A DYNAMIC MODEL OF FINAL SERVICE COMPETITION IN FIXED ELECTRONIC COMMUNICATIONS UNDER A CAPACITY INTERCONNECTION REGIME

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Abstract

The Spanish regulatory authority recently implemented a new interconnection regime based on capacity (and not per time) payments in fixed telecommunications. We propose a dynamic duopolistic model of final service competition in which the entrant first acquires a certain capacity at the local loop (at a fixed payment) from the incumbent and then both operators compete in prices for the final services. We introduce the entrant's possibility to assign efficiently the traffic he offer between different hours along the day as well as the possibility for him to use the capacity based model as well as the per-unit-of-time interconnection regime vis à vis the incumbent. The results show that, the simultaneous use of both interconnection models (instead of only the one based on time) leads always to tougher competition (more aggressive pricing) in the final service market and efficiency (internal to the firm and allocative) gains. Nevertheless, the entrant needs a minimum scale before this new model can be a viable alternative. In addition, once reached this scale, its convenience for the entrant will depend on certain conditions.

Resumen

Recientemente el regulador español ha implementado un nuevo modelo de interconexión en redes fijas de telecomunicaciones basado en la compra de capacidad que podrá ser utilizado en combinación con el modelo habitual de interconexión por tiempo. Se propone un modelo dinámico de competencia en duopolio en el cual el entrante puede comprar *ex ante* un cierto nivel de capacidad al incumbente para luego ambos operadores competir en precios. Se introduce la posibilidad de que el entrante pueda asignar eficientemente el tráfico entre distintas franjas horarias. Los resultados muestran que el uso simultáneo de ambos modelos genera una competencia más agresiva en precios y puede llegar a producir significativas ganancias de eficiencia (asignativa y otras internas a la firma). De cualquier forma, el entrante necesita alcanzar una masa crítica antes de que el nuevo modelo pueda representar una alternativa para él. Además, una vez alcanzado este nivel, su conveniencia para el entrante dependerá de determinadas condiciones.

Key words: Interconnection per time, interconnection by capacity, dynamics of the competition, regulation, network dimension.

JEL classification: C72, L13, L51, L96.

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I am grateful for useful comments and suggestion from Iñigo Herguera and Francisco Alvarez. The usual disclaimer applies. Financial support from the Consejo Nacional de Ciencia y Tecnología de México (CONACYT) is gratefully acknowledged.
Some of the ideas commented in this document have been published by the author in *Industrial Economy*, 337. March 2002, under the name "Evaluación de los efectos dinámicos de la introducción del modelo de Interconexión por Capacidad".

1. Introduction

Until recently, practically anywhere in the world, the only used method of interconnection (understood like an intermediate service) between fixed telecommunication networks, was the interconnection based on the number of units-of-time (minutes) acquired by the entrants (*IxT*). This regime involves the imposition on the newcomers of a cost structure (function) for the local infrastructure that is diametrically opposite to the incumbent one, and in fact, far from real. The incumbent operators don't face relatively high variable per-unit-of-time cost of usage in the local loop, while entrants have to pay this variable cost without being able to exploit any economy of scale, or scope, across time for the use of the infrastructure.

Then, under the *IxT* model, the new operators are not capable to emulate the supplies of the once monopolist, and have few options to reduce the asymmetries that differentiate them unless they can build their own infrastructure. Furthermore, the entrant's incentives to increase traffic are not so evident, because his average cost stays constant.

The Comisión del Mercado de las Telecomunicaciones (CMT), Spanish regulatory authority, recently has implemented an alternative to the *IxT* which consists in that the entrants can acquire interconnection capacity blocks[2] and use it freely. It would have sense to think that a model of interconnection based on capacity (*IxC*) and not based on per-unit-of-time usage, will allow the newcomers to create services and choose tariff schemes nearer to those expected in effective competition. In addition, they will be able to make use of the infrastructure, at least proportionally similar as the incumbent does because **part of the variable costs of these new operators would become fixed costs**.

With the introduction of the *IxC* model (both coexisting), the entrant's cost structure changes importantly. Now, they can achieve greater economies of scale (limited by the maximum capacity they rent) across the hours of the day, switching traffic from one hour (peak) to another (off-peak) as well as improving his corporate image when expanding his range of services. This change in the cost structure is also reflected in a more aggressive price competition. Then, it is not so evident that the *IxC* be the best alternative for the entrant, at least before getting a critical mass. Additionally, one would hope that the capacity payment regime allows the entrants to implement non-linear tariffs structures to the final customers, with all the efficiency gains that this may bring about in the market.

The CMT hopes that this new model brings on to the entrants: extra marginal income, smaller marginal costs, and an improvement of the traffic distribution, among other benefits.

[2] With the purpose of avoiding possible confusions, throughout the document will be used the expression "capacity block" instead of "capacity unit" (without a better alternative). This is because the term "unit" could be associated with "smallness".

It is important to mention that the regulator doesn't try to make attractive the *IxC* model by means of the interconnection prices itself but with the possibility of internalizing the management performance potential, this, as well, will transfer a certain risk to the entrants.

The *IxC* regime is already known in other markets, e.g., Internet backbones, international settlement traffic among operators, and so forth. Even though, the payments mechanisms at play in each market differ greatly. We want top focus on the effects on competition, tariff structures and dynamic effects of the capacity interconnection regime recently introduced in fixed telephony.

In this document, we proposed an approach to the design of an *IxC* model within the habitual frame of the unidirectional fixed telecommunications, in the sense that the incumbent is the only one who provides interconnection. The paper is structured as follows: in section 2, the basic model is exposed, including intrinsic and particular characteristics of the *IxC* model (and therefore nonexistent in an *IxT* model) and these are compared with those of the *IxT* model. In addition, we emphasized the possibility of using the (potential) management of resources that we call "network dimension" as a generating "spark plug" of a more aggressive price competition (instead of using for this purpose the interconnection prices itself). Section 3 shows the equilibrium prices and profits under different tariff schemes and under both models. Some welfare implications are commented in section 4. Finally, section 5 contains the main conclusions.

2. The model

The proposed model is a modification of that used by Bijl and Peitz (reference 2) to characterize a situation of competition in operator's selection and pre-selection. In this model is assume that the newcomer doesn't have (his own) infrastructure with the exception of "the backbone"[3], that is, he needs to buy interconnection to the incumbent to be able to reach the end users. It maintains the essence of seminal results of the literature on the economic theory of network interconnection like those of Armstrong (1998), Carter and Wright (1999), and Laffont, King, and Tirole (1998a, b), among others.

In this document two minor modifications have been done to the original, so that here: 1) the incumbent's marginal costs are zero (a particularization), and 2) the operators can charge a subscription fee[4] (a generalization). One of the consequences that would be expected with the introduction of an *IxC* model is the increase in the entrant's incentives for the establishment of flat tariffs.

[3] It is common, at least during the first stage of the competition, that the new operators have an incipient infrastructure (in proportion to the incumbent one).

Anyway, the contribution consists of the following changes: maintaining the original model (with the modifications previously suggested) and taking it as a reference, it is created other based on interconnection capacity (and not per time) payments. Both models represent the options to compete for the entrant at the beginning of every period.

Consider the following general assumptions: there are two operators, the incumbent, and the entrant. They play a dynamic game because the sequence of stages is repeated over an arbitrary number of periods. The incumbent is the only source of interconnection, moreover, at the beginning of the game, he has all the market, and at the beginning of each period, he has all the productive resources of the economy, i.e., the capacity is only usable for the effective period[5]. (these characteristics partially define him). Suppose that his capacity is large enough to satisfy his own demand and that of the newcomer (in this sense, capacity restrictions don't exist). Assume also that the operators have total coverage, i.e., they compete at any moment for all the market (e.g. a province).

2.1. Sequence of stages

The situation analyzed consists of the following sequences of stages in every period[6]:

Stage a) the regulator determines the prices of both interconnection methods: per time (a) and by capacity (P)[7]. To make the analysis simple assume that these prices are constant throughout time[8]. In addition, the prices are proportionally equivalent, (we will justify this assumption later). The incumbent offers the interconnection contract menu to the entrant.

Stage b) the entrant decides the way to compete for completely satisfy his estimated demand of incoming and outgoing traffic. He selects a contract mixture between *IxT* and *IxC*[9], and if this last contract is included, he must decide how many blocks to acquire and finally buy them. The entrant's behavior can be intuitively understood as follow: he anticipates the equilibrium prices and profits for each of the possible alternatives and then chooses that that maximize his benefits based on which the rival chooses (backward induction).

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[4] See reference 2, section 6.4, 7.2 and 7.3. There, the authors review the implications of the two-part tariffs but they only do it when both operators compete with complete infrastructure (two-way access).

[5] Otherwise, the entrant doesn't take any risk. In addition, the regulator has established fines for wrong estimation of the resources requirements. Anyway, we only consider that since the entrant cannot obtain income from the excess capacity but expenses, this one act like a fine by itself (although only for the case of overload). In other words, the cost incurred by the incumbent because of the entrant's bad planning is ignored.

[6] There doesn't exist a unique way to link the periods with the real time. Anyway, the parameters have been considered in monthly terms. This could not be in agreement with some other model elements, then, with the pertinent changes the duration of the period could be linked to, e.g.: the price revisions (operator's and regulator prices); the speed with which the entrant gains market share in the simulations; or the form in which it has been defined the growth of h.

[7] We don't modelize the regulatory decision. The players take it as exogenous.

[8] Notice that those prices are independent of the volume of contracted capacity.

[9] The mixed schemes will be viable at a global level and supported by the principle of complementariness between methods. Despite of this, the operators compete in pure strategies because, although the entrant can choose a contract mixture, chooses it with probability one.

Stage c) the operators compete "à la Bertrand" to determine their (equilibrium) retail prices. The solution concept is the subgames perfect Nash equilibrium in every period (the operators are myopic because don't consider that today's prices will determine tomorrow's initial market shares). In the price determination is taken into account the consumer behavior (it will be defined later) and that, if necessary, the entrant will buy on a per-time basis, all the interconnection that he requires to completely satisfy his demand. This is the only way in which it will be allowed to use both interconnection models.

Stage d) observing the prices and his particular switch cost, each consumer subscribes to one operator[10] (they only can decide if to remain or to change) and later makes his telephone calls[11].

Stage e) the rest of the variables are obtained. The market shares in t will be taken as the initials in $t+1$.

Now, the particular way in which the stages elements are carried out is exposed.

2.2. The consumer

The market has a fixed size of n consumers[12]. They are homogenous in the sense that they have the same utility and demand parameters. However, in order to allow for a realistic transition of market shares over time, we introduce heterogeneity: a consumer who wants to switch from one operator to the other incurs a switching cost (*SC*). It emerges because capturing market share from a well-known, established firm requires great marketing efforts and substantially better price-quality combinations by the entrants. Hence, although the minimum negotiable unit of final good is homogenous, for a particular consumer, calling from any operator in view of the fact that he is affiliated to one of them, is not.

Several price structures are contemplated. In particular, let the retail per-minute-price be denoted by p. At that price, an individual consumes $x(p)$ call minutes and derives $u(x(p))$ of

[10] Therefore, the service demand (but not the call demand) is inelastic whenever the participation restriction is not violated.

[11] Since no information is added from one decision to the other, these could be taken as simultaneous. We consider it separately only for the benefit of the exposition.

[12] In the real market, the operator problem consists of maximizing the benefits derived from a collection of interconnection points, each one with a certain estimated traffic. Thus, it could be possible to transfer traffic from a point to another one by geographic, technological or another type of relation if this were in the operator's interest. Although the analysis of the strategic complementarity among interconnection points (alternative routing) can be very interesting, in this work the problem is simplified too much. We did not think in terms of interconnection points (therefore we did not take into account its strategic interaction) but in terms of capacity requirements. Then, n is the number of consumers arbitrarily related with the generation of certain traffic volume. We suppose that the aggregate problem is obtained fitting the parameters. Being strict, many others aspects exist that the model doesn't consider: e.g., congestion and overloads, effects of the sharing or not of resources, possibility of discounts, type of interconnection point (optical, electrical, etc.), "clock stop" of evolution of the project, types of traffic or levels of interconnection (only access and completion are considered).

utility. This function is expressed in monetary units and satisfies $u'(x) > 0$ and $u''(x) < 0$ $\forall\ x \in [0, \tilde{x}]$[13], where $\tilde{x} = \arg\max_x \{u(x)\}$. As usual, the individual demand arises from $u'(x) = p$.

The consumer problem is $v = \max_x \{h + u(x) - xp - m\}$[14]. Two new terms have been introduced by the previous expressions, the subscription fee, *m*, and h. Let h be an exogenous component of the utility intrinsically related to subscribing to an operator and not to the rival. It can be seen like loyalty to the company, better quality, confidence, etc. It makes sense to think that whereas the incumbent already has formed it and maintains it at a constant level, the newcomers are constructing it throughout time (e.g. supposed a learning curve). Without this, the operators are "excessively symmetric" and an entrant could gain market "too fast" while in reality, entrants typically increase quality levels over time.

Summarizing, the consumers make two decisions: they choose an operator and an amount of minutes of use[15]. They do it anticipating the utility they would obtain with each operator (resolving their problem) according to his indirect utility function, *v*, which provides all the information they need.

2.3. The market

In order to simplify the notation subscript 1 will be related to the incumbent and 2, to the entrant. Superscripts will refer to time. Other elements will be introduced along the paper.

At the beginning of the game, all the operators have a market share of f (initial endowment). Unless another thing was said, the game begins in a moment in which $f_1^0 = 1$, and $f_2^0 = 0$[16].

a is the inverse of the substitution degree between players. At the beginning of period *t* the consumers of operator *i*, have a *SC* uniformly distributed in $[0, s f_i^{t-1}]$. Then, the indifferent consumer between remain with the incumbent and leave him in favor of the entrant will be that for which $n_2^t - \hat{SC} = n_1^t$ (where $0 \le \hat{SC} \le s f_1^{t-1}$), that implies that all those consumers with $SC \in [0, n_2^t - n_1^t]$, leave the incumbent. Consequently, the percentage of incumbent consumers who remain is $1 + \left(\frac{v_1^t - v_2^t}{s f_1^{t-1}}\right)$ whose proportion in a market level gives the operator share in *t*. This is,

$$f_1^t = f_1^{t-1} + \frac{v_1^t - v_2^t}{s}. \tag{1}$$

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[13] In others words the utility function is continuous, quasiconcave and strictly monotone.
[14] Following Laffont, Rey, and Tirole (1998b), this implies that the consumers don't restrain to call when the prices increase but simply call less or make shorter calls.
[15] Of a single type, let us say local.
[16] Of this form is avoided any possible complication related to justify (others) initial values. Moreover, the entrance process of present and later operators is ignored.

The market share is an essential element of the dynamic model because it is the link between periods (the state variable).

2.4. Specifics introduced by the *IxC* model

In reality, the managed traffic by an operator doesn't follow strictly a uniform distribution along the day and most of the time is below the saturation point (e.g. the network is underused in off-peak hours). So, assume the existence of leftover resources because the network is not used at any moment and at full capacity[17]. We introduce the following definitions **exclusively related to one capacity block** [18]:

- Let *h* be the hour with the highest traffic (most congested hour) of the day as a month's average (e.g. 10-11 a.m.).
- Let *r* be the monthly traffic in *h* (e.g. 22,000 minutes).
- *d* is defined as the maximum number of minutes that an operator is (really) able to manage in a month (the network dimension).
- *r/d* is the weight of the hour with the highest traffic in the total potential traffic.
- *K* is defined as the number of minutes that an average operator is able to manage in a month (then, a particular value of *d*). The regulator decides it.
- *P* is the price. The regulator decides it.

2.4.1. The average management by capacity block *(K)*

Every capacity block that the entrant acquires will be related with a potential transmission of 2 Mbit/s[19]. The regulator must solve the problem of how many minutes an (average) operator is able to manage with one block, to later determine the (unique for all operators) block price.

The top average traffic that is transmitted with a connection of 2 Mbit/s, composed of 30 usable channels of 64 Kbit/s, has been determined by the CMT as follows: It is fixed a quality service level of 0.5% (equal for all operators) that corresponds with 63% as recommendable maximum level of traffic by connection in *h*[20]. This is equivalent to 19 channels each one transmitting 60 minutes (other combinations are possible). It gives a total of 1,142 minutes. In order to take this amount to a monthly level, it is multiplied by 25 (5 days less per month due to the weekends) and by 11/12 (considering the effect of the smaller traffic during the summer

[17] At least at the recommended capacity.
[18] More extensive definitions of these and other related concepts can be found in references 1, 7, and 8.
[19] Although the minimum amount of interconnection that an entrant can acquire is a circuit of 2 Mbit/s, the capacity can be contracted for multiples of smaller units, particularly, channels of 64 Kbit/s, being able to contract the rest per time. However, and without modifying the essence of the document, we take a circuit of 2 Mbit/s, as the minimum amount of capacity that can be contracted.
[20] According to Erlang B table.

months) then we have r=26,171. We take this value as a technical restriction and the same for all the operators**.**

The CMT has determined that *r/K* =.13. Then, finally, we have a monthly average of 201,314 minutes by a connection of 2 Mbit/s, i.e., the regulator has defined K=201,314 (then, K is a parameter in the model). **Nevertheless, *r/K*=.13 is an average estimation, thus, the operators could be able to manage more or less traffic based on other factors**[21].

The network sizing is determined by r, function of the quality service level, whereas the convenience of one or another model is determined by volume of estimated minutes.

When deciding the price of the capacity block (P), the regulator deliberately has tried that both models were practically price equivalent, that is, $a = \frac{P}{K}$, as we can see if we compare the third columns for each model in Table 1[22]. In this fashion, he tries to distort as little as he cans, the total average cost of the operators (economic continuity between models). Although, doing this, he doesn't grant the new operators the possibility of using the price advantage as a direct promoter of the *IxC* model. The regulator considers that the assignment of this objective rests on the idea of network dimension, which we comment in the following section.

	Interconnection per time			*Interconnection by capacity**		
Type of traffic	**p/p/h**	**p/op/h**	**pond/p****	**K**	**(2Mbit/s)/P**	**e/p/p**
	euro-cents / min			minutes	euro-cents	euro-cents / min
Local	0.757	0.457	**0.661**	201,314	132,611	**0.659**
Metropolitan	1.046	0.625	**0.836**	201,314	167,521	**0.832**
Simple	1.16	0.697	**1.012**	201,314	202,758	**1.007**
Double	2.218	1.334	**1.935**	201,314	387,380	**1.924**

* Combined capacity (voice + Internet).

** It is considered that a 68 % of the traffic correspond to normal tariffs and a 32 % to the reduced one (except metropolitan traffic where it is taken to equal parts).

p/p/h : price per minute in peak hours; p/op/h : price per minute in off-peak hours; pond/p : pondered price; (2Mbit/s)/P : price per capacity block; e/p/p : equivalent price per minute;

source: Own elaboration with CMT data.

Table 1: Price comparison between models.

2.4.2. The network dimension by capacity block *(d)*

The network dimension concept is only crucial for those entrants who choose the *IxC* contract. They will have the added responsibility to plan his interconnection resources to be able to use efficiently the service offered by the incumbent. Those that prefer the per-time contract are indifferent to dimension matters because the incumbent provides to them on the spot the required interconnection. In this sense, it would not be necessary to make an anticipated and precise forecast for the interconnection requirements.

[21] Evidently, the ideal for an operator is that his traffic was perfectly distributed (that is, $r/d = 1/24 \approx 4.17\%$).

The argument that gives sustenance to the network dimension concept is that the interconnection capacity contracted by an operator doesn't have to be equal to the traffic indeed attended to him, which will be function of his performance that, as well, will be function of his owns characteristics. Hence, it is not an exogenous variable. Unlike the *IxT*, the entrant has the possibility of being able to manage the capacity in an efficient way ($d>K$) and the risk of its inefficient management ($d<K$)[23]. Anyway, it is necessary to say that, in the model, once chosen the way to compete, the only decision variables of an operator are his retail prices; then, given these, he knows his network dimension (there are no exogenous factors as demand shocks). Therefore, it would have to verify

$$d = f\left(p_1, m_1, p_2, m_2, \mathrm{q}\right).$$

Where **q** is the initial value vector. Particularly, in a first approach, we have wanted to evaluate the consequences of considering the network dimension dependent of the operator's market size (then, operators do the best their market size allows them). An operator influences the market by means of his prices. For him, more aggressiveness in prices produces, *ceteris paribus*, a greater market share and this facilitates a greater network dimension that, as well, allows taking advantage of much bigger economies of scale. Consequently, this variable is internalized.

According to the CMT, will be the ability to motivate the demand, the correct management of the interconnection traffic, and the incursion towards new models of businesses as resale, and the sharing of interconnection points, that that will make possible the optimal use of the contracted interconnection capacity[24]. We put emphasis in the two firsts of the previous activities and considered appropriate to relate them with the operator's market size. The reason is, and this is the hypothesis, that proportionally as an operator is gaining market size, will be in better disposition to extend his package of services[25] (local telephony, long distance, Internet, data, and so forth) and/or to model properly his subscribers demand, through the design of his supply (publicity, packages, discounts in certain calls, etc.) increasing his traffic in off-peak hours as a result (this is independent of h). That is, we suppose that it is in the operator's interest be more

[22] According to the data, an average difference of .5% exists but it is ignored throughout the rest of the paper. See that this equivalence is at unitary level but not at aggregate level where is satisfied only when the bought units under *IxT* are a multiple of *K*. In any other case, the cost under *IxC* is higher.
[23] Strictly speaking, in this version this risk is omitted (except when the entrant is sufficiently small) since the operators "don't mistake" when planning their capacity necessities. Although the Oferta de Interconexión de Referencia (OIR) 2001 establishes a group of formal procedures about capacity requests and the fines by lack of fulfillment of the same ones, these will not have implication in the model because complete information exist (although imperfect because the decisions will be taken simultaneously). It is necessary to take into account that these penalties would act like added incentive to increase the traffic or to make a moderate planning with the purpose of avoiding them.
[24] Notice that in the design of a resale (secondary) market, the excess capacity of an operator will be useful for another one, if and only if, is managed by the first one. This is because it is indeed his own management that makes possible this excess capacity.
[25] We only see the traffic volume and its distribution. Then, it doesn't matter that there are only one good in the market.

generalist and to have a diversified supply, which is easy to observe in the real market. In conclusion, he obtains more flexibility. Nevertheless, we will take only an average price (the equilibrium price). Therefore, given that the entrant has acquired, for example, one capacity block will be easier for him to reach more than *K* minutes, obtaining a smaller unit cost.

Particularly, we have settled that the network dimension for operator *i* satisfies the following equality

$$d_2 = Z + Ynf_2, \tag{2}$$

that depends on the operator's number of subscribers[26], and the parameters *Z*, and *Y* (*>0*). *Z* is the management capacity when f_2=0, e.g., at the beginning of the competition, his intrinsic characteristics allow the operator to manage *Z* minutes. It fulfills $r<Z<K$. The contribution of each consumer in the better network management is *Y*.

It is important to remember that when buying a capacity block, the operator doesn't get a fixed amount of minutes, but a transmission possibility, where ***K* is only a reference when trying to obtain an equivalence with the *IxT* model**. Nevertheless, the block price is fixed and equal to *P*. Then, the internalization of the network dimension is an added incentive to compete more aggressively in prices. This is the bet of the regulator when introducing the *IxC* model.

We have $\frac{r}{d_2} = \frac{26{,}171}{Z + Ynf_2}$, then, the operator has an incentive to diminish his *r/d* (given that the block price is fixed *ex ante*) but, as defined in page 8, the regulator has fixed the numerator, so, the problem is to try to increase *d* ($r/d<(>).13$ ® $d>(<)K$). The only way to satisfy it, is competing more aggressively (ignoring the strategic effect that this could have in the rival). For whom that cannot modify his traffic pattern over the level fixed by the regulator (*K*), the adoption of the *IxC* model will not be profitable.

As *d* represents a maximum value (maximum management capacity), if the newcomer considers it appropriate (if this maximize his benefits) he would choose not to operate his dimension at that level. Nevertheless, given a set of prices, the entrant's best strategy is to maximize the use of his capacity. Then, when speaking of network dimension, we will refer to the maximum possible level, *d*[27].

2.5. The operators under *IxT*

The operators maximize their benefits in a non-cooperative way in every period. Simplifying, from now on, assign w_j to the amount of total call minutes (minutes of traffic), demanded to

[26] Of this form, the equilibrium prices are independent of *n*.
[27] Anyway, this is checked in the simulations. Look at section 2.4.2 and Appendix 3 for a more detailed analysis of the concept of network dimension.

operator j ($w_j=nf_jx_j$), and m_j to the total of his affiliated ($m_j=nf_j$). If the entrant decided to compete under *IxT*, the incumbent's benefits function that the literature habitually suggests is

$$p_1 = w_1(p_1 - c_a) + w_2(a - c_b) + m_1(m_1 - f_1).$$

The operator obtains income derived from the sale of call minutes to his affiliated (in its case), as well as of the subscription fee received from them (in its case). Moreover, he obtains income from the sale of interconnection to the entrant. A particular version for the (extreme) case of interest in which the incumbent's marginal costs or costs related to the traffic, c_a, c_b (cost *on net* and *off net*, respectively, of a call minute, a interconnection minute or a transmission unit) were sufficiently near to zero[28] (zero at the margin), is

$$p_1 = w_1 p_1 + w_2 a + m_1(m_1 - f_1) - F_1. \tag{3}$$

In counterpart, it has been added a fixed cost (F_1) related to previous investments that will be considered prorated *ex ante* and constant throughout time[29]. According to the prior specification, this term will not affect the equilibrium prices, so we can ignore it. As a result, an approach to the cost structure that was commented in earlier sections has been obtained.

On the other hand, the entrant's benefits function is

$$p_2 = w_2(p_2 - a) + m_2(m_2 - f_2), \tag{4}$$

which similarly, includes the possibility of obtaining a rent by the same first two concepts already commented for the incumbent but has the disadvantage of the variable cost related to the purchase of interconnection[30].

2.6. The operators under *IxC*[31]

If the entrant decided to compete under *IxC*, the incumbent's benefits function is

$$p_1 = w_1 p_1 + iP + (w_2 - id_2)a + m_1(m_1 - f_1) - F_1, \tag{5}$$

where, from now on, i will be the number of capacity blocks acquired by the entrant. The fundamental change with respect to the *IxT* model (equation 3) is that now, part of the income

[28] That $a>c_b=0$ occurs, could be imposed by the regulator to allow the recovery of some previous investment related to the network operation (e.g. access deficit related to the universal service provision).

[29] Take into account that many of the future conclusions could be determined by this cost assumption.

[30] Since the entrant doesn't have his own network, in a rigorous sense he will require two interconnection units by call minute that he serves (one related with the calling part and other with the receiving part). For the same reason, the interconnection networks requirements will be multiplied by two (although it could be considered that an important amount of calls would be terminated in others interconnection points, the one at issue would receive an equivalent amount of minutes of the others ones, assuming isotropic calling pattern). Anyway and without losing the essence of which it is tried to develop, a per-minute interconnection cost will be a and the value of d will serve to manage d call minutes.

[31] It is opportune to say that has not been designed an structure that faithfully reflects the technical aspects of a *IxC* model, but rather, one that develops an alternative form to describe the entrant's purchase of interconnection. Look at Appendix 1 for a more detailed analysis of the definition of the entrant benefits in the *IxC* model.

derived from the sale of call minutes of interconnection, $w_2 a$, which is direct function of the (present) prices, is replaced by the income derived from the capacity blocks that the entrant acquire, iP, which is indirect function of the (present) prices, remaining the preceding structure solely for the adjustment units (assume those are few in proportion). The explanation is the following: the component in *IxT*, $w_2 a$, is relevant in the determination of both operators' prices and under *IxC* it is not. Finally, under the *IxT* regime, the competition relaxes because both operators include a in their prices. The entrant does it fully and forced and the incumbent partially and at expenses of the entrant. The component in the case of *IxC*, iP, doesn't have influence in the prices (but the prices has influence in iP), thus**, it is eliminated a strategic component in the incumbent's behavior** (the *IxC* partly tries to obtain less dependency of the newcomer to the incumbent) and simultaneously a menu of options is offer to the entrant. In any case, the final decision taken by the entrant will depend of the present and previous prices. For this operator, the benefit function[32] is, for i=1,2,3...

$$p_2 = \begin{cases} w_2 p_2 - iP + m_2(m_2 - f_2) & \text{if} \quad w_2 \leq id_2, \\ id_2 p_2 + (w_2 - id_2)(p_2 - a) - iP + m_2(m_2 - f_2) & \text{if} \quad w_2 > id_2. \end{cases} \tag{6}$$

He maximizes this function choosing $\{i, p_2, m_2\}$. As it is observed, the amount demanded and the amount contracted are two different variables. If the amount demanded to the entrant is smaller to his capacity of total management, he will have excess capacity. On the contrary, if the amount demanded is bigger that this capacity, he will require extra units of interconnection to satisfy it. The benefit functions are summarized in the following table

If	p_1	p_2
IxT	$w_1 p_1 + w_2 a + m_1(m_1 - f_1) - F_1$	$w_2(p_2 - a) + m_2(m_2 - f_2)$
IxC and $w_2$² id_2	$w_1 p_1 + iP + (w_2 - id_2)a + m_1(m_1 - f_1) - F_1$	$w_2 p_2 - iP + m_2(m_2 - f_2)$
IxC and $w_2 > id_2$	$w_1 p_1 + iP + (w_2 - id_2)a + m_1(m_1 - f_1) - F_1$	$id_2 p_2 + (w_2 - id_2)(p_2 - a) - iP + m_2(m_2 - f_2)$

Table 2: The operator's problem.

The entrant chooses between *IxT* and *IxC*. That w_2£ id_2 or $w_2 > id_2$ occurs is a consequence of the competition process.

[32] Notice that this function can be simplified to $p_2 = w_2 p_2 - iP + m_2(m_2 - f_2) - \max\{0, a(w_2 - id_2)\}$. Nonetheless, we don't use this short version because is less illustrative.

3. Tariff schemes and equilibriums

Three different tariff schemes are contemplated[33]: linear tariffs, plain (flat) tariffs, and two-part tariffs (per-unit price and subscription fee). Evidently, in the case of linear tariffs, $m_1=m_2=0$ and in the case of flat tariffs, $p_1=p_2=0$[34]. Before continuing, we show the particular expressions that we have used to obtain the equilibrium values. Let us consider, $u(x)=Ax-\frac{Bx^2}{2g} \ \ \forall x\in\left[0,\ \frac{gA}{B}\right]$ where A, B >0; a quadratic function that satisfy section 2.2. A demand of the type $x=\frac{g(A-p)}{B}$ is derived. Therefore, the indirect utility function is $v(p)=h-m+\frac{g(A-p)^2}{2B}$. g is an exogenous demand shock, inactive until now.

The parameters have been arbitrarily fixed so that an operator with f >.08 (and sufficient amount demanded) is able to obtain added benefits when contracting by capacity, i.e., $d_2\big|_{f_2=.08}=K$. Particularly we take Z= 186,402=7.12r and Y= 37.28 (see Appendix 3). Next, we review the equilibrium results for the different tariff schemes[35].

3.1. Linear tariffs (per-minute price) only

IxT. The equilibrium prices are[36]

$$p_1^*=\frac{Bx_1\left(sf_1+ax_2\right)}{sf_1+Bx_1^2}, \tag{7a}$$

$$p_2^*=a+\frac{Bx_2sf_2}{sf_2+Bx_2^2}, \tag{7b}$$

IxC. The equilibrium prices are

$$p_1^*=\frac{Bx_1sf_1}{sf_1+Bx_1^2}, \tag{8a}$$

[33] Nevertheless, strictly speaking, despite the scheme structure, there is only a single type of tariff evaluated in the extreme cases. It is necessary to indicate that these tariff schemes are not exactly equivalent to a payment by amount of transmitted data (in contrast to the habitual prices per time). Moreover we did not take into account other price elements (e.g. an initial charge per call) other way to see it is to consider it uniformly distributed in this one. Then, is irrelevant the relation of units of measurement with the call duration. If these elements were separately considered, the model would increase in complexity (in principle, it would be necessary to introduce a probability distribution for the duration of the calls).

[34] A flat tariffs is a unique payment per period, independent (at least in a direct way) of the amount demanded by consumer. Nonetheless, it is necessary to take into account, that the fact that p_2=0, doesn't imply that p_2 is independent of d_2.

[35] Although losing generality, in order to gain clarity, not only we will assume $f_1=f_2=0$ and g=1 in the simulations, but in addition, from now on, we will omit them in the equations.

[36] It is necessary to notice that, in the next expressions, the equilibrium prices are not result of an exhaustive clearance. Thus, those are only streamlined expressions derived of the FOC. Moreover, the expressions to the right of the following equalities are evaluated in the equilibrium prices, nevertheless, trying to gain clarity in the document, we omit the abusive use of "*".

$$p_2^* = \begin{cases} \dfrac{Bx_2 s f_2}{s f_2 + Bx_2^2}, & \text{if} \quad w_2<id_2, \\ a + \dfrac{Bx_2(s f_2 - iaY)}{s f_2 + Bx_2^2}, & \text{if} \quad w_2>id_2. \end{cases} \tag{8b}$$

It is observed that for the same market share under both models, the prices in *IxC* are always smaller to the prices in *IxT*.

IxT. The equilibrium benefits are

$$p_2^* = \frac{nsBf_2^2 x_2^2}{s f_2 + Bx_2^2}, \tag{9}$$

IxC. The equilibrium benefits are[37]

$$p_2^* = \begin{cases} \dfrac{nsBf_2^2 x_2^2}{s f_2 + Bx_2^2} - iaK, & \text{if} \quad w_2<id_2, \\ \dfrac{nsBf_2^2 x_2^2 + iYf_2^2 as}{s f_2 + Bx_2^2} + ia(Y-K), & \text{if} \quad w_2>id_2. \end{cases} \tag{10}$$

3.2. Plain tariffs (subscription fee) only

IxT. The equilibrium prices are

$$m_1^* = s f_1 + ax_2(0), \tag{11a}$$

$$m_2^* = s f_2 + ax_2(0), \tag{11b}$$

IxC. The equilibrium prices are

$$m_1^* = s f_1 + ax_2(0) - iaY, \tag{12a}$$

$$m_2^* = \begin{cases} s f_2, & \text{if} \quad w_2<id_2, \\ s f_2 + ax_2(0) - iaY, & \text{if} \quad w_2>id_2. \end{cases} \tag{12b}$$

It is observed that for the same market share under both models, the prices in *IxC* are always smaller to the prices in *IxT*.

IxT. The equilibrium benefits are

[37] The entrant price just in the crossing of the curves is omitted for all the tariffs.

$$p_2^* = nsf_2^2, \quad (13)$$

IxC. The equilibrium benefits are

$$p_2^* = \begin{cases} nsf_2^2 - iaK, & \text{if} \quad w_2 < id_2, \\ nsf_2^2 + ia(Y-K), & \text{if} \quad w_2 > id_2. \end{cases} \quad (14)$$

3.3. Two-part tariffs

IxT. The equilibrium prices are

$$p_1^* = 0, \quad (15a)$$

$$p_2^* = a, \quad (15b)$$

$$m_1^* = sf_1 + ax_2(a), \quad (15c)$$

$$m_2^* = sf_2, \quad (15d)$$

IxC. The equilibrium prices are

$$p_1^* = 0, \quad (16a)$$

$$p_2^* = \begin{cases} 0, & \text{if} \quad w_2 < id_2, \\ a, & \text{if} \quad w_2 > id_2. \end{cases} \quad (16b)$$

$$m_1^* = sf_1 + ax_2(a) - iaY, \quad (16c)$$

$$m_2^* = \begin{cases} sf_2, & \text{if} \quad w_2 < id_2, \\ sf_2 - iaY, & \text{if} \quad w_2 > id_2. \end{cases} \quad (16d)$$

It is observed that for the same market share under both models, the prices in *IxC* are always smaller to the prices in *IxT*.

IxT. The equilibrium benefits are

$$p_2^* = nsf_2^2, \quad (17)$$

IxC. The equilibrium benefits are

$$p_2^* = \begin{cases} nsf_2^2 - iaK, & \text{if} \quad w_2 < id_2, \quad (18a) \\ nsf_2^2 + ia(Y-K), & \text{if} \quad w_2 > id_2. \quad (18b) \end{cases}$$

It can be seen that under *IxC,* the equilibrium prices are smaller whatever the type of tariff (which is fulfilled for an ample rank of values of the parameters); nevertheless, this doesn't give much light about if these will be finally the market prices.

Next, some results related to the different types of tariffs are comment. In the case of linear tariffs, the equilibrium benefits under *IxC* generate a plus ($\frac{iYf_2^2 a s}{s f_2 + Bx_2^2}$, when comparing with *IxT*) reason why it could be easier that it becomes the best entrant's option.

This doesn't happen in two-part tariffs. The operator prefers to be aggressive and choose a price that generates at least a demand equal to his capacity because he knows he can buy in per-time basis the differential. Nevertheless, he will not be "too" aggressive as to incur in a situation in which it would fail to take advantage of a certain percentage of his contracted capacity. With two-part tariffs, the entrant will require more incentives to choose *IxC* because in equilibrium everything depends on the market share gain. If this is not sufficient in *IxC* (as finally it happens under two-part tariffs), he doesn't choose this model even though he would be able at any moment to manage more minutes than those initially proposed by the regulator (or that the regulator decrease the price of the capacity unit). This because in any case the fixed cost compensates the possible income increase (the previous commentaries are valid for changes inferiors to 10%). Then, starting in the typical *IxT* model the entrant benefices by the possibility of using non-linear tariffs (although in damage not only of the incumbent but also of the consumers). Nevertheless, under two-part tariffs, the *IxC* hardly could represent an option, because the market share variations when changing from *IxT* to *IxC* are less significant under this type of tariff.

With two-parts tariffs, the entrant fixes a retail price equal to his marginal cost (real and perceived). Conversely, the incumbent, due to his cost structure, chooses a price equal to zero (plain tariffs). It is evident that the latter would prefer to move to a situation of flat tariffs. This would avoid that the entrant recovered the variable cost (retail price); at the same time, the incumbent would be able to increase his subscription fee. If the market share was the same in both tariffs structures, the entrant is indifferent between the application of flat tariffs and the application of a two-part tariffs (he obtains equal benefits). Nonetheless, as it were seen, incumbent would be wishing a policy of flat tariffs. It is clear also that, if the market share was the same in both tariffs, both operators prefer non-linear tariffs.

4. Welfare implications

Evidently, the use of different tariff schemes has implications in the welfare. The welfare measures used in this document are defined in the following way

$$CS = n\left(v_1 f_1 + v_2 f_2 - \frac{(v_1 - v_2)^2}{2s} \right), \tag{19a}$$

$$PS = p_1 + p_2 , \tag{19b}$$

$$W = CS + PS . \tag{19c}$$

The efficiency measurement used to compare the models is the aggregate welfare throughout the periods (t=35)[38]. Next, we show a graph that illustrates the essence under *IxT*.

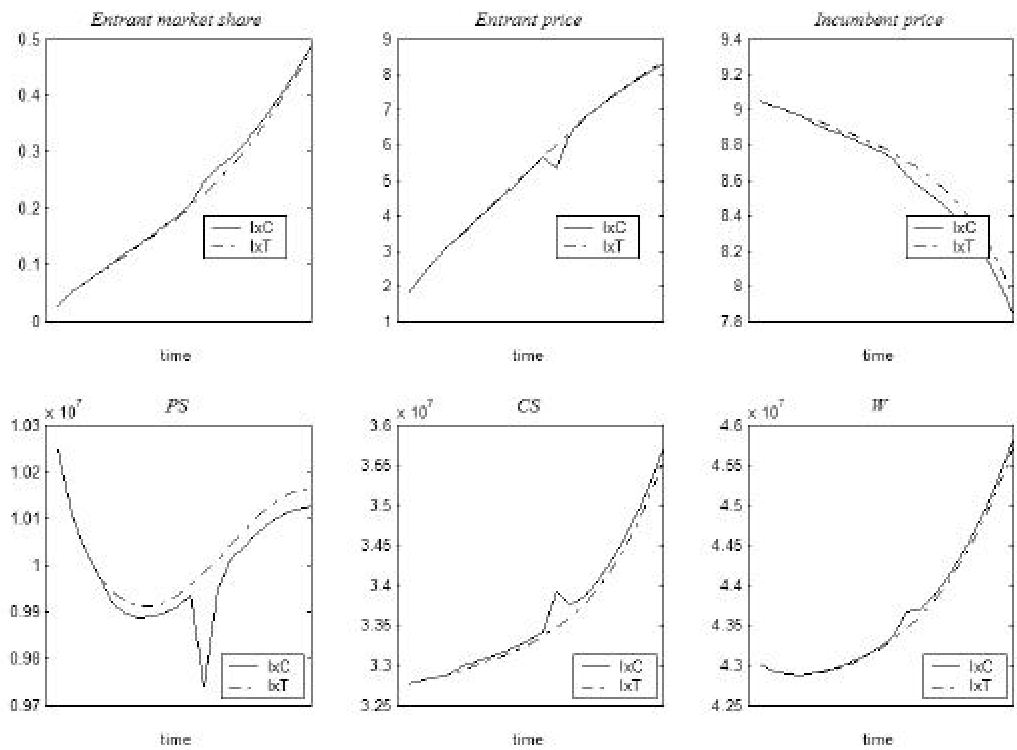

Figure 1: Competition evolution under linear tariffs.

Using the parameters shown in *Appendix 5*, starting in *IxT* and comparing whit the *IxT-IxC* option we obtain (for linear tariffs) and aggregate decrease of .73% in *PS* but an increase of .53% *CS*. Finally an increase of .24% in *W* is obtained.

5. Concluding remarks

In this paper, we have reviewed a proposal to design the recently introduced interconnection by capacity for the case of fixed networks of telecommunications; the main results are the following:

[38] We consider T=35. With this value, the changes in the equilibrium values from one period to the next are practically null.

The first conclusion emerges in a very natural way: if the newcomers are not able to influence the dimension of their network and if this dimension were fixed and equal to *K*, that is, $d=K$, the *IxT* model dominates the model of *IxC* (see Appendix 4 for a formal demonstration). Therefore, the *IxC* (taking into account the proportionality in the prices that the CMT has decided) doesn't offer any added advantage to the entrant but can bring to them the added risk to have to plan the necessities of interconnection in a precise way. The previous remark is valid independently of the entrant's size (market share).

The second conclusion is that: the entrants will have to reach a critical mass before they will be interested in the *IxC* model. If their market share is sufficiently small, choosing a contract of *IxC* could generate a very aggressive reaction in the incumbent that would not allow the entrant to obtain (and even lose) market share. This, independently of the commitments the entrant could incur due to the payment of the fixed cost. Therefore, it is not so evident that the *IxC* is the way in which the small entrants (in the initial phase of entrance) can begin to grow in the market.

Designing the network dimension as it has been done (direct function of the market share) and taking into account the values of the used parameters, the entrant prefers to be aggressive and choose the *IxT* model but only when linear tariffs apply. With two-part tariffs, the entrant will require more incentives to choose *IxC* because in equilibrium everything depends on the market share gain. If this is not sufficient in *IxC* (as finally it happens under two-part tariffs) he doesn't choose this model even though he would be able at any moment to manage more minutes than those initially proposed by the regulator (or that the regulator decrease the price of the capacity unit), because in not case the fixed cost compensates the possible income increase (the previous commentaries are valid for changes inferiors to 10%). Then, under two-part tariffs, the *IxC* hardly could represent an option, because the market share variations when changing from *IxT* to *IxC* are less significant under this type of tariff.

Hence, welfare gains are obtained under *IxC*, although these ones are dissipated through time.

It is necessary to remember that much of the results depend on the particular form in which the competition model has been designed.

Appendix 1

In agreement with the form in which the entrant's benefits have been defined under *IxC* is reciprocal that the incumbent's benefits correspond with the following function[39].

$$p_1 = \begin{cases} w_1 p_1 + iP + m_1(m_1 - f_1) - F_1 & \text{if} \quad w_2 \le id_2, \\ w_1 p_1 + iP + m_1(m_1 - f_1) + (w_2 - id_2)a - F_1 & \text{if} \quad w_2 > id_2. \end{cases}$$

Where he only chooses a set of prices and accept de *i* chosen by the entrant. Nevertheless, it generates a significant problem when finding equilibriums. Let us start analyzing the which's and the why's about the active benefits curves for each operator.

Given a **q** and an incumbent's price, the entrant's benefits function for the relevant values of his price draws a continuous (not differentiable in a point) curve in form of "v" inverted, that is, the active functions are the interiors ones (see Appendix 2) so, the maximum tends to the threshold. Deepening a little more, we could say than the entrant doesn't have excessive interest in raising his price because that would means not to optimize the investment incurred *ex ante*. But at the same time he doesn't have too much interest in diminishing his price either, because although small reductions could help him to improve the distribution of his traffic, an important reduction would imply to exceed the invested capacity and to have to buy on a per-time basis the non-covered interconnection, which, in cost terms, doesn't represent any added benefit to him.

Instead, given a **q** and an entrant price, the incumbent's benefits function for the relevant values of his price, draws a continuous (not differentiable in a point) curve in form of "m", that is, the active functions are the exteriors ones. The incumbent faces a trade off. On the one hand, is good for him to increase his price and, of this form, to transfer more income to *IxT*, but on the other, he has an incentive to lower it, and thus, increase his own amount demanded, making impossible for the entrant to recover the investment he has incurred as an added result. In other words, the incumbent could have incentives to make fail the entrant in his capacity estimations by defect as much as by excess.

In most of the cases this function will have a clear global maximum, nevertheless, for certain price combinations, infinitesimal changes in the entrant's price can cause that the global maximum jump of one "hump" to the other, i.e., a discontinuity occurs in the incumbent's reaction function.

Considering the difficulty of economically justify a function of this type[40], aside from the mentioned complications finding equilibrium values (since generally the optimization code finds

[39] Notice that this function can be simplified to $p_1 = w_1 p_1 + iP + m_1(m_1 - f_1) - F_1 + \max\{0, a(w_2 - id_2)\}$.

[40] Although a intuitively justification exist to this jump in the reaction function. When the entrant reach a size such that in a certain period the decision to choose or not an extra unit of capacity is relevant, if the entrant is very aggressive,

local maximums) we have decided to work with only one part (of two) of the function to describe the incumbent's behavior. Next, we comment the pros and cons of each possibility as well as the final decision.

Doing $p_1 = w_1 p_1 + iP + m_1(m_1 - f_1)$ implies that the incumbent doesn't receive extra income by the units that the entrant decides to buy on a per-time basis. Then, it would be necessary to justify that the entrant is able to contract interconnection coming from another supplier different of the incumbent (which contradict the model specifications in which the incumbent is the only source of interconnection in the market) as well as justify why the incumbent cannot sell interconnection in these conditions. Finally, when modeling of this form, the incumbent's equilibrium price turns out to be so much aggressive because it doesn't include the variable component. In these conditions, occurs a hard competition to conquer the market and produces that the *IxC* model could be a non-attractive option to the entrant unless the regulator fixed *K* sufficiently below to his real possibilities, this means $a >> \frac{P}{K}$[41].

Doing $p_1 = w_1 p_1 + iP + m_1(m_1 - f_1) + (w_2 - id_2)a$ implies that the income referred to the units bought under the *IxT* model is taken into account, but this same factor will represent a negative component of the incumbent's benefits when $w_2 < id_2$. Since to be too aggressive would generate just this situation it seems reasonable to see this negative element as a kind of fine that the regulatory authority imposes to him to avoid disloyal competition. It would work as follows: if the entrant has invested sufficiently to obtain a capacity management level and he really has obtained it (under the assumption that the entrant doesn't speculate) the fact that he finally doesn't obtain the corresponding amount demanded would be adjudged to the opportunistic behavior of the incumbent. Anyway, the newcomer doesn't have interest to cause a situation of this type either, because important investments that must be amortized are in game. Instead of the one in the previous paragraph, this alternative generates sensibly superior incumbent's prices.

Evaluating the advantages and disadvantages of each of the previous options it has been chosen to work with the last one. In any case, the decision is not trivial.

the incumbent chooses high price to cause a greater demand under the *IxT* regime. In the opposite case, the incumbent is not requested by added interconnection then, he puts all his efforts in maintaining the market share and in addition he causes an entrant excess capacity.

[41] Look that is not enough that $d > K$ because if $a = \frac{P}{K}$ remains also the *IxT* is affected by the cut of prices.

Appendix 2

The following is valid whatever the type of tariff.

In this appendix, we show (in separate graphs mode, for a particular **q** and a p_1) the strategic behavior intrinsic to the definition of the entrant's benefit curves under *IxC*, for several different situations. The following cases are compared: a) when he only could manage *K* minutes ($d_2=K$), and, b) when he is able to determine his network dimension according to $d_2=Z+Ynf_2$.

We suppose that p_2' satisfy $w_2=id_2$ when $d_2=K$, similarly, p_2'' satisfy $w_2=id_2$ when $d_2=Z+Ynf_2$. Prices superior to p_2' and p_2'', in each case, imply that the active benefits function will be that defined for w_2£ id_2. Also, prices inferior to p_2' and p_2'', imply that the active benefits function will be that defined for $w_2>id_2$. The price that finally maximizes the entrant's benefits (and therefore the one he chooses) will be p_2^* (review the graphics).

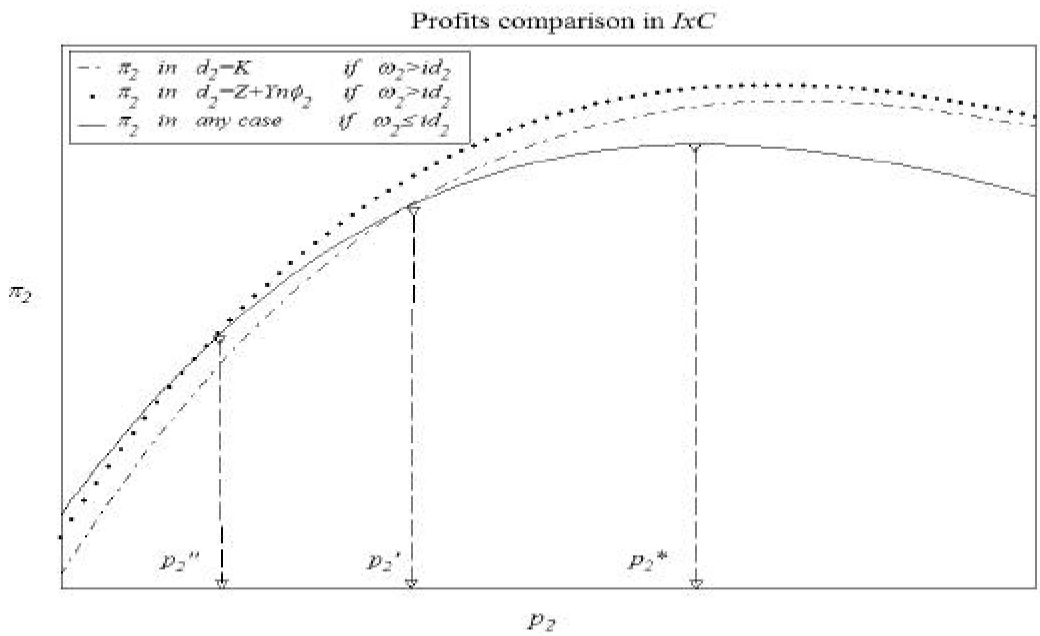

Figure 2: Passive condition (I). Direct and indirect effects.

As it is show in Figure 2, if the operator is sufficiently small (this is the cause according to the model but actually the reasons could be very diverse) could occur that the possibility of internalize the network dimension doesn't have any effect in the entrant's decision. The operator maximizes his benefits at p_2^*, that is, his capacity of management is superior to the amount demanded to him[42], i.e., w_2£ id_2. An aggressive behavior (in prices) doesn't represent any added benefit to him. The entrant will have excess capacity.

[42] This happens because the entrant has not reached a critical mass. Then the cause is that w_2 is small and not that d_2 is great.

In Figure 3, the operator chooses p_2^* so that, at the end, his capacity of management is superior to the amount demanded to him ($w_2 < id_2$). However, if the operator had had a fixed (and equal to K) dimension would have selected p_2' (where $p_2' > p_2^*$ and $p_2' < p_2^*$). It is necessary to indicate that the entrant doesn't take full advantage of the opportunity that the dimension provides to him. Anyway, the possibility of determining the network's size urges him (partially) to a more aggressive price competition. He will have excess capacity.

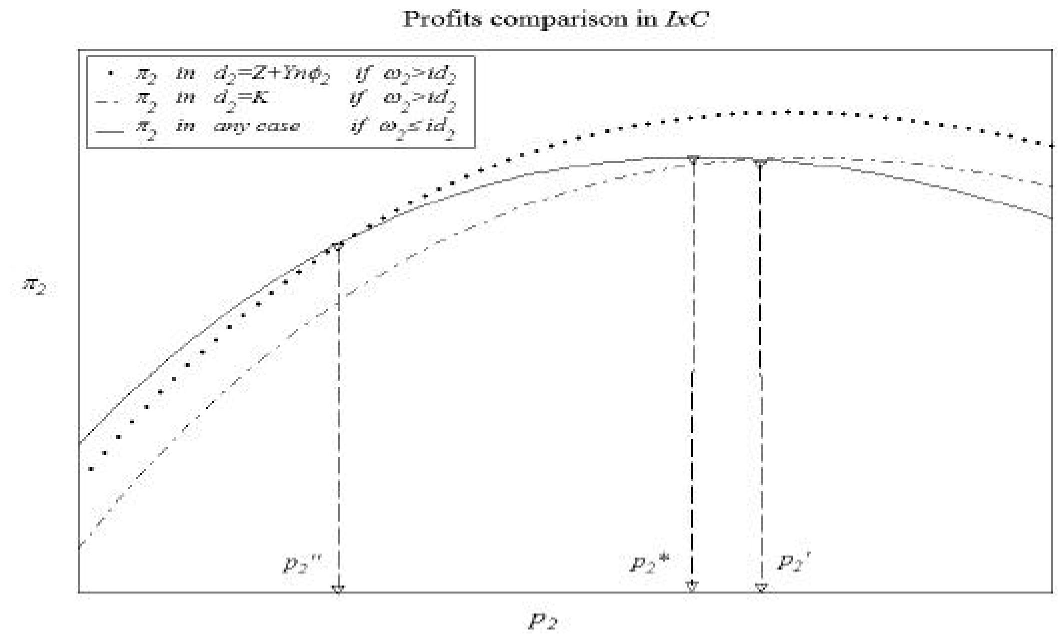

Figure 3: Active condition (partially). Direct effect.

As Figure 4 shown, the operator increases his income using the maximum of his network's size possibilities[43] (he has incentives to increase the variety of his services supply). Besides, the consumers enjoy the decrease in p_2 that change from p_2' to p_2''. It is interesting to observe that the threshold agrees with p_2^*. The entrant equals his capacity to his demand.

Figure 5 represent the situation in which to use the dimension network at the maximum level is the entrant's best choice. Nevertheless, it is not clear if this means an increase or a decrease in p_2 with respect to the situation where $d_2=K$, that will depend on the relation between the maximums (in this case implies a decrease). As it is observed, the thresholds are irrelevant in the determination of p_2^*. Anyway, in Figure 5 the equilibrium price implies that the operator will require added interconnection.

[43] Is obvious that the new situation must be better than that, that he leaves ($p_2' < p_2''$). Otherwise, he did not do the change (since complete information exists). Then, the network dimension give to the entrant a maximum level but, if he believe that is not recommendable exploit it, could fix smaller values (being more passive as far as its supply of services for example). According to the values of the parameters, this is not the case.

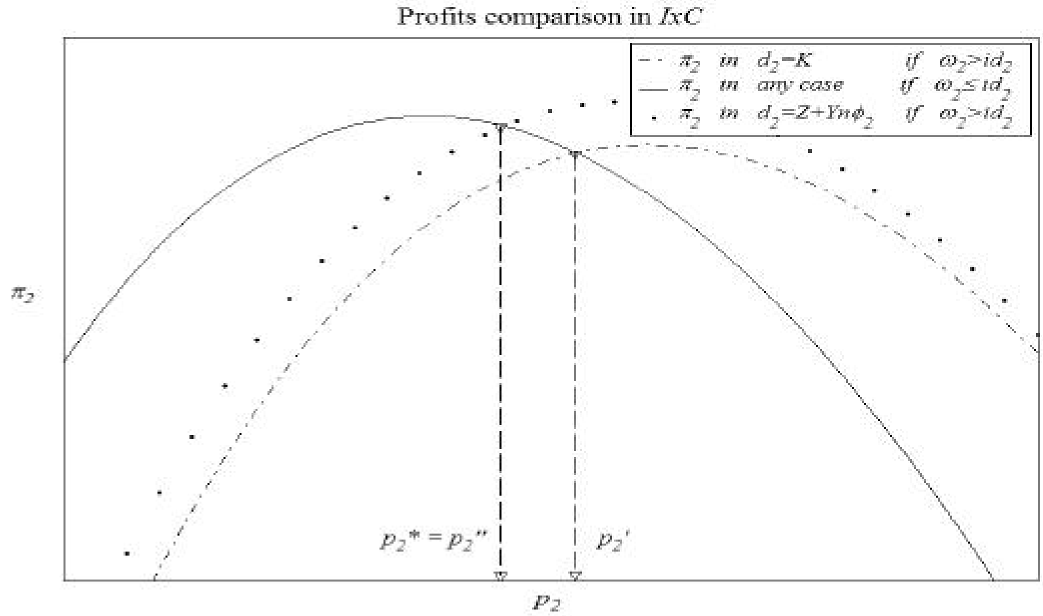

Figure 4: Active condition (totally). Direct effect.

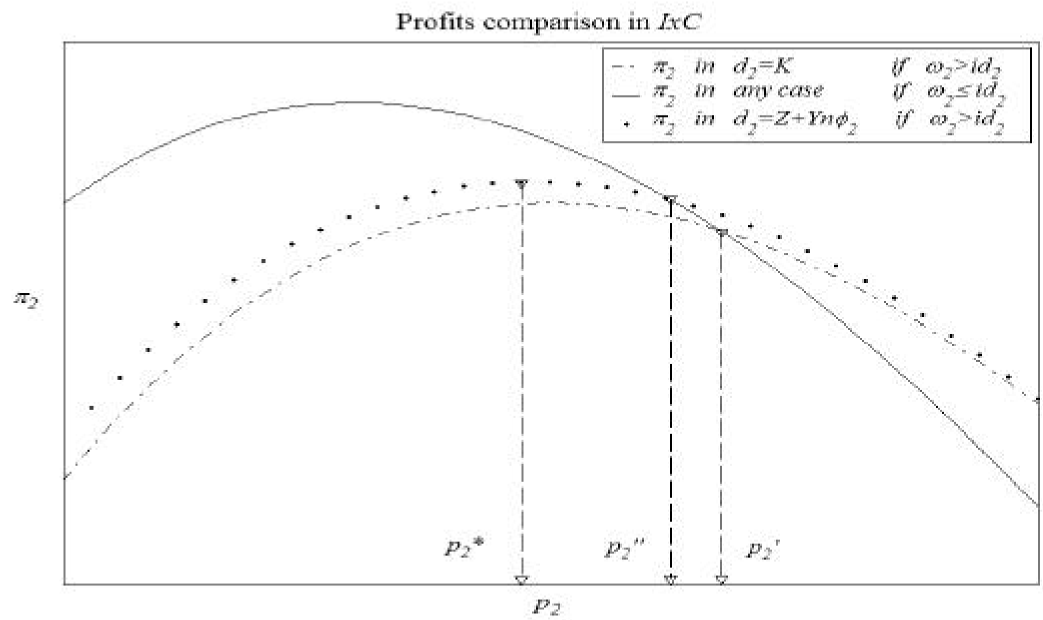

Figure 5: Passive condition (II). Direct effect.

The previous figures only include the direct effects generated by the possibility of network dimension[44]. To completely understand the competition process is necessary to take into account the strategic effects triggered by the rival (indirect effects) as illustrated in Figure 6 (that is the same situation that it is show in Figure 4 but including all the effects) and Figure 7.

[44] Although changing of a situation "a" to a situation "b" (see page 21) implies a transformation in the entrant benefit functions and therefore in his best respond function, can occur the case that the changes in the reaction occur in values sufficiently remote to the equilibrium (then, remaining constant). This is the case shown in Figure 2.

It is useful to suppose that we started in an equilibrium situation under $d_2=K$. But now the entrant is able to determine his network dimension in such a way that his benefit functions ends up moving, evidently, also does his reaction function, changing from $R_2'(p_1)$ to $R_2^o(p_1)$ (note that the reaction functions, Figure 7, correspond to those of strategic substitutes operators). In this new situation the entrant fix p_2^o, where $p_2'' < p_2^o < p_2'$, with which he increase his benefits. At this price, the incumbent's best response is p_1^o that fulfills $p_1^o < p_1'$.

The increase in the aggressiveness will cause a general decrease in the entrant's benefits function when $w_2 > id_2$ (for an ample parameter rank). This is because he doesn't obtain the awaited market share but also because he has incurred *ex ante* in a fixed investment, reason why he is himself forced to make another marginal reduction in p_2 ("to recover part of the pie") to which the incumbent responds with a marginal decrease in p_1. In a final stage (the process finishes when the curves are totally adjusted, i.e., when no one operator wish to change his decisions given the decision of the rival) the entrant fix his price at $p_2^*=p_2''$ and the incumbent responds with $p_1^*=p_1''$.[45]

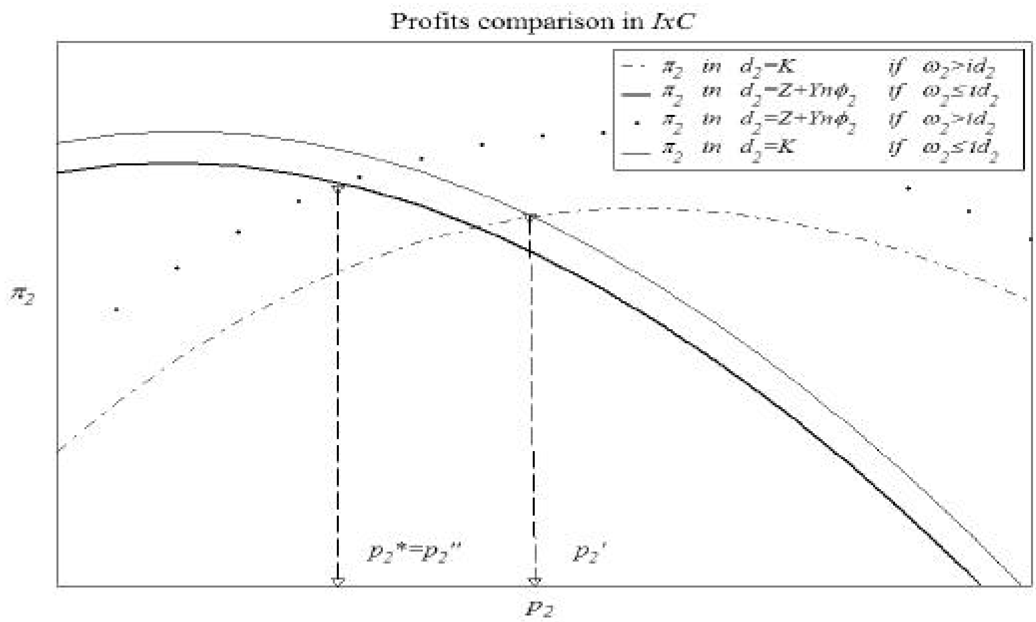

Figure 6: Active condition (totally). Direct and indirect effects.

45 In order to illustrate the competition process it has been tried to separate the effects in time and in its influence. Obviously, that happens simultaneously and affects the aggregate of functions.

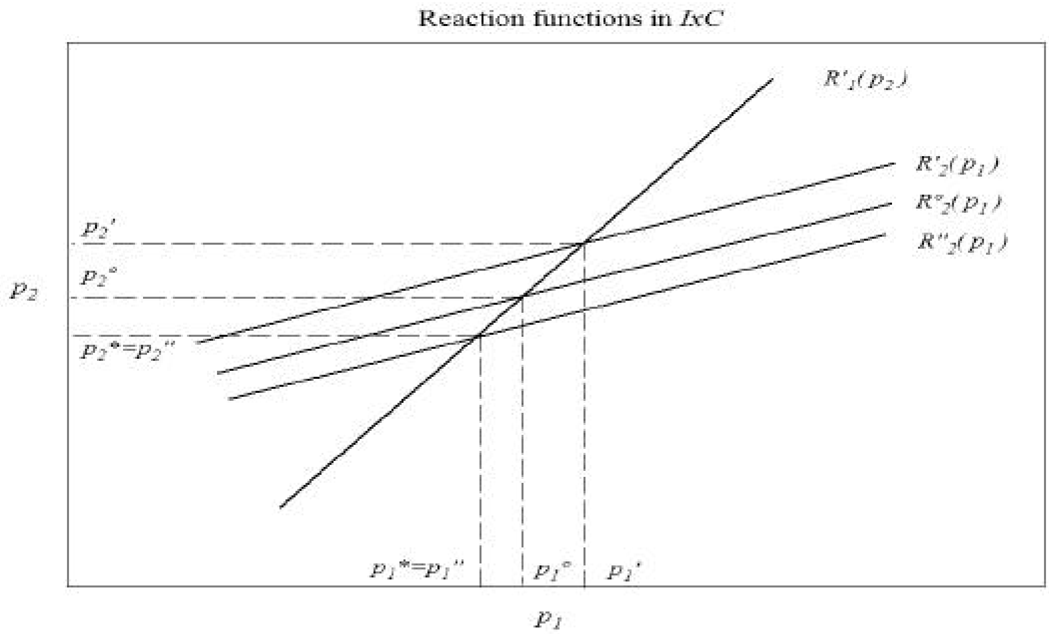

Figure 7: The competition process.

Up to here, have been reviewed the problem related with how is chosen the competition price since the operator has acquired a certain level of capacity. A different problem is the way in which the entrant chooses a particular model.

Appendix 3

In Table 3, is showing an example[46] of the way in which the code works when choosing the entrant's best strategy. A "*Yes*" implies that the number of acquired blocks of capacity satisfies the amount demanded and a *"No"*, the opposite one.

#	*Situation*	*IxT*	*IxC*			
			1 block	*2 blocks*	*3 blocks*	*4 blocks*
1	$d_2<\times_2$	-	*No*	-	-	-
2	$\times_2{}^2d_2$	-	*Yes*	-	-	-
3	$2d_2<\times_2$	-	-	*No*	-	-
4	$\times_2{}^2 2d_2$	-	-	*Yes*	-	-
5	$3d_2<\times_2$	-	-	-	*No*	-
6	$\times_2{}^2 3d_2$	-	-	-	*Yes*	-
7	$4d_2<\times_2$	-	-	-	-	*No*
8	$\times_2{}^2 4d_2$	-	-	-	-	*Yes*

Table 3: An example of the code operation.

In the "situations" preparation has been taken into account that if $d_2(j_2) = Z + Ynf_2$, and if the entrant's market share fluctuates according to $0 \le j_2 \le .6$, then the network dimension fluctuates according to $Z \le d_2 \le Z + .6nY$.

	Technical (maximum)	**Observed by the regulator (CMT)**	**Used in the paper**
d	$[r,\ \ 24r]$ *	$[5r,\ \ 10r]$	$[7.12r,\ \ 10.68r]$
r/d	[1, .042]	[.2, .1]	[.1404, .0936]

Table 4: Dimension network limits

*We supposed the entrants always have at least technically saturated one most congested hour.

The viable technical rank is very ample and is not observed in reality. In fact the CMT has indicated that the operators who handle business voice or residential Internet have approximately *r/d*=.18 whereas more generalist operators can even reach an *r/d*=.10. The incumbent can even be below this last value. The parameters *Z* and *Y* have been determined taking into account $f_2 \in$ [0, .6] and *r/d*=[.2, .1] as the relevant ranges. Nevertheless as a result we obtain *K* is very near to the superior limit of the rank, so it would be necessary that the entrant gained a significant market share to reach it, so we arbitrarily has settle *r/d*= [.1404, .0936].

[46] It only shows the relevant information according to the used values of the parameters.

Appendix 4

Next, we show how, under certain circumstances, "to choose *IxT*" dominates "to choose *IxC*". Some of these conditions have been satisfied voluntarily by the regulation. Then, it is evident that the authority trusts in other kind of "solutions" in order to promote the *IxC*. Particularly we will prove the following proposition:

> *Independently of the tariff scheme used, taking the incumbent's prices as given, in the presence of both interconnection models, with interconnections prices proportionally equivalents, and without the possibility of modifying the productivity of the contracted capacity ($d_2=K$), the decision "to choose IxT" dominates the decision " to choose IxC".*

Given the incumbent's prices we can rewrite equations 4 and 6 as

$$p_2^T(p_2,m_2)=w_2(p_2-a)+m_2(m_2-f_2),$$

$$p_2^C(p_2,m_2/w_2>iK)=iKp_2+(w_2-id_2)(p_2-a)-iP+m_2(m_2-f_2),$$

where exceptionally superscripts *T* and *C* have been used to distinguish between *IxT* and *IxC* respectively. Expanding the second of those expressions, grouping and remembering that $a=\frac{P}{K}$, is easy to arrive at

$$p_2^C(p_2,m_2/w_2>iK)=w_2(p_2-a)+m_2(m_2-f_2),$$

thus,

$$p_2^T(p_2,m_2)\equiv p_2^C(p_2,m_2/w_2>iK).$$

That means that the entrant's benefits when choosing *IxT* are the same ones he obtains when choosing *IxC* given $w_2>id_2$. Then, if the option "to choose *IxC*" were attractive to him, that would have to happen when $w_2 \leq id_2$, receiving (according to equation 6)

$$p_2^C(p_2,m_2/w_2\leq iK)=w_2p_2-iP+m_2(m_2-f_2),$$

but using $a=\frac{P}{K}$ the previous expression can be rewrite it as

$$p_2^C(p_2,m_2/w_2\leq iK)=w_2p_2-iaK+m_2(m_2-f_2),$$

that is to say, he has expenses of iaK, nevertheless, when contracting *IxT* the expenses are aw_2 (assume in both cases that $f_2=0$). Taking into account that this part of the analysis is valid when $w_2 \leq iK$, there is no doubt that the expenses when contracting capacity are greater than those when contracting time, and only in the margin (when $w_2=iK$) is equivalent. In other words, adding earlier results

$$p_2^T \equiv p_2^C\left(p_2, m_2 / w_2 > iK\right)$$

$$p_2^T \geq p_2^C\left(p_2, m_2 / w_2 \leq iK\right)$$

This result is independent of the way in which is defined the incumbent's benefits function. (See Appendix 1).

Appendix 5

Simulations have been running in *Matlab 6*. The code has mainly used optimization and numerical methods algorithms. Table 5 contains a list of the values of the used parameters.

USED VALUES OF THE PARAMETERS		
Cost parameters	**Utility parameters**	**Network dimension parameters**
f_1=0	$h_1^t = 5,000$	*r*=26,171
f_2=0	h_2^t *	*K*=201,314
F_1=2'206,460	**Demand parameters****	*Z*= 186,402=7.12*r*
Interconnection price parameters	*A*=20	*Y*= 37.28
Œ= 1.096	*B*=.039	*d*=[186,401.85, 279,602.78]
P=220,646	•=1	*r/d*=[.1404, .0936]
Number of periods	**Competitiveness level**	**Market parameters**
T=35	s =12,000	$j_1^0 = 1$
		$j_2^0 = 0$
		n=5,000

Table 5: Parameters.

*$h_2^t = \frac{h_1^t}{2} + \frac{h_1^t}{2} \min\left\{\left(\frac{(t-1), k}{k}\right)\right\}$ where *k* is the number of periods needed by the entrant to build a "track record" of quality. We arbitrarily fix *k*=16.

**The elasticity of demand $(e = -\frac{p}{A-p})$ is less than one for the equilibrium per-minute-price, whatever the tariff scheme used.

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