COORDINATING SHORT- AND LONG-RUN PUBLIC INVESTMENT POLICIES*

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ABSTRACT

Modelling the accumulation rule evolving public investment is an issue of utmost interest among economists and politicians. The present paper extends the Barro (1990) model of productive government expenditure by considering a time-adapted rule for the public investment/output ratio. The rule allows a particular target on the public investment ratio to be achievable in the long-run. Additionally, throughout the transition, the government may adjust its period-by-period public investment/output ratio in response to the current productivity of public capital relative to its long-run level. The degree of this response depends on a short-run policy instrument, which is decided by the fiscal authority simultaneously to the long-run target ratio. That way, the government problem could be interpreted as a coordination problem between short- and long-term policies. In comparison with a constant-ratio rule, and under alternative taxing scenarios, important welfare improvements are found when coordinating the short- and the long-run policy instruments in an optimal way.

Keywords: Public investment rule, policy coordination, transitional dynamics, Endogenous growth

JEL Classification: E0, E6, O4.

1. Introduction

Modelling the accumulation rule evolving public investment, and its effect on growth and welfare, has been extensively analyzed by economists, specially since the empirical work by Aschauer (1989) and Munell (1990) and the theoretical paper by Barro (1990). At the same time, the coordination between long-run policy targets and short-run policy interventions is being the subject of very detailed consideration by governments. When the public sector announces a determined long-run policy,[1] this long-run measure must be coordinated with a particular active policy throughout the transition for convergence to that target to be credible and achievable.

The relationship between short- and long-run policies has been widely studied in the monetary policy literature [see Svensson (1999) and Taylor (1999), among many others]. The monetary authority is assumed to follow a particular policy rule, with a *long-run target* (generally on inflation, nominal-GDP or money growth) as well as with a *short-run active rule* response to the state of the economy. However, little work has been done regarding the coordination between long- and short-run policies in the fiscal policy literature and, more specifically, concerning the public investment rule. The present paper attempts to make some progress along this line. To this purpose, we extend the Barro (1990) model of productive government expenditure by considering the possibility of a time-adapted rule for the public investment/output ratio.

The fiscal rule most often considered reduces to keeping a constant public expenditure/GNP ratio [as in Barro (1990), Glomm and Ravikumar (1994), Turnovsky (1996, 2000), among many others]. In the paper, we consider a more flexible rule for the evolution of public investment, although, by simplicity, we keep the constant-ratio rule for unproductive public expenditures. The rule will allow a particular target on the public investment ratio to be achievable in the long-run. This long-run target will be attained in just one period under the constant-size rule. But, if the public investment ratio reacts along the transition accordingly to the state of the economy, the fiscal authority will face with a continuum of alternative paths for the public investment/output ratio leading all of them to the pursued long-run target. That way, the government problem could be interpreted as a coordination problem between short- and long-term policies.

In a very simple case, we will assume that the government may adjust its period-by-period public investment/output ratio in response to the current productivity of public capital relative to its long-run level. The degree of this response will depend on a short-run policy instrument, which is decided by the fiscal authority simultaneously to the long-run target ratio. Doing that, the paper focus upon: (a) the importance of a given long-run policy target in the performance of a short-run policy rule, (b) the joint determination of the short-run policy instrument and the long-run target to maximize welfare and (c) the sensitivity of previous results to alternative financing scenarios.

[1] For instance, to homogenize future members, the Maastricht Treaty on European Union imposed several *long-run* targets to be achieved by any country attempting to be a potential member of the Union. As an additional example, the International Monetary Fund gives policy guidelines to countries with economic troubles in order to improve *long-run* sustained economic growth.

In general, the steady-state of the economy changes when a government implements a policy aimed to a new long-run target. Hence, if the economy displays no transition, there is no interesting role for short-run policies.[2] On the other hand, a short-run policy might allow for reducing the convergence speed substantially. That way, and in terms of welfare, the long-run effect of a policy aimed to a long-run target is somewhat misleading, since the steady-state may be reached only after a long time. Consequently, the relationship between the convergence speed and the short-run policy performance turns out to be crucial when measuring the impact on welfare of coordinating short- and long-run policies.

In comparison with the constant-ratio rule, and under alternative taxing scenarios, we find an important welfare improvement when using our more flexible policy rule. However, this welfare improvement depends upon whether coordination involves a long-run downsizing or upsizing in public investment, and when the resultant convergence to that target is fast or slow. In this respect, taxes and several structural parameters, such as the output elasticities of private and public capital, the unproductive public expenditure/output ratio and the elasticity of substitution, are found to be crucial.

Section 2 describes the framework for analysis. In section 3, the competitive equilibrium and a numerical method to solve the transitional dynamics along the competitive equilibrium are described. In section 4, the main issues of the paper are approached. Section 5 carry out a simple sensitivity analysis. Finally, section 6 offers main conclusions and extensions.

2. The basic framework

In this section we describe a simple endogenous growth model with public and private capital, and three economic agents: households, firms and a government.

2.1. Firms

There exists a continuum of firms, indexed by $[0,1]$, producing the single commodity good in the economy. Private capital, $\tilde{k}_t$, and labor, $\tilde{l}_t$, are lent by households to the firms to produce $\tilde{y}_t$ units of output.

The total amount of physical capital used by all the firms in the economy, $\tilde{K}_t$, is taken as a proxy for the index of knowledge available to each firm [as in Romer (1986)]. Additionally, public capital, $\tilde{K}_t^g$, affects the production process of all individual firms. Except for these externalities, the private production technology is a standard *Cobb-Douglas* function presenting constant returns to scale in the private inputs and increasing returns in the aggregate. For any firm,

$$\tilde{y}_t = f(\tilde{l}_t, \tilde{k}_t, \tilde{K}_t, \tilde{K}_t^g) = F\tilde{l}_t^{1-\alpha}\tilde{k}_t^{\alpha}\tilde{K}_t^{\theta_k}\left(\tilde{K}_t^g\right)^{\theta_g}, \ \theta_g, \ \alpha \in (0,1), \ \theta_k \geq 0, \tag{2.1}$$

[2] At least, in a deterministic setting. In a stochastic steady-state, a short-run active policy will affect the stochastic structure of the implied time series and the empirical density function of any variable, including welfare.

where α is the share of private capital in gross output, θ_g and θ_k are the elasticities of output with respect to public capital and the knowledge index, respectively, and F is a technological scale factor.

Since firms are identical, from (2.1), aggregate output, $\tilde{Y}_t$, is produced according to,

$$\tilde{Y}_t = F\tilde{L}_t^{1-\alpha}\tilde{K}_t^{\alpha+\theta_k}\left(\tilde{K}_t^g\right)^{\theta_g}, \tag{2.2}$$

where $\tilde{L}_t$ is aggregate labor.

During period t, each firm pays the competitive-determined wage $\tilde{w}_t$ on the labor it hires and the rate r_t on the capital it rents. The profit maximizing problem of the typical firm turns out to be static,

$$\underset{\{l_t,k_t\}}{Max}\ f(\tilde{l}_t,\tilde{k}_t,\tilde{K}_t,\tilde{K}_t^g) - \tilde{w}_t\tilde{l}_t - r_t\tilde{k}_t.$$

Optimality leads to the usual marginal productivity conditions:

$$r_t = f'_{\tilde{k}} = \alpha F\tilde{l}_t^{1-\alpha}\tilde{k}_t^{\alpha-1}\tilde{K}_t^{\theta_k}\left(\tilde{K}_t^g\right)^{\theta_g} = \alpha\frac{\tilde{y}_t}{\tilde{k}_t} = \alpha\frac{\tilde{Y}_t}{\tilde{K}_t}, \tag{2.3}$$

$$\tilde{w}_t = f'_{\tilde{l}} = (1-\alpha)F\tilde{l}_t^{-\alpha}\tilde{k}_t^{\alpha}\tilde{K}_t^{\theta_k}\left(\tilde{K}_t^g\right)^{\theta_g} = (1-\alpha)\frac{\tilde{y}_t}{\tilde{l}_t} = (1-\alpha)\frac{\tilde{Y}_t}{\tilde{L}_t}, \tag{2.4}$$

where we have used the fact that each firm treats its own contribution to the aggregate capital stock as given, rents the same amounts of the private inputs and produces the same amount of output.

2.2. Households

The representative consumer chooses the fraction of time to spend as leisure. She is the owner of physical capital, and allocates her resources between consumption, $\tilde{C}_t$, and investment in physical capital, $\tilde{I}_t^k$. Private physical capital accumulates over time according to

$$\tilde{K}_{t+1} = (1-\delta^k)\tilde{K}_t + \tilde{I}_t^k, \tag{2.5}$$

where δ^k is the depreciation factor for private capital, between zero and one. Zero population growth is assumed and the time endowment is normalized to one. Decisions are made each period to maximize the discounted aggregate value of the time separable utility function,[3]

$$\sum_{t=0}^{\infty}\beta^t u(\tilde{C}_t,h_t) = \sum_{t=0}^{\infty}\beta^t\frac{\left[\tilde{C}_t^{\rho}(1-h_t)^{1-\rho}\right]^{1-\theta}-1}{1-\theta},\ \rho\in[0,1],\ \theta>0,\ \theta\neq 1, \tag{2.6}$$

$$= \sum_{t=0}^{\infty}\beta^t\left[\rho\ln\tilde{C}_t + (1-\rho)\ln(1-h_t)\right],\ \rho\in[0,1],\ \theta=1,$$

[3] A CES representation is assumed for the single period utility function, capturing cross-substitution between leisure and consumption [King et al. (1988)].

where h_t is the fraction of time devoted to production, β is the discount factor, between zero and one, $1/\theta$ is the elasticity of substituting consumption intertemporally and ρ characterizes the importance of consumption relative to leisure.

Her single period budget constraint is

$$(1+\tau_t^c)\tilde{C}_t + \tilde{K}_{t+1} + \tilde{X}_t \leq \tilde{W}_t h_t + \tilde{K}_t \left[1-\delta^k + r_t\left(1-\tau_t^k\right)\right], \tag{2.7}$$

where $\tilde{K}_{t+1}$ denotes the stock of physical capital at the end of time t, with $\tilde{K}_0 > 0$, τ_t^k and τ_t^c are the tax rates applied to capital income and private consumption, respectively, and $\tilde{X}_t$ is a net transfer made by households to the public sector, which could be negative.

The representative household faces a discrete dynamic programing problem, in which corner solutions are avoided and restrictions hold with equality due to the special form of the instantaneous utility function and the fact that consumption and leisure are normal goods. Optimality conditions are standard: the consumption-saving decision (2.8), the consumption-leisure choice (2.9), and the budget constraint (2.7),

$$\frac{\tilde{C}_{t+1}}{\tilde{C}_t} = \left\{\beta\left(\frac{1-h_{t+1}}{1-h_t}\right)^{(1-\rho)(1-\theta)}\left[1-\delta^k + r_{t+1}\left(1-\tau_{t+1}^k\right)\right]\right\}^{\frac{1}{1-\rho(1-\theta)}}, \tag{2.8}$$

$$\frac{\rho}{1-\rho} = \frac{\tilde{C}_t}{(1-h_t)}\frac{(1+\tau_t^c)}{\tilde{w}_t}. \tag{2.9}$$

Additionally constraints are: $\tilde{C}_t > 0$ and $\tilde{K}_{t+1} > 0$, $h_t \in (0,1)$ and the transversality condition, that places a limit on the accumulation of capital,[4]

$$\lim_{t\to\infty} \beta^t \tilde{K}_{t+1}\frac{\partial u(\tilde{C}_t, h_t)}{\partial \tilde{C}_t} = 0. \tag{2.10}$$

2.3. The public sector

The government is characterized as a fiscal authority. A broad classification of public expenses is considered: unproductive public expenses, $\tilde{C}_t^g$, which do not directly affect the productive process or consumers' welfare, and public investment, $\tilde{I}_t^g$, which positively affects production. Public capital is accumulated according to

$$\tilde{K}_{t+1}^g = \tilde{I}_t^g + (1-\delta^g)\tilde{K}_t^g, \tag{2.11}$$

where δ^g is the public capital depreciation factor, between zero and one.

The government is assumed to be restricted to financing a given unproductive public consumption/output ratio, $\varkappa_c$, which remains constant over time (i.e., due to inefficient bureaucratic or administrative costs, or the payment of interest on outstanding debt),

$$\varkappa_c = \tilde{C}_t^g/\tilde{Y}_t,\ \varkappa_c \in [0,1). \tag{2.12}$$

[4]Consumption and physical capital are strictly positive because marginal utility at the origin is equal to infinity.

Additionally, the government commits to following a pre-announced rule for $\tilde{I}_t^g$:

$$\varkappa_{it} = \tilde{I}_t^g / \tilde{Y}_t = \bar{\varkappa}_i \left(\frac{\tilde{Y}_t / \tilde{K}_t^g}{\overline{Y/K^g}} \right)^{\eta}, \ \varkappa_{it} \in [0, 1 - \varkappa_c) \text{ for all } t, \tag{2.13}$$

where $\overline{Y/K^g}$ denotes the average product of public capital in the long-run, which, as we will see, is constant in our setting.

The government chooses the values of $\bar{\varkappa}_i$ and η, which will induce a path for $\varkappa_{it}$. In general, this path will also depend on preferences, technology, the level of $\varkappa_c$ and the tax system used to finance total public expenditures.

According to (2.13), $\bar{\varkappa}_i$ is a *long-run target* for $\varkappa_{it}$. Along the transition, the ratio $\varkappa_{it}$ will respond to the current productivity of public capital relative to its long-run value. This reaction depends upon the magnitude and the sign of η, which can be considered as a *short-run policy instrument*: (*a*) if $\eta = 0$, $\varkappa_{it} = \bar{\varkappa}_i$ every period and (2.13) becomes a passive rule; in all other cases, (2.13) is an active rule: (*b*) when $\eta > 0$, $\varkappa_{it}$ raises above (falls below) $\bar{\varkappa}_i$ when the average product of public capital is higher (lower) than its long-run level, while (*c*) the public investment ratio exhibits the opposite relationship when $\eta < 0$.

We assume that tax revenues finance total public expenses every period. For the sake of simplicity, just two polar tax financing alternatives are considered, both with constant tax rates: (*i*) capital income taxes, under which the government budget constraint is:

$$\tilde{C}_t^g + \tilde{I}_t^g = \tilde{X}_t + \tau^k \tilde{K}_t r_t, \tag{2.14}$$

and (*ii*) consumption taxes, and the government budget constraint now becomes:

$$\tilde{C}_t^g + \tilde{I}_t^g = \tilde{X}_t + \tau^c \tilde{C}_t. \tag{2.15}$$

In both cases, transfers $\tilde{X}_t$ are used to balance the government budget every period when a transitory surplus or deficit ($\tilde{X}_t < 0$ or $\tilde{X}_t > 0$, respectively) is generated.[5]

3. Competitive equilibrium and the government's problem

The competitive equilibrium and the balanced growth path are characterized in this section. A numerical procedure to solve for the dynamics of variables in levels and the government's problem is also discussed.

[5]In our setting, since labor is elastically supplied, taxing its rent might distort saving, but by less than when taxing directly capital income. On the other hand, income taxes is a mixture between labor and capital income taxes. Hence, results under alternative tax scenarios (total income taxes, labor income taxes or some mixture of them) should be expected to fall between those reached under private capital and consumption taxes.

3.1. The competitive equilibrium and the balanced growth path

Starting from an initial state, $\tilde{K}_0, \tilde{K}_0^g > 0$, the *competitive equilibrium* is a set of allocations $\left\{\tilde{C}_t, h_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{I}_t^k, \tilde{Y}_t, \tilde{C}_t^g, \tilde{K}_{t+1}^g, \tilde{I}_t^g\right\}_{t=0}^{\infty}$, a set of prices $\tilde{p} = \{r_t, \tilde{w}_t\}_{t=0}^{\infty}$ and a fiscal policy $\tilde{\pi} = \left(\bar{\varkappa}_i, \eta, \tau^k, \tau^c, \left\{\tilde{X}_t\right\}_{t=0}^{\infty}\right)$, such that, given $\tilde{p}$ and $\tilde{\pi}$: (*i*) $\{\tilde{C}_t, h_t, \tilde{K}_{t+1}\}_{t=0}^{\infty}$ maximize households' welfare [satisfying (2.7)-(2.10)]; (*ii*) $\{\tilde{K}_{t+1}, \tilde{L}_t\}_{t=0}^{\infty}$ satisfy the profit-maximizing conditions [(2.3)-(2.4)], and $\tilde{I}_t^k$ accumulates according to (2.5); (*iii*) $\{\tilde{C}_t^g, \tilde{K}_{t+1}^g, \tilde{I}_t^g\}_{t=0}^{\infty}$ evolve according to (2.12)-(2.11); (*iv*) the budget constraint of the public sector [either (2.14) under capital income taxes or (2.15) under consumption taxes] and the technology constraint (2.2) to produce $\tilde{Y}_t$ hold; finally, (*iv*) markets clear every period,

$$\tilde{L}_t = h_t, \tag{3.1}$$
$$\tilde{Y}_t = \tilde{C}_t + \tilde{C}_t^g + \tilde{I}_t^k + \tilde{I}_t^g. \tag{3.2}$$

A *balanced growth path (bgp)* is defined as an equilibrium path along which aggregate variables either stay constant or grow at a constant rate. Barro (1990) and Jones and Manuelli (1997), among others, have shown that cumulative inputs must present constant returns to scale in the private production process (i.e., $\alpha + \theta_g + \theta_k = 1$). Additionally, r_t must be constant and high enough for the equilibrium to display positive steady-growth (hereinafter, variables with bar "–" denote values along the *bgp*). From now on, we will focus on the special case in which $\alpha + \theta_g + \theta_k = 1$. Under these conditions, it is easy to show from the equilibrium conditions, that $\tilde{Y}_t$, $\tilde{C}_t$, $\tilde{K}_t$, $\tilde{K}_t^g$, $\tilde{C}_t^g$ and $\tilde{X}_t$ must all grow at the same constant rate along the *bgp*, denoted by $\bar{\gamma}$ hereinafter, while bounded variables, such as tax rates, r_t and h_t, must be constant.

From (2.8), a positive long-term growth rate is achieved whenever

$$\bar{\gamma} = \left\{\beta\left[1 - \delta^k + (1 - \tau^k)\bar{r}\right]\right\}^{\frac{1}{1-\rho(1-\theta)}} - 1 > 0 \Leftrightarrow \bar{r} > \frac{1 - \beta(1 - \delta^k)}{(1 - \tau^k)\beta}. \tag{3.3}$$

However, even though $\bar{\gamma}$ will then be positive, it cannot get so high that allow households to follow a chain-letter action [(2.10) must hold on the *bgp*], i.e.,

$$\lim_{t\to\infty} \frac{\rho(1 - \bar{h})^{(1-\rho)(1-\theta)}\tilde{K}_0\,(1 + \bar{\gamma})\,\beta^t\,(1 + \bar{\gamma})^t}{\left[\tilde{C}_0(1 + \bar{\gamma})^t\right]^{1-\rho(1-\theta)}} = 0 \Leftrightarrow \beta(1 + \bar{\gamma})^{\rho(1-\theta)} < 1, \tag{3.4}$$

which also ensures that time-aggregate utility (2.6) remains finite.

3.2. The financing system

The aim of the paper is to discuss the relevance, in terms of welfare gains, of considering a short-run policy instrument, instead of maintaining a constant level of the public investment/output ratio for the whole transition. To this end, and to isolate the impact of a given choice of η on welfare from that produced by possible changes in distorting taxes (either in τ^k or τ^c), tax rates are assumed to be constant.

The level of the tax rate is such that lump-sum transfers will be zero along the *bgp*, so $\bar{X} = 0$. Therefore, by combining (2.14) and (2.15) with (2.13), (2.12) and (2.3), tax rates under capital and private consumption taxes are:

$$\tau^c = 0 \text{ and } \tau^k = (\bar{\varkappa}_i + \varkappa_c)/\alpha, \quad (3.5)$$

$$\tau^c = (\bar{\varkappa}_i + \varkappa_c)\overline{Y/C} \text{ and } \tau^k = 0, \quad (3.6)$$

where $\overline{Y/C}$ is constant along the *bgp*. Along the transition, any deviation from the long-run target is assumed to be financed through lump-sum transfers. Hence, under capital income taxes, plugging (3.5) into (2.14) and using (2.12) and (2.13),

$$\tilde{X}_t = (\varkappa_{it} - \bar{\varkappa}_i)\tilde{Y}_t, \quad (3.7)$$

while under consumption taxes, by setting (3.5) into (2.14) and using also (2.12) and (2.13),

$$\tilde{X}_t = \left[(\varkappa_c + \varkappa_{it}) - (\varkappa_c + \bar{\varkappa}_i)\left(\frac{\tilde{C}_t/\tilde{Y}_t}{\overline{C/Y}}\right)\right]\tilde{Y}_t. \quad (3.8)$$

Summing up, changes in the short-run policy instrument, η, can only alter the public investment/output ratio and the tax base along the transition, but they do not affect the associated tax rate. Although far from realistic, this simple financing rule is convenient for addressing the main issues in this paper, as well as being an appropriate benchmark for future extensions.

3.3. The benchmark calibration

The calibration roughly captures the main *bgp* characteristics of industrialized economies over the eighties. The time unit is considered to be one quarter. It reproduces on the *bgp* an annual after-tax net capital rate of return of 6%, an average proportion of working time, $\bar{h}$, of 1/3, and a 2.5% annual growth rate.

Parameter values are: $\delta^k = .025$, $\delta^g = .0125$, $\theta = 1.20$, $\alpha = .36$, $\theta_g = .20$, and $\theta_k = .44$, so that $\alpha + \theta_g + \theta_k = 1$.[6] Schuknecht and Tanzi (1997) is followed to set $\varkappa_c$ equal to 17%. Following Easterly and Rebelo (1993) and Barro and Sala-i-Martin (1995), total public investment is about 30% of total investment, which leads to an average public investment/output ratio of 7%. This is taken as the initial level of $\bar{\varkappa}_i$. Finally, $\beta = .996$, and ρ and F are chosen to maintain the after-tax net capital rate of return, the level of $\bar{h}$ and the benchmark growth rate at the values mentioned above. Transfers are assumed to be zero initially and the associated tax rate is computed residually from (3.5) and (3.6). As a result, under private consumption taxes, $\rho = .35$, $F = .24$, $\tau^k = 0$ and $\tau^c = .52$, while under capital income taxes, $\rho = .34$, $F = .59$, $\tau^c = 0$ and $\tau^k = .67$.

[6] In the empirical literature, estimations of θ_g vary from .06 in Ratner (1983), to .20 in Lynde and Richmond (1993) and to the more questionable .39 in Aschauer (1989).

The main difference between the two alternative tax systems is the value of the scale factor F, which is adjusted to replicate the 2.5% annual growth rate. Consequently, welfare paths between the alternative tax scenarios considered are not comparable.

3.4. Solving for the dynamics of level variables

The competitive equilibrium cannot be analytically solved, so a numerical solution is required. However, numerical techniques are designed to solve for the dynamics of variables with a well defined steady-state, which is not the case for variables in levels in endogenous growth models. An alternative approach is usually to deal with normalized level variables. Hereinafter, Z_t will denote the normalized level of $\tilde{Z}_t$, $Z_t = \tilde{Z}_t/(1+\bar{\gamma})^t$, which will grow at a zero rate along the *bgp*. But the steady-state value of Z_t is not well defined, so standard numerical methods applied directly to normalized variables cannot be used either.

In that setting, it is fairly simple to solve for stationary ratios, but that strategy precludes the possibility of analyzing welfare issues, for which level variables are needed. We describe next a procedure that uses the dynamics of stationary ratios to recover the equilibrium path for normalized level variables, starting from a given initial state of the economy.[7]

(*i*) In terms of stationary ratios: $c_t = \tilde{C}_t/\tilde{K}_t$, $k_t^g = \tilde{K}_t^g/\tilde{K}_t$, $y_t = \tilde{Y}_t/\tilde{K}_t$ and $k_{t+1} = \tilde{K}_{t+1}/\tilde{K}_t$, competitive equilibrium conditions (2.7)-(2.15) can be reduced to a system of 6 equations in c, k^g, y, k, r and h,

$$k_{t+1}\frac{c_{t+1}}{c_t} = \left\{\beta\left(\frac{1-h_{t+1}}{1-h_t}\right)^{(1-\rho)(1-\theta)}\left[1-\delta^k+(1-\tau^k)r_{t+1}\right]\right\}^{\frac{1}{1-\rho(1-\theta)}}, \tag{3.9}$$

$$\frac{\rho}{1-\rho} = \frac{(1+\tau^c)\,c_t}{(1-\alpha)y_t}\frac{h_t}{1-h_t}, \tag{3.10}$$

$$y_t = c_t + \varkappa_c y_t + k_{t+1} - (1-\delta^k) + \bar{\varkappa}_i\bar{y}\left(\frac{y_t}{\bar{y}}\right)^{1+\eta}\left(\frac{k_t^g}{\bar{k}^g}\right)^{-\eta}, \tag{3.11}$$

$$r_t = \alpha y_t, \tag{3.12}$$

$$y_t = F h_t^{1-\alpha}\left(k_t^g\right)^{\theta_g}, \tag{3.13}$$

$$k_{t+1} = \frac{1}{k_{t+1}^g}\left[\left(1-\delta^g\right)k_t^g + \bar{\varkappa}_i\bar{y}\left(\frac{y_t}{\bar{y}}\right)^{1+\eta}\left(\frac{k_t^g}{\bar{k}^g}\right)^{-\eta}\right], \tag{3.14}$$

where (3.9) corresponds to (2.8); (3.10) comes from combining (2.9) with (2.4) and (3.1); (3.11) combines (2.7) (holding as an equality) with (2.3), (2.4), (2.12), (2.13) and either (2.14) under capital income taxes or (2.15) under consumption taxes; (3.12) and (3.13) come directly from (2.3) and (2.2), respectively; finally, (3.14) combines (2.13) with (2.11).[8]

[7] Novales et al. (1999) describes an alternative method to solve for the dynamics of level variables in endogenous growth models.

[8] The system might be reduced to a system of three equations in c_t, h_t and k_t^g, but that does not significantly simplify the computations.

(ii) Use values for structural parameters to solve (3.9)-(3.14) for the *bgp*.

(iii) Log-linearize (3.9)-(3.14) around the *bgp*. A log-linear based approach is used to solve for the dynamics of stationary ratios [Uhlig (1999)]. $V(t)$ includes the beginning-of-period state variables, just k_t^g in our case; $Q(t)$ is the vector of real variables $(c_t, y_t, r_t, h_t, k_{t+1})$. Their values on the *bgp* are denoted by $\bar{V}$ and $\bar{Q}$, and $\hat{v}(t)$ and $\hat{q}(t)$ denote log-deviations of $V(t)$ and $Q(t)$ around $\bar{V}$ and $\bar{Q}$, respectively. More compactly, a log-linear approximation to conditions (3.9)-(3.14) can be rewritten as:

$$\begin{aligned} 0 &= A\hat{v}(t+1) + B\hat{v}(t) + C\hat{q}(t), &(3.15)\\ 0 &= F\hat{v}(t+2) + G\hat{v}(t+1) + H\hat{v}(t) + J\hat{q}(t+1) + K\hat{q}(t), &(3.16) \end{aligned}$$

where those conditions showing dynamics of any variable in $Q(t)$ (i.e., (3.9) in our case) are included in (3.16), and matrices A, B, C,..., are functions of all structural and fiscal policy parameters.[9]

(iv) The following log-linear law of motion for $Q(t)$ and $V(t)$ is assumed:

$$\begin{aligned} \hat{v}(t+1) &= P\hat{v}(t), &(3.17)\\ \hat{q}(t) &= S\hat{v}(t), &(3.18) \end{aligned}$$

where P and S are free matrices of dimension (1x1) and (5x1). Next, conditions (3.15)-(3.16) are directly solved by the undetermined coefficients method, imposing the eigenvalues of P to be inside the unit circle, since otherwise $Q(t)$ and $V(t)$ would present explosive paths.

(v) Starting at (K_0, K_0^g), we have $k_0^g = K_0^g/K_0$, and values of C_0, Y_0, r_0, h_0 and k_1 are directly obtained from

$$\begin{pmatrix} C_0 \\ Y_0 \\ r_0 \\ h_0 \\ k_1 \end{pmatrix} = \begin{pmatrix} K_0 & 0 & 0 & 0 & 0 \\ 0 & K_0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} Q(0), \qquad (3.19)$$

where, from (3.18), $Q(0) = \exp\left[S\left(\ln k_0^g - \ln \bar{k}^g\right) + \ln \bar{Q}\right] = (c_0\ y_0\ r_0\ h_0\ k_1)'$, and k_1^g is obtained from (3.17).

(vi) K_1 and K_1^g (the state of the economy next period) are easily recovered from k_1:

$$K_1 = \frac{K_0 k_1}{(1+\bar{\gamma})} \text{ and } K_1^g = k_1^g K_1.$$

(vii) Time series for normalized variables for successive periods are recovered by going recursively through steps (v) and (vi). Stability of the resulting time series is guaranteed since appropriate stability conditions were implemented when solving (3.15)-(3.16).

[9] Details for the log-linearization of (3.9)-(3.14), together with matrices A, B, C,..., are shown in the appendix.

3.5. Solving for the government's problem

The economy is assumed to start on the *bgp* associated to the benchmark calibration, with an initial public investment/output ratio of 7% and a public capital stock, K_0^g, of 100.[10] The public sector chooses $\bar{\varkappa}_i$ and η to maximize the representative household's welfare along the competitive equilibrium allocation. A standard search method is used to numerically handle this control problem.

Given the tax system, the initial state (K_0, K_0^g) and a pair $(\bar{\varkappa}_i, \eta)$: (a) (3.9)-(3.14) is solved for the *bgp* and the level of $\bar{\gamma}$ is obtained; (b) the process described in the previous subsection allows to recovering time series of C_t and h_t from the log-linear approximation (3.15)-(3.16); (c) the utility of the representative consumer is evaluated[11]

$$\sum_{t=0}^{\infty} \left\{ \frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^t \left[C_t^{\rho}(1-h_t)^{1-\rho}\right]^{1-\theta}}{1-\theta} - \frac{\beta^t}{1-\theta} \right\}; \tag{3.20}$$

(e) the process is repeated for any feasible policy $(\bar{\varkappa}_i, \eta)$, and the one maximizing (3.20) will be the welfare-maximizing choice.

To evaluate the infinite sum in (3.20), a truncated version with T^* periods is used, where T^* is chosen so that equilibrium time series are close enough to the *bgp*.[12] For each policy, time series $\{C_t, h_t\}_{t=0}^{T^*}$ are used to estimate welfare up to period T^*:

$$\sum_{t=0}^{T^*} \left\{ \frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^t \left[C_t^{\rho}(1-h_t)^{1-\rho}\right]^{1-\theta}}{1-\theta} \right\} - \frac{1}{(1-\beta)(1-\theta)}. \tag{3.21}$$

After period T^*, the economy is considered to be *close enough* to the *bgp* associated to the implemented policy. Therefore, according to (3.20), since $\beta(1+\bar{\gamma})^{\rho(1-\theta)} < 1$ [by (3.4)], the term

$$\begin{aligned} & \sum_{t=T^*+1}^{\infty} \frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^t \left[C_{T^*}^{\rho}(1-\bar{h})^{1-\rho}\right]^{1-\theta}}{1-\theta} \\ = \ & \left[C_{T^*}^{\rho}(1-\bar{h})^{1-\rho}\right]^{1-\theta} \frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^{T^*+1}}{(1-\theta)\left[1-\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]} \end{aligned} \tag{3.22}$$

approximates aggregate utility after period T^*, which is added-up to the numerical value obtained from (3.21).[13]

[10] The initial state is $K_0^g = 100$ and $K_0 = 100/\bar{k}^g$. The welfare-maximizing policy is shown to be invariant to this choice.

[11] The feasible range of welfare is bounded because: (i) the numerical procedure imposes the competitive equilibrium to be on the stable manifold and hence, C_t and h_t eventually stabilize, and (ii) $\bar{\gamma}$ is bounded from above by (3.4). Moreover, since the single time-period utility is continuous and strictly concave and the choice set is convex, there exists at most one interior solution to the government problem.

[12] T^* is chosen so that $\left|X_{T^*} - \bar{X}\right| < 10^{-3}$, with $T^* < 1500$, due to computational restrictions.

[13] Notice that (3.21) and (3.22) must be computed simultaneously, because T^* and C_{T^*} depend on the whole transitional dynamics up to period T^*.

In the following sections, the main results of the policy experiment and the sensitivity of the main conclusions to the choice of some parameter values in the economy are discussed.

4. The public investment rule

The public sector commits to following the public investment rule (2.13). Therefore, there exists a whole continuum of potential paths for $\varkappa_{it}$ consistent with reaching the long-run target $\bar{\varkappa}_i$, depending upon the selected short-run policy instrument, η. Several aspects are discussed in this section: (a) the effect of η over the transitional dynamics of main macroeconomic variables, for a given level of $\bar{\varkappa}_i$; (b) the welfare-maximizing policy when simultaneously choosing $\bar{\varkappa}_i$ and η and (c) the sensitivity of the main results to the choice of the tax system.

4.1. Transitional dynamics and the public investment rule

To evaluate the welfare incidence of any policy, we need to consider the effects, over the whole transition process, on: (i) the long-run growth rate, (ii) the initial impact on private consumption and leisure, (iii) the convergence speed to the new *bgp* and (iv) the accumulation of productive factors along the transition. Obviously, (i) and (ii) are important elements for characterizing welfare effects. In relation to (iii), the convergence speed provides a relative weight to policy effects along the transition versus long-term effects.[14] Finally, the accumulation of capital factors in the short-run stimulates output and consumption along the transition. In relation to the public investment rule (2.13), the choice of η does not have long-run effects, but it may influence (ii), (iii) and (iv), while $\bar{\varkappa}_i$ might affect all the described features.

Table 4.1 shows the convergence speed and the initial impact on the main macroeconomic variables when alternative long-run targets for $\bar{\varkappa}_i$ of either 4% (below the starting 7% level) or 10% are pursued by the government, combined with alternative levels of the short-run policy instrument (η equal to -1.20, 0 and 1.20), under consumption and capital income taxes.[15] Figures 4.1-4.4 show time paths for the public investment/output ratio, $\varkappa_{it}$, normalized consumption, C_t, working time, h_t, and the gross private capital growth rate, κ_t, under each tax system and parameterization considered in table 4.1. Several conclusions can be drawn from this policy experiment.

[INSERT TABLE 4.1 ABOUT HERE]

The long-run target on $\varkappa_{it}$, $\bar{\varkappa}_i$, is reached in just one period when $\eta = 0$.[16] Figure 4.1

[14] The convergence speed is measured here as the number of periods to cover half the distance between the initial and the final equilibrium *bgp* [as in Barro and Sala-i-Martin (1992)]. Chamley (1981) and Auerbach et al. (1983) have already discussed the importance of the convergence speed in welfare analysis.

[15] Qualitative results are quite robust to the magnitude of η, the sign of η being the relevant aspect.

[16] However, the economy displays transition. In addition to $\eta = 0$, δ^k and δ^g need to be equal to one for the economy to jump to the new steady-state in just one period.

shows that $\varkappa_{it}$ initially overshoots $\bar{\varkappa}_i$ when $\eta = 1.20$ ($\eta > 0$, in general), but convergence to the new *bgp* occurs rapidly, precisely because of the nature of the policy rule (2.13); on the other hand, $\varkappa_{it}$ never overshoots $\bar{\varkappa}_i$, and convergence to $\bar{\varkappa}_i$ is much slower. Moreover, for sufficiently negative levels of η, it could be the case that $\varkappa_{it}$ would initially move in opposite direction to $\bar{\varkappa}_i$, before approaching very slowly (i.e., see figure 4.1 under capital income taxes, for $\eta = -1.20$ and either $\bar{\varkappa}_i = 10\%$ or $\bar{\varkappa}_i = 4\%$).

Convergence speed to the new *bgp* reduces significantly when η takes negative values, independently of the financing system and the level of $\bar{\varkappa}_i$ (see table 4.1), approaching zero when η takes highly negative values.[17]

The initial impact on $\tilde{C}_t$ and h_t highly depends on the downsizing/upsizing nature of $\bar{\varkappa}_i$, but also on the tax system considered. Under consumption taxes (see table 4.1 and figures 4.2-4.4), a reduction in $\bar{\varkappa}_i$, from 7% to 4% in the example, has an initial positive impact on $\tilde{C}_t$, h_t and $\tilde{I}_t^k$. Under these circumstances, a positive level of η emphasizes this initial positive impact on $\tilde{C}_t$ and on $\tilde{I}_t^k$, while h_t is less stimulated, since as $\varkappa_{it}$ starts falling below $\bar{\varkappa}_i$, more resources are available to consume and save. However, $\tilde{I}_t^g$ suffers from an important initial reduction. An opposite behavior is found when $\bar{\varkappa}_i$ is chosen above its initial level.

Under capital income taxes (see table 4.1 and figures 4.2-4.4), a reduction in $\bar{\varkappa}_i$ below 7% implies an immediate decline of the capital tax rate [from condition (3.5)], thus savings turning now more attractive for households. For a significant reduction of $\bar{\varkappa}_i$, the substitution effect on private consumption might become even more important than the income effect and, consequently, $\tilde{C}_t$ might initially decline. In addition, h_t and $\tilde{I}_t^k$ initially raise. Moreover, as it was mentioned above, when a policy of reducing $\bar{\varkappa}_i$ is combined with a negative level of the short-run policy instrument, $\varkappa_{it}$ may be above 7% in the early periods of the transition, $\tilde{I}_t^g$ initially increasing, in spite of the reduction on the long-run public investment ratio (26.3% for $\eta = -1.20$ versus -41.8% for $\eta = 0$ and -73.8% for $\eta = 1.20$). An opposite behavior is found when $\bar{\varkappa}_i$ rises above its initial level.

4.2. Welfare-maximizing short-run policy instrument for a given level of $\bar{\varkappa}_i$

We turn now our attention to the welfare-maximizing level of the short-run policy instrument, $\hat{\eta}$, conditional on a given level of the long-run target $\bar{\varkappa}_i$. We assume that the fiscal authority pursues a given level of $\bar{\varkappa}_i$, and it must choose the best way to reach it among a continuum of alternative paths, by deciding on the value of η in (2.13).

Under our assumption on public financing, the tax rate is affected by $\bar{\varkappa}_i$ but not by η. Therefore, changes in η will not aid the government to reduce the associated tax rate, which is, potentially, the most important *negative effect* of an increase in $\bar{\varkappa}_i$. However, certain values for η might emphasize the *positive impact* of increasing the productive investment ratio, allowing for $\varkappa_{it}$ to be well above $\bar{\varkappa}_i$ along the transition, and this transitory increment being financed through lump-sum taxes as described in (3.7) and (3.8).

[17] In that case, matrix P in (3.17) has an eigenvalue close to one, which would imply unfeasible explosive trajectories for normalized time series.

In general, we will have $\varkappa_{it} > \bar{\varkappa}_i$ along the transition when either (i) $\bar{\varkappa}_i > 7\%$ and $\eta > 0$ or (ii) $\bar{\varkappa}_i < 7\%$ and $\eta < 0$. Figure 4.5 shows the welfare maximizing short-run policy instrument $\hat{\eta}$ as a function of $\bar{\varkappa}_i$, revealing that, indeed, $\hat{\eta} > 0$ when $\bar{\varkappa}_i > 7\%$, while $\hat{\eta} < 0$ for any $\bar{\varkappa}_i < 7\%$, with independence of the tax system considered. In the previous subsection, we showed that these strategies imply an initial utility loss, since private consumption and leisure are initially discouraged or, at least, less stimulated than under alternative policies. Consequently, given $\bar{\varkappa}_i$, the level of $\hat{\eta}$ must compensate this initial utility loss with more important welfare gains along the transition. Since η does not affect the long-run growth rate, the ability to compensate the initial utility loss must come through an stimulus on the short-run accumulation of private and/or public capital, which will allow for the initial negative response of output, private consumption and leisure to become positive after a short enough number of periods.

Table 4.2 shows the main dynamic characteristics of the transition corresponding to values of $\hat{\eta}$ associated to $\bar{\varkappa}_i = 4\%$ and $\bar{\varkappa}_i = 10\%$. Under capital and consumption taxes, $\hat{\eta}$ is equal to -1.51 and -1.64, respectively, when $\bar{\varkappa}_i = 4\%$, making the economy to converge to the *bgp* at its *lowest* speed. On the other hand, $\hat{\eta}$ is equal to 1.25 and 3.06 when $\bar{\varkappa}_i = 10\%$.

In the table $\triangle U$ shows the percent increment on welfare under $\hat{\eta}$ with respect to be $\eta = 0$, for a common level of $\bar{\varkappa}_i$. For instance, when $\bar{\varkappa}_i = 4\%$, $\triangle U$ is equal to 23.3% and 21.8% under capital and consumption taxes, respectively, being just 1.61% and .78% when $\bar{\varkappa}_i = 10\%$. The intuition behind this significant difference on welfare improvement comes from the implied speed of the adjustment process. Convergence is quite fast when $\hat{\eta} > 0$. Hence, the positive impact of the short-run policy instrument on welfare cannot be very important, since its effect operates only along the transition. On the other hand, convergence is very slow when $\hat{\eta}$ is highly negative, so that the positive effect of being $\eta \neq 0$ extends throughout a large number of periods.

[INSERT TABLE 4.2 ABOUT HERE]

4.3. Coordination between short- and long-run policy instruments

The remainder of the section is devoted to jointly determining the welfare-maximizing levels of $\bar{\varkappa}_i$ and η, denoted by $\bar{\varkappa}_i^+$ and η^+. The endogenous public investment rule considered in Barro (1990), Glomm and Ravikumar (1994) and Turnovsky (1996, 2000), among many others, is a special case of (2.13) with $\eta = 0$. Obviously, the flexibility to simultaneously decide on η and $\bar{\varkappa}_i$ must lead to a welfare improvement in relation to the constant-size rule. Hence, the aim of this section is to characterize the magnitude of this welfare gain.

Table 4.3 summarizes the main results obtained for the baseline calibration. Welfare obtained when the economy stays on the initial *bgp* is compared with that obtained: (i) when η and $\bar{\varkappa}_i$ are chosen simultaneously to maximize welfare (the optimal coordinating policy) and (ii) when $\bar{\varkappa}_i$ is the welfare-maximizing level under the restriction $\eta = 0$.

[INSERT TABLE 4.3 ABOUT HERE]

First, independently of the tax system considered, $\bar{\varkappa}_i^+$ is lower than the optimal choice of $\bar{\varkappa}_i$ under $\eta = 0$ (1.13% versus 3.28% under capital income taxes and 29.0% versus 29.6% under consumption taxes). The intuition of this result is the following: this smaller level of $\bar{\varkappa}_i$ has a positive impact on welfare, since the associated tax rate becomes lower along the whole transition, but it has a negative effect through the reduction of public capital accumulation along the *bgp*. Nevertheless, this negative impact could be compensated by setting a level of η that allows for the public investment ratio to be above the final level of $\bar{\varkappa}_i$ along the transition, the implied deficit being financed through lump-sum taxes. This way, $\bar{\varkappa}_i^+$ is below the one obtained under a short-run passive policy.

Second, the welfare-maximizing coordination between $\bar{\varkappa}_i$ and η allows for the public investment ratio to be higher than $\bar{\varkappa}_i^+$ along the transition, with independence of whether $\bar{\varkappa}_i^+$ is above or below its initial value, slowly declining towards the chosen long-run target.

Third, the welfare improvement of choosing η and $\bar{\varkappa}_i$ simultaneously is greater when the implied convergence process is slow, as already discussed in the previous subsection. This is achieved by coordinating a certain downsizing of $\bar{\varkappa}_i$ with a negative level of η. This way, the government is able to maintain the public investment ratio above $\bar{\varkappa}_i$ during a long number of periods, financing this gap with lump-sum taxes, and extending this positive transitory effect throughout a long interval of time.

Fourth, the optimality of reducing the long-run public investment ratio strongly depends on the cost of raising resources to finance public investment, for which the tax base becomes crucial.

The welfare gain associated to a policy coordination scheme highly depends on the tax system considered. In the baseline calibration, downsizing $\bar{\varkappa}_i$ from 7% until 1.13%, coordinated with $\eta = -1.49$, maximizes welfare under capital income taxes. This level of $\bar{\varkappa}_i^+$ is far below the optimal 3.28% obtained under $\eta = 0$. Relative to keeping the economy on the initial *bgp*, with $\varkappa_{it} = 7\%$ for any period, aggregate utility increases by 15.3% when $\eta = 0$ and $\bar{\varkappa}_i = 3.28$, while it increases by up to 60.4% under the welfare-maximizing coordination policy. The slow convergence induced by the associated negative level of η produces this large difference. On the other hand, under consumption taxes, $\bar{\varkappa}_i^+ = 29.0\%$, with $\eta^+ = 1.27$, only slightly lower than the optimal level of $\bar{\varkappa}_i$, 29.6%, under $\eta = 0$. The faster convergence induced by the welfare-maximizing coordination makes aggregate utility to increase very similarly: by 29.2% when $\eta = 0$ and $\bar{\varkappa}_i = 29.6\%$, and by 29.9% when choosing η jointly with $\bar{\varkappa}_i$.

Summarizing, the simple public investment rule assumed in (2.13) allows the government to decide on several types of paths for $\varkappa_{it}$:

(*i*) long-run downsizing or upsizing, with a constant ratio $\varkappa_{it}$ throughout the whole transition ($\bar{\varkappa}_i < 7\%$ or $\bar{\varkappa}_i > 7\%$, with $\eta = 0$),
(*ii*) long-run downsizing, initial overshooting of $\varkappa_{it}$ and fast-convergence ($\bar{\varkappa}_i < 7\%$ and $\eta > 0$),
(*iii*) long-run downsizing, no initial overshooting of $\varkappa_{it}$ and slow-convergence ($\bar{\varkappa}_i < 7\%$ and $\eta < 0$),
(*iv*) long-run upsizing, initial overshooting of $\varkappa_{it}$ and fast-convergence ($\bar{\varkappa}_i > 7\%$ and

$\eta > 0$), and
(v) long-run upsizing, no initial overshooting of $\varkappa_{it}$ and slow-convergence ($\bar{\varkappa}_i > 7\%$ and $\eta < 0$).

For the baseline calibration, (iii) is the welfare-maximizing strategy under capital income taxes (more distorting taxes), while (iv) is the optimal strategy under consumption taxes (less distorting taxes). However, in the present context, the optimality of reducing the long-run public investment ratio is not specific to the capital income tax scenario, since changes in some structural parameters could reverse this result. Next section shows this possibility.

5. Sensitivity analysis

It was shown in the previous section that reducing the long-run public investment/output ratio maximizes welfare under capital income taxes, but not under consumption taxes. We now perform a sensitivity analysis showing that there are factors that could reverse that result.

In particular, a high enough level of the output elasticity of private capital, α, makes long-run downsizing to be optimal under private consumption taxes, the opposite being true under capital income taxes. Moreover, independently of the tax system considered, downsizing improves welfare when the elasticity of intertemporal substitution of private consumption, $1/\theta$, and/or the output elasticity of public capital, θ_g, are low enough, or when the unproductive public expenditure/output ratio, $\varkappa_c$, is sufficiently high.

The sensitivity analysis modifies parameters independently, considering first $\alpha = .75$,[18] second $\theta_g = .06$ [the value fitted in Ratner (1983)], third $\varkappa_c = .20$ (only slightly higher than the benchmark .17, since otherwise the initial steady-state growth rate would become negative) and finally $\theta = 3$ (a relatively high value for the risk aversion parameter). Table 4.4 summarizes the main results from this sensitivity analysis.

[INSERT TABLE 5.1 ABOUT HERE]

For $\alpha = .75$, the shape of the welfare-maximizing coordination policy under both tax systems reverses. Now, under capital income taxes, $\bar{\varkappa}_i^+ = 11.0\%$ and $\eta^+ = 7.55$, while under private consumption taxes, $\bar{\varkappa}_i^+ = 2.5\%$ and $\eta^+ = -1.41$. The increment on welfare, relative to staying on the initial *bgp*, is .71% and 2.70%, respectively. Under capital income taxes, given $\varkappa_c$, a higher level of α provokes that a lower tax rate on capital income is needed to finance any given level of $\bar{\varkappa}_i$ [from (3.5)], since now private capital is more productive. Therefore, taxing private capital accumulation becomes less harmful for welfare the larger is α, and a higher level of $\bar{\varkappa}_i$ maximizes aggregate utility. On the other hand, under consumption taxes, the crowding-out on consumption caused by an increase in the public investment ratio is more important the higher is α, since

[18] This level is quite higher than the standard .36. However, it would be a reasonable value if private capital is interpreted in a broad sense, for example, including physical as well as human capital (Romer (1987)).

now savings becomes more attractive. Consequently, a lower level of $\bar{\varkappa}_i$ would maximize welfare.

Several aspects must be emphasized relative to the remaining cases. First, a larger level of $\varkappa_c$ implies a higher taxation cost of increasing $\bar{\varkappa}_i$, since the associated tax rate is now higher every period. Second, a lower θ_g reduces the benefit of raising public capital, since now public capital is less productive, and the externality factor θ_k increases.[19] Finally, a higher level of θ (i.e., a lower level of the elasticity of intertemporal substitution) increases the importance of short-run effects on welfare. Hence the implemented policy rule must punish less initial consumption, in exchange of lower long-run growth. Consequently, the welfare-maximizing value of $\bar{\varkappa}_i$ is lower than in the baseline calibration, independently of the tax system considered. For instance, for $\theta_g = .06$, $\bar{\varkappa}_i$ is lower than the initial 7% independently of the taxing scenario considered.

6. Conclusions

The coordination between short- and long-run policies has been extensively studied in the monetary policy literature. However, little work has been done regarding the coordination between long- and short-run fiscal policy measures. In this paper, it is assumed that, given a long-run target for the public investment/output ratio, the government follows a particular rule that makes the public investment/output ratio to react along the transition according to the current productivity of public capital relative to its long-run level. This public investment rule is more flexible than those commonly considered in the literature [Barro (1990), Turnovsky (1996, 2000), among many others], where the public investment/output ratio is assumed to remain constant over time.

In comparison with the Barro-type rule, and under alternative taxing scenarios, we find significant welfare gains when coordinating the long- and the short-run policies. We showed that the optimal coordination allows for the public investment ratio to be higher than its long-run target along the transition, no matter whether it is higher or lower than the initial level of 7%, to then decline monotically towards it. This way, the government maintains the public investment ratio above its long-run target, financing this gap with lump-sum taxes, and extending this positive transitory effect throughout the transition.

Therefore, an optimal policy coordination inducing a low convergence would achieve a higher welfare improvement than alternative fast convergence policies, since the positive transitory impact would extend throughout a larger number of periods. On that score, for the baseline calibration, downsizing from 7% to 1.13%, coordinated with a low convergence short-run policy, maximizes welfare under capital income taxes, while a long-run upsizing from 7% to 29.0%, combined with a fast convergence short-run policy, would be optimal under consumption taxes.

The optimality of downsizing the long-run public investment ratio is not specific to the capital income tax scenario, since changes in some structural parameters reverse this result. For instance, a sufficiently high productivity of private capital makes the

[19] Recall that the condition $\alpha + \theta_g + \theta_k = 1$ is always imposed.

combination of downsizing and a low-convergence short-run policy to be optimal under consumption taxes. Additionally, for sufficiently high levels of the unproductive public expenditure/output ratio, and low enough levels of the public capital elasticity and the elasticity of substitution, long-run downsizing becomes optimal, independently of the tax system.

Coordinating short- and long-run fiscal policies is an issue of utmost interest that requires extending the analysis made in this paper in several directions. First, more complex and realistic financing structures should be considered to escape from the neutrality of financing the short-run active policy, allowing that changes in the short-run policy affect the tax rate. Another extension to the paper regards the particular shape of the public investment rule. Still more interesting active policy rules could be considered. For instance, the public sector could have several short-run policy tools at its disposal, allowing for the government to select among an even higher set of public investment plans. In addition, since politicians increasingly make decisions according to the evolution of *real* macroeconomic variables, specifying a reasonable policy rule would be a convenient way to describe the performance of public expenditure plans.

7. Appendix: Log-linear optimality conditions

Uhlig (1999) proposes a procedure where optimal conditions are log-linearized without need of differentiating. Let $\hat{v}_t$ and $\hat{z}_t$ denote the variables in log-deviation to their steady-state, $\hat{v}_t = \ln\left(V_t/\bar{V}\right)$ and $\hat{z}_t = \ln\left(Y_t/\bar{Y}\right)$. X^a can be approximated:

$$\left(\frac{V}{\bar{V}}\right)^a = \exp\left(a\ln\left(\frac{V}{\bar{V}}\right)\right) = \exp(av) \simeq (1+a\hat{v}) \Rightarrow V^a \simeq \bar{V}^a(1+a\hat{v}). \tag{7.1}$$

In addition, $\hat{v}\hat{z} \simeq 0$ if variables are close enough to their steady-state values. Log-linearized versions of (3.9)-(3.14) are (all variables in log-deviations about steady-state):

$$\begin{aligned}
\hat{c}_{t+1} - \hat{c}_t + \hat{k}_{t+1} + \frac{\tilde{\rho}\tilde{\theta}\bar{h}}{1-\bar{h}}(\hat{h}_{t+1} - \hat{h}_t) - \frac{\tilde{\theta}\bar{r}^k(1-\bar{\tau}^k)}{\bar{R}}\hat{r}_{t+1} &= 0, & (7.2)\\
\hat{c}_t - \hat{y}_t + \frac{1}{1-\bar{h}}\hat{h}_t &= 0, & (7.3)\\
\bar{y}\hat{y}_t - \bar{c}\hat{c}_t - \varkappa_i\bar{y}(1+\eta)\hat{y}_t - \varkappa_i\bar{y}\eta\hat{v}_t - \varkappa_c\bar{y}\hat{y}_t - (1+\bar{\gamma})\hat{k}_{t+1} &= 0, & (7.4)\\
\bar{r}\hat{r}_t - \alpha\bar{y}\hat{y}_t &= 0, & (7.5)\\
\hat{y}_t - (1-\alpha)\hat{h}_t - \theta_g\hat{k}_t^g &= 0, & (7.6)\\
\left(\hat{k}_{t+1} + \hat{k}_{t+1}^g\right)(1+\bar{\gamma})\bar{k}^g - (1-\delta^g)\bar{k}^g\hat{k}_t^g - \varkappa_i\bar{y}(1+\eta)\hat{y}_t - \varkappa_i\bar{y}\eta\hat{k}_t^g &= 0, & (7.7)
\end{aligned}$$

where $\hat{k}_{t+1} = \left[\ln\left(\frac{\tilde{K}_{t+1}}{\tilde{K}_t}\right) - \ln(1+\bar{\gamma})\right]$, $\tilde{\theta} = \frac{1}{1-\rho(1-\theta)}$, $\tilde{\rho} = (1-\rho)(1-\theta)$, $\bar{\gamma} = \varkappa_i F\bar{h}^{1-\alpha}\left(\bar{k}^g\right)^{\theta_g - 1} - \delta^g$, $\bar{y} = F\bar{h}^{1-\alpha}\left(\bar{k}^g\right)^{\theta_g}$, $\bar{r} = \alpha\bar{y}$, and $\bar{R} = \left[1 + (1-\bar{\tau}^k)\bar{r} - \delta^k\right]$. Tax-rates are constant. Steady-state conditions have been used to simplify these expressions. The endogenous state variable is $\hat{k}_t^g = \ln k_t^g - \ln\bar{k}^g$, denoted in general terms by $\hat{v}(t)$. Endogenous control variables are included in $\hat{z}(t)' = \left(\hat{y}_t, \hat{c}_t, \hat{h}_t, \hat{r}_t, \hat{k}_{t+1}\right)$. From (7.2)-(7.7), log-linear conditions can be rewritten:

$$\begin{aligned}
A\hat{v}_{t+1} + B\hat{v}_t + C\hat{y}_t &= 0, & (7.8)\\
F\hat{v}_{t+2} + G\hat{v}_{t+1} + H\hat{v}_t + J\hat{y}_{t+1} + K\hat{y}_t &= 0, & (7.9)
\end{aligned}$$

where

$$\begin{aligned}
A &= \begin{pmatrix} 0 & 0 & 0 & 0 & (1+\bar{\gamma})\bar{k}^g \end{pmatrix}',\ B = \begin{pmatrix} 0 & -\varkappa_i\bar{y}\eta & 0 & -\theta_g & \left(-(1-\delta^g)\bar{k}^g - \varkappa_i\bar{y}\eta\right) \end{pmatrix}',\\
C &= \begin{pmatrix} -1 & 1 & \frac{1}{1-\bar{h}} & 0 & 0\\ \bar{y}(1-\varkappa_i(1+\eta)-\varkappa_c) & -\bar{c} & 0 & 0 & -(1+\bar{\gamma})\\ -\alpha\bar{y} & 0 & 0 & \bar{r} & 0\\ 1 & 0 & -(1-\alpha) & 0 & 0\\ -\varkappa_i\bar{y}(1+\eta) & 0 & 0 & 0 & (1+\bar{\gamma})\bar{k}^g \end{pmatrix},\\
F &= 0,\ G = 0,\ H = 0,\\
J &= \begin{pmatrix} 0 & 1 & \frac{\tilde{\theta}\tilde{\rho}\bar{h}}{1-\bar{h}} & -\frac{\tilde{\theta}\bar{r}(1-\bar{\tau}^k)}{\bar{R}} & 0 \end{pmatrix},\ K = \begin{pmatrix} 0 & -1 & -\frac{\tilde{\theta}\tilde{\rho}\bar{h}}{1-\bar{h}} & 0 & 1 \end{pmatrix}.
\end{aligned}$$

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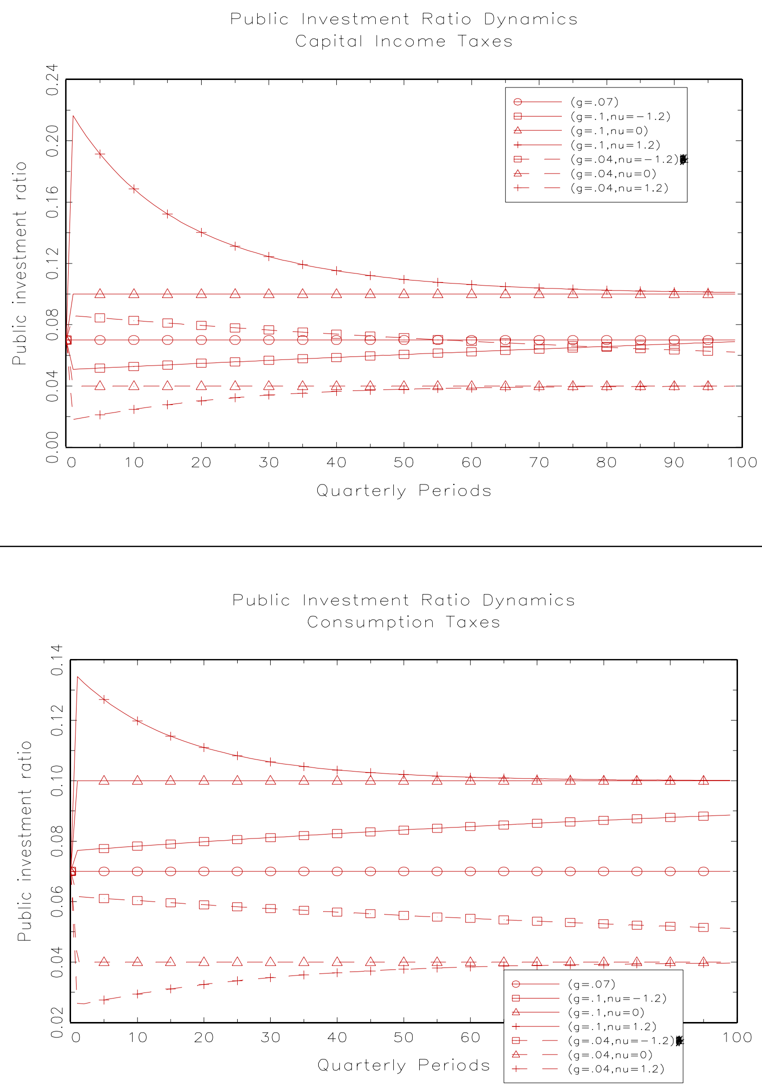
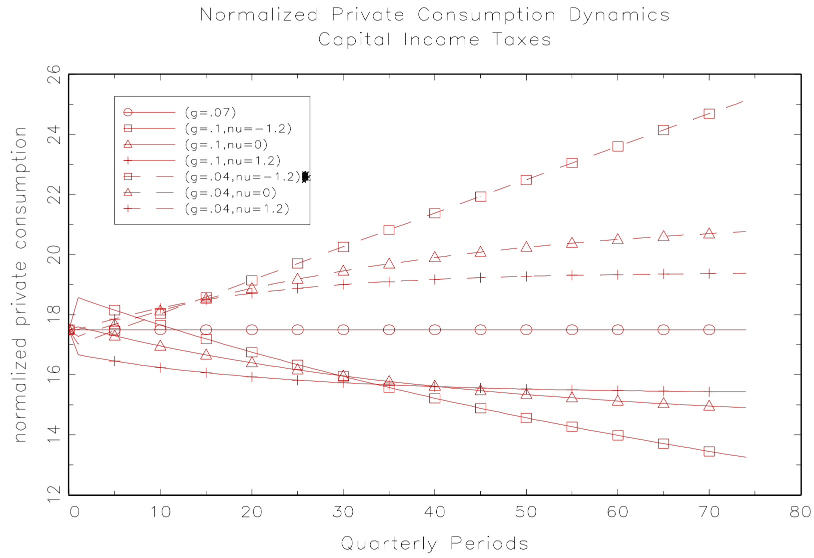
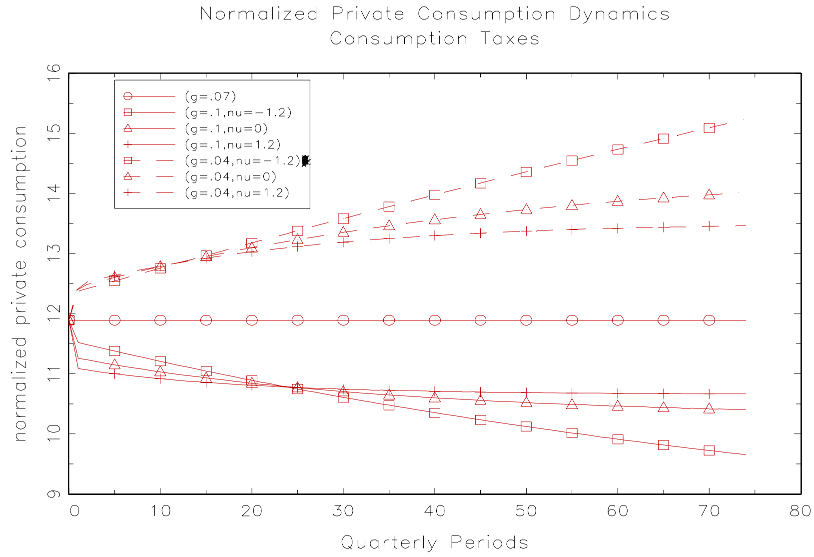
Figure 4.1: The dynamics of the public investment/output ratio



Note: each line shows the transitional dynamics of the corresponding variable for 100 periods, starting at a common state. In the legend of each graph, g=$\bar{\varkappa}_i$ and nu=η. The initial public investment/output ratio is always 7%. Lines with triangles keep η=0, lines with crosses correspond to η=1.20 and lines with squares to η= -1.20. Additionally, the dotted-lines correspond to $\bar{\varkappa}_i$=.04, while the solid lines correspond to $\bar{\varkappa}_i$=.10. The solid line with circles shows the path when the economy stays on the initial balanced growth path.

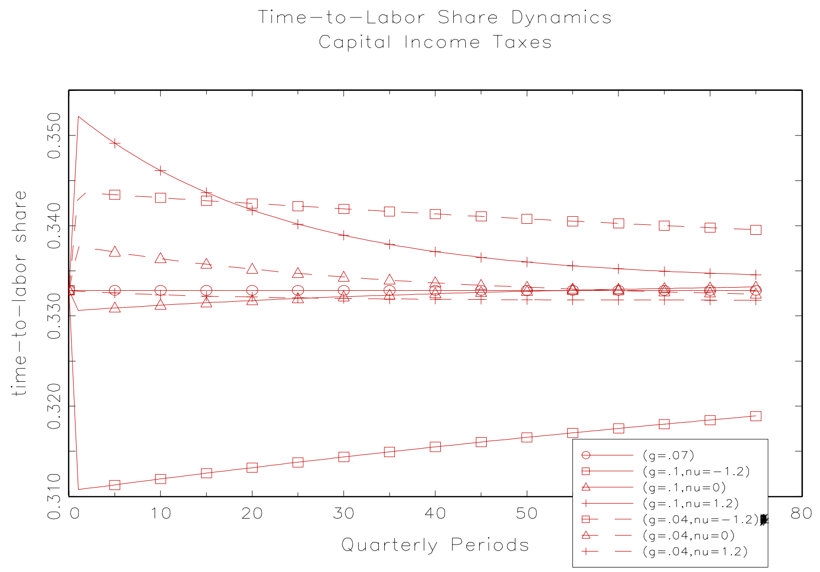
Figure 4.2: The dynamics of normalized consumption

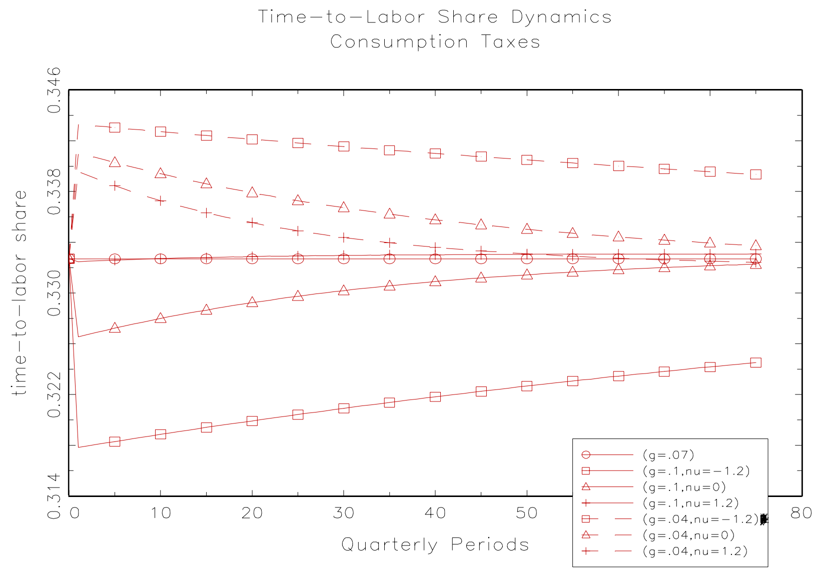




Note: see footnote to figure 4.1.

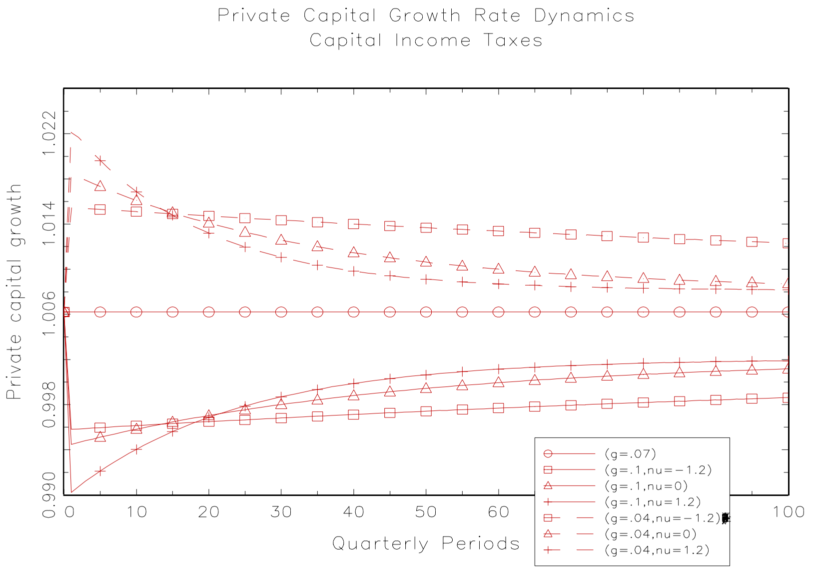
Figure 4.3: The dynamics of time-to-labor share

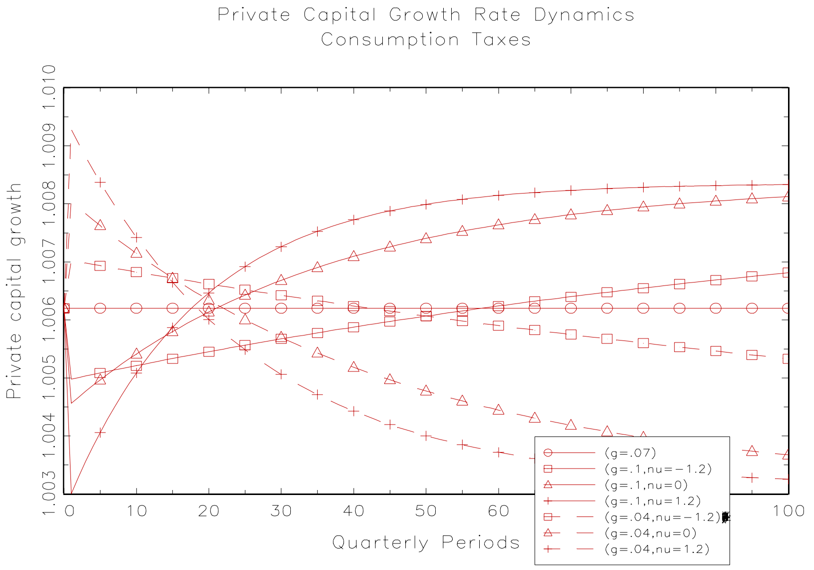




Note: see footnote to figure 4.1.

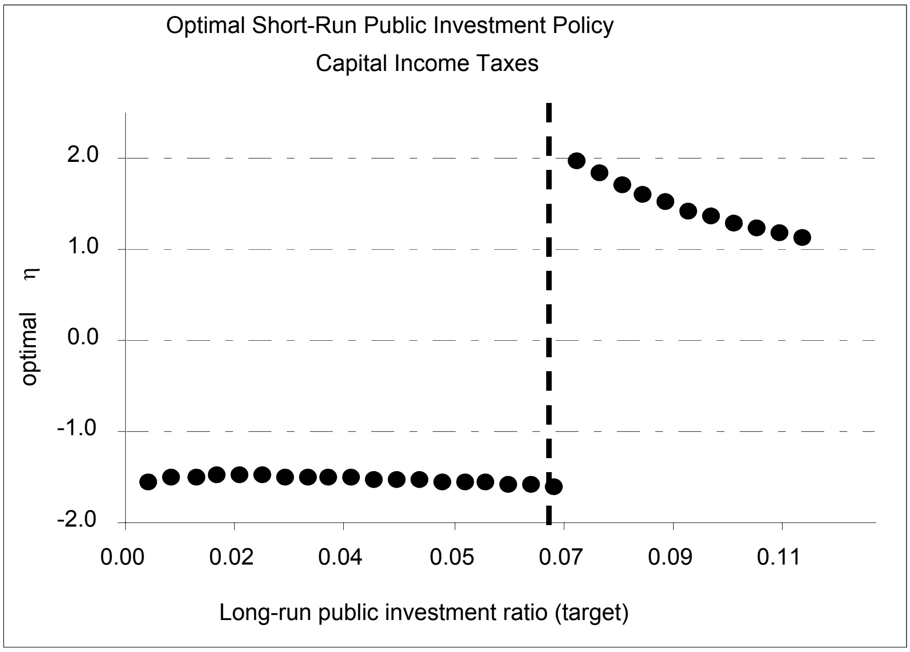
Figure 4.4: The dynamics of private capital growth rate

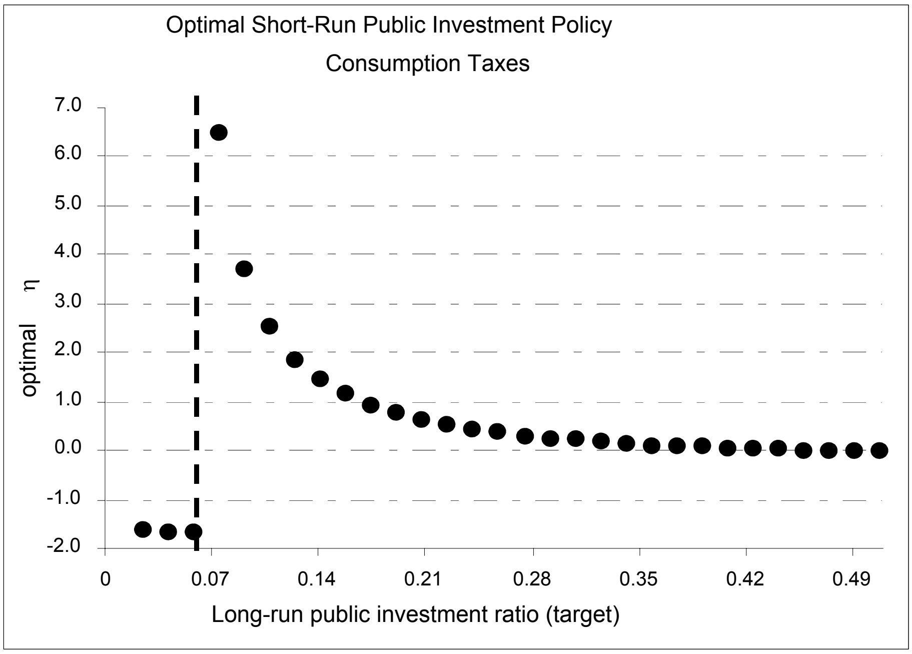




Note: see footnote to figure 4.1.

Figure 4.5: Optimal levels of η for a committed long-run public investment ratio





Note: each line shows the welfare-maximizing level of η, $\hat{\eta}$, as a function of the long-run public investment ratio target, $\bar{\varkappa}_i$. The economy starts with an initial public investment/output ratio of 7%.

List of tables

Table 4.1: Transitional dynamics and steady-state properties

CAPITAL INCOME TAXES											
Public investment rule		Convergence speed	Percent initial impact					Steady-state values			
η	κ_i(%)	Periods	C	I^k	I^g	h	Y	γ(%)	h	I^k/Y	C/Y
-1.20	4.0	122.6	-1.9	33.4	26.3	3.3	2.9	0.81	0.33	0.08	0.65
	10.0	114.2	6.3	-31.1	-30.3	-6.6	-4.1	0.21	0.34	0.13	0.66
0	4.0	24.0	-0.4	45.9	-41.8	1.5	1.8	0.81	0.33	0.08	0.65
	10.0	31.9	0.8	-34.4	42.5	-0.7	-0.2	0.21	0.34	0.13	0.66
1.20	4.0	12.7	0.8	64.4	-73.8	-0.0	0.8	0.81	0.33	0.08	0.65
	10.0	15.9	-4.5	-44.0	221.3	5.8	3.9	0.21	0.34	0.13	0.66
CONSUMPTION TAXES											
Public investment rule		Convergence speed	Percent initial impact					Steady-state values			
η	κ_i(%)	Periods	C	I^k	I^g	h	Y	γ(%)	h	I^k/Y	C/Y
-1.20	4.0	120.4	4.4	3.8	-9.8	3.2	2.4	0.31	0.32	0.31	0.48
	10.0	87.4	-2.3	-2.6	7.7	-4.5	-2.1	0.83	0.34	0.32	0.41
0	4.0	30.9	4.9	7.6	-41.7	2.5	1.9	0.31	0.32	0.31	0.48
	10.0	24.5	-4.5	-3.8	42.3	-1.9	-0.4	0.83	0.34	0.32	0.41
1.20	4.0	17.3	5.4	12.5	-62.6	2.1	1.6	0.31	0.32	0.31	0.48
	10.0	12.6	-6.0	-8.2	93.7	-0.1	0.8	0.83	0.34	0.32	0.41

Note: The column labelled *speed* shows the number of periods needed to cover half the distance between the initial state and the final $\underline{bgp}$. Benchmark calibration sets initially $\bar{\varkappa}_i$=.07, $\bar{\gamma}$=.62% and $\bar{\text{h}}$=.33. Initially, $\overline{C/Y}$=.65 and $\overline{I^k/Y}$=.11 under capital income taxes, while $\overline{C/Y}$=.44 and $\overline{I^k/Y}$=.32 under consumption taxes.

Table 4.2: Welfare-maximizing levels of η for a committed level of $\bar{\varkappa}_i$

CAPITAL INCOME TAXES										
Public investment rule		Convergence speed	Percent initial impact							
κ_i(%)	η	Periods	C	I^k	I^g	h	Y	ΔU(*)	γ(%)	♦
4.0	-1.51	$10e^{10}$	-2.4	31.3	54.3	3.8	3.3	23.3	0.81	0.58
10.0	1.25	15.6	-4.8	-44.6	232.8	6.1	4.1	1.6	0.20	0.75
CONSUMPTION TAXES										
Public investment rule		Convergence speed	Percent initial impact							
κ_i(%)	η	Periods	C	I^k	I^g	h	Y	ΔU(*)	γ(%)	♦
4.0	-1.64	$10e^{10}$	3.8	2.7	5.2	3.4	2.5	21.8	0.34	0.44
10.0	3.06	6.6	-7.3	-18.1	219.5	1.5	1.8	0.8	0.83	0.65

Note: see footnote of Table 4.1.
(*) It shows the percent increment in welfare under $\hat{\eta}$ with respect to the level obtained when implementing a passive rule ($\eta = 0$) for a common value of $\bar{\varkappa}_i$.

Table 4.3: Optimal simultaneous choice of η and $\bar{\varkappa}_i$ under the benchmark calibration

CAPITAL INCOME TAXES										
	Public investment rule		Convergence speed	Percent initial impact						
	✠$_i$	♒	Periods	C	I^k	I^g	h	Y	☟U(*)	♑(%)
Simultaneous choice	1.13	-1.49	10e10	-1.8	56.6	116.0	2.7	2.2	60.4	0.48
Optimal ✠$_i$ given ♒=0	3.28	0	24.1	-0.9	59.7	-52.1	2.0	2.1	15.3	0.80
CONSUMPTION TAXES										
	Public investment rule		Convergence speed	Percent initial impact						
	✠$_i$	♒	Periods	C	I^k	I^g	h	Y	☟U(*)	♑(%)
Simultaneous choice	29.0	1.27	15.1	-49.3	-21.6	450.8	-1.3	0.7	29.9	1.59
Optimal ✠$_i$ given ♒=0	29.6	0	19.3	-45.3	-15.9	298.1	-11.4	-5.9	29.2	1.60

Note: see footnote in table 4.1.
(*) percent welfare gains under the jointly optimal values of $\bar{\varkappa}_i$ and η with respect to a situation in which the economy stays along the initial bgp.

Table 5.1: Optimal simultaneous choice of η and $\bar{\varkappa}_i$. Sensitivity analysis

CAPITAL INCOME TAXES											
	Public investment rule		Convergence speed	Percent initial impact							
Alternative parameters	✠$_i$	♒	Periods	C	I^k	I^g	h	Y	☟U(*)	♑(%)	⧫
♋=0.75	11.0	7.55	0.59	10.1	-54.0	4,309	1.6	12.5	0.7	0.12	0.37
✠$_c$=0.20	0.98	-1.55	$10e^{10}$	-2.4	76.1	197.0	3.0	2.0	143.6	0.0	0.58
❑$_g$=0.06	0.20	-1.10	$10e^{10}$	0.8	63.0	109.4	0.6	1.7	79.7	0.01	0.47
❑=3.0	0.11	-1.33	$10e^{10}$	0.5	33.9	69.0	-0.2	0.2	7.9	0.00	0.50
CONSUMPTION TAXES											
Alternative parameters	Public investment rule		Convergence speed	Percent initial impact							
	✠$_i$	♒	Periods	C	I^k	I^g	h	Y	☟U(*)	♑(%)	⧫
♋=0.75	2.5	-1.41	$10e^{10}$	37.7	8.8	10.6	3.9	8.6	2.7	0.04	1.52
✠$_c$=0.20	26.5	0.35	14.7	-45.6	-21.3	420.3	-0.1	1.5	28.5	0.02	2.15
❑$_g$=0.06	2.5	-1.16	$10e^{10}$	7.9	3.4	5.1	3.7	3.2	3.7	0.01	0.40
❑=3.0	16.5	0.04	26.6	-15.8	-10.8	135.7	-3.3	-1.4	2.6	0.01	0.87

Note: See footnote to table 4.1.
(*) percent welfare gains under the jointly optimal values of $\bar{\varkappa}_i$ and η with respect to a situation in which the economy stays along the initial bgp.

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