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ABSTRACT

Long memory in the volatility of individual return series and in the volatility of equal-weighted portfolios constituted by these individual return series is analyzed to see if the memory characteristic of the volatility representation is correlated with the portfolio characteristics of size, standard deviation of returns, and firm's beta. The sample is also split in two to determine whether these characteristics change over time. A U-shaped pattern emerges from the analysis of the portfolios created on the basis of size, standard deviation and beta. Portfolios constructed with individual return series whose portfolio characteristics lay in the top quintile generally displayed long memory in volatility. Similarly, the portfolios made up of individual return series whose portfolio characteristics lay in the bottom quintile also displayed long memory in volatility, though to a lesser degree. Finally, the medium portfolios displayed short memory in volatility.

JEL Classifications: C43, G10, G30, C40

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On the Nature of the Dependence in the Volatility of U.S. Stock Returns

by

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Abstract

Long memory in the volatility of individual return series and in the volatility of equal-weighted portfolios constituted by these individual return series is analyzed to see if the memory characteristic of the volatility representation is correlated with the portfolio characteristics of size, standard deviation of returns, and firm's beta. The sample is also split in two to determine whether these characteristics change over time, since it is expected that a market is more informationally efficient as it develops. A U-shaped pattern emerges from the analysis of the portfolios created on the basis of size, standard deviation and beta. Portfolios constructed with individual return

[*]Many thanks to my primary advisor, Pedro J. F. de Lima. All errors are mine alone.

series whose portfolio characteristics lay in the top quintile generally displayed long memory in volatility. Similarly, the portfolios made up of individual return series whose portfolio characteristics lay in the bottom quintile also displayed long memory in volatility, though to a lesser degree. Finally, the medium portfolios displayed short memory in volatility. Over time, the dependence in volatility increases for the top quintile portfolios, and decreases for the other portfolios. Results for the individual return series also suggested that individual return series in the top and middle mid-sample size category displayed long memory, whereas the individual return series in the lowest grouping tended to display short memory in volatility. The dependence in volatility for the individual return series increased over time.

1 Introduction

The potential predictability of stock market returns is an area that is actively researched by academics and people seeking to "beat the market" alike. Until recently the thrust of this interest has revolved around the first moment, or the expected value of stock market returns. With the onset of such notions as risk management and derivative securities (e.g., options), the volatility of stock market returns has garnered its own following. This interest in volatility has honed researchers' understanding of the nature of market efficiency. While in general it has been accepted that the best forecast of tomorrow's stock price is today's price, conditional on the past price or return series, or that weak-form market efficiency holds, it has been shown that stronger hypotheses of market efficiency are not supported by the empirical evidence[1]. These stronger hypotheses are refuted given the presence of dependence in the higher moments of stock returns.

A manifestation of this sort of dependence is found in the slow decay of the autocorrelation function of measures of volatility of stock market return indices. The slow decay of the autocorrelation function is called long memory, and its presence in the volatility of stock market return indices implies that the movement of returns is not perfectly unpredictable or entirely random. This characteristic of stock market return indices has been well-documented by Ding, Granger, and Engle (1993), de Lima and Crato (1994), Bollerslev and Mikkelsen (1993) and Harvey (1993). Barnes (1998) finds evidence of long memory in the volatility of individual return series as well.

In this study, the predictability of the volatility of returns, and, consequently, the idea of market efficiency are further explored. The size (market capitalization), beta, and standard deviations of returns of a firm's security are used as a basis for studying the nature of the dependence in the volatility of stock returns. A sample of return series is extracted from the Center for Research in Security Prices (CRSP) and is sorted by these three statistics. Long-memory measures and tests are performed

[1]Namely those hypotheses which posit a martingale with independent disturbances.

on the volatility of individual returns sorted by mid-sample size, and on portfolios constructed with returns of securities which have particular portfolio characteristics according to these three measures.

Size is chosen because there is evidence that it may play a role in the autocorrelation of both the levels and the volatility of return series (c.f., de Lima *et al.* (1998)). Friedman (1976) suggests that variability in price changes could affect the ability of the price system to direct market activity and could increase the cost of assimilating information. These manifestations of variability in inflation have implications for the informational efficiency of the stock market. Therefore, the standard deviation of nominal returns could affect the dependence in the volatility of returns. Beta is analyzed because it is a measure of firm risk relative to market risk, and because the model behind the firm's beta is often used as a model for normal returns. Abnormal returns are measured from such models, and are an important element of the efficient market hypothesis.

The patterns found in the dependence in the volatility of returns is dissimilar to that found in the mean of returns. In the levels of returns, the autocorrelation of returns is strongest for the low market capitalization portfolio and weakest for the high market capitalization portfolio. Too, the autocorrelations tend to weaken over time.[2] In contrast, there is a nonlinear relationship between portfolio size (low,

[2] The results for the levels are surveyed in Campbell, Lo and MacKinlay (1997).

medium and high) and dependence in volatility, and the dependence in the volatility of the high portfolios becomes stronger over time, as evidenced in the two sub-samples. The medium portfolios tend to display the least dependence in volatility across the entire sample, while the low and high portfolios displayed long memory in volatility, producing a U-shaped pattern. For the low and medium portfolios dependence in volatility decreases over the sample period.

An equal-weighted index of all return series included in this study also displays long memory in volatility, and the dependence in the volatility increased in the second half of the sample. Individual return series tend to exhibit long memory in volatility according to the nonparamteric measure and test, and this dependence increases over time too. When these individual return series are grouped into three groups according to their mid-sample market capitalization, the average measure and statistic of long memory for the volatility of all individual return series within that group increases with market capitalization. When the sample is split in two, it is evident that the persistence again increases over time. Again, this is in stark contrast to the results for the levels.

Section two discusses the source of the data, how the series are filtered, how the portfolio measures are calculated, and how the indices are constructed. Concepts of dependence are provided in section three. The methodology of the analysis and the results are developed in the fourth section, and section five concludes.

2 Data

The basic data are daily individual return series extracted from CRSP for which there were at least 1029 observations available December 31, 1994.[3] To be selected these return series also had to trade on the NYSE, and information on market capitalization had to be available. Portfolios are created from these individual return series according to the CRSP methodology of portfolio ranking by a security's market capitalization, standard deviation and beta. Yearly portfolio rankings based on market capitalization are available from CRSP for these sampled firms, but portfolio numbers ranked by standard deviation of returns and beta are not available. Instead, these last two yearly measures are calculated separately for the individual return series and thus the rankings are relative only to other securities in this particular sample. Market capitalization rankings, however, are available both relative to this sample, and relative to the entire sample available from CRSP. All return series are filtered before being tested or added to a portfolio using the method developed by Tauchen and Gallant (1994) which is described in appendix A. The standard deviation of returns is the usual unbiased estimate, and beta is calculated as in CRSP: $\beta_i = \frac{\sum(ret_{i,t} * mret3_t) - (1/n)*(\sum ret_{i,t})*(\sum mret3_t)}{\sum(mret_t * mret3_t) - (1/n)*(\sum mret_t)*(\sum mret3_t)}$, where $ret_{i,t} = \log(1+$return for security i on day $t)$, $mret_t = \log(1+$[value-weighted] market return for security i on day $t)$,

[3] This number of observations allows for the remainder of 1024 observations, a fourier frequency, after the series are filtered for AR(5).

$mret3_t = mret_{t-1} + mret_{t-2} + mret_{t-3}$ (a three day moving market window), and $n =$ (number of observations for the year). Beta is thus the regression coefficient of the market return regressed on the individual security's return.

3 Some Definitions, Tests and Measures of Long Memory

3.1 Some Definitions: Long and Short Memory

When the autocorrelation function (ACF), $\rho(h)$, of a weakly stationary process is geometrically bounded, the process is said to display short memory, while if it decays hyperbolically it is said to display long memory. The notion of geometric decay found in the ACF of weakly stationary long-memory processes can be expressed formally as:

$$\rho(h) \sim Ch^{2d-1} \text{as } h \to \infty, \text{ where C} \neq 0 \text{ and } d < 0.5. \tag{1}$$

The correlate for short memory is

$$\mid \rho(h) \mid \leq Cr^{|h|} \text{ for some } C > 0,\ 0 < r < 1. \tag{2}$$

In the frequency domain, a process has long memory if its spectrum, $f(\lambda)$, is undefined near the origin, or

$$f(\lambda) \sim C \mid \lambda \mid^{-2d} \text{ as } \lambda \to 0 \text{ with } d \neq 0. \tag{3}$$

An AutoRegressive Fractionally Integrated Moving Average (ARFIMA) process is defined to be a process with the following representation

$$\begin{aligned}(1-B)^d\,\phi(B)\,\nu_t &= \theta(B)\,\eta_{t,}\\ \eta_t &\sim i.i.d.N.\left(0,\sigma_\eta^2\right).\end{aligned} \tag{4}$$

B is the usual backshift operator, $\phi(B)$ is the AR polynomial, $\theta(B)$ is the MA polynomial, and d is the order of fractional integration. When $d = 0$, this represents a short-memory process, when $d = 1$, this represents an ARIMA process. It has long memory when $0 < d \leq 0.5$, and it is nonstationary for $d > 0.5$.

3.2 Long Memory Tests

For comparability with the existing literature, only two statistical tests for and two measures of long memory are used here: The semiparametric Geweke and Porter-Hudak (1983) test, which is further developed by Robinson (1993), and the associated estimate of d, the differencing parameter, and the Lo (1991) modifications of the Hurst (1951) nonparametric tests which are the R/S statistic, known as the normalized rescaled range, and the Hurst exponent.[4]

[4] Pagan (1996) suggested the use of parametric tests. One of the problems with the tests used here is that the choices of truncation points and the number of autocovariances, q, to include in the R/S test are arbitrary. Pagan's (p. 16) criticism of the choice of q is "if you want to find long range dependence, keep q short; if you don't, make q long."

The null hypothesis of short memory for the Geweke and Porter-Hudak test is that the slope, d, in the regression of the natural log of the periodogram at low frequencies on some function of these frequencies is zero. Fourier frequencies of the periodogram up to m_u are analyzed. The reason for this truncation is that it is desirable to have an estimate of d that is undisturbed by the short-memory dynamics of the process. To avoid bias, Robinson proposed truncating the initial frequencies up to m_l. Use of this test involves experimentation, to determine how many frequencies should be used to calculate d. The null hypothesis of the t-test for short memory and antipersistence is that $\widehat{d} \leq 0$, against long-memory alternatives, $\widehat{d} > 0$. For $\mid \widehat{d} \mid > \frac{1}{2}$, the series is said to be nonstationary because it has infinite variance. If $\widehat{d} = 0$, it has short memory. When $-\frac{1}{2} < \widehat{d} < 0$, the series is antipersistent, and when $0 < \widehat{d} < \frac{1}{2}$, it is said to be a long-memory series. An antipersistent series is a series for which all autocorrelations and partial correlations are negative. When $\widehat{d} = -\frac{1}{2}$, the series is stationary but not invertible; when $\widehat{d} = \frac{1}{2}$, the series is invertible but nonstationary.

The normalized rescaled range statistic (R/S-statistic) is

$$Q(n, q) = \frac{R(n)}{S(n, q)}, \tag{5}$$

where $R(n)$, the adjusted range, is defined to be

$$R(n) = \max_{1 \leq \kappa \leq n} \left\{ \sum_{i=1}^{\kappa} X_i - \kappa \overline{X} \right\} - \min_{1 \leq \kappa \leq n} \left\{ \sum_{i=1}^{\kappa} X_i - \kappa \overline{X} \right\}, \tag{6}$$

and $S(n,q)$, the normalization factor, is the square root of the estimate for variance, $S^2(n,q)$,

$$S^2(n,q) = \sum_{j=-q}^{q} w_q(j)\,\widehat{\gamma}(j)\,. \tag{7}$$

$\overline{X}$ is the sample mean, $\widehat{\gamma}(j)$ are the estimated autocovariances and the weights, $w_q(j)$, come from the Bartlett window. When $q = 0$ the classical R/S-statistic is obtained. The Hurst exponent J is estimated by

$$\widehat{J}(n,q) = \frac{\ln(Q(n,q))}{\ln(n)}. \tag{8}$$

Mandelbrot and Taqqu (1979) prove that if the series has long memory this exponent should converge to a number larger than $\frac{1}{2}$. The relationship between $\widehat{J}$ and $\widehat{d}$ is that $\widehat{J} = \widehat{d} + \frac{1}{2}$.

Using Monte Carlo experiments, de Lima, Breidt, and Crato (1998) look at the finite sample performance of the spectral regression test and the R/S test statistic, when the generated series are either short memory, which they model as an AutoRegressive Stochastic Volatility (ARSV) process or long memory, which they model as a Long-Memory Stochastic Volatility (LMSV) process. They find that $m_u = 0.45$ appears to be a good choice for the upper truncation point because the size of the standard t-test is closest to the nominal significance level of 5%. de Lima *et al.* (1998) uses the R/S statistic in related simulations and shows that long and short memory are distinct, and that Andrew's (1991) data-dependent rule for choice of q, $q*$, lessens the bias of the classical Hurst exponent. These findings are used as a guidline in this study for determining the presence of long memory in a series, and only tests and measures with these paramaters ($m_u = 0.45$, and $q = q*$) will be reported.

4 Methodology and Results

The volatility of the individual return series, the volatility of the total aggregate of all series sampled, and the volatilities of portfolios created on the basis of size, standard deviation, and beta are all tested for long memory. As many researchers have found, the null hypothesis of short memory can be rejected for the volatility of the equal-weighted aggregate (the index of all individual series sampled) using both the nonparametric and the semiparametric measures and tests for long memory. Both the nonparametric and semiparametric tests reject the short-memory null. The semiparametric measure rejects the short-memory null for $m_u = 0.45$, with a t-statistic of 2.22. The associated estimate of d, $\widehat{d}$, is 0.33. The estimated Hurst coefficient, $\widehat{J}$, and the associated R/S-statistic are 0.639 and 2.62, respectively, for $q = q*$. From Table 1 it is evident that the quality of the memory in the volatility of the aggregate index changes over time. The dependence in the volatility of the aggregate index increases over the second half of the sample on average for the semiparametric measure, although from a statistical point of view, the null of short memory can not be rejected over either subsample. The estimated d, $\widehat{d}$, goes from 0.352 to 0.484, but the associated t-statistic goes from 1.54 to 1.36. The nonparametric estimate of the Hurst coefficient and the R/S-statistic shows a strong movement towards long memory over the two sample halves. The estimated Hurst coefficient, J, $\widehat{J}$ goes from 0.578 to 0.596, and the associated R/S-statistic increases from 1.63 to 1.82.

Series	$m_{\mathbf{u}}$=*0.45*	*q=q**
	$\widehat{d}$, *(t-stat)*	$\widehat{J}$, *(R/S-stat)*
AGGREGATE INDEX	0.330	0.639
12/13/90-12/31/94	(2.22)	(2.62)
AGGREGATE INDEX	0.352	0.578
12/13/90-12/21/92	(1.54)	(1.63)
AGGREGATE INDEX	0.484	0.596
12/22/92-12/31/94	(1.36)	(1.82)

Table 1: *GPH Estimate of d (and Assoc. t-statistic), and Lo's Est. Hurst Coeff., (and Assoc. R/S-statistic) Performed on Equal-weighted Aggregate Index. Over the entire sample, from 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94.*

On an individual basis, the short-memory null can not be rejected on average using the Geweke, Porter-Hudak t-statistic, but can be rejected using the R/S-statistic. The semiparametric estimate of d is 0.125 with an associated t-statistic of 0.610. In contrast, the nonparametric test rejects the null with an R/S-statistic of 1.94. The estimated Hurst coefficient, J, is 0.589. Table 2 catalogues these results. The two types of measures studied here, the estimated Hurst coefficient and d, have differing conclusions about the quality of memory in volatility over time, too.[5] When the sample is split in two on December 22, 1992, the average semiparametric measure of long memory of the individual return series moves slightly from 0.0576 to 0.0401, with the associated t-statistic moving from 0.235 to 0.159. However, the nonparametric measure and associated R/S-statistic move further away from short memory, towards long memory, albeit only slightly. The estimated Hurst coefficient increases from 0.562 to 0.564, and the associated R/S-statistic moves from 1.53 to 1.56. These movements are also found in Table 2.[6]

The individual return series are then grouped into low, medium and high categories, depending upon whether their mid-sample market capitalization measures lay in the bottom, mid or top fifths of mid-sample market capitalization values. As noted above, two different rankings are provided for market capitalization. Market capi-

[5] As noted in Barnes (1998), the semiparametric measure tends to be indecisive about the quality of memory in the volatility of individual return series (as well as for indices) as indicated by the wide confidence bands around this measure. The measure indicates anything from antipersistance to long memory, so it is not too surprising that short memory is found on average.

[6] Barnes (1996) found strong evidence of long memory in the volatility of individual return series using the R/S-statistic.

talization I is ranked relative to all return series available from CRSP, and market capitalization II is ranked relative to those return series extracted from CRSP for this study. Although the rankings are different, the results of these tests and measures on the volatility of the various portfolios are quite similar.

The nonparametric and semiparametric measures and tests again reveal different patterns in the data using either of the two different size measures (market capitalization I and II) and over time. Over the entire sample, the semiparametric test cannot reject the short-memory null on average for those series which are placed into any of the three categories, low, medium or high. The average t-statistic for these portfolios sorted on market capitalization II are 0.672, 0.499, and 0.608 respectively for the entire sample. These results are very similar for the categories formed by individual returns sorted by market capitalization I. The semiparametric measure and test move towards dependence in the memory of the volatility over time for the medium category, though the null of short memory still cannot be rejected. In general, though, the mean statistics and measures suggest that there is less dependence in the volatility over time (across the two sample halves). These results are in Tables 3, 4, and 5. As is noted above, they are similar across the two different rankings of market capitalization.

In contrast, the results obtained using the nonparametric measure and test illustrate dissimilar patterns. The average estimated Hurst coefficient and R/S-statistic averaged across the R/S-statistics for the volatility of the individual returns for those return series with the lowest quintile of market size indicate that the volatility displayed short memory over the entire sample. ($\widehat{J}$ and the associated R/S-statistic were 0.569 and 1.69, respectively, over the entire sample for market capitalization I. The short-memory null can not be rejected as the critical value is 1.74 at the 5% level of significance.) According to these same statistics, the volatility of those returns grouped into the middle or the highest quintile of mid-sample market capitalization displayed long memory over the entire sample, with R/S-statistics of 1.88 for the medium category and 2.38 for the high category. Thus, the evidence for long memory in volatility is strongest for the highest quintile. These patterns are exacerbated over time: the dependence in volatility weakens for the low size category (the R/S-statistic moves down from 1.51 to 1.04), and strengthens for the medium and high size categories. The R/S-statistic increases from 1.52 to 1.55 for the medium category over time, and it increases from 1.54 to 2.22 for the high category over time. The $\widehat{J}$ are increasing over time too. Again, the results can be found in Tables 3, 4, and 5, and indicate that there is a predictable component in the volatility of return series with particular market capitalization characteristics. The results show that such predictability is stronger, the larger is the mid-sample market capitalization of the firm. The predictable component becomes stronger over time for the medium and high categories, too. These results hold for both market capitalization I and II.

The individual series are then sorted according to their values of market capitalization, standard deviation and beta, and placed into three different portfolios. That is, the values of the portfolio characteristics of market capitalization, standard deviation of returns, and beta were sorted and grouped into five consecutive sub-groupings, for three different portfolio rankings. Returns which were ranked in the bottom quintile for a particular year were placed in a low portfolio for that year, returns which were ranked in the middle quintile were placed in a medium portfolio, and firms which were ranked in the highest quintile were placed in a high portfolio. Thus, a given security could belong to one portfolio for year t, and to another portfolio for year $t+1$. The measures and tests of long memory are performed on the volatility of the resultant portfolios, which are equal-weighted aggregates of the returns within the portfolio.

A stronger and more consistent pattern emerges from the nonparametric measure and test across different portfolio characteristics. For the three portfolios sorted by market capitalization II, the estimated Hurst coefficient and associated R/S-statistic increase in magnitude as the portfolios increase from low to high, and the R/S-statistic rejects the short-memory null in favor of long memory for the medium and high portfolios (2.01 and 3.34 respectively).[7] When the sample is split in two, the

[7] The results for market capitalization I are also in Table 4. For comparison across different portfolio characteristics, market capitalization II is emphasized in the text. In general, the nonparametric results are somewhat different for these six portfolios, with the three constructed with market capitalization II displaying more evidence of long memory in volatility according to the nonparametric

estimated Hurst coefficient and R/S-statistic move left in the second half of the sample for the low and medium portfolios. The estimated Hurst coefficient (and associated R/S-statistic) evaluated at $q = q*$ moves from 0.585 (1.70) to 0.518 (1.12) for the low portfolio, and from 0.598 (1.84) to 0.569 (1.54) for the medium portfolio. This pattern reverses itself for the high portfolio, with the estimated Hurst coefficient (and R/S-statistic) moving to the right over time from 0.611 (2.00) to 0.658 (2.69). These results are in Tables 6, 7, and .

The semiparametric measure and test suggest that the short-memory null can not be rejected for the volatility of either the low, medium or high portfolios. The estimate of d (and the associated t-statistic) are 0.141 (0.909), 0.269 (1.46), and 0.268 (1.36) for the low, medium and high portfolios respectively. These measures also suggest opposite movements over time to that of the nonparametric measure for the low and high portfolios: Tables 6, 7, and 8 illustrate that the estimate of d moves to the right over time for the low portfolio, and moves to the left for the medium and high portfolios.

The portfolios sorted on beta were sorted on betas computed with five different market measures: the CRSP value-weighted index with dividends (beta1), the CRSP value-weighted index without dividends (beta3), the CRSP equal-weighted index with

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measure. The semiparametic measure and t-statistic overwhelmingly suggest short memory for the volatility of portfolios formed from sorting on either market capitalization I or II.

dividends (beta2), the CRSP equal-weighted index without dividends (beta4), and the SP 500 (beta5). de Lima, Breidt, and Crato, (1998) found less evidence for long memory in the volatility of the equal-weighted CRSP index than for the value-weighted CRSP index. As these different indices appear to have differing dependence characteristics in volatility, this study seeks to further elucidate such apparent differences.

The memory characteristics of the volatility of the different portfolios are quite similar for portfolios sorted on beta1 and beta2, or the two betas constructed with the two value-weighted CRSP indices, indicating that dividends make little difference in the results. Therefore only the memory characteristics of the volatility of those portfolios constructed with beta1 are discussed here. Differences across beta1 and beta2 are noted when they occur. Using either the semiparametric or the nonparametric measures, it is evident that the lowest portfolio sorted on beta1 has long memory in volatility over the entire sample. As reported in Tables 9, 10, and 11, $\widehat{d}$ (and the associated t-statistic) are 0.633 and (2.18). Similarly, Table 3.6 shows that $\widehat{J}$ (and the associated R/S-statistic) are 0.661 and (3.05). The medium portfolio displays short memory in volatility ($\widehat{d}$ and the associated t-statistic are 0.0307 and 0.132, while $\widehat{J}$ and the associated R/S-statistic are 0.573 and 1.66) and the highest portfolio shows the strongest evidence for long memory in volatility.[8] For the highest portfolio, $\widehat{d}$ and

[8] The medium portfolio created by sorting on beta2 has more evidence for long-memory in volatility, but still a U-shaped pattern emerges.

the associated t-statistic are 0.475 and 2.62, while $\widehat{J}$ and the associated R/S-statistic are 0.689 and 3.71. Thus, semiparametric and nonparametric measures display similar U-shaped patterns in the measure of long memory over the entire sample.

The change in the dependence characteristic of the volatility of these portfolios over time is different across the semiparametric and nonparametric measures of long memory, however. The nonparametric measure decreases, or moves away from long memory, over time for the low and medium portfolios. For the high portfolio, the dependence in volatility becomes stronger over the second half of the sample. For the low portfolio, the R/S-statistic moves from 1.82 to 1.60 over time, and the same statistic for the volatility of the medium portfolio dcreases from 1.70 to 0.917. In contrast, the R/S-statistic on the volatility of the high portfolio increases from 1.40 to 2.86 over time. These results are in Table 12, 13, and 14. The semiparametric measure of long memory tends to move towards short memory over time for the medium portfolio, and towards long memory over time for the low and high portfolios. The t-statistic for the medium portfolio moves from 1.38 to 0.113, while the t-statistics on the low and high portfolios move from 0.729 to 1.85 and from 0.840 to 1.31 respectively. These statistics are reported in Tables 9, 10, and 11. Thus, for the low portfolio, the movement over time is reversed for the semiparametric and nonparametric measures of long memory. The semiparametric measure suggests a movement towards long memory over time, while the nonparametric measure moves away from long memory in volatility and towards short memory over time.

The quality of the dependence in the volatility of portfolios sorted by all the different betas is relatively similar for the entire sample period when judged by the nonparametric measure and test. For those portfolios sorted by beta3, beta4 and beta5, the pattern across portfolios over the entire sample is the familiar U-shape elucidated above. These results are also in Tables 12, 13, and 14. $\widehat{J}$ (and the associated R/S-statistic) move from 0.678 (3.42) to 0.491 (0.939) to 0.684 (3.58) as the portfolio moves from low to medium to high for beta3. Similar movements are witnessed in $\widehat{J}$ (and the associated R/S-statistic) for beta4 and beta5. For beta4, the measure and test statistic go from 0.674 (3.34) to 0.504 (1.03) to 0.685 (3.61). These same measurements move from 0.664 (3.11) to 0.629 (2.44) to 0.634 (2.53) for beta5. The null is rejected for the volatility of the low and high portfolios, but tends to be accepted, or the evidence for long memory is weakest, for the medium portfolio. The pattern is slightly different for the portfolios sorted by beta3, beta4, and beta5 according to the semiparametric measure. As is illustrated in Tables 9, 10, and 11, the familiar U-shaped pattern is found for beta4, with the semiparametric measure and test statistic moving from 0.331 (1.23) to -0.0544 (-0.344) to 0.0616 (0.320), as the portfolios move from low to high. For beta3, there is a tendency for there to be less memory in volatiliy as the portfolios go from low to high. In light of the evidence above, it is not surprising that the volatility of the medium portfolio for beta5 displays the weakest memory over the entire sample. The volatility of the low portfolio for beta5 displays the strongest dependence (the short-memory null is rejected), and the volatility of the high portfolio has short memory, but is more dependent than that found in the volatility of the medium portfolio, again evincing a U-shaped pattern.

Over time there are some differences which appear from the nonparametric measure and test. The volatility of all three portfolios ranked by beta3 display less dependence in the second half of the sample. For beta4 and beta5, however, the movement over time in the quality of the memory in the volatility of the portfolios mirrors the movement discovered for beta1 and beta2. That is, dependence in volatility seems to decrease over time for the low and medium portfolios, but increases for the volatility of the high portfolio. Using the semiparametric measure and test, it is evident that both sets of portfolios sorted by beta3 and beta4 move similarly over time: the dependence in the volatility of the low and high portfolios increases over time, while the dependence in the volatility of the medium portfolio decreases over time. For the volatility of the portfolios created by sorting on beta5, though, the movement in the memory characteristic of the volatility is away from long memory over time.

From these sets of results, three general patterns emerge: 1) A U-pattern in the measurement of memory in the volatility of the portfolios as the portfolios move from low to high for different definitions of the market and for both the nonparametric and semiparametric measures and tests; 2) Dependence in volatility tends to decrease over time for the low and medium portfolios and increase for the high portfolio according to the nonparametric measure and test; and 3) Dependence in volatility tends to increase over time for the low and high portfolios and decrease for the medium portfolios according to the semiparametric measure and test.

The relative strength of the measures across portfolios and their movements over time are more congruous for the semiparametric and nonparametric measures and tests for the three portfolios sorted by standard deviation of returns. As can be seen from Tables 9, 10, 11, 12, 13 and 14, the measures of long memory move towards long memory as the portfolios go from low to high for both the estimate of d and the estimated Hurst coefficient. Both measures also indicate similar rightward movements over time for the low portfolio, and leftward movements over time for the high portfolio. Where they differ is in their statistical conclusion about the rejection of the short-memory null, and in their movement over time for the medium portfolio. The movement over time in these measures for the medium portfolio is rightward using the semiparametric measure and leftward using the nonparametric measure. Also, the null of short memory can be rejected only for the high portfolio using the semiparametric test, whereas the same null can be rejected for the medium and high portfolios using the nonparamentric test. This pattern of more dependence in volatility as the standard deviation of returns increases lends credence to the idea that greater variability in prices leads to less efficient markets.

5 Conclusions

This study has elucidated the nature of the predictability found in the volatility of stock returns by uncovering changes across portfolios created by sorting on measures of size, standard deviation and beta, along with movement over time in the characteristic of the dependence in the volatility of these portfolios. The nature of the movement over time of the dependence in the volatility of the total aggregate index of all sampled series is also explored, as it is for the individual return series themselves. The dependence in the volatility of the individual return series is scrutinized by grouping the individual return series into three groups on the basis of mid-sample market capitalization.

While the levels of returns tend to exhibit less dependence over time and less dependence as portfolios move from low to high, the volatility of returns shows no such linear tendency for less predictability. Although the picture which emerges from the results in this paper are not completely without ambiguities, general patterns do arise, particularly when the nonparametric measure and test are given more weight, as is suggested in Barnes (1998), due to apparent problems with the Geweke Porter-Hudak methodology under aggregation. A U-shaped pattern emerges: the high portfolios tend to display long memory in volatility across the entire sample, the low portfolios tend to display somewhat less long memory in volatility, and the medium portfolios tend to display short memory in volatility. Over time, the dependence in the volatility of the low and medium portfolios decreases, and it increases for the volatility of the high portfolios.

The equal-weighted average of all sampled return series corroborates the existing evidence in the literature: there is long memory in the volatility of the total aggregate index. In fact, this dependence increases over the second half of the sample when the sample is split in two. On an individual basis, the volatility of returns can not reject the short-memory null on average using the Geweke, Porter-Hudak t-statistic, but can rject the null using the R/S-statistic. These two types of measures, the estimated Hurst coefficient and the estimate of d, have differing conclusions about the quality of memory in volatility over time, too. When the sample is split in two on December 22, 1992, the average semiparametric measure of long memory of the individual return series moves slightly towards a stronger short-memory result. However, the nonparametric measure and associated R/S-statistic move further away from short memory, towards long memory, albeit only slightly.

When these same series are grouped into low, medium and high categories according to market capitalization, and the same measures are performed on the volatility of the individual series within each group, different results emerge from the different semiparametric and nonparametric measures and tests. Over the entire sample, the semiparametric test cannot reject the short-memory null on average for those series which are placed into any of the three categories, low, medium or high. The semiparametric measure and test move towards dependence in the memory of the volatility over time for the medium category, though the null of short memory still cannot be rejected. In general, though, the mean statistics and measures suggest that there is less dependence in the volatility over time (across the two sample halves). The results employing the nonparametric measure and test show that such predictability is stronger, the larger is the mid-sample market capitalization of the firm. The predictable component becomes stronger over time for the medium and high categories, too.

6 Appendix A: Filtering The Data

As in Gallant *et al.* (1994), the individual return series are filtered for seasonal, calendar and short-run autocorrelation effects, by estimating the following regressions:

$$\begin{aligned} y_t &= \mathbf{X}_t \beta + u_t \qquad \text{(mean)} \\ \mid \widehat{u}_t \mid &= \mathbf{X}_t \gamma + w_t \quad \text{(scale)}, \end{aligned} \tag{9}$$

where y_t is the individual return series and $\mathbf{X}_t$ is constituted by

1. A constant term: ($\mathbf{X}[t, 1] = 1.$).

2. Dummy variables for days of the week such that:

   $\mathbf{X}[t, 2] = 1$ if the trading day is a Tuesday,

   $\mathbf{X}[t, 3] = 1$ if the trading day is a Wednesday,

   $\mathbf{X}[t, 4] = 1$ if the trading day is a Thursday,

   $\mathbf{X}[t, 5] = 1$ if the trading day is a Friday.

3. Dummy variable for the square root of the number of calendar days between trading days:

   $\mathbf{X}[t, 6] = \sqrt{gap}$.

4. Dummy variables for the months March through November:

   $\mathbf{X}[t, 7] = 1$ if the month is March,

   $\mathbf{X}[t, 8] = 1$ if the month is April,

   $\mathbf{X}[t, 9] = 1$ if the month is May,

$\mathbf{X}[t, 10] = 1$ if the month is June,

$\mathbf{X}[t, 11] = 1$ if the month is July,

$\mathbf{X}[t, 12] = 1$ if the month is August,

$\mathbf{X}[t, 13] = 1$ if the month is September,

$\mathbf{X}[t, 14] = 1$ if the month is October,

$\mathbf{X}[t, 15] = 1$ if the month is November.

5. Dummy variables for the weeks of January and December:

$\mathbf{X}[t, 16] = 1$ if the month is January and $1 \leq day \leq 7$,

$\mathbf{X}[t, 17] = 1$ if the month is January and $8 \leq day \leq 14$,

$\mathbf{X}[t, 18] = 1$ if the month is January and $15 \leq day \leq 21$,

$\mathbf{X}[t, 19] = 1$ if the month is January and $22 \leq day \leq 31$,

$\mathbf{X}[t, 20] = 1$ if the month is December and $1 \leq day \leq 7$,

$\mathbf{X}[t, 21] = 1$ if the month is December and $8 \leq day \leq 14$,

$\mathbf{X}[t, 22] = 1$ if the month is December and $15 \leq day \leq 21$,

$\mathbf{X}[t, 23] = 1$ if the month is December and $22 \leq day \leq 31$.

Once the scale equation is estimated, the AR(5)-filtered y_t is adjusted for the calendar effects in the mean and scale, and the subsequent analysis is performed on

$$z_t = \frac{\widehat{u}_t}{\widehat{\sigma}_t} = \frac{(y_t - \mathbf{X}_t \boldsymbol{\beta})}{\widehat{\sigma}_t} \tag{10}$$

where $\widehat{\sigma}_t = \mathbf{X}_t \widehat{\gamma}$ is the conditional variance.

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Series	$m_{\mathbf{u}}$=*0.45*	*q=q**
	$\widehat{d}$, *(t-stat)*	$\widehat{J}$, *(R/S-stat)*
Ind. Ret.: Hurst Coeff.	0.125	0.589
12/13/90-12/31/94	(0.231)	(0.0447)
Ind. Ret.: R/S-statistic	0.610	1.94
12/13/90-12/31/94	(1.10)	(0.603)
Ind. Ret.: Hurst Coeff.	0.0576	0.562
12/13/90-12/21/92	(0.269)	(0.0442)
Ind. Ret.: R/S-statistic	0.235	1.53
12/13/90-12/21/92	(1.07)	(0.423)
Ind. Ret.: Hurst Coeff.	0.0401	0.564
12/22/92-12/31/94	(0.275)	(0.0464)
Ind. Ret.: R/S-statistic	0.159	1.56
12/22/92-12/31/94	(1.11)	(0.464)

Table 2: *GPH Estimate of d, the Assoc. t-statistic, Lo's Est. Hurst Coeff., and Assoc. R/S Statistic Performed on Individual Return Series, Mean (Standard Deviation). Over the entire sample, from 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94.*

Series	*MCI*($\widehat{d}; t$)	*MCII*($\widehat{d}; t$)	*MCI*($\widehat{J}; R/S$)	*MCII*($\widehat{J}; R/S$)
LOW: $\widehat{d}$ *or* $\widehat{J}$	0.143	0.136	0.568	0.569
12/13/90-12/31/94	(0.223)	(0.244)	(0.0464)	(0.0454)
LOW: R/S or t-stat.	0.721	0.672	1.69	1.69
12/13/90-12/31/94	(1.12)	(1.18)	(0.568)	(0.556)
LOW: $\widehat{d}$ *or* $\widehat{J}$	0.114	0.0752	0.559	0.560
12/13/90-12/21/92	(0.252)	(0.257)	(0.0450)	(0.0457)
LOW: R/S or t-stat.	0.441	0.310	1.50	1.51
12/13/90-12/21/92	(1.12)	(1.06)	(0.438)	(0.442)
LOW: $\widehat{d}$ *or* $\widehat{J}$	0.0366	0.0613	0.506	0.505
12/22/92-12/31/94	(0.283)	(0.260)	(0.0229)	(0.0216)
LOW: R/S or t-stat.	0.121	0.221	1.05	1.04
12/22/92-12/31/94	(1.08)	(1.02)	(0.150)	(0.139)

Table 3: *Geweke Porter-Hudak Estimate of d, the Associated t-statistic, Lo's Estimated Hurst Coefficient, and the Assoc. R/S-statistic Performed on Individual Return Series Sorted into Categories by Market Capitalization I and II (Low, Med, High), Mean (Standard Deviation). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Contents continued in next table.)*

Series	*MCI*($\widehat{d}; t$)	*MCII*($\widehat{d}; t$)	*MCI*($\widehat{J}; R/S$)	*MCII*($\widehat{J}; R/S$)
MED: $\widehat{d}$ *or* $\widehat{J}$	0.113	0.0965	0.591	0.586
12/13/90-12/31/94	(0.209)	(0.278)	(0.0418)	(0.0397)
MED: R/S or t-stat.	0.518	0.499	1.96	1.88
12/13/90-12/31/94	(0.992)	(1.33)	(0.577)	(0.514)
MED: $\widehat{d}$ *or* $\widehat{J}$	0.0534	0.0168	0.561	0.560
12/13/90-12/21/92	(0.255)	(0.289)	(0.0487)	(0.0481)
MED: R/S or t-stat.	0.244	0.102	1.53	1.52
12/13/90-12/21/92	(1.03)	(1.15)	(0.447)	(0.454)
MED: $\widehat{d}$ *or* $\widehat{J}$	0.0605	0.0244	0.570	0.569
12/22/92-12/31/94	(0.265)	(0.278)	(0.0231)	(0.0203)
MED: R/S or t-stat.	0.273	0.139	1.57	1.55
12/22/92-12/31/94	(1.12)	(1.07)	(0.219)	(0.193)

Table 4: *Geweke Porter-Hudak Estimate of d, the Associated t-statistic, Lo's Estimated Hurst Coefficient, and the Assoc. R/S-statistic Performed on Individual Return Series Sorted into Categories by Market Capitalization I and II (Low, Med, High), Mean (Standard Deviation). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Contents continued in next table.)*

Series	$MCI(\hat{d};t)$	$MCII(\hat{d};t)$	$MCI(\hat{J};R/S)$	$MCII(\hat{J};R/S)$
HIGH: $\hat{d}$ *or* $\hat{J}$	0.125	0.124	0.620	0.621
12/13/90-12/31/94	(0.210)	(0.213)	(0.0344)	(0.0333)
HIGH: R/S or t-stat.	0.608	0.608	2.36	2.38
12/13/90-12/31/94	(0.994)	(1.00)	(0.557)	(0.549)
HIGH: $\hat{d}$ *or* $\hat{J}$	0.0718	0.0705	0.562	0.564
12/13/90-12/21/92	(0.282)	(0.274)	(0.0440)	(0.0440)
HIGH: R/S or t-stat.	0.283	0.282	1.53	1.54
12/13/90-12/21/92	(1.06)	(1.04)	(0.431)	(0.432)
HIGH: $\hat{d}$ *or* $\hat{J}$	0.0434	0.0512	0.624	0.625
12/22/92-12/31/94	(0.258)	(0.260)	(0.0283)	(0.0265)
HIGH: R/S or t-stat.	0.168	0.210	2.20	2.22
12/22/92-12/31/94	(1.09)	(1.10)	(0.394)	(0.376)

Table 5: *Geweke Porter-Hudak Estimate of d, the Associated t-statistic, Lo's Estimated Hurst Coefficient, and the Assoc. R/S-statistic Performed on Individual Return Series Sorted into Categories by Market Capitalization I and II (Low, Med, High), Mean (Standard Deviation). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Cont.)*

Series	$MCI(\hat{d};t)$	$MCII(\hat{d};t)$	$MCI(\hat{J};R/S)$	$MCII(\hat{J};R/S)$
LOW: $\hat{d}$ *or* $\hat{J}$	-0.350	0.141	0.568	0.576
12/13/90-12/31/94				
LOW: R/S or t-stat.	-1.74	0.909	1.60	1.70
12/13/90-12/31/94				
LOW: $\hat{d}$ *or* $\hat{J}$	-0.671	-0.301	0.555	0.585
12/13/90-12/21/92				
LOW: R/S or t-stat.	-2.26	-0.882	1.41	1.70
12/13/90-12/21/92				
LOW: $\hat{d}$ *or* $\hat{J}$	-0.0841	-0.0507	0.560	0.518
12/22/92-12/31/94				
LOW: R/S or t-stat.	-0.404	-0.154	1.45	1.12
12/22/92-12/31/94				

Table 6: *Geweke Porter-Hudak Estimate of d, the Associated t-statistic, Lo's Estimated Hurst Coefficient, and the Assoc. R/S-statistic Performed on Portfolios Ranked by Market Capitalization I or II (Low, Med, High). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Contents continued in next table.)*

Series	$MCI(\widehat{d};t)$	$MCII(\widehat{d};t)$	$MCI(\widehat{J};R/S)$	$MCII(\widehat{J};R/S)$
MED: $\widehat{d}$ *or* $\widehat{J}$ *12/13/90-12/31/94*	-0.0136	0.269	0.547	0.601
MED: R/S or t-stat. 12/13/90-12/31/94	-0.0695	1.46	1.39	2.01
MED: $\widehat{d}$ *or* $\widehat{J}$ *12/13/90-12/21/92*	0.583	0.324	0.546	0.598
MED: R/S or t-stat. 12/13/90-12/21/92	2.06	1.75	1.33	1.84
MED: $\widehat{d}$ *or* $\widehat{J}$ *12/22/92-12/31/94*	-0.0162	0.262	0.569	0.569
MED: R/S or t-stat. 12/22/92-12/31/94	-0.0746	1.01	1.54	1.54

Table 7: *Geweke Porter-Hudak Estimate of d, the Associated t-statistic, Lo's Estimated Hurst Coefficient, and the Assoc. R/S-statistic Performed on Portfolios Ranked by Market Capitalization I or II (Low, Med, High). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Contents continued in next table.)*

Series	*MCI*($\widehat{d}; t$)	*MCII*($\widehat{d}; t$)	*MCI*($\widehat{J}; R/S$)	*MCII*($\widehat{J}; R/S$)
HIGH: $\widehat{d}$ *or* $\widehat{J}$	-0.143	0.268	0.591	0.674
12/13/90-12/31/94				
HIGH: R/S or t-stat.	-0.557	1.36	1.87	3.34
12/13/90-12/31/94				
HIGH: $\widehat{d}$ *or* $\widehat{J}$	0.178	0.345	0.528	0.611
12/13/90-12/21/92				
HIGH: R/S or t-stat.	0.571	1.36	1.19	2.00
12/13/90-12/21/92				
HIGH: $\widehat{d}$ *or* $\widehat{J}$	0.0878	0.0886	0.551	0.658
12/22/92-12/31/94				
HIGH: R/S or t-stat.	0.343	0.341	1.37	2.69
12/22/92-12/31/94				

Table 8: *Geweke Porter-Hudak Estimate of d, the Associated t-statistic, Lo's Estimated Hurst Coefficient, and the Assoc. R/S-statistic Performed on Portfolios Ranked by Market Capitalization I or II (Low, Med, High). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Cont.)*

Series	*Beta1*	*Beta2*	*Beta3*	*Beta4*	*Beta5*	*S.D.*
LOW: $\widehat{d}$	0.633	0.611	0.288	0.331	0.534	0.128
12/13/90-12/31/94						
LOW: t-stat.	2.18	2.75	1.67	1.23	2.73	0.557
12/13/90-12/31/94						
LOW: $\widehat{d}$	0.162	0.268	0.091	0.072	0.225	-0.244
12/13/90-12/21/92						
LOW: t-stat.	0.729	1.16	0.421	0.257	1.46	-1.00
12/13/90-12/21/92						
LOW: $\widehat{d}$	0.500	0.428	0.099	0.096	0.093	-0.009
12/22/92-12/31/94						
LOW: t-stat.	1.85	2.34	0.451	0.466	0.325	-0.037
12/22/92-12/31/94						

Table 9: *Geweke Porter-Hudak Estimate of d and the Associated t-statistic Performed on Portfolios Ranked by Beta1-Beta5 and Standard Deviation of Returns (S.D.), (Low, Med, High). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Contents continued in next table.)*

Series	*Beta1*	*Beta2*	*Beta3*	*Beta4*	*Beta5*	*S.D.*
MED: $\widehat{d}$	0.031	0.192	0.071	-0.055	-0.004	0.291
12/13/90-12/31/94						
MED: t-stat.	0.132	0.770	0.444	-0.344	-0.018	1.38
12/13/90-12/31/94						
MED: $\widehat{d}$	0.403	0.388	0.305	0.416	0.310	0.269
12/13/90-12/21/92						
MED: t-stat.	1.38	1.40	0.578	1.25	0.725	0.917
12/13/90-12/21/92						
MED: $\widehat{d}$	0.031	0.106	0.07	0.039	0.212	0.262
12/22/92-12/31/94						
MED: t-stat.	0.113	0.359	0.180	0.103	0.676	1.24
12/22/92-12/31/94						

Table 10: *Geweke Porter-Hudak Estimate of d and the Associated t-statistic Performed on Portfolios Ranked by Beta1-Beta5 and Standard Deviation of Returns (S.D.), (Low, Med, High). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Contents continued in next table.)*

Series	*Beta1*	*Beta2*	*Beta3*	*Beta4*	*Beta5*	*S.D.*
HIGH: $\widehat{d}$	0.475	0.488	-0.114	0.062	0.095	0.344
12/13/90-12/31/94						
HIGH: t-stat.	2.62	2.32	-0.592	0.320	0.444	2.13
12/13/90-12/31/94						
HIGH: $\widehat{d}$	0.139	0.073	-0.057	-0.106	0.299	0.192
12/13/90-12/21/92						
HIGH: t-stat.	0.840	0.436	-0.249	-0.455	0.822	0.581
12/13/90-12/21/92						
HIGH: $\widehat{d}$	0.337	0.339	0.111	0.295	-0.127	0.022
12/22/92-12/31/94						
HIGH: t-stat.	1.31	1.30	0.337	0.967	-0.538	0.061
12/22/92-12/31/94						

Table 11: *Geweke Porter-Hudak Estimate of d and the Associated t-statistic Performed on Portfolios Ranked by Beta1-Beta5 and Standard Deviation of Returns (S.D.), (Low, Med, High). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Cont.)*

Series	*Beta1*	*Beta2*	*Beta3*	*Beta4*	*Beta5*	*S.D.*
LOW: $\widehat{J}$ *12/13/90-12/31/94*	0.661	0.663	0.678	0.674	0.664	0.537
LOW: R/S-stat. 12/13/90-12/31/94	3.05	3.08	3.42	3.34	3.11	1.29
LOW: $\widehat{J}$ *12/13/90-12/21/92*	0.596	0.596	0.626	0.606	0.596	0.484
LOW: R/S-stat. 12/13/90-12/21/92	1.82	1.82	2.20	1.93	1.82	0.904
LOW: $\widehat{J}$ *12/22/92-12/31/94*	0.575	0.572	0.529	0.526	0.560	0.525
LOW: R/S-stat. 12/22/92-12/31/94	1.60	1.57	1.20	1.18	1.45	1.17

Table 12: *Lo's Modified Hurst Estimate and the Associated R/S-statistic Performed on Portfolios Ranked by Beta1-Beta5 and Standard Deviation of Returns (S.D.), (Low, Med, High). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Contents continued in next table.)*

Series	*Beta1*	*Beta2*	*Beta3*	*Beta4*	*Beta5*	*S.D.*
MED: $\widehat{J}$ *12/13/90-12/31/94*	0.573	0.584	0.491	0.504	0.629	0.617
MED: R/S-stat. 12/13/90-12/31/94	1.66	1.78	0.939	1.03	2.44	2.25
MED: $\widehat{J}$ *12/13/90-12/21/92*	0.585	0.607	0.545	0.542	0.632	0.589
MED: R/S-stat. 12/13/90-12/21/92	1.70	1.95	1.32	1.30	2.28	1.75
MED: $\widehat{J}$ *12/22/92-12/31/94*	0.486	0.479	0.504	0.493	0.533	0.550
MED: R/S-stat. 12/22/92-12/31/94	0.917	0.875	1.02	0.957	1.23	1.37

Table 13: *Lo's Modified Hurst Estimate and the Associated R/S-statistic Performed on Portfolios Ranked by Beta1-Beta5 and Standard Deviation of Returns (S.D.), (Low, Med, High). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Contents continued in next table.)*

Series	*Beta1*	*Beta2*	*Beta3*	*Beta4*	*Beta5*	*S.D.*
HIGH: $\widehat{J}$	0.689	0.684	0.684	0.685	0.634	0.701
12/13/90-12/31/94						
HIGH: R/S-stat.	3.71	3.57	3.58	3.61	2.53	4.01
12/13/90-12/31/94						
HIGH: $\widehat{J}$	0.554	0.576	0.657	0.650	0.549	0.630
12/13/90-12/21/92						
HIGH: R/S-stat.	1.40	1.61	2.67	2.55	1.36	2.25
12/13/90-12/21/92						
HIGH: $\widehat{J}$	0.668	0.668	0.654	0.672	0.599	0.487
12/22/92-12/31/94						
HIGH: R/S-stat.	2.86	2.86	2.61	2.93	1.86	0.921
12/22/92-12/31/94						

Table 14: *Lo's Modified Hurst Estimate and the Associated R/S-statistic Performed on Portfolios Ranked by Beta1-Beta5 and Standard Deviation of Returns (S.D.), (Low, Med, High). From 12/13/90, through 12/31/94, as well as over the two sub-samples, 12/13/90-12/21/92 and 12/22/92-12/31/94. (Cont.)*