Growth and Welfare: Distorting versus Non-Distorting Taxes*

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Abstract

In an infinitely-lived framework, taxing capital income may be *growth and welfare enhancing* when it allows for correcting distorting externalities in the competitive equilibrium allocation. This is the case when public capital is subject to congestion by private capital or total income [Fisher and Turnovsky (1998)] or when government expenditure exerts an external effect on physical capital [Corsetti and Roubini (1996)]. However, none of these features appear in simple one-sector endogenous growth models with public capital. Alternatively, we consider certain realistic *fiscal policy constraints* in a simple one-sector growth model with *productive* and *unproductive* public expenditures, to show that raising revenues through factor income taxes may be preferred to using lump-sum taxes.

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1. Introduction

How should public expenses be financed? Should existing tax systems be reorganized and income taxes be substituted by less distorting taxes? How high should public investment be as a percentage of GNP? Issues like these, regarding the effects of fiscal policy on growth and welfare, have been widely debated in the public finance literature. Most work has focused on the incidence of a second-best tax structure in dynamic settings, in the line of the seminal paper by Ramsey (1927). Along this line, pioneer work by Judd (1985), Chamley (1986) and Lucas (1990) emphasized the negative incidence of capital income taxes on welfare in general equilibrium models. Judd (1999) argued for a zero tax rate on physical and human capital income in the long-run. Jones et al. (1997) and Milesi-Ferretti and Roubini (1998) extended the zero tax rate result to labor and consumption taxes in models with human capital, although pointing out that certain *public revenue constraints* could imply that taxing productive factors positively in the long-run might be optimal in a second-best sense. Nevertheless, the general presumption drawn from previous work is that raising revenues through lump-sum taxes is more favorable to welfare than taxing income from productive factors.

A parallel renewed line of research emerged with Barro (1990), who combined the public finance literature, the study of productive public expenditures and the endogenous growth literature.[1] Among others, Futagami et al. (1993), Glomm and Ravikumar (1994) and Turnovsky (1996, 2000) are extended versions of Barro (1990). In them, public revenues come from proportional income taxes and the government chooses the welfare-maximizing ratio of productive public expenditures-to-output. However, in that setting, not much work has been done regarding the optimal simultaneous choice of a *productive public expenditure/output ratio* and a *financing rule*.[2] In this paper, we aim to make some progress along this line. In a Barro-type setting we characterize the welfare-maximizing *productive* public expenditure/output ratio under alternative tax scenarios, which are, for the sake of simplicity, those considered in Fisher and Turnovsky (1998): (i) a distortionary tax on total income and (ii) a lump-sum tax, in both cases with a period-by-period balanced budget. We find that, under certain realistic *fiscal policy constraints*, raising revenues through taxes on total income might be a preferred strategy to raising revenues through lump-sum taxation.

An argument in favor of taxing in a distorting fashion arises from distributive considerations. Generally, the positive effect of income distribution between generations makes a zero tax rate on capital income not to be optimal: see Jones and Manuelli (1992) in an overlapping generations setting with production, and Caballé (1998) in an infinitely-lived framework with altruistic preferences. Alternatively, in an infinitely-lived framework, distorting taxation may become optimal when the competitive equilibrium allocation involves a harmful externality that could be corrected, at least partially, by taxing factors income.

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[1] Empirical work by Ratner (1983), Aschauer (1989) and Munnell (1990) emphasizes the positive link between public expenses and the private production process.

[2] Exceptions are, among others, Fisher and Turnovsky (1998) in a neoclassical growth model and Corsetti and Roubini (1996) in a two-sector endogenous growth setting.

For example, Chamley (2001) shows that borrowing constraints may induce individuals to over-accumulate capital in the long-run when insuring against idiosyncratic shocks. In models with public capital, productive public expenditures are sometimes assumed to exert a positive external effect on physical capital [as in Corsetti and Roubini (1996)], while private capital over-congests public capital in competitive equilibrium in Fisher and Turnovsky (1998). In these environments, taxing capital income in the long-run would be optimum, since otherwise physical capital would over-accumulate in competitive equilibrium.

In this paper we consider more fundamental reasons for distorting taxes to be a preferred revenue raising strategy in a *one-sector* endogenous growth model with *non-congested* public capital. An explicit fiscal policy constraint is assumed,[3] in the form of a constant *unproductive* public expenditure/output ratio (i.e., due to inefficient bureaucratic or administrative costs, or to the payment of interest on outstanding debt), which will limit the choice of the *productive* public investment/output ratio when maximizing growth or welfare. In a Barro-type setting, a tax increase can be used to finance a higher level of productive public expenditures, which has a direct and positive impact on growth and welfare. If the source of revenues is income taxes, private capital accumulation is discouraged, and the positive effect on growth and welfare of the increase in productive public expenditures is partly neutralized. On the other hand, although lump-sum tax financing is not harmful for private capital accumulation, private consumption will be strongly affected, since it will experience most of the implied crowding-out effect.

Therefore, choosing between distorting and non-distorting taxes represents a trade-off between current and future consumption. Discouraging private capital accumulation, distorting taxes have a long-run effect on future growth and consumption. Alternatively, taxing lump-sum has a strong short-run effect on consumption, with no disincentive on private capital accumulation. Which tax system is preferred will depend on the relative size of both effects on welfare. What is specific to our model is that financing unproductive expenditures through lump-sum taxes produces a strong crowding-out impact on current consumption which may sharply reduce the possibilities for public investment. This distorts the accumulation of public capital, relative to private capital, limiting the growth stimulus achievable through public investment. It turns out that this restriction can be more damaging for growth and welfare than the disincentive created on private capital accumulation when taxing productive factors' income.

The paper is organized as follows. In section 2 the basic framework is described. In section 3 the competitive equilibrium and the long-run equilibrium path are characterized. In sections 4 the growth- and welfare-maximizing public investment policies under income and lump-sum taxes are compared, and a simple numerical example is presented. Finally, section 5 ends with main conclusions and extensions.

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[3]The fiscal policy rigidity is *"based on informational and political constraints which are not explicitly modeled"* [Jones et al. (1997)].

2. A basic framework for analysis

The model draws on work by Barro (1990), Futagami et al. (1993) and Glomm and Ravikumar (1994). It differs from the non-congested version in the Glomm-Ravikumar setting because of the presence of an spillover factor in production and the fact that not all public expenditure is productive. The economy consists of a continuum of firms indexed by [0,1], a fiscal authority and a representative household.

2.1. Firms

Firms are identical, rent the same amount of physical capital k_t and labor l_t from households, and produce y_t units of the consumption commodity at a given period t. The capital stock used in the aggregate by all firms, K_t, is taken as a proxy for the index of knowledge available to each single firm [as in Romer (1986)]. Additionally, public capital, K_t^g, is exogenous to the private production process and affects all individual firms in the same way. Except for these externalities, the private production technology can be represented by a standard *Cobb-Douglas* function presenting constant returns to scale. For any firm,

$$y_t = F(l_t, k_t, K_t, K_t^g) = \mathsf{F}\, l_t^{1-\alpha} k_t^\alpha K_t^\phi \left(K_t^g\right)^\theta, \; \alpha, \theta \in (0,1),\, \phi \in [0,1], \qquad (2.1)$$

where α is the share of private capital in output, θ and ϕ are the constant elasticities of output with respect to public capital and the knowledge index, and F is a technological scale.

Since all firms are identical, we can aggregate on (2.1), to obtain total output in the economy, Y_t,

$$Y_t = \mathsf{F}\, L_t^{1-\alpha} K_t^{(\alpha+\phi+\theta)} \left(\frac{K_t^g}{K_t}\right)^\theta, \qquad (2.2)$$

where L_t, K_t are the total amounts of labor and physical capital used by all the firms in the economy. During period t, each firm pays the competitive-determined wage w_t on the labor it hires and the rate r_t on the capital it rents. The profit maximizing problem of the typical firm turns out to be static,

$$\underset{\{l_t, k_t\}}{Max}\; F(l_t, k_t, K_t, K_t^g) - w_t l_t - r_t k_t,$$

leading to the usual marginal product conditions,

$$r_t \;=\; F_{k_t} = \alpha \mathsf{F}\, l_t^{1-\alpha} k_t^{\alpha-1} K_t^\phi \left(K_t^g\right)^\theta = \alpha \frac{y_t}{k_t} = \alpha \frac{Y_t}{K_t}, \qquad (2.3)$$

$$w_t \;=\; F_{l_t} = (1-\alpha)\mathsf{F}\, l_t^{-\alpha} k_t^\alpha K_t^\phi \left(K_t^g\right)^\theta = (1-\alpha)\frac{y_t}{l_t} = (1-\alpha)\frac{Y_t}{L_t}, \qquad (2.4)$$

where we have used the fact that each firm treats its own contribution to the aggregate capital stock as given. From these optimality conditions we have the standard result on income distribution,

$$r_t K_t + w_t L_t = Y_t. \qquad (2.5)$$

2.2. The public sector

The public sector collects taxes to finance its expenditures, which are distributed between public investment, I_t^g, and public services, C_t^g, the latter not entering as an argument in consumers' utility or in the production function, but being required for the public sector to exist. The government is assumed to follow a policy that maintains constant ratios, $\varkappa_i$ and $\varkappa_c$, of both types of public expenditures to total output,

$$I_t^g = \varkappa_i Y_t, \ \varkappa_i \geq 0, \tag{2.6}$$
$$C_t^g = \varkappa_c Y_t, \ \varkappa_i \geq 0, \tag{2.7}$$

with $\varkappa_i + \varkappa_c < 1$. Public capital accumulates according to,

$$K_{t+1}^g = I_t^g + (1 - \delta^g)K_t^g. \tag{2.8}$$

Tax revenues finance total public expenses every period. Two alternative tax scenarios are considered [as in Fisher and Turnovsky (1998)]: (i) a distorting taxation scheme, where total income is taxed at a rate τ_t, while transfers, X_t, are zero, with a government budget constraint,

$$I_t^g + C_t^g = \tau_t Y_t, \tag{2.9}$$

and (ii) a non-distorting tax scenario, with $\tau_t = 0$, and the government financing its expenditures through lump-sum taxes,

$$I_t^g + C_t^g = X_t. \tag{2.10}$$

2.3. Households

We assume zero population growth and normalize population size to one. The representative consumer is the owner of physical capital, and allocates her resources between consumption, C_t, and investment in physical capital, I_t. Private physical capital accumulates over time according to,

$$K_{t+1} = I_t + (1 - \delta)K_t, \tag{2.11}$$

and decisions are made each period to maximize the discounted aggregate value of the time separable, logarithmic utility function

$$\underset{\{C_t, K_{t+1}\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} \beta^t \ln(C_t), \tag{2.12}$$

subject to the resource constraint,

$$C_t + K_{t+1} - (1 - \delta)K_t \leq (1 - \tau_t)(w_t L_t + r_t K_t), \tag{2.13}$$

under income taxes and

$$C_t + K_{t+1} - (1 - \delta)K_t + X_t \leq w_t L_t + r_t K_t, \tag{2.14}$$

under lump-sum taxes. K_{t+1} denotes the stock of physical capital at the end of time t, with $K_0 > 0$, and β is the discount factor, between zero and one.

The consumer takes fiscal policy and factor prices as given when deciding how to split her current income between consumption and savings. When the government finances its operations through income taxes, the optimality condition for the consumer is

$$\frac{C_{t+1}}{C_t} = \beta \left[(1 - \delta) + (1 - \tau_t)r_t \right], \tag{2.15}$$

while in the case of lump-sum taxation, the optimality condition becomes,

$$\frac{C_{t+1}}{C_t} = \beta \left[(1 - \delta) + r_t \right] \tag{2.16}$$

together with the budget constraint, either (2.13) or (2.14), the transversality condition,

$$\lim_{t \to \infty} \beta^t K_{t+1} \frac{\partial U}{\partial C_t} \equiv \lim_{t \to \infty} \beta^t K_{t+1} \frac{1}{C_t} = 0, \tag{2.17}$$

and $K_{t+1} \geq 0$, $C_t \geq 0$, for any period t.

3. Equilibrium conditions and the balanced growth path

3.1. The competitive equilibrium

A particular fiscal policy π is characterized by two parameter values, $\varkappa_i$, $\varkappa_c$, and two time sequences, $\{\tau_t, X_t\}_{t=0}^{\infty}$. Following Glomm and Ravikumar (1994), we define a π-*competitive equilibrium* (π-CE):

Definition 1. *Given initial conditions K_0, $K_0^g > 0$, a π-CE for the overall economy is a set of allocations $\left\{ C_t, C_t^g, K_{t+1}, K_{t+1}^g, I_t, I_t^g, L_t, Y_t \right\}_{t=0}^{\infty}$, a set of prices $\{r_t, w_t\}_{t=0}^{\infty}$ and a fiscal policy π, such that, given $\{r_t, w_t\}_{t=0}^{\infty}$: (i) $\{L_t, K_{t+1}\}_{t=0}^{\infty}$ solve the profit maximizing problem of firms [i.e., (2.3)-(2.4) hold], (ii) $\{C_t, K_{t+1}\}_{t=0}^{\infty}$ maximize the utility of households [i.e., (2.17), $C_t, K_{t+1} \geq 0$ and either (2.13) and (2.15) under income taxes or (2.14) and (2.16) under lump-sum taxes hold], (iii) the public sector budget constraint [either (2.9) or (2.10)], together with (2.6), (2.7) and the technology constraints (2.2), (2.11), (2.8) hold and (iv) markets clear every period:*

$$L_t = 1, \tag{3.1}$$
$$Y_t = C_t + C_t^g + I_t + I_t^g. \tag{3.2}$$

In fact, marginal utility at the origin equal to infinity guarantees that strict inequalities will hold for $K_{t+1} > 0$, $C_t > 0$ at all time periods, which we use in what follows.

3.2. The balanced growth path

The balanced growth path is a π-CE trajectory along which aggregate variables grow at a zero or positive constant rate. Barro (1990), Rebelo (1991) and Jones and Manuelli (1997), among others, have shown that cumulative inputs must present constant returns to scale in the private production process (i.e., $\alpha + \theta + \phi = 1$) and r_t must be constant and high enough, for the equilibrium to display positive and steady growth in our Barro-type setting. From now on, we will focus on the special case in which[4] $\alpha + \theta + \phi = 1$.

Under these conditions, it is easy to show from the equilibrium conditions that Y_t, C_t, K_t, K_t^g, C_t^g and X_t must all grow at the same constant rate, denoted $\bar{\gamma}$ hereinafter, along the balanced growth path, while bounded variables, such as τ_t and r_t, must be constant. Therefore, the ratios $c_t = C_t/K_t$, $k_t^g = K_t^g/K_t$, $y_t = Y_t/K_t$, $c_t^g = C_t^g/K_t$ and $x_t = X_t/K_t$ are constant along the balanced growth path.

In terms of these ratios, π-CE conditions can be particularized for a balanced growth path equilibrium to the following system in $\bar{\gamma}$, $\bar{c}$, $\bar{k}^g$, $\bar{y}$, $\bar{r}$, $\bar{c}^g$, $\bar{x}$ and $\bar{\tau}$ (letters with bar refer to values along the balanced growth path):

$$\bar{\gamma} + \delta = (1 - \varkappa_i - \varkappa_c)\bar{y} - \bar{c}, \tag{3.3}$$

$$\bar{\gamma} + \delta^g = \varkappa_i \bar{y}(\bar{k}^g)^{-1}, \tag{3.4}$$

$$\bar{r} = \alpha\bar{y}, \tag{3.5}$$

$$\bar{y} = F(\bar{k}^g)^\theta, \tag{3.6}$$

$$\bar{c}^g = \varkappa_c \bar{y}, \tag{3.7}$$

and either

$$\bar{x} = 0, \tag{3.8}$$

$$\bar{\tau} = \varkappa_i + \varkappa_c, \tag{3.9}$$

$$1 + \bar{\gamma} = \beta\left[(1 - \delta) + (1 - \bar{\tau})\bar{r}\right], \tag{3.10}$$

under income taxes, or

$$\bar{\tau} = 0, \tag{3.11}$$

$$\bar{x} = (\varkappa_i + \varkappa_c)\bar{y}, \tag{3.12}$$

$$1 + \bar{\gamma} = \beta(1 - \delta + \bar{r}), \tag{3.13}$$

under lump-sum taxes.

Condition (3.3) comes from the global constraint of resources, (3.4) is the public investment rule, (3.5) is the gross return on capital accumulation, (3.6) is the production function, (3.7) is the public consumption rule and, depending on the tax system considered, either (3.10) or (3.13) refer to the intertemporal substitution of consumption relationship and either (3.9) or (3.12) is the government budget constraint.

In parallel to Definition 1, for a particular *stationary fiscal policy* $\bar{\pi} = \{\varkappa_i, \varkappa_c, \bar{\tau}, \bar{x}\}$, the $\bar{\pi}$-*balanced growth path* ($\bar{\pi}$-BP) is defined:

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[4]The presence of the spillover effect allows us to change the values of the α, θ parameters independently from each other.

Definition 2. *A $\bar{\pi}$-BP is a vector $\bar{\bar{\Pi}} = \{\bar{\gamma}, \bar{c}, \bar{k}^g, \bar{y}, \bar{r}, \bar{c}^g\}$ and a stationary fiscal policy $\bar{\pi}$ satisfying: (3.3)-(3.7), either (3.8)-(3.10) under income taxes or (3.11)-(3.13) under lump-sum taxes, the transversality condition (2.17) and non-negativity conditions $\bar{c} > 0$ and $\bar{k}^g > 0$.*

The common growth rate property allows us to write the balanced growth version of the transversality condition (2.17) as, $\lim_{t \to \infty} \beta^t (1 + \bar{\gamma}) \frac{1}{\bar{c}} = 0$, which will be satisfied by any $\bar{\pi}$-BP.

3.3. The full depreciation $\bar{\pi}$-BP

For simplicity, we initially assume that both types of capital fully depreciate each period, so that $\delta = \delta^g = 1$ in (3.3)-(3.13). This assumption enable us to obtain an analytical characterization of the π-CE and the $\bar{\pi}$-BP allocations, so that our results can be compared with those obtained in previous research.[5]

Proposition 1 shows the existence of a single $\bar{\pi}$-BP under $\delta = \delta^g = 1$,[6] provided that $\varkappa_i$ and $\varkappa_c$ are such that a positive amount of resources is left to the consumer every period.[7]

Proposition 1. *If $\varkappa_i + \varkappa_c < 1$, there is a single $\bar{\pi}$-BP under income taxes. Under lump-sum taxes $\varkappa_i + \varkappa_c < 1 - \alpha\beta$ is required for a $\bar{\pi}$-BP to exist. Then, the $\bar{\pi}$-BP is also unique.*
Proof. *(i) Under income taxes, if $\varkappa_i + \varkappa_c < 1$, the set of equations (3.3)-(3.10), particularized to the case of full depreciation, has a single solution,*

$$\bar{k}^{gd} = \frac{\varkappa_i}{\alpha\beta(1 - \varkappa_i - \varkappa_c)}, \tag{3.14}$$

$$\bar{r}^d = F\alpha^{1-\theta}\left[\frac{\varkappa_i}{\beta(1 - \varkappa_i - \varkappa_c)}\right]^{\theta}, \tag{3.15}$$

$$\bar{\gamma}^d = F\varkappa_i^{\theta}[\alpha\beta(1 - \varkappa_i - \varkappa_c)]^{1-\theta} - 1, \tag{3.16}$$

$$\bar{y}^d = F\left[\frac{\varkappa_i}{\alpha\beta(1 - \varkappa_i - \varkappa_c)}\right]^{\theta}, \tag{3.17}$$

$$\bar{c}^d = (1 - \varkappa_i - \varkappa_c)^{1-\theta}(1 - \alpha\beta)F\left(\frac{\varkappa_i}{\alpha\beta}\right)^{\theta}, \tag{3.18}$$

$$\bar{c}^{gd} = \varkappa_c F\left[\frac{\varkappa_i}{\alpha\beta(1 - \varkappa_i - \varkappa_c)}\right]^{\theta}. \tag{3.19}$$

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[5] Together with the assumption that leisure does not enter in the utility function, which is of the logarithmic type. In section 4, full depreciation is shown not to be crucial for the conclusions relating to the growth-maximizing public expenditures policies, although it may be relevant for welfare analysis.

[6] In the Appendix (part 1), we show this statement for any $\delta, \delta^g \in [0, 1]$.

[7] Hereinafter, a d-uppercase denotes a variable under distortionary taxation, while an n-uppercase denotes the value of the variable under non-distortionary taxes.

Since $\alpha\beta < 1$, it is clear that $\bar{c}^d, \bar{k}^{gd} > 0$, so the vector $\bar{\Pi}^d = \{\bar{\gamma}^d, \bar{c}^d, \bar{k}^{gd}, \bar{y}^d, \bar{r}^d, \bar{c}^{gd}\}$ defined by (3.14)-(3.19), will be a $\bar{\pi}$-BP.

(ii) Similarly, under lump-sum taxes, combining (3.3)-(3.7) with (3.11)-(3.13) leads to,

$$\bar{k}^{gn} = \frac{\varkappa_i}{\alpha\beta}, \tag{3.20}$$

$$\bar{r}^n = F\alpha^{1-\theta}\left(\frac{\varkappa_i}{\beta}\right)^\theta, \tag{3.21}$$

$$\bar{\gamma}^n = F\varkappa_i^\theta (\alpha\beta)^{1-\theta} - 1, \tag{3.22}$$

$$\bar{y}^n = F\left(\frac{\varkappa_i}{\alpha\beta}\right)^\theta, \tag{3.23}$$

$$\bar{c}^n = F\left(\frac{\varkappa_i}{\alpha\beta}\right)^\theta [(1 - \varkappa_i - \varkappa_c) - \alpha\beta], \tag{3.24}$$

$$\bar{c}^{gn} = \varkappa_c F\left(\frac{\varkappa_i}{\alpha\beta}\right)^\theta. \tag{3.25}$$

Furthermore, $\bar{c}^n, \bar{k}^{gn} > 0$ so long as $\varkappa_i + \varkappa_c < 1 - \alpha\beta$. Under that condition, the vector $\bar{\Pi}^n = \{\bar{\gamma}^n, \bar{c}^n, \bar{k}^{gn}, \bar{y}^n, \bar{c}^{gn}, \bar{r}^n\}$, which is uniquely defined by conditions (3.20)-(3.25) above, will be a $\bar{\pi}$-BP ∎

3.4. Characterizing the full depreciation π-CE

The simplicity of the model allows for the π-CE to be analytically characterized. Regarding the decision rules for C_t and K_{t+1}, we make a linear guess for their dependence on output: $C_t = aY_t$ and $K_{t+1} = bY_t$. Under income taxes, taking these linear rules to: (2.13), (2.15), (2.9), (2.6), (2.7), (2.3) together with $L_t = 1$, we get,

$$C_t^d = [(1 - \alpha\beta)(1 - \varkappa_i - \varkappa_c)]F\left(K_t^{gd}\right)^\theta \left(K_t^d\right)^{1-\theta}, \quad t = 0, 1, 2, ..., \tag{3.26}$$

$$K_{t+1}^d = \alpha\beta(\varkappa_i + \varkappa_c)F\left(K_t^{gd}\right)^\theta \left(K_t^d\right)^{1-\theta}, \quad t = 0, 1, 2, ..., \tag{3.27}$$

while taking to (2.14), (2.16), (2.10), (2.6), (2.7) (2.3) the proposed linear rules for capital and consumption, together with $L_t = 1$, under lump-sum taxes, we obtain,

$$C_t^n = [(1 - \varkappa_i - \varkappa_c) - \alpha\beta]F(K_t^g)^\theta (K_t^n)^{1-\theta}, \quad t = 0, 1, 2, ..., \tag{3.28}$$

$$K_{t+1}^n = \alpha\beta F(K_t^{gn})^\theta (K_t^n)^{1-\theta}, \quad t = 0, 1, 2, ... \tag{3.29}$$

Finally, under both tax systems, combining (2.2), (2.6) and $L_t = 1$, we get

$$K_{t+1}^g = \varkappa_i F(K_t^g)^\theta K_t^{1-\theta}, \quad t = 0, 1, 2, ... \tag{3.30}$$

which is, under either tax system, a set of three equations characterizing the propagation mechanism for C_t, K_{t+1} and K_{t+1}^g along a π-CE.

The characteristics of the dynamics of C_t, K_{t+1} and K_{t+1}^g are given by the eigenvalue structure of the coefficient matrix of the state-space representation of the system above, in logs.[8] Independently of the tax system, that matrix has a zero eigenvalue and a second eigenvalue equal to one. The zero eigenvalue reflects the absence of transitional dynamics, while the unit eigenvalue is inherent to sustained growth models,[9] implying that the ratios K_t^g/K_t, C_t/K_t converge to constant levels, $\bar{k}^g$ and $\bar{c}$. Since households face a convex control problem, existence and uniqueness of the π-CE is guaranteed. Consequently, given K_0, $K_0^g > 0$, the previous system provides us with the values of C_0, K_1 and K_1^g under either tax policy, the three variables growing from that time on at the common rate $\bar{\gamma}$ given by (3.16) and (3.22).

4. Income versus lump-sum taxes

In this section, we discuss the possibility that the steady-state growth rate as well as the level of welfare might be higher under income than under lump-sum taxes, when the government chooses the public investment/output ratio, $\varkappa_i$, to maximize either growth or welfare. The government is assumed to be constrained by the need to finance a fixed ratio of *unproductive* public expenses to output, $\varkappa_c \geq 0$. From Proposition 1, we already know that any $\varkappa_c < 1$ is feasible under income taxes, while $\varkappa_c < 1 - \alpha\beta$ is the feasibility condition under lump-sum taxes. Hence, we just consider parameterizations in

$$\Omega \equiv \left\{ \omega = (\varkappa_c, \alpha, \beta, \theta) \in \Re^4 : \alpha, \beta, \theta \in (0,1), \ \alpha + \theta \leq 1, \ \varkappa_c \in [0, 1 - \alpha\beta] \right\}.$$

4.1. Maximizing steady-state growth

In an economy without transitional dynamics, welfare is determined by the growth-rate and the initial consumption level, so the influence of the steady-state growth rate on welfare is obvious. We characterize in this section conditions under which a given tax system produces higher growth, leaving the discussion on the implied welfare levels for the next section. From (3.16), given a value of $\varkappa_c$, $\bar{\gamma}$ is strictly concave in $\varkappa_i$, with an interior maximum at

$$\varkappa_i^{d*} = \theta(1 - \varkappa_c), \tag{4.1}$$

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[8]Under either tax system, the state-space representation of the decision rules and policy function with variables in logs [$\hat{c} = \ln(C)$, $\hat{k} = \ln(K)$, $\hat{k}^g = \ln(K^g)$] is,

$$\hat{c}_t = d_1 + \theta\hat{k}_t^g + (1 - \theta)\hat{k}_t,$$

$$\begin{pmatrix} \hat{k}_{t+1}^g \\ \hat{k}_{t+1} \end{pmatrix} = \begin{pmatrix} d_2 \\ d_3 \end{pmatrix} + \begin{pmatrix} \theta & 1 - \theta \\ \theta & 1 - \theta \end{pmatrix} \begin{pmatrix} \hat{k}_t^g \\ \hat{k}_t \end{pmatrix},$$

where d_1, d_2 and d_3 are constants.

[9]See King and Rebelo (1988) and Caballé and Santos (1993).

which gives us the growth-maximizing level[10] of $\varkappa_i$ under *income taxes*.[11] Condition (4.1) includes as special cases the result in section I in Barro (1990), Glomm and Ravikumar (1994) and Futagami et al. (1993), who obtain $\varkappa_i^{d*} = \theta$ when working with $\varkappa_c = 0$.

Under *lump-sum taxes*, (3.22) shows that the steady-state growth rate $\bar{\gamma}$ is monotonically increasing and concave in $\varkappa_i$, the level of $\varkappa_i$ being bounded by

$$\varkappa_i^{n*} = 1 - \varkappa_c - \alpha\beta. \tag{4.2}$$

This upper bound on $\varkappa_i$, which is inversely related to $\varkappa_c$, α and β, restricts the choice set of the government under lump-sum taxes.

Comparing (3.16) with (3.22), we see that the growth rate $\bar{\gamma}$ will be strictly higher under lump-sum than under income taxes for a *common* value of $\varkappa_i$. This is due to the disincentive effect that a positive capital tax rate exerts on the accumulation of private capital, relative to taxing lump-sum, when financing a given level of $\varkappa_i + \varkappa_c$. However, a government interested in maximizing long-run growth would choose a productive public investment ratio $\varkappa_i^{d*}$ under income taxes, choosing an investment ratio as close as possible to $\varkappa_i^{n*}$ under lump-sum taxes.

Plugging (4.1), (4.2) in (3.16), (3.22), the implied growth rates, denoted by $\bar{\gamma}^{d*}$ and $\bar{\gamma}^{n*}$, respectively, satisfy

$$\frac{1 + \overline{\gamma}^{d*}}{1 + \overline{\gamma}^{n*}} = \left(\frac{\varkappa_i^{d*}}{\varkappa_i^{n*}}\right)^{\theta} \left(1 - \varkappa_i^{d*} - \varkappa_c\right)^{1-\theta} = \left(\frac{\theta}{1 - \varkappa_c - \alpha\beta}\right)^{\theta} \left(1 - \varkappa_c\right)\left(1 - \theta\right)^{1-\theta}, \tag{4.3}$$

which increases with $\varkappa_c$ for $\varkappa_c > 1 - \frac{\alpha\beta}{1-\theta}$. This is the same condition guaranteeing that $\varkappa_i^{d*} > \varkappa_i^{n*}$, which emerges from (4.3) as a necessary condition for $\overline{\gamma}^{d*} > \overline{\gamma}^{n*}$. Hence, for sufficiently high levels of the non-productive public expenditure ratio, any ordering between the implied growth rates might arise, and Figure 4.1 shows that, indeed, growth could be higher under distortionary taxes.

The relationship between growth rates can be written,

$$\bar{\gamma}^{d*} > \bar{\gamma}^{n*} \Leftrightarrow \Psi(\omega) \equiv 1 - \frac{\beta\alpha}{(1 - \varkappa_c)\left\{1 - \theta\left[(1 - \varkappa_c)(1 - \theta)\right]^{\frac{1-\theta}{\theta}}\right\}} < 0, \tag{4.4}$$

with $\Psi(\omega)$ a function defined over Ω. For any given values of α, β, θ in Ω, Proposition 2 guarantees the existence of a single and positive level of $\varkappa_c$, denoted by $\varkappa_c^{\gamma}$, for which $\Psi(\varkappa_c^{\gamma}, \alpha, \beta, \theta) = 0$ and $\bar{\gamma}^{d*} = \bar{\gamma}^{n*}$. Values of $\varkappa_c$ above $\varkappa_c^{\gamma}$ will *significantly reduce* the government's choice set for $\varkappa_i$ under lump-sum taxes so that $\varkappa_i^{n*}$ will fall to the left of A in Figure 4.1, making $\bar{\gamma}^{d*}$ to be higher than $\bar{\gamma}^{n*}$.[12] Above $\varkappa_c^{\gamma}$, the value of $\varkappa_c$ places an

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[10] Along the paper, an asterisk denotes a value obtained under a growth-maximizing public investment strategy.

[11] Condition (4.1) is equivalent to that in section IV of Barro (1990).

[12] In the non-congested Glomm-Ravikumar (1994) setting, where $\varkappa_c = 0$, lump-sum taxes would always produce faster growth than income taxes, a special case of our result.

upper bound on the choice of productive public investment under lump-sum taxes which is more damaging for growth than the distortion introduced by income taxes.

Finally, Corollary 1 shows that $\varkappa_c^\gamma$ is inversely related to α, β and θ. Therefore, in a Barro-type setting, it is more likely that the maximum achievable growth rate will be higher under income than under lump-sum taxes in economies with high values of the unproductive public consumption/output ratio, high private and public capital productivity (high α and θ) and where households weight heavily future consumption.

Proposition 2. *There is a critical value of $\varkappa_c$, $1 - \frac{\alpha\beta}{1-\theta} < \varkappa_c^\gamma < 1 - \alpha\beta$, above (below) which the maximum achievable growth rate is higher (lower) under income than under lump-sum taxes.*

Proof. *From the expression of $\Psi(\omega)$ in (4.4), it is easy to check that: (a) $\Psi(\omega)$ is continuous on Ω; (b) for any given values of α, β, θ, $\Psi\left(\varkappa_c = 1 - \frac{\alpha\beta}{1-\theta}, \alpha, \beta, \theta\right) = 1 - \frac{1-\theta}{\left(1-\theta(\alpha\beta)^{\frac{1-\theta}{\theta}}\right)} > 0$, while $\lim\limits_{\varkappa_c \to (1-\alpha\beta)^-} \Psi(\omega) = 1 - \frac{1}{1-\theta[\alpha\beta(1-\theta)]^{\frac{1-\theta}{\theta}}} < 0$; and (c) $\partial\Psi(\omega)/\partial\varkappa_c = -\beta\alpha\frac{1-\theta}{(1-\varkappa_c)^2}\frac{1-[(1-\varkappa_c)(1-\theta)]^{\frac{1-\theta}{\theta}}}{\left\{1-\theta[(1-\varkappa_c)(1-\theta)]^{\frac{1-\theta}{\theta}}\right\}} < 0$ on Ω. Therefore, there exists a single level of $\varkappa_c$ in $\left[1-\frac{\alpha\beta}{1-\theta}, 1-\alpha\beta\right)$, $\varkappa_c^\gamma$, such that $\Psi(\omega) < 0$ if and only if $\varkappa_c > \varkappa_c^\gamma$. From (4.4), that implies $\bar{\gamma}^{d*} > \bar{\gamma}^{n*}$, the opposite being true if $\varkappa_c \leq \varkappa_c^\gamma$ ∎*

Corollary 1. *$\varkappa_c^\gamma$ is inversely related to α, θ and β.*

Proof. *$\Psi(\omega) = 0$ defines an implicit function which is C^2 on Ω. Let us denote $B = (1-\varkappa_c)\left\{1 - \theta\left[(1-\varkappa_c)(1-\theta)\right]^{\frac{1-\theta}{\theta}}\right\} > 0$. From Proposition 2, $\partial\Psi(\omega)/\partial\varkappa_c < 0$. Consequently, by the implicit function theorem: (a) $\frac{\partial\varkappa_c^\gamma}{\partial\alpha} = -\frac{\partial\Psi(\omega)/\partial\alpha}{\partial\Psi(\omega)/\partial\varkappa_c} < 0$, since $\frac{\partial\Psi(\omega)}{\partial\alpha} = \frac{-\beta}{B} < 0$; (b) $\frac{\partial\varkappa_c^\gamma}{\partial\beta} = -\frac{\partial\Psi(\omega)/\partial\beta}{\partial\Psi(\omega)/\partial\varkappa_c} < 0$, since $\frac{\partial\Psi(\omega)}{\partial\beta} = \frac{-\alpha}{B} < 0$; and (c)$\frac{\partial\varkappa_c^\gamma}{\partial\theta} = -\frac{\partial\Psi(\omega)/\partial\theta}{\partial\Psi(\omega)/\partial\varkappa_c} < 0$, since $\frac{\partial\Psi(\omega)}{\partial\theta} = \frac{\alpha\beta[(1-\varkappa_c)(1-\theta)]^{\frac{1-\theta}{\theta}}\ln[(1-\varkappa_c)(1-\theta)]}{(1-\varkappa_c)\theta\left\{1-\theta[(1-\varkappa_c)(1-\theta)]^{\frac{1-\theta}{\theta}}\right\}^2}$, which is negative because $\ln\left[(1-\varkappa_c)(1-\theta)\right] < 0$ ∎*

Since the economy displays no transition, at $t = 1$ variables are already on their balanced growth path, growing at the steady-state rate unless any policy or structural change occurs. Given K_0, K_0^g, (3.27), (3.29) and (3.30) show that state variables at $t = 1$ under the growth-maximizing policies satisfy,

$$K_1^{d*}/K_1^{n*} = \varkappa_i^{d*} + \varkappa_c = \theta + (1-\theta)\varkappa_c < 1,$$
$$K_1^{gd*}/K_1^{gn*} = \varkappa_i^{d*}/\varkappa_i^{n*} = \frac{\theta(1-\varkappa_c)}{1-\varkappa_c-\alpha\beta},$$

so that the disincentive created by income taxes leads to a private capital stock below that accumulated under lump-sum taxes. The distortion on private capital accumulation decreases for high values of $\varkappa_c$, since the K_1^{d*}/K_1^{n*}-ratio depends positively on $\varkappa_c$. On the other hand, K_1^{gd*}/K_1^{gn*} increases with $\varkappa_c$ and $K_1^{gd*} > K_1^{gn*}$ for $\varkappa_c > 1 - \frac{\alpha\beta}{1-\theta}$ (i.e., when $\varkappa_i^{d*} > \varkappa_i^{n*}$). The economy reacts to taxes on private capital with higher public capital

accumulation, and this shift of resources from private to public capital accumulation can be enough to produce higher long-run growth.

The K_1^{gd*}/K_1^{gn*}-ratio increases with β, α and θ and, additionally, the K_1^{d*}/K_1^{n*}-ratio increases with θ, which explains the result in Corollary 1. The higher the output elasticities of either type of capital, α, θ, the lower will be the loss of future resources produced by distortionary taxation, so it will be more likely that this system produces higher growth.

Growth in this economy arises from the accumulation of both, private and public capital. A higher preference for future utility increases the desire to save much more under non-distortionary taxation since then, the return to private capital is not taxed. Not leaving many resources for consumption, the government has a limit on their own productive investment expenditures, which restricts the possibilities for future growth. The distortion of unproductive public expenditures in this model is reflected in the fact that under lump-sum taxes too much private capital is accumulated relative to public capital. This disequilibrium is negative for long-run growth, which explains the likely superiority of distortionary taxation.

Theoretical results in Proposition 2 are easy to show because both types of capital fully depreciate every period. However, full depreciation is not needed for the qualitative statements in this proposition. Maintaining the assumption of linear depreciation rates for both types of capital, it is shown in the Appendix (part 1): (a) there exists a single and well-defined balanced growth path; (b) the growth-maximizing public investment ratio under income taxes, $\varkappa_i^{d*}$, is given by (4.1) for any δ, δ^g; (c) the result in Proposition 2 remains valid: the $\varkappa_c$-threshold, $\varkappa_c^{\gamma}$, for any $\delta^g \in [0,1]$ and $\delta = 1$ is the same as with $\delta^g = \delta = 1$, while it is higher[13] than that value when $\delta \in [0,1)$.

4.2. Maximizing welfare

We now extend the analysis to discuss the possibility that the maximum level of welfare might be higher under income than under lump-sum taxes. Given $\varkappa_c \geq 0$ and $K_0, K_0^g > 0$, we assume that the government chooses $\varkappa_i$ to maximize the welfare of the representative household over the set of π-CE allocations. Since under full depreciation of both types of capital the economy displays no transition, the problem reduces to choosing $\varkappa_i$ such that,

$$\max_{0 \leq \varkappa_i \leq 1} V(C_0, \bar{\gamma}) = \left[\frac{1}{1-\beta} \ln C_0 + \frac{\beta}{(1-\beta)^2} \ln(1+\bar{\gamma}) \right], \qquad (4.5)$$

subject to either (3.16) and (3.26) under income taxes or to (3.22) and (3.28) under lump-sum taxes. For $0 < \beta < 1$, $V(C_0, \bar{\gamma})$ is strictly concave and bounded and the choice set is convex and compact, so that the optimization problem (4.5) has a single solution. Moreover, since $\lim_{\varkappa_i \to 0^+} V = \lim_{\varkappa_i \to 1^-} V = -\infty$, the welfare-maximizing level of $\varkappa_i$ falls strictly inside the interval $(0,1)$.

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[13]So, for given β, α, θ, the range of $\varkappa_c$-values leading to higher growth under income taxes is smaller when private capital depreciation is not complete. A numerical example is also provided in table 6.1.1 in the appendix (part 1), showing that, in fact, $\varkappa_c^{\gamma}$ varies very little for $\delta^g, \delta \in [0,1]$.

In fact, the welfare maximizing levels of $\varkappa_i$ under income taxes, $\varkappa_i^{d+}$, and under lump-sum taxes, $\varkappa_i^{n+}$, are given by:[14]

$$\varkappa_i^{d+} = \beta\theta(1 - \varkappa_c) = \beta\varkappa_i^{d*}, \tag{4.6}$$

$$\varkappa_i^{n+} = \frac{\beta\theta}{1 - \beta(1-\theta)}(1 - \varkappa_c - \alpha\beta) = \frac{\beta\theta}{1 - \beta(1-\theta)}\varkappa_i^{n*}, \tag{4.7}$$

with $\varkappa_i^{d*}$ and $\varkappa_i^{n*}$ being the growth-maximizing investment ratios defined in (4.1) and (4.2).

Since $\frac{\beta\theta}{1-\beta(1-\theta)} < 1$, the growth-maximizing public investment/output ratio is strictly higher than the welfare-maximizing ratio under both tax rules,[15] the difference between them being larger under the less distorting tax system. Under income taxes, the level of $\varkappa_i^{d+}$ differs from $\varkappa_i^{d*}$ by the factor β, as in Glomm and Ravikumar (1994), since the public capital stock only becomes productive next period and so the representative household discounts the positive effect of public investment on welfare by β. Under lump-sum taxes, the discrepancy between $\varkappa_i^{n*}$ and $\varkappa_i^{n+}$, measured by $\frac{\beta\theta}{1-\beta(1-\theta)}$, is smaller than β, but it increases with β and θ. The welfare-maximizing productive public investment ratio is higher under income than under lump-sum taxation for $\varkappa_c > \tilde{\varkappa}_c = 1 - \frac{\alpha}{1-\theta}$.[16]

To discuss conditions under which welfare could be higher under income than under lump-sum taxes, we evaluate (4.5) under $\varkappa_i = \varkappa_i^{n+}$ and under $\varkappa_i = \varkappa_i^{d+}$, to obtain maximized levels of welfare V^n and V^d, respectively. Their difference, $D = V^n - V^d$, can be written as[17]

$$\begin{aligned}
(1-\beta)D(\omega) \;=\; & \ln\left(\frac{1-\beta}{1-\alpha\beta}\right) + \frac{1 - \beta(1-\theta)}{1-\beta}\ln\left[\frac{1 - \varkappa_c - \alpha\beta}{1 - \beta(1-\theta)}\right] \\
& - \frac{1}{1-\beta}\ln(1 - \varkappa_c) - \frac{1-\theta\beta}{1-\beta}\ln(1 - \beta\theta),
\end{aligned} \tag{4.8}$$

where $D(\omega)$ is defined on Ω.

Even though there is no possibility of finding explicit conditions implying $D(\omega) < 0$, for given values of α, β, θ, Proposition 3 guarantees existence of a single positive threshold $\varkappa_c^v$ for the non-productive investment ratio, above (below) which taxing the income of productive factors is welfare-superior (-inferior) to taxing lump-sum. Lemmas 1 and 2 show two intermediate results.

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[14]See Appendix (part 2). Along the paper, a "+" uppercase denotes a value obtained under a welfare-maximizing public investment strategy.

[15]$\varkappa_i^{d+}$ and $\varkappa_i^{d*}$ are equal to each other in the no-spillover model of Barro (1990), while $\varkappa_i^{d+} < \varkappa_i^{d*}$ in Futagami et al. (1993) and in Glomm and Ravikumar (1994).

[16]Since $\varkappa_i^{d+} > \varkappa_i^{n+}$ for $\varkappa_c > 1 - \frac{\alpha}{1-\theta}$, we will have equality between $\varkappa_i^{d+}$ and $\varkappa_i^{n+}$ when $\varkappa_c = 0$ and $\alpha = 1 - \theta$ (i.e., no spillover in the productive process). Precisely under these conditions, Corsetti and Roubini (1996) show that the welfare-maximizing public investment ratio is independent of the tax system considered in a standard Barro-type framework.

[17]See Appendix (part 2) for more details on this point.

Lemma 1. $D\left(0; \alpha, \beta, \theta\right) > 0$ for any α, β, θ in Ω.
Proof. *See Appendix (part 3)* ∎

Lemma 2. *For any given values* $\alpha_0, \beta_0, \theta_0$ *in* $(0, 1)$ *with* $\alpha_0 + \theta_0 \leq 1$, *the function* $h(\varkappa_c)$ *defined by* $h(\varkappa_c) = D(\varkappa_c, \alpha_0, \beta_0, \theta_0)$ *in (4.8) is concave, and has a single maximum at* $\tilde{\varkappa}_c = 1 - \frac{\alpha_0}{1 - \theta_0}$.
Proof. *It is easy to see from (4.8) that, given* $\alpha_0, \beta_0, \theta_0$ *in* Ω, $h(\varkappa_c)$ *is differentiable, the single solution to* $\partial h / \partial \varkappa_c = \partial D(\varkappa_c; \alpha_0, \beta_0, \theta_0) / \partial \varkappa_c = 0$ *is* $\tilde{\varkappa}_c = 1 - \frac{\alpha_0}{1 - \theta_0}$, *and* $\partial^2 D(\varkappa_c; \alpha_0, \beta_0, \theta_0) / \partial \varkappa_c^2 \mid_{\varkappa_c = \tilde{\varkappa}_c} < 0$ ∎

Proposition 3. *There is a critical value of* $\varkappa_c$, $\varkappa_c^v \in (\tilde{\varkappa}_c, 1 - \alpha\beta)$, *so that income taxes are a preferred alternative to lump-sum taxes from the point of view of welfare if and only if* $\varkappa_c > \varkappa_c^v$.
Proof. (a) $D(\omega)$, *defined by (4.8) is continuous in* Ω; (b) *we have seen in Lemma 1 that* $D\left(0; \alpha, \beta, \theta\right) > 0$ *for any* α, β, θ *in* Ω, *while Lemma 2 shows that* $\tilde{\varkappa}_c = \arg\max D\left(\varkappa_c; \alpha, \beta, \theta\right)$, *so that* $D\left(\tilde{\varkappa}_c; \alpha, \beta, \theta\right) > 0$; (c) *at the highest feasible value of* $\varkappa_c$, $\lim\limits_{\varkappa_c \to (1-\alpha\beta)^-} D(\omega) = -\infty$.
Therefore, there exists a single and well-defined threshold for $\varkappa_c$, $\varkappa_c^v \in (\tilde{\varkappa}_c, 1 - \alpha\beta)$, *such that* $D(\omega) < 0$ *for any* $\varkappa_c > \varkappa_c^v$ ∎

Corollary 2 shows that $\varkappa_c^v$ declines with α, the output elasticity of private capital. We also show that there is a threshold θ^*, such that $\varkappa_c^v$ declines with the output elasticity of public capital θ, provided $\theta > \theta^*$. For standard parameterizations, the implied value of θ^* is low enough so that we can safely consider $\partial \varkappa_c^v / \partial \theta < 0$. Finally, a numerical exploration shows that $\varkappa_c^v$ declines with β when this parameter is close to one, as considered in standard calibrations. This should be expected, since (4.5) shows that as β approaches 1, maximizing long-run growth and maximizing welfare become equivalent. These results suggest that, as it was the case when comparing growth, high levels of β, α and θ increase the likelihood that taxing total income could be a welfare-superior alternative to taxing lump-sum.

Corollary 2. $\varkappa_c^v$ *is inversely related to* α, *while* $\partial \varkappa_c^v / \partial \theta < 0$ *for* $\theta \in (\max\{0, \theta^*\}, 1 - \alpha]$, *with* $\theta^* = -\frac{\varkappa_c - \beta(1-\alpha)}{\beta(2 - \varkappa_c - \alpha\beta)}$, *and* $\partial \varkappa_c^v / \partial \theta > 0$ *otherwise.*
Proof. *See Appendix (part 4)* ∎

Corollary 3 shows that $\varkappa_i^{d+}$ needs to be bigger than $\varkappa_i^{n+}$ for the maximum level of welfare to be higher under income than under lump-sum taxes, which is a condition similar to that found when comparing growth rates.

Corollary 3. *A necessary condition for maximum welfare to be larger under income than under lump-sum taxes is that the welfare-maximizing public investment ratio be higher under income than under lump-sum taxes.*
Proof. *Let us assume that maximum welfare is larger under income than under lump-sum taxes. Then, by Proposition 3,* $\varkappa_c > \varkappa_c^v$. *But* $\varkappa_c^v > \tilde{\varkappa}_c$, *so that* $\varkappa_c > \tilde{\varkappa}_c$, *which implies, from (4.6) and (4.7),* $\varkappa_i^{d+} > \varkappa_i^{n+}$ ∎

Lack of transitional dynamics allows us to write welfare as in (4.5), a combination of initial consumption and long-term growth. In fact, in choosing one versus the other tax system, we are trading-off current versus future consumption. Taxing income from productive factors disincentives private capital accumulation and reduces long-run growth, with a moderate immediate consumption sacrifice. Alternatively, lump-sum taxation produces an important consumption sacrifice initially, to the possible benefit of higher long-term growth. From (3.26), (3.28), we have, under the welfare-maximizing public investment ratios,

$$\frac{C_0^{d+}}{C_0^{n+}} = \frac{(1 - \alpha\beta)(1 - \varkappa_c)(1 - \beta\theta)}{(1 - \beta)(1 - \varkappa_c - \alpha\beta)}[1 - \beta(1 - \theta)],\qquad(4.9)$$

from which it is not hard to show that initial consumption is always higher under distortionary taxation.[18] In fact, for standard parameterizations, initial consumption can easily be ten or twenty times bigger under income taxes than under lump-sum taxes, even for low $\varkappa_c$-ratios.

On the other hand, for small levels of the non-productive public expenditure ratio $\varkappa_c$, growth is higher under non-distortionary taxes and there are two competing effects in (4.5). As we saw in the previous section, it is when $\varkappa_c$ increases that growth may become higher under distortionary taxation. Hence, any condition favoring higher growth under income taxes will also tend to induce that distortionary taxation might be preferred from the point of view of welfare. In fact, if growth ever gets higher under distortionary taxation, it is clear from (4.5) that this type of taxes will also be preferred in terms of welfare.

In particular, we have already seen that higher output elasticities of either type of capital attenuate the negative effects of the disincentive to accumulate capital, making more likely that long-run growth may be higher under distortionary than under non-distortionary taxation. It is therefore not surprising that they also favor that maximum welfare may be higher under distortionary taxation. Finally, maximizing welfare amounts to maximizing long-run growth for high values of β, so that an increase in the value of β in that range increases the likelihood that distortionary taxation may lead to a higher level of welfare.

4.3. A numerical illustration

The purpose of this section is to illustrate some of the main findings discussed above with some numerical examples. Specifically, for alternative calibrations (i.e., specific values for β, α and θ), we find the solutions to $D(\varkappa_c; \beta, \alpha, \theta) = 0$ and $\Psi(\varkappa_c; \beta, \alpha, \theta) = 0$. Assuming annual data, four alternative parameterizations (bench1, bench2, bench3 and bench4) are considered in table 4.1, all of them sharing a value $\beta = 0.99$. In bench1, we assume a standard parametrization: $\alpha = 0.4$, $\theta = 0.15$ so that $\phi = 1 - \alpha - \theta = 0.45$.

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[18]Which is easy to see when $\varkappa_c = 0$. Since the ratio of initial consumptions is increasing in $\varkappa_c$, the general result follows.

Relative to bench1, bench2 is a small variation, with $\alpha = 0.75$. This parameterization agrees with a broad interpretation of aggregate capital in the private producing process, K_t including human and physical capital [see Romer (1987)]. In bench3 we consider the possibility of a technology intensive in productive public capital by setting $\theta = 0.35$ [close to the value estimated in Aschauer (1989)], and $\alpha = 0.4$, as in bench1. Finally, bench4 assumes a high productive technology in both types of capital, with $\alpha = 0.7$ and $\theta = 0.3$, implying absence of spillover effects.

For each parameterization considered in table 4.1, table 4.2 shows numerical values for: (i) the two thresholds: $\varkappa_c^\gamma$ and $\varkappa_c^v$, the zeroes to $\Psi(\omega)$ and $D(\omega)$, (ii) for $\varkappa_c = \varkappa_c^\gamma$, the growth-maximizing public investment/output ratio under income taxes [$\varkappa_i^{d*}$ from (4.1)] and the level of $\varkappa_i^{n*}$ under lump-sum taxes (4.2), and (iii) for $\varkappa_c = \varkappa_c^v$, the welfare-maximizing public investment/output ratios under income taxes [$\varkappa_i^{d+}$ in (4.6)] and under lump-sum taxes [$\varkappa_i^{n+}$ from (4.7)].

As expected, both thresholds decrease when we increase α or θ, although it seems that changes in the elasticity of private capital are more relevant. For instance, from bench1, the level of $\varkappa_c^\gamma$ decreases from 60.4% to 59.2% when θ increases from .15 to .35, but it falls down to 24.9% when α is increased from .40 to .75. A similar observation applies to $\varkappa_c^v$. The threshold above which growth is higher under distortionary taxation is always higher than the one above which distortionary taxation leads to higher welfare, so the latter situation arises more often, as we pointed out in the previous section.

Productive public investment ratios maximizing either growth or welfare under lump-sum taxes are well below those obtained under distortionary taxation. Finally, under either tax system, investment ratios maximizing welfare are very similar to those maximizing long-run growth. Under distortionary taxation, the $\varkappa_i$-ratio maximizing welfare is always below that maximizing growth, the opposite result arising under lump-sum taxes. This is again due to the fact that initial consumption is higher under distortionary taxation.

The presence of the spillover externality allows us to change the values of the output elasticities of private and public capital, α and θ, independently from each other, but that externality plays a minor role in the model. In fact, the proofs to all results in the previous section remain valid for the case $\phi = 0$, so long as we maintain the restriction $\alpha + \theta = 1$.

5. Conclusions

The superiority of non-distorting versus distorting taxation in endogenous growth economies has been challenged from different perspectives. In particular, it is already well known that positive taxation on the income of productive factors may be growth and welfare enhancing when it corrects a negative externality in the competitive equilibrium allocation.

Along this line, we have shown that existence of a significant level of non-productive public investment, as a percentage of output, is a sufficient condition for income taxes to lead to higher long-run growth and welfare than lump-sum taxes. Specifically, we have characterized a threshold for the *unproductive* public expenditure/output ratio, above

(below) which taxing productive factors' income results in a welfare-superior (inferior) financing alternative to taxing lump-sum. The higher the output elasticities of public and private capital, the lower the threshold for the unproductive public expenditure/output ratio, favoring income taxes to be preferred to lump-sum taxes for steady-state growth and welfare.

A given level of non-productive public expenditures works as a negative externality, which is more damaging under non-distortionary taxation. In that case, accumulation of private capital is stronger because the return on capital is not being taxed. As a consequence, less resources are left for public investment, which limits the scope for future growth. A higher time discount factor increases the desire to accumulate capital, intensifying the effect of this externality and making more likely that distortionary taxes may be preferred to lump-sum taxes.

Welfare analysis has been relatively easy to make under the assumption that both types of capital fully depreciate every period. This assumption allows us to solve for the competitive equilibrium analytically, although in a framework too simple to establish strong policy recommendations. Regarding long-run growth, we have shown that full depreciation is not determinant for the qualitative and quantitative conclusions, since the analysis is of the steady-state type. Less than full depreciation in public and private capital would alter the model significantly, since the steady-state would no longer be reached in just one period. Welfare analysis then requires a full characterization of the transitional dynamics, which can only be done through numerical solution methods. That would be an interesting extension of this paper.

6. Appendix:

6.1. Part 1: Partial depreciation of capital stocks

Conditions (3.10) and (3.13) can be jointly written,

$$\bar{\gamma} = \beta\left[(1 - \delta) + (1 - \bar{\tau})\bar{r}\right] - 1, \tag{6.1}$$

with $\bar{\tau} = 0$ under lump-sum taxes and $\bar{\tau} = \varkappa_i + \varkappa_c$ under income taxes.

Using this together with (3.3)-(3.6), which are valid under either tax system, we get,

$$\Phi\left(\bar{k}^g\right) = F\left(\bar{k}^g\right)^{\theta-1}\left(\varkappa_i - \beta\alpha(1 - \bar{\tau})\bar{k}^g\right) + 1 - \delta^g - \beta(1 - \delta) = 0, \tag{6.2}$$

whose positive roots are potential candidates to be steady-state values of $\bar{k}^g$. Since $\Phi\left(\bar{k}^g\right)$ is continuous and decreasing in $\bar{k}^g$, with $\lim_{\bar{k}^g \to 0^+} \Phi\left(\bar{k}^g\right) = +\infty$ and $\lim_{\bar{k}^g \to +\infty} \Phi\left(\bar{k}^g\right) = -\infty$, there exists a single $\bar{k}^g > 0$ such that $\Phi\left(\bar{k}^g\right) = 0$, which defines the steady-state of the economy.

Lemma 3. *Under either tax system, the steady-state level of $\bar{k}^g$ is inversely related to the productive public investment ratio $\varkappa_i$.*

Proof. Applying the implicit function theorem to the C^2-mapping $\Phi(\cdot)$ and using the fact that under income taxes $\bar{\tau} = \varkappa_i + \varkappa_c$, we get,

$$\frac{\partial \bar{k}^{gd}}{\partial \varkappa_i} = -\frac{\partial \Phi\left(\bar{k}^{gd}\right)/\partial \varkappa_i}{\partial \Phi\left(\bar{k}^{gd}\right)/\partial \bar{k}^{gd}} = \frac{1 + \beta\alpha\bar{k}^{gd}}{(1 - \theta)\frac{1}{\bar{k}^{gd}}\varkappa_i + \alpha\beta\theta(1 - \varkappa_i - \varkappa_c)} > 0, \tag{6.3}$$

while under lump-sum taxes,

$$\frac{\partial \bar{k}^{gn}}{\partial \varkappa_i} = -\frac{\partial \Phi\left(\bar{k}^{gn}\right)/\partial \varkappa_i}{\partial \Phi\left(\bar{k}^{gn}\right)/\partial \bar{k}^{gn}} = \frac{1}{(1 - \theta)\frac{1}{\bar{k}^{gn}}\varkappa_i + \alpha\beta\theta} > 0 \; \blacksquare \tag{6.4}$$

We prove next that, *under income taxes,* expression (4.1) for the growth-maximizing ratio $\varkappa_i^{d*} = \theta(1 - \varkappa_c)$ remains valid for any $\delta, \delta^g \in [0, 1]$.

Lemma 4. *Under income taxes, the growth-maximizing ratio is $\varkappa_i^{d*} = \theta(1 - \varkappa_c)$ for any $\delta, \delta^g \in [0, 1]$.*

Proof. From (3.4) and (3.3) we get: $\lim_{\varkappa_i \to 0} \bar{\gamma} = -\delta^g < 0$ and $\lim_{\varkappa_i \to 1^-} \bar{\gamma} = -\varkappa_c F\left(\bar{k}^{gd}\right)^{\theta} - \bar{c} - \delta < 0$, so that positive values of $\bar{\gamma}$ can be attained only for values of $\varkappa_i^{d*}$ in the open interval $(0, 1)$. Taking derivatives in (3.4) and combining

$$\frac{\partial \bar{\gamma}}{\partial \varkappa_i} = F\left(\bar{k}^{gd}\right)^{\theta-2}\left(\bar{k}^{gd} - \varkappa_i(1 - \theta)\frac{\partial \bar{k}^{gd}}{\partial \varkappa_i}\right) = 0, \tag{6.5}$$

with (6.3) implies $\varkappa_i^{d*} = \theta(1 - \varkappa_c)$ for any δ and δ^g ∎

The qualitative result in Proposition 2 holds for $\delta^g < 1$. The $\varkappa_c^\gamma$-threshold is the same as that characterized in the proposition, so long as $\delta = 1$. If $\delta < 1$, the $\varkappa_c^\gamma$-threshold is higher than that in Proposition 2.

Lemma 5. *There is a critical value of $\varkappa_c$, $\hat{\varkappa}_c^\gamma$, above (below) which the maximum achievable growth rate is higher (lower) under income than under lump-sum taxes. For any $\delta^g \in [0,1]$ and $\delta = 1$, this $\varkappa_c$-threshold is the same as with $\delta^g = \delta = 1$, while it is higher than that value when $\delta \in [0,1)$.*

Proof. Since the growth rate under lump-sum taxes is increasing in $\varkappa_i$ for any $\delta^g, \delta \in [0,1]$, the highest feasible growth rate that can be achieved *under lump-sum taxes* is again obtained for a productive investment ratio of $\varkappa_i^{n*}$, the level of $\varkappa_i$ implying $\bar{c}^n = 0$. Let us denote by $\hat{\varkappa}_c^\gamma$ the $\varkappa_c$-threshold above which the maximum growth rate under income taxes becomes higher than growth under $\varkappa_i^{n*}$, and by $\bar{k}^{gd*}$, $\bar{k}^{gn*}$, the steady-state values of the $\bar{K}_t^g/\bar{K}_t$ ratio obtained under $\varkappa_i^{d*}, \varkappa_i^{n*}$, the growth maximizing investment policies under income and lump-sum taxes, respectively, when $\varkappa_c = \hat{\varkappa}_c^\gamma$.

Thus, by definition, $\hat{\varkappa}_c^\gamma$, $\varkappa_i^{n*}$, $\bar{k}^{gd*}$ and $\bar{k}^{gn*}$ must verify: (1) $\Phi(\bar{k}^{gd*}) = 0$ under income taxes for $\varkappa_i = \varkappa_i^{d*}$, (2) $\Phi(\bar{k}^{gn*}) = 0$ under lump-sum taxes for $\varkappa_i = \varkappa_i^{n*}$, (3) $\bar{c}^n = 0$ under lump-sum taxes for $\varkappa_i = \varkappa_i^{n*}$, and (4) for $\varkappa_c = \hat{\varkappa}_c^\gamma$, the growth rate obtained under $\varkappa_i^{d*}$, must be equal to the upper bound on growth rates under lump-sum taxes, *i.e.*, the growth rate that would be obtained under $\varkappa_i^{n*}$.

These conditions reduce to:

$$F\left(\bar{k}^{gd*}\right)^{\theta-1}\left(1 - \hat{\varkappa}_c^\gamma\right)\left[\theta - \beta\alpha(1-\theta)\bar{k}^{gd*}\right] + 1 - \delta^g - \beta(1-\delta) = 0, \qquad (6.6)$$

$$F\left(\bar{k}^{gn*}\right)^{\theta-1}\left(\varkappa_i^{n*} - \beta\alpha\bar{k}^{gn*}\right) + 1 - \delta^g - \beta(1-\delta) = 0, \qquad (6.7)$$

$$\left(1 - \varkappa_i^{n*} - \hat{\varkappa}_c^\gamma\right)F\left(\bar{k}^{gn*}\right)^\theta - \varkappa_i^{n*}F\left(\bar{k}^{gn*}\right)^{\theta-1} - \delta + \delta^g = 0, \qquad (6.8)$$

$$\varkappa_i^{n*} = \theta\left(1 - \hat{\varkappa}_c^\gamma\right)\left(\frac{\bar{k}^{gn*}}{\bar{k}^{gd*}}\right)^{1-\theta}. \qquad (6.9)$$

Subtracting (6.6) from (6.7) and combining with (6.8),

$$\frac{\bar{k}^{gn*}}{\bar{k}^{gd*}} = \left[(1-\theta)\left(1 - \hat{\varkappa}_c^\gamma\right)\right]^{1/\theta}. \qquad (6.10)$$

Plugging (6.10) into (6.9),

$$\varkappa_i^{n*} = \theta\left(1 - \theta\right)^{\frac{1-\theta}{\theta}}\left(1 - \hat{\varkappa}_c^\gamma\right)^{\frac{1}{\theta}}. \qquad (6.11)$$

On the other hand, adding up (6.7) and (6.8),

$$F\left(\bar{k}^{gn*}\right)^\theta\left(1 - \varkappa_i^{n*} - \hat{\varkappa}_c^\gamma - \beta\alpha\right) + (1-\beta)(1-\delta) = 0. \qquad (6.12)$$

If $\delta = 1$ then, for any $\delta^g \in [0, 1]$, we have from (6.12): $\varkappa_i^{n*} = 1 - \hat{\varkappa}_c^\gamma - \beta\alpha$, since $\bar{k}^{gn*} > 0$, and plugging (6.11) into this relationship, we obtain,

$$\hat{\varkappa}_c^\gamma - 1 + \frac{\beta\alpha}{1 - \theta\left[(1 - \hat{\varkappa}_c^\gamma)(1 - \theta)\right]^{\frac{1-\theta}{\theta}}} = 0, \tag{6.13}$$

which has the same roots as the Ψ-function defined in (4.4). Consequently, $\hat{\varkappa}_c^\gamma = \varkappa_c^\gamma$, the same value obtained under full depreciation of both types of capital. Alternatively, if $\delta \in [0, 1)$ we would have $(1 - \beta)(1 - \delta) > 0$ in (6.12), which implies that, for any $\delta^g \in [0, 1]$, we have $\varkappa_i^n > 1 - \hat{\varkappa}_c^\gamma - \beta\alpha$, and hence $\hat{\varkappa}_c^\gamma > \varkappa_c^\gamma$ ∎

Even though the $\varkappa_c^\gamma$-threshold changes with the depreciation rate of private capital, this effect is minor. Table 6.1 summarizes a numerical example for $\theta = .15$, $\alpha = .75$ and $\beta = .99$. System (6.6)-(6.9) is solved for this calibration to obtain $\hat{\varkappa}_c^\gamma$, which turns out to vary just between 24.9% and 25.5% for $\delta^g, \delta^k \in [0, 1]$.

6.2. Part 2: The welfare-maximizing public investment ratio

Under income taxes: plugging (3.26) and (3.16) into (4.5), we get the optimization problem,

$$\max_{0 \le \varkappa_i \le 1} \frac{1}{1 - \beta} \left(\begin{array}{c} \ln\left[(1 - \varkappa_i - \varkappa_c)(1 - \alpha\beta)\right] + \\ + \frac{\beta}{(1-\beta)} \theta\left[\ln \varkappa_i + (1 - \theta)\ln(1 - \varkappa_i - \varkappa_c)\right] + \Delta \end{array} \right), \tag{6.14}$$

where $\Delta = \frac{\beta}{1-\beta}\ln F + \frac{\beta}{1-\beta}(1-\theta)\ln(\alpha\beta) + \ln\left(F\left(K_0^g\right)^\theta K_0^{1-\theta}\right)$. Hence, the welfare-maximizing ratio, $\varkappa_i^{d+}$, is the solution to,

$$\frac{-1}{1 - \varkappa_i^{d+} - \varkappa_c} + \frac{\beta}{1 - \beta}\left(\frac{\theta}{\varkappa_i^{d+}} - (1 - \theta)\frac{1}{1 - \varkappa_i^{d+} - \varkappa_c}\right) = 0, \tag{6.15}$$

leading to: $\varkappa_i^{d+} = \beta\theta(1 - \varkappa_c)$.

In a similar way, under lump-sum taxes: plugging (3.28) and (3.22) into (4.5),

$$\max_{0 \le \varkappa_i \le 1} \frac{1}{1 - \beta}\left(\ln\left[(1 - \varkappa_i - \varkappa_c) - \alpha\beta\right] + \frac{\beta}{(1-\beta)}\theta\ln\varkappa_i + \Delta\right), \tag{6.16}$$

and the welfare-maximizing ratio, $\varkappa_i^{n+}$, the solution to

$$\frac{-1}{1 - \varkappa_i^{n+} - \varkappa_c - \alpha\beta} + \frac{\beta}{1 - \beta}\theta\frac{1}{\varkappa_i^{n+}} = 0 \tag{6.17}$$

is: $\varkappa_i^{n+} = \frac{\beta\theta}{1 - \beta(1-\theta)}(1 - \varkappa_c - \alpha\beta)$.

Given $K_0^g, K_0 > 0$ as initial conditions, to obtain a closed form for the welfare function under income taxes, we evaluate (4.5) under $\varkappa_i = \varkappa_i^{d+}$ and $\bar{\tau} = \varkappa_c + \varkappa_i^{d+}$, obtaining V^d:

$$\begin{aligned} V^d &= \left[\frac{1}{1 - \beta}\ln C_0^{d+} + \frac{\beta}{(1 - \beta)^2}\ln(1 + \bar{\gamma}^{d+})\right], \text{ with} \\ C_0^{d+} &= (1 - \varkappa_c)(1 - \beta\theta)(1 - \alpha\beta)Y_0, \tag{6.18} \\ 1 + \bar{\gamma}^{d+} &= F(1 - \varkappa_c)(\beta\theta)^\theta\left[\alpha\beta(1 - \beta\theta)\right]^{1-\theta}, \tag{6.19} \end{aligned}$$

so that,

$$(1-\beta)V^d = \ln\left(1-\varkappa_c\right) + \ln\left(1-\beta\theta\right) + \ln\left(1-\alpha\beta\right) + \ln Y_0 + \qquad (6.20)$$
$$+\frac{\beta}{(1-\beta)}\left\{\begin{array}{l}\ln F + \ln\left(1-\varkappa_c\right) + \theta\ln(\beta\theta)+ \\ +\left(1-\theta\right)\left[\ln\left(\alpha\beta\right) + \ln\left(1-\beta\theta\right)\right]\end{array}\right\}.$$

Under lump-sum taxes, we evaluate (4.5) under $\varkappa_i = \varkappa_i^{n+}$ and $\bar{\tau} = 0$, obtaining V^n:

$$V^n = \left[\frac{1}{1-\beta}\ln C_0^{n+} + \frac{\beta}{(1-\beta)^2}\ln(1+\bar{\gamma}^{n+})\right], \text{ with}$$

$$C_0^{n+} = \frac{(1-\beta)\left(1-\varkappa_c-\alpha\beta\right)}{1-\beta(1-\theta)}Y_0, \qquad (6.21)$$

$$1+\bar{\gamma}^{n+} = F\left[\frac{\beta\theta}{1-\beta(1-\theta)}\left(1-\varkappa_c-\alpha\beta\right)\right]^\theta (\alpha\beta)^{1-\theta}, \qquad (6.22)$$

so that,

$$(1-\beta)V^n = \ln(1-\beta) + \ln\left(1-\varkappa_c-\alpha\beta\right) - \ln\left(1-\beta(1-\theta)\right) + \ln Y_0 + \qquad (6.23)$$
$$+\frac{\beta}{(1-\beta)}\left\{\begin{array}{c}\ln F + \theta\left[\ln(\beta\theta) + \ln\left(1-\varkappa_c-\alpha\beta\right) - \ln\left(1-\beta(1-\theta)\right)\right]+ \\ +\left(1-\theta\right)\ln\left(\alpha\beta\right)\end{array}\right\}.$$

Finally, from (6.20) and (6.23), it is easy to show that $(1-\beta)\left(V^n - V^d\right)$ is given by (4.8).

6.3. Part 3: Proof of Lemma 1

Proof. From (4.8), we have $\frac{\partial D(0;\alpha,\beta,\theta)}{\partial\alpha} = -\beta\frac{\theta\beta}{(1-\alpha\beta)(1-\beta)} < 0$ at $\varkappa_c = 0$. It suffices to show that for the highest feasible value of α, $\alpha = 1-\theta$, $D(0;1-\theta,\beta,\theta) = \ln\left(\frac{1-\beta}{1-(1-\theta)\beta}\right) - \frac{1-\theta\beta}{1-\beta}\ln\left(1-\beta\theta\right) > 0$, to conclude that $D(0;\alpha,\beta,\theta) > 0$ for any α,β,θ in Ω.

To show that $D(0;1-\theta,\beta,\theta) > 0$, let us denote by

$$f(\theta;\beta) = \ln\left(1-\beta\right) - \ln\left[1-\beta\left(1-\theta\right)\right]; \ g(\theta;\beta) = \frac{1-\beta\theta}{1-\beta}\ln\left(1-\theta\beta\right).$$

We want to show that $w(\theta;\beta) \equiv f(\theta;\beta) - g(\theta;\beta) > 0$.

Second derivatives are:

$$\frac{\partial^2 f}{\partial\theta^2} = \frac{\beta^2}{\left[1-\beta\left(1-\theta\right)\right]^2}; \ \frac{\partial^2 g}{\partial\theta^2} = \frac{\beta^2}{\left(1-\beta\right)\left(1-\beta\theta\right)},$$

so that

$$\frac{\partial^2 w}{\partial\theta^2} = \beta^2 \frac{\left(1-\beta\right)\left(1-\beta\theta\right) - \left[1-\beta\left(1-\theta\right)\right]^2}{\left[1-\beta\left(1-\theta\right)\right]^2 \left(1-\beta\right)\left(1-\beta\theta\right)}.$$

Since $0 < \beta < 1$ and $0 < \theta < 1$, we have:

$$sgn\left(\frac{\partial^2 w}{\partial \theta^2}\right) = sgn\left[(1-\beta)(1-\beta\theta) - [1-\beta(1-\theta)]^2\right] =$$
$$= sgn\left[-\beta\theta^2 - 3(1-\beta)\theta + (1-\beta)\right].$$

Let us denote: $\varphi(\theta;\beta) = \beta\theta^2 + 3(1-\beta)\theta - (1-\beta)$, a second degree polynomial in θ for each given β, with $\varphi(0;\beta) < 0, \varphi(1;\beta) > 0$. Its two roots are given by:

$$\theta_\beta = \frac{-3(1-\beta) \pm \sqrt{9(1-\beta)^2 + 4\beta(1-\beta)}}{2\beta},$$

where it is clear that $\theta_\beta^- < 0$, while it is not hard to show that $\theta_\beta^+ < 1$.

Hence, in the range of feasible values of θ, the [0,1]-interval, the $\varphi(\theta;\beta)$ polynomial moves from negative to positive, crossing the θ-axis just once. Equivalently, $w(\theta;\beta)$ is convex between $\theta = 0$ and the positive root, $\theta = \theta_\beta^+$, and concave to the right of that point, being differentiable on $[0,1]$. Together with $w(0;\beta) = 0$, $w(1;\beta) = 0$, $w'(0;\beta) = 0$ and $w'(1;\beta) < 0$, all these conditions imply that $w(\theta;\beta)$ reaches a single local maximum between θ_β^+ and $\theta = 1$, taking always positive values on the interval $[0,1]$ ∎

6.4. Part 4: Proof of Corollary 2

Proof. $D(\omega)$ in (4.8) defines an implicit function in ω. Since $D(\omega) \in \mathbb{C}^2$ on its domain, Ω, the implicit function theorem applies. In addition, we know from Lemma 2 that $\partial D(\omega)/\partial \varkappa_c < 0$ for $\varkappa_c \geq \varkappa_c^v$. Hence:

$$(a) \; \partial \varkappa_c^v/\partial\alpha = -\frac{\partial D(\omega)/\partial\alpha}{\partial D(\omega)/\partial \varkappa_c} < 0, \text{ since}$$

$$\frac{\partial D(\omega)}{\partial\alpha} = -\beta\frac{\varkappa_c(1-\beta) + \beta\theta(1-\beta\alpha)}{(1-\alpha\beta)(1-\beta)(1-\varkappa_c-\alpha\beta)} < 0,$$

$$(b) \; \partial \varkappa_c^v/\partial\theta = -\frac{\partial D(\omega)/\partial\theta}{\partial D(\omega)/\partial \varkappa_c}, \text{ with}$$

$$\partial D(\omega)/\partial\theta = \frac{\beta}{1-\beta}\ln\left[\frac{(1-\varkappa_c-\alpha\beta)(1-\beta\theta)}{1-\beta(1-\theta)}\right].$$

Let us consider the function $h(\theta;\beta,\varkappa_c,\alpha) = \frac{(1-\varkappa_c-\alpha\beta)(1-\beta\theta)}{1-\beta(1-\theta)} - 1$. It is easy to verify that: (i) $\partial h(\cdot)/\partial\theta = (1-\varkappa_c-\alpha\beta)\beta\frac{-2+\beta}{(1-\beta(1-\theta))^2} < 0 \; \forall\theta$, since $\varkappa_c < 1-\alpha\beta$ and (ii) $h(\cdot)$ has a single root at $\theta^* = -\frac{\varkappa_c-\beta(1-\alpha)}{\beta(2-\varkappa_c-\alpha\beta)} < 1-\alpha$, which shows the statement of the corollary ∎

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Table 4.1: Alternative benchmark calibrations				
	bench1	_bench2_	_bench3_	_bench4_
β	0.99	0.99	0.99	0.99
α	0.40	0.75	0.40	0.70
θ	0.15	0.15	0.35	0.30
ϕ	0.45	0.10	0.25	0.00

Table 4.2: $\varkappa_c$-thresholds under growth and welfare-maximizing policies				
	bench1	_bench2_	_bench3_	_bench4_
$\varkappa_c^{\gamma}$	60.4	24.9	59.2	25.9
$\varkappa_i^{d*}$	5.9	11.3	14.3	22.2
$\varkappa_i^{n*}$	0.0	0.9	1.2	4.8
$\varkappa_c^{v}$	60.3	24.3	58.8	24.5
$\varkappa_i^{d+}$	5.9	11.2	14.2	22.4
$\varkappa_i^{n+}$	0.1	1.4	1.6	6.0

Note: For values of $\varkappa_c$ above $\varkappa_c^{\gamma}$ ($\varkappa_c^{v}$), the maximum long-run growth rate (welfare) is higher under income than under lump-sum taxes. $\varkappa_i^{d*}$ and $\varkappa_i^{d+}$ are the productive public investment-to-output ratios maximizing growth and welfare, respectively, under income taxes and for $\varkappa_c = \varkappa_c^{\gamma}$. $\varkappa_i^{n*}$ is the supremum of the public investment ratio under lump-sum taxes for $\varkappa_c = \varkappa_c^{v}$. Growth is monotonic on $\varkappa_i$ in that case. $\varkappa_i^{n+}$ is the value of the ratio maximizing welfare under lump-sum taxes.

Table 6.1: $\hat{\varkappa}_c^{\gamma}$ (%) under alternative $\delta^g, \delta \in [0,1]$						
$\delta^g \backslash \delta$	0	.20	.40	.60	.80	1
0	25.48	25.32	25.20	25.09	24.90	24.90
.20	25.50	25.35	25.20	25.10	25.00	24.90
.40	25.50	25.36	25.21	25.10	25.00	24.90
.60	25.51	25.37	25.22	25.10	25.00	24.90
.80	25.52	25.38	25.24	25.10	25.01	24.90
1	25.53	25.39	25.30	25.10	25.01	24.90

Fig. 4.1: Growth-maximizing policies under lump-sum and income taxes
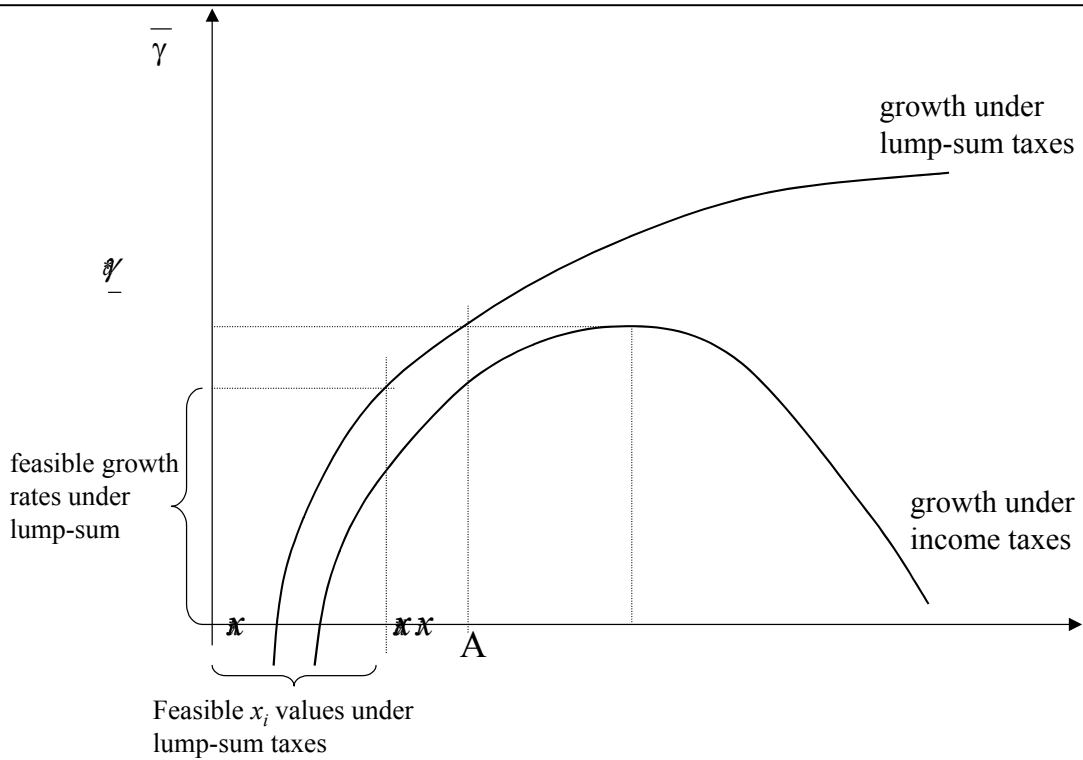
Note: $\varkappa_i^{d*}$ maximizes $\bar{\gamma}$ under income taxes and $\varkappa_i^{n*}$ is such that $\bar{c} = 0$ under lump-sum taxes. If $\varkappa_i^{n*}$ is located to the left of A, a higher growth rate could be achieved under income than under lump-sum taxes.