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Discussion Paper 2010-34

Institut de Recherches Économiques et Sociales
de l'Université catholique de Louvain



On the Recovery Path during a Liquidity Trap: Do Financial Frictions Matter for Fiscal Multipliers?*

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September 2, 2010

Abstract

This paper investigates the effects of a fiscal stimulus when financial frictions and a liquidity trap are present. These two conditions make a government spending expansion and a reduction in capital income taxes more efficient in stimulating output. In contrast, a reduction in labor income taxes may aggravate the economic conditions. In addition, small implementation delays in government spending may result in big spending multipliers in the short run. All of these results rely partly on the dynamic interaction between inflation and the external finance premium. Lastly, simulations of the ARRA stimulus package predict that the output gains due to the presence of financial frictions may lie between 1.3 % and 2.5 % of GDP.

Keywords: Zero Lower Bound, Financial Accelerator, Fiscal Policy

JEL codes: E31; E44; E52; E58

*We are grateful to Roberto Billi, Tommaso Monacelli, Gert Peersman, and Luca Pensirosso for their helpful comments. We also thank the participants in economic seminars at the University of Lille, Ghent University, the Midwest Macroeconomics Meeting 2010, the Canadian Economic Association Annual Conference 2010, the Conference on Monetary and Fiscal Policy for Macroeconomic Stability 2010, and the Econometric Society World Congress for their input.

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1 Introduction

The severe recession of 2008-2010 has prompted monetary and fiscal authorities to undertake drastic policies. One is the reduction of the target short run nominal interest rate by different central banks to record low levels. In the United States, for instance, the fed funds rate has been set to between 0 and 0.25 per cent since December 2008. In addition, various governments have implemented important fiscal packages aimed at stimulating their economies. In U.S., the *American Recovery and Reinvestment Act* (ARRA) plans to distribute 787 billion dollars (around 5.5 per cent of 2009 GDP) in the course of the next few years. In a context of financial turmoil, the role of financial frictions might be crucial in the analysis of monetary and fiscal policy. Indeed, a few studies suggest that financial accelerator mechanisms – in which firms balance sheet positions affect the cost of external finance – are relatively more important during recessions than during booms.¹

In this paper, we investigate the effect of fiscal stimulus packages on an economy featuring financial frictions that enters into a liquidity trap. Specifically, we are interested in the potential implication that financial frictions may have in the fiscal multiplier. The fiscal stimulus package that we consider consists of either an increase in government spending, or a cut in distortionary labor and capital income taxes.

The effect of government spending expansions in a zero nominal interest rate environment has been explored in the literature. For instance, Christiano *et al.* (2009a) argue that the impact of government spending on output can be large when the zero lower bound is binding. Indeed, the well-known crowding-out of investment by government spending is neutralized when the interest rate stays close to zero for an extended period of time. The

¹For instance, Christiano *et al.* (2003) used a financial accelerator model à la Bernanke *et al.* (2000) to study the Great depression in the U.S.. Gertler *et al.* (2007) employed a similar strategy for the Korean economy to the Asian financial crisis of 1997. Finally, Peersman and Smets (2005), using European industry data, showed that the financial accelerator mechanism can explain the asymmetric effects of monetary policy during booms and recessions.

reason behind this result lies in the fact that the rise in inflation, led by the fiscal stimulus, reduces the real interest rate when the nominal rate remains fixed. Erceg and Lindé (2010) showed that the size of the fiscal stimulus matters for the value of the fiscal multiplier. They demonstrated that massive government spending expansions might substantially reduce the duration of the liquidity trap that automatically turns on the crowding-out effect on investment.² Furthermore, Fernández-Villaverde (2010) assessed the impact of a fiscal stimulus in an economy featuring financial frictions. He concluded that credit market imperfections magnify the effect of government spending on output. This holds since, by increasing inflation, government spending indirectly improves the balance sheet position of firms, which in turn reduces the external finance premium.

Given these results, we suspected that the presence of financial frictions and of zero short-term interest rates would reinforce the final effects of a fiscal stimulus. In this environment, our contribution is to explore how fiscal multipliers react to different fiscal policy regimes. This allows the policy responses that are best-suited to an economy suffering from a liquidity trap and credit market imperfections to be identified. To pursue our analysis, we use a financial accelerator model with nominal and real rigidities, plus a zero lower bound constraint on the nominal interest rate. We use the financial frictions model *à la* Bernanke *et al.* (2000), enriched by the nominal-denominated debt contracts introduced by Christiano *et al.* (2009b) and Fernández-Villaverde (2010).³ The latter introduced an additional transmission device, which is similar to the debt-deflation channel that Fisher (1933) used

²Corsetti *et al.* (2010) show that the positive impact of the so-called “spending reversal” on output can be large when the zero lower bound is binding, once it is undertaken sufficiently late on the recovery path. Other authors have assessed the effects of government spending expansion under the zero lower bound. Focusing on changes in policy regimes, Eggertsson (2006) argues that monetary and fiscal policy coordination can induce a shift in expectations, that helps the economy to recover when the short-term nominal interest rate is close to zero.

³The financial accelerator model *à la* Bernanke *et al.* (2000) has been extensively employed to explore the amplification effects of financial frictions (see Gilchrist and Leahy, 2002; Meier and Müller, 2006; Faia, 2007; Christensen and Dib, 2008). More recently, some authors have investigated the relative importance of financial shocks over the business cycle by estimating the financial accelerator model (Christiano *et al.*, 2003, 2009b; Fuentes-Albero, 2009; Gilchrist *et al.*, 2009; Nolan and Thoenissen, 2009; Queijo von Heideken, 2009).

to explain the worsening of economic conditions during the Great Depression.

In this framework, a hypothetical recession is generated from a sudden decrease in the net worth of entrepreneurs and a shift from spending to saving by households (i.e., a preference shock). Then, we explore the role of different fiscal policies aimed at either expanding government purchases or cutting the income tax rates. We assess the role of financial frictions and the liquidity trap on the fiscal multipliers by carrying out a set of counterfactual exercises. In addition, we pay attention to the role of different fiscal regimes on government spending multipliers. First, we consider automatic stabilizers and debt stabilization by allowing for dynamic income tax rates. Second, we stress the impact of delays in the implementation of government spending expansion on the fiscal policy efficiency. Finally, we perform a quantitative exercise by introducing the ARRA fiscal stimulus of 2009 as approximated by Cogan *et al.* (2010).

Our main results are as follows. First, the Fisher effect, along with financial frictions, plays a crucial role in the efficiency of fiscal policy. For instance, a government spending expansion and a capital income tax cut see their effects amplified, because these measures increase inflation, which in turn reduces the external finance premium. On the contrary, a cut in the labor income tax rate can result in short-term negative multipliers. Indeed, this policy increases the labor supply that lowers real wages and in turn inflation. Consequently, the debt-deflation channel operates in the opposite direction reducing investment.

Second, we find that when there is financial friction, announcing the path of government spending a few quarters in advance can increase the size of the fiscal multiplier in the short run. This is due to the fact that agents adjust their inflation expectations upwards from the present period. This activates the debt-deflation channel by increasing net worth and decreasing the risk premium, since inflation starts rising from today.

Lastly, we emphasize that the ARRA fiscal stimulus has more efficient effects in the presence

of financial frictions than without. Its effect on GDP was greatest in 2010, with an increase in GDP of about 0.62%. This effect is even stronger when the zero lower bound lasts for 12 quarters instead of 6.

The rest of the paper is as follows. Section 2 introduces the model. Section 3 elaborates on its calibration and explains the solution method when the interest rate is equal to zero. Section 4 analyzes the effects of government spending expansion and income tax cuts. Section 5 assesses the importance of fiscal policy regimes on the spending multiplier. Section 6 simulates the ARRA fiscal stimulus in the model. Finally, some concluding remarks are offered in Section 7.

2 The Model

The framework is based on a standard New-Keynesian model with real and nominal rigidities, which is enriched with frictions in the credit market *à la* Bernanke *et al.* (2000). The model also features the Fisher's debt-deflation channel, since we assume that debt contracts are denominated in nominal terms.

In addition, we incorporate the zero lower bound constraint on the nominal interest rate, which adds an important non-linearity to the model. Regarding fiscal policy, distortionary tax rules on labor and capital income are included. Consequently, a fiscal stimulus package is undertaken by expanding government spending or by lowering income tax rates.

2.1 Households

Preferences The economy is inhabited by a continuum of differentiated households, indexed by $i \in [0, 1]$. A typical household selects a sequence of consumption, wages, and savings that are deposited in a financial intermediary that pays the riskless rate of return. Households differ the specific labor type they are endowed with, which gives them monop-

olistic power to set their own wage. Household i 's objective is to maximize her expected lifetime utility

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ \varepsilon_t \mathbb{U}(c_t - bc_{t-1}) - \mathbb{V}(\ell_{i,t}^h) \right\},$$

subject to the sequence of constraints

$$c_t + \frac{d_{t+1}}{R_t} \leq (1 - \tau_t^w)[w_{i,t}\ell_{i,t}^h + \text{div}_t] + \frac{d_t}{1 + \pi_t} + \Upsilon_t, \quad (1)$$

where E_t is the expectation operator conditional on the information available in period t ; $\beta \in (0, 1)$ is the subjective discount factor; $b \in [0, 1)$ is the habit parameter; τ_t^w is the time-varying labor income tax rate; c_t denotes real consumption; P_t is the price of final goods; $w_{i,t} \equiv W_{i,t}/P_t$ and $\ell_{i,t}^h$ denote the real wage and the labor supply of type- i household at period t ; $\pi_t = P_t/P_{t-1} - 1$ is the inflation rate; d_t equals D_t/P_{t-1} , where D_t denotes the nominal deposits carried over from period $t - 1$ and maturing in period t ; R_t denotes the riskless gross nominal interest rate; div_t are real profits redistributed by monopolistic firms; and Υ_t denotes real lump-sum taxes adjusted according to the rule specified below. In addition, ε_t denotes a preference shock which follows an autorregressive process of the form

$$\log(\varepsilon_t) = \rho_\varepsilon \log(\varepsilon_{t-1}) + \epsilon_{\varepsilon,t},$$

where $\rho_\varepsilon \in (0, 1)$, and $\epsilon_{\varepsilon,t} \sim \text{i.i.d.}(0, \sigma_\varepsilon)$. The first order conditions with respect to consumption and deposits are quite standard and are omitted for simplicity.⁴

Wage Setting Type- i household is a monopoly supplier of type- i labor. Following Erceg *et al.* (2000), we assume that the set of differentiated labor inputs, indexed by $i \in [0, 1]$, is aggregated into a single labor input ℓ_t^h by a competitive labor intermediary. Her profit

⁴The log-linear model is described in appendix A.

maximization yields the labor demand functions

$$\ell_{i,t}^h = \left(\frac{W_{i,t}}{W_t} \right)^{-\theta_w} \ell_t^h, \quad (2)$$

where $\theta_w > 1$ is the elasticity of substitution between any two labor types and W_t is the aggregate wage level.⁵

Following Calvo (1983), we assume that at each point in time a household has a probability $(1 - \alpha_w)$ of re-optimizing its wage. Wages that are not allowed to re-optimize at time t can be partially indexed to the most recently available inflation measure, π_{t-1} . Let $W_{i,t}^*$ denote the nominal wage rate chosen by type- i household at time t , and $\ell_{i,t+k}^{h*}$ the hours worked k periods after the last period during which the type- i household re-optimized its wage. Type- i household selects $W_{i,t}^*$ in order to maximize her expected lifetime utility with respect to its budget constraint and labor demand at each period. The first order condition is given by

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \alpha_w)^{T-t} \ell_{i,t+k}^{h*} \left\{ \lambda_{t+k} \delta_{t+k}^w (1 - \tau_t^w) w_{i,t}^* - \mu_w \mathbb{V}_\ell \left(\ell_{i,t+k}^{h*} \right) \right\} = 0, \quad (3)$$

where λ_t is the Lagrangian multiplier associated the budget constraint; $\mathbb{V}_\ell(\cdot)$ denotes the derivative of $\mathbb{V}(\cdot)$ w.r.t. ℓ^h ; $w_{i,t}^* \equiv W_{i,t}^*/P_t$ is the time t optimal real wage; $\mu_w = \theta_w / (\theta_w - 1)$ denotes the wage mark-up; and δ_{t+k}^w equals $\prod_{j=t}^{T-1} (1 + \pi)^{1-\gamma_w} (1 + \pi_j)^{\gamma_w}$ when $k > 0$ and 1 otherwise, where $\gamma_w \in (0, 1)$ is the degree of partial indexation, and π is the steady-state inflation rate.

⁵The labor intermediary production function is

$$\ell_t^h = \left(\int_0^1 [\ell_{i,t}^h]^{(\theta_w-1)/\theta_w} di \right)^{\theta_w/(\theta_w-1)}.$$

2.2 Entrepreneurs

Optimal Financial Contract There is a continuum of risk neutral entrepreneurs, indexed by $e \in [0, 1]$. At time t , type- e entrepreneur purchases at price Q_t the stock of capital $\tilde{k}_{e,t+1}$ for use in $t + 1$. Capital expenditure is financed from internal resources and debt. Let $n_{e,t+1}$ be the available real net worth of type- e entrepreneur at the end of period t and $b_{e,t+1}$ the amount of real debt owed to the financial intermediary (or lender). Accordingly,

$$q_t \tilde{k}_{e,t+1} = b_{e,t+1} + n_{e,t+1}, \quad (4)$$

where $q_t \equiv Q_t/P_t$. The lender is also risk neutral and obtains its funds from the households, to whom she pays back the principal plus interest earnings according to the riskless real gross rate of return, $r_t = R_t/E_t(1 + \pi_{t+1})$. Following Bernanke *et al.* (2000), it is assumed that the *ex-post* real gross return on capital for type- e entrepreneur, $r_{e,t+1}^k$, is affected by an idiosyncratic disturbance, denoted by $\omega_{e,t+1}$. The latter is an i.i.d. random variable across time and types, with a continuous and once-differentiable c.d.f., $F(\omega)$, over a non-negative support. It is assumed that $\omega_{e,t+1}$ is unknown to both the entrepreneur and the lender prior to the investment decision, with $E(\omega) = 1$ and $V(\omega) = \sigma_\omega^2$.

In the spirit of Townsend (1979), lenders pay a fixed monitoring cost to observe the borrowers' realized return, while borrowers observe it for free. For simplicity, it is assumed that the monitoring cost is a proportion $\mu \in [0, 1]$ of the realized gross payoff to the entrepreneur's capital, i.e., $\mu \omega_{e,t+1} r_{e,t+1}^k q_t \tilde{k}_{e,t+1}$.

The type- e entrepreneur chooses the value of her project's capital, $q_t \tilde{k}_{e,t+1}$, and the associated level of borrowing, $b_{e,t+1}$, prior to the realization of $\omega_{e,t+1}$. The optimal contract is characterized by a gross non-default loan rate, $r_{e,t+1}^g$, and a threshold value of the idiosyncratic shock, $\bar{\omega}_{e,t+1}$, such that for values of $\omega_{e,t+1}$ greater than or equal to $\bar{\omega}_{e,t+1}$, the

entrepreneur repays the debt at rate $r_{e,t+1}^g$. Thus, $\bar{\omega}_{e,t+1}$ and $r_{e,t+1}^g$ are defined by

$$\bar{\omega}_{e,t+1} r_{e,t+1}^k q_t \tilde{k}_{e,t+1} = r_{e,t+1}^g b_{e,t+1}.$$

When $\omega_{e,t+1} < \bar{\omega}_{e,t+1}$, the entrepreneur declares bankruptcy and the lender pays the monitoring cost to audit the entrepreneur. To avoid any misreport temptation by the borrower, it is assumed that once the lender is forced to audit, she keeps all of the borrower's actual returns.

The lender participates in the contract as long as an expected loan return equal to the opportunity costs of its funds, represented by r_t , the risk free rate is assured.

Let $\tilde{r}_{e,t} = E_t\{r_{e,t+1}^k/r_t\}$ be the expected discounted return on capital. The optimal lending contract consists of choosing $\tilde{k}_{e,t+1}$ and $\bar{\omega}_{e,t+1}$ to maximize the entrepreneurs expected returns subject to the participation constraint of the lender.⁶ The first order conditions of the problem imply that, at equilibrium, the discounted return on capital will be equal to the marginal cost of external finance, i.e.

$$\tilde{r}_{e,t} = x \left(\frac{q_t \tilde{k}_{e,t+1}}{n_{e,t+1}} \right), \quad (5)$$

with $x'(\cdot) > 0$, for $n_{t+1} < q_t \tilde{k}_{t+1}$. The intuition behind function $x(\cdot)$ is quite simple: other things being equal, the cost of external finance should increase whenever the leverage ratio, $q_t \tilde{k}_{t+1}/n_{t+1}$, increases. This is because low levels of net worth (or collateral) increase the probability of default on the loan. Consequently, the lender asks to be compensated with a higher cost of borrowing in order to participate in the contract. This is the key feature of the financial accelerator model, and allows us to treat the expected discounted return on capital, $\tilde{r}_{e,t}$, as the external finance premium.

⁶A detailed description of the financial contract is provided in a technical appendix, available upon request.

Entrepreneurs in General Equilibrium Type- e entrepreneur, that owns the stock of capital $\tilde{k}_{e,t}$, provides capital services $k_{e,t}$ to intermediate firms, according to

$$k_{e,t} = u_{e,t} \tilde{k}_{e,t},$$

where $u_{e,t} > 0$ is the individual rate of capital utilization. At the beginning of period t , after observing all the shocks, entrepreneurs choose how intensively to use their capital. They rent capital services to intermediate firms, and once goods have been produced, they sell the remaining un-depreciated stock of capital to the capital producer. Thus, the gross return to holding a unit of capital from $t - 1$ to t can be written as

$$r_{e,t}^k \equiv \frac{(1 - \tau_t^z) u_{e,t} z_t + (1 - \delta(u_{e,t})) q_t}{q_{t-1}}. \quad (6)$$

where z_t is the real payment for capital services taxed at a rate of τ_t^z per cent, and $\delta(u) \in [0, 1]$ is a convex depreciation function. Like Queijo von Heideken (2009), we consider a function with $\delta(0) = 0$, $\lim_{u \rightarrow \infty} \delta(u) = 1$, and with a steady-state value of $\delta(1) = \delta$. Entrepreneurs choose the rate of capital utilization by maximizing Equation (6) with respect to $u_{e,t}$. Notice that, since z_t and q_t are aggregate prices, all entrepreneurs will choose exactly the same rate of capital utilization, independently of their own capital holdings. This implies that $r_{e,t}^k = r_t^k$, $u_{e,t} = u_t$, $\check{r}_{e,t} = \check{r}_t \forall e$, and

$$z_t = \delta'(u_t) q_t.$$

Following Bernanke *et al.* (2000) and Carlstrom and Fuerst (1997), entrepreneurs participate in the general labor market by supplying one unit of labor every period, earning the nominal wage W_t^e . Finally, each entrepreneur has a probability of exiting the economy of $1 - \gamma_t$. This assumption captures the idea that entrepreneurs are not allowed to accumulate enough

wealth to be fully self-financed.⁷

The aggregate real net worth of entrepreneurs at the end of period t , n_{t+1} , is given by

$$n_{t+1} = \gamma_t v_t + (1 - \tau_t^w) w_t^e \quad (7)$$

where $(1 - \tau_t^w) w_t^e$ denotes the after-tax real wage earned by entrepreneurs. Gross revenues from capital holdings from $t - 1$ to t less borrowing repayments (i.e. the entrepreneurs' equity) are given by v_t , where

$$v_t = r_t^k q_{t-1} \tilde{k}_t (1 - \mu G(\bar{\omega}_t)) - r_{t-1} (q_{t-1} \tilde{k}_t - n_t) \quad (8)$$

Entrepreneurs that fail in t , consume the residual net worth, $c_t^e = (1 - \gamma_t) \varrho v_t$, where the complementary fraction $(1 - \varrho)$ is transferred in lump-sum taxes to households.

Finally, we assume that the parameter γ_t follows

$$\log(\gamma_t) = \rho_\gamma \log(\gamma_{t-1}) + (1 - \rho_\gamma) \log(\gamma) + \epsilon_{\gamma,t},$$

where $\rho_\gamma \in (0, 1)$, and $\epsilon_{\gamma,t} \sim \text{iid}(0, \sigma_\gamma)$. Christiano *et al.* (2009b) interpret variations in γ_t as movements in the value of assets that are not obviously linked to movements in fundamentals.⁸ Consequently, a drop in γ_t can be viewed as a decrease in the value of entrepreneurs' assets, which will have spillover effects on the credit market, the external finance premium, and the rest of the economy.

⁷It is assumed, though, that the rate of birth of entrepreneurs equals the mortality rate, in order to keep the number of entrepreneurs constant.

⁸Nolan and Thoenissen (2009) appeal to the former Federal Reserve chairman Alan Greenspan's remark about "irrational exuberance", concerning the stock market boom in the U.S in 1996.

2.3 Capital Producer

At the end of period t , after production has taken place, a competitive capital producer buys the existing capital stock in order to combine it with a portion of final goods, denoted as aggregate investment i_t , and produces the new stock of capital to be used in period $t + 1$, \tilde{k}_{t+1} . Following Christiano *et al.* (2009b), we assume that the capital producer faces investment adjustment costs denoted by $\Phi(i_t/i_{t-1})$, where $\Phi(\cdot)$ is an increasing and concave function and $\Phi(0) = 0$. Thus, \tilde{k}_{t+1} evolves as follows

$$\tilde{k}_{t+1} = (1 - \delta(u_t)) \tilde{k}_t + \left[1 - \Phi\left(\frac{i_t}{i_{t-1}}\right) \right] i_t. \quad (9)$$

The new capital stock is sold to the entrepreneurs, yielding the profit maximization problem for the capital producer

$$\max_{i_t} \mathbf{E}_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left\{ Q_t \tilde{k}_{t+1} - Q_t (1 - \delta(u_t)) \tilde{k}_t - P_t i_t \right\}, \text{ subject to (9).}$$

The presence of adjustment costs allows for a variable price of capital, which in turn contributes to the volatility of the net worth. In equilibrium, the relative price of capital, $q_t \equiv Q_t/P_t$, is given by

$$q_t = \left[1 - \Phi\left(\frac{i_t}{i_{t-1}}\right) + \beta \mathbf{E}_t \left\{ \frac{q_{t+1}}{q_t} \Phi' \left(\frac{i_{t+1}}{i_t} \right) \left[\frac{i_{t+1}}{i_t} \right]^2 \right\} \right]^{-1}. \quad (10)$$

2.4 Final Good Producers

The final good, y_t , used for consumption and investment, is produced in a competitive market by combining a continuum of intermediate goods indexed by $j \in [0, 1]$, via a typical Dixit-Stiglitz aggregator. The maximization of profits yields a sequence of input demand

functions of the form

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_p} y_t, \quad (11)$$

where $y_{j,t}$ denotes the overall demand addressed to the producer of intermediate good j , θ_p is the input demand elasticity, and $P_{j,t}$ is the price of the intermediate good produced by firm j .⁹

2.5 Intermediate Good Sector

Production Function Type- j intermediate firm produces a differentiated good by assembling services of labor and capital, $\ell_{j,t}$ and $k_{j,t}$, respectively. Capital services are rented from the entrepreneur who owns the capital stock. Type- j firm's total labor input, $\ell_{j,t}$ is composed of household labor, $\ell_{j,t}^h$, and entrepreneurial labor, $\ell_{j,t}^e$, according to

$$\ell_{j,t} = [\ell_{j,t}^h]^\Omega [\ell_{j,t}^e]^{1-\Omega}.$$

Type- j intermediate good is produced with the constant return to scale technology

$$y_{j,t} = \ell_{j,t}^{1-\alpha} k_{j,t}^\alpha. \quad (13)$$

Each monopolistic firm chooses capital and labor services to minimize its real production cost subject to the production technology (13), taking w_t , w_t^e and z_t as given.

Price Setting Prices are sticky in the sense introduced by Calvo (1983). That is, at each period of time a firm faces a constant probability, $1 - \alpha_p$, of being able to re-optimize its

⁹The production function is of the form

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\theta_p-1}{\theta_p}} dj \right)^{\frac{\theta_p}{\theta_p-1}}. \quad (12)$$

price. Even if the price cannot be re-optimized, it can be partially indexed to the most recently available inflation measure, π_{t-1} .

Let $P_{j,t}^*$ denote the nominal price chosen in time t and $y_{j,t+k}^*$ the demand for good j , k quarters after the last price re-optimization. Therefore, firm j selects $P_{j,t}^*$ so as to maximize the present discounted sum of profit streams, subject to its production technology and its input demand function. The first order condition is given by

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \alpha_p)^k \lambda_{t+k} \frac{y_{j,t+k}^*}{P_{j,t}^*} \left\{ \frac{\delta_{t+k}^p p_{j,t}^*}{1 + \pi_{t+k}} - \mu_p s_t \right\} = 0, \quad (14)$$

where $p_{j,t}^* \equiv P_{j,t}^*/P^*$, $1 + \pi_{t+k} \equiv P_{t+k}/P_t$, $\mu_p \equiv \theta_p/(\theta_p - 1)$ denotes the mark-up of the monopolistic firm, and δ_{t+k}^p equals $\prod_{j=t}^{T-1} (1 + \pi)^{1-\gamma_p} (1 + \pi_j)^{\gamma_p}$ when $k > 0$ and 1 otherwise, where $\gamma_p \in (0, 1)$ is the degree of partial indexation.

2.6 Monetary Policy

The nominal interest rate follows a Taylor rule whenever such rule prescribes a non-negative level for the central bank's target interest rate. If this is not the case, then the central bank simply fixes its target rate equal to zero. Following Reifschneider and Williams (2000) and Bodenstein *et al.* (2009), we introduce the concept of a notional nominal interest rate, R_t^{not} in gross terms, which is subject to the rule

$$\frac{R_t^{not}}{\bar{R}} = \left(\frac{R_{t-1}^{not}}{\bar{R}} \right)^{\rho_R} \left[\left(\frac{1 + \pi_t}{1 + \pi} \right)^{a_\pi} \left(\frac{y_t}{y_t^f} \right)^{a_y} \right]^{1-\rho_R}. \quad (15)$$

Here $\rho_R \in (0, 1)$ denotes the interest rate smoothing parameter, a_π is the elasticity of R_t^{not} w.r.t. inflation deviations, a_y is the elasticity of R_t^{not} w.r.t. the output gap, and y_t^f is the output level that would prevail in the absence of nominal rigidities and financial frictions. Notice that \bar{R} denotes the steady-state level of the gross nominal interest rate, determined

by $(1 + \pi)\beta^{-1}$. The actual short-term gross nominal interest rate implemented by the central bank, R_t , is chosen according to

$$R_t = \max(1, R_t^{not}). \quad (16)$$

2.7 Fiscal Policy

The fiscal authority purchases final goods (g_t), raises distortionary labor income taxes (τ_t^w), capital income taxes (τ_t^z), lump-sum taxes (Υ_t), and issues debt (d_{t+1}), consisting of one-period nominal discount bonds. The period t government budget constraint is

$$\frac{d_{t+1}}{R_t} = \frac{d_t}{1 + \pi_t} + g_t - \Upsilon_t - \tau_t^w (w_t \ell_t^h + w_t^e \ell_t^e) - \tau_t^z z_t k_t.$$

Like Galí *et al.* (2007) and Bilbiie *et al.* (2009), we assume that the government budget constraint is balanced in the steady-state, meaning that the level of public debt is zero in the long run ($d \equiv 0$). In order to stabilize the public debt, we assume that the steady-state percent deviations of lump-sum taxes (or transfers) follow the rule

$$\hat{\Upsilon}_t = \tilde{a}_d \hat{d}_t + \tilde{a}_g \hat{g}_t, \quad (17)$$

where $\hat{d}_t \equiv d_t/y$ denotes the debt-to-GDP ratio, and other hatted variables denote percentage deviations from the steady-state. Finally, \tilde{a}_d and \tilde{a}_g are positive constants. In addition, we assume that the steady-state percent deviations of the income tax rates may have three

different patterns

$$\hat{\tau}_t^s = \begin{cases} 0 & \text{so } \tau_t^s = \tau^s \\ \chi_d^s \hat{d}_t + \chi_y^s \hat{y}_t & \text{pro-cyclical taxation} \\ \rho_{\tau_s} \hat{\tau}_{t-1}^s + \epsilon_{\tau_s,t} & \text{fiscal stimulus} \end{cases} \quad \text{for } s = \{w, z\} \quad (18)$$

where $s = \{w, z\}$ refers to the labor and capital income, respectively, χ_d^s and $\chi_y^s \geq 0$ are positive constants, ρ_{τ_s} defines a persistence parameter, and $\epsilon_{\tau_s,t}$ is tax rate shock. The first specification simply dictates that the type- s tax rate is constant. The second specification indicates that the type- s tax rate is pro-cyclical and it is adjusted to debt ratio deviations. Finally, the third specification appeals to a fiscal stimulus based on a type- s tax rate cut. Last but not least, we assume that the steady-state percentage deviations of government spending evolve exogenously according to a $ARMA(p, q)$ process

$$A(L)\hat{g}_t = B(L)\epsilon_{g,t}$$

where $\epsilon_{g,t}$ is the government spending shock. The $A(L)$ lag-polynomial allows us to model \hat{g}_t as a strict monotonic process (as an $AR(1)$), or as a hump-shaped sequence (as an $AR(2)$). The $B(L)$ lag-polynomial allows implementation delays in government spending to be modeled. We provide further details about the sequence of government spending in the following sections.

2.8 Resource Constraint and Equilibrium

The production of the final good is allocated to investment, total private consumption by households and entrepreneurs, public spending, and monitoring costs paid by lenders,

$$y_t = i_t + c_t + c_t^e + g_t + \mu G(\bar{\omega}_t) r_t^k q_{t-1} \tilde{k}_t.$$

In the symmetric equilibrium, all entrepreneurs, households, and firms are identical and make the same decisions. In addition, equilibrium on the labor market yields $\int_0^1 \ell_{j,t} dj = \ell_t^h$. The symmetric equilibrium is characterized by an allocation $\{y_t, c_t, c_t^e, i_t, \ell_t, k_t, \tilde{k}_t, n_t\}$ and a sequence of price and co-state variables $\{\pi_t, r_t, r_t^k, q_t, \pi_t^w, z_t, \lambda_t\}$ that satisfies the optimization conditions in each sector, the monetary and fiscal rules, and the stochastic shocks.

3 Methodology

3.1 Calibration

The model's parameters are calibrated to fit the quarterly frequency. Table 1 presents the calibrated values for the parameters related to households, firms, and the economic authorities.

[insert Table 1 here]

The subjective discount factor, β , is set to 0.99, which implies an annual real interest rate of 4 per cent. The Frisch elasticity of labor supply, $\varpi_w^{-1} \equiv \nabla_{\ell} / (\ell \nabla_{\ell \ell})$, is set to unity. The degree of habit on consumption, b , is set to 0.63, while the inverse of the inter-temporal elasticity of substitution, σ , is set to 0.2. All these values are taken from Christiano *et al.* (2009b).

Regarding production, the capital share in the intermediate sector, α , is set to 0.35; the depreciation capital rate, δ , equals 0.03, as in Christiano *et al.* (2009b); the investment adjustment cost, $\varkappa \equiv \Phi''(1)$, is calibrated to 5.86, following Smets and Wouters (2007); the elasticity of the utilization rate of capital, $\vartheta_u \equiv u\delta''(u)/\delta'(u)$, is calibrated to 0.31^{-1} ,

similar to Queijo von Heideken (2008). When it comes to price setting, we assume that the elasticity of substitution between intermediate goods, θ_p , is 11, which implies a price mark-up of 10 per cent. Similarly, the elasticity of substitution between labor types, θ_l , is set to 21, which translates into a wage mark-up of 5 per cent. The degrees of price and wage rigidities, α_p and α_w , are set at 0.67 and 0.68, respectively, implying that the average time between price or wage re-optimization is six months. Price and wage indexation parameters, γ_p and γ_w , are set to 0.75 and 0.70, respectively. All these values have been estimated by Christiano *et al.* (2009b). The steady-state inflation, π , is zero.

Table 2 shows the calibrated values of the parameters related to the financial sector, which are taken from Bernanke *et al.*'s (2000) results.

[insert Table 2 here]

The proportion of entrepreneurial wages in terms of income is set to 0.01, implying a value of $\Omega = 0.9846$. The steady-state share of capital investment that is financed by the entrepreneur's net worth, $x = \tilde{k}/n$, is calibrated to 2, meaning that the steady-state leverage ratio amounts to 50 per cent. The steady-state external finance premium, $\tilde{r} = r^k/r$, is set to $1.02^{0.25}$, corresponding to an annual risk spread of 200 basis points, equal to the sample average spread between the business prime lending rate and the three-month Treasury bill rate. Finally, the annual business failure rate, $F(\bar{\omega})$, is set to 3 per cent. It is assumed that the idiosyncratic productivity shock, ω_t , has a log-normal distribution with positive support, and an unconditional expectation of 1. These moments help to determine the steady-state survival probability of entrepreneurs, γ , which is set to 0.98, the monitoring costs to realized payoffs ratio, μ , at 0.12, the steady-state variance of the entrepreneurs' idiosyncratic shock, σ_ω , which is 0.28, and the steady-state idiosyncratic threshold of 0.50.¹⁰

¹⁰In technical terms, $\bar{\omega}$, σ_ω , γ and μ are chosen so as to satisfy the following system of steady-state

Table 3 shows the calibrated values of the parameters for monetary and fiscal policies.

[insert Table 3 here]

The interest rate smoothing parameter, ρ_R , is calibrated to 0.88; the elasticity of the notional interest rate with respect to inflation, a_π , is set to 1.85; and the elasticity of the interest rate with respect to the output gap, a_y , is set to 0.313/4. Once again, these values are taken from the estimations of Christiano *et al.* (2009b). When it comes to the fiscal policy, the elasticity of lump-sum taxes with respect to debt and government spending are set to 0.33 and 0.10 respectively. These values corresponds to Galí *et al.*'s estimates (2007). The steady-state labor and capital income tax rates are set at 0.28 and 0.36 respectively, as suggested by Drautzburg and Uhlig (2010). They are assumed to be constant in the benchmark calibration ($\psi_y = \psi_d = 0$). The steady-state share of government purchases in total output is calibrated to 0.19, which corresponds to the last decade's historical average.

3.2 Zero Lower Bound: Solution Strategy

The zero lower bound constraint, described in Equation (16), introduces an important non-linearity into the system. Had this constraint not appeared, we could have proceeded to analyze the dynamics of the economy using the linear rational expectations solution that can be derived from the system described above. In fact, all the model equations can be linearized except for the nominal interest rate, which imposes different dynamics depending on whether the zero lower bound constraint is binding or not.

We used the piecewise-linear approach described in Bodenstein *et al.* (2009) to solve for the

equations:

$$F(\bar{\omega}) = 0.03/4; \quad x = 1 + \Gamma_\omega(\bar{\omega})[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]/[[1 - \Gamma(\bar{\omega})][\Gamma_\omega(\bar{\omega}) - \mu G_\omega(\bar{\omega})]],$$

$$(x - 1)/x\check{r} = \Gamma(\bar{\omega}) - \mu G(\bar{\omega}); \quad n = \gamma[nr^k x[1 - \mu G(\bar{\omega})] - rn(x - 1)] + (1 - \tau)w^e.$$

model dynamics, which is numerically equivalent to the method employed by Eggertson and Woodford (2003).¹¹ To be specific, we linearized all the model equations around the non-stochastic steady-state, except for the monetary policy representation.¹² We then assumed that a set of exogenous shocks hit the economy and depressed the nominal interest rate so that the zero lower bound was reached at Period 2, and remained in place for T periods. The zero lower bound horizon T is determined by the time- T value of the notional interest rate, which must satisfy the condition

$$R_T^{not} < 1 \leq R_{T+1}^{not}. \quad (19)$$

In terms of percentage deviation from the steady-state, this condition becomes $\hat{R}_T^{not} < -R \leq \hat{R}_{T+1}^{not}$, where $R = 1/\beta$, which is the steady-state level of the gross nominal interest rate.¹³ We solved the log-linear rational expectations model by using the AIM algorithm (see Anderson and Moore, 1985) and we ensured that Blanchard and Kahn's (1980) conditions was fulfilled. The structural model was therefore transformed into a state space transition system that could be used to compute the model's dynamics. The piecewise-linear system conforms to two different dynamic structures. First, before Period 2, the nominal interest rate follows a Taylor rule. From Period 2 to T , the zero lower bound constraint is binding and the interest rate is equal to $-R$. The dynamics were derived using backward-induction of the state-space system. This implies that the model's impulse response functions (IRFs) are deterministic, in the sense that agents make their decisions knowing that in period $T + 1$ the interest rate will follow the Taylor rule path. Second, from period $T + 1$, the zero

¹¹There are different approaches to fixing the non-linearity problem resulting from the zero lower bound constraint. For instance, the level of the nominal interest rate could be augmented by additive disturbances that ensure that the zero lower bound constraint is hit for T periods (Reifschneider and Williams, 2000). In this paper, we adopt the view that agents have perfect foresight on the duration of the liquidity trap. This methodology has the advantage that agents can adjust their decisions to fiscal expansions which are announced several quarters in advance.

¹²See appendix B.

¹³According to the calibration of the preceding section, this equals 4 per cent in annual terms.

lower bound constraint is not binding and the Taylor rule operates. The dynamics are then derived by using the VAR representation of the model. Condition (19) is used to pick up the value of T .¹⁴

4 Effects of a Fiscal Stimulus during a Deep Recession

The purpose of this section is to highlight to what extent a fiscal stimulus can help the economy to recover from financial turmoil. First of all, we have to characterize both the ingredients of the deep recession and the instruments of the fiscal stimulus packages. The economic downturn is driven by two simultaneous shocks: a temporal 10 per cent decrease in the entrepreneurs' survival probability (e.g. γ_t falls from 0.98 to 0.88), and a negative preference shock, ε_t . We chose to combine these shocks for two reasons. First, the presence of the two disturbances increases the probability that the interest rate hits its zero bound.¹⁵ Second, these two shocks generate both a reduction in consumption and investment that characterizes a deep recession. Indeed, a negative preference shock reflects an increase in the household's desire to save, which in turn increases the demand for bonds and puts downward pressure on consumption, output, and the nominal interest rate.¹⁶ As for the negative financial shock, it reduces the net worth, increasing in turn the external finance premium and depressing investment. The size of the two shocks was selected to ensure that the nominal interest rate hit its zero-floor at Quarter 2.

¹⁴The methodology is detailed in the appendix. Bodenstein *et al.* (2009) also present a detailed description of this solution algorithm.

¹⁵Reifschneider and Williams (2000), Schmitt-Grohé and Uribe (2007), and Amano and Shukayev (2009), among others, argue that in order to engage the zero lower bound constraint, single shocks would have to be quite big with respect to what is usually estimated. Thus, the probability that the zero lower bound constraint binds with a single shock is very low.

¹⁶Bodenstein *et al.* (2009), Christiano *et al.* (2009b), and Drautzburg and Uhlig (2010) used shocks with these characteristics to induce the zero lower bound in their analyses. Drautzburg and Uhlig (2010) assumed that there was an interest rate spread shock between the risk-free rate and the rate perceived by households. A shock of this kind has exactly the same effect as a preference shock, not only because both shocks appear in the Euler equation in a similar way, but because both of them create an increase in the desire to save inducing a drop in consumption.

For our fiscal stimulus packages, we consider two types of policy: *a)* the government decides to increase its public expenditures and; *b)* it decides to decrease either the labor or capital income tax rates.

4.1 Government Spending Expansion

In this subsection, we first assess the model dynamics resulting from an exogenous increase in government purchases, and second, we quantitatively investigate the factors that modify the efficiency of this fiscal policy on output.

4.1.1 Model dynamics

Figure 1 compares the IRFs of some macro-variables in the context of two scenarios. The *baseline* scenario illustrates the severe recession without any fiscal stimulus package. The *gov. spending* scenario corresponds to the baseline case enriched by a positive government spending shock. We assume that government spending follows an $AR(1)$ process, with a persistence coefficient equal to $\rho_g = 0.945$, as suggested by Christiano *et al.* (2009b). Since the recession is particularly deep in the baseline scenario, for the sake of simplicity we assume a substantial increase in government spending, rising from 19 per cent of GDP, its steady-state value, to 23 per cent of GDP during the impact period.¹⁷

[insert Figure 1 here]

In the baseline scenario, the duration of the liquidity trap is 13 quarters. In our example, the government spending expansion shortens the duration of the zero lower bound by one quarter, and the fiscal stimulus effectively reduces the drop in production. However, as the

¹⁷In this section, we assume that government spending increases at the same time as the recessionary shocks hit the economy. In the next section, we relax this assumption to allow implementation delays in fiscal policy.

effects of the stimulus fade out, output eventually converges towards its baseline path in the absence of the stimulus. The rise in the debt-to-GDP ratio is financed by an increase in lump-sum taxes (or equivalently, a reduction in lump-sum transfers). The public deficit increases at impact, due to both the rise in government spending and the decrease in income tax revenues that follows the recession.

The most emblematic effect of government spending in the model is its impact on the external finance premium and the rest of the financial sector. An increase in government spending expands aggregate demand, which translates into a lesser decrease of inflation with respect to the baseline scenario. As noted earlier, a lower drop in inflation implies that the entrepreneurs' real debt increases by less, due to Fisher's debt-deflation channel. This has a positive effect on the net worth of entrepreneurs, which reduces the moral hazard problem in the financial sector. The lower decrease in the net worth induces a smaller increase in the external finance premium, which provides positive incentives to invest.¹⁸ Notice that the real interest rate is actually lower under the *gov. spending* scenario. This is explained by the fact that the nominal interest rate is constrained by its zero lower bound, and is thus not responsive to the lower decrease in inflation caused by the fiscal stimulus. As a result, investment is not crowded out but is actually stimulated by government spending, since the external finance premium is lower. In sum, investment is cheaper under the *gov. spending* scenario than in the baseline one, and does not fall as much when the fiscal stimulus is in place.

When it comes to consumption, Figure 1 does not show a sizable impact of government spending. On the one hand, the lesser decrease in output might also help consumption to fall by less. On the other hand, the economy is subject to the Ricardian equivalence, and thus an increase in government spending crowds out consumption. These two effects offset

¹⁸A similar explanation can be found in Fernández-Villaverde (2010).

each other in the final response of consumption, yielding the apparent lack of changes in this variable.

To conclude, Fisher’s debt-deflation channel is a main component in explaining the effects of government spending on output and investment. In addition, the crowding-out effect on investment following a positive government spending shock is lessened by the presence of the zero lower bound constraint on the nominal interest rate. To disentangle the role of these two factors in the effect of a government spending expansion on output, we resort to the spending multiplier.

4.1.2 Counterfactual Exercises on the Spending Multiplier

We now determine how the spending multiplier is altered by the presence of credit market imperfections and a zero nominal interest rate. The impact spending multiplier in period $t + k$ is determined by the marginal change in output in period $t + k$ due to an increase in government spending in period t , i.e. $\frac{dy_{t+k}}{dg_t}$.¹⁹ The upper panel of Figure 3 displays the determinants of the government spending multiplier. The *benchmark* spending multiplier is the *gov. spending* scenario displayed in Figure 1, i.e. where the nominal interest rate binds its zero lower bound and financial frictions are present.

[insert Figure 3 here]

¹⁹The impact spending multiplier can be computed by using the IRFs of the baseline and *gov. spending* scenarios. Let \hat{x}_t^0 denote the response of variable x_t , in terms of percentage deviations from the steady-state, given the baseline scenario. Similarly, let \hat{x}_t^{fis} denote the response of the same variable under the gov. spending scenario. Since x_t can be approximated by $x(1 + \hat{x}_t)$, where x is the steady state value of x_t , then the net effect of a fiscal expansion on x_t is given by $x\hat{x}_t^{net}$, where $\hat{x}_t^{net} \equiv \hat{x}_t^{fis} - \hat{x}_t^0$. The Quarter k impact spending multiplier is thus given by

$$\frac{dy_{t+k}}{dg_t} \equiv \frac{y}{g} \frac{\hat{y}_{t+k}^{net}}{\hat{g}_t^{net}}.$$

It is worth noticing that this “net fiscal multiplier” corresponds exactly to the multiplier that would be obtained in the absence of negative financial and preference shocks. Consequently, it can be compared to the literature since it measures the effect of a spending expansion on output.

The benchmark spending multiplier is slightly larger than that at impact, with a value of 1.09.²⁰ Two years after the stimulus, the benchmark multiplier reaches 0.60. It appears that output increases by slightly more than a one-by-one basis with respect to government spending when both the zero lower bound and financial frictions are present. The intuition behind this result is given by the positive net effect on investment provided by the government spending expansion, as explained in the preceding section.

Role of the Zero Lower Bound One may ask if the benchmark spending multiplier is greater than one due only to the presence of the zero lower bound. The line *FA with no-ZLB* in Figure 3 tackles this point. In such a case, the fiscal multiplier equals 0.87 at impact and 0.33 after two years. The lower-than-one value of the multiplier can be explained as follows. A fiscal stimulus, like any other positive shock to aggregate demand, tends to increase inflation. If the nominal interest rate is allowed to be negative during the deep economic recession, it would be relatively less negative after the stimulus. Thus, the real interest would increase more sharply than when the nominal rate is not constrained by its zero-floor. Thus, investment would become relatively more expensive, following the classic crowding-out effect. This example illustrates the claim that the fiscal multiplier tends to be larger when the zero lower bound constraint is binding (Christiano *et al.*, 2009a).

Role of Financial Frictions What is the impact of credit market imperfections on the efficiency of government spending? Figure 3 shows that the government spending multiplier is reduced in comparison with the benchmark configuration when we consider an economy that does not feature financial imperfections (shown by the line *No-FA with ZLB*). Indeed, it equals 1.01 at impact and reaches 0.43 after two years. This result can be explained by the

²⁰In a model without financial frictions and the zero lower bound constraint, this value reaches 0.85. Zubairy (2009) estimates a value greater than one, but her model embeds deep habit formation in public and private consumption. Our result is consistent with Cogan *et al.* (2010).

fact that in the no-financial accelerator model, entrepreneur decisions are not conditional on the external financial premium. In this case, government spending has no relevant impact on the net worth or the risk premium. The Fisher debt-deflation channel is shut down, and thus the potency of government spending to reduce the risk premium is nil, which also forecloses the additional incentives for investment coming from this mechanism. This result is consistent with Cogan *et al.* (2010).

Extended zero-rate duration We also tackle the possibility that the monetary and fiscal authorities undertake a policy mix corresponding to a zero-interest-rate commitment and a fiscal expansionary policy. To do so, we assume that the economic authorities implement an increase in government purchases along with the announcement of keeping the future nominal interest rate at zero for a longer period than prescribed by the monetary policy rule (16).²¹ We assume that the central bank commits to keeping the nominal interest rate at zero for 20 periods, that is 6 periods more than the recommendation of the Taylor rule. Figure 3 shows that the *net effect* of government spending under this policy mix is greater, as shown by the spending multiplier *FA with Zero-rate commitment*.²² In particular, the multiplier equals 1.18 at impact and 0.75 after two years, with a peak value of 1.23 in the second quarter. Since agents expect a low interest rate for a longer period of time, they respond to the adverse economic conditions by smoothing their consumption and investment. This yields an even lower drop in inflation than in the *gov. spending* scenario of Figure 1, which in turn produces a lower increase in the real interest rate. On the one hand, a smaller

²¹Eggertsson and Woodford (2003) discuss the implications for the economic dynamics of managing public expectations about the future path of the interest rate. The argument, is that economic agents make their decisions taking into account their expectations about the future policy that the central bank is likely to implement. The bottom line of their discussion is that, according to their optimal path for monetary policy, the central bank should announce and maintain the nominal interest rate at very low levels for longer than would normally be prescribed by a strict inflation targeting rule. This is explained by the fact that, when agents expect a period of abundant liquidity, accompanied by a rise in inflation expectations, they will start to increase consumption and investment from today.

²²The net effect of fiscal policy is computed by subtracting the pure effects of the zero-rate commitment policy from the final effects on economic activity that result from the policy mix.

increase in the real interest rate implies that service charges on the debt that entrepreneurs have to pay are lower. On the other hand, a smaller decrease in inflation rises by less the real debt of entrepreneurs through the debt-deflation channel. These two effects lessen the crowding-out effect on investment and make the recession milder. Therefore, this result tells us that a policy mix, characterized by a zero interest rate commitment and an increase in government spending, is the most effective measure to increase output.

4.2 Distortionary Tax Cut

We now turn to investigating the impact of a temporal reduction in the labor and capital income tax rates. We assume in this case that the percentage deviation of the income tax rate, $\hat{\tau}_t^s$, for $s = \{w, z\}$, follows an $AR(1)$ process, with a persistence parameter ρ_τ equal to 0.80. We assume that the labor income tax rate temporarily shifts from 0.28, its steady-state value, to 0.25, while the capital income tax rate shifts from 0.36 to 0.31. These variations correspond to a decrease of 15% in both tax rates.

4.2.1 Model dynamics

Figure 2 compares the IRFs of an economy that suffers from a deep recession with and without an income tax cut policy. Notice that lowering income tax rates does not affect the zero lower bound duration, which still binds for 13 quarters. Income tax policy does, however, have a different effect on inflation and output, as can be seen by comparing the *labor tax* and *capital tax* scenarios in Figure 2.

[insert Figure 2 here]

In particular, we find that a reduction in the capital income tax rate increases the returns to capital which in turn increases the net worth. Investment is thus stimulated, which entails a

positive impact on inflation. The damped decrease in inflation stimulates the economy even further through the debt-deflation channel. Consequently, the net increase in investment implies that the recession is milder.

In contrast, the effects on inflation and output of a labor income tax cut are reversed. A labor income tax cut increases the returns on labor, leading households to increase their labor supply. In turn, this implies a reduction in nominal wages. This diminishes the labor costs that firms are facing, which will tend to decrease inflation even further than under the baseline scenario. This makes the cost of borrowing higher, through the debt-deflation channel. Therefore, investment decreases by more when the government undertakes a labor income tax cut policy, yielding a worse recession.

Finally, notice that the debt ratio rises by more when a labor income tax cut is performed. This is due to the high share of labor tax revenues on the government budget constraint.

4.2.2 Counterfactual Exercises on the Tax Multipliers

We now disentangle the role of financial frictions and the zero lower bound on the economic efficiency of the previously proposed tax cut policies. To do so, we employ the Quarter k impact total tax revenue multiplier, which is determined by the marginal change in output in period $t+k$ due to a reduction in the total tax revenue ($T_t = \tau_t^w(w_t \ell_t^h + w_t^e \ell_t^e) + \tau_t^z z_t k_t$) in period t , i.e. $\frac{dy_{t+k}}{dT_t}$.²³ This fiscal multiplier measure, used by Blanchard and Perotti (2002) and Zubairy (2009), is useful for making comparable statements about the effect on output of different tax policies. For the sake of simplicity, we will refer to the total tax revenue multiplier that is driven by a shock in the labor income tax rate as the *labor tax multiplier*.

²³The total tax multiplier is computed as

$$-\frac{dy_{t+k}}{dT_t} \equiv -\frac{\hat{y}_{t+k}^{net}}{\tau^z z k / y(\hat{\tau}_t^{z,net} + \hat{z}_t^{net} + \hat{k}_t^{net}) + \tau^w w \ell^h / y(\hat{\tau}_t^{w,net} + \hat{w}_t^{net} + \hat{\ell}_t^{h,net}) + \tau_t^w w^e / y(\hat{\tau}_t^{w,net} + \hat{w}_t^{e,net})}.$$

A similar logic applies to our *capital tax multiplier* measure. The upper-right and bottom panels of Figure 3 display the labor and capital tax multipliers for different configurations. As in the previous section, the *benchmark* fiscal multipliers correspond to the *labor tax* and *capital tax* scenarios displayed in Figure 2, i.e. when the zero lower bound and financial frictions are present.

Interestingly, the benchmark labor tax multiplier is negative at impact, and becomes positive at Quarter 6, with a peak of 0.12 in Quarter 12. In contrast, the benchmark capital tax multiplier is positive at impact and it reaches a peak of 0.84 in Quarter 5. It is worth noting, however, that the value of these tax multipliers are small in comparison with the benchmark spending multiplier. This can be explained by the fact that, even if they provide more incentives to consume and invest, these policies do not have the straightforward impact of an aggregated demand shock that directly fuels the economy.

Role of the Zero Lower Bound It appears that the labor tax multiplier is higher than the benchmark when the zero lower bound constraint is not hit. Indeed, it becomes positive, equaling 0.07 at impact, with a peak at 0.26 after two years. As suggested earlier, a reduction in the labor tax rate drives down inflation through the reduction in nominal wages. Consequently, all other things being equal, the real interest rate should increase. However, by allowing the nominal interest rate to follow the Taylor rule, and thus become negative, the real interest rate would be actually lower than in the *labor tax* scenario of Figure 2, thus stimulating the economy. The zero lower bound constraint prevents this happening, and this is why a labor income tax cut policy is not effective during a liquidity trap.

When it comes to the capital tax multiplier, the previous mechanism is reversed, since this fiscal policy has a positive net effect on inflation. Therefore, as for a government spending expansion, the absence of the zero lower bound constraint decreases the size of the multiplier, meaning that the real interest rate is higher than in the *capital tax* scenario of Figure 2.

The capital tax multiplier is thus lower than the benchmark, equaling just 0.08 at impact and peaking at 0.41 in Quarter 12.

Role of Financial Frictions Interestingly, when financial frictions are omitted, the labor tax multiplier is bigger than the benchmark. Again, this can be explained by the effect of a labor income tax cut on inflation. When financial frictions are present, the stronger decrease in inflation due to this policy is followed by a "negative" debt-deflation effect that increases the value of debt for entrepreneurs, reducing the value of collateral and increasing the external finance premium. In the absence of financial frictions, this recessionary effect on investment and output is shut down.

On the other hand, the capital income tax multiplier is smaller than the benchmark in the absence of financial frictions. This is due to the fact that, without financial frictions, the role of collateral is irrelevant for the cost of credit, and does not alter the dynamics of investment.

Extended zero-rate duration We now turn to assessing how a zero-nominal-interest-rate commitment modifies the value of the fiscal multipliers.²⁴ The change in the size of the capital tax multiplier during the policy mix are negligible in comparison to the benchmark case. However, the labor tax multiplier is, as expected, lower than the benchmark when the period of the liquidity trap is extended. In this case, consumers and investors expect to see high real interest rates for a longer period of time, due to the extended zero-rate commitment and the stronger decrease in inflation put in place by the labor tax policy. Thus, investment is discouraged even further than in the benchmark case.

²⁴We follow a similar strategy to that in Section 4.1.2 to compute the net effect of fiscal policy.

5 Government Spending Multiplier and Fiscal Strategies

The goal of this section is to further investigate how credit market imperfections affect the size of the government spending multiplier, when the government adopts different fiscal strategies. To do so, we proceed in two steps. First, we look at the size of the spending multiplier when distortionary taxes respond to debt and output. Second, we allow for delays in the implementation of the spending expansion.

5.1 The Spending Multiplier with Different Taxation Policies

Financing a large increase in government spending can require the raising of distortionary taxes that are known to reduce the effectiveness of fiscal policy, since the incentives for consumption and investment may be weakened.

In this section, we re-compute the government spending multiplier for two different distortionary tax rules. The results are displayed in Figure 4, in which the upper-left panel corresponds to a model in which both financial frictions and the zero lower bound are present; the upper-right panel excludes financial frictions but maintains the zero lower bound constraint; and, the bottom panel removes the zero lower bound while it maintains financial frictions. The *benchmark* spending multiplier corresponds to the case where both the labor and capital income tax rates are constant, implying that $\tau_t^w = 0.28$ and $\tau_t^z = 0.36$ at each date. For the *time-varying labor tax* spending multiplier we assume that the capital income tax rate is constant, while the labor income tax rate follows the rule defined in Equation (18), in which the tax rate responds to the level of debt and is pro-cyclical. Finally, for the *time-varying capital tax* spending multiplier, we assume that the labor income tax rate is constant, while the capital income tax rate follows Rule (18). Using Zubairy's (2009) estimates, we assume that $\chi_y^w = 0.11$ and $\chi_d^w = 0.02$, while $\chi_y^z = 0.13$ and $\chi_d^z = 0.01$.

[insert Figure 4 here]

We show that the value of the government spending multiplier is smaller when the government raises capital income taxes to finance its debt, instead of keeping them constant. For instance, the spending multiplier in the baseline is 1.05, while it is 0.92 when τ_t^z follows Rule (18). This can easily be explained. Capital income taxes increase due to the rise in output (driven by the government spending stimulus) and the rise in debt. This reduces the rental rate of capital and therefore magnifies the crowding-out effect in investment generated by the positive government spending shocks. Consequently, investment is reduced and output increases by less after the rise in government purchase. Another interesting result emerges from Figure 4. The size of the spending multiplier is larger when the labor income taxes follows Rule (18). It reaches 1.17 at impact and peaks at 1.20 in Quarter 2. This indicates that a government spending expansion can be more efficient when it is financed by labor income taxes. In our case, the labor income tax strongly increases, led by the expansionary effect of the government spending expansion. As noted before, the rise in the labor income taxes drives inflation up, due to the increase in nominal wages. This increase in inflation reduces the real interest rate more and makes the debt deflation channel more active. These two standard effects stimulate investment and improve the efficiency of the government spending policy. This result contrasts with the finding of Erceg and Lindé (2010). Unlike these authors, we assume that labor tax responds both to debt and to output. Therefore, the increase in labor income taxes is sufficiently strong to raise inflation, stimulating investment through the zero lower bound and the debt-deflation channel.²⁵

This interpretation is confirmed by the middle and right panels of Figure 4. For a time-

²⁵We carried out the same exercise by assuming $\chi_y = 0$ and $\chi_y = 0.02$, consistent with the Erceg and Lindé (2010) rule. We obtained results similar to theirs. Indeed, since labor income taxes only respond to debt, they increased slightly. Therefore, the negative wealth effect dominated the increase in nominal wages. This implies that inflation is slightly smaller with constant wage taxation.

varying labor income tax, we show that the spending multiplier is close to the baseline case when the model does not feature financial frictions and it becomes smaller when the nominal interest rate can reach negative values. This suggests that the rise in inflation driven by both the government spending expansion and the rise in labor income tax helps to rescue the economy through the debt-deflation channel and the zero lower bound. When it comes to the capital income tax, the role of these two factors is different. After a positive government spending shock, inflation increases by less when capital income taxes increase. In the absence of financial frictions and a zero lower bound, this smaller increase in inflation is less painful, implying that the spending fiscal multipliers are close to the baseline case. To conclude, we have shown in this section that time-varying labor income taxes can improve the efficiency of the government spending shock, when they are pro-cyclical and the zero lower bound constraint is active. On the contrary, time-varying capital income taxes always reduce the value of the spending multiplier, due to their negative impact on inflation.

5.2 Delays in the Implementation of Government Spending

The time scale over in which a fiscal stimulus is implemented is a major concern when studying the final effects of fiscal policy on output. Ramey (2010) shows that government spending expansions are usually anticipated, at least several months in advance. The consensus among economists is that the longer it takes to increase government spending, the less effective it will be in raising output. The intuition behind this statement is quite simple. If the strong pulse of the stimulus comes at a time when the economy has already started to recover, the increase in government spending may raise the short and long run interest rates, eventually crowding-out investment.²⁶ This subsection aims at reviewing the effects of lagged implementation in government spending with and without financial frictions, while

²⁶For instance, Erceg and Lindé (2010) show, using a model featuring no financial frictions and a liquidity trap, that the fiscal multiplier can even be negative when the government spending expansion is undertaken too late.

the zero lower bound constraint on the nominal interest rate is active.

For this purpose, we assume that government spending, as a percentage deviation from its steady state-value, follows an $ARMA(2, q)$ process

$$\hat{g}_t = \rho_{g1}\hat{g}_{t-1} + \rho_{g2}\hat{g}_{t-2} + b_0\epsilon_{g,t} + b_1\epsilon_{g,t-1} + \dots + b_q\epsilon_{g,t-q}.$$

The two autoregressive terms allow \hat{g}_t to display a (concave) hump-shaped pattern, which reflects the fact that any realistic fiscal stimulus would need some time to reach its peak.

The moving average terms allow us to determine in which period the increase in government spending occurs.²⁷ For instance, if we assume that $b_4 > 0$ while all other $b_{q'}$ are equal to zero, this means that government spending will increase a year after its official announcement.

This in turn implies that, from period $t = 0$, all agents in the economy make their decisions knowing that in period $t = 4$ increases in government purchases will take place.²⁸

Taking into account implementation lags in government spending deter us from using the impact fiscal multiplier introduced in Section 4.1.2, since in most cases the percentage deviation of government spending will be zero at the impact period. Thus, we follow Zubairy (2009) and Uhlig (2010) by introducing an alternative measure of efficiency of government spending, known as the present value multiplier k -periods ahead (PVM_k) defined by

$$PVM_k = \frac{\mathbb{E}_t \sum_{j=0}^k \beta^j y_{t+k}^{net}}{\mathbb{E}_t \sum_{j=0}^k \beta^j g_{t+k}^{net}} = \frac{y}{g} \frac{\mathbb{E}_t \sum_{j=0}^k \beta^j \hat{g}_{t+k}^{net}}{\mathbb{E}_t \sum_{j=0}^k \beta^j \hat{g}_{t+k}^{net}},$$

where PVM_k measures the total discounted net effect of government spending on output k -periods ahead in time, from the perspective of an agent in period t . We introduce the

²⁷Schmitt-Grohé and Uribe (2008) adopt a similar specification to model anticipated shock. They show that this type of shocks explains a significant part of the business cycle.

²⁸Since the model solution method implies a deterministic pattern for the economic dynamics, an $ARMA(p, q)$ process for government spending allows us to introduce future shocks that are known by everybody from the initial period.

same negative net worth and preference shocks described in Section 4 in order to generate a deep recession in two versions of the model: one in which financial frictions are present, and another in which they are absent. The persistence parameters of government spending are set to $\rho_{g1} = 1.4$ and $\rho_{g2} = -0.45$.

Figure 5 displays the pattern of various government spending shocks that we assume in the analysis.

[insert Figure 5 here]

All of these processes are rescaled so that the total amount of public expenditure for every lagged structure is equal in present value terms. In order to keep the duration of the zero lower bound constraint constant across all implementation delays, we consider a modest increase in government spending. This assumption allow us to focus only on the effect of the delays themselves.

We first focus on the long run effects of a government spending expansion on output with different delays on its implementation. Table 4 shows the value of PVM_k for $k = 50$, for different lag structures and model environments.

[insert Table 4 here]

In accord with the results reported in Section 4.1.2, a government spending expansion always has a greater impact on output when financial frictions are present. Following the same logic as in the preceding sections, this is a signal that credit market imperfections along with the debt-deflation channel help to minimize the crowding-out effect of investment. A second result is that, at least in the long run, implementation delays decrease the size of the government spending multiplier. This might seem to confirm the standard view regarding the importance of implementing a government spending expansion as quickly as possible. In

the short run, however, the presence of financial frictions brings a completely new message about how this implementation should take place.

Figure 6 displays the value of PVM_k for different short run horizons for the two versions of the model.

[insert Figure 6 here]

First, consider the model without financial frictions. In that case, Figure 6 again shows that implementation delays decrease the value of the multiplier, with values that may even be negative when the government spending expansion is undertaken too late (close to the end of the liquidity trap). This result is in line with the results of Erceg and Lindé (2010).

Interestingly, in the model with financial frictions this result is reversed. Short delays in the implementation of government spending increase the value of the multiplier in the very short run. This apparently counter-intuitive result can be explained as follows. When government spending is expected to increase in, say, one year from now, agents adjust their inflation expectations upwards from the present period. This activates the debt-deflation channel by increasing the net worth and decreasing the risk premium. Thus, investment and output start increasing from today, even before the fiscal stimulus physically takes place, which eventually entails a positive effect on inflation. In a year from now, when government spending actually rises, output has already accumulated some gains from the expected stimulus, which makes the size of the present value multiplier much greater than one. Eventually, the value of the multiplier decreases in order to establish the ordering found in the long run, as summarized by Table 4. It is worth noticing, however, that even if short delays may imply big multipliers in the short run, long delays yield small multipliers. Thus, the presence of financial frictions tells us that, in addition to ensuring short delays when increasing public expenditures, the government should also announce with precision when, and by how much, it will increase expenditure. By doing so, the fiscal authority can exploit

an expectations effect that might provide incentives to the economy using the debt-deflation channel.

6 ARRA Implementation

In this section, we simulate the path of the ARRA in order to assess the importance of the output gains derived from the presence of financial frictions and the alternative duration periods of the liquidity trap. We refer to Cogan *et al.*'s (2010) ARRA approximations regarding the path of government spending from 2009 to 2013. As in the preceding sections, we assume that agents know in advance the sequence of government spending. In addition, we simulate the ARRA stimulus package for two different durations of the liquidity trap. In the first case, we assume that the nominal interest rate remains at its zero-floor from 2009Q1 to 2010Q2, that is 6 quarters in total. In the second case, we assume that the liquidity trap starts in 2009Q1 and remains in place until 2011Q4, or 12 quarters in total. Figure 7 displays the path of government spending according to Cogan *et al.* (2010), along with its impact on output, consumption, and investment, for the two assumed paths of the liquidity trap. In this exercise, we consider again two versions of the model: one with financial frictions (FA), and another without (NFA).

[insert Figure 7 here]

According to the models, the maximum impact of the ARRA fiscal stimulus on output occurs during 2010. To be precise, the rise in GDP reaches 0.61% in the FA model when the liquidity trap lasts for 6 quarters, and 0.87% when it lasts for 12 quarters. The NFA model, in which financial frictions are omitted, shows a smaller effect of the fiscal stimulus on output. This is explained by the different predictions that the two models make for the

impact on investment, as can be seen in the middle panels of Figure 7. As expected, the model with financial frictions predicts a higher impact on investment. The reasons behind this effect have been explored extensively in the preceding sections, in which the impact of the stimulus on inflation and the debt-deflation channel play a crucial role. In the case of consumption, the two models have predicted quite similar results.

The FA model also predicts that the accumulated gains in output that can be attributed to the presence of financial frictions are of the order of 1.26% of GDP, in present value terms, from 2009 to 2013 when the liquidity trap lasts for 6 quarters, and of 2.46% when it lasts for 12 quarters. Taken together, these results confirm the importance that financial frictions and the debt-deflation channel have on the efficiency of fiscal policy.

7 Conclusion

This paper investigates how fiscal stimulus packages can help an economy to recovery, when a deep recession has make the zero lower bound on the nominal interest rate binding. The recession is driven by a negative financial shock, characterized by a negative net worth shock, and a negative preference shock. The fiscal stimulus package that we consider consists of either an increase in government spending, or a cut in distortionary labor and capital income taxes.

In the first step, we analyzed whether credit market imperfections and the zero nominal interest rate affect the efficiency of these policies. We show that financial frictions combined with the debt-deflation channel play a major role in the fiscal policy's efficiency. For instance, a government spending expansion or a capital income tax cut dampen the reduction in inflation, decreasing the nominal debt of entrepreneurs and stimulating investment. In contrast, a labor income tax cut makes the drop in inflation stronger, raising the nominal debt of entrepreneurs and worsening the recession.

In the second step, we examine different fiscal policy strategies to assess the importance of financial frictions on the government spending multiplier. We show that the spending multiplier can be higher when it is financed by labor income taxes. Indeed, the rise in labor income taxes dampens the drop in inflation, stimulating investment and output. Secondly, we show that when financial frictions are present, implementing a government spending expansion with a lag can have positive effects on the short run fiscal multiplier. This can be explained by expectations of higher inflation that stimulate investment through the debt-deflation channel.

Finally, we simulated the path of ARRA in our model. We show that the rise in GDP due to the ARRA package is between 1.26% in the absence of financial frictions and 2.46% in their presence.

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Appendix A: log-linearized model

Household

$$(1 - \beta b) \sigma \hat{\lambda}_t = \beta b \mathbf{E}_t \{ \hat{c}_{t+1} \} - (1 + \beta b^2) \hat{c}_t + b \hat{c}_{t-1} + \sigma \hat{\varepsilon}_t - \beta b \sigma \mathbf{E}_t \{ \hat{\varepsilon}_{t+1} \}, \quad (20)$$

where $\sigma^{-1} = -\mathbb{U}_{cc}c/\mathbb{U}_c$.

$$\hat{\lambda}_t - \hat{R}_t = \mathbf{E}_t \{ \hat{\lambda}_{t+1} - \hat{\pi}_{t+1} \} \quad \text{and} \quad \hat{r}_t = \hat{R}_t - \mathbf{E}_t \{ \hat{\pi}_{t+1} \}. \quad (21)$$

$$\hat{\pi}_t^w - \gamma_w \hat{\pi}_{t-1} = \frac{(1 - \alpha_w)(1 - \beta \alpha_w)}{\alpha_w(1 + \omega_w \theta_w)} \left[\omega_w \hat{\ell}_t^h - \hat{\lambda}_t - \hat{w}_t + \hat{\tau}_t \right] + \beta \mathbf{E}_t \{ \hat{\pi}_{t+1}^w - \gamma_w \hat{\pi}_t \}, \quad (22)$$

where $\varpi_w = \ell^h \nabla_{\ell\ell} / \nabla_{\ell}$.

$$\hat{\pi}_t^w = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t. \quad (23)$$

Intermediate Good Sector

$$\hat{y}_t = (1 - \alpha) \hat{\ell}_t + \alpha \hat{k}_t \quad \text{and} \quad \hat{\ell}_t = \Omega \hat{\ell}_t^h. \quad (24)$$

$$\hat{w}_t = \hat{s}_t + \hat{y}_t - \hat{\ell}_t^h, \quad \hat{w}_t^e = \hat{s}_t + \hat{y}_t, \quad \text{and} \quad \hat{z}_t = \hat{s}_t + \hat{y}_t - \hat{k}_t \quad (25)$$

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \frac{(1 - \alpha_p)(1 - \beta \alpha_p)}{\alpha_p} \hat{s}_t + \beta \mathbf{E}_t \{ \hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t \}, \quad (26)$$

Entrepreneur

$$\hat{x}_t = \hat{q}_{t-1} + \hat{k}_t - \hat{n}_t, \quad \text{and} \quad \hat{r}_t = \hat{r}_t^k - \hat{r}_{t-1} \quad (27)$$

$$\mathbf{E}_t\{\widehat{r}_{t+1}\} = \mathbf{E}_t\{\widehat{\Lambda}_{t+1}\} [1 - \check{r} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]], \quad (28)$$

$$\mathbf{E}_t\{\widehat{\Lambda}_{t+1}\} = \mathbf{E}_t\{\widehat{\omega}_{t+1}\} \bar{\omega} \left[\frac{\Gamma_{\omega\omega}(\bar{\omega})}{\Gamma_{\omega}(\bar{\omega})} - \frac{\Gamma_{\omega\omega}(\bar{\omega}) - \mu G_{\omega\omega}(\bar{\omega})}{\Gamma_{\omega}(\bar{\omega}) - \mu G_{\omega}(\bar{\omega})} \right], \quad (29)$$

$$\hat{x}_{t+1} = \mathbf{E}_t\{\widehat{r}_{t+1}\} [x - 1] + \mathbf{E}_t\{\widehat{\omega}_{t+1}\} \bar{\omega} \check{r} x [\Gamma_{\omega}(\bar{\omega}) - \mu G_{\omega}(\bar{\omega})]. \quad (30)$$

where

$$\begin{aligned} \Gamma(\bar{\omega}_{t+1}) &= \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f(\omega) d\omega + \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega, \quad \text{and} \\ \mu G(\bar{\omega}_{t+1}) &= \mu \int_0^{\bar{\omega}_{t+1}} \omega f(\omega) d\omega. \end{aligned}$$

$$\mathbf{E}_t\{\hat{r}_{t+x}^k\} = \mathbf{E}_t\{\hat{z}_{t+1} - \hat{r}_t\} \frac{z(1-\tau)}{r^k} + \mathbf{E}_t\{\hat{q}_{t+1}\} \frac{(1-\delta)}{r^k} - \hat{q}_t. \quad (31)$$

$$\hat{n}_{t+1} \frac{1}{x r^k} = (\hat{\gamma}_t + \hat{v}_t) \gamma [1 - \Gamma(\bar{\omega})] + (\hat{w}_t^e - \hat{r}_t) \left[\frac{1}{r^k x} - \gamma [1 - \Gamma(\bar{\omega})] \right], \quad (32)$$

$$\hat{v}_t [1 - \Gamma(\bar{\omega})] = \hat{r}_t^k [1 - \mu G(\bar{\omega})] + \hat{n}_t [1 - \Gamma(\bar{\omega})] + \hat{x}_t \left[1 - \frac{1}{\check{r}} - \mu G(\bar{\omega}) \right] - \hat{r}_{t-1} \frac{1}{\check{r}} \left[1 - \frac{1}{x} \right] - \widehat{\omega}_t \bar{\omega} \mu G_{\omega}(\bar{\omega}), \quad (33)$$

$$\hat{c}_t^e \frac{c^e}{\check{k} r^k} = (\hat{v}_t (1 - \gamma) - \hat{\gamma}_t \gamma) \varrho [1 - \Gamma(\bar{\omega})]. \quad (34)$$

Capital Producer

$$\widehat{k}_{t+1} \frac{1}{\delta} = \hat{i}_t + \widehat{k}_t \left[\frac{1}{\delta} - 1 \right] - \hat{u}_t \frac{z}{\delta}. \quad (35)$$

$$\hat{q}_t \frac{1}{\varkappa} = \hat{i}_t [1 + \beta] - \hat{i}_{t-1} - \beta \mathbf{E}_t\{\hat{i}_{t+1}\}, \quad (36)$$

where $\varkappa = \Phi''(1)$.

$$\hat{k}_t = \hat{u}_t + \hat{\tilde{k}}_t \quad \text{and} \quad \hat{z}_t = \vartheta_u \hat{u}_t + \hat{q}_t, \quad \text{where} \quad \vartheta_u = u\delta''(u)/\delta'(u). \quad (37)$$

Resource Constraint

$$\hat{y}_t = \hat{c}_t \frac{c}{y} + \hat{i}_t \frac{i}{y} + \hat{c}_t^e \frac{c^e}{y} + \hat{g}_t \frac{g}{y} + \left[\hat{r}_t^k + \hat{q}_{t-1} + \hat{\tilde{k}}_t \right] \left[\mu G(\bar{\omega}) r^k \frac{\tilde{k}}{y} \right] + \hat{\bar{\omega}}_t \bar{\omega} \mu G_\omega(\bar{\omega}) r^k \frac{\tilde{k}}{y}. \quad (38)$$

Unconstrained Monetary Policy

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) a_\pi \hat{\pi}_t + (1 - \rho_R) a_y \hat{y}_t \quad (39)$$

Fiscal Policy

$$\beta \hat{d}_{t+1} = \hat{d}_t + \gamma_g \hat{g}_t - \gamma_\Upsilon \hat{\Upsilon}_t - \tau(\hat{\tau}_t - \hat{y}_t), \quad (40)$$

$$\hat{\Upsilon}_t = a_d \hat{d}_t + a_g \hat{g}_t, \quad (41)$$

$$\tau \hat{\tau}_t = \chi_d \hat{d}_t + \chi_y \hat{y}_t, \quad (42)$$

$$\widehat{deficit}_t = \gamma_g \hat{g}_t - \gamma_\Upsilon \hat{\Upsilon}_t - \tau(\hat{\tau}_t - \hat{y}_t). \quad (43)$$

Appendix B: Solving the model with a ZLB constraint

The methodology used in this paper to solve the model in the presence of a zero lower bound constraint follows Bodenstein *et al.* (2009). All the equations of the model are loglinearized, except for the ZLB constraint given by Equation (16).

The loglinearized model was solved by using the AIM algorithm (see Anderson and Moore, 1985). The equilibrium conditions are written in the matrix form

$$\mathbf{H}_{-1}\mathbf{y}_{t-1} + \mathbf{H}_0\mathbf{y}_t + \mathbf{E}_t\{\mathbf{H}_1\mathbf{y}_{t+1}\} + \mathbf{G}_0\varepsilon_t = 0, \quad (44)$$

where \mathbf{y}_t is a vector ($n \times 1$) of variables with n being the number of variables (including shocks), \mathbf{H}_i refers to structural coefficient matrices ($n \times n$), ε_t is a vector ($k \times 1$) of innovations with k as the number of innovation and \mathbf{G}_0 is a matrix ($n \times k$).

To impose the zero lower bound constraint on this model, we proceeded in three steps. In System (44), let us assume that the line associated with R_t is the first line while R_t^{not} is associated to the second line. We assume that the zero lower bound constraint is hit from period T^{low} to T^{up} .

Let consider the solution of Equation (44) for $t \geq T^{up} + 1$,

$$\mathbf{y}_t = \mathbf{F}\mathbf{y}_{t-1} + \mathbf{C}\varepsilon_t, \quad (45)$$

Then, let recursively solve the system for $T^{low} < t \leq T^{up}$. The system (44) is re-written as

$$\tilde{\mathbf{H}}_0\mathbf{y}_t = -\mathbf{H}_{-1}\mathbf{y}_{t-1} - \mathbf{E}_t\{\mathbf{H}_1\mathbf{y}_{t+1}\} - \mathbf{G}_0\varepsilon_t - \mathbf{g}, \quad (46)$$

where $\tilde{\mathbf{H}}_0$ is an ($n \times n$) matrix, and \mathbf{g} is an ($n \times 1$) vector. The difference between $\tilde{\mathbf{H}}_0$ and \mathbf{H}_0 is that we set $\tilde{\mathbf{H}}_0(2, 2) = 0$, implying that $R_t = R_{not}$. In addition, we impose $\mathbf{g}(2, 1) = R$,

implying that the notional interest rate is set to its steady state value.

If $t = T^{up}$, then, plugging Equation (45) into Equation (46) yields

$$\begin{aligned}\tilde{\mathbf{H}}_0 \mathbf{y}_{T^{up}} &= -\mathbf{H}_{-1} \mathbf{y}_{T^{up}-1} - \mathbf{H}_1 \mathbf{F} \mathbf{y}_{T^{up}} - \mathbf{g}, \\ \Leftrightarrow \mathbf{y}_{T^{up}} &= \Theta_1 \mathbf{y}_{T^{up}-1} + \tilde{\mathbf{g}}_1,\end{aligned}\tag{47}$$

where $\Theta_1 \equiv -(\tilde{\mathbf{H}}_0 + \mathbf{H}_1 \mathbf{F}) \mathbf{H}_{-1}$ and $\tilde{\mathbf{g}}_1 = -(\tilde{\mathbf{H}}_0 + \mathbf{H}_1 \mathbf{F}) \mathbf{g}$.

Then we can solve the model from $T^{up} - 1$ to T^{low} , by backward induction. For $t = T^{up} - 1$, we can write Equation (46) as

$$\mathbf{y}_{T^{up}-1} = -\tilde{\mathbf{H}}_0^{-1} \mathbf{H}_{-1} \mathbf{y}_{T^{up}-2} - \tilde{\mathbf{H}}_0^{-1} \mathbf{H}_1 \mathbf{y}_{T^{up}} - \tilde{\mathbf{H}}_0^{-1} \mathbf{g},\tag{48}$$

and using Equation (47) yields

$$\mathbf{y}_{T^{up}-1} = \Theta_2 \mathbf{y}_{T^{up}-2} + \tilde{\mathbf{g}}_2.\tag{49}$$

where $\Theta_2 \equiv -\left[\mathbf{I}_n + \tilde{\mathbf{H}}_0^{-1} \mathbf{H}_1 \Theta_1\right]^{-1} \tilde{\mathbf{H}}_0^{-1} \mathbf{H}_{-1}$ and $\tilde{\mathbf{g}}_2 \equiv -\left[\mathbf{I}_n + \tilde{\mathbf{H}}_0^{-1} \mathbf{H}_1 \Theta_1\right] \left[\tilde{\mathbf{g}}_1 - \tilde{\mathbf{H}}_0^{-1} \mathbf{g}\right]$.

We generalize the expressions so that $\Theta_i \equiv -\left[\mathbf{I}_n + \tilde{\mathbf{H}}_0^{-1} \mathbf{H}_1 \Theta_{i-1}\right]^{-1} \tilde{\mathbf{H}}_0^{-1} \mathbf{H}_{-1}$ and $\tilde{\mathbf{g}}_i \equiv -\left[\mathbf{I}_n + \tilde{\mathbf{H}}_0^{-1} \mathbf{H}_1 \Theta_{i-1}\right] \left[\tilde{\mathbf{g}}_{i-1} - \tilde{\mathbf{H}}_0^{-1} \mathbf{g}\right]$.

Consequently, for $2 \leq i \leq T^{up} - T^{low}$, computing the dynamics of \mathbf{y}_{T-i} amounts to compute Θ_t and $\tilde{\mathbf{g}}_t$ so that

$$\mathbf{y}_{T^{up}-i} = \Theta_{i+1} \mathbf{y}_{T^{up}-i-1} + \tilde{\mathbf{g}}_{i+1}.\tag{50}$$

The next step is to compute the dynamics for $0 \leq t < T^{low}$. For $T^{up} - T^{low} + 1 \leq i < T^{up}$, the dynamics are expressed as previously, except that $\tilde{\mathbf{H}}_0^{-1}$ is replaced by \mathbf{H}_0^{-1} .

Finally, for $i = T^{up}$, the impact response of the variables is given by

$$\mathbf{y}_0 = \tilde{\Theta}_{T^{up}+1} \varepsilon_0 + \tilde{\mathbf{g}}_{T^{up}+1}. \quad (51)$$

where $\tilde{\Theta}_{T^{up}+1} = -[\mathbf{I}_n + \mathbf{H}_0^{-1} \mathbf{H}_1 \Theta_{T^{up}}]^{-1} \mathbf{H}_0^{-1} \mathbf{G}_0$.

The choice of T , for $T = \{T^{low}, T^{up}\}$ is determined by computing R_T^{not} and R_{T+1}^{not} and ensuring that $\hat{R}_T^{not} < -R \leq \hat{R}_{T+1}^{not}$.

Table 1. Calibrated Parameters

Preferences and Technology		Value
β	Discount factor	0.99
b	Degree of habit on consumption	0.63
σ	Inv. of the elasticity of intertemporal substitution	0.20
ω_w	Elasticity of labor disutility	1.00
ϕ	Elasticity of value added wrt capital	0.35
δ	Capital depreciation rate	0.03
\varkappa	Investment adjustment cost	5.86
ϑ_u	Utilization rate of capital parameter	1/0.31
Nominal Rigidities		
θ_p	Elasticity of substitution of goods	11.00
α_p	Degree of price stickiness	0.67
γ_p	Degree of price indexation	0.75
θ_w	Elasticity of substitution of labor	21.00
α_w	Degree of wage stickiness	0.69
γ_w	Degree of wage indexation	0.70

Table 2. Calibrated Parameters

Financial Accelerator Mechanism		Value
Ω	Proportion of household labor in aggr. labor	0.9846
x	Steady-state ratio of capital to net worth	2.00
\tilde{r}	Steady-state risk spread	1.02 ^{0.25}
γ	Survival rate of entrepreneurs	0.9785
$\bar{\omega}$	Threshold value of idiosyncratic shock	0.4982
σ_ω	Standard error of idiosyncratic shock	0.2764
$(1 - \varrho)$	Transfers from failed entrepreneur to households	0.99
μ	Monitoring cost	0.1175

Table 3. Calibrated Parameters

Monetary Policy		Value
ρ_R	Interest rate smoothing	0.88
a_π	Elasticity of the interest rate wrt inflation	1.85
a_y	Elasticity of the interest rate wrt output gap	0.08
Fiscal Policy		Value
g/y	Share of government expenditure in output	0.19
ϕ_b	Elasticities of lump-sum taxes wrt debt	0.33
ϕ_g	Elasticities of lump-sum taxes wrt government spending	0.10
ψ_y	Elasticities of distortionary taxes wrt output	[0; 0.05]
ψ_d	Elasticities of distortionary taxes wrt debt	[0; 0.05]
τ	Income tax rate	[0; 0.30]

Table 4. Value of PVM_k , with $k = 50$

	No lag	2 lags	4 lags	6 lags	8 lags	10 lags	12 lags
With financial frictions	1.04	0.99	0.93	0.83	0.71	0.56	0.42
Without financial frictions	0.51	0.48	0.46	0.44	0.40	0.37	0.36

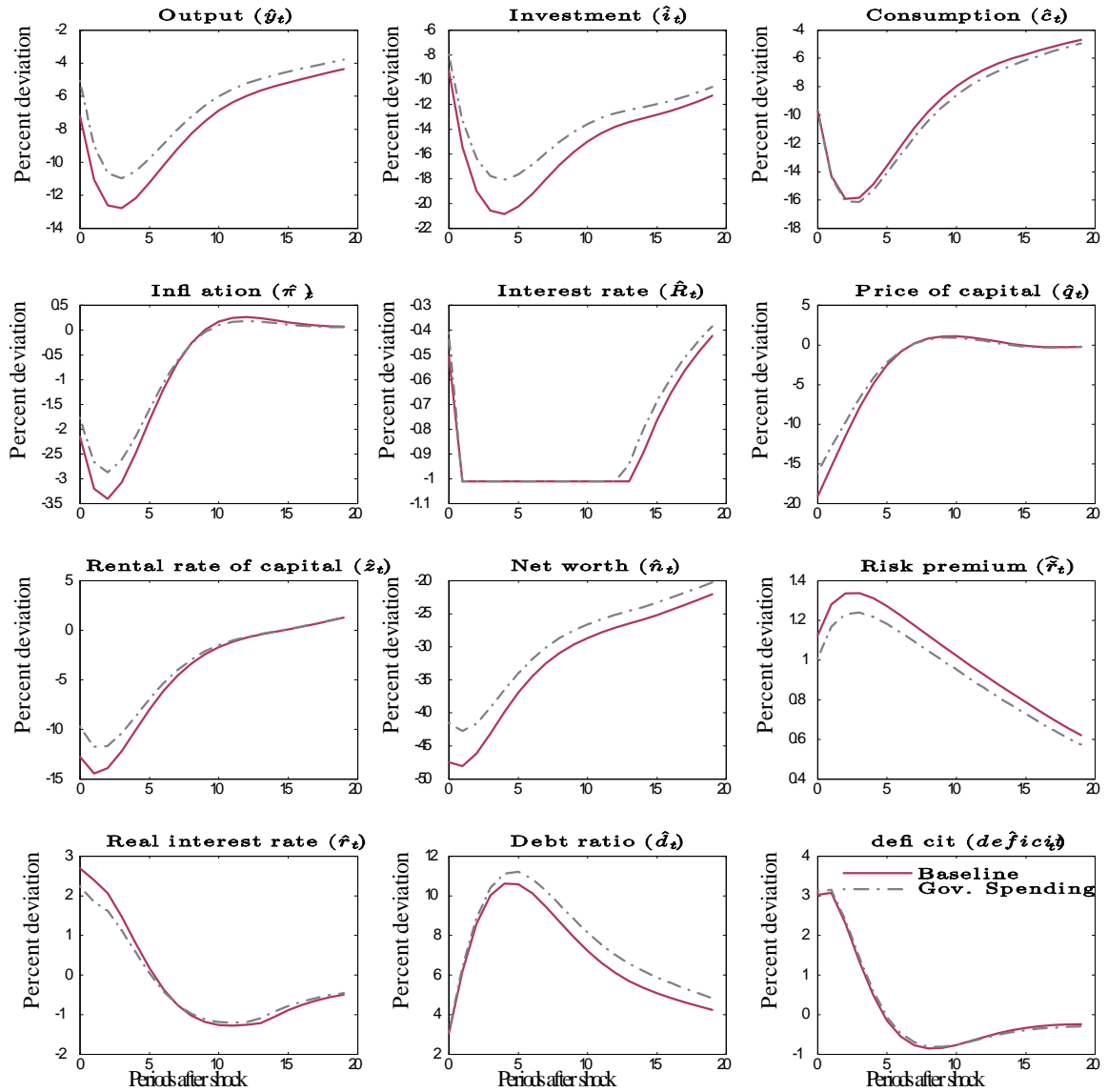


Figure 1: IRFs multiplied by 100 to a negative financial shock (decrease in γ_t) and a negative preference shock (decrease in ε_t). The solid lines correspond to the baseline model's dynamics under the deep recession and without fiscal expansion. The dashed lines represent the IRF to an increase in government spending under the deep recession.

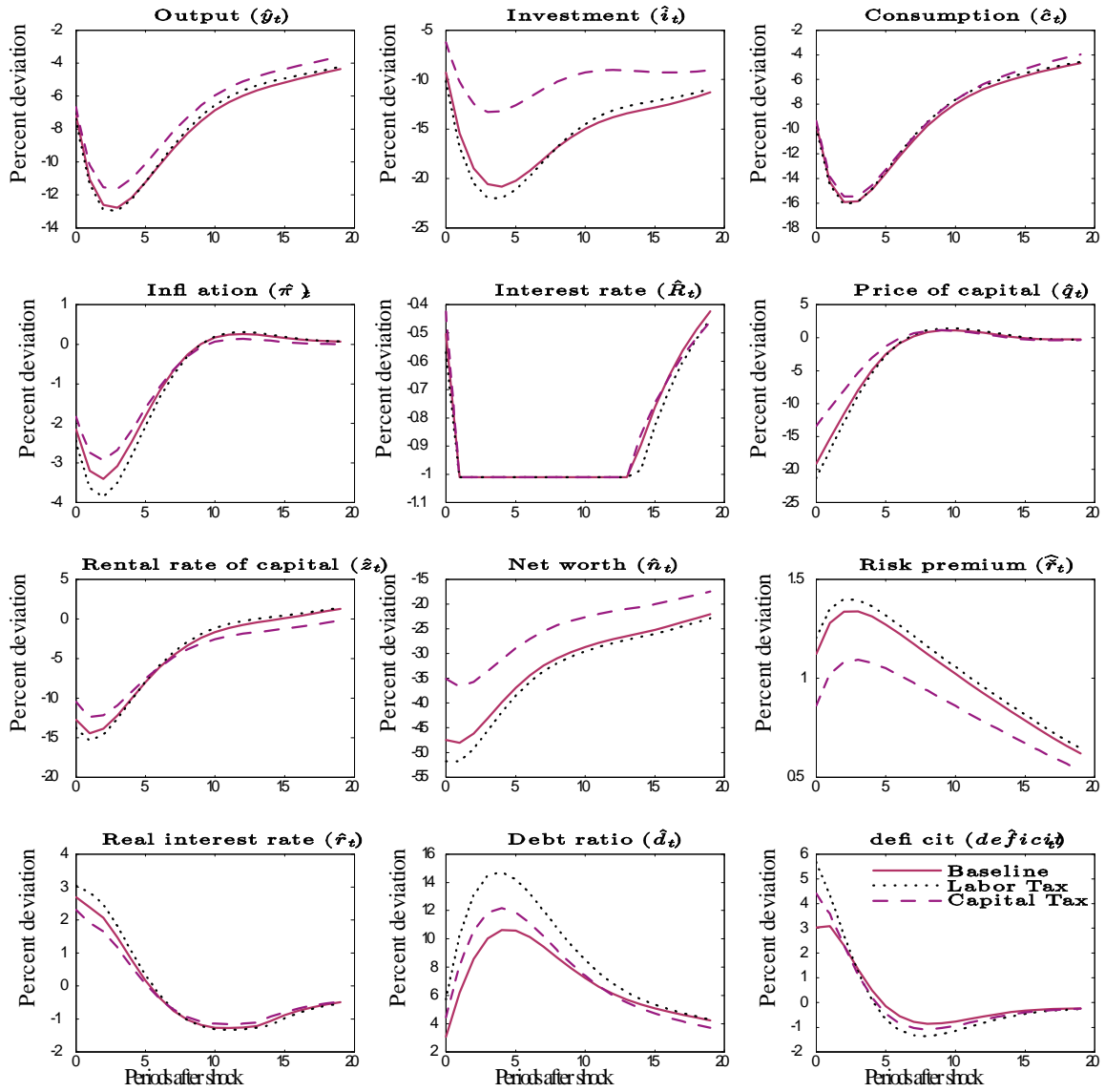


Figure 2: IRFs multiplied by 100 to a negative financial shock (decrease in γ_t) and a negative preference shock (decrease in ε_t). The solid lines correspond to the baseline model's dynamics under the deep recession without fiscal expansion. The dotted lines represent the IRFs to a deep recession and a labor income tax cut. The dashed lines represent the IRFs to a deep recession and a capital income tax cut.

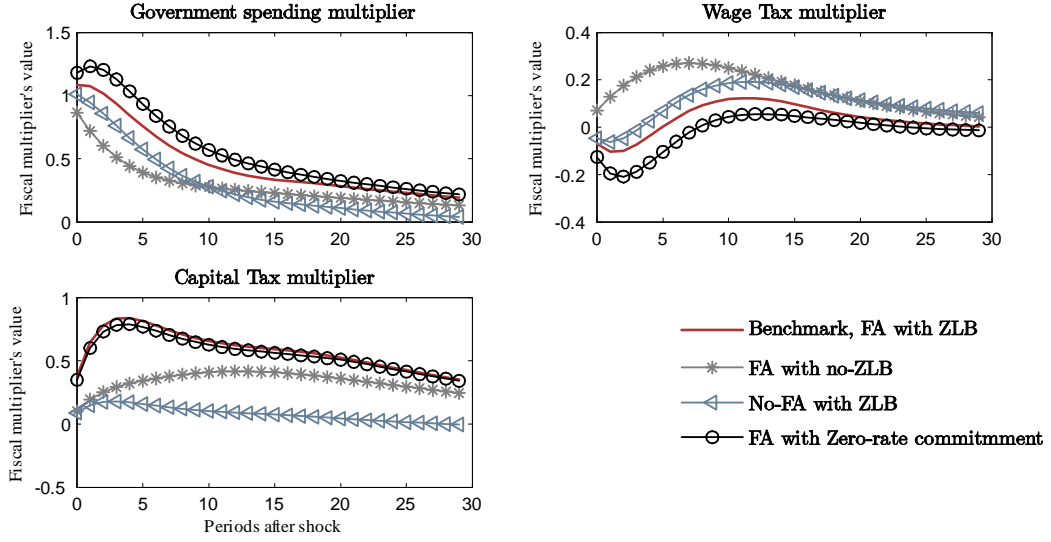


Figure 3: Fiscal multiplier for various model specifications. The solid line shows the benchmark scenario. The starred line corresponds to a model without ZLB constraint. The line with triangles corresponds to a model without financial frictions. The line with circles corresponds to the benchmark case with a zero interest rate period of 20 quarters.

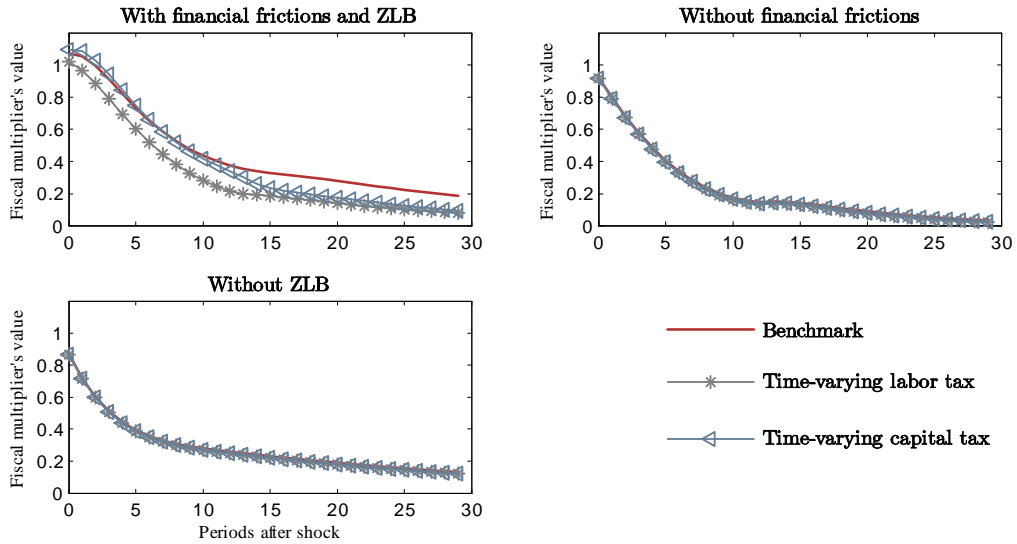


Figure 4: Gov. spending multiplier for various taxation policies. The solid line represents the benchmark case. The starred line corresponds to a model with labor income taxes that vary with output and debt. The line with triangles corresponds to a model with capital income taxes that vary with output and debt. The left upper panel shows to the benchmark model with financial frictions and ZLB, while the right upper panel illustrates a model without financial frictions and the left lower panel corresponds to a model without financial frictions.

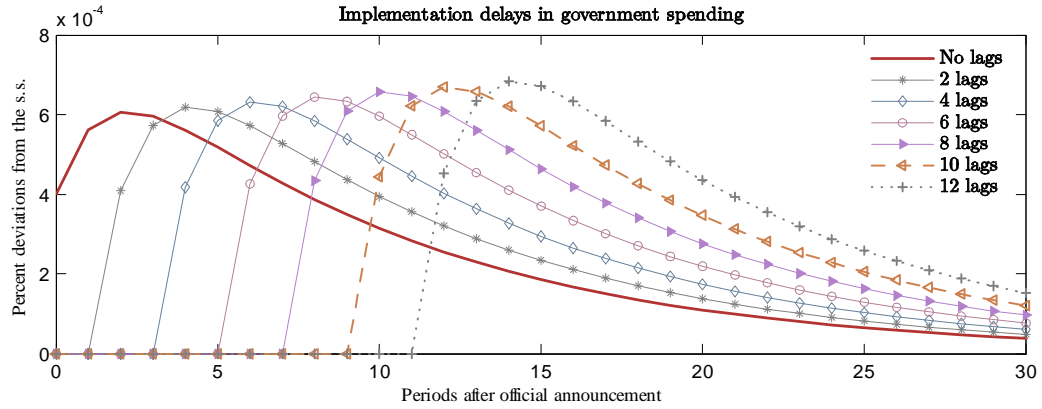


Figure 5: IRFs of government spending: the effect of various delays in the implementation.

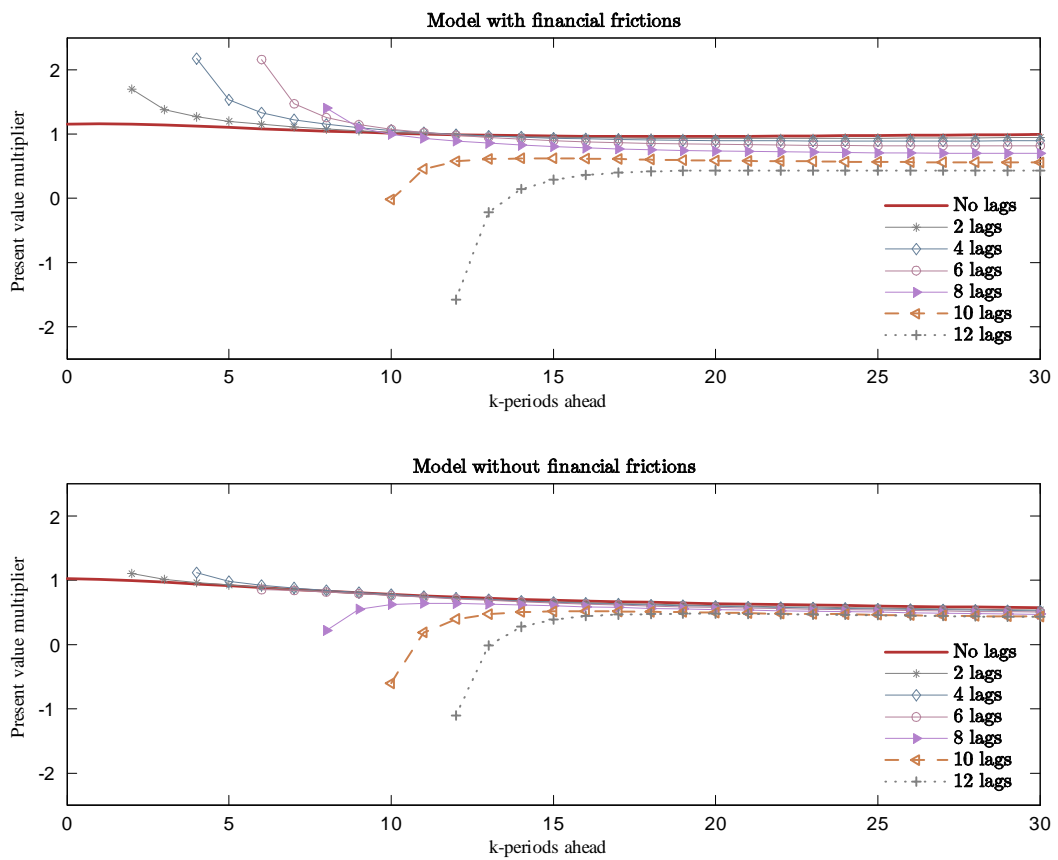


Figure 6: Present value multiplier k-quarters ahead for government spending.

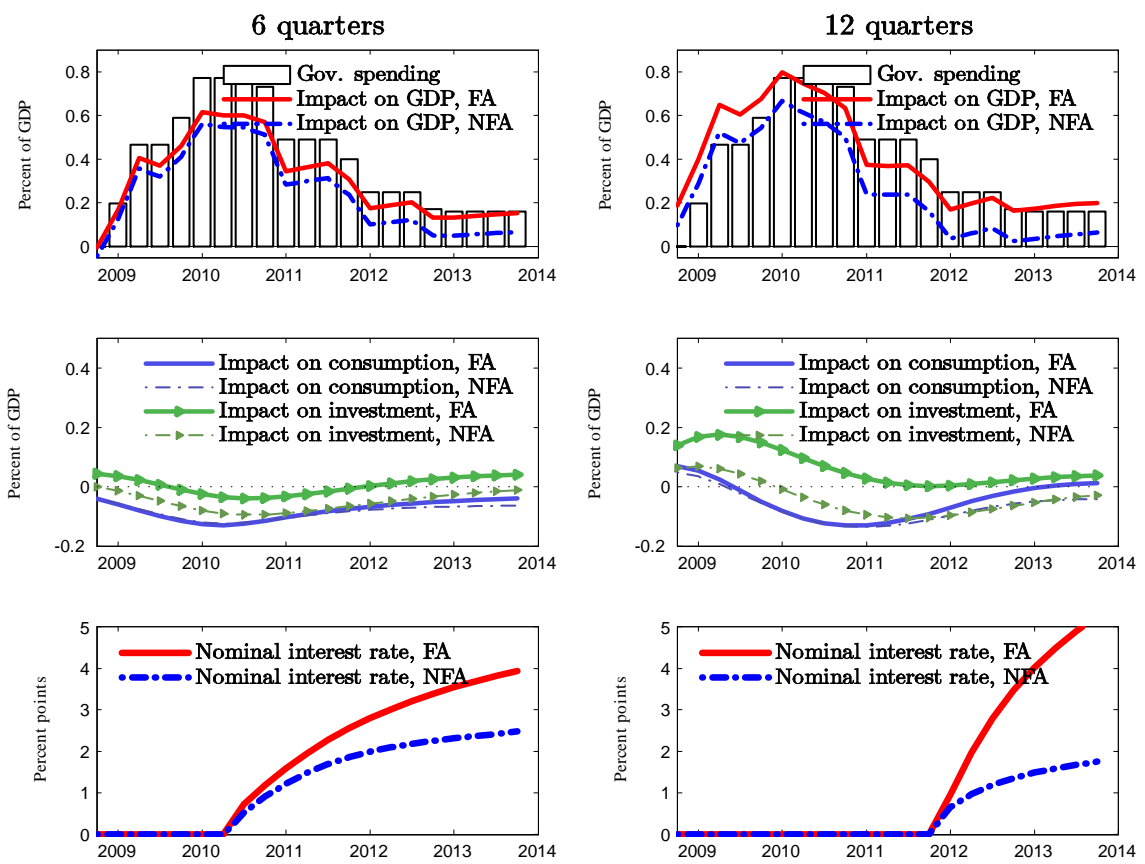


Figure 7: ARRA simulations for different durations of the liquidity trap

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