

# Counter Marginalization of Information Rents: Implementing Negatively Correlated Compensation Schemes for Colluding Parties\*

Gorkem Celik<sup>†</sup>

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## Abstract

A principal contracts with a productive agent whose production cost is private information and with an insurer who can insure the principal against variations in the payment to the agent. The insurer and the agent can collude in their responses to the principal's contract. Non-cooperative play of the principal's contract constitutes the outside option for the colluding parties. In this setup, we characterize the implementable outcomes for the principal. We then identify the optimal implementable outcome under the assumption that the principal faces a budget constraint. The optimal outcome provides the principal with *partial* insurance: For higher realizations of the production cost, the budget may not be exhausted even though the principal is not directly concerned with the unspent portion of the monetary funds.

**Key Words:** Collusion, mechanism design.

**JEL Classification:** D82, C72.

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<sup>†</sup>Department of Economics, the University of British Columbia, #997-1873 East Mall, Vancouver, BC, V6T 1Z1 Canada. e-mail: celik@interchange.ubc.ca

# 1 Introduction

In hidden information settings, contracting with an informed player leaves the uninformed designer of the contract with an uncertain cash flow. Employing a third party, who is uninformed but who has access to the capital markets, can provide insurance against this uncertainty. As an example of this situation, consider a local government's procurement of a public service, such as, garbage collection, the construction and maintenance of roads, or the creation of parks and recreational facilities, from a private contractor. The cost of providing such a service may be the private information of the contractor. The literature on adverse selection suggests that the government should respond to the asymmetric information by designing an incentive contract, which stipulates varying service and monetary compensation levels for the contractor across different cost realizations. The variation in the monetary dimension of this contract is problematic for the government if the marginal cost of raising public funds is increasing or if the government's budget allocates no more than a fixed amount for the public service in question. In this case, the government can benefit from involving an outside financier.<sup>1</sup>

In such a scheme, the payoff to the party providing the insurance depends on the choices of the party providing the actual service. This being the case, the insurer would like to collude with the productive agent to affect his choices. In other words, the insurer would like to create an incentive scheme for the productive agent apart from the designer's initial contract. In this paper, we investigate the incentives resulting from this collusion potential. We discuss how the designer should design the initial contract to account for these collusive incentives.<sup>2</sup>

Our model builds on an adverse selection framework, where a *principal* designs a contract for two players: a productive *agent* with private information on the production cost and an uninformed *insurer* whose task is to insure

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<sup>1</sup>Similarly, a patient visiting her physician is not perfectly informed about the nature or monetary cost of the treatment she requires. The patient, or her employer acting on her behalf, can obtain insurance against variations in healthcare expenses by contracting with a Health Maintenance Organization (HMO).

<sup>2</sup>The term "collusion" here does not necessarily point to an illegal activity. In the context of our public service procurement example, the financier may be a shareholder or a bondholder of the contractor firm and, therefore, has some legal control over the choices of management (Lewis and Sappington, 1995 and Martimort, 2006). Similarly, HMOs use legal financial incentives to induce primary care physicians to cut the number of referrals to specialists and medical screening tests (Gaynor, Rebitzer, and Taylor, 2004 and Grumbach et al., 1998).

the principal for the variation in cash-flow.<sup>3</sup> This contract determines the compensation level for each player, a transfer for the agent and a wage for the insurer, as a function of the production level. In the absence of collusion, the relationship between the compensation and the production levels is governed by the incentive compatibility and the individual rationality constraints for every player. These constraints lay out the outcomes available to the principal in a *collusion free* setup. When the players have the capacity to collude in their responses to the contract, the principal's options in the determination of the compensation levels are restricted further. This paper explores the extent to which the principal can link the insurer's wage to the performance of the productive agent. This leads to the characterization of outcomes that are feasible under collusion. This characterization is used to determine the optimal contract for a principal whose monetary transfers are constrained by budget considerations.

In line with the earlier literature, collusion is modeled here as a side contract between the insurer and the agent. If this collusion were fully efficient, then the resulting outcome would always be on the Pareto frontier of the colluding parties and these players would act as a single composite player from the principal's perspective. In such a case, finding the incentive constraints for this composite player would be sufficient for characterizing the feasible outcomes for the principal. However, since the productive agent's private information is unknown to the insurer, collusion takes place under asymmetric information. Thus, generally, collusion falls short of achieving full efficiency. This presents the opportunity for the implementation of outcomes which are unattainable under fully efficient collusion. The inefficiency in the collusion process may exhibit itself in two different forms, which will be introduced in the following two paragraphs.

Regardless of whether there is an insurer present, the principal must guarantee that the agent is receiving a transfer contingent on the production level in order to induce production. This transfer not only covers the production cost of the agent but also leaves him an information rent. As is documented by the adverse selection literature, for the agent not to overstate his production costs, his information rent should be decreasing in the cost level.<sup>4</sup> When there is a collusive insurer present, there are different ways for the principal to deliver this transfer to the agent. For instance, the principal may choose not to provide any direct incentive to the agent but to delegate this task to

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<sup>3</sup>We use masculine pronouns for the principal as well as the agent, and feminine pronouns for the insurer.

<sup>4</sup>See Baron and Myerson (1982) among others.

the insurer. Under this strictly hierarchical structure, the principal does not deliver the information rent to the agent himself, but motivates the insurer to do so. For the insurer to be willing to leave a larger information rent to the agent whenever the agent has a low production cost, the insurer's own payoff must also be decreasing in the production cost. From the principal's perspective, this corresponds to leaving an additional information rent to the insurer on top of the agent's information rent. Both components of the information rent are decreasing in the production cost. This phenomenon is referred to as the *double marginalization* of information rents.<sup>5</sup> Under double marginalization, the principal cannot use the insurer's wage to offset the variations in the agent's transfer.

The fact that hierarchies are prone to double marginalization has been established in the literature. The current paper complements this finding with the identification of a different phenomenon in collusive setups. To see the emergence of this contrasting effect, suppose that the principal foregoes the hierarchical structure above and decides to provide direct incentives to the agent. The insurer still has the opportunity to offer a collusive side contract to the agent. However, this time the agent has the option of refusing the collusive offer and responding to the principal's contract non-cooperatively. This outside option provides the agent with a reservation utility at the collusion stage. Moreover, since different agent types respond to the principal's contract differently, this reservation utility can depend on the realized production cost. If the reservation utility is decreasing in the production cost, some types of agents may find it profitable to understate their cost to increase restitution of the forgone outside option, rather than overstating the cost to increase the compensation for production. A consequence of this incentive reversal of the agent is that the main concern for the insurer may turn out to be the deterrence of the understatement of the production cost rather than its overstatement. This would be consistent with leaving a wage to the insurer that is increasing in the production cost as opposed to the decreasing wage under double marginalization. This is the phenomenon we name the *counter marginalization* of information rents.

Outlining the linkage between the insurer's wage and the production levels, and, therefore, the characterization of the feasible outcomes, requires identifying the extent to which the double marginalization and counter marginalization can be employed in the principal's contract design. This is demonstrated in Proposition 1. When the principal faces a budget constraint, he would

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<sup>5</sup>See McAfee and McMillan (1995), and Melumad, Mookherjee, and Reichelstein (1995) for double marginalization in the delegation setup.

like the insurer's wage to be negatively correlated with the agent's transfer.<sup>6</sup> Therefore, the principal's optimal outcome exhibits counter marginalization of information rents. This optimal outcome, which is identified in Propositions 3 and 4, provides the principal with *partial* insurance: For higher cost realizations, the budget may not be exhausted even though the principal does not receive a direct utility from the unspent portion of the monetary funds.

There is a rapidly expanding literature on collusion between multiple agents. The methodology of modeling collusion as a side contract between asymmetrically informed agents was developed by Laffont and Martimort (1997, 2000).<sup>7</sup> More recently, Che and Kim (2006a) show that there exists a contract that is robust to any side contract between the agents and that achieves the optimal collusion free expected payoff for the principal as long as his preferences are quasilinear in money. This result does not apply to our analysis, since we do not impose quasilinearity of the principal's payoff. In Section 5, we relate our characterization result to Che and Kim's (2006a) and to other papers studying collusion in adverse selection settings.<sup>8</sup>

In our application with budget constraints, the insurer is valuable since she can be used to balance the principal's budget even when the agent is supposed to receive a larger payment than the budget. The need for a similar *budget-breaking* third party arises in environments where agents exert *uncontractible* effort. Transfers to this third party can support ex-post inefficient punishments for the agents and achieve the first best solution even when rene-

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<sup>6</sup>Relative performance evaluation schemes and rank order contracts also use negatively correlated compensation levels for multiple agents. These schemes are considered to be non-robust to collusion (Che and Yoo, 2001 and Malcomson, 1986). The current paper suggests a way to reconcile collusion proofness with relative performance measures.

<sup>7</sup>See also Caillaud and Jehiel (1998) for collusion in second price auctions under negative externalities, Severinov (2008) for collusion between the producers of substitutable products, Faure-Grimaud, Laffont, and Martimort (2003) and Celik (2007) for collusion between a supervisor and an agent, Quesada (2004) for collusion initiated by an informed party, Kofman and Lawarree (1993) for collusion when the contract is signed ex-ante (before the agents are informed), and Baliga and Sjostrom (1998) for collusion in a moral hazard setup. Inefficiency of collusion can also be sustained by assuming exogenous transaction costs for colluding parties. See Tirole (1992) for an extensive survey of the literature following this alternative approach.

<sup>8</sup>Che and Kim's (2006a) model is expansive enough to cover an arbitrary number of colluding agents, an arbitrary production technology for the agents and an arbitrary distribution of types, allowing for correlation, as long as some regularity condition is satisfied. However, the contract they identify is optimal only under the assumption of quasilinear preferences. When this assumption fails, as it does under the budget constraints, we need a characterization of all the feasible outcomes to identify the optimal outcome. The current paper derives this characterization result for a limited setup, where there is only one agent holding private information.

gotiation is possible. Baliga and Sjoström (2007) study the agents' collusion with such a third party in the relationship-specific investment and team production environments. They show that the possibility of collusion does not impose any further cost and, therefore, the first best is still available to the agents. The main difference between their moral hazard setup and the adverse selection setup here is the fact that their budget-breaker is only needed to support the off the equilibrium path punishments. In contrast, in our application, the insurer receives or makes non-zero payments depending on the state of the world. We show that the insurer is still valuable to the principal, but not as valuable as she would have been without the possibility of collusion.<sup>9</sup>

Lewis and Sappington (1995) and Martimort (2006, Section 4) use a regulator's and a local government's risk preference to motivate the assumption of principal's risk aversion. As in the current paper, they argue that a risk neutral third party would improve the principal's rent extraction. They represent the principal's risk aversion by a concave utility function in both the production level and money. In this setup, the principal is fully insured if he receives a constant ex-post payoff in all states of nature. This is possible if the principal delegates to the risk neutral third party. We diverge from this approach by accounting for the principal's need for insurance against variations in the monetary transfer stream rather than in his ex-post payoff.<sup>10</sup> We envision a situation where the agent's production generates consumption value for the principal but it is not readily convertible to money for its own finance, such as, garbage collection, public roads, and recreational facilities. The principal needs the third party because he prefers not to, or cannot, raise the necessary funds for production himself.

The organization of the rest of the paper is as follows: We present the model in Section 2. In Section 3, we characterize the outcomes that are feasible under the threat of collusion. In Section 4, we turn to an application where the total monetary compensation by the principal is restricted by a budget constraint and we identify the optimal outcome. In Section 5, we discuss the existing literature in connection with our results. We conclude in Section 6. The

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<sup>9</sup>In Baliga and Sjoström's (2007) construction, the existence of messages, which can be sent to the designer but which are not contractible at the collusion stage, is crucial for the implementation of the first best. In our model, the production level will be the only contractible variable for both the principal's contract and the collusive side contract.

<sup>10</sup>Della Vigna and Malmendier (2006) also make a distinction between the motives of minimizing the variance of payment and minimizing the variance of payoff (footnote 24). They suggest that the former motive may justify gym users' preference for monthly or yearly *memberships*, which are disadvantageous in expectation given their usage patterns. Minimizing the variance of payoff would instead favor the price per visit contracts.

Appendix contains the proofs and the characterization of the optimal output levels for a specific parametrization of the application in Section 4.

## 2 The Model

The focus of this paper is collusion in multiparty interactions. Nevertheless, we begin our analysis with a bilateral adverse selection setup. This will establish the basic structure for studying the three player setup and also serve as a useful benchmark. We then introduce the insurer as an additional player and formalize the collusion procedure.

### 2.1 The Bilateral Setup

The principal ( $P$ ) is the residual claimant of a good produced by the agent ( $A$ ). The constant unit cost of production ( $\theta$ ) is observed by  $A$ , but unknown to  $P$ . We also refer to the variable  $\theta$  as the type of  $A$ . The variable  $\theta$  is continuously distributed on the support  $[\underline{\theta}, \bar{\theta}]$ , where  $0 < \underline{\theta} < \bar{\theta}$ . This distribution is governed by the cumulative distribution function  $F(\cdot)$  with a probability density function  $f(\cdot)$ . There exists  $\epsilon > 0$  such that  $f(\theta) \geq \epsilon$  for all  $\theta$ . To impose *monotone hazard rate* conditions on the distribution, we require  $\frac{d}{d\theta} \left( \frac{F(\theta)}{f(\theta)} \right)$  and  $\frac{d}{d\theta} \left( \frac{F(\theta)-1}{f(\theta)} \right)$  to be well defined and non-negative for all  $\theta$ .<sup>11</sup>

To induce  $A$ 's production,  $P$  commits to a **transfer schedule**  $T(\cdot)$  that maps output levels to monetary transfers from  $P$ .<sup>12</sup>  $A$ 's utility is quasilinear in this monetary transfer, i.e., it can be expressed as  $T(x) - \theta x$ , where  $x$  is the output level and  $\theta$  is  $A$ 's type.  $P$ 's preferences are defined over the output and transfer levels, and the type of the agent. These preferences are represented by the payoff function  $P(x, T, \theta)$ . In most applications in the literature, it is assumed that  $P$ 's payoff is quasilinear in money. In the current specification, the quasilinear payoff is a special case such that  $P(x, T, \theta) = p(x, \theta) - T$ . For the first part of the paper, where the focus is on the characterization of the feasible outcomes,  $P$ 's preferences will not be relevant. For the application

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<sup>11</sup>A standard monotone hazard rate condition would only demand  $\frac{d}{d\theta} \left( \frac{F(\theta)-1}{f(\theta)} \right)$  to be non-negative. The additional requirement is commonplace in the literature on type dependent reservation utility.

<sup>12</sup>The only contractible variable is the output level and  $P$  is restricted to a tariff rather than a more complex mechanism making use of messages. In the bilateral case, this restriction does not inflict a cost on  $P$ .

in Section 4, where the optimal outcome is identified, we assume a particular functional form for  $P$ 's utility.

After observing the transfer schedule  $T(\cdot)$ ,  $A$  chooses the output level that maximizes his utility conditional on his type. This decision induces an **output profile**  $x(\cdot)$  and a **transfer profile**  $t(\cdot)$ , both of which are functions of the type of  $A$ .

**Definition 1** *The output - transfer profile  $\{x(\cdot), t(\cdot)\}$  is **incentive compatible** (through  $T(\cdot)$ ) if there exists a transfer schedule  $T(\cdot)$  such that*

$$\begin{aligned} x(\theta) &\in \arg \max_x \{T(x) - \theta x\}, \\ t(\theta) &= T(x(\theta)) \end{aligned}$$

for all  $\theta \in [\underline{\theta}, \bar{\theta}]$ .

Define  $u(\theta) = t(\theta) - \theta x(\theta)$  as the *information rent* for type  $\theta$ . The following first order and monotonicity conditions are necessary and sufficient for the incentive compatibility constraints:

$$u(\theta_2) - u(\theta_1) = - \int_{\theta_1}^{\theta_2} x(\theta) d\theta \text{ for all } \theta_1, \theta_2 \in [\underline{\theta}, \bar{\theta}], \quad (1)$$

$$x(\theta) \text{ is weakly decreasing.} \quad (2)$$

Condition (1) reveals that  $A$ 's information rent is decreasing (strictly if  $x(\theta) > 0$ ) in  $\theta$ . In the absence of a decreasing information rent schedule,  $A$  has an incentive to overstate  $\theta$ , the production cost, to increase his compensation from  $P$  for the production costs he incurs. The information rent schedule above precludes such a misrepresentation of  $A$ 's type by making imitation of higher types less desirable. After integrating by parts, we rewrite condition (1) as

$$t(\theta_2) - t(\theta_1) = \int_{\theta_1}^{\theta_2} \theta dx(\theta) \text{ for all } \theta_1, \theta_2 \in [\underline{\theta}, \bar{\theta}], \quad (3)$$

where the term on the right hand side is the Stieltjes integral of  $\theta$  with respect to the function  $x(\theta)$ . It is clear from this representation that the transfer profile is decreasing (strictly if  $x(\cdot)$  is not constant) in  $\theta$ .

Incentive compatibility outlines the set of output - transfer profiles available to  $P$ , conditional on  $A$ 's consent to participate in production under the proposed transfer schedule. Securing  $A$ 's participation requires leaving him non-negative utility regardless of his type. This is stipulated by the following



individual rationality constraint:

$$IR(\theta) : t(\theta) - \theta x(\theta) \geq 0 \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}].$$

Since incentive compatibility implies the monotonicity of  $t(\theta) - \theta x(\theta)$ , the individual rationality constraint for the highest type,  $IR(\bar{\theta})$ , is sufficient to satisfy the other  $IR(\theta)$  constraints.

$P$  chooses the output - transfer profile that maximizes his expected payoff among the incentive compatible and individually rational profiles. This maximization problem can be expressed as:<sup>13</sup>

$$\begin{aligned} & \max_{\{x(\cdot), t(\cdot)\}} \int_{\underline{\theta}}^{\bar{\theta}} P(x(\theta), t(\theta), \theta) f(\theta) d\theta \text{ s.t.} & (4) \\ & \{x(\cdot), t(\cdot)\} \text{ is incentive compatible,} \\ & IR(\bar{\theta}) : t(\bar{\theta}) - \bar{\theta} x(\bar{\theta}) \geq 0. \end{aligned}$$

## 2.2 Collusion with the Insurer

In this section, we introduce the insurer ( $N$ ) as a third party to the principal - agent interaction.  $N$  does not incur any production cost or enjoy any direct benefit from production. Her ex-post utility is equal to the monetary payment she receives.<sup>14</sup> Nevertheless,  $P$  may still deem  $N$  useful, since her monetary payment can be conditioned on the production of  $A$ .

In this extended setup,  $P$  is the mechanism designer with the commitment power as in the bilateral setup. A *grand contract* offered by  $P$  has two components: As before,  $T(\cdot)$  is the transfer schedule to  $A$ . The new component,  $W(\cdot)$ , is the **wage schedule** for  $N$ . Both schedules are functions of the level of output produced by  $A$ . In order to use the results from optimal control theory, we assume for the rest of the paper that functions  $T(\cdot)$  and  $W(\cdot)$  are continuous.

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<sup>13</sup>This representation of the mechanism design problem does not allow  $P$  to offer a *stochastic* mechanism that maps  $A$ 's type to probability distributions over output and transfer pairs. However, if  $P$ 's payoff is concave in output and transfer pairs on the relevant domain, as is the case in the application we consider in Section 4, any stochastic mechanism is dominated by a *deterministic* mechanism. This is an implication of the *linearity* of the agent's payoff in the output level. When the agent's payoff is non-linear, Strausz (2005) demonstrates that a stochastic mechanism does not need to satisfy the monotonicity of the expected output levels and, therefore, may be undominated by deterministic mechanisms.

<sup>14</sup>Risk neutrality of the insurer simplifies the analysis significantly. However, the potential for insurance exists as long as the principal is not risk neutral and the insurer is not infinitely risk averse in monetary funds.

After the introduction of the new player  $N$ , there is no change in  $A$ 's utility function. He chooses an output level that maximizes his type dependent utility. In order to compare bilateral contracting and the setup with the insurer, we assume that  $P$  cares only about the total payment he makes, and not about the allocation of this payment between  $A$  and  $N$ . Accordingly,  $P$ 's utility function is written as  $P(x, T + W, \theta)$ .

As a result of  $A$ 's optimization, the grand contract  $\{T(\cdot), W(\cdot)\}$  yields an output profile  $x(\cdot)$ , a transfer profile  $t(\cdot)$ , and a **wage profile**  $w(\cdot)$ , all of which are functions of  $\theta$ . We refer to  $\{x(\cdot), t(\cdot), w(\cdot)\}$  as an **outcome**.

As before,  $A$ 's participation in the grand contract is guaranteed by the  $IR(\theta)$  constraints.  $N$  does not know the type of  $A$ , but is informed about the type distribution. Therefore,  $N$ 's participation is assured by the following ex-ante individual rationality constraint:

$$IR - N : \int_{\underline{\theta}}^{\bar{\theta}} w(\theta) f(\theta) d\theta \geq 0.$$

This constraint reveals that the expected value of  $w(\theta)$  is at least 0. Therefore, existence of the insurer does not help in reducing the expected value of the total payment  $P$  must make to the other players ( $t(\theta) + w(\theta)$ ). If  $P$ 's payoff is a function of the *expected* level of the total payment but does not depend on its other distributional characteristics (which is the case with a quasilinear payoff), then the existence of an insurer is not relevant for  $P$ 's design problem. In that case, implementing zero wages ( $W(x) = 0$  for all  $x$ ), i.e., *firing* the insurer and interacting with  $A$  bilaterally, is optimal. On the other hand, if  $P$ 's utility depends on the allocation of the payments in different states of nature, then  $P$  can use the wages to the insurer to insure himself against variations in the transfers to  $A$ .

If there were no possibility of collusion between  $A$  and  $N$ , constraint  $IR - N$  would be the only relevant constraint in the determination of  $w(\theta)$ . In such a *collusion free* setup,  $P$  maximizes his expected payoff subject to  $A$ 's incentive compatibility and individual rationality constraints and to  $N$ 's  $IR - N$  constraint. As is demonstrated shortly, collusion further restricts the set of available outcomes.

In this environment, making any use of the insurer's existence requires that her compensation is affected by  $A$ 's production level. This relation between the payoff of  $N$  and the production choice of  $A$  introduces the question of collusion between these two players. Collusion between the two parties is modeled as a bilateral contractual relation, where  $N$  has full bargaining power. After  $P$ 's announcement of the grand contract  $\{T(\cdot), W(\cdot)\}$ ,  $N$  commits to a *side*

contract,  $B(\cdot)$ , which specifies the *bribe* she pays  $A$  as a function of the output level.<sup>15</sup> The timing of the resulting game is as follows.

T0:  $\theta$  is selected by nature and observed by  $A$ .

T1:  $P$  announces a grand contract  $\{T(\cdot), W(\cdot)\}$  to  $N$  and  $A$ . Each of them accepts or rejects the grand contract. If both accept, the game proceeds to the next stage. Otherwise, the game ends without any production or monetary payment.<sup>16</sup>

T2:  $N$  announces a side contract  $B(\cdot)$  to  $A$ .  $A$  accepts or rejects the side contract.<sup>17</sup>

T3:  $A$  chooses  $x$ , the level of production.  $P$  pays  $T(x)$  to  $A$ , and  $W(x)$  to  $N$ . If  $A$  accepted the side contract,  $N$  pays  $B(x)$  to  $A$ .<sup>18</sup> If  $A$  rejected the side contract,  $N$  does not make him any payment.

If  $A$  accepts  $N$ 's side contract, his output choice is affected by both the direct transfer he receives from  $P$ , and the bribe he receives from  $N$ . Accordingly, the resulting output - transfer profile is

$$\begin{aligned} x(\theta) &\in \arg \max_x \{T(x) + B(x) - \theta x\}, \\ t(\theta) &= T(x(\theta)) + B(x(\theta)) \end{aligned}$$

for all  $\theta \in [\underline{\theta}, \bar{\theta}]$ . In this setup,  $t(\theta)$  is defined as the net transfer to type  $\theta$  agent which includes the bribe he receives from  $N$ . Similarly,  $w(\theta)$  is the net wage for  $N$ ; net of the bribe she pays to the type  $\theta$  agent.

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<sup>15</sup>In this paper, the side contract is assumed to be enforceable as is the grand contract. Martimort (1999), Abdulkadiroglu and Chung (2003), and Khalil and Lawarree (2006) relax this enforceability assumption.

<sup>16</sup>Alternatively, we can assume that the transfer schedule  $T(x)$  is in effect even if  $N$  rejects the grand contract. Since  $N$ 's rejection is an off equilibrium path event, this alternative assumption does not change the results.

<sup>17</sup>Equivalently,  $N$  can be restricted to make only non-negative bribe commitments to  $A$ . In this case,  $A$  would not have an incentive to reject any such side contract.

<sup>18</sup>It is important to note our reliance on the indirect tariff functions  $T(\cdot)$ ,  $W(\cdot)$ , and  $B(\cdot)$  when modeling the mechanisms available to the parties, instead of more general contracts utilizing messages. This restriction does not inflict any cost in bilateral setups, such as the one studied in the previous section. However, in our collusion setup, where both  $P$  and  $N$  design their own contracts for a *common agent*, communication through messages may be beneficial to the designer of the contract (See Peters, 2001, and Martimort and Stole, 2002 for common agency). For instance, it is conceivable that  $N$  changes her beliefs about  $A$ 's type after the side contracting stage. These updated beliefs may affect the message she would have submitted to  $P$  if messages were allowed in a grand contract. Felli (1996) provides an example to how the principal can manipulate these updated beliefs to eliminate the entire cost of collusion potential. The current setup bypasses these considerations by restricting attention to indirect contracts.

The profile  $\{x(\cdot), t(\cdot)\}$  satisfies the conditions above only if it is incentive compatible through  $T(\cdot) + B(\cdot)$ . Conversely, if  $\{x(\cdot), t(\cdot)\}$  is incentive compatible, for any transfer schedule  $T(\cdot)$ , there exists some bribe schedule  $B(\cdot)$  such that the above conditions are satisfied. Therefore, the above conditions on  $\{x(\cdot), t(\cdot)\}$  reduce to the incentive compatibility condition we discussed in the previous section.

The side contract of  $N$  must also provide the incentive for  $A$  to collude with  $N$ .<sup>19</sup> If  $A$  rejects  $N$ 's collusive offer, he responds *noncooperatively* to the grand contract and receives a type dependent utility of  $\max_x \{T(x) - \theta x\}$ . To guarantee  $A$ 's participation, his ex-post utility from colluding with  $N$  must be greater than this reservation utility for each  $\theta$ .

These incentive compatibility and participation constraints outline the available output - transfer profiles for  $N$  at the collusion stage. By choosing a side contract, she picks the available profile that maximizes her expected surplus. Provided that  $\{T(\cdot), W(\cdot)\}$  is the grand contract, and  $N$  induces  $\{x(\cdot), t(\cdot)\}$  through her side contract, her ex-post surplus is  $T(x(\theta)) + W(x(\theta)) - t(\theta)$  as a function of  $\theta$ . Accordingly,  $N$ 's side contract selection problem is the following:

$$\begin{aligned} \max_{\hat{x}(\cdot), \hat{t}(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [T(\hat{x}(\theta)) + W(\hat{x}(\theta)) - \hat{t}(\theta)] f(\theta) d\theta \text{ s.t.} \quad (5) \\ \{\hat{x}(\theta), \hat{t}(\theta)\} \text{ is incentive compatible,} \end{aligned}$$

$$\text{Participation } (\theta) : \hat{t}(\theta) - \theta \hat{x}(\theta) \geq \max_x \{T(x) - \theta x\} \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}].$$

For  $\{x(\cdot), t(\cdot), w(\cdot)\}$  to be a feasible outcome under the threat of insurer - agent collusion,  $x(\cdot)$  and  $t(\cdot)$  must constitute a solution to the above problem and  $w(\cdot)$  must identify the ex-post utility of  $N$ , net of the bribe she pays.

**Definition 2** *The outcome  $\{x(\cdot), t(\cdot), w(\cdot)\}$  is **collusion feasible**<sup>20</sup> (through  $\{T(\cdot), W(\cdot)\}$ ) if there exists a transfer schedule  $T(\cdot)$  and a wage schedule  $W(\cdot)$  such that*

$$i) \{x(\cdot), t(\cdot)\} \text{ is a solution to (5),} \quad (6)$$

$$ii) t(\theta) + w(\theta) = T(x(\theta)) + W(x(\theta)) \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}]. \quad (7)$$

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<sup>19</sup>Any outcome that results from  $A$ 's rejection of the side contract can also be achieved by  $A$ 's acceptance of an expanded side contract that induces  $A$ 's non-cooperative behavior as an additional choice for  $A$ . Therefore, there is no loss of generality in considering only the outcomes that result from  $A$ 's acceptance of the side contract.

<sup>20</sup>It should be noted that the outcomes defined here are feasible under the possibility of collusion between  $N$  and  $A$ . These outcomes need not be renegotiation proof for  $P$  and  $A$ .

Once again,  $P$ 's mechanism design problem reduces to choosing the collusion feasible and individually rational (for both  $A$  and  $N$ ) outcome that maximizes his expected payoff:

$$\begin{aligned} & \max_{\{x(\cdot), t(\cdot), w(\cdot)\}} \int_{\underline{\theta}}^{\bar{\theta}} P(x(\theta), t(\theta) + w(\theta), \theta) f(\theta) d\theta \text{ s.t.} & (8) \\ & \{x(\cdot), t(\cdot), w(\cdot)\} \text{ is collusion feasible,} \\ IR(\bar{\theta}) & : t(\bar{\theta}) - \bar{\theta}x(\bar{\theta}) \geq 0, \\ IR - N & : \int_{\underline{\theta}}^{\bar{\theta}} w(\theta) f(\theta) d\theta \geq 0. \end{aligned}$$

As long as the output - transfer profile  $\{x(\cdot), t(\cdot)\}$  satisfies the incentive compatibility and individual rationality constraints, setting the wage profile  $w(\cdot)$  uniformly to zero satisfies the remaining constraints of the problem in (8). Therefore, it is not possible for  $P$  to be worse off with the introduction of third party insurance. Of course, a more interesting question is whether  $P$  is strictly better off in the presence of insurance.

The difficulty in dealing with the principal's maximization problem in (8) results from collusion feasibility. This condition demands that any candidate for a solution to the maximization problem in (8) must induce a solution to  $N$ 's maximization problem in (5) as well. In the following section, we show that the collusion feasibility condition can be simplified significantly.

If the output - transfer profile  $\{x(\cdot), t(\cdot)\}$  is induced by a collusion feasible outcome, it follows from the definition of collusion feasibility that  $\{x(\cdot), t(\cdot)\}$  is incentive compatible. This is sufficient to identify  $t(\cdot)$  up to a constant, given any weakly decreasing  $x(\cdot)$ . The characterization of the collusion feasible outcomes is complete with the identification of a  $w(\cdot)$  that is consistent with an incentive compatible output - transfer profile. This is the agenda for the following section.

### 3 Collusion Feasible Outcomes

Finding the collusion feasible outcomes requires analyzing the side contract selection problem in (5). This problem is different from the optimization problem of a principal contracting with a single agent as in (4) due to the outside option provided for the agent if he rejects the side contract offer. In the bilateral setup, the reservation utility of an agent is exogenously set to 0, regardless of the type of the agent. In contrast, in problem (5), the agent's continuation

payoff from rejecting the contract is  $\max_x \{T(x) - \theta x\}$ , which varies by type. For this reason, the side contract selection process is a design problem with a type dependent reservation utility. Unlike in the bilateral setup, determination of the relevant participation constraints of (5) is not immediate. Depending on how the reservation utility responds to  $\theta$ , the participation constraint can be slack for the highest type  $\bar{\theta}$  and/or binding for types lower than  $\bar{\theta}$ .

The effect of the type dependent reservation utility at the collusion stage can also be observed by examining the incentives that govern  $A$ 's behavior. When the reservation utility is uniformly zero, as we have seen in the bilateral setup,  $A$  has an incentive to overstate his type in order to increase the compensation he receives for the production costs. However, type dependent reservation utility may be a source of an additional incentive that *countervails* the original one: If the reservation utility is declining in type,  $A$  may prefer to understate his type in order to increase the restitution for the forgone reservation utility. The optimal mechanism depends on which of these incentives is dominant for each type. Mechanism design problems of this particular nature were introduced by Lewis and Sappington (1989), and studied in detail by Maggi and Rodriguez-Clare (1995), and Jullien (2000). Following these studies, we transform problem (5) into an optimal control program, where  $x(\theta)$  is the control variable and  $t(\theta) - \theta x(\theta)$  is the state variable.<sup>21</sup> The solution to this program is identified by the result below.

**Proposition 1**  $\{x(\cdot), t(\cdot), w(\cdot)\}$  is collusion feasible if and only if  $\{x(\cdot), t(\cdot)\}$  is incentive compatible and there exists a function  $\gamma(\cdot)$  defined on  $[\underline{\theta}, \bar{\theta}]$  such that  $w(\cdot)$  satisfies the first order condition

$$w(\theta_2) - w(\theta_1) = \int_{\theta_1}^{\theta_2} \left( \frac{F(\theta) - \gamma(\theta)}{f(\theta)} \right) dx(\theta) \text{ for all } \theta_1, \theta_2 \in [\underline{\theta}, \bar{\theta}], \quad (9)$$

and

- a)  $\gamma(\cdot)$  is weakly increasing,
- b)  $\gamma(\cdot)$  assumes values in  $[0, 1]$ ,
- c)  $\theta_2 + \frac{F(\theta_2) - \gamma(\theta_2)}{f(\theta_2)} \geq \theta_1 + \frac{F(\theta_1) - \gamma(\theta_1)}{f(\theta_1)}$  if  $x(\theta_2) < x(\theta_1)$ .

The proofs of this proposition, and all other proofs, are relegated to the Appendix.

Unlike the first order condition for incentive compatibility (3), the first order condition for collusion feasibility (9) does not reveal the rate of change

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<sup>21</sup> Another problem where these techniques have proven to be useful is mechanism design subject to renegotiation. See the working paper version of Jullien (2000) for an application regarding renegotiation proofness.

in  $w(\cdot)$  as a function of  $x(\cdot)$  alone. The function  $\gamma(\cdot)$  is also relevant for the identification of the wage profile. For instance, whenever  $x(\cdot)$  is differentiable at  $\theta$ ,  $w(\cdot)$  is also differentiable and its derivative is equal to  $\frac{F(\theta)-\gamma(\theta)}{f(\theta)}x'(\theta)$ . Note that the wage profile  $w(\cdot)$  does not have to be monotonic as does the transfer profile  $t(\cdot)$ . The rate of change in the wage profile at  $\theta$  has the same sign as  $\gamma(\theta) - F(\theta)$ .

For any weakly decreasing output profile  $x(\cdot)$  and any  $\gamma(\cdot)$  that satisfies the conditions in Proposition 1, there exists a transfer profile  $t(\cdot)$  and a wage profile  $w(\cdot)$  that are collusion feasible together with  $x(\cdot)$ . This indicates that  $\gamma(\cdot)$  can be considered as a choice variable for  $P$ . Moreover, the selection of  $x(\cdot)$  and  $\gamma(\cdot)$  describes profiles  $t(\cdot)$  and  $w(\cdot)$  up to a constant.

Jullien (2000) interprets  $\gamma(\theta)$  as the *shadow value* associated with the uniform marginal reduction of the reservation utility for all types between  $\underline{\theta}$  and  $\theta$ . In essence, through the selection of function  $\gamma(\cdot)$ , the principal determines the weight of each participation constraint in the insurer's side contract selection problem (5). To illustrate this point, suppose  $\gamma(\theta)$  equals  $F(\theta)$  for all  $\theta$ . In this case, the relevance of the participation constraints in problem (5) is determined solely by the probability distribution. This situation arises if  $N$  is indifferent to the announced type of  $A$ . In other words, by setting  $\gamma(\theta)$  equal to  $F(\theta)$ ,  $P$  can implement a wage profile that is constant (possibly zero) in  $\theta$ . This can be considered to be the replication of a bilateral contract between  $P$  and  $A$ .

Proposition 1 implies that  $P$  can set  $\gamma(\theta)$  to values other than  $F(\theta)$  as well. This suggests that there are implementable outcomes other than the replications of the bilaterally implementable ones. For instance, by setting  $\gamma(\cdot)$  smaller than  $F(\cdot)$  for a certain type  $\theta$ ,  $P$  can induce a wage profile  $w(\cdot)$  which is decreasing at  $\theta$ . Following Jullien's interpretation of the shadow values,  $\gamma(\theta) < F(\theta)$  indicates that the participation constraints for types higher than  $\theta$  have more weight in  $N$ 's optimization problem than do the participation constraints for types lower than  $\theta$ . Therefore, the dominant incentive for the type  $\theta$  agent is to exaggerate the production cost to increase his compensation (as in the bilateral setup). The agent with type  $\theta$  should be left an information rent to preclude such a misrepresentation of his type. This is where the specifics of  $N$ 's wage profile play an important role. Since  $N$ 's wage is decreasing in  $\theta$ , she is willing to leave the information rent to  $A$  to prevent an overstatement of the cost. In this case, both the transfer profile for  $A$  and the wage profile for  $N$  are decreasing in  $\theta$ . From  $P$ 's perspective, this is equivalent to leaving an information rent to  $N$  in addition to the information rent that is already left to  $A$ , where both components of the information rent are decreasing in  $\theta$ .

Following the previous literature, we refer to this phenomenon as the *double marginalization* of the information rents.

On the other hand, if  $\gamma(\theta)$  is larger than  $F(\theta)$ , then the wage profile  $w(\cdot)$  is increasing at  $\theta$ . When this happens, the participation constraints for types lower than  $\theta$  have more weight in  $N$ 's optimization problem than do the participation constraints for types higher than  $\theta$ . This indicates the existence of a type lower than  $\theta$  that has a reservation utility large enough for his participation constraint to matter. In this case, the countervailing incentive for the type  $\theta$  agent dominates the original incentive: The type  $\theta$  agent must be motivated not to understate his type. Such an understatement of  $A$ 's type reduces the compensation for  $N$  since her wage is increasing in  $\theta$ . Therefore, she is willing to provide the motivation to prevent  $A$  from understating  $\theta$ . Notice that, when  $\gamma(\theta)$  is larger than  $F(\theta)$ , the transfer and the wage profiles are moving in the opposite direction at  $\theta$ . We coin the term *counter marginalization* of the information rents to describe this latter situation.

Double marginalization is experienced most intensely when  $\gamma(\theta) = 0$  for all  $\theta$ . Then,  $w(\cdot)$  decreases in  $\theta$  with the highest possible rate allowed by collusion feasibility. In this case, the only relevant participation constraint is that of the highest type  $\bar{\theta}$  in the side contract selection problem (5). Hence,  $P$  does not need to provide  $A$  with a type dependent reservation utility. This corresponds to a decentralized organizational form, where  $P$  delegates to  $N$  the authority to contract with  $A$ . The opposite case, where counter marginalization is profound and the only relevant participation constraint is that of the lowest type  $\underline{\theta}$ , is represented by  $\gamma(\theta) = 1$  for all  $\theta$ .

It should be noted that double marginalization and counter marginalization both relate to the direction and the rate of change of the wage profile but not to its expected value. When constraint  $IR - N$  is binding in problem (8), any payment that is made to  $N$  is passed on to  $A$  *in expectation*. If  $P$ 's payoff is assumed to be quasilinear in money, i.e.  $P(x, T + W, \theta) = p(x, \theta) - T - W$ , then the expected value of the wage profile is the only relevant piece of information for problem (8). In this case, since  $P$  is not interested in the rate of change of the wage profile, neither double marginalization nor counter marginalization bears any significance for his payoff maximization. However, when  $P$ 's preferences cannot be represented by a quasilinear payoff function, specifically when  $P$  is not indifferent to the variations in the wage profile, then the principal might find it optimal to choose a function  $\gamma(\cdot)$  that gives rise to double marginalization or counter marginalization. For instance, if  $P$  desires to have a negative correlation between the compensation levels he provides for the other players, then counter marginalization is likely to arise in his optimal contract. In the following section, we study an environment where the need



for such a negative correlation emerges due to budget considerations.

## 4 Optimal Outcome under a Budget

In this section, we use Proposition 1 of the previous section to search for the optimal outcome for an environment characterized by an upper bound on the *total* payment that  $P$  can make to  $A$  and  $N$ . Whatever the type of the agent, the sum of the transfer and the wage levels cannot exceed  $P$ 's budget, which is denoted by  $M$ . Once this budget constraint is satisfied,  $P$ 's payoff is uniquely determined by the output. We represent these preferences with the following payoff function:

$$P(x, T + W, \theta) = \begin{cases} p(x) & \text{if } T + W \leq M \\ -\infty & \text{otherwise} \end{cases} .$$

The function  $p(\cdot)$  is continuously differentiable, strictly increasing, concave and satisfies properties  $\lim_{x \rightarrow 0} p'(x) = \infty$  and  $\lim_{x \rightarrow \infty} p'(x) = 0$ . The number  $-\infty$  can be considered a real number small enough to preclude  $P$  from implementing an outcome where the ex-post total payment exceeds  $M$ .<sup>22</sup>

The payoff function above stands in contrast to more common specifications, where money enters directly into the principal's objective function as a consumption good with positive marginal utility everywhere.<sup>23</sup> However, in the public good procurement interpretation of our model, this payoff function may arise naturally: The principal can be thought as (a department of) a local government whose budget for the public project in question is fixed (by the local legislature) and who is required to relinquish any unspent portion of this budget. This particular payoff function is useful for our analysis of counter marginalization since it gives rise to an analytically tractable setup, where

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<sup>22</sup>The principal never chooses to implement an outcome where the total payment level exceeds  $M$ . For lower total payment levels, the principal's payoff does not respond to the payment level and it is strictly concave in the output level. As discussed in footnote 13, allowing for stochastic mechanisms would not change the optimal outcome.

<sup>23</sup>With a considerable algebraic burden, we could impose an inherent disutility from spending money for the principal in addition to the budget constraints. In that case, the principal's preferences would be represented by the payoff function

$$P(x, T + W, \theta) = \begin{cases} p(x) - T - W & \text{if } T + W \leq M \\ -\infty & \text{otherwise} \end{cases} .$$

If no budget constraint is binding, then the specification above reduces to the quasilinear specification. If there is a binding budget constraint, then all the results that follow qualitatively remain.

the principal has an interest in sustaining a negative correlation between the compensation levels of the parties involved.

Recall that incentive compatibility implies a weakly decreasing transfer profile for  $A$ . If  $P$  did not have access to insurance, the budget constraint for the lowest type ( $t(\underline{\theta}) \leq M$ ) would be sufficient to satisfy all the other budget constraints. In contrast, when the insurer  $N$  is present, not only can the transfer to  $A$  vary with  $A$ 's type, but so can the wage to  $N$ . In this case,  $P$  can construct  $N$ 's wage profile to distribute the burden of the budget constraint over types other than the lowest type. To elucidate this point, we first consider the collusion free setup, where  $N$  does not have the capacity to collude with  $A$ . Later, we extend the analysis to account for collusion.

#### 4.1 Optimal Outcome in the Collusion Free Setup

In the absence of collusion, the only constraint restricting  $N$ 's wage is the ex-ante individual rationality constraint,  $IR - N$ , which stipulates that her expected wage must be non-negative.  $P$  can use the variation in  $N$ 's wage to distribute the burden of the budget constraint over the states of nature where  $A$ 's type is not  $\underline{\theta}$ . As a result,  $P$  can design her mechanism for  $A$  as though the budget constraint is imposed ex-ante:

$$\begin{aligned} & \max_{\{x(\cdot), t(\cdot)\}} \int_{\underline{\theta}}^{\bar{\theta}} p(x(\theta)) f(\theta) d\theta \text{ s.t.} & (10) \\ & \{x(\cdot), t(\cdot)\} \text{ is incentive compatible,} \\ IR(\bar{\theta}) & : t(\bar{\theta}) - \bar{\theta}x(\bar{\theta}) \geq 0, \\ BB & : \int_{\underline{\theta}}^{\bar{\theta}} t(\theta) f(\theta) d\theta \leq M. \end{aligned}$$

The next step involves solving for the transfer profile  $t(\cdot)$  in terms of the output profile  $x(\cdot)$ , and reformulating the problem such that the only choice variable for  $P$  is the output profile.

**Proposition 2** *If  $x^*(\cdot)$  is a solution to problem (10) together with  $t^*(\cdot)$ , then*

$$x^*(\cdot) \in \arg \max_{x(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} p(x(\theta)) f(\theta) d\theta \text{ s.t.} \quad (11)$$

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta x(\theta) f(\theta) d\theta + \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) F(\theta) d\theta \leq M, \quad (12)$$

and  $x(\cdot)$  is weakly decreasing.

When we ignore the monotonicity constraint, the problem above reduces to an optimization problem with a single constraint. Piecewise maximization of the Lagrangian function for this problem yields

$$p'(x^*(\theta)) = \lambda \left( \theta + \frac{F(\theta)}{f(\theta)} \right) \quad (13)$$

for all  $\theta$ , where  $\lambda$  is the Lagrange multiplier. Given the monotone hazard rate condition and the concavity of the function  $p(\cdot)$ , the output profile characterized by the above equation satisfies the monotonicity constraints as well. The magnitude of  $\lambda$  depends on  $P$ 's utility function  $p(\cdot)$ , the distribution function  $F(\cdot)$ , and the size of the principal's budget  $M$ .

Once the optimal output profile is identified, constraint  $IR(\bar{\theta})$  and the first order condition (3) reveal the optimal transfer profile  $t^*(\cdot)$ . Since the budget is exhausted in every state of nature, the corresponding wage levels are  $w^*(\theta) = M - t^*(\theta)$  for all  $\theta$ .

## 4.2 Optimal Outcome under Collusion

In this section, we study the budget constrained environment under the possibility of collusion between  $A$  and  $N$ . In this setup,  $N$ 's wage profile is constrained not only by her ex-ante individual rationality, but also by the collusion feasibility requirement. Unlike in the collusion free setup,  $P$  cannot freely choose the rate of change of the wage profile to offset the variation in the transfer profile. However, the first order condition (9) for collusion feasibility provides some latitude for  $P$  in the determination of the rate of change in the wage profile through the selection of function  $\gamma(\cdot)$ . The design problem for  $P$  in this setup is given as

$$\begin{aligned} & \max_{\{x(\cdot), t(\cdot), w(\cdot)\}} \int_{\underline{\theta}}^{\bar{\theta}} p(x(\theta)) f(\theta) d\theta \text{ s.t.} & (14) \\ & \{x(\cdot), t(\cdot), w(\cdot)\} \text{ is collusion feasible,} \\ IR(\bar{\theta}) & : t(\bar{\theta}) - \bar{\theta}x(\bar{\theta}) \geq 0, \\ IR - N & : \int_{\underline{\theta}}^{\bar{\theta}} w(\theta) f(\theta) d\theta \geq 0, \\ BB(\theta) & : t(\theta) + w(\theta) \leq M \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}]. \end{aligned}$$

The collusion feasibility condition constrains the rates of change of the output, transfer, and wage levels; but it is silent for the fixed components of these variables, which are determined by the remaining constraints.

In what follows, we discuss the wage, output, and transfer profiles that constitute the solution to this problem. We start with a result revealing the optimal  $\gamma(\cdot)$ , and consequently identify the optimal wage profile given the optimal output and transfer profiles.

**Proposition 3** *If  $\{x^*(\cdot), t^*(\cdot), w^*(\cdot)\}$  is a solution to problem (14), then  $w^*(\cdot)$  is derived by the binding budget constraint at type  $\underline{\theta}$  and the first order condition (9) with  $\gamma^*(\theta) = \min\{F(\theta) + f(\theta)\theta, 1\}$  or identically with*

$$\gamma^*(\theta) = \begin{cases} F(\theta) + f(\theta)\theta & \text{if } \theta < \theta^* \\ 1 & \text{if } \theta \geq \theta^* \end{cases}, \quad (15)$$

where  $\theta^*$  is defined as the solution to  $F(\theta) + f(\theta)\theta = 1$  on the interval  $[\underline{\theta}, \bar{\theta}]$ , if such a solution exists and as  $\underline{\theta}$  if there is no solution.<sup>24</sup>

An example of the optimal transfer and wage profiles is depicted in Figure 1. It is not surprising that the budget constraint  $BB(\theta)$  is binding for the lowest cost  $\underline{\theta}$  in the optimal outcome. It follows from condition (3) that the transfer to  $A$  reaches its maximum at this cost level. Also note that  $P$  does not receive any direct utility from the unspent portion of his budget. Therefore,  $P$  tries to use all of his funds to induce production as permitted by the budget and the collusion feasibility constraints. The first order conditions (3) and (9) pin down the total payment by  $P$  as

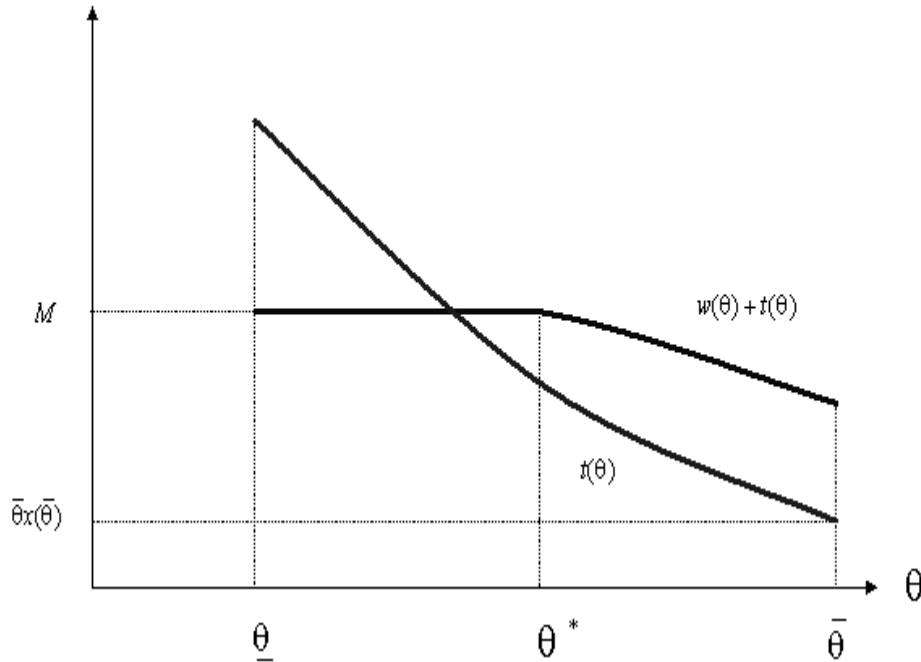
$$t^*(\theta) + w^*(\theta) = t^*(\underline{\theta}) + w^*(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \left( s + \frac{F(s) - \gamma^*(s)}{f(s)} \right) dx^*(s) \quad (16)$$

for cost level  $\theta$ . The budget is balanced at  $\underline{\theta}$  so that  $t^*(\underline{\theta}) + w^*(\underline{\theta}) = M$ . By setting  $\gamma^*(\theta)$  equal to  $F(\theta) + f(\theta)\theta$  for  $\theta < \theta^*$ ,  $P$  ensures that the integrand in (16) is 0. By doing so, he balances the budget for all values of  $\theta$ . However, for  $\theta > \theta^*$ ,  $F(\theta) + f(\theta)\theta$  is larger than 1, which is the upper bound on  $\gamma^*(\theta)$ . Therefore, for these values of  $\theta$ ,  $\gamma^*(\theta)$  is set to 1 in order to come as close as possible to balancing the budget. The properties of the optimal outcome regarding the budget constraint are summarized by the following corollary to Proposition 3.

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<sup>24</sup>To see the uniqueness of  $\theta^*$ , note that the solution to  $F(\theta) + f(\theta)\theta = 1$  is also a solution to  $\theta + \frac{F(\theta)-1}{f(\theta)} = 0$ . The monotone hazard rate condition implies that the left hand side of this second equation is strictly increasing in  $\theta$ .

Figure 1: The optimal transfer – wage profile under budget constraints



**Corollary 1** *In the optimal outcome, the budget constraint  $BB(\theta)$  is binding for  $\theta \leq \theta^*$ . In contrast, the budget constraint  $BB(\theta)$  is slack for  $\theta > \theta^*$  as long as  $x^*(\theta) < x^*(\theta^*)$ .*

Considering that  $P$  has no value for the unspent funds in his budget, it is rather unexpected that the budget is not always exhausted under the optimal contract.  $P$  must implement variations in the transfer to  $A$  in order to separate different agent types.<sup>25</sup> Nevertheless, as we have seen in the collusion free environment,  $P$  could have used  $N$ 's wage to offset these variations and to balance the budget if there were no possibility of collusion. However, the threat of collusion between  $N$  and  $A$  limits the opportunities of insurance for  $P$ . In other words, the optimal contract's failure to balance the budget is an indication of the designer's concern over collusion.

Even under the possibility of collusion, the existence of the insurer is valuable for  $P$  because collusion is not Pareto efficient under asymmetric information. The coalitional inefficiency is most apparent for cost levels lower than  $\theta^*$ , where the optimal outcome induces a flat total payment of the amount  $M$  from

<sup>25</sup>See Levaggi (1999) for another example (with a binary type space for the agent) where a principal fails to balance the budget in the absence of a third party.

*P.* The  $N - A$  coalition could have reduced the output level (and, therefore, the total production cost) without changing his total payment to the coalition members. If  $N$  were informed of the type of  $A$ , she would have offered him a bribe to encourage him to reduce his production level. Lacking information on  $A$ 's type,  $N$  is unwilling to make such an offer because reducing the output level of type  $\theta$  makes it easier for all types higher than  $\theta$  to imitate type  $\theta$ . This would require a higher information rent from  $N$  to all of these types in order to preclude such imitations. In contrast, when the unit production cost is high ( $\theta > \theta^*$ ), the coalitional inefficiency is not large enough to sustain a flat payment to the coalition. This is the result of the relatively smaller measure of types higher than  $\theta$  which may consider imitating  $\theta$  following a decline in the output level.<sup>26</sup>

At the solution to problem (14),  $\gamma(\theta)$  is larger (strictly when  $\theta < \bar{\theta}$ ) than  $F(\theta)$ . Therefore, the optimal outcome exhibits counter marginalization of the agent's and the insurer's information rents. The optimal wage profile is weakly increasing in  $\theta$ , whereas the incentive compatible transfer profile is weakly decreasing. This is the only method for the principal to use  $N$ 's wage to offset the variations in  $A$ 's transfer.

After the identification of the optimal wage profile, we turn our attention to the output levels. To simplify the analysis, we assume  $f(\underline{\theta})\underline{\theta} < 1$  and therefore,  $\left[\underline{\theta} - \frac{\gamma^*(\underline{\theta})}{f(\underline{\theta})}\right]$  equals 0. This assumption allows us to employ the pointwise maximization technique used in the collusion free setup.

**Proposition 4** *Suppose  $f(\underline{\theta})\underline{\theta} < 1$ . If  $\{x^*(\cdot), t^*(\cdot), w^*(\cdot)\}$  is a solution to problem (14), then*

$$x^*(\cdot) \in \arg \max_{x(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} p(x(\theta)) f(\theta) d\theta \quad s.t. \quad (17)$$

$$\left[ \int_{\underline{\theta}}^{\theta^*} [\theta f(\theta) + F(\theta)] x(\theta) d\theta + \int_{\theta^*}^{\bar{\theta}} \left[ 1 + [1 - F(\theta)] \left[ 1 + \frac{d}{d\theta} \left( \frac{F(\theta)-1}{f(\theta)} \right) \right] \right] x(\theta) d\theta \right] \leq M, \quad (18)$$

and  $x(\cdot)$  is weakly decreasing.

Following the analysis in the collusion free setup, first we ignore the monotonicity constraint and determine the first order conditions of the piecewise maxi-

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<sup>26</sup>The magnitude of the distortion from coalitional Pareto efficiency declines in the type of the agent and vanishes for the *least efficient* type  $\bar{\theta}$ . This is in contrast with the “no distortion for the *most efficient* type” result under the uniform reservation utility. This distinction arises from  $A$ 's incentive reversal under the type dependent reservation utility.

mization of the Lagrangian function:

$$p'(x(\theta)) = \begin{cases} \lambda \left( \theta + \frac{F(\theta)}{f(\theta)} \right) & \text{for } \theta < \theta^* \\ \lambda \left[ \frac{1}{f(\theta)} + \frac{1-F(\theta)}{f(\theta)} \left[ 1 - \frac{d}{d\theta} \left( \frac{1-F(\theta)}{f(\theta)} \right) \right] \right] & \text{for } \theta \geq \theta^* \end{cases} \quad (19)$$

where  $\lambda$  is the Lagrange multiplier. For small values of  $\theta$ , the first order condition is that of the collusion free case. However, for large values of  $\theta$ , the virtual cost of production depends on the magnitude of the hazard rate  $\frac{1-F(\theta)}{f(\theta)}$  as well as its rate of change. The monotone hazard rate condition is not sufficient to rule out the non-monotonicity of the output profile outlined in (19). The solution to problem (17) may require the *bunching* of several types at the same output level to preserve monotonicity as in Guesnerie and Laffont (1984) .

Once the optimal output profile is identified, constraint  $IR(\bar{\theta})$  and the first order condition (3) reveal the optimal transfer profile  $t^*(\cdot)$ . Moreover, as stipulated by Proposition 3, the optimal wage profile  $w^*(\cdot)$  is determined by the binding budget constraint for type  $\underline{\theta}$  and the first order condition (9) where function  $\gamma(\cdot)$  is determined by (15).

In the Appendix, we provide an example of a particular utility function  $p(x)$  and a type distribution  $F(\theta)$ , and solve for the optimal output levels. We use this example to compare  $P$ 's expected payoff from the optimal outcome to his bilateral contracting and collusion free payoff levels. The comparison reveals that the optimal contract under collusion is an improvement over the bilateral contract even though it does not perform as well as the collusion free contract.

## 5 Literature Revisited

In this section, we study some recent developments in the collusion literature in light of the characterization result in Proposition 1. The models in the literature surveyed here differ from the current model in many respects such as the number of the colluders, the productive and informative tasks of colluders, and the timing of collusion. Therefore, our characterization result cannot be directly exported to these different environments. Nevertheless, the outcomes shown to be implementable by these papers are analogous (in the setup of the current paper) to feasible outcomes that reflect certain properties (identified in the current paper).

As discussed in the introduction, Che and Kim (2006a) consider a very general model and construct a grand contract that reproduces the collusion

free payoff for a principal who has *quasilinear preferences*. Their construction is based on the idea that the principal sells the firm to the agents. Under the proposed grand contract, the principal receives a constant ex-post payoff regardless of the state. For this reason, any manipulation of the agents' behaviors under collusion does not affect the principal's payoff. Unlike the application of the previous section, Che and Kim's (2006a) optimal grand contract does not utilize the manipulation of the outside options of the agents at the side contracting stage. Actually, their optimal outcome can also be implemented by delegating to an uninformed party and not directly interacting with the agents. With the notation of the current paper, this optimal outcome corresponds to an outcome that can be supported by  $\gamma(\theta) = 0$  for all  $\theta$ .

Mookherjee and Tsumagari (2004) study collusion between two productive agents. They examine a stronger version of collusion where the agents are allowed to collude *prior* to their participation in the mechanism. Given this timing, the *selling the firm* mechanism of Che and Kim (2006a) does not achieve the collusion free payoff since there are states of nature where the agents will collectively choose not to participate after learning each other's type. In this setup, Mookherjee and Tsumagari (2004) construct a grand contract which outperforms delegation to one of the agents. Under the characterization of the grand contract, they show that the participation constraints for all types can be replaced by a single ex-ante participation constraint that requires leaving  $A$  a predetermined *expected* utility level. Using the notation of the present paper, this corresponds to outcomes that can be supported by  $\gamma(\theta) = \lambda F(\theta)$  for all  $\theta$ , where  $\lambda \in [0, 1]$ . In this environment, Mookherjee and Tsumagari (2004) show that  $P$  can reduce the effect of double marginalization by choosing  $\lambda$  larger than 0 and, therefore, improve over delegation (which corresponds to  $\lambda = 0$ ).

Pavlov (2006) and Che and Kim (2006b) examine the existence of a grand contract reproducing the collusion free payoff under the stronger notion of collusion employed by Mookherjee and Tsumagari (2004). The environment they consider is an auction setup where the bidders collude prior to their participation in the auction.<sup>27</sup> They find conditions for which the collusion free payoff is attainable to the auctioneer. Unlike in the environment studied by Che and Kim (2006a), the optimal payoff here cannot be supported by

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<sup>27</sup>Pavlov (2006) considers a single bidding ring consisting of all the bidders who are assumed to be ex-ante symmetric. These bidders can collude on their bids and make side transfers to each other. Che and Kim (2006b) study possibly asymmetric bidders and possibly multiple bidding rings, each equipped with the capacity to reallocate the auctioned object within the ring members. Furthermore, the bidding rings can coordinate the bids and make side transfers.



delegation. Actually, the implementation of the collusion free payoff calls for the extreme form of counter marginalization, where the only relevant collusion participation constraint is that of the highest valuation type. This corresponds to an outcome supported by  $\gamma(\theta) = 1$  for all  $\theta$ .<sup>28</sup>

## 6 Conclusion

Mechanism design theory studies the outcomes available to a principal designing a contract for the other players. The possibility of collusion between these players complicates the design problem. In this paper the problem investigated was that of a principal who deals with a productive agent and an insurer with a *deep pocket*. In our setup, the principal was able to utilize the services of the insurer to sustain inefficiencies in the insurer's collusion with the agent. These inefficiencies feed on the asymmetric information between the colluding parties.

One well-studied form of coalitional inefficiency in the multi-player design setting is the double marginalization of the information rents. Double marginalization is especially observed in strictly hierarchical structures, where each tier of the hierarchy contracts with only an immediate subordinate. In the context of our problem, this corresponds to the principal's contracting with only the insurer and delegating to her the task of motivating the agent's production. Under double marginalization, the insurer's payoff is increasing in the information rent that is left to the agent.

In this paper, we focused on an alternative form of coalitional inefficiency, which we named the counter marginalization of the information rents. Sustaining this second type of inefficiency requires the principal to actively contract with all the colluding parties and to provide them with an outside option to collusion. The principal can create an incentive reversal for the agent through the manipulation of these outside options. When the principal has a budget constraint and, therefore, wishes to induce negatively correlated compensations for the colluding parties, we established that his contract should exploit counter marginalization.

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<sup>28</sup>An earlier example where the collusion participation constraints are relevant only for the most efficient type is provided by Caillaud and Jehiel (1998). They study collusion between bidders, each of which suffers a negative externality if some other bidder receives the auctioned object. They restrict their attention to second price auctions with a reserve price. The optimal reserve price for the auctioneer induces coalitional inefficiency. See Celik (2007) for an example of countervailing incentives for an agent colluding with a supervisor who is partly informed of the agent's productivity.

## 7 Appendix

### 7.1 Proof of Proposition 1

We start with a lemma analogous to Theorem 1 of Jullien (2000). We then use the lemma to prove the necessity and sufficiency parts of the proposition.

**Lemma 1** *Given  $W(\cdot)$  and  $T(\cdot)$ , an incentive compatible output - transfer profile  $\{x(\cdot), t(\cdot)\}$  satisfying Participation  $(\theta)$ , for all  $\theta$ , is a solution to  $N$ 's maximization problem (5) if and only if there exists a function  $\gamma(\cdot)$  defined on  $[\underline{\theta}, \bar{\theta}]$  such that*

- i)  $\gamma(\cdot)$  is weakly increasing,*
- ii)  $\gamma(\cdot)$  is constant on any interval where the participation  $(\theta)$  constraints are slack,*
- iii)  $\gamma(\bar{\theta}) \leq 1$  (with equality if Participation  $(\bar{\theta})$  is slack),  $\gamma(\underline{\theta}) \geq 0$  (with equality if Participation  $(\underline{\theta})$  is slack), and*

$$x(\theta) \in \arg \max_x \left\{ T(x) + W(x) - \theta x - \frac{F(\theta) - \gamma(\theta)}{f(\theta)} x \right\} \quad (20)$$

for all  $\theta \in [\underline{\theta}, \bar{\theta}]$ .<sup>29</sup>

**Proof.** First we ignore the monotonicity requirement (2) of incentive compatibility and later show that it is satisfied. After defining  $u(\theta) = t(\theta) + \theta x(\theta)$ , we can write maximization problem (5) as

$$\max_{x(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [T(x(\theta)) + W(x(\theta)) - \theta x(\theta) - u(\theta)] f(\theta) d\theta \text{ s.t.} \quad (21)$$

$$u'(\theta) = -x(\theta) \text{ almost every } \theta, \quad (22)$$

$$u(\theta) \geq \max_x \{T(x) - \theta x\} \text{ for all } \theta. \quad (23)$$

This is an optimal control problem with  $x(\theta)$  as the control variable,  $u(\theta)$  as the state variable, and the participation constraints (23) as the state constraints. Seierstadt and Sydsaeter (1987) provide the necessary and sufficient conditions for this type of problem in Theorems 3 and 4 of Chapter 5. Their results imply that (20) is necessary and sufficient for maximization, where  $\gamma(\cdot)$  satisfies conditions (i), (ii) and (iii) as stated in the lemma.

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<sup>29</sup>Technically, there are output profiles that constitute solutions to (5) other than the one described by (20) for all  $\theta$ . However, any such output profile satisfies (20) *almost everywhere*, and, therefore, is *essentially* equivalent to  $x(\cdot)$  which satisfies this condition for *all*  $\theta$ .

In order to complete the proof, we must show that the solution to (20) yields a weakly decreasing output profile. If the participation constraint (23) is slack at  $\theta$ , then  $\gamma(\cdot)$  is constant at  $\theta$ . The monotone hazard rate conditions imply that  $\frac{d}{d\theta} \left( \frac{F(\theta) - \gamma}{f(\theta)} \right)$  is non-negative when  $\gamma$  is a real number on the interval  $[0, 1]$ . Since  $\theta + \frac{F(\theta) - \gamma(\theta)}{f(\theta)}$  is strictly increasing,  $x(\theta)$  solving (20) is weakly decreasing for constant  $\gamma$ . Alternatively, if (23) is binding at  $\theta$ , then

$$x(\theta) = -u'(\theta) = -\frac{d}{d\theta} \max_x \{T(x) - \theta x\} = x^*(\theta), \quad (24)$$

where  $x^*(\theta) \in \arg \max_x \{T(x) - \theta x\}$ . Therefore,  $x(\theta) = x^*(\theta)$  is weakly decreasing. ■

• **Necessity**

Suppose  $\{x(\cdot), t(\cdot), w(\cdot)\}$  is collusion feasible. Lemma 1 implies the existence of a weakly increasing function  $\gamma(\cdot)$ , taking values on  $[0, 1]$ , such that

$$\begin{aligned} & T(x(\theta_2)) + W(x(\theta_2)) - \theta_2 x(\theta_2) - \frac{F(\theta_2) - \gamma(\theta_2)}{f(\theta_2)} x(\theta_2) \\ & \geq T(x(\theta_1)) + W(x(\theta_1)) - \theta_2 x(\theta_1) - \frac{F(\theta_2) - \gamma(\theta_2)}{f(\theta_2)} x(\theta_1) \end{aligned} \quad (25)$$

for all  $\theta_1, \theta_2 \in [\underline{\theta}, \bar{\theta}]$ . Substitute (7) in the above inequality:

$$\begin{aligned} & t(\theta_2) + w(\theta_2) - \theta_2 x(\theta_2) - \frac{F(\theta_2) - \gamma(\theta_2)}{f(\theta_2)} x(\theta_2) \\ & \geq t(\theta_1) + w(\theta_1) - \theta_2 x(\theta_1) - \frac{F(\theta_2) - \gamma(\theta_2)}{f(\theta_2)} x(\theta_1) \end{aligned} \quad (26)$$

$$t(\theta_2) + w(\theta_2) - t(\theta_1) - w(\theta_1) \geq \left( \theta_2 + \frac{F(\theta_2) - \gamma(\theta_2)}{f(\theta_2)} \right) (x(\theta_2) - x(\theta_1)). \quad (27)$$

Changing the roles of  $\theta_1$  and  $\theta_2$ , and merging the inequalities yield

$$\left( \theta_1 + \frac{F(\theta_1) - \gamma(\theta_1)}{f(\theta_1)} \right) (x(\theta_2) - x(\theta_1)) \geq t(\theta_2) + w(\theta_2) - t(\theta_1) - w(\theta_1) \quad (28)$$

$$t(\theta_2) + w(\theta_2) - t(\theta_1) - w(\theta_1) \geq \left( \theta_2 + \frac{F(\theta_2) - \gamma(\theta_2)}{f(\theta_2)} \right) (x(\theta_2) - x(\theta_1)). \quad (29)$$

Since  $f(\theta)$  is bounded away from 0,  $\theta + \frac{F(\theta) - \gamma(\theta)}{f(\theta)}$  is a bounded function of  $\theta$ . Thus, the total payment  $t(\theta) + w(\theta)$  is absolutely continuous with respect to the measure generated by  $x(\theta)$ . It follows from the Radon - Nikodym theorem<sup>30</sup> that  $t(\theta) + w(\theta)$  can be written as a Stieltjes integral with respect to the function  $x(\theta)$ :

$$t(\theta_2) + w(\theta_2) - t(\theta_1) - w(\theta_1) = \int_{\theta_1}^{\theta_2} \left( \theta + \frac{F(\theta) - \gamma(\theta)}{f(\theta)} \right) dx(\theta) \quad (30)$$

for all  $\theta_1, \theta_2 \in [\underline{\theta}, \bar{\theta}]$ . Finally, equation (9) follows from this last equation and the first order condition (3). Moreover, the inequalities in (28) and (29) imply that function  $\gamma(\cdot)$  satisfies condition (c) stated in the proposition.

### • Sufficiency

We prove sufficiency by constructing a grand contract  $\{T(\cdot), W(\cdot)\}$  such that all the participation constraints are binding,  $x(\theta)$  satisfies (20), and  $W(x(\theta)) + T(x(\theta)) = w(\theta) + t(\theta)$  for all  $\theta$ .

Let us first define function  $x^{-1}(\cdot)$  given a weakly decreasing output profile  $x(\cdot)$  as

$$x^{-1}(x) = \begin{cases} \sup \{\theta : x(\theta) \geq x\} & \text{if } x \leq x(\underline{\theta}) \\ 0 & \text{otherwise} \end{cases} \quad (31)$$

Note that if  $x(\cdot)$  has an inverse, its inverse equals the function defined in (31) on the relevant domain. The function  $x^{-1}(\cdot)$  is weakly decreasing and continuous except for the countably many output levels where  $x(\cdot)$  is constant.

Now consider the following  $T(\cdot)$  and  $W(\cdot)$  such that  $T(x(\bar{\theta})) = t(\bar{\theta})$ ,  $W(x(\bar{\theta})) = w(\bar{\theta})$ ,

$$T(x_2) - T(x_1) = \int_{x_1}^{x_2} x^{-1}(x) dx \quad \text{and} \quad (32)$$

$$W(x_2) - W(x_1) = \int_{x_1}^{x_2} \frac{F(x^{-1}(x)) - \gamma(x^{-1}(x))}{f(x^{-1}(x))} dx \quad (33)$$

for all  $x_1$  and  $x_2$  pairs. Since  $x^{-1}(\cdot)$  is weakly decreasing, (32) implies that  $T(x)$  is concave and

$$\begin{aligned} x(\theta) &\in \arg \max_x \{T(x) - \theta x\} \\ t(\theta) - \theta x(\theta) &= \max_x \{T(x) - \theta x\} \end{aligned}$$

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<sup>30</sup>See Kolmogorov and Fomin (1970).

for all  $\theta \in [\underline{\theta}, \bar{\theta}]$ . The last condition ensures that all the participation constraints are binding if  $\{x(\cdot), t(\cdot)\}$  is a solution to  $N$ 's maximization problem (5), so that condition (ii) of Lemma 1 is trivially satisfied.

Similarly, condition (c) implies that  $x^{-1}(x) + \frac{F(x^{-1}(x)) - \gamma(x^{-1}(x))}{f(x^{-1}(x))}$  is weakly decreasing and  $T(\cdot) + W(\cdot)$  is concave. Accordingly,  $x(\theta)$  satisfies (20). It follows from Lemma 1 that  $\{x(\cdot), t(\cdot)\}$  is a solution to (5). It is immediate from the construction of the grand contract  $\{T(\cdot), W(\cdot)\}$  and the first order conditions (3) and (9) that  $w(\theta) + t(\theta) = W(x(\theta)) + T(x(\theta))$  for all  $\theta$ .<sup>31</sup>

## 7.2 Proof of Proposition 2

The objective functions in problems (10) and (11) are the same, and they depend only on the output profile. Therefore, in order to prove the proposition, it is sufficient to show that

- i) the constraints of the former problem imply the constraints of the latter,
- ii) for any output profile satisfying the constraints of the latter problem, we can find a transfer profile satisfying the constraints of the former.

**Part (i):** The first order condition (3) for incentive compatibility, integration by parts, and  $IR(\bar{\theta})$  imply that

$$\begin{aligned}
 t(\theta) &= t(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} a dx(a) \\
 t(\theta) &= t(\bar{\theta}) - \bar{\theta}x(\bar{\theta}) + \theta x(\theta) + \int_{\theta}^{\bar{\theta}} x(a) da \\
 t(\theta) &\geq \theta x(\theta) + \int_{\theta}^{\bar{\theta}} x(a) da
 \end{aligned} \tag{34}$$

for all  $\theta$ . When we take the expectation of both sides of the inequality and integrate the right hand side by parts once more, we get

$$\begin{aligned}
 \int_{\underline{\theta}}^{\bar{\theta}} t(\theta) f(\theta) d\theta &\geq \int_{\underline{\theta}}^{\bar{\theta}} \theta x(\theta) f(\theta) d\theta + \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\bar{\theta}} x(a) da f(\theta) d\theta \\
 \int_{\underline{\theta}}^{\bar{\theta}} t(\theta) f(\theta) d\theta &\geq \int_{\underline{\theta}}^{\bar{\theta}} \theta x(\theta) f(\theta) d\theta + \int_{\underline{\theta}}^{\bar{\theta}} F(\theta) x(\theta) d\theta.
 \end{aligned} \tag{35}$$

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<sup>31</sup>The grand contract  $\{T(\cdot), W(\cdot)\}$  is a *collusion proof* contract, since the resulting transfer and wage levels are the same as the transfer and wage levels that  $A$  and  $N$  would receive if there were no opportunity for them to collude.

The inequality above and the constraint  $BB$  together imply constraint (12). Monotonicity of the output profile is a requirement of incentive compatibility.

**Part (ii):** Given a weakly decreasing output profile  $x(\cdot)$  satisfying constraint (12), define the transfer profile  $t(\cdot)$  such that constraint  $IR(\bar{\theta})$  is binding and the first order condition (3) is satisfied. Then, the expected value of  $t(\cdot)$  satisfies (35) as an equality. Therefore, it follows from (12) that constraint  $BB$  is also satisfied as an equality.

### 7.3 Proof of Proposition 3

The proof of the proposition follows from the lemmas below. The first lemma establishes that the definitions for the optimal  $\gamma(\cdot)$  provided in the statement of the proposition are indeed identical.

**Lemma 2** *Suppose  $\theta^*$  is defined as the solution to  $F(\theta) + f(\theta)\theta = 1$  on the interval  $[\underline{\theta}, \bar{\theta}]$  if such a solution exists and as  $\underline{\theta}$  if there is no such solution. Then,*

$$\min \{F(\theta) + f(\theta)\theta, 1\} = \begin{cases} F(\theta) + f(\theta)\theta & \text{if } \theta < \theta^* \\ 1 & \text{if } \theta \geq \theta^* \end{cases} \quad (36)$$

for  $\theta \in [\underline{\theta}, \bar{\theta}]$ .

**Proof.** Notice that  $F(\theta) + f(\theta)\theta$  is continuous and takes the value  $1 + f(\bar{\theta})\bar{\theta} > 1$  at  $\bar{\theta}$ . Therefore, if no solution exists to  $F(\theta) + f(\theta)\theta = 1$  and  $\theta^* = \underline{\theta}$ , it must be that  $F(\theta) + f(\theta)\theta > 1$  on the relevant domain. Accordingly,  $\min \{F(\theta) + f(\theta)\theta, 1\} = 1$  for  $\theta \in [\underline{\theta}, \bar{\theta}]$ , confirming (36). Now suppose a solution exists to  $F(\theta) + f(\theta)\theta = 1$ . Let us examine the behavior of the left hand side of this equality for values of  $\theta < \theta^*$ . The first term ( $F(\theta)$ ) is strictly increasing. To check the derivative of the second term ( $f(\theta)\theta$ ), first recall that the monotone hazard rate condition implies that  $\frac{d}{d\theta} \left( \frac{F(\theta)-1}{f(\theta)} \right) = \frac{(f(\theta))^2 + (1-F(\theta))f'(\theta)}{(f(\theta))^2}$  is non-negative. This reveals that  $f'(\theta) \geq -\frac{(f(\theta))^2}{1-F(\theta)}$ . Now consider the derivative

$$\begin{aligned} \frac{d(f(\theta)\theta)}{d\theta} &= f(\theta) + f'(\theta)\theta \geq f(\theta) - \frac{(f(\theta))^2}{1-F(\theta)}\theta \\ \frac{d(f(\theta)\theta)}{d\theta} &\geq \frac{f(\theta)}{1-F(\theta)}(1-F(\theta) - f(\theta)\theta). \end{aligned} \quad (37)$$

This yields

$$\frac{d(F(\theta) + f(\theta)\theta)}{d\theta} > 0 \text{ for } F(\theta) + f(\theta)\theta \leq 1, \quad (38)$$

proving  $F(\theta) + f(\theta)\theta$  passes through the value 1 at  $\theta^*$  from below and takes on values larger than 1 for  $\theta > \theta^*$ , confirming (36) again. ■

Now we show that the optimal wage profile solving (14) is also a solution to a different maximization problem.

**Lemma 3** *Suppose  $\{x(\cdot), t(\cdot), w(\cdot)\}$  is a solution to problem (14). Then  $w(\cdot)$  is also a solution to the constrained maximization problem (39) defined below:*

$$\begin{aligned} \max_{\hat{w}(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} \hat{w}(\theta) f(\theta) d\theta \quad s.t. \quad (39) \\ \{x(\cdot), t(\cdot), \hat{w}(\cdot)\} \text{ is collusion feasible,} \\ BB(\theta) : t(\theta) + w(\theta) \leq M \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}]. \end{aligned}$$

**Proof.** Since  $\{x(\cdot), t(\cdot), w(\cdot)\}$  is a solution to (14),  $w(\cdot)$  satisfies the constraints in (39) together with  $x(\cdot)$  and  $t(\cdot)$ . Suppose  $w(\cdot)$  is not a solution to (39). Then there exists  $\tilde{w}(\cdot)$  such that

$$\begin{aligned} \{x(\cdot), t(\cdot), \tilde{w}(\cdot)\} \text{ is collusion feasible,} \\ BB(\theta) : t(\theta) + \tilde{w}(\theta) \leq M \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}], \text{ and} \\ \int_{\underline{\theta}}^{\bar{\theta}} \tilde{w}(\theta) f(\theta) d\theta = \int_{\underline{\theta}}^{\bar{\theta}} w(\theta) f(\theta) d\theta + \Delta, \text{ where } \Delta > 0. \end{aligned}$$

Now consider the outcome  $\{x(\cdot) + \Delta/\bar{\theta}, t(\cdot) + \Delta, \tilde{w}(\cdot) - \Delta\}$ , which is constructed by adding or subtracting constant terms to the output, transfer, and wage profiles in the outcome  $\{x(\cdot), t(\cdot), \tilde{w}(\cdot)\}$ . Next, we establish that the constructed outcome satisfies the constraints of (14) and gives a higher value of the objective function.

- Collusion feasibility of  $\{x(\cdot) + \Delta/\bar{\theta}, t(\cdot) + \Delta, \tilde{w}(\cdot) - \Delta\}$  follows from the fact that  $\{x(\cdot), t(\cdot), \tilde{w}(\cdot)\}$  is collusion feasible, and the uniform shifts on the output, transfer, and wage profiles do not affect the statements of the first order conditions (3) and (9) or the monotonicity of the output profile.
- $IR(\bar{\theta}) : t(\bar{\theta}) + \Delta - \bar{\theta}[x(\bar{\theta}) - \Delta/\bar{\theta}] \geq 0$  follows from  $t(\bar{\theta}) + \Delta - \bar{\theta}[x(\bar{\theta}) - \Delta/\bar{\theta}] = t(\bar{\theta}) - \bar{\theta}x(\bar{\theta}) \geq 0$ , where the last inequality reflects the fact that  $IR(\bar{\theta})$  is satisfied for the outcome  $\{x(\cdot), t(\cdot), w(\cdot)\}$ .

- $IR-N$  :  $\int_{\underline{\theta}}^{\bar{\theta}} [\tilde{w}(\theta) - \Delta] f(\theta) d\theta \geq 0$  follows from  $\int_{\underline{\theta}}^{\bar{\theta}} [\tilde{w}(\theta) - \Delta] f(\theta) d\theta = \int_{\underline{\theta}}^{\bar{\theta}} \tilde{w}(\theta) f(\theta) d\theta - \Delta = \int_{\underline{\theta}}^{\bar{\theta}} w(\theta) f(\theta) d\theta \geq 0$ , where the last inequality reflects the fact that  $IR-N$  is satisfied for the outcome  $\{x(\cdot), t(\cdot), w(\cdot)\}$ .
- $BB(\theta)$  :  $t(\theta) + \Delta + \tilde{w}(\theta) - \Delta \leq M$  for all  $\theta \in [\underline{\theta}, \bar{\theta}]$  follows from  $t(\theta) + \Delta + \tilde{w}(\theta) - \Delta = t(\theta) + \tilde{w}(\theta) \leq M$ , where the last inequality reflects the fact that  $\{x(\cdot), t(\cdot), \tilde{w}(\cdot)\}$  satisfies  $BB(\theta)$  for all  $\theta \in [\underline{\theta}, \bar{\theta}]$ .
- Finally, the objective function is higher under the output profile  $x(\cdot) + \Delta/\bar{\theta}$  than it is under the profile  $x(\cdot)$ , i.e.,  $\int_{\underline{\theta}}^{\bar{\theta}} p(x(\theta) + \Delta/\bar{\theta}) f(\theta) d\theta > \int_{\underline{\theta}}^{\bar{\theta}} p(x(\theta)) f(\theta) d\theta$ , since function  $p(\cdot)$  is strictly increasing.

Therefore,  $\{x(\cdot), t(\cdot), w(\cdot)\}$  cannot be a solution to (14), unless  $w(\cdot)$  is a solution to (39). ■

In light of the previous lemma, to prove the proposition, it suffices to show that the wage profile  $w^*(\cdot)$  provided in the statement of the proposition is the essentially unique solution to (39) given  $\{x^*(\cdot), t^*(\cdot)\}$  is incentive compatible. The first step is showing that  $w^*(\cdot)$  satisfies the constraints of (39).  $BB(\theta)$  for all  $\theta \in [\underline{\theta}, \bar{\theta}]$  is established in the text (after the statement of Proposition 3). We prove collusion feasibility by showing that  $\gamma^*(\cdot)$  in (15) satisfies conditions (a), (b), and (c) as stated in Proposition 1. Condition (a) follows from (38). Condition (b) is immediate from the statement of  $\gamma^*(\cdot)$  in the proposition. For condition (c), notice that  $\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)}$  is equal to 0 for  $\theta < \theta^*$  and to  $\theta + \frac{F(\theta) - 1}{f(\theta)}$  otherwise. It follows from the monotone hazard rate assumption that  $\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)}$  is weakly increasing for all  $\theta$ , proving condition (c).

Since  $w^*(\cdot)$  satisfies the constraints in (39), for  $w^*(\cdot)$  not to be the essentially unique solution to (39), there must exist an alternative profile  $w^{**}(\cdot)$  such that

$$\begin{aligned} & w^{**}(\cdot) \text{ is essentially different from } w^*(\cdot), \\ & \{x^*(\cdot), t^*(\cdot), w^{**}(\cdot)\} \text{ is collusion feasible,} \\ & BB(\theta) : t^*(\theta) + w^{**}(\theta) \leq M \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}], \end{aligned}$$

and

$$\int_{\underline{\theta}}^{\bar{\theta}} w^{**}(\theta) f(\theta) d\theta \geq \int_{\underline{\theta}}^{\bar{\theta}} w^*(\theta) f(\theta) d\theta.$$

The last inequality together with the fact that the wage profiles  $w^*(\cdot)$  and  $w^{**}(\cdot)$  are essentially different imply the existence of a type  $\theta' \in [\underline{\theta}, \bar{\theta}]$  such



that  $w^{**}(\theta') > w^*(\theta')$ . First, recall that  $t^*(\theta) + w^*(\theta)$  equals  $M$  for  $\theta \leq \theta^*$ . Therefore,  $\theta'$  cannot be smaller than or equal to  $\theta^*$ . Otherwise,  $BB(\theta')$  would fail under  $w^{**}(\cdot)$ . Now suppose  $\theta'$  is larger than  $\theta^*$ . It follows from the collusion feasibility of  $\{x^*(\cdot), t^*(\cdot), w^{**}(\cdot)\}$  that there exists a function  $\gamma^{**}(\cdot)$  satisfying the first order condition (9) with  $w^{**}(\cdot)$ , as well as conditions (a), (b), and (c) as stated in Proposition 1. Accordingly,

$$\begin{aligned}\tilde{w}^{**}(\theta^*) &= w^{**}(\theta') + \int_{\bar{\theta}}^{\theta^*} \left( \frac{F(\theta) - \gamma^{**}(\theta)}{f(\theta)} \right) dx^*(\theta) \\ &> w^*(\theta') + \int_{\bar{\theta}}^{\theta^*} \left( \frac{F(\theta) - 1}{f(\theta)} \right) dx^*(\theta) = w^*(\theta^*),\end{aligned}\quad (40)$$

where the strict inequality follows from  $\gamma^{**}(\theta) \leq 1$  for all  $\theta$  and  $w^{**}(\theta') > w^*(\theta')$ . This, however, implies that  $BB(\theta^*)$  fails under  $w^{**}(\cdot)$ , which constitutes a contradiction to the existence of profile  $w^{**}(\cdot)$ .

## 7.4 Proof of Proposition 4

The proof of Proposition 4 follows similar steps as that of Proposition 2. The objective functions are the same in problems (14) and (17), and they depend only on the output profile. Proposition 3 provides the information on the wage profile solving problem (14). To prove the current proposition, it suffices to show that

i) any outcome which is a solution to problem (14) satisfies the constraints of problem (17),

ii) for any output profile satisfying the constraints of problem (17), there exists transfer and wage profiles satisfying the constraints of (14).

**Part (i):** Suppose  $\{x^*(\cdot), t^*(\cdot), w^*(\cdot)\}$  is a solution to (14). The constraints of problem (14) include the first order condition (3) and  $IR(\bar{\theta})$ . Therefore,  $\{x^*(\cdot), t^*(\cdot)\}$  satisfies inequality (34) derived in the proof of Proposition 2. We start by writing (34) for  $\underline{\theta}$ :

$$t^*(\underline{\theta}) \geq \underline{\theta}x^*(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} x^*(\theta) d\theta. \quad (41)$$

Once we identify the lower bound on  $t^*(\underline{\theta})$ , constraint  $BB(\underline{\theta})$  reveals the upper

bound on  $w^*(\underline{\theta})$  as

$$\begin{aligned} w^*(\underline{\theta}) &\leq M - t^*(\underline{\theta}) \\ &\leq M - \underline{\theta}x^*(\underline{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} x^*(\theta) d\theta. \end{aligned} \quad (42)$$

Now we can write down the wage level in terms of the output profile by using (9) and the upper bound on  $w^*(\underline{\theta})$ .

$$\begin{aligned} w^*(\theta) &= w^*(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \frac{F(a) - \gamma^*(a)}{f(a)} dx^*(a) \\ &\leq M - \underline{\theta}x^*(\underline{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} x^*(\theta) d\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} x^*(\theta) \\ &\quad - \frac{F(\underline{\theta}) - \gamma^*(\underline{\theta})}{f(\underline{\theta})} x^*(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} x^*(a) d\left(\frac{F(a) - \gamma^*(a)}{f(a)}\right) \\ &\leq M - \left[\underline{\theta} - \frac{\gamma^*(\underline{\theta})}{f(\underline{\theta})}\right] x^*(\underline{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} x^*(\theta) d\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} x^*(\theta) \\ &\quad - \int_{\underline{\theta}}^{\theta} x^*(a) \frac{d}{da} \left(\frac{F(a) - \gamma^*(a)}{f(a)}\right) da \\ &\leq M - \int_{\underline{\theta}}^{\bar{\theta}} x^*(\theta) d\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} x^*(\theta) \\ &\quad - \int_{\underline{\theta}}^{\theta} x^*(a) \frac{d}{da} \left(\frac{F(a) - \gamma^*(a)}{f(a)}\right) da, \end{aligned} \quad (43)$$

where the second line follows from integration by parts and the third line from the absolute continuity of function  $\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)}$  given function  $\gamma^*(\theta)$  described in Proposition 3.

Next, we take the expectation over both sides of the last inequality:

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} w^*(\theta) f(\theta) d\theta &\leq M - \int_{\underline{\theta}}^{\bar{\theta}} x^*(\theta) d\theta + \int_{\underline{\theta}}^{\bar{\theta}} \left[ \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} \right] x^*(\theta) f(\theta) d\theta \\
&\quad - \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} x^*(a) \frac{d}{da} \left( \frac{F(a) - \gamma^*(a)}{f(a)} \right) da f(\theta) d\theta \\
\int_{\underline{\theta}}^{\bar{\theta}} w^*(\theta) f(\theta) d\theta &\leq M - \int_{\underline{\theta}}^{\bar{\theta}} x^*(\theta) d\theta + \int_{\underline{\theta}}^{\bar{\theta}} \left[ \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} \right] x^*(\theta) f(\theta) d\theta \\
&\quad - \int_{\underline{\theta}}^{\bar{\theta}} (1 - F(\theta)) \frac{d}{d\theta} \left( \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} \right) x^*(\theta) d\theta \quad (44)
\end{aligned}$$

where the last line follows from integration by parts over the final term. Recalling that (15) implies

$$\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} = \begin{cases} -\theta & \text{for } \theta < \theta^* \\ \frac{F(\theta)-1}{f(\theta)} & \text{for } \theta \geq \theta^* \end{cases}$$

and

$$\frac{d}{da} \left( \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} \right) = \begin{cases} -1 & \text{for } \theta < \theta^* \\ \frac{d}{d\theta} \frac{F(\theta)-1}{f(\theta)} & \text{for } \theta \geq \theta^* \end{cases} ,$$

we can rewrite the last inequality as

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} w^*(\theta) f(\theta) d\theta &\leq M - \int_{\underline{\theta}}^{\theta^*} [\theta f(\theta) + F(\theta)] x^*(\theta) d\theta \\
&\quad - \int_{\theta^*}^{\bar{\theta}} \left[ 1 + [1 - F(\theta)] \left[ 1 + \frac{d}{d\theta} \left( \frac{F(\theta) - 1}{f(\theta)} \right) \right] \right] x^*(\theta) d\theta. \quad (45)
\end{aligned}$$

Constraint  $IR-N$  and inequality (45) imply constraint (18). Monotonicity of the output profile follows from collusion feasibility.

**Part (ii):** Given a weakly decreasing output profile  $x(\cdot)$  satisfying constraint (18), define the transfer profile  $t(\cdot)$  such that constraint  $IR(\bar{\theta})$  is binding and the first order condition (3) is satisfied. Similarly, define the wage profile  $w(\cdot)$  such that  $BB(\underline{\theta})$  is binding and the first order condition (9) is satisfied with  $\gamma^*(\cdot)$  defined in (15). The resulting total compensation profile  $t(\cdot) + w(\cdot)$ , which is weakly decreasing, satisfies all the other budget constraints. Moreover, the expected value of  $w(\cdot)$  satisfies (45) as an equality.

Therefore, it follows from (18) that constraint  $IR - N$  is also satisfied as an equality.

## 7.5 The Example

In this part of the paper, we turn to the application with budget constraints. We introduce an example with a particular utility function for the principal and a type distribution for the agent. We identify the optimal output profile under different contractual arrangements and then compare the expected payoffs for  $P$  under these arrangements. For this example, we assume that the unit production cost  $\theta$  is uniformly distributed on the support  $[\frac{1}{2}, \frac{3}{2}]$ , i.e.,  $F(\theta) = \theta - \frac{1}{2}$  and  $f(\theta) = 1$  over the support.  $P$ 's utility from output is given as  $p(x) = \ln x$ . As in Section 4,  $P$  cannot spend more than  $M$  in any state of nature. He maximizes his expected utility  $\int_{1/2}^{3/2} \ln x(\theta) d\theta$  subject to the constraints resulting from the contractual arrangement.

### • Bilateral Contract

The first setup studied is a bilateral contract between  $P$  and the productive agent  $A$ . Following the discussion of the bilateral setup in Section 2.1, we can write  $P$ 's design problem as

$$\begin{aligned} \max_{\{x(\cdot), t(\cdot)\}} & \int_{\underline{\theta}}^{\bar{\theta}} \ln x(\theta) d\theta \text{ s.t.} \\ & \{x(\cdot), t(\cdot)\} \text{ is incentive compatible,} \\ IR(\bar{\theta}) & : t(\bar{\theta}) - \bar{\theta}x(\bar{\theta}) \geq 0, \\ BB(\underline{\theta}) & : t(\underline{\theta}) \leq M. \end{aligned} \tag{46}$$

Note that constraint  $BB(\underline{\theta})$  is sufficient for all the other budget constraints since incentive compatibility implies a monotonic transfer profile.

The next step is writing down the transfers in terms of the output levels and substituting them in the objective function. To this end, recall that the first order condition (3) and constraint  $IR(\bar{\theta})$  imply inequality (34), which is derived in the proof of Proposition 2, for type  $\underline{\theta}$ :

$$t(\underline{\theta}) \geq \underline{\theta}x(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) d\theta. \tag{47}$$

Together with  $BB(\underline{\theta})$ , this last inequality outlines a condition which must be fulfilled by any output profile satisfying the constraints of  $P$ 's bilateral design

problem in (46):

$$\underline{\theta}x(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) d\theta \leq M. \quad (48)$$

Moreover, given a weakly decreasing output profile  $x(\cdot)$  fulfilling (48), the binding  $IR(\bar{\theta})$  constraint and the first order condition (3) identify a transfer profile  $t(\cdot)$  satisfying the constraints of (46) together with  $x(\cdot)$ . Therefore, following the proofs of Propositions 2 and 4, we conclude that the optimal output profile is a solution to the problem

$$\max_{x(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} \ln x(\theta) d\theta \text{ s.t.} \quad (49)$$

$$\underline{\theta}x(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) d\theta \leq M, \quad (50)$$

and  $x(\cdot)$  is weakly decreasing.

Next, we show that the solution to problem (49) induces complete pooling, i.e.,  $x(\theta)$  is constant for all  $\theta$ . To see this, let  $x(\cdot)$  be a weakly decreasing output profile satisfying constraint (50). Now we construct the alternative profile where all types are assigned to the expected value of  $x(\theta)$  such that  $\hat{x}(\theta) = \hat{x} = \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) d\theta$ . This constant output profile trivially satisfies the weak monotonicity constraint. It also satisfies constraint (50):

$$\underline{\theta}\hat{x} + \int_{\underline{\theta}}^{\bar{\theta}} \hat{x} d\theta \leq \underline{\theta}x(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) d\theta \leq M \quad (51)$$

since  $\hat{x} = \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) d\theta \leq x(\underline{\theta})$ . Moreover, profile  $\hat{x}(\cdot)$  induces an improvement in the objective function since

$$\int_{\underline{\theta}}^{\bar{\theta}} \ln \hat{x} d\theta = \ln \hat{x} = \ln \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) d\theta \geq \int_{\underline{\theta}}^{\bar{\theta}} \ln x(\theta) d\theta, \quad (52)$$

where the last inequality is strict unless  $x(\cdot)$  is constant. This last step proves that the optimal output profile is constant.

Now what remains is identifying the constant output level that solves (49). The objective function is increasing in the level of output. The binding constraint (50) reveals this level as  $M/\bar{\theta}$ . This output level leaves  $P$  with a utility level of  $\ln M - \ln \frac{3}{2} \cong \ln M - 0.41$  in every state of nature.

- **Grand Contract in a Collusion Free Setup**

The second setup we consider is the collusion free setup with the insurer  $N$ . Following the analysis in Section 4.1, the first order condition revealing the output profile is determined as

$$\begin{aligned}\frac{1}{x(\theta)} &= \lambda \left( \theta + \frac{F(\theta)}{f(\theta)} \right), \\ x(\theta) &= \frac{1}{\lambda \left( 2\theta - \frac{1}{2} \right)}.\end{aligned}\tag{53}$$

Moreover, when we substitute in the value of  $x(\theta)$  into the binding constraint (12)

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\lambda \left( 2\theta - \frac{1}{2} \right)} \left( 2\theta - \frac{1}{2} \right) d\theta = M,\tag{54}$$

we identify the value of the Lagrange multiplier as  $\lambda = 1/M$ . So the optimal collusion free output levels are

$$x(\theta) = \frac{M}{2\theta - \frac{1}{2}}\tag{55}$$

for all  $\theta$ .

The transfer profile is given by the binding  $IR(\bar{\theta})$  constraint and the first order condition (3) as before.  $P$ 's budget is exhausted in every state of nature and, therefore, the wage level is given as  $w(\theta) = M - t(\theta)$  for all  $\theta$ . In this collusion free setup,  $P$ 's expected payoff is  $\ln M - \int_{1/2}^{3/2} \ln \left( 2\theta - \frac{1}{2} \right) d\theta = \ln M - 0.32$ . In words, with the inclusion of an insurer, the principal can use his budget as a means of separating the different types of the agent, even though he ends up with the same total payment level in every state of nature. As a result, he increases his expected payoff above the optimal bilateral contract payoff.

- **Grand Contract with Collusion**

In this setup,  $P$  can implement a wage payment to  $N$  contingent on the production level. However,  $N$  can collude with  $A$  to influence his output choice. The analysis of this final contractual arrangement requires employing the results derived in Section 4.2. After noticing that  $\theta^*$  equals  $3/4$  in this

setup, we can rewrite problem (17) as

$$\max_{x(\cdot)} \int_{1/2}^{3/2} \ln(x(\theta)) d\theta \text{ s.t.} \quad (56)$$

$$\int_{1/2}^{3/4} \left(2\theta - \frac{1}{2}\right) x(\theta) d\theta + \int_{3/4}^{3/2} (4 - 2\theta) x(\theta) d\theta \leq M, \quad (57)$$

and  $x(\cdot)$  is weakly decreasing.

We first establish that the optimal outcome exhibits pooling of the types on the interval  $[3/4, 3/2]$  at the same output level. To see this, let  $x(\cdot)$  be a weakly decreasing output profile satisfying constraint (57). Now we construct the alternative profile  $\hat{x}(\cdot)$  where all types larger than  $3/4$  are assigned to the expected value of  $x(\theta)$  conditional on  $\theta > 3/4$  and all types lower than  $3/4$  are left with the original output levels, such that

$$\hat{x}(\theta) = \begin{cases} x(\theta) & \text{if } \theta \leq 3/4 \\ \frac{1}{3/2-3/4} \int_{3/4}^{3/2} x(\theta) d\theta & \text{if } \theta > 3/4 \end{cases} \quad (58)$$

The alternative output profile  $\hat{x}(\cdot)$  is weakly decreasing. After changing the output profile to  $\hat{x}(\cdot)$ , the change in the value of the left hand side of constraint (57) can be expressed as

$$\begin{aligned} & \int_{3/4}^{3/2} (4 - 2\theta) E[x(\theta)] d\theta - \int_{3/4}^{3/2} (4 - 2\theta) x(\theta) d\theta \\ &= (3/2 - 3/4) [E[4 - 2\theta] E[x(\theta)] - E[(4 - 2\theta)x(\theta)]] \end{aligned} \quad (59)$$

where  $E[\cdot]$  stands for the expectation of the argument conditional on  $\theta$  on the interval  $[3/4, 3/2]$ . This expression equals  $-(3/2 - 3/4) Cov(4 - 2\theta, x(\theta))$  on the relevant interval, which is non-positive since both  $4 - 2\theta$  and  $x(\theta)$  are weakly decreasing.<sup>32</sup> Therefore, constraint (57) is satisfied with profile  $\hat{x}(\cdot)$ .

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<sup>32</sup>For a formal derivation, consider

$$\begin{aligned} Cov(4 - 2\theta, x(\theta)) &= E[(4 - 2\theta - E[4 - 2\theta])(x(\theta) - E[x(\theta)])] \\ &= -2E[(\theta - E[\theta])(x(\theta) - E[x(\theta)])] \\ &= -2E[(\theta - E[\theta])(x(\theta) - x(E[\theta]))] \\ &\quad -2E[(\theta - E[\theta])(x(E[\theta]) - E[x(\theta)])] \\ &= -2E[(\theta - E[\theta])(x(\theta) - x(E[\theta]))]. \end{aligned}$$

Moreover,  $\hat{x}(\theta)$  induces an improvement in the objective function since

$$\begin{aligned} \int_{3/4}^{3/2} \ln \hat{x}(\theta) d\theta &= (3/2 - 3/4) \ln E[x(\theta)] \\ &\geq (3/2 - 3/4) E[\ln x(\theta)] = \int_{3/4}^{3/2} \ln x(\theta) d\theta, \end{aligned} \quad (60)$$

where the inequality is strict unless  $x(\cdot)$  is constant on the relevant domain. This last step proves that the optimal output profile is constant on the interval  $[3/4, 3/2]$ .

When we impose pooling on types  $[3/4, 3/2]$ ,  $P$ 's optimization problem reduces to choosing function  $x(\cdot)$  on the interval  $[1/2, 3/4)$ , and choosing  $x_p$  for the pooling region:

$$\max_{x(\cdot), x_p} \int_{1/2}^{3/4} \ln(x(\theta)) d\theta + \int_{3/4}^{3/2} \ln(x_p) d\theta \text{ s.t.} \quad (61)$$

$$\int_{1/2}^{3/4} \left(2\theta - \frac{1}{2}\right) x(\theta) d\theta + \int_{3/4}^{3/2} (4 - 2\theta) x_p d\theta \leq M, \quad (62)$$

$$x(\cdot) \text{ weakly decreasing and } x(\theta) \geq x_p \text{ for all } \theta.$$

Ignoring the monotonicity constraints, pointwise maximization yields

$$x(\theta) = \frac{1}{\lambda \left(2\theta - \frac{1}{2}\right)} \quad (63)$$

for  $\theta \in [1/2, 3/4)$ , and

$$\begin{aligned} \frac{1}{x_p} (3/2 - 3/4) &= \lambda \int_{3/4}^{3/2} (4 - 2\theta) d\theta, \\ \frac{1}{x_p} \frac{3}{4} &= \lambda \frac{21}{16}, \\ x_p &= \frac{4}{7\lambda}. \end{aligned} \quad (64)$$

The binding constraint (62) reveals the value of the Lagrange multiplier as  $\lambda = 1/M$ . The resulting output profile satisfies the ignored monotonicity

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The right hand side and, therefore, the covariance term is non-negative since  $x(\cdot)$  is weakly decreasing.



constraints:

$$x(\theta) = \begin{cases} \frac{M}{(2\theta - \frac{1}{2})} & \text{if } \theta \leq 3/4 \\ \frac{4M}{7} & \text{otherwise} \end{cases} .$$

The budget constraints are binding for  $\theta \leq 3/4$ , but slack for the other types. It is optimal for  $P$  not to exhaust his budget in every state of nature, even though money does not directly enter into his utility function. The expected payoff for  $P$  under this contract is

$$\ln M - \int_{1/2}^{3/4} \ln \left( 2\theta - \frac{1}{2} \right) d\theta + \int_{3/4}^{3/2} \ln \frac{7}{4} d\theta = \ln M - 0.34. \quad (65)$$

As anticipated, this contract yields an expected payoff in-between the two expected payoff levels discussed earlier.<sup>33</sup>

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<sup>33</sup>The agent's payoff under each of these contractual arrangements depends on his realized type. The low cost types are more likely to prefer the existence of insurance to bilateral contracts and collusion free insurance to insurance under collusion.

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