Costly Capital Reallocation and Energy Use

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Abstract

In time series data, energy use does not change much with energy price changes. However, energy use is responsive to international differences in energy prices in cross-section data across countries. In this paper we consider a model of energy use in which production takes place at individual plants and capital can be used either to directly produce output or to reduce the energy required to run the planto We assume that reallocating capital from one use to another is costly. This turns out to be crucial for the quantitative properties of the model to be in conformity with the low short-mn and high long-run elasticities of energy use seen in data. Furthermore, our model displays variations in capacity utilization that are in line with those observed during the period of major oil price increases.

Key words: energy price, energy use, costly capital reallocation, number of plants.

JEL codes: E22, E23, Q43

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1 Introduction

There are two salient features of data on energy use and energy prices. On the one hand, energy use is not very responsive with energy prices in the time series, and energy expenditure varies very much with energy price changes. On the other hand, in cross-section data across countries, energy use is very responsive to international differences in energy prices. Also, there is a long lasting debate about the nature of the reallocation frictions that influcncc thc aggrcgatc rcsponsc to cncrgy pricc changcs [d. Davis and Haltiwanger (1999)]. Do the frictions mainly involve labor or capital reallocation? In this paper we consider a model of energy use to account for the short-run and long-run features aforementioned and in which the main channel of transmission of energy price changes is capital reallocation. Our model is a version of the neoclassical growth model augmented with a second type of physical capital that acts purely as an energy saving device. We interpret this capital good as induced energy-saving innovation and we will call it technological capital.'

There are several studies on the effect of changes in energy prices on aggregate variables that also focus their attention to the capital channel. For instance, Pindyck and Rotemberg (1983) build a model in the neoclassical tradition. In their model, capital and energy are highly complementary and capital is subject to adjustment costs. The response to an energy price shock predicted by their model is a sharp reduction in energy use which limits fluctuations in energy expenditure together with a big rise in the capital-energy ratio. As for the long mn behavior of the aggregates, persistent international differences in energy prices lead to large international differences in energy use and, due to the complementarity between capital and energy, they imply large differences in output. Just the opposite to what the data show.

Atkcson and Kchoc (1999) dcpart from thc ncoclassical framcwork and analyzc cncrgy intensity choice in a model of differentiated putty-clay capital goods. They consider a vintage technology according to which older vintages of capital have higher fixed energy requirements. Consequently, when the energy price rises, old vintages cannot be scrapped and converted into new, more efficient, capital goods. Adjnstment to energy price changes is only possible through investment in new capital units. Therefore, energy use remains fairly constant and energy expenditure varies very much with shocks. This approach is appealing but somewhat extreme. Indeed, in their model, the capital-energy ratio adjusts too slowly compared to what we observe in data. Nevertheless, their model predicts small international differences in output, in spite of large international disparity in energy prices. The reason is that countries in the long run can lower the aggregate use of energy investing in more energy efficient vintages[2]

1 **Newell (1999) test Hicks' induced innovation hypothesis that states that rising energy prices should have induced energy-saving innovatian. He finds evidence that energy prices have affected the energy efficiency oí models oí air conditioners and gas water heaters available on the market over the last four decades.**

3 Finn (1995) and Kim and Loungani (1992) aiso buiid models for energy use but their main focus

These studies focus on the effects of energy price changes on aggregate activity but abstract away from the response of the industrial organization of production to those price changes. In particular, we are interested in analyzing the effects of energy prices on the aggregate capacity utilization rate, specially during the period of major oil price increases, 1975-85. Bresnahan and Ramey (1993) provide some micro evidence that adjusting capacity utilization along the extensive margin is quantitatively important. They study weekly data from 1972 to 1983 for 50 automotive plants and provide statistics on how frequently the automobile industry uses various margins to adjust output. Plant shutdowns are by far the most common margin used, accounting for 65 percent of the output variance for thc pcriod *lD72-83.* To account for this fact wc follow Coolcy ct al. (1995) and build a model economy in which production takes place at individual plants that are subject to idiosyncratic technology shocks. That is, we do not assume an aggregate production function, as Pindyck and Rotemberg (1983) do. Nevertheless, value added in our framework has a ready representation as a constant returns to scale function of aggregate production factors. Consequently, we maintain our work within the neoclassical framework.

As in Atkeson and Kehoe (1999), we assume that there is a fixed energy requirement, here at the plant level, that cannot be changed in the short runo Differently from them, we do not use the vintage framework. We assume that plants are ex-ante identical and differ according to an idiosyncratic technology shock. Capital installed in a plant has two uses: one is directly productive, and the other is to reduce the energy required to run the planto Thus, for convenience, we will talk about two types of capital: Productive capital and technological capital. We should think about the latter as any device that reduces energy use, as adjustable speed motors, or what Doms and Dunne (1993) call advanced manufacturing technologies (AMTs).[3] These authors use data on energy use per unit of output at the plant level for plants built earlier than in 1989. They estimate that plants that use three to five AMTs use 13.8 percent less energy per unit of output than plants that use none. They also find that plants built between 1972 and 1983, the period right after the oil shocks, use 12.6 percent less energy than the youngest plants in the sample. In our model we abstract away from heterogeneity in energy efficiency across plants but retain the main implication of Doms and Dunne (1993): investment in technological capital varies with energy prices. Nevertheless, we assume that this investment is subject to adjustment costs. The existence of adjustment costs implies that energy use reacts very slowly to energy price changes since reallocating capital from its productive use to its technological use (converting productive capital into technological capital) as well as undertaking new investment in technological capital are costly[4] Clearly, though, this

is to improve the predictions of the standard real business cycle mode!. Hamilton (1988) and Davis and Haltiwanger (1999) analyze the labor market reallocation effects of energy price shocks. See Rotemberg and Woodford (1996) for a model of energy use that incorporates imperfect competition.

3These AMTs refer, for instance, to computer aided desing, flexible manufacturing systems, computers used on factory flúor, etc.

4 Newell (1999) report that "major tooling and redesign changes to incorporate energy-saving design **options in rnodels of heat pumps and air-conditioners require lead times of about 1.5-2 years for a single roadel and longer for an entire lineo A typical cycle for introducing new appliance models can be three**

specification willleave open the channels for capital and energy to be substitutable in the long runo

In addition to the fixed energy requirement, a fixed number of workers is also needed to operate the plant and the marginal product of additional workers beyond this number is zero. A particular plant is operated if, given its realized technology shock and the realized energy price, it is able to produce enough output to cover its labor and energy costs. Thus, in equilibrium, some plants will operate and others will not and so, capacity utilization rate will vary with changes in energy prices. The number of plants may vary overtime. Nevertheless, we assume that the number of plants cannot be changed readily to accommodate fluctuations in energy prices. Creating a plant takes time. We capture this idea assuming that capital is already allocated to the plant before the energy price and the idiosyncratic technology shock are known.

We evaluate the empirical performance of our model on US data. In doing so, we simulate the model feeding in the data on the energy price to obtain predictions for the time paths of energy use, energy expenditure, capital and output. We find that the time-series behavior of the aggregate variables in our model economy is very similar to that seen in data. Our findings are in line with those reported in Atkeson and Kehoe (1999) vintage model, but we improve their results on the behavior of the energy-capital ratio. In our model an energy price increase is followed by a reduction in the number of operating plants in the short run, which amounts to a fall in the capacity utilization rateo This is followed by a decrease in the number of plants. Technological capital adjust very slowly and, therefore, energy use does not change very much with energy prices. As a result, energy expenditure fluctuates very mucho The energy-capital ratio moves as in the data. Additionally, our model is able to account for the observed changes in capacity utilization, specially during the period 1975-85.

We also consider the effect on output of an energy tax that leads to a doubling of energy prices in this model. We find that a doubling of the energy price leads to a 1.75 percent fall in long rnn Olltput. This number is comparable to that estimated by Goulder (1992), Gouldcr (1003), GOllldcr (1005), and Jorgcnson and Wilcoxcn (1003). It is smallcr than that fOllnd by Atkcson and Kchoc (1999), rcflccting thc diffcrcncc bctwccn thc vintagc framework and our own.

The rest of the paper is organized as follows: Section 2 describes the economic environment, analyzes the decisions taken within the plant and defines the eqllilibrium concepto In Section 3 we review the calibration of the model and Section 4 discusses the results obtained from the simulation of the model. Section 5 concludes.

years."

2 The rnodel econorny

In this scction wc dcscribc thc cnvironmcnt and thc cquilibrium dcfinition. As wc havc said in thc Introduction, wc build upon Coolcy ct al. (1995) thcorctical framcwork and we introduce energy as an additional production factor. We will assume that energy is entirely bought in an international market at an exogenously given price *Pt.* Therefore, from the point of view of the economic agents the energy price follows a stochastic process.

In this economy, production takes place at individual plants that are ex-ante identical but differ in an idiosyncratic technology shock. The production of output requires capital, energy and labor. Capital has two uses: one which is directly productive and another that is to save energy. When used for energy saving purposes capital will be called technological capital. At the plant level energy and productive capital are complements. The amount of energy needed to run a plant is directly increasing with the amount of productive capital and decreases with the amount of technological capital. Additionally, there is a minimum number of workers required to run the plant and any number beyond that threshold has zero marginal productivity. It follows that, once the energy price and the idiosyncratic shock are known sorne plants will not operate since their level of produced output will not be enough to cover energy and labor costs. Therefore, sorne capital will be left idle in equilibrium.

Establishing a plant amounts to choosing the amount of capital installed before the energy price and the idiosyncratic shocks are realized. Thus, the plant's manager must forecast the input prices to choose the amount of productive and technological capital that maximizes expected profit. To compute this expected profit the manager of the prospective plant takes into account that the plant only will operate if the idiosyncratic shock is sufficiently high. The number of plants established will be such that maximum expected profit is zero. Subsections 1 and 2 analyze these issues in detai!.

Subsection 3 describes the household sector of this model economy. Households either work a fixed workweek or not work at all. After output is produced households decide how much to consume and how much to invest in each type of capital. Investing in technological capital is subject to adjustment costs. Subsection 4 defines the equilibrium concept as well as a quasi-social planner's problem whose solution is the competitive equilibrium allocation associated to this model economy.

2.1 Technology, measure of plants, and timing

2.1.1 Technology

Production of the unique final good is carried out at a continuum of autonomous plants with measure mt and indexed by a productivity parameter, *Sto* Output is produced with capital, labor and energy. A number r_j of workers are required to operate the plant and the marginal productivity of additional workers beyond r_j is zero. There are two uses for capital. When capital is used directly to produce output we call it productive capital and denote it as k_t. When capital is used to reduce the energy required to mn the plant we call it technological capital and denote it as a_to Productive capital is combined with energy and labor to produce the final good. The proportion in which productive capital and energy are combined depends on the amount of technological capital used. Specifically, we assume that the ratio productive capital to energy, k_t/e_t used in the plant cannot exceed the proportion $1/\gamma$ of the amount of technological capital. Thus, the amount of energy used in the plant should satisfy

$$\frac{k_t}{e_t} \leq \frac{1}{\gamma} a_t, \ \gamma > 0.$$

This specification implies that a plant can use many different production processes that differ in their energy intensity use, measured by its productive capital-energy ratio. A more efficient technology is one which has a lower energy intensity use. Adopting a more efficient technology requires a higher level of technological capital. We should think of technological capital as any engine or appliance that reduces the energy required to mn a planto

The output produced by a plant with k_t units of productive capital, a_t units of technological capital, d_t workers and e_t units of energy is given by,

$$y_t = \begin{cases} (z + s_t)\, B k_t^{\theta} h(s_t) & \text{if } d_t \geq \eta \text{ and } e_t \geq \gamma \frac{k_t}{a_t}, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

where s_t is a plant specific technology shock assumed to be independent and identically distributed across time and across plants. We assume that s_t is uniformly distributed in the interval $[-\sigma, \sigma]$. The function $h(s_t)$ represents the number of hours the plant is operated and it is restricted to either be equal to $h_0 > 0$ or zero. The parameters z and θ are both positive, with $\theta \in (0,1)$.[5] The scale parameter B is greater than zero. Plants are established by renting productive and technological capital from households.

2.1.2 Timing of decisions at the plant level

Any prospective plant must choose the amount of capital to be installed in the plant before the energy price and the idiosyncratic shocks are known. Thus, the manager chooses k_t and a_t given the aggregate stock of capital, K_t and A_t taking into account that the plant will be operated if it is able to produce enough output to cover, at least, its energy and labor costs. Once the capital choices have been made, the energy price and the idiosyncratic shock are revealed. Then the plant's manager decides whether the

[5] An economy wide technology shock could be introduced by using a random variable z_t instead of parameter z as in Cooley et al. (1995). This exceeds the scope of this paper notwithstanding.

plant is operated or noto If it is operated, workers are hired and energy is bought. Then production takes place. Figure 1 summarizes the timing of decisions at the plant level.

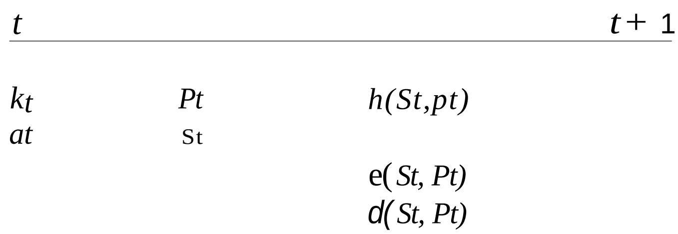

Figure 1: Timing of decisions at the plant level.

2.1.3 Timing at the aggregate level

At the beginning of a period *t* the measure of plants that can operate at that period, mt has been already established. The energy price *Pt* is observed and the idiosyncratic technology shocks at the plant level are realized. Plants decide whether to operate or noto This determines the fraction of plants operated during the period, which we denote as *nt.* Plants that operate hire labor and use energy to produce output. Households consume and save. Then the measure of plants that can operate next period, *mt+l,* is determined. Figure 2 summarizes the timing of events at the aggregate level.

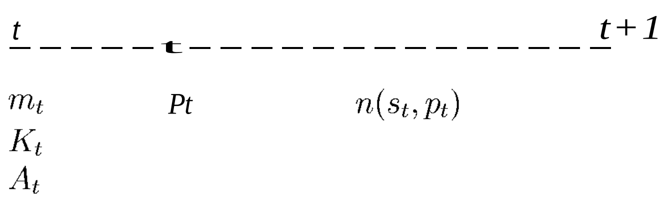

Figure 2: Timing of events at the aggregate level.

2.1.4 A plant manager's problem

Let us denote as $\Omega_t^* = \{K_t, A_t\}$ the information set available to the manager at the time of choosing the amount of capital to be located at the plant, before the energy price *Pt* and the idiosyncratic technology shock St are realized, and let $\Omega_t = \Omega_t^* \cup \{Pt\}$. Therefore, the plant manager's problem is

$$\max_{k_t,a_t} E\left\{E\left[\max_{h(s_t)\in\{0,1\}}\left[(s_t+z)Bk_t^{\theta}-w_t\eta-p_t\gamma\frac{k_t}{a_t}\right]h(s_t)\mid\Omega_t\right]-r_t^k k_t-r_t^a a_t\mid\Omega_t^*\right\} \quad (2)$$

The expression inside the straight brackets shows the problem that the plant's manager solves once the energy price and the idiosyncratic shock are revealedo The decision is whether to operate the plant or noto Taking into account that the plant will not operate for low values of the technology shock, the plant's manager chooses the amount of productive and technological capital that maximizes expected profit conditional on Ω_t^*; that is, before the energy price and the idiosyncratic technology shock are knowno

2.1.5 The measure of plants

Let us call $\Pi(K_t, A_t, k_t, a_t)$ the maximum value of the expected profit given in expression (2)0 Since all plants are ex-ante identical all of them will use the same amount of capital and hence, in equilibrium, $k_t = \frac{K_t}{m_t}$ and $a_t = \frac{A_t}{m_t}$ o Since the cost of establishing a new plant is zero, it follows that the number of plants at the beginning of period t, m_t, is that for which

$$\Pi\left(K_t, A_t, \frac{K_t}{m_t}, \frac{A_t}{m_t}\right) = 0.$$

The number (measure) of plants is well determined since capital has to be paid for independently of the plant being operated or noto Therefore, at the time the idiosyncratic shock is realized, capital cost is a fixed cost from the viewpoint of the planto The existence of this fixed cost is what prevents the measure of plants from being infiniteo

2.2 Capacity utilization

We now describe in more detail the determination of the measure of plants, m_t, the amount of productive and technological capital, k_t and a_t, assigned to a plant; and the fraction of plants that operate, n_to Consider first the problem faced by a plant's manager after s_t has been observed, the price p_t is known and m_t, k_t and a_t have already been determinedo First, the amount of energy used to mn the plant will be equal to $\gamma k_t/a_t$ since any higher amount beyond that level has zero marginal productivityo Likewise, the number of hired workers will be η. If the hourly wage is $w_t = W(K_t, A_t, p_t)$, where K_t is the aggregate stock of productive capital, A_t is the aggregate stock of technological capital, and p_t is the energy price, it will cost $w_t h_0 \eta + p_t \gamma k_t / a_t$ to operate the planto It is profitable to operate the plant only if the output produced by the plant exceeds this costo Hence, only plants with sufficiently large realized values of s_t will be operatedo That is,

$$h(s_t) = \begin{cases} h_0 & \text{if } (z + s_t) B k_t^{\theta} \geq w_t \eta + p_t \gamma \frac{k_t}{a_t}, \\ 0 & \text{otherwiseo} \end{cases}$$

Since aH plants are ex-ante identical and $\theta < 1$, aH plants will be assigned the same amount of productive and technological capital: $k_t = K_t/m_t$ and $a_t = A_t/m_t$. Consequently, in equilibrium, there exists a threshold level, $s(K_t, A_t, p_t, m_t)$, below which the plant will not be operated. This implies that the equilibrium value far $s(K_t, A_t, P_t, m_t)$ is given by the solution to the foHowing equation:

$$[z + s(K_t, A_t, p_t, m_t)]\, B\left(\frac{K_t}{m_t}\right)^{\theta} = w_t\eta + p_t\gamma\frac{K_t}{A_t}. \tag{3}$$

Since s_t is unifarmly distributed, the fraction of plants that will operate is

$$n_t \equiv n(K_t, A_t, p_t, m_t) = \int_{s(K_t, A_t, p_t, m_t)}^{\sigma} \frac{1}{2\sigma} ds = \frac{\sigma - s(K_t, A_t, P_t, m_t)}{2\sigma}. \tag{4}$$

Therefare, we can rewrite equation (3) as

$$[z + \sigma - 2\sigma n_t]\, B\left(\frac{K_t}{m_t}\right)^{\theta} = w_t\eta + p_t\gamma\frac{K_t}{A_t}, \tag{5}$$

which, in turn, determines the equilibrium value of n_t given m_t, W_t, P_t, K_t and A_t.

Next, we consider the problem of the manager of a prospective plant befare s_t has been realized and the energy price P_t is known. The manager must farecast the wage W_t, and the rental prices of both types of capital, $r_{kt} = r_k(K_t, A_t, p_t)$ and $r_{at} = r_a(K_t, A_t, p_t)$, respectively, to compute the plant's expected profit. Far a given value of the energy price expected profit is given by,

$$E\left[(z + s_t) B k_t^{\theta} h(s_t) - w_t\eta h(s_t) - p_t\gamma\frac{k_t}{a_t}h(s_t) \mid \Omega_t\right] - r_{kt}k_t - r_{at}a_t$$
$$\int_{s(K_t, A_t, p_t)} \left[(z + s_t) B k_t^{\theta} - w_t\eta - p_t\gamma\frac{k_t}{a_t}\right] h(s_t)\frac{ds}{2\sigma} - r_{kt}k_t - r_{at}a_t.$$

Where $\Omega_t = (K_t, A_t, p_t)$ denotes the infarmation set available to the plant's manager befare the idiosyncratic technology shock has been realized. After solving the integral, the plant's expected profit, given aH input prices, is

$$= (z + \sigma(1 - n_t))\, n_t B k_t^{\theta} h_0 - w_t\eta n_t h_0 - p_t\gamma\frac{k_t}{a_t}n_t h_0 - r_{kt}k_t - r_{at}a_t, \tag{6}$$

where we have used equation (4) to eliminate $s(K_t, A_t, P_t, m_t)$. The problem faced by the plant's manager befare the energy price and the idiosyncratic technology shock are both realized, shown in expression (2), can be written as

$$\max_{k_t,a_t} E\left\{(z+\sigma(1-n_t))\,n_tBk_t^{\theta}h_0 - w_t\eta n_th_0 - p_t\gamma\frac{k_t}{a_t}n_th_0 - r_{kt}k_t - r_{at}a_t \mid \Omega_t^*\right\},$$

The solution to this problem will determine the demand far both types of capital at the plant level. Since in equilibrium $k_t = K_t/m_t$ and $a_t = A_t/m_t$, we can use the first arder conditions of this problem to obtain the following equilibrium conditions far the rental prices of both types of capital:

$$E\left\{\theta\,(z+\sigma(1-n_t))\,n_tB\left(\frac{K_t}{m_t}\right)^{\theta-1}h_0 - p_t\gamma\frac{m_t}{A_t}n_th_0 - r_{kt} \mid \Omega_t^*\right\} = 0, \tag{7}$$

$$E\left\{p_t\gamma\frac{K_t}{A_t^2}m_tn_th_0 - r_{at} \mid \Omega_t^*\right\} = 0. \tag{8}$$

Taking into account (7) and (8), we can rewrite maximized expected profit in expression (6) as

$$\Pi\left(K_t, A_t, \frac{K_t}{m_t}, \frac{A_t}{m_t}\right) = E\left\{(1-\theta)\,(z+\sigma(1-n_t))\,n_tB\left(\frac{K_t}{m_t}\right)^{\theta}h_0 - w_t\eta n_th_0 - p_t\gamma\frac{K_t}{A_t}n_th_0 \mid \Omega_t^*\right\}.$$

Finally, we know that the number of established plants, m_t, at the beginning of period t will be that which makes

$$\Pi\left(K_t, A_t, \frac{K_t}{m_t}, \frac{A_t}{m_t}\right) = 0. \tag{9}$$

2.3 Households

The economy is populated by a large number of infinitely lived households and the total number of households is one. Households are ex-ante identical and seek to maximize expected discounted lifetime utility,

$$E_0\sum_{t=0}^{\infty}\beta^t\left(\log C_t + \alpha\log l_t\right),\ \beta \in (0,1),\ \alpha > 0, \tag{10}$$

where c_t denotes consumption at date t and l_t denotes leisure. Households are endowed with one unit of time that can be allocated to either work, h_t, or leisure, so $l_t = 1 - h_t$. Following Rogerson (1988) and Hansen (1985), labor is assumed to be indivisible; in a given period households can work either a shift of length h_a, or not at all, where h_a is an exogenous parameter. Households also accumulate both types of capital which they rent to individual plants to be used in production.[6] Capital of each type held at the beginning of the subsequent period is

$$K_{t+1} = (1-\delta)K_t + X_{kt}, \tag{11}$$

$$A_{t+1} = (1-\delta)A_t + X_{at}, \tag{12}$$

where X_{kt}, X_{at} denote investment in productive and technological capital, respectively, undertaken in period t. Investing one unit of resources in technological capital costs $1 + g\left(\frac{X_{at}}{A_t}\right)$ units of resources. That is, there are adjustment costs associated to changes in the stock of installed technological capital. There are two ideas that we want to capture introducing this specification of adjustment costs: changing the level of energy efficiency in the economy (augmenting the level oftechnological capital) and changing the use of capital (converting productive capital into technological capital) are costly. Our strategy here is not particularly concerned with the role of aggregate adjustment costs as determinants of investment demand; rather, it tries to stress the different nature of the two uses of capital in our modelo

The budget constraint of the representative household is

$$c_t + X_{kt} + X_{at}\left[1 + g\left(\frac{X_{at}}{A_t}\right)\right] \leq w_t h_t + r_{kt}K_t + r_{at}A_t + D_t, \tag{13}$$

which shows that total income in a given period consists of labor income, capital income from both types of capital and dividend payments, D_t, from the ownership of plants. Since households are ex-ante identical and there is a continuum of plants, they will diversify their portfolios in such a way that realized dividends are the same for all households.

2.4 Equilibrium and the quasi-social planner problem

2.4.1 Household's problem

Since labor is indivisible in this economy, we follow Rogerson (1988) and Hansen (1985) by allowing agents to trade employment lotteries. Given a wage rate, w_t, households choose

6We could have modelled investment at the plant level. We are implicitly assuming that managers of plants can completely ensure themselves against idiosyncratic risk at the plant level, and, hence, aggregate investment is not affected.

a probability of working h_0 hours, denoted π_t, in order to maximize expected utility. We assume that households are paid w_t per hour when they work (which happens with probability π_t) and that they have access to a market for unemployment insurance. Since preferences are additively separable in consumption and leisure households will insure themselves so that their consumption levels are independent of whether or not they work. Given this, the household's optimization problem can be written as follows:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t (\log c_t + \alpha \log(1 - h_0)\pi_t)$$

subject to (11) and (12), and

$$c_t + X_{kt} + X_{at}\left[1 + g\left(\frac{X_{at}}{A_t}\right)\right] \leq w_t \pi_t h_0 + r_{kt}K_t + r_{at}A_t + D_t, \tag{14}$$

where D_t is the dividend paid to the household as an owner of the plants locations. Given that there is a continuum of plants and households are risk averse and ex ante identical, they will diversify their portfolios in such a way that realized dividends are the same for all households.

2.4.2 Equilibrium

Definition 1 *An equilibrium for this economy, given $\{p_t\}$, is an allocation $\{c_t, \pi_t, K_t, A_t, m_t, n_t\}$ and a vector of prices $\{w_t, r_{kt}, r_{at}\}$ such that* (i) *$\{K_t, A_t, m_t, n_t\}$ satisfies* (4), (7), (8), (9) *given $\{w_t, r_{kt}, r_{at}, p_t\}$, (ii), $\{c_t, \pi_t, K_t, A_t, m_t, n_t\}$ solves the consumer's problem given $\{w_t, r_{kt}, r_{at}, p_t\}$, and* (iii) *which the labor market clears: $\pi_t = m_t n_t \eta$, for all t.*

2.4.3 The quasi-social planner's problem

It is convenient for computational purposes to write a quasi-social planner's problem whose solution is the competitive equilibrium allocation of this model economy. Obviously, we can not express a social planner problem since the energy price is exogenously given. We start deriving value added for this economy. To do this, we substitute equilibrium values in the household's budget constraint shown in (14). Aggregate realized dividends in this economy are

$$D_t = \left[(z + \sigma(1 - n_t))\, n_t B k_t^{\theta} h_0 - w_t \eta n_t h_0 - p_t \gamma \frac{k_t}{a_t} n_t h_0 - r_{kt}k_t - r_{at}a_t\right] m_t. \tag{15}$$

Taking into account that $\pi_t = m_t n_t \eta$, $K_t = k_t m_t$, and $A_t = a_t m_t$, we can substitute this expression into the household's budget constraint and its right hand side can be written as

$$(z+\sigma(1-n_t))\,n_t B K_t^{\theta} m_t^{1-\theta} h_0 - p_t\gamma\frac{K_t}{A_t} m_t n_t h_0. \tag{16}$$

The right hand side of this expression represents per capita value added at period t. It is easy to show that per capita value added can also be written in a more conventional way as a function of aggregate capital and labor. To do that we would need to solve the following problem:

$$F(K_t, A_t, p_t, H_t) = \max_{n_t}\left\{(z+\sigma(1-n_t))\,n_t B K_t^{\theta} m_t^{1-\theta} h_0 - p_t\gamma\tfrac{K_t}{A_t} m_t n_t h_0\right\}$$

$$\text{subject to } H_t = h_0 m_t n_t \eta,$$

whose solution is given by

$$F(K_t, A_t, p_t, H_t) = \psi_t B K_t^{\theta} H_t^{1-\theta} - p_t\tfrac{\gamma}{\eta}\tfrac{K_t}{A_t} H_t,$$

$$\psi_t = (z+\sigma(1-n_t))\,(n_t h_0)^{\theta}\,\eta^{\theta-1}.$$

This expression shows that value added displays constant returns to scale in the three factors of production. Notice that total energy use in this notation is $E_t = \frac{\gamma}{\eta}\frac{K_t}{A_t} H_t$. Two things are worth noting: first, factor shares are not constant in this environment, and second, total factor productivity defined as the Solow's residual depends on the fraction of operated plants.

We can define a quasi-social planner's problem recursively in two steps. First, the planner chooses the number of plants that maximizes the expected future value of discounted utility before the energy price is realized, conditional on the information set $\Omega_t^* = \{K_t, A_t\}$. Second, the planner decides the levels of consumption, investment in both types of capital, and the fraction of plants that will operate once the energy price shock is realized. Denoting with a tilde next period values, this problem can be written as:

Solving backwards, we first take as given the number of prospective plants Given the number of plants, the problem solved is

$$W(K, A, p, m) = \max_{C, Xk, Xa, n \in [0,1]} \{\log(c) + \alpha \log(1 - h_0) mn\eta + \beta V(K', A', p)\}$$

subject to

$$c + X_k + X_a\left[1 + g\left(\frac{X_a}{A}\right)\right] \leq (z + \sigma(1-n))\, nBK^{\theta}m^{1-\theta}h_0 - p\gamma\frac{K}{A}mnh_0,$$

$$K' = X_k + (1 - \delta)K,$$

$$A' = X_a + (1 - \delta)A.$$

where

$$V(K', A', p) = \max_{m'} E\{W(K', A', p', m') | p\}.$$

3 Calibration

We calibrate the model so that the steady state of the non-stochastic version of the model matches sorne stylized facts of the US economy. First of aU, we briefiy discuss a the data we use. We construct series for the energy price and energy use as weU as economic aggregates for the period 1960-99. Since we assume that aU energy is imported in our model economy, we need to construct measures of value added, investment and capital stock excluding, respectively, output, investment, and capital of energy producing sectors. To obtain an aggregate series on energy use for the U.S. economy, we construct a constant-price measure of the use of electricity, petroleum, coal, and natural gas. Correspondingly, our aggregate energy price is the ratio of energy use measured in current prices to energy used measured in constant prices. A fuU explanation of the sources and methods used in our data construction is given in the Appendix.

The time period is a year. In our model economy GDP, or value added, is gross output net of energy expenditures,

$$y = (z + \sigma(1-n))\, nBK^{\theta}m^{1-\theta}h_0 - p\gamma\frac{K}{A}mnh_0.$$

The share of aggregate capital in gross output is chosen to be 38.96 percent, which is equivalent to a share of 40 percent of value added. Energy expenditure as a fraction of gross output is 3.04 percent. Now we need to set a value for the capital-value added ratio. We take the view that technological capital is tangible capital; therefore, accordingly to our data, we set $\frac{K+A}{y} = 2.6824$. For the same reason, we assume that productive capital and technological capital depreciate at the same rateo This implies that the net return of both types of capital is the same, $r_k = r_a$. The investment share is chosen to be equal to 0.2387. Since we do not have a good idea of what fraction of total capital is used for energy saving purposes, we calibrate our model to match the observed ratio of energy expenditures to GDP for the period considered. This, along with the chosen capital-output ratio and the share of investment in GDP, will give us the steady state values of productive and technological capital. The adjustment cost function $g(.)$ is assumed to be homogeneous of degree zero in X_a and A. It satisfies that $g(\delta) = 0$ and $g'(\delta) = 0$, so that the steady state capital stock is not affected by the introduction of adjustment costs. We follow King and Rebelo (1993) and set equal to 15 the elasticity of marginal adjustment costs.

The parameters η and z are set equal to 1, and h_a is set equal to 0.38. The average of the time spent in the market is set equal to 0.31, $m_t n_t \eta h_0 = 0.31$, and the parameter B is chosen so that the level of GDP, or value added, y is equal to 1. Following Cooley et al. (1995)and Bresnahan and Ramey (1993), we define capacity utilization rate as level of output divided by the level of output that would be obtained if all plants produce at a given periodo Thus, the fraction of plants operated in equilibrium is chosen so the capacity utilization rate is 82 percent,

$$\frac{(z+\sigma(1-n))\, n\, B K^{\theta} m^{1-\theta} h_a - p\gamma \frac{K}{A}\, m\, n\, h_a}{B K^{\theta} m^{1-\theta} h_a - p\gamma \frac{K}{A}\, m\, h_a} = 0.82.$$

Finally, following Finn (1995), Kim and Loungani (1992) and Atkeson and Kehoe (1999), we estimate an ARMA(1,1) process for the international price of energy parameterized by

$$\log p_{t+1} = (1-p)\log \overline{p} + p \log p_t + \phi\epsilon_t + \epsilon_{t+1},$$

where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ and $\overline{p}$ is the average energy price in the data. Using annual energy price data from the 1960-1999 period, $p = 0.8872$, $\overline{p} = 0.8979$, $\phi = 0.3138$ and $\sigma_\epsilon = 0.0909$. The values obtained for the rest of the parameters are

β	α	σ	θ	γ	δ	B
0.9427	2.0149	1.0568	0.3896	0.0095	0.0890	1.8476

The steady state values of sorne aggregate variables,

y	K	A	m	n	E
1.0000	2.4730	0.2094	1.4951	0.5457	0.0350

4 Results

To assess the ability of our model to account for the time-series data on energy expenditure and energy use, we simulate it, feeding in the data on the energy price to obtain predictions for the time paths of energy use, energy expenditure, capital and output. Figure 3 shows the results of our model in energy use and energy expenditure. To measure the evolution of energy use we have plotted the logarithm of energy used per unit of value added. Energy expenditure is the logarithm of energy expenditure per unit of value added. Clearly, the model closely tracks the data. Figure 4 shows the evolution of the logarithm of the capital-energy ratio predicted by our model and compares it to the data. Figure 5 shows the predicted evolution of the capacity utilization rate and compares it to the corresponding series of total capacity utilization for the U.S. economy. Figure 6 compares the model capacity utilization rate to that of manufacturing in the U.S. economy. As we can see, our model does well at capturing the drop in the capacity utilization rate during the period of major oil price shocks, 1975-1981, and offers worse predictions for the later period 1981-1999. We need to keep in mind that we only have considered energy price shocks and that we have abstracted away from aggregate technology shocks. Consequently, energy price shocks are an important source of variability in capacity utilization in our model, which is consistcnt with thc micro cvidcncc providcd in Brcsnahan and Ramcy (1993).

The intuition about the evolution of our simulated data is the following: Since in the short mn capital and energy are complements, given the capital installed at the plant, the energy bill is also given and is independent of the productivity of the planto Thus, only plants with high productivity operate. Therefore, capacity utilization rate immediately falls. This is followed by a reduction in the number of plants established next periodo The reason for this fall in the number of plants is that the persistence in the energy price makes the managers of the prospective plants to expect a high energy price. But the reduction in the energy bill through this channel is very limited. The substitution of productive capital for technological capital is costly and takes time, as well as it is investing in new technological capital. Therefore, energy expenditure fiuctuates very much, whereas energy use does not.

To understand the role of variable capacity utilization and capital adjustment costs we also simulate our model without these features. The later turns out to be crucial for energy use being inelastic in time-series data and elastic in cross-section data whereas the former determines indeed most of the variability in capacity utilization. Figures 7, 8, 9, and **10** show the results of the model in such a case. In the absence of adjustment costs capital can be swiftly reallocated from its productive to its technological use. This is exactly what happens when energy price increases. The response of the capacity utilization rate is then negligible as far as we allow for the number of plants to be determined once the energy price shock is known. Basically, our model behaves as the Pindyck and Rotemberg *(1083)* putty-putty modcl with physical capital subjcct to adjustmcnt costs. Encrgy use is very responsive, whereas energy expenditure is noto The results for this case lead us to conclude that variable capital utilization is less important than costly capital reallocation at influencing the aggregate response to energy price shocks. Furthermore, these findings are in line with those reported by Cooley et al. (1995) according to which equilibrium business cycles with idle resources and variable capacity utilization are similar to those of a standard business cycle model.

Finally, we consider the effect on output of an energy tax that leads to a doubling of energy prices in this model. We assume that the revenue collected is spent on public goods that affect neither the steady-state real returns nor the steady-state marginal product of capital. In this model a doubling of the energy price leads to a 1.75 percent fall in output and a 2.66 percent drop in the capital stock. The revenue raised from the tax in the long run is 2.18 percent of long run GDP. These numbers are comparable to those estimated by Gouldcr (1002), Gouldcr (1003), Gouldcr (1995), and Jorgcnson and Wilcoxcn(1003). Thcy arc smallcr than thosc found by Atkcson and Kchoc (1999). Thc rcason is that in our model the ability to substitute energy for capital is greater than in the Atkeson and Kchoc (1999) framcwork.

5 Final comments

In this paper we have built a version of the neoclassical growth model augmented with a second type of physical capital that acts purely as an energy saving device. We interpret this capital good as induced energy-saving innovation and we will call it technological capital. The model is able to justify two salient features of the data: First, that in time series, energy use is not very responsive to energy price changes, whereas energy expenditure fluctuates mucho Secondly, in cross section data for different countries big international differences in energy prices do not lead to big differences in per capita output. These findings point to a very specific and potentially important friction that influence the aggregate response to energy price changes, namely, costly capital reallocation. Additionally, to any capital use corresponds a given energy requirement and production takes place at the plant level. The standard features of the neoclassical growth model were otherwise preserved. Consequently, we consider our model specification a promising tool for business cycle analysis.

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Appendix

In this Appendix we document the construction of the data series we use in the empirical part of the paper. We obtain data from two sources: Annual Energy Review (2000) and National Income and Product Accounts. The data we use can be accessed in the addresses: http://www.eia.doe.gov and http://www.bea.doc.gov. From now on we will refer to each source as AER, and NIPA, respectively. We follow Atkeson and Kehoe (1999) procedure to construct the data series for the period studied, 1960-99. Our updated data set is available upon request.

A Energy price, use, and expenditures series

The energy data covers the energy consumption of end users. We consider four forms of energy: coal, petroleum, natural gas and electricity. AER (Table 2.1) gives data on total energy consumption by end users measured in British termal units (BTUs) disaggregated into the four forms of energy considered. We denote these data on energy use for each type of energy by Q_{it}, where the index i denotes the form of energy.

This measure E_{it} is already net of energy use of the electricity sector. There are no corresponding data on the energy use by type of the other three energy-producing sectors. There are no data on energy consumption by the natural gas sector. According to Atkeson and Kehoe (1999), for the period 1960-94 the energy consumed by the coal and petroleum sectors is about 1/60 of total energy consumption reported in Table 2.1. We take this number as an estimate of total energy consumption by these two sectors for the period 1960-99 and assume that the BTUs consumed are divided among the four forms of energy according to the averages shares of the industrial sector. These shares are constructed from the data contained in Table 2.1.

We construct a constant-price measure of energy use. We choose the base year to be 1987 and define energy use to be $E_t = \sum_i Q_{it}P_{i0}$, where P_{i0} is the price in dollars per BTUs of energy type in 1987 from AER. For coal, natural gas and petroleum we use the production price series (AER, Table 3.1). For electricity, we use the retail price of electricity sold by electric utilities (see AER, Table 8.13). All prices are real prices in dollars of 1996. In Table 8.13 the price for electricity is in cents per kilowatt-hour. We use AER Table 13.6 to convert the price to cents per BTUs.

We construct the energy price defiator as

$$P_t = \frac{\sum_i Q_{it}P_{it}}{\sum_i Q_{it}P_{i0}}.$$

Finally, nominal expenditure is $P_t \cdot E_t = \sum_i Q_{it} P_{it}$.

B Output, consumption, investment, and the capital stock

We follow the method described by Cooley and Prescott (1995) to construct broad measures of output, consumption, investment, and the capital stock. For output, investment, and capital we subtract from each of these series the corresponding series for the energy producing sector. To calculate the output of the energy sector, we sum the value added of the coal, petroleum, electricity and natural gas sectors. The value added of each sector is assumed to be equal to the value of domestic production of that sector. The series of domestic production are in the AER (Table 5.1 for oil; Table 6.1 for natural gas; Table 7.2 for coal; and Table 8.1 for electricity). Real gross output is the sum of value added and the expenditure on energy. The investment in the energy sectors is defined as the sum of total investment in sectors defined as coal mining, oil and gas extraction, and electric and gas services. Similarly, we have subtracted from the aggregate capital stock that corresponding to the sectors mentioned. The data used for these series is the historical data on investment and net stock by industry that can be accessed in the NIPA page.

C FOCs of the quasi-social planner's problem

We have

$$\frac{\partial W}{\partial c} = u'(c) - \lambda = 0,$$

$$\frac{\partial W}{\partial n} = \alpha \log(1 - h_0) m\eta + \lambda \left[(z + \sigma - 2\sigma n) B \left(\frac{K}{n}\right)^{\theta} h_a - p\gamma \frac{K}{A} h_a \right] \geq 0,$$

$$\frac{\partial W}{\partial X_k} = -\lambda + \mu_k = 0,$$

$$\frac{\partial W}{\partial X_a} = -\lambda \left(1 + g\left(\frac{X_a}{A}\right) + \frac{X_a}{A} g'\left(\frac{X_a}{A}\right)\right) + \mu_a = 0,$$

$$\frac{\partial W}{\partial K'} = -\mu_k + \beta \frac{\partial V(K', A', p)}{\partial K'} = 0,$$

$$\frac{\partial W}{\partial A'} = -\mu_a + \beta \frac{\partial V(K', A', p)}{\partial A'} = 0,$$

$$\frac{\partial V(K', A', p)}{\partial K'} = E_p \left\{ U'(C') \cdot \theta (z + \sigma(1 - n'))n'R \left(\frac{K'}{m'}\right)^{\theta - 1} h_0 - p'\gamma \frac{m'}{A'} n' h_0 + (1 - \delta) \right\},$$

$$\frac{\partial V(K', A', p)}{\partial A'} = E_p \left\{ U'(C') \left[p'\gamma \frac{K' m'}{(A')^2} n' h_0 + (1 - \delta) - g' \left(\frac{X'_a}{A'}\right) \left(\frac{X'_a}{A'}\right)^2 \right] \right\},$$

$$\frac{\partial V(K', A', p)}{\partial m'} = E_p \left\{ \text{cdog}(1 - h_0) n' \eta + U'(C') \left[(1 - \theta)(z + \sigma(1 - n'))n'R \left(\frac{K'}{m'}\right)^{\theta} h_0 - p'\gamma \frac{K'}{A'} n' h_0 \right] \right\}$$

$$= 0.$$

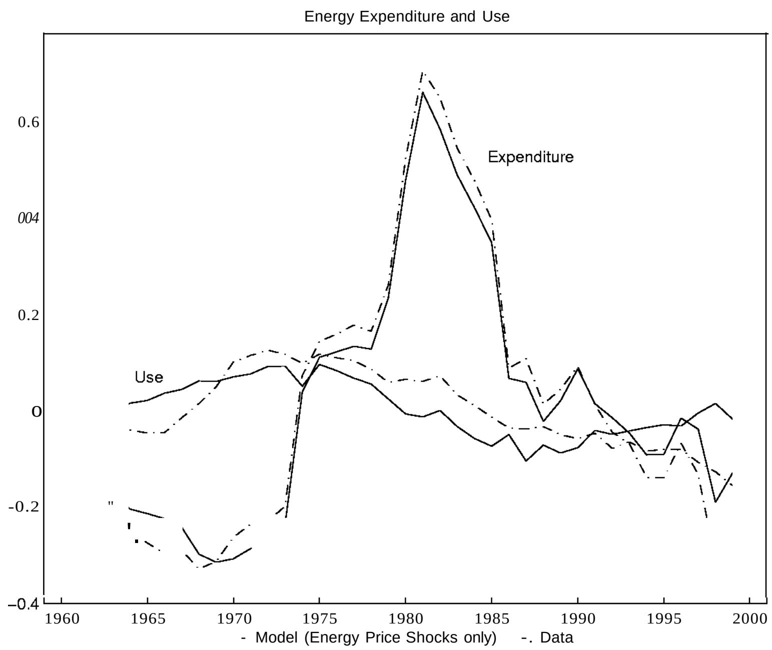

Figure 3: Energy use and energy expenditure.

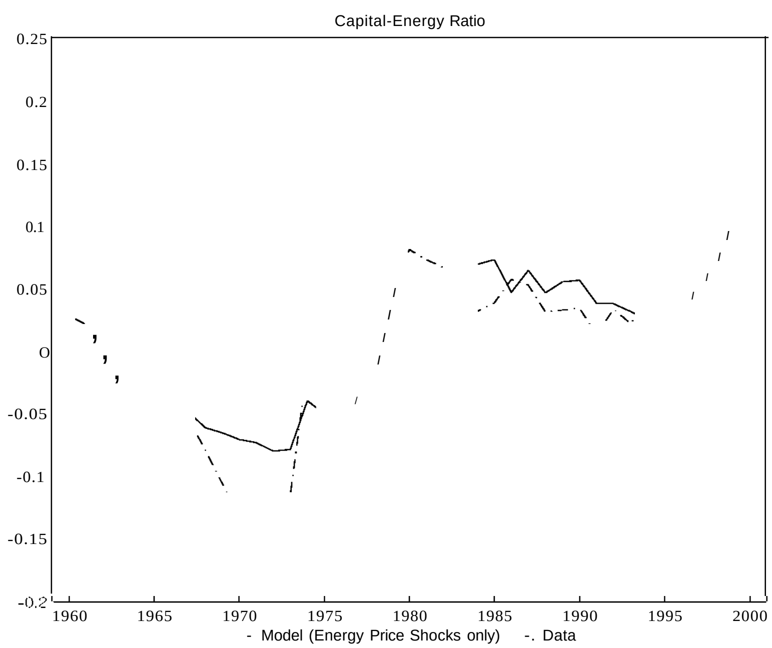

Figure 4: Evolution of the capital-energy ratio.

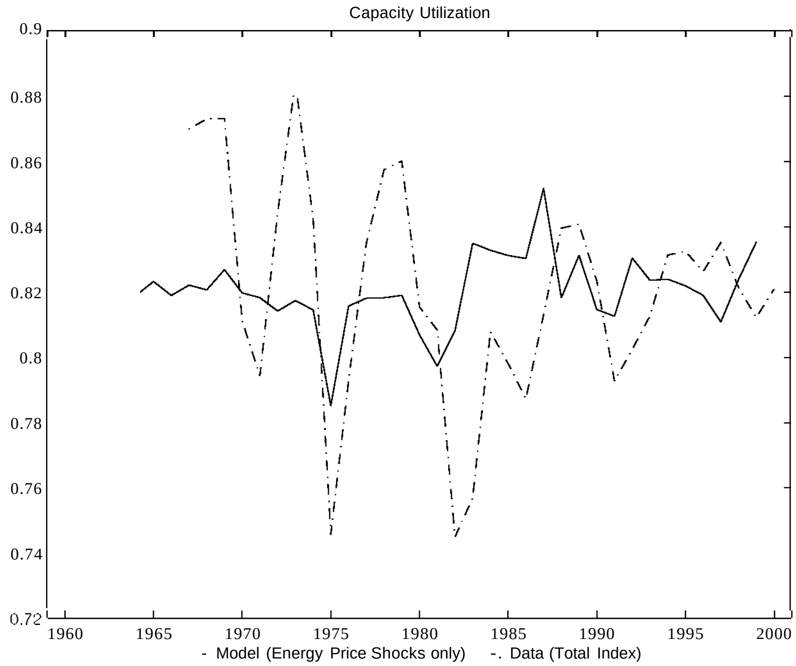

Figure 5: Capacity utilization (total).

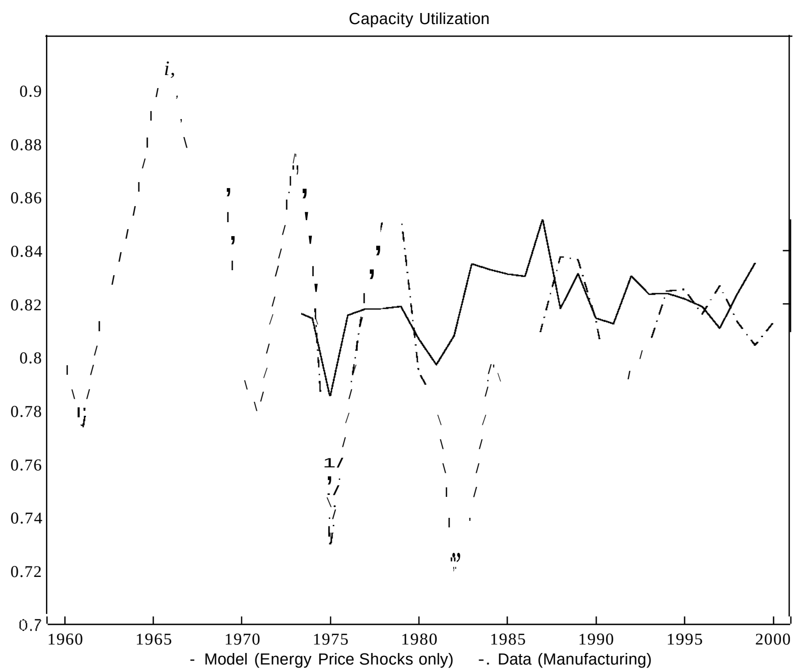

Figure 6: Capacity utilization rate (manufacturing).

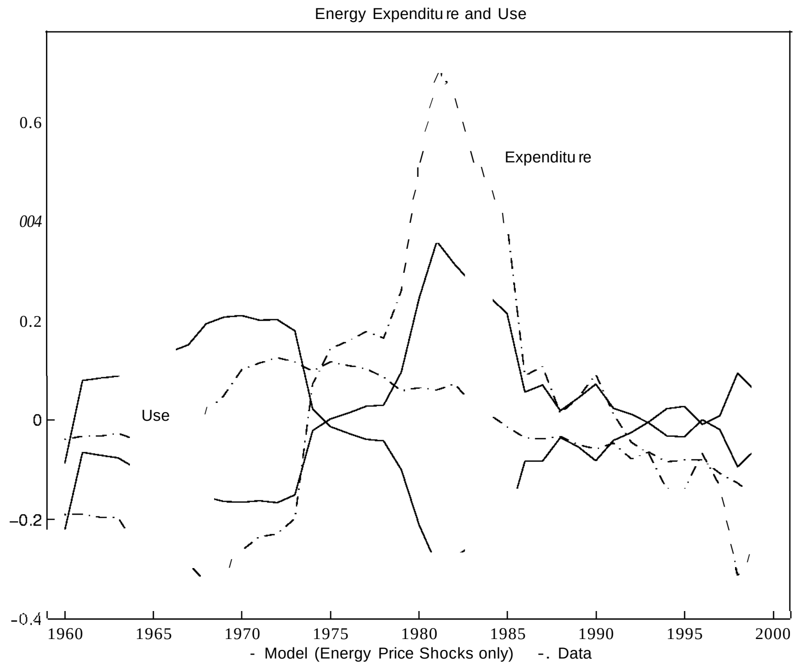

Figure 7: Energy use and energy expenditure with a fixed number of plants and no adjustment costs .

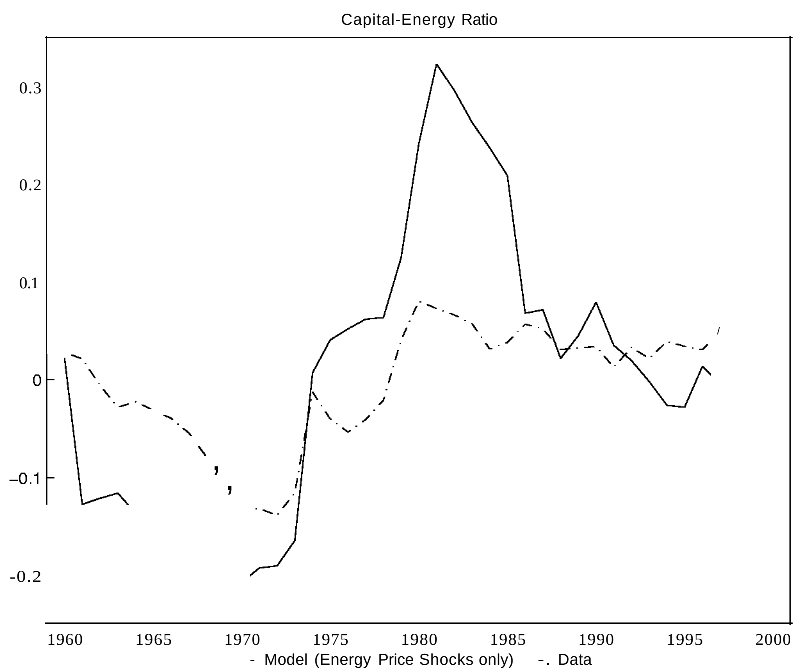

Figure 8: Evolution of the capital-energy ratio with a fixed number of plants and no adjustment costs.

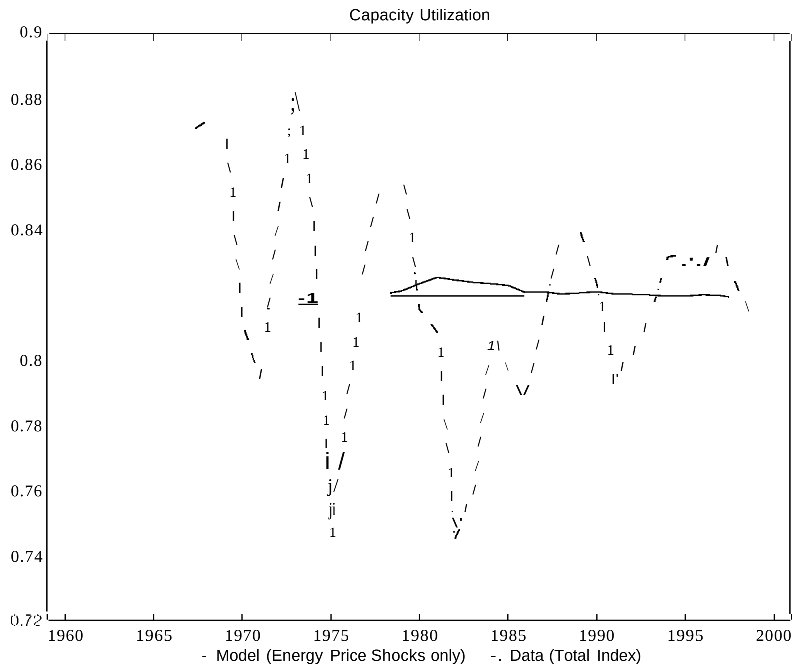

Figure 9: Capacity utilization (total) with a fixed number of plants and no adjustment costs.

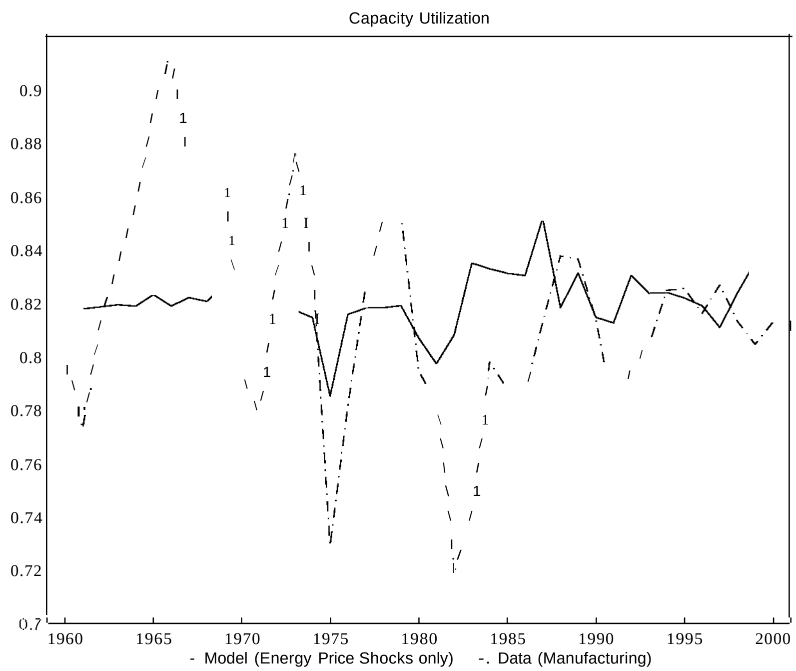

Figure 10: Capacity utilization rate (manufacturing) with a fixed number of plants and no adjustment costs.