, \gamma : \Omega^m \to \triangle(\mathcal{A})\}$. These mechanisms are *not* direct mechanisms in the formal sense since they do not allow the agents to report their full types. On the other hand, it is natural to model competition in Γ^D since a principal operating by himself will always be able to find his best mechanism by searching in Γ^D. Observe that when the agents have no private information about their own preferences, then the set of direct mechanisms is simply the set of probability distributions over incentive schemes.[4]

Suppose that $\Gamma^D \subset \Gamma$ for some large set of mechanisms Γ. Then (Peters 2001b) shows that if there is only a single agent, then every pure strategy equilibrium relative to Γ^D is also a pure strategy equilibrium relative to Γ, and if the payoff functions u_i and v_j satisfy the no externalities assumption (described below) then every allocation that is supported by some equilibrium relative to Γ can also be supported as equilibrium relative to Γ^D.

2 Pure Strategies

To begin, we consider the pure strategy theorem. This example illustrates a pure strategy equilibrium in simple incentive schemes that does not survive when principals are allowed to communicate with agents.

There are two principals and two agents in the example. The principals

[4]At this stage, we do not want to rule out the possibility that principals want to associate random outcomes with agent efforts. This might be useful for incentive reasons if agents are risk averse.

cannot contract on agents' effort and simply choose between one of two simple actions called A and B. Agents have no private information and simply choose one of two effort levels. To help interpretation, it is possible to interpret the efforts as agent choice among the two principals, but this is not essential. To make the example relevant, the game has to involve at least four players, so it is somewhat difficult to write the payoffs. In the following table, each cell corresponds with a pair of actions chosen by the principals. Each of the cells itself contains a table with payoffs corresponding to the effort levels taken by each of the agents. The first payoff is the payoff to principal 1 who chooses the row in the outer table. The second payoff is the payoff to principal 2 who chooses the column in the outer table. The third payoff is to agent 1 who chooses the row in the inner table, and similarly for the last payoff. So, for example, if both principals choose the simple action B, and both agents choose effort 1 (or go to principal 1), then the payoff is 3 for principal 1, 0 for principal 2 and -1 for each agent.

		A		B	
		1	2	1	2
A	1	$*$	$*$	$\left(\frac{7}{8}, \frac{7}{8}, -\frac{3}{2}, \frac{5}{4}\right)$	$\left(\frac{7}{8}, \frac{7}{8}, \frac{5}{4}, -\frac{3}{2}\right)$
	2	$*$	$*$	$\left(\frac{7}{8}, \frac{7}{8}, \frac{5}{4}, -\frac{3}{2}\right)$	$\left(\frac{7}{8}, \frac{7}{8}, -\frac{3}{2}, \frac{5}{4}\right)$
B	1	$\left(0, 0, -\frac{3}{2}, \frac{5}{4}\right)$	$\left(0, 0, \frac{5}{4}, -\frac{3}{2}\right)$	$(3, 0, -1, -1)$	$(1, 1, 1, 1)$
	2	$\left(0, 0, \frac{5}{4}, -\frac{3}{2}\right)$	$\left(0, 0, -\frac{3}{2}, \frac{5}{4}\right)$	$(1, 1, 1, 1)$	$(0, 3, -1, -1)$

We focus on the pure strategy equilibrium for this game in which both principals use action B. No attempt is made to characterize the entire set of equilibria in this table, the point is simply to show that pure strategy equilibria might not survive if principals are allowed to communicate with agents before they select their action. So no attempt is made to fill in the box where principals both use action A.

Since the principals are unable to write contracts contingent on the agents efforts, a simple incentive contract for each principal is just a specification of the action that they plan to take. Suppose that the continuation equilibrium is such that when both principals announce action B, the agents coordinate their actions with agent 1 choosing principal 2 and agent 2 choosing principal 1. Everyone's payoff in this case is 1. If principal 1 deviates to action A, (while the other continues to offer action B) the agents face the following continuation payoff matrix

	1	2
1	$\left(-\frac{3}{2},\frac{5}{4}\right)$	$\left(\frac{5}{4},-\frac{3}{2}\right)$
2	$\left(\frac{5}{4},-\frac{3}{2}\right)$	$\left(-\frac{3}{2},\frac{5}{4}\right)$

as indicated in the upper right hand box of the diagram. This subgame has a unique mixed strategy equilibrium in which both agents mix equally over both actions. The payoff in this equilibrium is $-\frac{1}{8}$ to both agents. Whatever this continuation equilibrium happens to be, the payoff to the principal who deviates to action A is $\frac{7}{8}$. So it is clear that having both principals offer action B with agents then coordinating their choice over principals is a subgame perfect equilibrium for this process.

In this example, there is another equilibrium for the subgame that occurs when both principals offer the action B. In this equilibrium the agents fail to coordinate their choices, and instead mix equally between the two principals. It is straightforward that the continuation payoff that the agents receive in this case is zero - so they much prefer the outcome that does prevail in which each of them gets payoff 1. The principals, however, do better with this mixed continuation equilibrium and receive a payoff of $\frac{5}{4}$ because the large payoff 3 occurs with a high enough probability to compensate them for the 0 payoff that occurs when no buyers choose them.

The point of this example is to show that if one of the principals can negotiate with the agents before choosing his action, this outcome can no longer be supported. The reason is that the deviating principal can offer a contract that eliminates the coordination equilibrium that the agents play, leaving only mixed continuation equilibrium in which the principal's payoff exceeds 1.

The deviation involves a mechanism γ' in which the principal promises to make his action contingent on a message sent by the agent. The mechanism works as follows: the agents can send either the message A or the message B. If exactly one of the two agents (it doesn't matter which one) sends the message A, then the principal will use action A, otherwise the principal commits himself to action B.

The claim is that the deviating principal is better off along every continuation equilibrium path associated with this offer than he is in the original equilibrium. Showing this is a bit tedious because there are a number of potential continuation equilibrium depending on which combinations of messages and efforts the agents use with positive probability. However, the idea behind the deviation can be see by focusing on just one of them - the completely mixed equilibrium.

To see the argument, note first that in the original equilibrium action B is used by both principals for sure, and everyone receives payoff 1. The problem with reproducing this outcome with prior communication is that the actions and efforts of all players are perfectly predictable. Suppose that agent 1 believes that agent 2 reports B and uses effort 1 for sure, as in the original equilibrium. Further suppose that principal 1 deviates and offers the communications mechanism where agents are asked to report either A or B. To reproduce the outcome from the original equilibrium, the agents would have to coordinate on a predictable report, say B. If this occurred, the agent 1 could profitably deviate by sending the message A to the deviating principal then choosing effort level 2. Principal 1 would then switch his action to A. Since communication and effort are chosen simultaneously, agent 2 would not be expecting this to occur - agent 1's payoff rises to $\frac{5}{4}$ while agent 2's payoff falls to $-3/2$. It would appear that the coordinated outcome in which all players receive payoff 1 can no longer be supported as an equilibrium with prior communication.

What are the continuation equilibria associated with this offer? It is straightforward, but tedious to show that there are three continuation equilibrium (up to permutations of messages). In one of the equilibria the agents coordinate for sure on message B (or equivalently, coordinate for sure on message A since both things lead to action B by the deviating principal), but then randomize equally over efforts. This yields payoff 0 to the agents and payoff $5/4$ to each of the principals. It doesn't pay either agent to deviate and send message A in this case, because if he does, principal 1 changes his action to A and both agents' payoffs fall to $-\frac{1}{8}$. Call this equilibrium E_1.

In the continuation game that they play, agents have four pure actions made up by combining each of the reports A or B with a different action 1 or 2. Let a_i refer to the pure action where the agent reports A to principal 1 then takes action i, and similarly for b_i. A second equilibrium occurs when the agents randomize using each of their pure actions $\{a_1, a_2, b_1, b_2\}$ with equal probability. The payoff to the principal in this case is $\frac{17}{16}$ which again exceeds his payoff in the original pure strategy equilibrium. Refer to this as equilibrium E_2.

Finally, there is a third equilibrium in which each agent randomizes equally over pure actions a_1 and b_2 (or equivalently b_1 and a_2). In this case the payoff to the principal is $\frac{19}{16}$. This is equilibrium E_3.

Claim 1 *The continuation game in which one principal offers the mechanism γ' while the other offers the pure action B has exactly three equilibria described by E_1, E_2 and E_3.*

Proof. First observe that there can be no equilibria in which either agent takes a single effort for sure. For example, suppose that agent 2 takes effort 1 for sure. Then no matter how the agents randomize over messages, agent 1 must take action 2 for sure. Let p be the probability with which agent 1 sends message A in equilibrium. Agent 2 has payoff $(1-p)-3/2$ if she sends message B and payoff $p-(1-p)\frac{3}{2}$ if she sends message A and uses pure effort 1. Suppose that $p \geq \frac{1}{2}$ $(\leq \frac{1}{2})$. Then the payoff if agent 2 sends message B (message A) and switches to effort 2 is

$$p\frac{5}{4}-(1-p)$$

so there must exist a profitable deviation. By symmetry the argument is identical for the other agent and the other action.

Now suppose there is a mixed equilibrium in which one of the players sends a single message with probability 1. Then it is straightforward to check that whatever mixed strategy that player is using to choose his action, the other player will strictly prefer to send the message that induces the action B by principal 1. Thus apart from equilibrium E_1 (and its variant where both agents send the message A with probability 1), both players must mix over messages in every equilibrium.

This leaves equilibria in which agents mix over all pure actions, and equilibria in which they mix over alternate signal effort pairs. It is straightforward to use the indifference relations in mixed strategy equilibria and symmetric of the payoff matrices to show that the only equilibria of this type are E_2 and E_3. ■

The important point in all this is that the deviating principal's payoff is strictly higher in each of these equilibria than it was in the initial pure strategy equilibrium. As a consequence, the pure strategy equilibrium is not robust to an expansion of the set of feasible mechanisms.

3 No Externalities

The no externalities assumption in (Peters 2001b) requires two things: that the agent's ranking of the actions of each principal conditional on the agent's effort is independent of the actions of the other principals, and that principals' payoffs are affected only by their own actions and the agents efforts. Any common agency problem in which the principal transfers (or receives) money from the agent while the agent takes some effort that the principal observes satisfies the no-externalities assumption. This makes the theorem quite useful since all but one of the most widely cited papers on common agency have this property.

When no externalities holds, then equilibrium allocations supported by competition in mechanisms in a common agency problem can always be supported as equilibrium allocations when principals simply compete in incentive contracts. The argument works as follows: for any equilibrium in indirect mechanisms, the agent is offered a menu of incentive contracts by the principal. Whichever incentive contract the agent chooses from each menu, imagine that the principals simply offer these incentive contracts to the agent directly without giving him any choice. The effort the agent chose in the initial equilibrium must still be optimal for him in this new situation, and so is an appropriate continuation equilibrium response for the agent. Replacing indirect mechanisms with incentive contracts in this way clearly preserves the payoffs of the agent and all the principals.

This argument by itself does not prove the theorem. The real problem is to show that a continuation equilibrium can be constructed for each possible deviation so that no principal has any incentive to deviate from this configuration by offering some alternative incentive scheme. Now suppose that some principal deviates by offering an alternative incentive scheme. This deviation would have been feasible in the initial equilibrium, so suppose that the agent responds by choosing the same effort in response to this deviation as he would have used in response to the same deviation in the original equilibrium. It is here that the no externalities assumptions first comes into play. In the original game, it is possible that this deviation would induce the agent to change his selection from the other principals' menus. It could be exactly this that makes the deviation unprofitable. Yet by no externalities, the principal cares only about his own action and the agent's effort, so provided the agent uses the same effort as in the original continuation equilibrium, the principal's payoff must be the same as it was when he made the same deviation in the original equilibrium. In other words, it must be unprofitable.

It remains only to argue that the effort level the agent chose in the original equilibrium is, in fact, a best response the principal's deviation in incentive contracts. For a second time the no externalities assumption plays a role. To see how, it helps to have a bit of notation. In the original equilibrium all the principals offer menus of incentive contracts, the agent selects one from each of the principals. This defines a series of incentive contracts $\{\gamma_1^*, \dots \gamma_n^*\}$. When one of the principals, say principal j deviates to an incentive contract γ in the original equilibrium, the agent responds by selecting effort e and choosing a new set of incentive contracts, not necessarily γ_{-j}^* from the menus offered by the non-deviators. We want to show that it is optimal for the agent to choose effort e when he is offered the simple incentive schemes $\left\{\gamma, \gamma_{-j}^*\right\}$ by the principals. If it is, then this describes the continuation equilibrium that will make the deviation unprofitable.

Suppose the agent can do strictly better that $\left\{\gamma(e), \gamma_{-j}^*(e)\right\}$ by choosing some level of effort other than e. Whatever effort the agent uses to improve upon this, the outcome he achieves would certainly have been feasible in the original continuation equilibrium in which the non-deviating agents offered menus. So it can be no better than some outcome $\{\gamma(e), a_{-j}\}$ where the actions a_{-j} are attainable with effort e by appropriately selecting incentive contracts from each of the non-deviating principals' menus. Of course, the actions a_{-j}need not coincide with $\gamma_{-j}^*(e)$. Here is where the no externalities assumption comes into play. For each non-deviating principal, the action $\gamma_k^*(e)$ must be the best available action for the agent who takes effort e, and since this best action is independent of what the other principals do, the outcome $\{\gamma(e), a_{-j}\}$ can be no better for the agent than $\left\{\gamma(e), \gamma_{-j}^*(e)\right\}$. This contradicts the assumption that the agent can improve upon this outcome.

This argument assumes that agents use pure actions throughout and have no private information. Nonetheless the argument can be generalized to show that the allocations and payoffs associated with any pure strategy equilibrium in mechanisms can be supported as equilibria in which principals compete in simple incentive schemes. This is very useful since it is very common to restrict attention to pure strategy equilibria. Once you do, all equilibrium allocations can be characterized by assuming simple competition in incentive schemes.

So what is the appropriate generalization of the no-externalities condition in the multiple agent case? Part of the extension would say that the payoff to each principal depends only on his own action and the actions of all the agents. If this doesn't hold, then it will be possible to support some equilibrium allocations with menus by having agents play a different continuation equilibrium (resulting in changes in other principals incentive schemes) when there is a deviation than when there isn't. The most obvious extension for the agent is that conditional on the effort level used by each of the agents, each agent ranks the actions of each principal in a way that is independent of the actions of the other principals.

This definition allows the possibility that the way an agent ranks the actions of one principal depend on the *efforts* of other agents. These efforts will implicitly depend on the actions that other principals plan to take. So this condition will evidently not be strong enough to support the theorem. A counter example to the no-externalities theorem under these conditions is given in (Epstein and Peters 1999). So we wish to go a step beyond this here and extend the no-externalities condition to make the agents ranking of each principals actions independent of both the actions of the other principals and the efforts of other agents. This assumption may at first glance appear quite strong, but it is consistent with, for example, Prat and Rustichini (Prat and Rustichini 2000) who assume that principals simply make monetary transfers to agents, while agents suffer some additively separable cost of exerting effort.

Formally

Definition 2 *The no-externalities assumption holds if*

(i) for each principal j, for each array of actions $(y_1, \ldots, y_{j-1}, y_j, y_{j+1}, \ldots y_n)$ and for each array of effort levels $e \in E^m$ and types $\omega \in \Omega^n$ for the agents;

$$v(y_1, \ldots y_j, \ldots y_n, e, \omega) = \tilde{v}(y_j, e, \omega);$$

and

(ii) similarly for each agent i, each principal j, each effort level $e_i \in E$, and for any subset $B \subset Y$ there is a $y \in B$ such that,

$$U_i(y, y_{-j}, e_i, e_{-i}, \omega') \geq U_i(y', y_{-j}, e_i, e_{-i}, \omega')$$

for all $y' \in B$; $\omega' \in \Omega^m$; $y_{-j} \in Y^{n-1}$ and $e_{-i} \in E^{m-1}$.

The most effective way to think about this assumption is to imagine the framework of (Prat and Rustichini 2000) in which the principal makes a transfer to each agent (so the agent's payoff has to depend on the actions of all the principals). For any set of possible transfers, the agent always prefers the largest one no matter what transfers are offered by the other principals no matter what efforts the agents make.

The objective in all this is to try to show conditions under which prior communication with agents can be ignored. The principal wants to communicate with the agent to learn market information, for example, to learn when one of the other principals has deviated from the equilibrium path. When agents' rankings of the various options offered by principals is independent of this information, there will be no way for principals to accomplish this in an incentive compatible way. This intuition is simply inadequate in the multiple agent setting. The following example illustrates this.

The payoffs for the principals and agents are given in the following table, which is interpreted exactly as the table above. If principal 1 uses action A, principal 2 uses action B, agent 1 uses effort 1 while agent 2 uses effort 2 the payoffs to principal 1 and principal 2 is -1 the payoff to agent 1 is 0, while the payoff to principal 2 is -1. It is completely straightforward, but also completely tedious to check that the no-externalities condition holds. For example, the entry at $A, B, 1, 1$ must have the same first component as $A, A, 1, 1$ in order that principal 1's payoff conditional on the efforts of the agents is independent of principal 2's action.

		A			B	
		1	2		1	2
A	1	$(1,1,1,1)^*$	$(-1,-1,1,-1)$	1	$(1,2,0,0)^*$	$(-1,-1,0,-1)$
	2	$(-1,-1,-1,1)$	$(1,1,-1,-1)$	2	$(-1,-1,-1,0)$	$(1,0,-1,-1)$
		1	2		1	2
B	1	$(2,1,0,0)^*$	$(-1,-1,0,-1)$	1	$(2,2,-2,-2)$	$(-1,-1,-2,-1)$
	2	$(-1,-1,-1,0)$	$(0,1,-1,-1)$	2	$(-1,-1,-1,-2)$	$(0,0,-1,-1)^*$

Principals are not able to write contracts contingent on the agents efforts in this example, and agents have no private information. So the set of direct mechanisms is simply the set of take it or leave it offers in which each principal proposes one of the two actions to the agent. There are two pure Nash equilibrium for this game occurring when the principals use different actions while the agents both take effort 1.

To see this, observe that the form of the no-externalities assumption being employed here makes the continuation equilibrium for the agents trivial. For each pair of actions chosen by the principals each agent has a unique optimal effort that is independent of what the other agent chooses to do. The continuation equilibrium is simply the outcome where each of the agents chooses this optimal action. So, for example, when both principals offer the action B, then the unique continuation equilibrium occurs when both agents take effort 2. The continuation equilibrium outcomes are starred (*) in the table to make it a bit easier to read.

Notice, however, that the optimal effort level for the agents does depend on the simple actions taken by the principals. The agents want to take effort 1 unless both principals use action B in which case they both switch and want effort 2. This effort level provides the principals with much lower payoffs than effort 1 does.

Using these unique continuation equilibria, the induced normal form game between the principals has payoffs as given in the following table

	A	B
A	$1, 1, 1, 1$	$1, 2, 0, 0$
B	$2, 1, 0, 0$	$0, 0, -1, -1$

from this table, it is clear that the pure equilibrium in direct mechanisms occur when the principals use different actions. In particular the pair of actions AA, which is particularly good for the agents, cannot be supported.

This latter outcome where both principals use action AA, can be supported when principals are allowed to use more complex contracts. In particular, suppose that both principals start by allowing agents to send either the message A or B. If both agents send the message A, the principal uses action A, otherwise, the principal uses action B. In this case, both the agents send both the principals the message A, then choose effort 1, resulting in payoffs $(1, 1, 1, 1)$.

If either principal deviates from this, then the agents use the following strategies - if the deviating contract provides one or more pairs of messages that the agents can send that will induce action A, then the agents coordinate on one of these pairs. In this case they both send the non-deviator the message A as well and take effort 1. In this case, of course, the outcome does not change. If there are no messages that can be used to induce the deviator to take action A, then both agents send the message B to the non-deviator and take action 2. These strategies ensure that the deviator's payoff will either fall (in the case where the action A is not supported by the deviator) or stay the same (when A is supported).

It remains to check the strategies of the agents. It is straightforward that no agent wants to unilaterally send messages that induce either or both of the principals to change their actions from A to B. The payoff $1 + \varepsilon$ can only be attained when both principals take action A. Otherwise the agents payoffs will be lower no matter what happens in the continuation.

In the case where the deviator offers a contract that makes it impossible for the agents to induce action A, the agents could send messages that induce the non-deviating principal to stick with that action. Neither of them does so, however, because they believe the other agent is bound to send a message that will induce B anyway (since the non-deviator only requires a single report B to switch). Given this belief, neither of them can do better than to send the message B to the non-deviator and choose effort 2.

The many examples provided in the literature to illustrate why mechanisms will generally lead to new equilibrium outcomes share the property that the contract ensures that deviators cannot move unilaterally. When the try to switch actions, the non-deviator responds with a change in actions that punishes. In the common agency case this is accomplished using externalities in the payoff functions. The deviator's change in actions causes the agents preference ranking over actions of the non-deviator to change in a way that hurts the deviator. This is not happening in this example because the agent's ranking of the non-deviator's actions do not depend in any way on the actions of the deviator. Furthermore, the deviator doesn't care per se what action the non-deviator uses - that is ruled out by the no-externalities assumption.

The punishment is accomplished here by a change in the joint efforts of the agents. The agents change their efforts because they expect the other agent to send a signal that changes the non-deviator's action in a way that makes the new effort level optimal. This change in effort is what hurts the deviator.[5]

4 Conclusion

The literature on competing mechanisms with multiple agents makes ad hoc assumptions about the set of mechanisms that are feasible for the principals

[5]To give a slightly less abstract story about what this example is doing, imagine that agents can take either high or low effort which generates outcomes for two different principals. If either of the principals uses a high powered incentive scheme (uses action A) then both agents want to exert high effort. Since it only takes one high powered incentive scheme to induce the high effort, each principal would like to free ride on the incentive scheme of the other, which explain the pair of asymmetric equilibria. The signalling game between the principals and the agents has the property that when one of the principals deviates and tries to adopt the low powered incentive scheme, the agents send messages to the other principal which induce him to do so as well.

- for example they may be allowed to use *direct mechanisms* in which agents report information about their preferences.[6] Though there is some support for this approach in the case of common agency, it appears to be unjustified with multiple agents for two reasons. First, equilibria with direct mechanisms in which type is improperly defined are not robust to the possibility that principals might allow agents to send more complex signals. Second, there are equilibrium allocations that are supportable when principals can use complex mechanisms which cannot be supported when principals are restricted to naive direct mechanisms.

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[6]An example would be (McAfee 1993).