Volatility Trends and Optimal Portfolios: the Case of Agricultural Commodities

Antonios Antypas* Phoebe Koundouri† Nikolaos Kourogenis‡§

Abstract

While the financial world is experiencing a crisis, the prices of most agricultural commodities have remained high, although exhibiting extreme volatilidy. Motivated by evidence showing that volatility trends are present in agricultural commodity prices, we analyze stochastic processes whose unconditional variance changes with time. This analysis suggests a semi-parametric model for capturing the trending behavior of second moments, in which these moments are polynomial-like functions of time. Based on this model, we formulate the portfolio problem faced by an investor when the variances and the covariances of the returns of the available assets are trending. Then, we obtain an approximate solution of the problem, which is based on the consistent estimation of the order of variance-covariance growth and apply it for the construction of an optimal portfolio of agricultural commodities. It is shown that the performance of this portfolio is superior to those of alternative portfolios which are formed by employing methods not accounting for the presence of volatility trends in commodity returns.

Keywords: Volatility trends; polynomial-like trending variance; order of variance growth; optimal portfolios; agricultural commodities.

JEL Classification: G11, C22, Q14

*Department of Banking and Financial Management, University of Piraeus, 80 Karaoli and Dimitriou str., 18534 Piraeus, Greece. Email: aantypas@unipi.gr. Tel: +302104142142. Fax: +302104142341.

†DIEES, Athens University of Economics and Business, 76, Patission str., 10434, Athens, Greece. Email: pkoundouri@aueb.gr. Tel: +302108203455. Fax: +302108214122.

‡Department of Banking and Financial Management, University of Piraeus.

§*Correspondence to:* Nikolaos Kourogenis, Department of Banking and Financial Management, University of Piraeus, 80 Karaoli and Dimitriou str., 18534 Piraeus, Greece. Email(1): nkourogenis@yahoo.com Email(2): nkourog@unipi.gr. Tel: +302104142142. Fax: +302104142341.

1 Introduction

Recently the price behavior of major agricultural commodities has attracted a lot of interest, because it has been displaying unusually high volatility. For example, the price of wheat increased sharply between January 2007 and June 2008, then went down and has been rising again since April 2009. Whether this rise in volatility has been driven by global supply and demand factors or is the result of excess speculation in the futures markets for these commodities, is a source of current debate in the literature.

The intertemporal behavior of volatility of returns of several asset classes, such as stocks, bonds or commodities, has been extensively investigated in the last twenty five years or so, by both the academic and investment communities. Currently, there is widespread agreement among researchers that this volatility has not remained constant over time. Various models for describing the time variation in volatility have been proposed in the literature, such as the well known GARCH and stochastic volatility models. These models treat the observed "volatility clustering" as non-linear dependence arising through the conditional variance of returns. This interpretation permits the underlying stochastic process, $\{R_t\}$, generating the returns to be strictly, or even second-order stationary, since a time-varying conditional variance can coexist with a time-invariant unconditional variance. In other words, the observed time variation in the volatility of asset returns may be consistent with a stationary $\{R_t\}$, which exhibits second-order temporal dependence.

However, the aforementioned models are not capable of capturing all empirical characteristics of the volatility of asset returns. For example, there are quite a few studies presenting evidence of variance breaks in $\{R_t\}$ (see, for example, Lamoureux and Lastrapes (1990), Stărică and Granger (2005)). In fact, the high degree of persistence observed in the conditional variance process of returns may be the result of shifts in the unconditional variance of an otherwise locally stationary $\{R_t\}$, which (the shifts) have not been taken into account in the estimation of the conditional variance. The presence of variance breaks in $\{R_t\}$ implies that apart from conditional heteroscedasticity (non-linear dependence), the returns process is also characterized by unconditional heteroscedasticity (local time heterogeneity).

Campbell et al. (2001) suggest that the type of non-stationarity displayed by the process generating stock returns is more "global" than that implied by variance breaks. Specifically, these authors present evidence showing that the idiosyncratic component of the unconditional variance of the returns of individual firms exhibits a large positive linear trend over a 35-year period. The presence of such a trend is likely to dominate the behavior of the total firm volatility, thus producing a returns process which exhibits global non-stationarity. The latter is meant to imply that the marginal distributions of $\{R_t\}$ do not display intervals of time homogeneity (as in the case of local stationarity implied by variance breaks) but instead are continuously changing. This change, however, is not patternless but is governed by a systematic evolution of the variances of the marginal distributions of $\{R_t\}$.

Apart from stock returns, commodity returns have been found to exhibit non-stationary volatilities as well. One of the earliest studies that presents evidence on increasing volatilities is the classical study of Kendall (1953) on the Chicago wheat series. Kendall's conclusion is the following: "We have here an interesting and rather unusual case of a time-series for which the mean remains constant but the variance appears to be increasing" (1953, p. 15). Also, recent empirical literature suggests the presence of volatility trends in commodity prices. Yang Haigh and Leatham (2001) present evidence suggesting that the volatility of three major grain commodities, namely corn, soybeans and wheat has increased over time as a result of radical changes in agricultural liberalization policy (see also Ray et. al. 1998). Focusing solely on wheat, Crain and Lee (1996) show that depending on the type of government farm program, the volatility of wheat price changes might be either increasing or decreasing. Pindyck (2001) reports evidence showing that once the major volatility spikes in 1986 and 1991 (caused by Saudi Arabia's over-supply and Iraqi invasion of Kuwait, respectively) are removed, the volatility series of crude oil, heating oil and especially gasoline display trending behavior. Cuddington and Liang (2003) relate the behavior of commodity price volatility to the type of the exchange rate regime that is in place. In particular, they show that the volatility of returns of agricultural raw materials, beverages, food and metals, is much higher after the collapse of the Bretton Woods fixed rate system in the early 1970s than it was before. Moreover, closer inspection of their reported volatility graphs suggests the presence of positive trends, even within the post-1973 period alone, especially for the case of beverages and metals. In a very recent paper, Calvo-Gonzales Shankar and Trezzi (2010) study thoroughly the behavior of volatility of 45 individual commodity prices, from the end of the 18th century until today and report strong evidence on volatility breaks. Some of these breaks are followed by prolonged periods within which the volatility displays trending behavior. The overall behavior of volatility (over the full sample) for some important commodities such as copper, corn and wheat is clearly trending (see Chart 1 p. 10). In line with the results of Cuddington and Liang (2003) discussed above, these authors find that one of the subperiods during which the volatility for most commodities was increasing, is the period of flexible exchange rates (see also Chu and Morisson 1984, Reinhart and Wicham 1994). The volatility rise of dollar denominated commodity prices after 1970 is likely to reflect the increasing volatility in nominal exchange rates which in turn is the result of the presence of an increasing number of floating-rate regimes in this period. Cashin and McDermott (2001) find that the amplitude of commodity price changes and the frequency of large price changes have increased after the early 1900s and 1970, respectively. A recent report of the American Gas Foundation by Henning, Sloan and de Leon (2003) finds that natural gas has exhibited huge increases in price volatility over the last fifteen years or so, which stems from three primary causes namely supply and demand factors, effects of commodity trading techniques and market imperfections. On the other hand, Jacks et. al. (2009) and Moledina et. al. (2004) report evidence supporting the alternative hypothesis, namely that the volatility in commodity prices displays no consistent trending behavior across time. Apart from the academic literature, there seems to be a consensus in the financial industry that the volatility of commodity prices has been increasing under the growing influence of financial derivatives in commodity markets. Financial involvement in the futures markets for agricultural products, which took the form of the so-called index trading, is likely to have contributed significantly to the rise in volatility. For example, in a recent report of the consulting company "Accenture" we read that "the introduction of financial derivatives has fueled speculation in global commodity prices, creating tremendous price volatility".

The present paper focuses exclusively on agricultural commodities. The presence of volatility trends in agricultural commodity prices affects both production and investment decisions. For farmers, for example, positive volatility trends make the cost of hedging, through options, increasingly higher (see Calvo-Gonzales et. al 2010). Nonfarm investors interested in including agricultural commodities in their portfolios (possibly through exchange traded funds - ETFs) are also affected by volatility trends. More specifically, their investment decisions are likely to be severely distorted if trends in the covariance matrix of the commodity returns are not accounted for. Indeed, one of the main purposes of the present paper is to analyze in detail how investors should construct optimal portfolios for cases in which the second moments of asset returns display trending behavior.

The purpose of this paper is three-fold: First, we provide evidence showing that variance trends are present in the returns series of major agricultural commodities. Second, motivated by this evidence, we investigate plausible stochastic structures together with their properties that are likely to capture adequately the trending behavior of volatility. We show that such structures arise quite naturally and are simple to describe. Moreover, we show that similar structures have already been employed extensively in the recent time series literature on unit roots. We also show that the presence of volatility trends does not necessarily imply explosive asymptotic behavior of the underlying process, $\{R_t\}$, but instead it is consistent with convergence-in-law of $\{R_t\}$ to an infinite-variance random variable with a well defined distribution. In fact, trending volatility may be thought of as providing a link between the bounded and infinite variance cases analyzed in the literature, since it permits the variances to be finite for any $t < \infty$, tending to infinity (not necessarily monotonically) as t grows larger. These first two tasks, namely the empirical and theoretical motivation of volatility trends are analyzed in Section 2. Third, we formulate and solve the portfolio problem faced by an investor when the variances and the covariances of the returns of the available assets are polynomial functions of time of order k. We derive a consistent estimator of k and apply this estimator to construct an optimal portfolio consisting of four agricultural commodities, namely corn, soybeans, sugar and wheat. Finally, we compare the performances of this portfolio with the performances of portfolios constructed by alternative strategies, not accounting for volatility trends. These issues are analyzed in Section 3. The last Section concludes the paper.

2 Motivation

2.1 Empirical Motivation

In this section, we show that the monthly percentage changes, R_{it}, in the price of four major agricultural commodities, namely corn, soybeans, sugar and wheat, are characterized by trending variances[1]. We employ an index of the spot price for each of these commodities generated by Standard and Poors (S&P GSCI^{TM}) for the period 1990m1 to 2009m8[2]. Figures 1 and 2 report recursive and rolling estimates respectively of the residual variance of an AR(1) model for R_{it}.

FIGURE 1 AROUND HERE

It can be seen that a clear upward, albeit non-monotonic trend is evident in all the four series under consideration. It must be noted that although the overall long-run volatility behavior is upward trending there are nevertheless periods in which this trend is disrupted. This in turn implies that modelling such a behavior in terms of just including a linear trend in the variance equation is clearly inappropriate. Instead, modelling such a complex behavior requires a parametric model which is flexible enough to account not only the long run upward trend but the variations around this trend, as well.

[1] We assume that the investor holds the spot commodity in his portfolio, thus treating it as any other financial asset (see Arthur, Carter and Abizadeh 1988, for a similar approach).

[2] Note that three alternative S&P GSCI^{TM} indices are published for each crop: excess return, total return and spot indices. The excess return index measures the returns accrued from investing in uncollateralized nearby commodity futures, the total return index measures the returns accrued from investing in fully-collateralized nearby commodity futures, and the spot index measures the level of nearby commodity prices. All the three alternative definitions give similar results.

FIGURE 2 AROUND HERE

This discussion suggests that a possible model for the unconditional variance of R_{it} might take the polynomial-like form

$$\begin{aligned} R_{it} &= \sqrt{f_i(t)}\nu_{it} \\ f_i(t) &= t^{k_i} + g_i(t),\ k_i \geq 0 \end{aligned} \tag{1}$$

where

$$g_i(t) = o\left(t^{k_i}\right) \tag{2}$$

and ν_{it} are zero-mean, second-order stationary processes. The flexibility of this specification arises from the fact that it aims at capturing the long-run volatility trends, as determined by the values of k_i, under a wide range of possible functional forms for $g_i(t)$. For example, consider the second-order polynomial-like function

$$f_i(t) = c_{i,0} + c_{i,1}t + c_{i,2}t^2 + c_{i,3}(\sin(\frac{t}{d}) + 1) + c_{i,4}t(\sin(\frac{t}{d}) + 1),\ d, c_{i,j} > 0. \tag{3}$$

This equation falls into the class of functions defined by (1) - (2). The two extra terms, $(\sin(\frac{t}{d}) + 1)$ and $t(\sin(\frac{t}{d}) + 1)$ in (3) capture the potentially oscillating behavior of volatility around the long-run upward trend as is empirically documented in the agricultural commodity series under consideration (see Figure 1). Note that the parameter, d, controls for the number of the sinusoidal cycles that are likely to be present in a sample of T observations.

It must be noted that the specification (1) assumes implicitly that R_{it} is a serially uncorrelated process. In the case that R_{it} exhibits linear temporal dependence, the volatility trends may be introduced via the error sequence driving R_{it}. For example, if R_{it} follows an AR(1) process with volatility trends, model (1) should be replaced by

$$R_{it} = \rho R_{it-1} + \varepsilon_{it}$$

where $\varepsilon_{it} = \sqrt{f_i(t)}\nu_{it}$.

2.2 Theoretical Motivation and Connection to Existing Literature

The preceding subsection has offered evidence that crops returns exhibit volatility trends. In this section, we describe plausible stochastic structures that display unconditional heteroscedasticity of the sort introduced in (1). We begin by analyzing a stochastic structure that is quite familiar from the literature on unit-root processes. More specifically, certain Gaussian processes have covariance matrices that accommodate the simple symmetric random walk, as a special case. These processes can also give rise to autoregressive models with trending error variances. For example, consider the process $\{Y_t\}_{t\geq 0}$ with $Y_0 = 0$ a.s., defined by:

$$\begin{bmatrix} Y_t \\ Y_{t-1} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_0^2 t & r_t\sigma_0^2\sqrt{t(t-1)} \\ r_t\sigma_0^2\sqrt{t(t-1)} & \sigma_0^2(t-1) \end{bmatrix}\right), \quad r_t \geq 0 \tag{4}$$

where r_t is the correlation coefficient of Y_t and Y_{t-1}. Then

$$E[Y_t|Y_{t-1}] = r_t\sqrt{\frac{t}{t-1}}Y_{t-1} \tag{5}$$

and

$$Var(Y_t|Y_{t-1}) = (1-r_t^2)\sigma_0^2 t \tag{6}$$

Let us consider the case where

$$E[Y_t|Y_{t-1}] = \rho Y_{t-1}, \ -1 < \rho \leq 1 \tag{7}$$

Taking into account (5), we have that

$$r_t = \rho\sqrt{\frac{t-1}{t}} \tag{8}$$

Therefore,

$$\begin{bmatrix} Y_t \\ Y_{t-1} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_0^2 t & \rho\sigma_0^2(t-1) \\ \rho\sigma_0^2(t-1) & \sigma_0^2(t-1) \end{bmatrix}\right)$$

Given that $\sigma_1 = \rho\sigma_0^2$, we have

$$\begin{bmatrix} Y_t \\ Y_{t-1} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_0^2 t & \sigma_1(t-1) \\ \sigma_1(t-1) & \sigma_0^2(t-1) \end{bmatrix}\right). \tag{9}$$

Note that since $-1 \leq r_t \leq 1$ and (8) holds for every t, we conclude that $|\rho| \leq 1$. Therefore, $|\sigma_1| \leq \sigma_0^2$. Moreover, from (6), (8) and the definition of σ_1 we have that

$$Var(Y_t|Y_{t-1}) = \frac{\sigma_0^4 - \sigma_1^2}{\sigma_0^2}t + \frac{\sigma_1^2}{\sigma_0^2} \tag{10}$$

So the class of processes, described by (9), can be divided in two disjoint subclasses: the first representing the case of $|\sigma_1| = \sigma_0^2$ (which corresponds to the case $\rho = 1$) and the second representing the case $\sigma_1 < \sigma_0^2$ (which corresponds to the case $\rho < 1$).

Let now the process $\{u_t\}$ be defined by

$$u_t = Y_t - E[Y_t|Y_{t-1}] \tag{11}$$

Then, by virtue of (7), (11), (10) and (9) we have that

$$Y_t = \rho Y_{t-1} + u_t \tag{12}$$

where

$$u_t \sim NI\left(0, \frac{\sigma_0^4 - \sigma_1^2}{\sigma_0^2} t + \frac{\sigma_1^2}{\sigma_0^2}\right) \tag{13}$$

From (12) and (13), we conclude that the two disjoint subclasses that form the class described by (9) correspond to:

i)

$$\left\{ \begin{array}{l} Y_t = \rho Y_{t-1} + u_t,\ \rho = 1 \\ u_t \sim NI\left(0, \sigma_0^2\right) \end{array} \right\} \tag{14}$$

and

(ii)

$$\left\{ \begin{array}{l} Y_t = \rho Y_{t-1} + u_t,\ |\rho| < 1 \\ u_t \sim NI\left(0, \left(1 - \rho^2\right) t + \rho^2 \sigma_0^2\right) \end{array} \right\}. \tag{15}$$

It is obvious that the model (15) which is a stable AR(1) model with trending error variance can be used to describe a stochastic process which exhibits linear temporal dependence and unconditional heteroscedasticity. Nevertheless, this model is restrictive in the sense that it allows only linear volatility trends. More general trending behavior may be captured quite naturally by replacing the linear specification with a more general one, such as (1) in which the error variance is a polynomial-like function of time. To sum up, in this section we show that an autoregressive model with trending error variance derives from a similar but more general econometric structure, than the one of a unit root model.

2.3 Volatility Trends: Asymptotic Explosive Behavior or Convergence-in-Law?

Model (15) is driven by a noise sequence, $\{u_t\}$, with a changing variance that tends to infinity. This might be regarded as an undesirable feature of the model, since it may be thought to imply that in the long run the underlying process exhibits explosive behavior. Since such a behavior is not observed in practice, the question arises whether model (15) is a reasonable model. However, this concern is not well-founded since an infinite variance does not necessarily imply explosive behavior. More specifically, the assumption of infinite limiting variance does not preclude the possibility that the sequence $\{u_t\}$ converges in distribution to a well defined random variable. As an example, consider a random variable X_c, $c > 0$ that takes values in the interval $[-c, c]$, according to a truncated Cauchy distribution with zero median and half width at half maximum equal to b[3]. The probability measure, induced by X_c, is defined as follows:

$$P[X_c = -c] = P[X_c = c] = \int_{-\infty}^{-c} h(x)dx = \int_{c}^{\infty} h(x)dx = \frac{1}{2} - \frac{1}{\pi}\arctan\left(\frac{c}{b}\right)$$

and for $x \in (-c, c)$

$$P[X_c \le x] = P[X_c = -c] + \int_{-c}^{x} h(s)ds$$

Then

$$E[X_c] = 0$$

and

$$\begin{aligned}
Var(X_c) &= c^2\left(1 - \frac{2}{\pi}\arctan\left(\frac{c}{b}\right)\right) + \frac{b}{\pi}\int_{-c}^{c}\frac{x^2}{b^2+x^2}dx \\
&= c\left[c\left(1 - \frac{2}{\pi}\arctan\left(\frac{c}{b}\right)\right)\right] + \frac{b}{\pi}\int_{-c}^{c}dx - \frac{b^2}{\pi}\int_{-c}^{c}\frac{b}{b^2+x^2}dx \\
&= c\left[c\left(1 - \frac{2}{\pi}\arctan\left(\frac{c}{b}\right)\right)\right] + \frac{2bc}{\pi} - \frac{2b^2}{\pi}\arctan\left(\frac{c}{b}\right) \\
&= c\left[c\left(1 - \frac{2}{\pi}\arctan\left(\frac{c}{b}\right)\right) + \frac{2b}{\pi}\right] - \frac{2b^2}{\pi}\arctan\left(\frac{c}{b}\right)
\end{aligned}$$

Moreover,

[3] This random variable may be denoted as $C(0, b)$. Its density function is given by $h(x) = \frac{1}{\pi}\frac{b}{b^2+x^2}$.

$$\lim_{c\longrightarrow\infty} c\left(1-\frac{2}{\pi}\arctan\left(\frac{c}{b}\right)\right) = \lim_{c\longrightarrow\infty}\frac{1-\frac{2}{\pi}\arctan\left(\frac{c}{b}\right)}{\frac{1}{c}} = \lim_{c\longrightarrow\infty}\frac{-\frac{2}{\pi}\frac{b}{b^2+c^2}}{-\frac{1}{c^2}} = \frac{2b}{\pi}$$

Set now

$$A_c = c\left(1-\frac{2}{\pi}\arctan\left(\frac{c}{b}\right)\right)+\frac{2b}{\pi} \text{ and } Z_c = \frac{1}{\sqrt{A_c}}X_c$$

Then,

$$\lim_{c\longrightarrow\infty}\sqrt{A_c} = 2\sqrt{\frac{b}{\pi}}$$

and the probability measure, corresponding to Z_c, is given by

$$P[Z_c \le x] = P[X_c \le \sqrt{A_c}x] = \frac{1}{2}+\frac{1}{\pi}\arctan\left(\frac{\sqrt{A_c}x}{b}\right)$$

for $-\frac{1}{\sqrt{A_c}}c \le x < \frac{1}{\sqrt{A_c}}c$ and

$$P\left[Z_c \le \frac{1}{\sqrt{A_c}}c\right] = 1\ .$$

Therefore

$$Z_c \overset{L}{\to} C\left(0, \frac{2}{\sqrt{b\pi}}\right) \text{ as } c\to\infty.$$

Since b is an arbitrary positive number, then for every random variable, Z, following a zero median Cauchy distribution, we can construct a stochastic process Z_c with $Var(Z_c) < \infty$ that converges to Z. The variance of Z_c is given by

$$Var(Z_c) = r(c) = c - \frac{\frac{2b^2}{\pi}\arctan\left(\frac{c}{b}\right)}{c\left(1-\frac{2}{\pi}\arctan\left(\frac{c}{b}\right)\right)+\frac{2b}{\pi}} = c+q(c) \text{ with } q(c) = O(1)\ .$$

Next, let us assume that c is a function of t of the form, $c \equiv c_t = t^k$, $k > 0$, and set $u_t \equiv Z_{c_t}$, $f(t) \equiv r(c_t) = t^k + q(t^k)$ and, $v_t = \frac{1}{\sqrt{f(t)}}u_t$. Observe that $f(t)$ is a polynomial-like function. Moreover, it is obvious that $Var(u_t) = Var(Z_{c_t}) = r(c_t) = f(t)$. In the context of this example, the order k determines the rate at which the sequence $\{u_t\}$ converges in distribution to the Cauchy random variable. These considerations will motivate the model proposed in the next section.

3 Optimal Portfolios of Assets with Trending Volatilities

The next logical question concerns the implications of the trending variance hypothesis for optimal portfolio construction. In particular, assume that $\{\mathbf{R}_t\}$ denotes a $n-$dimensional vector stochastic process of the returns, R_{it}, $i = 1, 2, ..., n$ of n assets. The standard Markowitz procedure assumes that $\{\mathbf{R}_t\}$ is an independent and identically distributed (*iid*) process with mean vector, $\boldsymbol{\mu}$, and covariance matrix Σ. Based on the *iid* assumption, the portfolio $\mathbf{w} = [w_1, w_2, \ldots, w_n]'$ that minimizes the risk for a given level of expected return is time-invariant. The assumption of trending variances in stock returns violates the *iid* assumption, thus requiring a re-formulation of the optimization problem in the new framework. In the specification that follows, we shall retain the assumption of independence of $\{\mathbf{R}_t\}$ for reasons of simplicity. Specifically, we have,

$$\mathbf{R}_t = [R_{1t}, R_{2t}, \ldots, R_{nt}]' ,$$

$$E[\mathbf{R}_t] = \boldsymbol{\mu} = [\mu_1, \mu_2, \ldots, \mu_n]' \ .$$

and

$$\mathbf{R}_t = \boldsymbol{\mu} + \mathbf{u}_t,$$

with $\mathbf{u}_t = [u_{1t}, u_{2t}, \ldots, u_{nt}]'$. The stochastic properties of $\{\mathbf{R}_t\}$ are determined by those of $\{\mathbf{u}_t\}$. In particular, we assume that $\{\mathbf{u}_t\}$ is an independent process with $E[\mathbf{u}_t] = \mathbf{0}$. Moreover, we assume that the covariance matrix, Q_t, of $\mathbf{u}_t$ changes with time according to

$$Q_t = (q_{ij,t})_{1 \leq i,j \leq n} = F_t \bullet \Sigma \ , \tag{16}$$

$$F_t = (f_{ij}(t))_{1 \leq i,j \leq n} \, , \, \Sigma = (\sigma_{ij})_{1 \leq i,j \leq n}$$

where "$\bullet$" denotes the element-wise Hadamard product and $f_{ij}(t)$, $1 \leq i, j \leq n$ are functions of time, yet to be specified. This means that

$$q_{ij,t} = f_{ij}(t)\sigma_{ij}, \ 1 \leq i, j \leq n.$$

Note that Q_t is also the covariance matrix of $\mathbf{R}_t$.

More specifically, we postulate the following model for the time-heterogeneity structure of $\mathbf{u}_t$:

$$\left.\begin{array}{c} \mathbf{u}_t = \mathbf{A}(t)\mathbf{v}_t \ , \\ \mathbf{A}(t) = diag\left\{\sqrt{f_1(t)}, \sqrt{f_2(t)}, \ldots, \sqrt{f_n(t)}\right\} , \\ f_i(t) = t^{k_i} + o\left(t^{k_i}\right) , \ k_i \geq 0 \\ \mathbf{v}_t \sim iid(0, \Sigma) \\ \max_i E[|v_{it}|^r] \leq B < \infty \text{ a.s.}, \, r > 2. \end{array}\right\} \tag{17}$$

The model (17) implies that both the variances and the (absolute values of the) covariances of $\mathbf{R}_t$ are, in general, increasing functions of time. As a result, the optimal portfolio weights will vary over time as well. Specifically, assume that at period T, the typical investor wishes to determine the portfolio $\mathbf{w}_{pT} = [w_{1T}, w_{2T}, \ldots, w_{nT}]'$ that, for a given level of expected returns, minimizes the portfolio risk for period $T+1$. The solution of this optimization problem produces the following portfolio (vector of weights):

$$\mathbf{w}_{pT} = \left(\frac{C\mu_p - A}{D}\right) Q_{T+1}^{-1}\boldsymbol{\mu} + \left(\frac{B - A\mu_p}{D}\right) Q_{T+1}^{-1}\mathbf{1} \ , \tag{18}$$

where

$$\begin{aligned} A &= \mathbf{1}'Q_{T+1}^{-1}\boldsymbol{\mu} \ , \\ B &= \boldsymbol{\mu}'Q_{T+1}^{-1}\boldsymbol{\mu} \ , \\ C &= \mathbf{1}'Q_{T+1}^{-1}\mathbf{1} \ , \\ D &= BC - A^2 \text{ and} \\ \mathbf{1} &= [1, 1, \ldots, 1]' \in \mathbb{R}^n \ . \end{aligned}$$

For the practical implementation of solution (18), we need to obtain consistent estimates of $\boldsymbol{\mu}$ and Q_{T+1}.To this end, note that the standard sample covariance matrix estimator $\frac{1}{T}\sum_{t=1}^{T}\hat{\mathbf{u}}_t\hat{\mathbf{u}}_t'$ diverges to infinity, where $\hat{\mathbf{u}}_t = \mathbf{R}_t - \hat{\boldsymbol{\mu}}$, and $\hat{\boldsymbol{\mu}} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{R}_t$. The first step towards solving the estimation problem at hand, is to obtain a consistent estimator of k_i, $i = 1, 2, \ldots, n$. Such an estimator is provided by the following theorem:

Theorem 1: Under the specification (17), the statistic $\hat{k}_i$ given by

$$\hat{k}_i = \frac{1}{\ln 2} \ln\left(\frac{\sum_{t=1}^{T}\hat{u}_{i,t}^2}{\sum_{t=1}^{[T/2]}\hat{u}_{i,t}^2}\right) - 1 \tag{19}$$

is a strongly consistent estimator of k.

Proof: See Appendix A.

Next, we must obtain a consistent estimator of Σ. Let us denote by '$\xrightarrow{P}$' the convergence in probability. Theorem 1 allows us to obtain the following estimator for the covariance matrix Σ:

Proposition 1: Under the specification (17), and if $k_i - \widehat{k}_i = o_p\left(\frac{1}{\ln T}\right)$, $i = 1, 2, \ldots, n$,

$$S_T \bullet \sum_{t=1}^{T} \hat{\mathbf{u}}_t \hat{\mathbf{u}}_t' \xrightarrow{P} \Sigma \tag{20}$$

where

$$S_T = (s_{ij,T})_{1 \le i,j \le n} = \left(\frac{\left(\widehat{k}_i + \widehat{k}_j\right)/2+1}{T^{\left(\widehat{k}_i + \widehat{k}_j\right)/2+1}} \right)_{1 \le i,j \le n}.$$

Proof: See Appendix A.

Proposition 1 allows us to estimate Σ, which in turn implies that a consistent estimate $\widetilde{Q}_t$ of Q_t is feasible, in the sense that $\widetilde{Q}_t - Q_t \xrightarrow{P} 0$, provided that the exact form of F_t were known. However, we assume no a-priori knowledge of the exact functional forms f_i, $1 \le i \le n$, except from the fact that the degree k_i of the polynomial part, t^{k_i}, can be consistently estimated through (19). Since $f_i - t^{k_i} = o(t^{k_i})$, we are allowed to consider as an adequate approximation of $f(t+1)$ the value of $(t+1)^{\widehat{k}_i}$. A direct application of this approximation and Proposition 1 to (18) yields a feasible approximation of the optimal portfolio based on the extrapolated covariance matrix $\widehat{Q}_{T+1}$:

Corollary: The optimal portfolio, $\mathbf{w}_{pT}$, which minimizes the quantity

$$Var(r) = \mathbf{w}_T' Q_{T+1} \mathbf{w}_T \tag{21}$$

subject to

$$E[r] = \mathbf{w}_T' \boldsymbol{\mu} = \mu_p. \tag{22}$$

and

$$\mathbf{1}' \mathbf{w}_T = 1$$

is approximated by

$$\widehat{\mathbf{w}}_{pT} = \left(\frac{\widehat{C}\mu_p - \widehat{A}}{\widehat{D}}\right)\widehat{Q}_{T+1}^{-1}\widehat{\boldsymbol{\mu}} + \left(\frac{\widehat{B} - \widehat{A}\mu_p}{\widehat{D}}\right)\widehat{Q}_{T+1}^{-1}\mathbf{1} \ ,$$

where

$$\begin{aligned}
\widehat{Q}_{T+1} &= \left(\hat{q}_{ij,T+1}\right)_{1\le i,j\le n} \ , \\
\text{with } \hat{q}_{ij,T+1} &= \hat{\sigma}_{ij}\,(T+1)^{\left(\hat{k}_i+\hat{k}_j\right)/2} \\
&= \left(1+\frac{1}{T}\right)^{\left(\hat{k}_i+\hat{k}_j\right)/2} \frac{\left(\left(\hat{k}_i+\hat{k}_j\right)/2+1\right)}{T}\sum_{t=1}^{T}\hat{u}_{it}\hat{u}_{jt} \\
\widehat{A} &= \mathbf{1}'\widehat{Q}_{T+1}^{-1}\widehat{\boldsymbol{\mu}} \ , \\
\widehat{B} &= \boldsymbol{\mu}'\widehat{Q}_{T+1}^{-1}\widehat{\boldsymbol{\mu}} \ , \\
\widehat{C} &= \mathbf{1}'\widehat{Q}_{T+1}^{-1}\mathbf{1} \ , \\
\widehat{D} &= \widehat{B}\widehat{C} - \widehat{A}^2 \text{ and} \\
\mathbf{1} &= [1,1,\ldots,1]' \in \mathbb{R}^n \ .
\end{aligned}$$

The optimization problem defined above yields the one-period ahead optimal portfolio for a specific level of expected portfolio returns, thus determining the one-period ahead efficient frontier. Estimating Q_{T+1} by means of the sample covariance matrix $\widetilde{Q}_{T+1}$ produces misleading estimates of the second moments of $\mathbf{R}_{T+1}$, since it ignores the presence of variance trends. In particular, if $k > 0$ then $\widetilde{Q}_T \to \infty$. More specifically, let us assume that the investor ignores the presence of unconditional heteroscedasticity and decides to estimate the supposedly time-invariant covariance matrix by means of the standard sample covariance matrix estimator, $\widetilde{Q}_{T+1} = \left(\widetilde{q}_{ij,T+1}\right)_{1\le i,j\le n}$ with

$$\widetilde{q}_{ij,T+1} = \frac{1}{T}\sum_{t=1}^{T}\hat{u}_{it}\hat{u}_{jt} \ .$$

In such a case, the investor would have erroneously concluded that the optimal weights are given by

$$\begin{aligned}
\widetilde{\mathbf{w}}_{pT} &= \left(\frac{\widehat{C}\mu_p - \widehat{A}}{\widehat{D}}\right)\widetilde{Q}_{T+1}^{-1}\widehat{\boldsymbol{\mu}} + \left(\frac{\widehat{B} - \widehat{A}\mu_p}{\widehat{D}}\right)\widetilde{Q}_{T+1}^{-1}\mathbf{1} \\
&= \widetilde{Q}_{T+1}^{-1}\widehat{Q}_{T+1}\widehat{\mathbf{w}}_{pT}
\end{aligned}$$

with $\widehat{\mathbf{w}}_{pT}$ being the truly optimal weights. The investor suffers zero loss only in the case that $\widetilde{Q}_{T+1}^{-1}\widehat{Q}_{T+1}$

is the identity matrix. However, this is not so, since for every $1 \leq i, j \leq n$,

$$\frac{\hat{q}_{ij,T+1}}{\widetilde{q}_{ij,T+1}} = \left(1 + \frac{1}{T}\right)^{\left(\hat{k}_i + \hat{k}_j\right)/2} \left(\left(\hat{k}_i + \hat{k}_j\right)/2 + 1\right),$$

which tends to 1 only in the case where $k_i = k_j = 0$, i.e. when all the returns have constant variances. These omitted trending variance effects are likely to explain an undesirable feature of the implementation of the standard optimization procedure often reported in the literature, namely that the estimated weights vary wildly with the selected sample. As DeMiguel, Garlappi and Uppal (2009) note "...the implementation of these portfolios with moments estimated via their sample analogues is notorious for producing extreme weights that fluctuate substantially over time and perform poorly out of sample" (2009, p. 1916).

Remarks:

1) The weights will not remain constant (and the optimal frontier too) due to time heterogeneity. This fact implies the importance of the stepwise recalculation of optimal weights. Moreover, since in the long run, the lower k(s) will yield significantly lower variances, an optimal portfolio chosen at time T with very long horizon, will consist only of the asset(s) that correspond to this lower k(s).

2) If $f_i(t) = t^{k_i} + o(1)$, $1 \leq i \leq n$, then from Proposition 1 we obtain $\widehat{Q}_{T+1} - Q_{T+1} \xrightarrow{P} 0$, which in turn implies that $\widehat{\mathbf{w}}_{pT} - \mathbf{w}_{pT} \xrightarrow{P} 0$ as $T \to \infty$.

It must be also noted that the presence of variance trends does not affect the consistency of the sample mean estimator $\hat{\mu}_i$ of μ_i, when $k_i < 1$. Indeed, we can see that

$$\sum_{t=1}^{T} \frac{Var(u_{it})}{t^2} = \sum_{t=1}^{T} \sigma_{ii} \frac{f_i(t)}{t^2} < \infty$$

only when $k_i < 1$. Then, an application of Theorem 20.11 of Davidson (1994) yields

$$\frac{1}{T} \sum_{t=1}^{T} u_{i,t} \xrightarrow{a.s.} 0,$$

and the strong consistency of $\hat{\mu}_i$ is proved by considering that $\hat{\mu}_i = \frac{\mathbf{1}}{\mathbf{T}} \sum_{t=1}^{T} R_{it} = \mu + \frac{1}{T} \sum_{t=1}^{T} u_{i,t}$.

4 An Empirical Application to Agricultural Commodities

In this section, we compare the out-of-sample performance of four alternative strategies of constructing "optimal" crop portfolios. First, we consider a set of four assets, namely corn, soybean, sugar and wheat whose returns have already been found to exhibit volatility trends. Second, at each point in time, we construct four alternative portfolios of these assets: The first portfolio, referred to as the "benchmark portfolio" (BP) is constructed by allocating 1/4 of wealth to each of these four assets that is available for investment at each rebalancing period. The second portfolio, referred to as the "Markowitz portfolio" (MP), is constructed by employing standard mean-variance optimization techniques, in which the population moments are estimated by their corresponding sample analogues, $\hat{\boldsymbol{\mu}}$ and $\widetilde{Q}_{T+1}$. The third portfolio, referred to as the Trending Volatility portfolio (TVP), is constructed as the Markowitz portfolio but using the trending variance estimator $\widehat{Q}_{T+1}$ instead of the static $\widetilde{Q}_{T+1}$. To fourth portfolio is constructed by assuming that conditional instead of unconditional heteroscedasticity is present in asset returns. More specifically, in this portfolio, referred to as the multivariate GARCH (MGARCH) portfolio the covariance matrix, Q_t^G, of asset returns is assumed to follow a multivariate GARCH process. This specification assumes that heteroscedasticity is a manifestation of non linear temporal dependence instead of time heterogeneity. It is well known that univariate GARCH type models are very popular for modeling the diagonal elements of Q_t^G, while multivariate GARCH models impose a GARCH type stochastic equation to all elements of Q_t^G (see, for example, Bauwens et. al. 2006). However these models become very demanding in computational effort for large dimensions. To overcome this problem, new models have been proposed that maintain the GARCH type structure for all variances in Q_t^G,while making additional assumptions on the dynamic behavior of correlation coefficients. Bollerslev (1990) suggested the Constant Conditional Correlation coefficient model and imposed the following structure for the off diagonal elements of Q_t^G:

$$Q^G_{(i,j),t} = \rho_{i,j}\sigma_{i,t}\sigma_{j,t}$$

where $\sigma_{i,t}$ and $\sigma_{j,t}$ are the roots of the GARCH type variances in the main diagonal of Q_t^G, while correlation coefficients are kept constant. Engle (2002) introduced the Dynamic Conditional Correlation coefficient in which case:

$$Q^G_{(i,j),t} = \rho_{(i,j),t}\sigma_{i,t}\sigma_{j,t}$$

imposing a specific parameterization of the updating equation for $\rho_{(i,j),t}$.

The out-of-sample performance of these portfolios is evaluated for the period 1996m1 - 2009m8 using standard Sharpe ratios, whereas the period 1990m1-1995m12 serves as the initial estimation period. The results may be summarised as follows:

(i) The estimates of k for both the full sample 1990m1-2009m8 and the estimation sample 1990m1-1995m12 are positive for all the four commodities under consideration. More specifically, the full-sample estimates, $\hat{k}$, are equal to 0.55, 0.70, 0.41 and 0.32 for corn, soybeans, sugar and wheat respectively.

(ii) The presence of non zero k's affects heavily the estimates of the covariance matrix of returns at each rebalancing period. For example, for the case of full sample, the standard sample covariance estimates $\widetilde{Q}_{T+1}$ and the trending-covariance estimates $\widehat{Q}_{T+1}$ are given by

$\widetilde{Q}_{T+1} =$

	corn	soybeans	sugar	wheat
corn	52.99	37.59	4.22	28.50
soybeans	37.59	52.43	6.77	24.56
sugar	4.22	6.77	84.82	4.36
wheat	28.50	24.56	4.36	53.61

and

$\widehat{Q}_{T+1} =$

	corn	soybeans	sugar	wheat
corn	82.48	61.28	6.23	41.00
soybeans	61.29	89.35	10.51	36.15
sugar	6.23	10.51	118.75	5.93
wheat	41.00	36.15	5.93	70.84

.

The presence of volatility trends as documented by the positive estimates of k for all the four assets under consideration results in an estimated covariance matrix $\widehat{Q}_{T+1}$ which is dramatically different than the standard $\widetilde{Q}_{T+1}$. Put it differently the elements of $\widetilde{Q}_{T+1}$ are significantly corrected to take into account the presence of polynomial like trend in the second moments. This correction is not the same for all the elements of the covariance matrix, but depends on the values of the estimated k. For example, the corrected estimates for the variance of corn, soybeans, sugar and wheat are 55%, 71%, 40% and 32.1% larger than their corresponding uncorrected estimates, reflecting the differences in the estimates of k among these four commodities.

(iii) The out-of-sample performance of TVP is much higher than that of the other three portfolios. In particular, the mean returns of TVP is 0.50, whereas the mean return of BP, MP and MGARCH is 0.28, 0.36 and 0.27 respectively. Moreover, the differences in the standard deviations of these four portfolios are negligible which together with the mean estimates produce Sharpe ratios equal to 0.080, 0.060, 0.048 and 0.044 for TVP, MP, BP and MGARCH, respectively.

5 Conclusions

In this paper we have focused on the volatility behavior of agricultural commodity prices. We have provided empirical evidence showing that the returns on major agricultural commodities such as corn, soybeans, sugar and wheat exhibit volatility trends. We have then shown analytically that: (a) the emergence of such volatility trends can arise quite naturally within the same class of stochastic processes that have been extensively analyzed in the unit root literature, and (b) such volatility trends are consistent with the empirical evidence on that stochastic process of returns do not display explosive behavior. These considerations led us to the development of a semi-parametric model of the behavior of the second moments of the return generating process. These second moments are assumed to exhibit unbounded heteroscedasticity in the form of polynomial-like functions of time, thus being asymptotically unbounded. We then solved the portfolio problem. It is shown that the optimal solution is a function of time, depending on the orders k_i, $i = 1, 2, ..., n$ at which the variances and covariances of asset returns grow over time. A feasible approximation to the optimal solution is obtained, which is based on the consistent estimator of k_i, also derived in this paper. This approximate solution is then applied to construct a portfolio consisting of the four agricultural commodities under consideration. The performance of this portfolio was found to be superior to those of alternative portfolios in which the covariance matrix of returns is estimated by traditional methods.

Appendix A: Proofs

Proof of Theorem 1: Let us define the sequence $F_{it} = u_{it}^2 - f_i(t)\sigma_{ii} = f_i(t)(v_{it}^2 - \sigma_{ii})$, $1 \leq t$. Then, set $a_{it} = t^{k_i+1}$. Then, since the process $\{u_{it}\}_{t\geq 1}$ satisfies our initial assumptions, $\{F_{it}\}_{t\geq 1}$ is a martingale difference process satisfying

$$E\left[|F_{it}|^{r/2}\right] \leq |f_i(t)|^{r/2}\left(B^{2/r} + \sigma_{ii}\right)^{r/2} = C_B|f_{ii}(t)|^{r/2} < \infty \ ,$$

for some $C_B > 0$. Therefore, from the definition of $f_{ii}(\cdot)$ we have that for some $C_{B'} > 0$,

$$\sum_{t=1}^{\infty} \frac{E\left[|F_{it}|^{r/2}\right]}{\left(t^{k_i+1}\right)^{r/2}} \leq C_{B'} \sum_{t=1}^{\infty} \frac{t^{k_i r/2}}{\left(t^{k_i+1}\right)^{r/2}} = C_{B'} \sum_{t=1}^{\infty} \frac{1}{t^{r/2}} < \infty,$$

since $r/2 > 1$. Therefore we can apply Theorem 20.11 of Davidson (1994) and conclude that

$$\frac{1}{T^{k_i+1}} \sum_{t=1}^{T} f_i(t)(v_{it}^2 - \sigma_{ii}) \stackrel{a.s.}{\to} 0 \ .$$

Then, we observe that

$$\frac{1}{T^{k_i+1}} \sum_{t=1}^{T} \sigma_{ii} f_i(t) \to \frac{\sigma_{ii}}{k_i+1} \Longrightarrow \frac{1}{T^{k_i+1}} \sum_{t=1}^{T} u_{it}^2 \stackrel{a.s.}{\to} \frac{\sigma_{ii}}{k_i+1} \ , \text{ as } T \to \infty \ . \tag{A.1}$$

Next note that since $\widehat{u}_{it} = u_{it} - \frac{1}{T}\sum_{l=1}^{T} u_{il}$, we can easily obtain

$$\frac{1}{T^{k_i+1}} \sum_{t=1}^{T} \widehat{u}_{it}^2 = \frac{1}{T^{k_i+1}} \sum_{t=1}^{T} u_{it}^2 - \frac{1}{T^{k_i+2}} \left(\sum_{t=1}^{T} u_{it}\right)^2 . \tag{A.2}$$

Since

$$\sum_{t=1}^{T} \frac{Var(u_{it})}{t^{k_i+2}} = \sum_{t=1}^{T} \frac{\sigma_{ii} f_i(t)}{t^{k_i+2}} < \infty,$$

we can apply again Theorem 20.11 of Davidson (1994) and obtain

$$\frac{1}{T^{k_i/2+1}} \sum_{t=1}^{T} u_{it} \stackrel{a.s.}{\to} 0, \text{ as } T \to \infty \ .$$

This, in turn, along with (A.1) and (A.2), implies that

$$\frac{1}{T^{k_i+1}} \sum_{t=1}^{T} \widehat{u}_{it}^2 \stackrel{a.s.}{\to} \frac{\sigma_{ii}}{k_i+1} \ , \text{ as } T \to \infty \ .$$

The same holds if we include only the first half of the sample, i.e.

$$\frac{1}{(T/2)^{k_i+1}} \sum_{t=1}^{[T/2]} \widehat{u}_{it}^2 \stackrel{a.s.}{\to} \frac{\sigma_{ii}}{k_i+1} \ .$$

Dividing these limits by parts we obtain

$$\frac{1}{2^{k_i+1}}\frac{\sum_{t=1}^{T}\widehat{u}_{it}^2}{\sum_{t=1}^{[T/2]}\widehat{u}_{it}^2}\overset{a.s.}{\rightarrow}1 \quad\Longrightarrow\quad \ln\left(\frac{\sum_{t=1}^{T}\widehat{u}_{it}^2}{\sum_{t=1}^{[T/2]}\widehat{u}_{it}^2}\right)\overset{a.s.}{\rightarrow}\ln 2^{k_i+1}=(k_i+1)\ln 2$$
$$\Longrightarrow\quad \frac{1}{\ln 2}\ln\left(\frac{\sum_{t=1}^{T}\widehat{u}_{it}^2}{\sum_{t=1}^{[T/2]}\widehat{u}_{it}^2}\right)-1\overset{a.s.}{\rightarrow}k_i \text{ as } T\to\infty.$$

Proof of Proposition 1: Since

$$q_{ij,t}=t^{(k_i+k_j)/2}\sigma_{ij}+o(t^{(k_i+k_j)/2})\ ,\ 1\le i,j\le n\ ,$$

As in the first part of Theorem 1, we set $F_{ijt}=u_{it}u_{jt}-\sqrt{f_i(t)f_j(t)}\sigma_{ij}=\sqrt{f_i(t)f_j(t)}(v_{it}v_{jt}-\sigma_{ij})$, $1\le t$. Then, following the same procedure we obtain

$$\frac{1}{T^{(k_i+k_j)/2+1}}\sum_{t=1}^{T}u_{it}u_{jt}\overset{a.s.}{\rightarrow}\frac{\sigma_{ij}}{(k_i+k_j)/2+1}\ ,\text{ as } T\to\infty\ .$$

Again, as in the proof of Theorem 1, we can easily derive that

$$\frac{1}{T^{(k_i+k_j)/2+1}}\sum_{t=1}^{T}\widehat{u}_{it}\widehat{u}_{jt}=\frac{1}{T^{(k_i+k_j)/2+1}}\sum_{t=1}^{T}u_{it}u_{jt}-\frac{1}{T^{(k_i+k_j)/2+2}}\left(\sum_{t=1}^{T}u_{it}\right)\left(\sum_{t=1}^{T}u_{jt}\right), \tag{A.3}$$

and conclude in

$$\frac{1}{T^{(k_i+k_j)/2+1}}\sum_{t=1}^{T}\widehat{u}_{it}\widehat{u}_{jt}\overset{a.s.}{\rightarrow}\frac{\sigma_{ij}}{(k_i+k_j)/2+1}\ ,\ 1\le i,j\le n\ .$$

Since, now, for every i, k_i is consistently estimated by $\widehat{k}_i$, if $k_i-\widehat{k}_i=o_p\left(\frac{1}{\ln T}\right)$, then $\frac{T^{(\widehat{k}_i+\widehat{k}_j)/2+1}}{T^{(k_i+k_j)/2+1}}\overset{P}{\rightarrow}1$

$$\frac{1}{T^{(\widehat{k}_i+\widehat{k}_j)/2+1}}\sum_{t=1}^{T}\widehat{u}_{it}\widehat{u}_{jt}\overset{P}{\rightarrow}\frac{\sigma_{ij}}{(k_i+k_j)/2+1}\ ,\ 1\le i,j\le n\ ,$$

which directly yields (20).

APPENDIX B: Figures

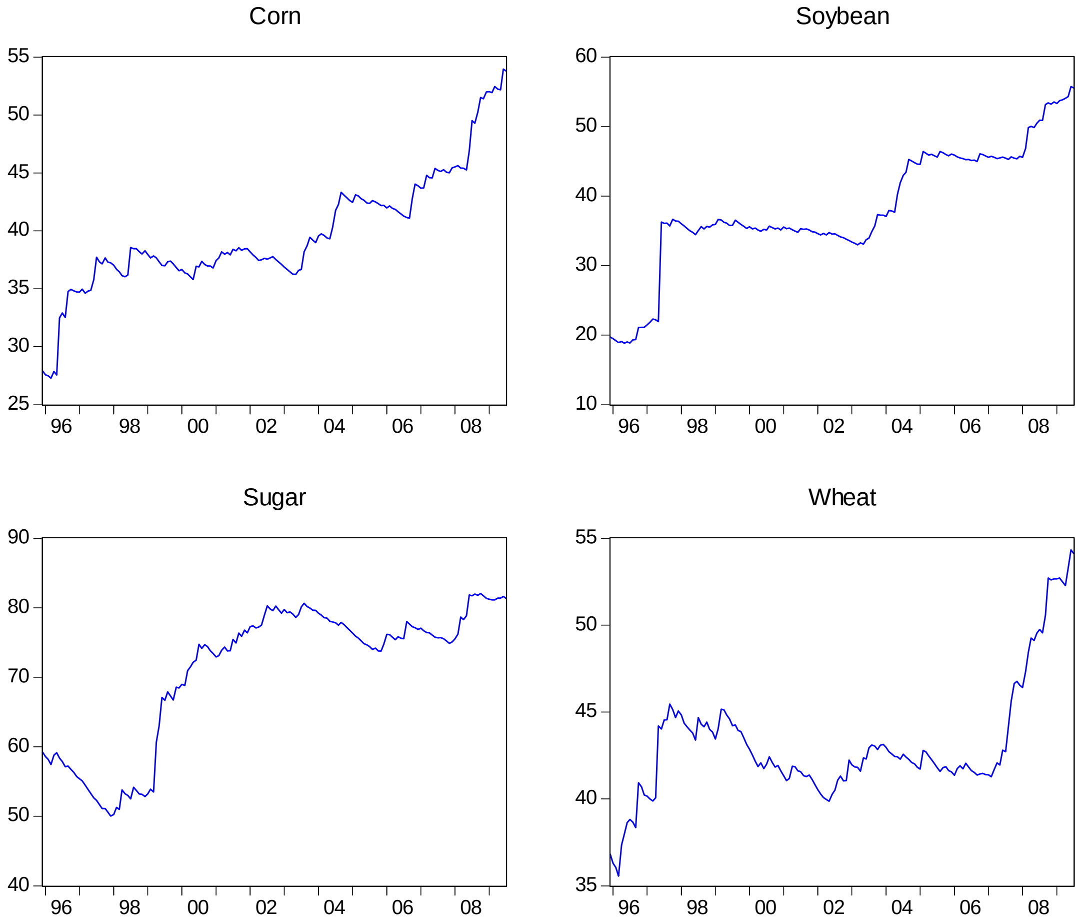

Fig. 1: Recursive Estimation of Residuals Variance from an AR(1) Model (Starting Period 1990M1-1995M12 (72 Obs)). Source: Bloomberg - S&P GS commodity indices - spot prices.

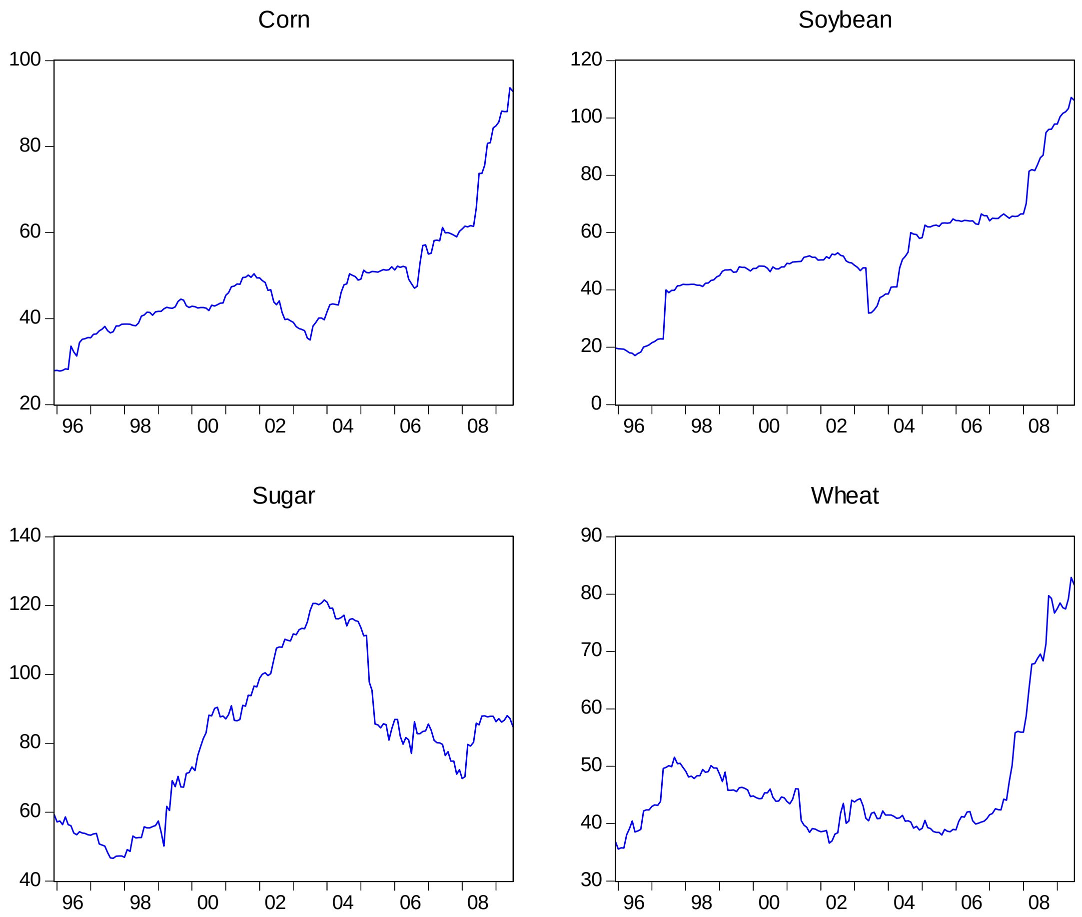

Fig. 2: Rolling Estimation of Residuals Variance from an AR(1) Model. Starting Period 1990M1-1995M12 (72 Obs). Source: Bloomberg - S&P GS commodity indices - spot prices.

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