CORE

# ELECTRONIC COMMERCE, CONSUMER SEARCH 

# AND RETAILING COST REDUCTION 

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#### Abstract

This paper explains four things in a unified way. First, how e-commerce can generate price equilibria, where physical shops either compete with virtual shops for consumers with Internet access, or alternatively, sell only to consumers with no Internet access. Second, how these price equilibria might involve price dispersion on-line. Third, why prices may be higher on-line. Fourth, why established firms can, but need not, be more reluctant than newly created firms to adopt ecommerce. For this purpose we develop a model where e-commerce reduces consumers' search costs, involves trade-offs for consumers, and reduces retailing costs.


## RESUMEN

Este trabajo explica, de manera unificada, cuatro cosas. Primero, cómo el comercio electrónico puede generar equilibrios de precios en los que las tiendas físicas compiten con las tiendas virtuales por los consumidores con acceso a Internet, o, alternativamente, venden sólo a los clientes que no tienen acceso a Internet. Segundo, cómo estos equilibrios de precios pueden generar dispersión de precios en Internet. Tercero, por qué los precios de las tiendas virtuales pueden ser más altos que los de las tiendas físicas. Y cuarto, por qué las empresas establecidas pueden tener menos incentivos que las empresas de nuevas creación para abrir tiendas virtuales. Para ello, desarrollamos un modelo en el que el comercio electrónic o reduce los costes de búsqueda de los consumidores, implica un trade-off para los consumidores, y disminuye los costes de producción de las empresas.

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JEL Classification: D43, D83, L13, L81, M31, O31, O33

[^0]
## 1 Introduction

The Internet allowed the creation of a new retailing technology: electronic commerce (e-commerce) ${ }^{2}$. Ecommerce has similarities with catalogue retailing. Without a physical shop, it offers products that cannot be physically inspected or immediately delivered, and are paid for usually with a credit card. But e-commerce also has unique attributes. The Internet allows to cheaply store, search, and disseminate information; is available anywhere, anytime, for anyone who can accede to it; allows interactivity; provides perceptual experiences superior to those of a catalogue, but inferior to those of physical inspection; and serves as a transactions and physical distribution medium for information goods ${ }^{3}$. Due to this last aspect, conceivably, it will be in markets for information goods that ecommerce will have a bigger impact, as the recent evolution of markets for stocks, mortgages, or life insurance suggests (Brown \& Goolsbee (2000)).

Four facts have emerged from ecommerce's short history. First, typically newly created, purely virtual firms, adopted e-commerce before established firms. In the book retail industry, new firms like Amazon adopted ecommerce before old firms like Barnes \& Noble, and in the stockbroking retail industry, new firms like E*trade also adopted e-commerce before old firms like Charles Schwab. Did this reflect an intrinsic reluctance of established firms to adopt e-commerce? If not, what did it depend on? Second, sometimes physical shops reacted to e commerce by lowering their prices, to compete with virtual shops for consumers that buy on-line; other times, they did not lower their prices, and concentrated on selling only to consumers that do not buy on-line. Charles Schwab lowered the off-line fee from $\$ 65$ to $\$ 30$ to match the on-line fee (New York Times, August 16, 1999), which suggests the first price regime. In 1999, Barnes \& Noble and Borders matched within hours an Amazon 50\% discount on best sellers on their virtual, but not on their physical shops (The New York Times, May 18, 1999). This suggests that established firms compete with new firms for consumers with Internet access on their virtual shops, but not on their physical shops, i.e., the second price regime. What explains these different price regimes? Third, there is price dispersion on-line. Brynjolfsson \& Smith (1999) find that established firms' virtual shops charge $8.7 \%$ more

[^1]than new firms' virtual shops, and that on-line the price range is $33 \%$ of the average price ${ }^{4}$. What explains price dispersion on-line, if supposedly, the Web gives consumers access to perfect information? Fourth, prices are lower on-line. Brynjolfsson \& Smith (1999) find that prices average 9-16\% less on virtual shops than on physical shops. Is this price reduction intrinsic to e-commerce? If not, what does it depend of?

We believe that these four facts are related, and develop a static, homogeneous product, partial equilibrium search model, that explains them in a unified way. The model has three important aspects: e-commerce reduces consumers' search costs, involves trade-offs for consumers, and reduces retailing costs. Ecommerce reduces consumers' search costs, because on the Web consumers can visit at a low cost virtual shops and learn prices ${ }^{5}$, or can use shopbots, software agents that automatically search for price information ${ }^{6}$. E-commerce involves trade-offs for consumers, because buying from a virtual shop does not require a shopping trip, but requires waiting for delivery. E-commerce reduces retailing costs, compared to physical shops, because virtual shops allow savings on property $\boldsymbol{c o s t s}$, i.e., leases and acquisition of shop and warehouse space, on labor costs, i.e., personnel to attend shops, and on inventory costs, i.e., inventories for showcasing or immediate delivery ${ }^{7}$.

In our model, firms decide whether to open virtual shops and set prices, and consumers search for prices. There are two consumer types: new consumers have Internet access, old consumers do not, or do not consider using the Internet an option. New consumers canvass prices through the Web, and then decide if they buy from a virtual or a physical shop. There are two firms: the old firm has a physical shop, the new firm does not. Virtual shops have lower marginal production costs than physical shops.

Since search and waiting for delivery are costly, new consumers accept prices above the minimum charged in the market. This gives firms market power.

The virtual shops' pricing behavior is simple. Virtual shops have the lowest cost and charge the lowest price. Thus, they are not constrained by consumer search, and charge their monopoly price.

[^2]The physical shop's pricing behavior depends of whether the old firm has a virtual shop, and on whether the new firm is in the market. Because new consumers have access to lower cost shops, and if waiting for delivery is not too costly, they only accept buying from a physical shop for a lower price than old consumers. When only the new firm opens a virtual shop, if the physical shop charges a lower price acceptable to both consumer types, it earns a lower per consumer profit, if it charges a higher price acceptable only to old consumers, it earns a higher per consumer profit. Thus, the physical shop trades-off volume of sales and per consumer profit; sometimes it chooses to sell to all consumers, and sometimes only to old consumers. When both firms open virtual shops, the old firm faces an additional effect, besides the volume of sales and per consumer profit effects. If its physical shop charges a lower price acceptable to both consumer types, half of the new consumers it sells to would otherwise buy from the old firm's virtual shop, where per consumer profit is higher. This causes the old firm to have its physical shop charge a lower price to attract new consumers, only if the virtual shops' cost reduction is small; otherwise it prefers to sell to new consumers only from its virtual shop. We argue that these price equilibria are different from others in search theory, where firms face consumers with different reservation prices.

If the new and old firms' virtual shops have different costs, there will be price dispersion on-line
Since information goods are more convenient to buy on-line, physical shops must charge lower prices than virtual shops to be able to sell them to new consumers.

The firms' incentives to open virtual shops depend on the virtual shops' cost reduction, and the new consumers' reservation price. If cost reduction is small, the new firm has more incentives to open a virtual shop; if cost reduction is large, and the new consumers' reservation price is high, this is no longer true. In fact, the old firm can choose to open a virtual shop when the new firm does not.

The model has two novel features. First, it captures some of the consumers' and firms' trade-offs regarding e-commerce. Second, the production and the search cost distributions are endogenous.

Section 2 presents the basic model, where reservation prices are exogenous, and section 3 characterizes its equilibria. Section 4 discusses the firms' incentives to open virtual shops. Section 5 allows the new and old firm to operate the new technology at different costs. Section 6 presents the model with endogenous reservation prices.

Section 7 discusses price equilibria for information goods. Section 8 discusses related literature. Proofs are in the Appendix.

## 2 The Basic Model

In this section we formalize the firms' opening of a virtual shop and pricing decisions, given consumers' reservation prices, as a 2 stage game. Later we will insert this Basic model in a larger game that includes a third stage, where reservation prices are determined.

## (a) The Setting

Consider a retail market for a homogeneous search good that opens for 1 period.
There are 2 alternative retailing technologies ${ }^{8}$ : a New, virtual shop based technology, and an Old, physical shop based technology. A Virtual Shop has a Web site, where consumers can observe prices and buy, and its logistics is based on the Web. A Physical Shop has a physical location, where consumers can observe prices and buy, and its logistics is based on the physical world. A physical shop may have a Web site, but only to post prices ${ }^{9}$. A firm is $\operatorname{Old}$ if it has a physical shop, opened before the game, and New if it does not.

The game has 2 stages. In stage 1 firms choose whether to open virtual shops. In stage 2 firms choose prices. Then consumers buy, delivery takes place, agents receive their payoffs and the market closes.

Subscript $j$ refers to firms and we index a new and an old firm by: $n, o$. Subscripts $t$ refers to shops and we index a new firm's virtual shop, an old firm's virtual shop, and a physical shop by: vn,vo, $p$.

## (b) Consumers

There is a unit measure continuum of risk neutral consumers of 2 types. New consumers, a proportion $\lambda \in(0,1]$, have Internet access; Old consumers do not. At price $p$ a consumer demands $D(p)$, where $D($.$) is a$ differentiable, decreasing, bounded function, with a bounded inverse.

Consumers ignore the prices of individual shops, and can only learn them by visiting the shops. Old consumers visit the physical shop's physical location, and if offered a price no higher than $r$, where $D(r) \equiv 0$, buy

[^3]and receive the product. When there are no virtual shops, new consumers behave similarly. Otherwise new consumers canvass prices through the Web ${ }^{10}$. They have the list of Web sites, obtained, e.g., from a search engine, but do not know to which type of shop the directions correspond ${ }^{11}$. We assume that:
(H.1) Each new consumers picks randomly which Web site to visit, from the set he has not sampled yet.

The new consumers' reservation price for a type $t$ shop is $\rho_{\mathrm{t}}$. When new consumers visit a new (old) firm's virtual shop, if offered a price no higher than $\rho_{\mathrm{vn}}\left(\rho_{\mathrm{vo}}\right)$, they buy, and wait for delivery; when they visit a physical shop's Web site, if offered a price no higher than $\rho_{\mathrm{p}}$, they go to the shop's physical location, buy, and receive the product; otherwise they reject the offer and search again ${ }^{12}$. Visiting a Web site or a physical shop's physical location, and waiting for delivery of the product bought from a virtual shop, involve costs which we will ignore until section 6 .

## (c) Firms

There are 2 risk neutral firms: a new and an old firm. If the new firm decides not to open a virtual shop, it exits the game with a 0 payoff. Opening a virtual shop involves a set-up cost, $K \in(0,+\infty)$. The probability with which firm $j$ opens a virtual shop is $a_{j}$; let $a=\left(\mathrm{a}_{n}, \mathrm{a}_{0}\right)$. At the end of stage $1 a$ is observed by all players. If at least 1 virtual shop opens, the physical shop creates its own Web site, where it posts its price.

Marginal production costs are constant for both shop types. The marginal cost of shop $t$ is $c_{t}$. A virtual shop has a lower marginal cost than a physical shop. Let $c_{p} \in(0, r)$ and $c_{v n}=c_{v o}=c_{v}=c_{p}-\Delta_{c}$, where $c_{p}$ is the common production cost, and $\Delta_{c} \in\left(0, c_{p}\right]$ is the production cost reduction induced by the new technology. All players know $\left(c_{p}, c_{v}\right)$.

The old firm can charge different prices at its 2 shops ${ }^{13}$. Shop $t$ 's price and per consumer profit are $p_{t}$ and $\pi\left(p_{t} ; c_{t}\right):=\left(p_{t}-c_{t}\right) D\left(p_{t}\right)$. Let $\hat{\mathrm{p}}_{\mathrm{t}}:=\operatorname{argmax}{ }_{\mathrm{p}} \pi\left(\mathrm{p} ; \mathrm{c}_{\mathrm{t}}\right)$. Assume that $\pi($.$) is strictly quasi-concave in p$, and that

[^4]even for the maximum cost reduction the physical shop can charge $\hat{p}_{v}$ without losses, i.e., $c_{p}<\hat{p}_{v}$ for $\Delta_{c}=c_{p}$. Shop $t$ 's expected consumer share and expected profit are: $\phi_{t}\left(p_{t}\right)$ and $\Pi\left(p_{t} ; c_{t}\right):=\pi\left(p_{t} ; c_{t}\right) \phi\left(p_{t}\right)$. The new and old firm's net expected profits are: $v^{n}:=\left[\Pi\left(p_{v n} ; c_{v}\right)-k\right]_{n}$ and $v^{0}:=\Pi\left(p_{p} ; c_{p}\right)+\left[\Pi\left(p_{v o} ; c_{v}\right)-k\right] a_{0}$.

A firm's stage 1 strategy, is a rule that for every firm type, says with which probability a firm should open a virtual shop. A firm's stage 2 strategy, is a rule that for each history and shop type, says which price a shop should charge. A firm's payoff is expected profit, net of the investment expenditure.

## (d) Equilibrium

A subgame perfect Nash Equilibrium is: an opening and a pricing rule, for each shop and firm type, $\left\{\left(a_{j}^{*}, p_{t}^{*}\right) j_{j=n, o ; t=v n, v o, p}\right\}$, such that:
(E.1) Given any $\rho_{\mathrm{t}}$ and $a$, firms choose $p_{t}^{*}$ to solve problems: $\max _{p_{p y}} V^{n}$ and $\max _{\left\{\mathrm{p}_{\mathrm{v},}, \mathrm{p}_{\mathrm{p}}\right\}} V^{\circ}$;
(E.2) Given any $\rho_{\mathrm{t}}$, and $p_{t}^{*}$, firms choose $a_{j}^{*}$ to solve problem: $\max _{a_{j}} V^{j}$.

## 3 Equilibrium of the Basic Model

In this section we construct the basic model's equilibrium by working backwards. First, given reservation prices and the profile of opening of virtual shops decisions, we derive the firms' equilibrium prices. Virtual shops charge their monopoly price. The physical shop charges sometimes the new consumers' reservation price, sometimes its monopoly price. Second, given reservation prices and equilibrium prices, we derive the firms' equilibrium opening of virtual shop's rule. Either firm sometimes opens a virtual shop, sometimes does not. There are 6 types of equilibria, depending on whether firms choose to open a virtual shop, and whether the physical shop sells to all or only to old consumers.

### 3.1 Stage 2: The Price Game

In this sub-section we characterize equilibrium prices.
The number of shops that charge a price acceptable to new consumers, i.e., $p_{t} \leq \rho_{\mathrm{t}}, t=\mathrm{vn}, \mathrm{vo}, \mathrm{p}$, is $\alpha$. If virtual shop $t$ charges a price higher than $\rho_{\mathrm{t}}$, it makes no sales; if it charges a price no higher than $\rho_{\mathrm{t}}$, given (H.1) and that there is a continuum of new consumers, its expected consumer share is $\lambda$ / $\alpha$. Thus, for $0<\alpha$ :

$$
\phi_{\mathrm{t}}\left(\mathrm{p} ; \rho_{\mathrm{t}}\right)=\left\{\begin{array}{ll}
0 & \Leftarrow \rho_{\mathrm{t}}<\mathrm{p} \\
\lambda / \alpha & \Leftarrow \mathrm{p} \leq \rho_{\mathrm{t}}
\end{array} \quad \mathrm{t}=\mathrm{w}, \mathrm{vo}\right.
$$

(we omit $\alpha$ and $\lambda$ in $\phi_{t}$ ).
If the physical shop charges a price higher than $r$, it makes no sales; if it charges a price higher than $\rho_{\mathrm{p}}$, but no higher than $r$, it sells to old consumers, $1-\lambda$; if it charges a price no higher than the $\rho_{\mathrm{p}}$, its expected consumer share is $\lambda$ / $\alpha+1-\lambda$. Thus, for $0<\alpha$ :

$$
\phi_{\mathrm{p}}\left(\mathrm{p} ; \rho_{\mathrm{p}}\right)= \begin{cases}0 & \Leftarrow \mathrm{r}<\mathrm{p} \\ 1-\lambda & \Leftarrow \rho_{\mathrm{p}}<\mathrm{p} \leq \mathrm{r} \\ \lambda / \alpha+1-\lambda & \Leftarrow \mathrm{p} \leq \rho_{\mathrm{p}}\end{cases}
$$

(we omit $\alpha, \lambda$ and $r$ in $\phi_{p}$ ).

We assume that $\rho_{p}<\hat{p}_{p}$, which rules out the uninteresting cases, where although virtual shops exist, the physical shop is able to sell to new consumers at $\hat{p}_{p}$, its monopoly price. We assume also that $\rho_{\mathrm{t}}$ is strictly higher than the lowest of the prices consumers can find if they search:
(H.2) $\min \left\{p_{t^{\prime}}\right\}<\rho_{t} \quad t^{\prime} \neq t$

This assumption rules out equilibria which are not subgame perfect in the larger model, if search and waiting for delivery are costly. It follows that costly search and impatience give firms market power ${ }^{14}$, since they lead new consumers to accept prices above the minimum charged in the market. By (H.2), $0<\alpha$.

When neither firm opens a virtual shop, $a=(0,0)$, the industry is a monopoly. The number of shops that charge a price acceptable to new consumers when firms play $\left(a_{n}, a_{o}\right)$ in stage 1 is $\alpha^{\mathrm{a}_{\mathrm{n}}{ }_{\mathrm{a}}} ; \alpha^{00}=1$.

Next we examine the case where only the new firm opens a virtual shop, and hence the industry's supply side consists of the physical shop, and the new firm's virtual shop. The value of $\rho_{\mathrm{p}}$ for which the old firm is indifferent between charging $p_{p}=\rho_{\mathrm{p}}$, and charging $p_{p}=\hat{\mathrm{p}}_{\mathrm{p}}$, given $a=(1,0)$ and $p_{v n} \leq \rho_{\mathrm{vn}}$, is $p_{o}^{s}$, i.e.

[^5]$\pi\left(p_{0}{ }^{s}(\lambda) ; c_{p}\right)[\lambda / 2+1-\lambda] \equiv \pi\left(\hat{p}_{p} ; c_{p}\right)(1-\lambda)$. We assume that when the old firm is indifferent between selling to both consumers types and selling only to old consumers, it chooses the latter.

Proposition 1: If $a=(1,0)$, then: (i) $p_{v n}^{*}=\hat{p}_{v} ;$ (ii)

$$
\mathrm{p}_{\mathrm{p}}^{*}=\left\{\begin{array}{l}
\rho_{\mathrm{p}} \Leftarrow \mathrm{p}_{\mathrm{o}}^{\mathrm{s}}(\lambda)<\rho_{\mathrm{p}} \\
\hat{\mathrm{p}}_{\mathrm{p}} \Leftarrow \rho_{\mathrm{p}} \leq \mathrm{p}_{\mathrm{o}}^{\mathrm{s}}(\lambda)
\end{array}\right.
$$

where $p_{o}^{s}($.$) is decreasing, and p_{o}^{s}(1)=c_{p}$.
Since the new firm's virtual shop charges the lowest price in the market, and given (H.2), it is never constrained by consumer search and always charges $\hat{p}_{v}$. The physical shop also benefits from the market power generated by costly search, and from being the only shop old consumers can buy from, by charging a higher price than the new firm's virtual shop. However, it is constrained by consumer search, if it is beneficial to sell to both consumer types ${ }^{15}$. Let $\hat{\lambda}\left(\rho_{p}\right):=\left(\rho_{o}^{s}\right)^{-1}\left(\rho_{p}\right)$. If $\rho_{\mathrm{p}}$ is high, i.e., $p_{o}^{s}(\lambda)<\rho_{p}$, or alternatively, if $\lambda$ is large, i.e., $\hat{\lambda}\left(\rho_{\mathrm{p}}\right)<\lambda$, the old firm wants to sell to both consumers types, so reduces its price below $\hat{p}_{p}$ and charges $\rho_{\mathrm{p}}$ (figure 1). If $\rho_{p}$ is low, i.e., $\rho_{\mathrm{p}}<\rho_{0}^{s}(\lambda)$, or alternatively, if $\lambda$ is small, i.e., $\left.\lambda<\hat{\lambda}_{( } \rho_{\mathrm{p}}\right)$, the old firm wants to sell only to old consumers and charges $\hat{p}_{p} .{ }^{16}$ The higher is $\lambda$, the more willing is the physical shop to lower its price to sell to new consumers. From Proposition 1:

$$
\alpha^{10}=\left\{\begin{array}{l}
2 \Leftarrow p_{\mathrm{o}}^{\mathrm{s}}(\lambda)<\rho_{\mathrm{p}} \\
1
\end{array} \Leftarrow \rho_{\mathrm{p}} \leq \mathrm{p}_{\mathrm{o}}^{\mathrm{s}}(\lambda), ~ l\right.
$$

When the old firm does not open a virtual shop and charges $\rho_{\mathrm{p}}$ instead of $\hat{p}_{p}$, it sells to $\lambda / 2$ new consumers, earning an additional $\pi\left(\rho_{p} ; c_{p}\right)(\lambda / 2)$, the Volume of Sales effect, but loses $-\left[\pi\left(\hat{p}_{p} ; c_{p}\right)-\pi\left(\rho_{p} ; c_{p}\right)\right]$ per old consumer, and a total of $-\left[\pi_{\left(\hat{p}_{p} ; c_{p}\right)}-\pi\left(\rho_{p} ; c_{p}\right)\right](1-\lambda)$, the per Consumer Profit effect. Thus, the physical shop trades-off volume of sales and per consumer profit ${ }^{17}$.

[^6]
## [Insert figure 1 here]

When the physical shop charges $\rho_{\mathrm{p}}$, new consumers search only once ${ }^{18}$; otherwise new consumers may search twice, until they find the virtual shop.

When only the new firm opens a virtual shop there can be 2 types of price equilibria. In both the virtual shop charges $\hat{p}_{v}$. The physical shop at a Competing equilibrium charges $\rho_{\rho}$, and at a Segmentation equilibrium charges $\hat{p}_{p}$. The Competing equilibrium occurs when $\left(\rho_{p}, \lambda\right)$ are large, and the Segmentation equilibrium occurs when $\left(\rho_{\mathrm{p}}, \lambda\right)$ are small.

Next we examine the case where both firms open virtual shops, and hence the industry's supply side consists of a physical shop and 2 virtual shops. The level of $\rho_{\mathrm{p}}$ for which the old firm is indifferent between its physical shop selling to both consumer types, and selling only to old consumers, given $a=(1,1)$ and $p_{t} \leq \rho_{\mathrm{t}}$, $t=v n, v_{0}$, is $p_{m}^{s}$, i.e., $\pi\left(\hat{p}_{v} ; c_{v}\right)(\lambda / 3)+\pi\left(p_{m}^{s}\left(\lambda, \Delta_{c}\right) ; c_{p}\right)[\lambda / 3+1-\lambda] \equiv \pi\left(\hat{p}_{v} ; c_{v}\right)(\lambda / 2)+\pi\left(\hat{p}_{p} ; c_{p}\right)(1-\lambda)$. We assume that when the old firm is indifferent between its physical shop charging $p_{p}=\rho_{\mathrm{p}}$, and charging $p_{p}=\hat{p}_{\mathrm{p}}$, it chooses the latter; and that for $\Delta_{c}=c_{p}, 2<\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right)$, which can be interpreted as the Large Cost Reduction Opportunities case. The value of $\Delta_{c}$ for which $\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right) \equiv 2$, is $\Delta_{c}^{c} .{ }^{c}{ }^{19}$

Proposition 2: If $a=(1,1)$, then: (i) $p_{v a}^{*}=p_{v o}^{*}=\hat{p}_{v}$; (ii)

$$
\hat{p}_{\mathrm{p}}^{*}= \begin{cases}\hat{p}_{\mathrm{p}} & \text { for } \Delta_{\mathrm{c}} \in\left[\Delta_{\mathrm{c}}^{c}, c_{\mathrm{p}}\right] \\
\left\{\begin{array}{l}
\rho_{\mathrm{p}} \\
\hat{p}_{\mathrm{p}}
\end{array} \mathrm{p}_{\mathrm{p}}^{\mathrm{s}}\left(\lambda, \Delta_{\mathrm{c}}\right)<\rho_{\mathrm{p}} \leq \mathrm{p}_{\mathrm{p}}^{s}\left(\lambda, \Delta_{\mathrm{c}}\right)\right. & \text { for } \Delta_{\mathrm{c}} \in\left(0, \Delta_{\mathrm{c}}^{c}\right)\end{cases}
$$

where $p_{m}^{s}($.$) is decreasing in \lambda$, increasing in $\Delta_{\mathrm{c}}, p_{o}^{s}(\lambda) \leq \mathrm{p}_{\mathrm{m}}^{s}\left(\lambda, \Delta_{\mathrm{c}}\right)$, and $p_{m}^{s}\left(1, \Delta_{\mathrm{c}}\right)<\hat{\mathrm{p}}_{\mathrm{p}}$.

Note that $p_{v n}^{*}=\rho_{v o}^{*}=\hat{p}_{v}$ is an expression of Diamond's (1971) paradox ${ }^{20}$. From Proposition 2:

[^7]\[

\alpha^{11}= $$
\begin{cases}2 & \text { for } \Delta_{\mathrm{c}} \in\left[\Delta_{\mathrm{c}}^{\mathrm{c}}, \mathrm{c}_{\mathrm{p}}\right] \\
\left\{\begin{array}{l}
3 \Leftarrow \mathrm{p}_{\mathrm{m}}^{\mathrm{s}}\left(\lambda, \Delta_{\mathrm{c}}\right)<\rho_{\mathrm{p}} \\
2 \Leftarrow \rho_{\mathrm{p}} \leq \mathrm{p}_{\mathrm{m}}^{\mathrm{s}}\left(\lambda, \Delta_{\mathrm{c}}\right)
\end{array} \text { for } \Delta_{\mathrm{c}} \in\left(0, \Delta_{\mathrm{c}}^{\mathrm{c}}\right)\right.\end{cases}
$$
\]

When the old firm opens a virtual shop, it faces another per Consumer Profit effect, now with respect to new consumers, besides the Volume of Sales effect, $\pi_{\left(\rho_{p} ; c_{p}\right)}\left(\lambda_{/ 6}\right)$, and the previous per Consumer Profit effect with respect to old consumers, $-\left[\pi\left(\hat{p}_{\mathrm{p}} ; c_{\mathrm{p}}\right)-\pi\left(\rho_{\mathrm{p}} ; c_{\mathrm{p}}\right)\right](1-\lambda)$. If its physical shop charges $\rho_{\mathrm{p}}$ instead of $\hat{p}_{p}$, half of the new consumers it sells to, $\lambda_{/ 6}$, would otherwise buy from its own virtual shop, causing a loss of $-\left[\pi\left(\hat{\rho}_{v} ; c_{v}\right)-\pi\left(\rho_{p} ; c_{p}\right)\right](\lambda / 6)$. This additional effect causes the old firm to only want to reduce its physical shop's price below $\hat{p}_{p}$ to attract new consumers, if cost reduction is small, i.e., $\Delta_{c} \leq \Delta_{c}{ }^{c}$. Otherwise, the old firm prefers to sell to new consumers only from its virtual shop. And when the old firm's physical shop does reduce its price to attract new consumers, it does so for higher reservation price values than when it does not open a virtual shop, $p_{o}^{s} \leq \mathrm{p}_{\mathrm{m}}^{s}$ (figure 2). Even when all consumers have Internet access, $\lambda=1$, if cost reduction is small, and $p_{m}^{s}<\rho_{\mathrm{p}}$, the old firm still sells from the physical shop, $p_{m}^{s}\left(1, \Delta_{c}\right)<\hat{p}_{p}$, since this allows it to have a new consumer share of $2 \lambda / 3$ instead of $\lambda / 2$.

## [Insert figure 2 here]

When both firms open virtual shops there is a Competing and a Segmentation equilibrium. A Competing equilibrium, exists when $\Delta_{c}$ is small and $\left(\rho_{p}, \lambda\right)$ are large, and a Segmentation equilibrium exists when either $\Delta_{c}$ takes intermediate values and $\left(\rho_{p}, \lambda\right)$ are small, or when $\Delta_{c}$ is large

The price equilibria of case $a=(1,1)$ are different from other search theory equilibria where firms must choose whether to sell only to high reservation price consumers, or to sell also to low reservation price consumers (e.g., Braverman (1980), Burdett \& Judd (1983), Rob (1985), Salop \& Stiglitz (1977), Varian (89), Wilde \& Schwartz (1979)), because the old firm's problem is not just whether to sell to low reservation price consumers, but also how to sell to them, since it can do so either through its virtual or its physical shop.

Next we examine the case where only the old firm opens a virtual shop, and hence the industry's supply side consists of the old firm's physical and virtual shops.

Proposition 3: If $a=(0,1)$, then: (i) $p_{v_{0}}^{*}=\hat{\rho}_{v}$; (ii) $\rho_{p}^{*}=\hat{\rho}_{\mathrm{p}}$.

Now since the old firm is alone in the industry, it has no incentive to reduce its physical shop's price below $\hat{p}_{p}$. Any new consumer its physical shop might attract is stolen from its virtual shop, where per consumer profit is no smaller. And, if all consumers have Internet access, $\lambda=1$, since $\rho_{\mathrm{p}}<\hat{\rho}_{\mathrm{p}}$, the physical shop has zero sales, which could be interpreted as Shutting Down. From Proposition 3: $\boldsymbol{\alpha}^{01}=1$.

When only the old firm opens a virtual shop there is a Segmentation equilibrium.
Table 1 summarizes the price equilibria's main features.

## [Insert table 1 here]

Next we order the equilibrium price distributions for the various stage 1 decision profiles. The market equilibrium price distribution when in stage 1 firms play $\left(a_{n^{\prime}} a_{o}\right)$ is $F^{a_{n}{ }^{a}{ }_{o(.)} ; F^{\prime}(.)} \prec F_{(.)}$means "distribution $F($. dominates distribution $F^{\prime}($.$) in the first-order stochastic sense". { }^{21}$

## Proposition 4: (i) $F^{I 0} \prec F^{0 l} \prec F^{00}$; (ii) $F^{l l} \prec F^{0 l}$

Since $F^{00}$ dominates all other distributions, if at least 1 virtual shop opens, prices fall due to 2 effects. The Cost Reduction effect is the fall on prices caused by the production cost reduction induced by e-commerce. The Price Competition effect is the fall on prices induced by the physical shop lowering its price to compete for new consumers with the new firm's virtual shop. If the new firm opens a virtual shop prices fall, since $F^{I 0} \prec F^{00}$ and $F^{1 l} \prec F^{01}$. If the old firm opens a virtual shop, prices only fall for sure if it is the only virtual shop, $F^{01} \prec F^{00}$. When the new firm opens a virtual shop, the old firm opening a virtual shop puts more weight on the left tail of the price distribution. But since $p_{o}^{s} \leq \mathrm{p}_{\mathrm{m}}^{\mathrm{s}}$, the physical shop charges a no lower price than when it has no virtual shop. Thus, $F^{11}$ and $F^{10}$ are not comparable in the first-order stochastic sense.

### 3.2 Stage 1: The Opening of Virtual Shops Game

In this sub-section we characterize the equilibrium opening rule and establish existence of equilibrium.

Firm $j$ 's net profit when in stage 1 firms play $\left(a_{n}, a_{o}\right)$, and after both firms and play optimally is $V_{a_{n} a_{o}}^{j}$. The difference between firm $j$ 's net profits when it opens a virtual shop, and when it does not, given that firm $j$ plays $\mathrm{d}=0,1$ in stage 1 is $\Delta_{1 \mathrm{~d}}^{j}$, e.g., $\Delta_{111}^{\circ}=\mathrm{v}_{11}^{\circ}-\mathrm{v}_{10}^{\circ}$ and $\Delta_{111}^{n}=\mathrm{v}_{11}^{n}-\mathrm{v}_{01}^{n}=\mathrm{v}_{11}^{n}$. Firm $j$ 's Expected Incremental Profit of opening a virtual shop is $\Sigma_{j}:=a_{j^{\prime}} \Delta_{111}^{j}+\left(1-a_{j^{\prime}}\right) \Delta_{100}^{j}, j^{\prime} \neq j$.

Firm $j$ 's optimal stage 1 decision is to open a virtual shop if its expected incremental profit is positive.

## Proposition 5: Equilibrium exists.

Given the range of values the $\Sigma_{j}$ 's can take, virtually any profile of decisions to open a virtual shop can be an equilibrium. In section 4 we describe equilibrium profiles for particular cases.

## 4 Incentives to Open Virtual Shops

In this section we discuss the firms' incentives to open virtual shops. We show that if cost reduction is small, the new firm opens a virtual shop when the old firm does, and also when the old firm does not; otherwise the old firm may open a virtual shop when the new firm does not.

We start by discussing how opening a virtual shop impacts the new and old firms' profits. Since the new firm has no physical shop, opening a virtual shop enables it to sell to new consumers. The Business Creating effect, is the increase in the new firm's profit from opening a virtual shop, $\pi\left(\bar{p}_{v} ; c_{v}\right)(\lambda / \alpha)$. The impact of opening a virtual shop on the old firm's profit can be decomposed in 3 effects. The old firm can sell to new consumers through its physical shop. But, if it opens a virtual shop it can sell to them at a lower cost. The Cost Reduction effect, is the increase in the old firm's profit from selling to new consumers through its virtual shop, instead of its physical shop, $\left[\pi\left(\widehat{p}_{v} ; c_{v}\right)-\pi\left(p_{p}^{*} ; c_{p}\right)\right]\left(\lambda / m_{c}\right)$, where $\lambda / m_{c}$ is the proportion of new consumers that buy from the old firm's virtual shop, but that would buy from the physical shop if the old firm did not open a virtual shop. By opening a virtual shop when the new firm also does, the old firm improves its ability to sell to new consumers. The Market Penetration effect, is the increase in the old firm's profit, due to the rise in its new consumers' share, from opening a virtual shop

[^8]when the new firm also does, $\pi\left(\bar{p}_{v} ; c_{v}\right)\left(\lambda / m_{p}\right)$, where $\lambda m_{p}$ is the proportion of new consumers that buy from the old firm's virtual shop, but that would buy from the new firm's virtual shop if the old firm did not open a virtual shop. By opening a virtual shop when the new firm also does, the old firm can sell to new consumers through its virtual shop, and have its physical shop sell only to old consumers. The Price Discrimination effect, is the increase in the old firm's profit from switching from a Competing to a Segmentation equilibrium, $\left.\left[\pi_{\left(\bar{p}_{p} ; c_{p}\right)}-\pi_{( } \rho_{p} ; c_{p}\right)\right](1-\lambda) .22$

The next lemma orders the firms incremental payoffs. The value of $\Delta_{c}$ for which $\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right) \equiv 3 / 2$, is $\breve{\Delta}_{\mathrm{c}}(3 / 2)$.

Lemma 1: (i) If $\Delta_{c} \in\left(0, \breve{\Delta}_{c}(3 / 2)\right)$, then $\Delta_{10}^{\circ}<v_{11}^{n}$, and thus $\max \left\{\Delta_{110}^{\circ}, \Delta_{1 \mid 1}^{\circ}\right\} v_{11}^{n} \leq v_{10}^{n}$. (ii) If $\Delta_{c} \in\left(\Delta_{c}^{c}, c_{p}\right]$ and $p_{o}^{s}(\lambda)<\rho_{\mathrm{p}}$, then $\mathrm{v}_{11}^{\mathrm{n}}=\mathrm{v}_{10}^{\mathrm{n}}<\Delta_{10}^{\circ}$, and thus $\Delta_{1 \mid 1}^{\circ}<\mathrm{v}_{10}^{n}<\Delta_{1 \mid 0}^{\circ}$. (iii) $\breve{\Delta}_{\mathrm{c}}(3 / 2)<\Delta_{c}^{\mathrm{c}}$.

Since the new firm's consumer share is no bigger when the old firm opens a virtual shop than when it does not: $V_{11}^{n} \leq \mathrm{v}_{10}{ }^{n}$. When both firms open a virtual shop, the Business Creating effect dominates the Market Penetration, Cost Reduction, and Price Discrimination effects: $\Delta_{1 \mid 1}^{\circ} \leq v_{11}^{n}$, and thus $\Delta_{111}^{\circ} \leq v_{11}^{n} \leq v_{10}{ }^{n}$.

Expression $V_{l d}^{n}-\Delta_{10}^{0}, d=0,1$, equals the difference between the Business Creating and the Cost Reduction effects. If $\Delta_{c}$ is small, i.e., $\Delta_{c}<\bar{\Delta}_{c}(3 / 2)$, the Business Creating effect dominates the Cost Reduction effect, $\Delta_{10}^{\circ}<v_{11}^{n}$, and thus: $\max \left\{\right.$ 110 $\left._{\circ}^{\circ}, \Delta_{11}^{\circ}\right\}<v_{11}^{n} \leq v_{10}{ }^{n}$. If however, $\Delta_{c}$ is large, i.e., $\Delta_{c}^{c}<\Delta_{c}$, and the physical shop competes for new consumers, i.e., $p_{o}^{s}(\lambda)<\rho_{\mathrm{p}}$ or alternatively $\hat{\lambda}\left(\rho_{\mathrm{p}}\right)<\lambda$, the $\boldsymbol{C o s t}$ Reduction effect dominates the Business Creating effect, $\mathrm{v}_{10}{ }^{\mathrm{n}}<\Delta_{10}^{\circ}$. Furthermore, for $\mathrm{a}=(1,0)$ the model has a Competing equilibrium, and for $a=(1,1)$ a Segmentation equilibrium. Thus, $V_{11}^{n}=\mathrm{V}_{10}^{\mathrm{n}}$, and consequently: $\Delta_{111}^{\circ}<\mathrm{V}_{11}{ }^{\mathrm{n}}=\mathrm{v}_{10}{ }^{\mathrm{n}}<\Delta_{10}^{\circ}$.

22 The Cost Reduction effect is present in $\Delta_{1_{10}}^{0}$, and in $\Delta_{1_{1} 1}^{0}$ if $p_{o}^{s} \leq \rho_{\mathrm{p}}$; the Market Penetration effect is present in $\Delta_{1_{1}}^{0}$ if $\rho_{\mathrm{p}}<p_{\mathrm{o}}^{\mathrm{s}}$ for $\Delta_{\mathrm{c}}^{\mathrm{c}}<\Delta_{\mathrm{c}}$, and if $\rho_{\mathrm{p}} \notin\left[\begin{array}{l}\mathrm{p}_{\mathrm{o}} \\ \text {, } \mathrm{p}_{\mathrm{m}}^{\mathrm{s}}\end{array}\right)$ for $\Delta_{c} \leq \Delta_{c}^{c}$; the Price Discrimination effect is present in $\Delta_{11}^{0}$ if $p_{o}^{s} \leq \rho_{p}$ for $\Delta_{c}^{c}<\Delta_{c}$, and if $\rho_{p} \in\left[\begin{array}{c}s \\ p_{o}^{s}, \\ p_{m}^{s}\end{array}\right)$ for $\Delta_{c} \leq \Delta_{c}^{c}$.

Next we characterize the opening of virtual shops equilibrium profiles, for Lemma 1's 2 cases. To focus on pure strategy equilibria, we assume that when a firm is indifferent between opening and not opening a virtual shop, it chooses the former ${ }^{23}$. We assume also that $0 \leq \max \left\{\Delta_{10}^{0}, v_{10}^{n}\right\}$, which rules out $a^{*}=(0,0)$. Let $w=\left(\lambda, c_{p}, \Delta_{c}, k, \rho_{p}\right)$

Proposition 6: (i) If $\Delta_{\mathrm{c}} \in\left(0, \breve{\Delta}_{\mathrm{c}}(3 / 2)\right)$, then:

$$
\mathrm{a}^{*}=\left\{\begin{array}{l}
(1,0) \Leftarrow\left\{\begin{array}{l}
w \mid \Delta_{1 p}^{\circ}<0 \\
(1,1) \Leftarrow\left\{\mathrm{w} 10 \leq \Delta_{1 n}^{\circ}\right.
\end{array}\right\}
\end{array}\right.
$$

(ii) If $\Delta_{c} \in\left(\Delta_{c}^{c}, c_{p}\right]$ and $p_{o}^{s}(\lambda)<\rho_{p}$, then:

$$
\mathbf{a}^{*}=\left\{\begin{array}{l}
(0,1) \Leftarrow\left\{\begin{array}{l}
\left.w \mid v_{10}^{n}<0 \leq \Delta_{10}^{\circ}\right\} \\
(1,0) \Leftarrow\left\{w \mid \Delta_{111}^{0}<0 \leq v_{10}^{n}\right\} \\
(1,1) \\
\models
\end{array}\right\} \\
\left\{w \mid 0 \leq \Delta_{11}^{\circ}\right\}
\end{array}\right.
$$

If $\Delta_{c}$ is small, the new firm opens a virtual shop when the old firm does, and also when the old firm does not: $\Delta_{111}^{0}<0$. We interpret this as the old firm having less incentive to open a virtual shop.

## [Insert figure $\mathbf{3}$ here]

For $(\lambda, \Delta) \in\left(0, \hat{\lambda}_{c}\left(\rho_{p}\right)\right) \times\left(0, \breve{\Delta}_{c}(3 / 2)\right), \Delta_{1 \mid 1}^{\circ}$ and $V_{l 0}^{n}$ are increasing in $\left(\lambda, \Delta_{c}\right)$, and decreasing in $K$ (figure 3 (i)). If $\left(\lambda, \Delta_{c}\right)$ are small, $\Delta_{1 \mid 1}^{\circ}<0 \leq v_{10}^{n}$, and thus $a^{*}=(1,0)$; for larger values of $\left(\lambda, \Delta_{c}\right), 0 \leq \Delta_{1 \mid 1}^{\circ}$, and thus $a^{*}=(1,1)$. And although the model is static, this could explain why typically new firms opened virtual shops before old firms. Initially, $\left(\lambda, \Delta_{c}\right)$ were small because few consumers had Internet access, and firms did not fully understand the new technology. Overtime, more consumers gained access to the Internet, and firms learned how to use the new technology.

If however, $\left(\lambda, \Delta_{c}\right)$ are large, the old firm may open a virtual shop when the new firm does not: $V_{10}^{n}<0 \leq \Delta_{10}^{\circ}$.

For $\left(\lambda, \Delta_{\mathrm{c}}\right) \in\left(\lambda_{\mathrm{c}}\left(\rho_{\mathrm{p}}\right), 1\right] \times\left(\Delta_{c}^{\mathrm{c}}, \mathrm{c}_{\mathrm{p}}\right], \Delta_{10}^{\circ}, \Delta_{111}^{\circ}$, and $V_{10}^{n}$, are increasing in $\left(\lambda, \Delta_{\mathrm{c}}\right)$, and decreasing in $K$ (figure
3: (ii)). If $\left(\lambda, \Delta_{\mathrm{c}}\right)$ are larger than $\left(\hat{\lambda}_{\mathrm{c}}\left(\rho_{\mathrm{p}}\right), \Delta_{\mathrm{c}}^{\mathrm{c}}\right)$ but not by much, $V_{10}^{n}<0 \leq \Delta_{10}^{\circ}$, and thus $a^{*}=(0,1)$; for larger values of $\left(\lambda, \Delta_{c}\right)$, again $0 \leq \Delta_{11}^{\circ}$, and thus $a^{*}=(1,1)$.

The model shows how the old firm might have less incentive to open a virtual shop if cost reduction is small. But, if cost reduction is large, the old firm need not have less incentive. In fact, it can choose to open a virtual shop when the new firm does not.

It has been claimed that an old firm may be reluctant to use e-commerce for fear of its virtual shop, with supposedly a lower per consumer profit, stealing business from its physical shop, i.e., Cannibalization ${ }^{24}$. Note first that cannibalization is part of the Cost Reduction effect. If a virtual shop has lower costs than a physical shop, and market power, this intra-firm transfer of new consumers is profitable ${ }^{25}$. Second, if an old firm opens a virtual shop it can increase its new consumers' share, Market Penetration effect, and price discriminate between new and old consumers, Price Discrimination effect, both of which are also profitable ${ }^{26}$. In the next section we add some comments on this issue.

## 5 Firm Asymmetry

In this section, we discuss how a possible asymmetry between the new and old firm with respect to the new technology, affects the firms' pricing behavior and incremental profits.

We assumed that the new and old firm are equally capable of achieving the new technology's cost reduction. However, if virtual shops require new forms of organization that take advantage of the new technology's low cost of information processing and transmission, if integrating virtual and physical shop retailing is hard, and if old firms' employees resist the new technology because it devalues their skills, the new firm might achieve larger

[^9]retailing cost reductions than the old firm. To model this asymmetry let $c_{v o}=c_{p}-(1-\varepsilon) \Delta_{c}$, where $\varepsilon \in[0,1]$ indicates the efficiency loss of the old firm relative to the new firm.

The firms' pricing behavior remains unchanged, except for $a=(1,1)$, where $p_{v o}^{*}=\min \left\{\boldsymbol{\rho}_{v o}, \hat{p}_{v o}\right\}$, $\hat{p}_{v o} \in\left[\hat{\mathrm{p}}_{\mathrm{v}}, \hat{\mathrm{p}}_{\mathrm{p}}\right]$. This implies that there may be price dispersion among virtual shops ${ }^{27}$.

Now $\Delta_{100}^{\circ}<v_{11}^{n}$, if $\Delta_{c}$ is small, or, if $\Delta_{c}$ is large but $\varepsilon$ is also large (figure 4). And $v_{10}^{n}<\Delta_{10}^{0}$, if $\Delta_{c}$ is large and $\varepsilon$ is small. And for $\varepsilon$ large enough, $\Delta_{1 \mid 0}^{\circ}<\mathrm{V}_{11}{ }_{11}$ for all values of $\Delta_{\mathrm{c}}$.

## [Insert figure 4 here]

The Cost Reduction effect becomes $\left[\pi\left(\min \left\{\rho_{v o}, \hat{p}_{v o}\right\} c_{v_{v}}\right)-\pi\left(p_{p}^{*} ; c_{p}\right)\right]\left(\lambda / m_{c}\right)$, which can be negative if $\rho_{v o}$ is small and $c_{v o}$ large, giving some justification to the fear of Cannibalization. The Market Penetration effect $\pi\left(\min \left\{\rho_{\mathrm{vo}}, \hat{p}_{\mathrm{vo}}\right\} ; c_{\mathrm{vo}}\right)\left(\lambda / \mathrm{m}_{\mathrm{p}}\right)$ also becomes smaller.

## 6 Endogenous Reservation Prices

In this section we add to the model, a third stage where reservation prices are determined, given consumers' search and waiting costs. The game consists of 3 stages. The first 2 unfold as in the basic model. In stage 3 consumers make their search and purchase decisions; then delivery takes place, agents receive their payoffs, and the market closes.

To complete the model we introduce the following costs. Visiting the physical shop involves cost, $\sigma \in(0,+\infty)$, which includes the opportunity cost of the time spent, and associated expenses like driving. Visiting a Web site involves cost, $\sigma-\Delta_{\sigma}$, which includes the opportunity cost of the time spent, and associated expenses like phone calls and Internet fees, and where, $\Delta_{\sigma} \in(0, \sigma)$, is the search cost reduction induced by the new technology. Waiting for delivery of a product bought from a virtual shop involves cost, $\delta$, that results from deferring consumption. Searching Web sites is instantaneous, a consumer may observe any number of prices, and may at any

[^10]time accept any offer received to date. Let $\mathrm{S}(\mathrm{p}):=\int_{\mathrm{p}}^{\infty} \mathrm{D}(\mathrm{t}) \mathrm{dt}$. The surplus of a consumer that buys from a type $t$ shop at price $p_{t}$, is $\mathrm{s}\left(\mathrm{p}_{\mathrm{t}}\right)-u_{\mathrm{t}}$, where $u_{t}=\sigma$ if $t=\mathrm{p}$, and $u_{t}=\delta$ if $t=\mathrm{vn}$, vo. Let $\Delta \mathrm{s}:=\mathrm{s}\left(\hat{\mathrm{p}}_{v}\right)-\mathrm{s}\left(\hat{\mathrm{p}}_{\mathrm{p}}\right)$.

Old consumers, and new consumers when $a=(0,0)$, visit the physical shop, and if offered a price $p \leq r$ buy and receive the product, getting a surplus of $S(p)-\sigma$. When $a \neq(0,0)$, new consumers first visit a Web site chosen at random: (H.1). ${ }^{28}$ Then, they decide if they accept the best offer at hand and terminate search; or if they reject it, retaining the option to recall it later, and visit one of the other shop's Web sites. If new consumers have visited all shops, they accept the offer with the highest surplus.

A consumer's information set just after his $k$-th search step, consists of all previously observed prices. A consumer's stage 3 strategy, $s$, is a stopping rule, that for any sequence of observations, says if search should stop or continue. A consumer's payoff is the expected consumer surplus, net of the search expenditure.

A subgame perfect Nash Equilibrium is: a stopping rule for new consumers, and an opening and a pricing rule, for each shop and firm type, $\left\{\left(a_{j}^{*}, p_{t}^{*}, s^{*}\right) j=n, o ; t=v n, v o, p\right\}$, such that:
(E.0) Given any $a$ and $p_{t}$, new consumers choose $s^{*}$ to maximize net expected surplus;
(E.1) Given any $a$, and $s^{*}$, firms choose $p_{t}^{*}$ to solve problems: $\max _{p_{v n}} V^{n}$ and $\max _{\left\{p_{v_{v}}, p_{p}\right\}} v^{0}$;
(E.2) Given $s^{*}$ and $p_{t}^{*}$, firms choose $a_{j}^{*}$ to solve problem: $\max _{a_{j}} V^{j}$.

Next we characterize the new consumers equilibrium search behavior for $a \neq(0,0)$.
When $a=(1,0)(0,1)$, new consumers' search may involve 3 steps. In step 3 , consumers know both prices, and the optimal strategy is to accept $p_{t}$, if $S\left(p_{t}\right)-u_{t} \geq \mathrm{s}\left(\mathrm{p}_{\mathrm{t}^{\prime}}\right)-u_{\mathrm{t}^{\prime}}$. In step 2, a consumer who was offered $p_{t}$ at the shop he choose to visit at random in step 1 , gains $\left[S\left(p_{t^{\prime}}\right)-u_{t^{\prime}}-s\left(p_{t}\right)+u_{t}\right]$ by searching. Search is optimal if and only if $\sigma-\Delta_{\sigma}<\left[s\left(p_{t^{\prime}}\right)-u_{t^{\prime}}-s\left(p_{t}\right)+u_{t}\right]$. Let $\rho^{a_{n}{ }^{\text {a }} o}$ equate the marginal search cost, $\sigma-\Delta_{\sigma}$, to the marginal benefit, when firms play $\left(a_{n}, a_{o}\right)$ in stages 1:

[^11]\[

$$
\begin{equation*}
\left[S\left(p_{t^{\prime}}\right)-u_{t}-s\left(\rho_{t}^{a_{n}{ }^{a}{ }_{0}}\right)+u_{t}\right]=\sigma-\Delta_{\sigma} \quad t^{\prime} \neq t \tag{1}
\end{equation*}
$$

\]

The new consumers' optimal search rule is to accept offer $p_{t}$ and terminate search, if $p_{t} \leq \rho_{\mathrm{t}}{ }^{\mathrm{a}_{\mathrm{n}}{ }^{\mathrm{a}}{ }^{0}}$, and reject offer $p_{t}$ and proceed to step 3, if $p_{t}>\rho_{\mathrm{t}}^{\mathrm{a}_{\mathrm{n}} \mathrm{a}_{0}} .{ }^{29}$ Equation (1) defines implicitly reservation price function, $\rho_{t}^{a_{n} \mathrm{a}_{0}}=\mathrm{R}_{\mathrm{t}}^{\mathrm{an}^{\mathrm{a}} 0}($.$) ,$ which is increasing in $p_{\mathrm{t}^{\prime}}$. Also, $R_{t}^{a_{n} a_{o}}\left(p_{p}, \sigma, \Delta_{\sigma}, \delta\right), t=\mathrm{vn}$, vo , are increasing in $\sigma$, and decreasing in $\Delta_{\sigma}$ and $\delta$, and $R_{p}^{a_{n}{ }^{a}}\left(p_{t}, \Delta_{\sigma}, \delta\right), t=\mathrm{vn}, \mathrm{vo}$, is increasing in $\delta$, and decreasing in $\Delta_{\sigma}$. It is straightforward to show that the maximum price for which new consumers accept the physical shop's offer in step 3, is strictly smaller than $\rho_{\mathrm{p}}^{10}$. Thus the physical shop cannot charge a price higher than $\rho_{\rho}^{10}$, expecting that it will be rejected in step 2 , but accepted in step 3 . If $p_{v n}^{*}=p_{v 0}^{*}, \rho_{v n}^{10}=\rho_{v o}^{01}$.


Lemma 2: (i) If $\Delta_{\sigma}<\delta$, then $p_{t}<\rho_{\mathrm{p}}^{\mathrm{a}_{\mathrm{a}}{ }^{\mathrm{a}}{ }^{0}}, t=v n$, vo. (ii) If $\delta<2 \sigma-\Delta_{\sigma}$, then $p_{p}<\rho_{\mathrm{t}}{ }^{\mathrm{a}_{\mathrm{a}}{ }^{\mathrm{a}}{ }^{\circ}}, t=v n$, vo. (iii) If

$$
\delta<\Delta \mathrm{S}+\Delta_{\sigma}, \text { then } \rho_{\mathrm{p}}^{\mathrm{ana}^{\text {a }} \mathrm{o}}<\hat{\mathrm{p}}_{\mathrm{p}} .
$$

From Lemma 2: (i)-(ii), (H.2) follows from search and waiting for delivery being costly. Since new consumers have access to lower cost shops, and from Lemma 2: (iii), if waiting for delivery is not too costly, new consumers only accept buying from the physical shop for a lower price than old consumers: $\rho_{\mathrm{p}}<\hat{\rho}_{\mathrm{p}}<r$. From now on let $\delta \in\left(\Delta_{\sigma}, \Delta_{\sigma}+\min \left\{\Delta s, 2\left(\sigma-\Delta_{\sigma}\right)\right\}\right.$.

When $a=(1,1)$, new consumers' search may consist of 4 steps. Steps 3 and 4 are similar to steps 2 and 3 of the previous 2 cases. In step 2 , there are 2 Web sites to sample. Let $S\left(p_{t^{\prime}}\right)-u_{t^{\prime}}<S\left(p_{t^{\prime}}\right)-u_{t^{\prime}}$. If $S\left(p_{t}\right)-u_{t}<S\left(p_{t^{\prime \prime}}\right)-u_{t^{\prime \prime}}$, a new consumer who is offered $p_{t}$, gains $\left[S\left(p_{t^{\prime}}\right)-u_{t^{\prime}}-S\left(p_{t}\right)+u_{t}\right]$ by sampling shop $t^{\prime}$, and gains $\left[S\left(p_{t^{\prime}}\right)-u_{t^{\prime}}-s\left(p_{t}\right)+u_{t}\right]$ by sampling shop $t^{\prime \prime}$. If $S\left(p_{t^{\prime \prime}}\right)-u_{t^{\prime \prime}} \leq S\left(p_{t}\right)-u_{t}<S\left(p_{t^{\prime}}\right)-u_{t^{\prime}}$, new consumers'
expect to gain $\left[S\left(p_{t^{\prime}}\right)-u_{t^{\prime}}-S\left(p_{t}\right)+u_{t}\right] / 2$ by searching, since they reject shop $t^{\prime \prime}$ 's offer. The optimal search rule is to hold reservation price $\rho^{11}$, which equates the marginal benefit to the marginal cost:

Equation (2) also defines implicitly reservation price function, $R_{t}^{l l}($.$) , which is non-decreasing in p_{\mathrm{t}^{\prime}}$ and $p_{\mathrm{t}^{\prime \prime}}$. As before, it is straightforward to show that the maximum price for which new consumers accept the physical shop's offer, is smaller in step $\tau$ than in step $\tau+1, \tau=2,3$. Lemma 2 holds, and if $\sigma<\left(s\left(\hat{p}_{p}\right)+\Delta_{\sigma}\right) / 2$, in equilibrium consumers always have a strictly positive net surplus.

Equilibrium prices are as in section 3 and Proposition 5 holds.
We conclude this section by explaining the role of the assumptions that consumers do not know beforehand to which type of shops the Web sites correspond, that the physical shop has a Web site, and that when there are virtual shops, new consumers must canvass prices through the Web. In Reinganum (1979) and Benabou (1993), firms have different costs, play pure strategies, and consumers do not know the firms' costs $c$. Thus, although consumers know the firms' equilibrium price rule $p^{*}($.$) , they do not know the firms' equilibrium prices, p^{*}(\mathrm{c})$, and have to search to learn them. In our model consumers know $\left(c_{p}, c_{v}\right)$. Thus, the first 2 assumptions ensure that meaningful search occurs. Alternatively, consumers could not know $\left(c_{p}, c_{v}\right)$. This would mean developing a more complicated incomplete information game. If when there are virtual shops, we allow new consumers to choose weather to canvass prices through the Web, the Competing Equilibrium becomes non-generic, unless we also make new consumers heterogeneous with respect $\mathfrak{0}$, e.g., $\delta$, which has an expository cost. Finally, although these assumptions are intended to ensure meaningful search in a simple setting, they are not without justification, since as we argue in footnotes 9 and 10, some consumers in some markets behave similarly.

## 7 Information Goods

In this section we discuss the price equilibria for the case where $\delta=0$. We argue that for this case, virtual shops' prices may be higher than the physical shop's price.

Restriction, $0<\Delta_{\sigma}<\delta$, potentially rules out information goods, for which $\delta$ is small, possibly zero. So assume $\delta=0$. From (1), $R_{p}^{a_{n} a_{o}}\left(p_{t}, \sigma, \Delta_{\sigma}, \delta\right)<p_{t}, t=v n, v o$. Lemma 2: (ii) and (iii) holds.

Equilibrium prices are as in section 3, with an important difference. When $a=(1,0)$ and $\rho_{0}^{s}<\rho_{\mathrm{p}}$, or $a=(1,1)$ and $p_{m}^{s}<\rho_{p}$, if $0<\Delta_{\sigma}<\delta, p_{t}^{*}=\hat{p}_{v}<\rho_{p}=p_{p}^{*}$, whereas if $\delta=0, p_{p}^{*}=\rho_{p}<\hat{p}_{v}=p_{\mathrm{t}}^{*}, t=v n, v o$. If buying from a virtual shop is more convenient than buying from a physical shop, the physical shop must charge a lower price than virtual shops to sell to new consumers.

Case $\delta=0$ illustrates an intuitive point. Functionally identical goods sold through different retailing technologies, acquire different attributes. Ecommerce reduces prices through the Price Competition and Cost Reduction effects. If in addition consumers value negatively e-commerce's attributes, relative to those of other retailing technologies, consumers will only buy on-line if compensated by lower prices, which pushes prices further down. If however, consumers value positively e-commerce's attributes, they will pay for the convenience of buying on-line, and the net effect can be such that prices are higher on-line than off-line.

## 8 Related Literature

This section inserts the paper on the literature. Our paper relates to 3 literature branches. First, to the ecommerce marketing literature: Alba, Lynch, Weitz, Janiszewski, Lutz, Sawyer \& Wood (1997), Bakos (1997), Lal \& Sarvary (1998), Peterson, Balasubramanian, \& Bronnenberg, (1997), Zettelmeyer (1997). Bakos (1997) presents a model of circular product dfferentiation, where consumers search for prices and product characteristics. All consumers have Internet access. If search costs for price and product information are separated, and if e-commerce lowers the former, prices decrease; if it lowers the latter, prices can increase.

Second, our paper relates to the literature that analyzes competition between alternative retailing technologies: Balasubramanian (1998), Bouckaert (2000), Friberg, Ganslandt \& Sandstrom (2000), Legros \& Stahl (2000), and Michael (1994). Balasubramanian (1998) and Bouckaert (2000) use a model of circular product differentiation to analyze competition between catalogue and physical shop retailing. Physical shops are located on the circumference, and catalogue firms at the center of the circle. The presence of a catalogue firm lowers prices, and the number of physical shops in the market.

Third, our paper relates to the advertising and markets for information literatures: Baye \& Morgan (1998), Caillaud \& Jullien (2000), Ellison \& Ellison (2001), Iyer \& Pazgal (2000), and Kephart \& Greenwald (1999). Baye \& Morgan (1998) examine the interaction between markets for information and the product market they serve. They show that the product market can exhibits price dispersion even if consumers are fully informed. Kephart \& Greenwald (1999) investigate the impact of shopbots on markets. Shopbots allow users to choose the number of searches, and make search cost depend only weakly on the number of searches, i.e., nonlinear, leading to a more extensive search.

## Appendix

In the appendix we prove the lemmas and propositions contained in the main text, and 2 auxiliary lemmas A1, and A2.

Lemma A1: If $\mathrm{a} \neq(0,0)$, then: (i) for $\mathrm{p}^{\prime} \in\left(\rho_{\mathrm{t}},+\infty\right)$

$$
\mathrm{p}_{\mathrm{t}}^{*}=\left\{\begin{array}{rl}
\min \left\{\rho_{\mathrm{t}}, \hat{p}_{\mathrm{v}}\right\} & \Leftarrow \mathrm{c}_{\mathrm{t}} \leq \rho_{\mathrm{t}} \\
& \Leftarrow \rho_{\mathrm{t}}<\mathrm{c}_{\mathrm{t}}
\end{array} \quad \mathrm{t}=\mathrm{v}, \quad \mathrm{vo}\right.
$$

(ii) for $p^{\prime \prime} \in\left(\rho_{p},+\infty\right)$

$$
\mathrm{p}_{\mathrm{p}}^{*}= \begin{cases}\rho_{\mathrm{p}} \text { or } \hat{\mathrm{p}}_{\mathrm{p}} & \Leftarrow \mathrm{c}_{\mathrm{p}} \leq \rho_{\mathrm{p}} \\ \mathrm{p}^{\prime \prime} & \Leftarrow \rho_{\mathrm{p}}<\mathrm{c}_{\mathrm{p}}\end{cases}
$$

(iii) $p_{v n}^{*}=p_{v o}^{*}<p_{p}^{*}$.

Proof: (i) First we establish the result for $p_{v n}^{*}$, and for $p_{v o}^{*}$ when $\rho_{p}<p_{p}^{*}$. We proceed in 2 steps. In step 1 we show that $\hat{p}_{t}^{*}=\min \left\{\rho_{1}, \hat{p}_{v}\right\}$ for $c_{v} \leq \rho_{t}$. Let $\hat{p}_{v}<\rho_{t}$. Suppose $p_{t}^{*} \neq \hat{p}_{v}$. If $p_{t}^{*}<\hat{p_{v}}$, there is a $\varepsilon>0$ sufficiently small such that $p_{t}^{*}+\varepsilon<\rho_{t}$. Thus, if a shop deviates and charges $p_{t}^{*}+\varepsilon$, it loses no customers, and by strict quasiconcavity of the per consumer profit, profit rises. Thus, $\hat{p}_{v} \leq p_{t}{ }^{*}$. If $\hat{p}_{v}<p_{t}{ }^{*}$, by definition of $\hat{p}_{v}$, if a shop deviates and charges $p_{t}^{*}=\widehat{p_{v}}$, it increases its profit. Thus, $p_{t}^{*} \leq \hat{p}_{v}$, and therefore, $p_{t}^{*}=\hat{p}_{v}$. Now let $\rho \leq \hat{p}_{v}$. Suppose $p_{t}^{*} \neq \rho_{t}$. Shops can make a non-negative profit so they never charge $\rho<p_{t}^{*}$. And, if they charge $p_{t}^{*}<\rho_{t}$, they could, as before, increase profit by rising price to $\rho_{\mathrm{t}}$. It follows that $p_{t}^{*}=\rho_{t}$ for $\rho \leq \hat{p}_{v}$, and therefore $p_{t}^{*}=\min \left\{\rho_{t}, \hat{p}_{v}\right\}$ for $c_{v} \leq \rho_{t}$.

In step 2 we show that $p_{t}^{*}=p^{\prime} \in\left(\rho_{t},+\infty\right)$ for $\rho<c_{v}$. If a shop charges $p^{\prime} \in\left(\rho_{t},+\infty\right)$ it has a zero profit; otherwise it has a negative profit.

Now we establish the result for $p_{v o}^{*}$ when $p_{p}^{*} \leq \rho_{p}$. The previous argument applies, since old firm's virtual shop only affects $\phi_{p}($.$) , if it charges \rho_{v 0}<p_{v 0}$ instead of $p_{v o} \leq \rho_{v 0}$, which is never optimal since $\pi_{\left(p_{v o} ; c_{v}\right)}(\lambda / 3)+\pi_{\left(p_{p} ; c_{p}\right)}[\lambda / 3+1-\lambda]-\pi_{\left(p_{p} ; c_{p}\right)}[\lambda / 2+1-\lambda]<0$.
(ii) Recall that by assumption $\rho_{\mathrm{p}}<\hat{\mathrm{p}}_{\mathrm{p}}$. Case $\rho_{\mathrm{p}}<c_{\mathrm{p}}$ is similar to (i). So Let $c_{p} \leq \rho_{\mathrm{p}}$. We proceed in 2 steps. In step 1 we observe that by the same arguments as in (i), $\rho_{\mathrm{p}} \leq \mathrm{p}_{\mathrm{p}}^{*} \leq \hat{\mathrm{p}}_{\mathrm{p}}$.

In step 2 we show that either $p_{p}^{*}=\rho_{\mathrm{p}}$ or $p_{p}^{*}=\hat{\rho}_{\mathrm{p}}$. Suppose $p_{p}^{*}=\mathrm{p} \in\left(\rho_{\mathrm{p}}, \hat{\mathrm{p}}_{\mathrm{p}}\right)$. The physical shop's profit is $\pi_{\left(\mathrm{p} ; \mathrm{c}_{\mathrm{p}}\right)}(1-\lambda)$. If a firm deviates and charges $p_{p}^{*}=\hat{\mathrm{p}}_{\mathrm{p}}$, it will loose no customers, and by strict quasi-concavity of the per consumer profit, profit rises.
(iii) Suppose $p_{p}^{*} \leq p_{t}^{*}, t=v n, v o$. Let $p_{p}^{*}<c_{p}$. By (ii) $\rho_{p}<p_{p}^{*} \leq p_{t}$, which violates (H.2). Let $c_{p} \leq p_{p}^{*}$. Consider $p_{p}^{*}=p_{t}^{*}$. By (i) and (ii), $\rho_{\mathrm{p}}=\mathrm{p}_{\mathrm{p}}^{*} \leq \mathrm{p}_{\mathrm{t}}^{*}=\min \left\{\boldsymbol{\rho}_{t}, \hat{p}_{v}\right\}$, which violates (H.2). Thus, $p_{t}^{*}<p_{p}^{*}$.

Proposition 1: For $\left(\rho_{p}, \lambda\right) \in\left[c_{p}, \hat{p}_{p}\right] \times(0,1], \quad \psi^{0}\left(\rho_{p}, \lambda\right):=\pi\left(\rho_{p} ; c_{p}\right)[\lambda / 2+1-\lambda]-\pi\left(\hat{p}_{p} ; c_{p}\right)(1-\lambda)$ and $\psi^{0}\left(p_{0}^{5}(\lambda) ; \lambda\right) \equiv 0$.
(i) Follows from (H.2) and Lemma A1: (iii) and (i).
(ii) We proceed in 3 steps. In step 1 we establish the existence of $p_{o}^{s}$. Since $\forall \lambda, \psi^{0}\left(c_{p} ; \lambda\right)<0<\psi^{0}\left(\bar{p}_{p} ; \lambda\right)$, and $\psi^{\circ}($.$) monotonic in \rho_{p}$, it follows from the intermediate value theorem that, $\forall \lambda \in(0,1), \exists{ }^{1} p_{0}^{s} \in\left(c_{p}, \hat{p}_{p}\right)$. The implicit function theorem implies that $p_{0}^{s}=p_{0}^{s}(\boldsymbol{\lambda})$ with $p_{o}^{s}($.$) decreasing.$

In step 2 we establish $p_{p}^{*}$. Follows from Lemma A1: (ii), and the definition of $p_{o}^{s}$.
In step 3 we establish $p_{0}^{s}(1)=c_{p}$. Follows from $\psi^{\circ}\left(c_{p} ; 1\right)=0$.
§

## Proposition 2:

For $\left(\rho_{p}, \lambda, \Delta_{c}\right) \in\left[c_{p}, \hat{p}_{p}\right] \times(0,1] \times\left(0, c_{p}\right], \psi^{m}\left(\rho_{p} ; \lambda, \Delta_{c}\right):=\pi\left(\hat{p}_{v} ; c_{v}\right)(\lambda / 3)+\pi\left(\rho_{p} ; c_{p}\right)[\lambda / 3+1-\lambda]-\pi\left(\hat{p}_{v} ; c_{v}\right)(\lambda / 2)-$ $\pi_{\left(\hat{p}_{p} ; c_{p}\right)}(1-\lambda)$ and $\psi^{m}\left(p_{m}^{s}\left(\lambda, \Delta_{c}\right) ; \lambda, \Delta_{c}\right) \equiv 0$.
(i) As in Proposition 1: (i).
(ii) We proceed in 7 steps. In step 1 we show that $\forall \Delta_{c} \in\left(0, \Delta_{c}^{c}\right] \pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right) \leq 2$ and $\forall \Delta_{c} \in\left(\Delta_{c}^{c}, c_{p}\right): \pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right)>2$. Since $\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right)$ is increasing in $\Delta_{c}$, for $\Delta_{c}=0$ $\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right)<2$, and for $\Delta_{c}=c_{p} 2<\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right)$, it follows from the intermediate value theorem that $\exists^{1} \Delta_{c}^{c} \in(0, c] \pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right) \equiv 2$. The result follows from the monotonicity of $\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right)$.

In step 2 we observe that $\psi^{m}($.$) is strictly increasing in \rho_{p}, \forall\left(\lambda, \Delta_{c}\right) \psi^{m}\left(c_{p} ; \lambda, \Delta_{c}\right)<0$ and $\left.\psi^{m}\left(\hat{p}_{p} ; \lambda, \Delta_{c}\right)=-\lambda \pi\left(p_{p}^{*} ; c_{p}\right)\left\{\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right)\right]-2\right\} \sigma$.

In step 3 we establish the existence of $p_{m}^{s}, \forall \Delta_{c} \in\left(0, \Delta_{c}^{c}\right]$. Given steps 1 and 2, it follows from the intermediate value theorem that $\forall(\lambda, \Delta) \exists^{1} p_{m}^{s} \in\left[c_{p}, \hat{p}_{p}\right]$. The implicit function theorem implies that $p_{\mathrm{m}}^{\mathrm{s}}=\mathrm{p}_{\mathrm{m}}^{\mathrm{s}}\left(\lambda, \Delta_{\mathrm{c}}\right)$, and $p_{m}^{s}($.$) is strictly decreasing in \lambda$, and strictly increasing in $\Delta_{\mathrm{c}}$.

In step 4 we show that $p_{p}^{*}=\hat{p}_{p}, \forall \Delta_{c} \in\left(\Delta_{c}^{c}, c_{p}\right]$. Follows from steps 1 and 2.
In step 5 we show that $p_{o}^{s} \leq p_{m}^{s}$. Note that $\psi^{m}=\psi^{0}-A, A=\lambda\left[\pi\left(\bar{p}_{v} ; c_{v}\right)+\pi\left(\rho_{p} ; c_{p}\right)\right] / 6$. Let $\psi:=\psi^{0}-\eta A, \eta \in[0,1]$. For $\eta=0 \psi=\psi^{0}$, and that for $\eta=1 \psi=\psi^{m}$. The result follows from $\partial \psi / \partial \eta<0$.

In step 6 we establish $p_{p}^{*}$. Follows from Lemma A1: (iii), step 4 the definition of $p_{m}^{s}$.
In step 7 we establish $\mathfrak{p}_{m}^{s}\left(1, \Delta_{c}\right)<\hat{p}_{p}$. Follows from $0<\psi^{m}\left(\hat{p}_{p} ; 1, \Delta_{c}\right), \forall \Delta_{c} \in\left(0, \Delta_{c}^{c}\right]$.
§

Proposition 3: Follows from (H.2), the definition of new and old consumers, $\pi\left(\widehat{p}_{p}, c_{p}\right)<\pi\left(\widehat{p}_{v}, c_{v}\right)$, and the old firm being the only firm.

Proposition 4: (i) We proceed in 2 steps. In step 1 we show that $F^{01} \prec F^{00}$. In both cases $p_{p}^{*}=\hat{p}_{\mathrm{p}}$, but for $\mathrm{a}=(0,1), p_{v o}^{*}=\hat{p}_{v}<\hat{p}_{\mathrm{p}}$. In step 2 we show that $F^{I 0} \prec F^{01}$. In both cases $p_{t_{v}}^{*}=\hat{p}_{v}$, but for $a=(1,0), p_{p}^{*} \leq \hat{p}_{p}$. (ii) For $a=(1,1)$, the physical shop charges a no higher price, and the old firm's virtual shop puts more weight on $\hat{p}_{v}$.

Proposition 5: Follows from Kakutani's fixed theorem, since firm $j$ ' $s$ best response correspondence, $A^{j}:[0,1] \times\left[\hat{p}_{v},+\infty\right) \times(0,1] \times\left(0, c_{\mathrm{p}}\right] \times(0,+\infty) \rightarrow[0,1]$ is upper-hemi continuous in $a_{j^{\prime}}:$
$\S$
For $\left(\Delta_{c}, q\right) \in\left[0, c_{p}\right) \times[1,2], \chi\left(\Delta_{c} ; q\right):=\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right)-q$ and $\chi\left(\breve{\Delta}_{c}(q) ; q\right) \equiv 0$.

Lemma A2: If $\Delta_{c} \leq \check{\Delta}_{c}(q)$, then $\chi\left(\Delta_{c} ; q\right) \leq 0$; if $\breve{\Delta}_{c}(q)<\Delta_{c}$, then $0<\chi\left(\Delta_{c} ;\right.$ q).
Proof: Since $\chi(0, q) \leq 0 \leq \chi\left(c_{p}, q\right)$ and $\chi($.$) is continuous in \Delta_{c}$, by the intermediate function theorem $\forall q \in$ $[1,2] \exists^{1} \Delta_{c}(q) \in\left(0, c_{p}\right]: \chi\left(\Delta_{c}(q), q\right) \equiv 0$. The result follows from the monotonicity of $\chi($.$) in \Delta_{c}$.

## Lemma 1:

(i) First note that $V_{11}^{n} \leq v_{10}{ }^{n}$ and $\Delta_{111}^{\circ} \leq v_{11}{ }^{n}$, and thus $\Delta_{1 \mid 1}^{\circ} \leq v_{11}^{n} \leq v_{10}{ }^{n}$, follow directly from the expressions of $V_{10}^{n}-\mathrm{v}_{11}^{\mathrm{n}}$ and $\mathrm{v}_{11}^{\mathrm{n}}-\Delta_{111}^{\circ}$. From the expression of $V_{11}^{n}-\Delta_{110}^{\circ}$ and Lemma A2, it follows that $\Delta_{100}^{\circ}<\mathrm{v}_{11}^{\mathrm{n}}$.
(ii) From the expression of $v_{10}^{n}-\Delta_{100}^{\circ}$ and Lemma A2, it follows that $V_{10}^{n}<\Delta_{10}^{\circ}$.

Proposition 6: (i) Let $\Delta_{111}^{\circ}<0$. If $\Delta_{100}^{\circ} \leq \Delta_{11}^{\circ}$, by assumption $0 \leq v_{10}^{n}$, and the old firm's dominant strategy is $a_{o}=0$. By (AI) the new firm's best response is $a_{n}=1$. If $\Delta_{111}^{\circ}<\Delta_{100}^{\circ}$, by Lemma 1: (i), $\Delta_{10}^{\circ}<v_{11}^{n} \leq v_{10}^{n}$. If $\Delta_{10}^{\circ}<0$, the argument is as before. If $0 \leq \Delta_{10}^{\circ}$, the new firm's dominant strategy is $a_{n}=1$. By (AI) the old firm's best response is $a_{o}=0$.
(ii) For $\mathrm{a}=(1,0)(1,1)$ the model has a competing and segmentation equilibrium, respectively. Thus, $V_{1 I}^{n}=v_{10}^{n}$. Let $V_{10}^{n}<0 \leq \Delta_{10}^{\circ}$. The new firm's dominant strategy is $a_{n}=0$. By (AI) the old firm's best response is $a_{0}=1$. Let
$\Delta_{11}^{0}<0 \leq \mathrm{v}_{10}{ }^{\mathrm{n}}$. The new firm's dominant strategy is $a_{n}=1$. By Lemma 1: (ii) and (AI) the old firm's best response is $a_{o}=0$. Let $0 \leq \Delta_{111}^{\circ}$. By Lemma 1: (ii), the firms' dominant strategies are: $a_{n}=\mathrm{a}_{\mathrm{o}}=1$.

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Figure 1 : Physical Shop' s Price


Figure 2: Price Equilibria


Figure 3 (i): Opening of Virtual Shops Equilibrium Profiles


Figure 3 (ii): Opening of Virtual Shops Equilibrium Profiles


Figure 4: Cost Asymmetry


Table 1: Summary of Model's Price Equilibria

| a |  |  | $\mathbf{p}_{\mathrm{p}}^{*}$ | Share vn | Share vo | Share p | Equilibrium |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1,0) | $p_{o}^{s}<\rho_{\mathrm{p}}$ |  | $\rho_{\mathrm{p}}$ | $\lambda / 2$ | - | $\lambda / 2+1-\lambda$ | Competing |
|  | $\rho_{\mathrm{p}} \leq \mathrm{p}_{\mathrm{o}}{ }^{\text {a }}$ |  | $\hat{p}_{p}$ | $\lambda$ | - | $1-\lambda$ | Segmentation |
| (0, 1) | - |  | $p_{p}$ | - | $\lambda / 2$ | $1-\lambda$ | Segmentation |
| $(1,1)$ | $\Delta_{c} \leq \Delta_{c}^{c}$ | $p_{m}^{s}<\rho_{\mathrm{p}}$ | $\rho_{\mathrm{p}}$ | $\lambda / 3$ | $\lambda / 3$ | $\lambda / 3+1-\lambda$ | Competing |
|  |  | $\rho_{\mathrm{p}} \leq \mathrm{p}_{\mathrm{m}}^{\mathrm{s}}$ | $\hat{p}_{p}$ | $\lambda / 2$ | $\lambda / 2$ | $1-\lambda$ | Segmentation |
|  | $\Delta_{c}^{c}<\Delta_{\text {c }}$ |  | $\widehat{p}_{p}$ | $\lambda / 2$ | $\lambda / 2$ | $1-\lambda$ | Segmentation |


[^0]:    ${ }^{1}$ We thank P.P. Barros, A. Cabrales, L. Corchón, R. Gatti, V. Karamychev, A. Kirman, S. Martin, J.L. Moraga, L. Moreno, T. Perez, and specially J. Sobel for useful comments, and E. Wasmer for giving us the Yahoo Internet Life example. The paper was presented at the Tinbergen Institute/Eramus University, I.S.E.G., U.A.B., C.E.M.F.I., WZB, XV Jornadas de Economia Industrial 1999, Stony Brook 2000, E.E.A. 2000, E.A.R.I.E. 2000, A.S.S.E.T. 2000, and "The Economics of The Software and Internet Industries" Toulouse 2001. We thank Universidad Complutense, "Proyecto de Investigacion Complutense, \# PR52/00-8892", for financial support.

[^1]:    2 Transacting products based on the processing and transmission of digitized data over the network of computers that use the transmission control protocol/Internet protocol, TCP/IP.
    ${ }^{3}$ Goods that can be digitized, i.e., expressed as zeros and ones.

[^2]:    ${ }^{4}$ Other empirical studies are: Brown \& Goolsbee (2000), Clemons, Hann \& Hitt (1999), Friberg, Ganslandt \& Sandstrom (2000), and Morton, Zettekmeyer \& Risso (2000).
    
    ${ }^{6}$ E.g., ClickTheButton, DealPilot, www.previewtravel.com for airfares, and www.microsurf.com for mortgages.
    ${ }^{7}$ On the Web, a banking transaction costs $\$ .1$, compared with $\$ .27$ at an ATM or $\$ .52$ over the phone, and processing an airline ticket costs $\$ 1$, compared with $\$ 8$ through a travel agent (The Economist, June 26, 1999). USA retailers with no physical presence in a state do not collect local sales taxes, $6 \%$.

[^3]:    ${ }^{8}$ Technologies that make products available for use or consumption. This concept is related to that of a distribution channel (see Kotler (1994)).
    9 Bailey (1998) and Brynjolfsson \& Smith (1999) found that, e.g., Cody's and Powell's Books, posted prices on the Web, but only sold at their physical locations.

[^4]:    10 In 2000, about half of the US car buyers will use the Internet. Most of them not to buy, but to obtain information to bargain lower prices out of local dealers (The Economist, February 14, 1998).
    11 At the end of section 6 we explain the role of the assumptions that consumers do not know beforehand to which type of shops the Web sites correspond, that the physical shop has a Web site, and that when there are virtual shops, new consumers canvass prices through the Web.
    12 As an alternative to sequential search new consumers could use shopbots, which in our context could be thought of as using a newspaper search technology (Braverman (1980), Salop \& Stiglitz (1977), Wilde \& Schwartz (1979)), i.e., perfect information at a fixed cost. This, however, contrasts with the reality of e-commerce (Brynjolfsson \& Smith (1999)). Shopbots are useful to limit the set of shops under consideration, but it is unclear that consumers will buy through them. Aspects like the return policy or post sale services, can only be learned at the firms' sites. In addition, some vendors block shopbots from searching their sites.
    13 Barnesandnoble.com charges different prices than Barnes and Noble's physical shops.

[^5]:    14 The ability to raise price above marginal cost.

[^6]:    15 And the threat of a second search by new consumers is credible, i.e., $\rho_{p}<\hat{\mathrm{p}}_{\mathrm{p}}$
    16 When $\rho_{\rho}<\rho_{p}^{*}$ the physical shop could shut its Web site.
    17 The physical shop could price discriminate between new and old consumers, by, e.g., offering coupons at its Web site. It might, however, be reluctant to do so, because when informed about them, old consumers could perceive these price differences as unfair. See Sinha (2000) for a discussion of this issue.

[^7]:    18 The option to search serves only as a credible, out of equilibrium threat, constraining the old firm's price decisions.
    19 Price $\hat{p}_{v}$ depends on $c_{v}$, and thus on $\Delta_{c}$. If $2<\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right), \Delta_{c}^{c} \leq c_{p}$, whereas if $\pi\left(\hat{p}_{v} ; c_{v}\right) / \pi\left(\hat{p}_{p} ; c_{p}\right) \leq 2, \Delta_{c}^{c}=c_{p}$.
    ${ }^{20}$ Low cost shops charge the monopoly price, regardless of how low the search cost is, and how many shops there are. See Davis \& Holt (1996) for experimental evidence.

[^8]:    ${ }^{21}$ Distribution $\mathrm{F}_{(.)}$Dominates distribution $\mathrm{F}^{\prime}($.$) in the First-Order Stochastic sense if F_{(.)} \leq \mathrm{F}^{\prime}($.$) , for all p$.

[^9]:    24 Toys"R"Us invested $\$ 80$ million to launch a virtual division, but Robert Mogg, the man in charge, resigned, claiming that the firm was afraid of competing with its own physical shops (El País, September 5, 1999). Alba et al. (1997): "E-commerce offers an advantage to retailers that have low penetration (...). On the other hand, companies with high penetration might experience significant cannibalization of its in-shop sales, making e-commerce less attractive".
    ${ }^{25}$ Baseball Express, claims that its Web site stole sales from its catalogue, but that selling on the Web is more profitable (The New York Times, September, 2, 1999).
    ${ }^{26}$ Ward \& Morganosky (2000) and Ward (1999) also arrive to a negative conclusion on cannibalization, although for different reasons.

[^10]:    ${ }^{27}$ See Baye \& Morgan (2000) and Ayer \& Pazgal (2000) for alternative ways of generating price dispersion on-line.

[^11]:    ${ }^{28}$ Thi
    This first step is usually absent in the search literature since it is assumed that consumers get their first price observation for free.

