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Heteroskedastic Factor Vector Autoregressive Estimation of Persistent and Non Persistent Processes Subject to Structural Breaks

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Abstract

In the paper the fractionally integrated heteroskedastic factor vector autoregressive (FI-HF-VAR) model is introduced. The proposed approach is characterized by minimal pretesting requirements and simplicity of implementation also in very large systems, performing well independently of integration properties and sources of persistence, i.e. deterministic or stochastic, accounting for common features of different kinds, i.e. common integrated (of the fractional or integer type) or non integrated stochastic factors, also featuring conditional heteroskedasticity, and common deterministic break processes. The proposed approach allows for accurate investigation of economic time series, from persistence and copersistence analysis to impulse responses and forecast error variance decomposition. Monte Carlo results strongly support the proposed methodology.

JEL classification: C22.

Key words: long and short memory, structural breaks, fractionally integrated heteroskedastic factor vector autoregressive model.

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1 Introduction

Recent developments in econometrics have dealt with the modelling of large systems of equations in the framework of factor vector autoregressive (FVAR) models. Following the lead of dynamic factor model (DFM) analysis proposed in Geweke (1977), it is assumed that a small number of structural shocks be responsible for the observed comovement in economic data; as the common factors are unobserved, accurate estimation may fail in the framework of small scale vector autoregressive (VAR) models, but succeed when cross-sectional information is employed to disentangle common and idiosyncratic features in economic time series.

Large VAR models are however subject to the curse of dimensionality: the FVAR approach can then be seen as a solution to the problem of over-parameterization, allowing for parsimonious modelling of large systems of dynamic equations in the framework of a (factor) augmented VAR model, where few common features account for the commonalities in a large set of economic time series.

Different approaches have been proposed in the literature so far, featuring both similarities and differences. For instance, most of the approaches rely on two-stage estimation, where the common features are estimated first, and then included in an augmented VAR, or VAR-X, model, estimated by OLS (Bernanke and Boivin, 2003; Bernanke et al., 2005; Favero et al., 2005).

One-stage Maximum Likelihood (ML) estimation, implemented in the Bayesian framework, through likelihood-based Gibbs sampling (Bernanke et al., 2005; Kose, Otrok and Whiteman, 2003), or in the classical framework, through the EM algorithm and Kalman filtering (Engle and Watson, 1981; Quah and Sargent, 1992; Doz et al., 2006, 2007), has also been proposed; yet, one-stage asymptotic efficiency could also be ensured through two-stage iterated estimation (Stock and Watson, 2005), bearing the interpretation of Quasi-ML estimation performed via the EM algorithm (Doz et al., 2007).

Differences also concern the estimation of the common factors, implemented either by means of principal components analysis (PCA) in the time domain (Stock and Watson, 1998) or in the frequency domain (Forni et al. 2000, 2002, 2004; Morana 2004; Beltratti and Morana, 2006), or by means of weighted averages of observed variables (Pesaran, 2006).

For instance, (unweighted) time domain PCA estimation is performed by Bernanke and Boivin (2003), Bernanke et al. (2005), Favero et al. (2005), Stock and Watson (2005), Morana (2009), Bagliano and Morana (2008), Banerjee and Marcellino (2009); weighted time domain PCA estimation is performed in Boivin and Ng (2004), with weights set equal to the inverse of the standard deviation of the estimated idiosyncratic components, featuring

Kalman filtering of the estimated factors in Doz et al. (2007) as well. Moreover, applications of frequency domain PCA can be found in Giannone et al. (2002, 2004) and Alessi et al. (2009), while the weighted average approach has so far been implemented in the Global VAR (GVAR) literature (Pesaran et al., 2004; Dees et al., 2010).

Estimation of common unobserved features by means of time domain PCA is promising, as recent asymptotic results, i.e. Bai (2003, 2004) and Bai and Ng (2004), have proved consistency and asymptotic normality under various conditions, covering the exact and approximate factor model case, with weakly stationary (short memory) or I(1) integrated processes, also featuring conditional heteroskedasticity; the validity of PCA for the intermediate case of long-memory processes has also been conjectured, and supporting Monte Carlo results are provided in Morana (2007).

Moreover, consistency for the Kalman filtering augmented PCA approach has been established by Doz et al. (2007), also showing that PCA is asymptotically equivalent to Quasi-ML estimation of the unobserved factors, when the approximating model is assumed to be static and the idiosyncratic components to be spherical.

Finally, differences can be found concerning the identification of the structural shocks; methods for the identification of selected shocks only are proposed and implemented in Stock and Watson (2005) and Dees et al. (2010), while a strategy for the identification of all the structural shocks, both common and idiosyncratic, is proposed in Morana (2009).

The paper, building on Morana (2009), contributes to the literature on FVAR modelling by introducing a new approach, the fractionally integrated heteroskedastic factor vector autoregressive (FI-HF-VAR) model, with minimal pretesting requirements for implementation, performing well independently of the integration properties of the data and of the sources of persistence, i.e. deterministic or stochastic, and therefore accounting for common features of different kinds, i.e. common integrated (of the fractional or integer type) or non integrated stochastic factors, also featuring conditional heteroskedasticity, and common deterministic break processes.

As data are modelled in deviations from the common features, accurate (and asymptotically normal and efficient) estimation can be achieved within the two-step iterated approach of Stock and Watson (2005), featuring therefore simplicity of implementation also in the case of large systems of dynamic equations. The two-step estimation procedure can also be implemented following the Granger and Jeon (2004) thick modelling strategy, providing median estimates of the parameters of interest and robust standard errors.

The proposed approach allows for an accurate investigation of the properties of the data, from persistence and copersistence analysis to impulse responses and forecast error variance decomposition for both common and idiosyncratic shocks, structuralized according to a double Choleski identification strategy of the type proposed in Morana (2009). Monte Carlo results strongly support the proposed methodology.

After this introduction, the paper is organized as follows. In section two the econometric methodology is presented, while in section three Monte Carlo analysis is performed; an empirical application is provided in section four, while conclusions are drawn in section five.

2 The FI-HF-VAR model

Consider the following fractionally integrated heteroskedastic factor vector autoregressive (FI-HF-VAR) model

$$\begin{aligned} x_t &= \Lambda_\mu \mu_t + \Lambda_f f_t + C(L)(x_{t-1} - \Lambda_\mu \mu_{t-1} - \Lambda_f f_{t-1}) + v_t \quad (1) \\ v_t &\sim iid(0, \Sigma_v) \\ P(L)D(L)f_t &= \eta_t = \sqrt{h_t}' \psi_t, \quad (2) \\ \psi_t &\sim iid(0, \Sigma_\psi) \\ M(L)\left(\eta_t^2 - w_t\right) &= N(L)(\eta_t^2 - h_t) \quad (3) \end{aligned}$$

where x_t is a n-variate vector of real valued integrated processes subject to structural breaks, $t = 1, ..., T$, L is the lag operator, f_t is a r-variate vector of heteroskedastic integrated, of order d in mean, and b in variance, common factors, with $0 \leq d_i \leq 1$, $0 \leq b_i \leq 1$, $i = 1, ..., r$, μ_t is an m-variate vector of common break processes, v_t is a n-variate vector of zero mean idiosyncratic i.i.d. shocks, with contemporaneous covariance matrix Σ_v, assumed consistent with the condition of weak cross-sectional correlation of the idiosyncratic components (Assumption E) stated in Bai (2003, p.143), ψ_t is a r-variate vector of common zero mean i.i.d. shocks, with covariance matrix $\Sigma_\psi = I_r$, $E[\psi_{it} v_{js}] = 0$ all i, j, t, s, Λ_f and Λ_μ are $n \times r$ and $n \times m$, respectively, matrices of loadings, $C(L)$ is a finite order stationary matrix of polynomials in the lag operator, i.e. $C(L) \equiv C_1 L + C_2 L^2 + ... + C_s L^s$, C_j $j = 1, .., s$ is a square matrix of coefficients of order n, $P(L)$ is a finite order stationary matrix of polynomials in the lag operator, i.e. $P(L) \equiv I_r + P_1 L + P_2 L^2 + ... + P_u L^u$, P_j, $j = 1, .., u$, is a square matrix of coefficients of order r.

$D(L)$, $M(L)$, and $N(L)$ are diagonal polynomial matrices in the lag operator of order r, specified according to the integration order of the mean and variance components of the common stochastic factors.

For the vector conditional mean process it is assumed:

$i)$ for the case of fractional integration (long memory) $(0 < d_i < 1)$[1] and the case of integration $(d_i = 1)$

$$D(L) \equiv diag\left\{(1-L)^{d_1}, (1-L)^{d_2}, ..., (1-L)^{d_r}\right\},$$

where $(1-L)^{d_i}$ is the differencing operator of fractional or integer order; the fractional differencing operator has a binomial expansion, which can be compactly written in terms of the Hypergeometric function, i.e.

$$\begin{aligned}(1-L)^{d_i} &= F(-d_i, 1, 1; L) \\ &= \sum_{k=0}^{\infty} \Gamma(k-d_i)\,\Gamma(k+1)^{-1}\,\Gamma(-d_i)^{-1} L^k \\ &= \sum_{k=0}^{\infty} \pi_k L^k, \end{aligned} \tag{4}$$

where $\Gamma(\cdot)$ is the Gamma function;

$ii)$ for the case of no integration (short memory) $(d_i = 0)$

$$D(L) \equiv I_r.$$

For the vector conditional variance process it is assumed that the common factors f_t are also conditionally orthogonal, i.e. $q_{f,t} = Cov(f_{i,t}, f_{j,s}|\Omega_{t-1}) = 0$ all i, j, t, s; moreover:

$$M(L) \equiv diag\{\phi_1(L), \phi_2(L), ..., \phi_r(L)\}$$

$$N(L) \equiv diag\{\delta_1(L), \delta_2(L), ..., \delta_r(L)\};$$

then,

$i)$ for the case of fractional integration (long memory) $(0 < b_i < 1)$ and the case of integration $(b_i = 1)$

$$\phi_i(L) \equiv (1 - \alpha_i(L) - \beta_i(L))(1-L)^{b_i}$$

$$\delta_i(L) \equiv (1 - \beta_i(L));$$

$ii)$ for the case of no integration (short memory) $(b_i = 0)$

[1] See Baillie (1996) for an introduction to long memory processes.

$$\phi_i(L) \equiv (1 - \alpha_i(L) - \beta_i(L))$$

$$\delta_i(L) \equiv (1 - \beta_i(L));$$

in both cases $\alpha_i(L) \equiv \alpha_{i,1}L + \alpha_{i,2}L^2 + ... + \alpha_{i,n}L^n$, $\beta_i(L) \equiv \beta_{i,1}L + \beta_{i,2}L^2 + ... + \beta_{i,m}L^m$, with all the roots of the $\phi_i(L)$ and $\delta_i(L)$ polynomials in the lag operator outside the unit circle; the conditional variance process $h_{i,t} \equiv Var(f_{i,t}|\Omega_{i,t-1})$ is therefore of the $FIGARCH\,(m, b_i, n)$ type (Baillie et al., 1996) or the $IGARCH\,(m, n)$ type (Engle and Bollerslev, 1986) for the fractionally integrated and integrated case, respectively, and of the $GARCH\,(m, n)$ type (Bollerslev, 1986) for the non integrated case.[2]

Finally, w_t is the long-term conditional variance process, or the time-varying unconditional variance process, or simply the break in variance process. Different specifications have been suggested in the literature for w_t , i.e. a quadratic or cubic spline function (Engle and Rangel, 2008), a Gallant (1984) flexible functional form (Baillie and Morana, 2009), a general smooth transition logistic function (Amado and Terasvirta, 2008), a Markov switching mechanism (Hamilton and Susmel, 2004); in all cases alternating regimes, recurrent or not recurrent, are allowed in the conditional variance equations. The advantage of the above break modelling approaches lies in the fact that the selection of the break process does not require pre-testing and the estimation of the break points; smooth transition across regimes, according to actual data properties, is also allowed for. The above mentioned mechanisms, may also be implemented for endogenous modelling of the break process in the mean of the series (μ_t).

2.1 The reduced fractional VAR form

Depending on persistence properties of the data, the vector autoregressive representation (VAR) for the factors f_t and the series x_t can be written as follows:

i) for the case of fractional integration (long memory) ($0 < d_i < 1$), by taking into account the binomial expansion in (4), it follows $P(L)D(L) \equiv I - \Pi(L)$, $\Pi(L) = \Pi_1 L^1 + \Pi_2 L^2 + ...$, where Π_i, $i = 1, 2, ...$, is a square matrix of coefficients of dimension r; by substituting (2) into (1), the infinite order

[2] This is just for convenience of presentation, as any combination of conditional mean and variance specification could be employed.

vector autoregressive representation for the factors f_t and the series x_t can then be written as

$$\begin{bmatrix} f_t \\ x_t - \Lambda_\mu \mu_t \end{bmatrix} = \begin{bmatrix} \Pi_f^*(L) & 0 \\ \Pi^*(L) & C(L) \end{bmatrix} \begin{bmatrix} f_{t-1} \\ x_{t-1} - \Lambda_\mu \mu_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix}, \quad (5)$$

$$\begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} I \\ \Lambda_f \end{bmatrix} [\sqrt{h_t}' \psi_t] + \begin{bmatrix} 0 \\ v_t \end{bmatrix},$$

where $\Pi_f^*(L) = \Pi(L)L^{-1}$ and $\Pi^*(L) = [I_n - C(L)L]\,\Lambda_f \Pi(L)L^{-1}$; since the infinite order representation cannot be handled in estimation, a truncation to a suitable large lag for the polynomial matrix $\Pi(L)$ is required.[3] Hence, $\Pi(L) = \sum_{j=1}^{p} \Pi_j L^j$;

ii) for the case of integration ($d_i = 1$), it should be firstly recalled that

$$\begin{aligned} P(L)D(L) &\equiv P(L)(1-L) \\ &\equiv (I_r - \rho L) - (P_1 L + P_2 L^2 + ... + P_u L^u)(1-L) \end{aligned}$$

with $\rho = I_r$.

The latter may be rewritten in the equivalent polynomial matrix form

$$I_r - \Gamma_1 L - \Gamma_2 L^2 - ... - \Gamma_{u+1} L^{u+1})$$

where Γ_i, $i = 1, ..., u+1$, is a square matrix of coefficients of dimension r, and

$$\Gamma_1 + \Gamma_2 + ... + \Gamma_{u+1} = \rho = I_r$$

$$P_i = -\left(\Gamma_{i+1} + \Gamma_{i+2} + ... + \Gamma_{u+1}\right) \quad i = 1, 2, ..., u.$$

Then, the (finite order) vector autoregressive representation for the factors f_t and the series x_t can be written as in (5), with $\Pi_f^*(L) = \Gamma(L)L^{-1}$ and $\Pi^*(L) = [I_n - C(L)L]\,\Lambda_f \Gamma(L)L^{-1}$;

[3] Monte Carlo evidence reported in Chan and Palma (1998) suggests that the truncation lag should increase with the sample size and the complexity of the ARFIMA representation of the long memory process, still remaining very small relatively to the sample size. For instance, for the covariance stationary fractional white noise case and a sample of 100 observations truncation can be set as low as 6 lags, while for a sample of 10,000 observations it should be increased to 14 lags; for the case of a covariance stationary ARFIMA (1,d,1) process and a sample of 1,000 observations truncation should be set to 30 lags.

iii) for the case of no integration (short memory) ($d_i = 0$), the (finite order) vector autoregressive representation for the factors f_t and the series x_t can still be written as in (5), but recalling that $D(L) \equiv I_r$, and therefore $P(L)D(L) = P(L)$, then, $\Pi_f^*(L) = P(L)L^{-1}$ and $\Pi^*(L) = [I_n - C(L)L]\,\Lambda_f P(L)L^{-1}$.

2.2 Estimation

Estimation of the model can be implemented following a two-stage iterative procedure, similar to Stock and Watson (2005), consisting of the following steps.

• **Step 1: persistence analysis.** Long memory and structural break tests are carried out on the series of interest in order to determine their persistence properties. Several approaches are available in the literature for structural break testing and estimation, as well as for fractional differencing parameter estimation.[4]

• **Step 2: initialization.** Conditional on the presence of structural breaks and long memory in the series investigated, an initial estimate of the unobserved deterministic (break processes b_t) and stochastic features (l_t) can be obtained by decomposing the series as $x_t = b_t + l_t$.

•• Then, the common break processes are estimated by means of Principal Components Analysis (PCA) implemented using the estimated break process $\hat{b}_t$, yielding an estimate of the $m \times 1$ vector of the standardized ($\hat{\Sigma}_{\hat{\mu}} = I_m$) principal components or common break processes $H_\mu \hat{\mu}_t = \hat{\Lambda}_b^{-1/2} \hat{A}' \hat{b}_t$, where $\hat{\Lambda}_b$ is the diagonal matrix of the estimated non zero eigenvalues of the reduced rank variance-covariance matrix of the (estimated) break processes $\hat{\Sigma}_{\hat{b}}$ (rank $m < n$), $\hat{A}$ is the matrix of the associated orthogonal eigenvectors, and H_μ ia an invertible square matrix of order m.

•• Next, the common long memory factors can be obtained by means of PCA implemented using the estimated break-free series $\hat{l}_t = x_t - \hat{b}_t$, yielding the estimate of the r common long memory factors $\hat{f}_t = H_{bf} \hat{\Lambda}_{bf}^{-1/2} \hat{B}' \hat{l}_t$, where $\hat{B}$ is the matrix of the estimated orthogonal eigenvectors associated with the r non-zero eigenvalues of the reduced rank variance-covariance matrix of the (estimated) break-free processes $\hat{\Sigma}_{bf}$ (rank $r < n$), and H_{bf} is an invertible square matrix of order r.

• **Step 3: starting the iterative procedure.** Conditional on the estimate of the deterministic and stochastic factors, the iterative procedure is started by computing a preliminary estimate of the polynomial matrix

[4] This literature is too vast to provide details in the current paper. See the empirical application for hints on some well known approaches.

$C(L)$ and the Λ_f factor loading matrix, by means of OLS estimation of the equation system in (1).

•• Then, a new estimate of the m deterministic factors and their factor loading matrix can be obtained by the application of PCA to the long memory-free series $x_t - \left[I - \hat{C}(L)L\right] \hat{\Lambda}_f \hat{f}_t$.

•• Next, conditional on the new common break processes and their factor loading matrix, the new common long memory factors can be obtained as the first r principal components of the set of break-free processes $x_t - \hat{\Lambda}_\mu \hat{\mu}_t$, and new estimates for the $C(L)$ polynomial matrix and the Λ_f factor loading matrix can also be obtained by means of OLS estimation of the equation system in (1).

•• The procedure described in step 3 is then iterated to improve efficiency.

• **Step 4: restricted estimation of the full model.** Once the final estimates of f_t and μ_t are available, for the non integration and integration case ($d_i = 0$ and $d_i = 1$), the polynomial matrix $\Pi_f^*(L)$ in (5) can be consistently estimated by OLS; for the fractionally integrated case ($0 < d_i < 1$), the fractional differencing parameter is estimated first, for each common factor, by means of a consistent semiparametric estimator, and the associated truncated infinite order VAR representation obtained by means of the binomial expansion in (4); then, $P(L)$ is estimated by OLS applied to the fractionally differenced series[5]; hence, the final estimate of the $\Pi_f^*(L)$ matrix polynomial is obtained through the product of the $\hat{P}(L)$ and $\hat{\Pi}(L)$ polynomials.[6]

By then employing the final estimate of the $\Pi_f^*(L)$ and $C(L)$ matrices, the restricted VAR in (5) can be estimated.

Also, following the thick modelling strategy of Granger and Jeon (2004), median estimates of the parameters of interest, impulse responses and forecast error variance decomposition, as well as of their confidence intervals, robust to model misspecification, can be obtained by means of simulation methods. The identification of the common and idiosyncratic shocks can be performed by means of a (double) Choleski approach, as discussed in the following section.

• **Step 5: conditional variance analysis.** Median factor estimated residuals can be firstly computed using the estimated median (me) parame-

[5] Alternatively, Non-Linear Least Squares estimation can be performed on the actual series, conditional to the estimated fractional differencing parameter.

[6] Alternatively, for the covariance stationary long memory case only, consistent estimation of the VARFIMA structure for the common long memory factors could be obtained by means of Conditional-Sum-of-Squares (Robinson, 2006), exact Maximum Likelihood (Sowell, 1992), or Indirect (Martin and Wilkins, 1999) estimation.

ters[7], i.e.

$$\hat{\eta}_t = \hat{f}_t - \hat{\Pi}^*(L)^{(me)} \hat{f}_{t-1};$$

then, a modified version of the O-GARCH model of Alexander (2002), in order to take into account either long memory or structural breaks in variance, or both, is implemented. The latter consists of the following steps:

i) firstly, conditional variance estimation is carried out factor by factor, yielding $\hat{h}_{i,t}$, $i = 1, 2, ..., r$;

ii) secondly, consistent with the assumptions of conditional and unconditional orthogonality of the factors, the conditional variance ($H_{x,t}$) and correlation ($R_{x,t}$) matrices for the actual series may be computed as

$$H_{x,t} = \Lambda_f H_t \Lambda_f',$$

where $H_t = diag\{h_{1,t}, h_{2,t}, ..., h_{r,t}\}$, and

$$R_{x,t} = H_{x,t}^{*-1/2} H_{x,t} H_{x,t}^{*-1/2},$$

where $H_{x,t}^* = diag\{h_{x_1,t}, h_{x_2,t}, ..., h_{x_n,t}\}$, respectively.

2.2.1 Reduced form and structural vector moving average representation of the FI-HF-VAR model

In the presence of unconditional heteroskedasticity, the computation of the impulse response functions and the forecast error variance decomposition (FEVD) should be made dependent on the estimated unconditional variance for each regime. In the case of (continuously) time-varying unconditional variance, policy analysis may then be computed at each point in time. For some of the conditional variance models considered in the paper, i.e. the FIGARCH and IGARCH processes, the population unconditional variance does not actually exist; in the latter cases the w_t component just bears the interpretation of *long term level* for the conditional variance; computation of policy analysis would still be feasible, yet subject to a different interpretation, FEVD referring, for instance, not to the proportion of forecast error (unconditional) variance accounted for by each structural shock, but to the proportion of forecast error *(conditional) long term variance* accounted for by each structural shock. With this caveat in mind, the actual computation of the above quantities is achieved in the same way as in the case of well defined population unconditional variance.

[7] This assumes that the Granger and Jeon (2004) procedure is inplemented; yet this is not necessary, and O-GARCH modelling can be carried out using the residuals obtained from the point, and not the median, estimated parameters.

Hence, the computation of the vector moving average (VMA) representation for the FI-HF-VAR model depends on the persistence properties of the standardized data $(x_{i,t}/\hat{h}_{x_i,t}^{1/2})$. The following distinctions should therefore be made.

For the short memory case, i.e. the zero integration order case ($d_i = 0$), the VMA representation for the $x_t - \Lambda_\mu \mu_t$ process can be written as

$$x_t - \Lambda_\mu \mu_t = G(L)\eta_t + F(L)v_t, \tag{6}$$

where $G(L) \equiv \Lambda_f D(L)^{-1}$ and $F(L) \equiv [I - C(L)L]^{-1}$.

For the long memory case ($0 < d_i < 1$) and the case of I(1) non stationarity ($d_i = 1$), the VMA representation should be computed for the differenced process $(1-L)(x_t - \Lambda_\mu \mu_t)$ yielding

$$(1-L)(x_t - \Lambda_\mu \mu_t) = G(L)^+ \eta_t + F(L)^+ v_t, \tag{7}$$

where $G(L)^+ \equiv \Lambda_f (1-L) D(L)^{-1} = (1-L) G(L)$ and $F(L)^+ \equiv (1-L)[I - C(L)L]^{-1} = (1-L) F(L)$.

Moreover, for the long memory case, the generic lag polynomial element in $G(L)^+$, i.e. $g_i(L)^+$ can be written in terms of the Hypergeometric function

$$\begin{aligned} g_i(L)^+ &= F(d_i - 1, 1, 1; L) \\ &= \sum_{k=0}^{\infty} \Gamma(k + d_i) \Gamma(k+1)^{-1} \Gamma(d_i)^{-1} L^k \\ &= \sum_{k=0}^{\infty} \lambda_k L^k. \end{aligned}$$

Impulse responses for the $x_t - \Lambda_\mu \mu_t$ process can then be finally computed as $I + \sum_{j=1}^{k} G_j^+$ and $I + \sum_{j=1}^{k} F_j^+$ $k = 1, 2, ...$

Identification of structural shocks The identification of the structural shocks in the FI-HF-VAR model can be carried out by means of a Choleski decomposition procedure, noting that, in the case of unconditionally hortogonal factor, factor residuals would already be structural (exactly identified), as $E[\eta_t \eta_t'] = I_r$; only the idiosyncratic shocks require to be identified.

The identification of the idiosyncratic shocks can then be achieved through the Choleski decomposition of the matrix Σ_v. By denoting ψ_t the n structural idiosyncratic shocks, the relation between reduced form and structural form idiosyncratic shocks can be written as $\psi_t = \Theta v_t$, where Θ is square and invertible. Hence, the identification of the structural idiosyncratic shocks amounts to the estimation of the elements of the Θ matrix. It is assumed that $E[\psi_t \psi_t'] = I_n$, and hence $\Theta \Sigma_v \Theta = I_n$.

The estimation of the lower triangular Θ matrix can then be carried out as follows:

1) regress $\hat{\varepsilon}_{x,t}$ on $\hat{\eta}_t$ by OLS and obtain $\hat{v}_t$ as the residuals;

2) then from $\psi_t = \Theta v_t$ it follows $E[\psi_t \psi_t'] = \Theta \Sigma_v \Theta = I$. Hence, $\hat{\Theta} = chol(\hat{\Sigma}_{\hat{v}})$.

The identification scheme performed allows for exact identification of the n structural idiosyncratic shocks, imposing $n(n-1)/2$ zero restrictions on the contemporaneous impact matrix $\hat{\Theta}^{-1}$.

In the case the common stochastic factors were not unconditionally hortogonal, as for instance when the common stochastic factors are extracted by means of PCA implemented on sub sets of variables (Bernanke et al., 2005; Bagliano and Morana, 2008), rather than on the whole set of variables, also factors' residuals would require structuralization. Denoting by ξ_t the vector of the r structural common shocks, the relation between reduced form and structural form common shocks can be written as $\xi_t = H\eta_t$, where H is square and invertible. Therefore, the identification of the structural common shocks amounts to the estimation of the elements of the H matrix. It is assumed that $E[\xi_t \xi_t'] = I_r$, and hence $H\Sigma_\eta H' = I_r$.

The lower triangular H matrix can then be estimated by the Choleski decomposition of the matrix $\hat{\Sigma}_\eta$, i.e. from $\xi_t = H\eta_t$ we have $E[\eta_t \eta_t'] = H\Sigma_\eta H' = I$, and hence, $\hat{H} = chol(\hat{\Sigma}_\eta)$. The identification scheme performed allows for exact identification of the r structural common shocks, imposing $r(r-1)/2$ zero restrictions on the contemporaneous impact matrix $\hat{H}^{-1}$.

The structural VMA representation can then be written as

$$x_t - \Lambda_\mu \mu_t = G^*(L)\xi_t + F^*(L)\psi_t, \qquad (8)$$

where $G^*(L) = G(L)H^{-1}$, $F^*(L) = F(L)\Theta^{-1}$, or

$$(1-L)(x_t - \Lambda_\mu \mu_t) = G^\circ(L)\xi_t + F^\circ(L)\psi_t, \qquad (9)$$

where $G^\circ(L) = G^+(L)H^{-1}$, $F^\circ(L) = F(L)^+\Theta^{-1}$, according to persistence properties, and $E\left[\psi_{i,t}\xi_{j,t}'\right] = 0$ any i, j, noting that, in the unconditionally hortogonal factor case, $H = I_r$ and $\xi_t = \eta_t$.

2.2.2 Asymptotic properties

The estimation procedure is multi-step, relying on PCA, OLS, and semiparametric/parametric estimation of the fractional differencing parameter for the fractionally integrated case, and iterated to improve efficiency. In the light of the arguments in Doz et al. (2007), the iterative procedure of Stock and Watson (2005) can be interpreted in terms Quasi-ML estimation, performed via the EM algorithm. Kalman filtering could also be added as additional step in the estimation of the common stochastic factors; yet, in the light of the Monte Carlo results provided in this study (see next section), it may be not necessary, particularly for the covariance stationary case, as the small sample performance of PCA is already highly satisfactory under many circumstances of empirical relevance.

Concerning the asymptotic properties of the proposed approach, consistency and asymptotic efficiency and normality can be conjectured on the ground of the following arguments.

Firstly, recent theoretical results validate the use of PCA in the case of both weakly (Bai, 2003) and strongly (Bai, 2004; Bai and Ng, 2004) dependent processes. In particular, under some general conditions, Bai (2003), given the invertible matrix Ξ, establishes $\sqrt{n}$ consistency and asymptotic normality of PCA for Ξf_t, at each point in time, for $n, T \to \infty$ and $\sqrt{n}/T \to 0$, when both the unobserved factors and the idiosyncratic components show limited serial correlation, and the latter also display limited heteroskedasticity in both their time-series and cross-sectional dimensions; under the same conditions he also establishes $\sqrt{T}$ consistency and asymptotic normality of PCA for $\Lambda_f \Xi^{-1}$, as well as $min\left\{\sqrt{n}, \sqrt{T}\right\}$ consistency and asymptotic normality of PCA for the unobserved common components $\Lambda_f f_t$, at each point in time (without requiring$\sqrt{n}/T \to 0$).

In Bai (2004) the above results are extended to the case of I(1) (non cointegrated) unobserved factors and I(0) idiosyncratic components, also featuring limited heteroskedasticity in both the time-series and cross-sectional dimensions for the latter components. In particular, under some general conditions, $\sqrt{n}$ consistency and asymptotic normality of PCA for Ξf_t is established, at each point in time, for $n, T \to \infty$ and $n/T^3 \to 0$; $\sqrt{T}$ consistency and asymptotic normality of PCA for $\Lambda_f \Xi^{-1}$, as well as $min\left\{\sqrt{n}, \sqrt{T}\right\}$ consistency and asymptotic normality for the unobserved common components $\Lambda_f f_t$, at each point in time, is established for $n, T \to \infty$. It is worthwile remarking that the above results assume that PCA is conducted using the level of the series, rather than their first differences.

While there are no asymptotic results for the application of PCA to fractionally integrated processes, supporting Monte Carlo evidence, on the validity of PCA in both the stationary and non stationary case, has however been provided by Morana (2007). Moreover, the use of PCA for the estimation of common deterministic trends has previously been advocated by Bierens

(2000).[8]

The estimation of the polynomial matrix $C(L)$ is conditional to the estimated common components which, as pointed out above, can be consistently estimated (stochastic component). Hence, also $(x_t - \Lambda_\mu \mu_t - \Lambda_f f_t)$ is consistently estimated, for $n, T \to \infty$, at the $min\left\{\sqrt{n}, \sqrt{T}\right\}$ rate; proceding then as it were observed, as $(x_t - \Lambda_\mu \mu_t - \Lambda_f f_t) \sim I(0)$, independent of the integration order of the actual series x_t, $\sqrt{T}$ consistent and asymptotically normal and efficient estimation of the polynomial matrix $C(L)$ can be obtained by the application of OLS on the estimated $(x_t - \Lambda_\mu \mu_t - \Lambda_f f_t)$ series (Hamilton, 1994; ch.3).

Similarly for the estimation of the polynomial matrix $\Pi_f^*(L)$ for the non integrated ($d_i = 0$) and integrated ($d_i = 1$) case, relying on the consistent estimates of Ξf_t, and therefore proceeding as the common stochastic factors were actually observed (Hamilton, 1994; ch.3 for the non integrated case; ch.17 for the integrated case).

Similarly also for the fractionally integrated cases ($0 < d_i < 1$), as the consistently estimated common factors, following the Box-Jenkins strategy, would be fractionally differenced, using a consistent (and asymptotically normal) semiparametric estimate of the long memory parameter, previous to the application of OLS for the estimation of the short memory dynamics.

Finally, Quasi-ML estimation of the conditional variance equations can be implemented, using the estimated residuals; the latter estimator has been shown, or conjectured, to yield consistent and asymptotically normal estimates in the framework considered (see Bollerslev, 1986; Baillie et al., 1996; Baillie and Morana, 2009).

Recent results of Pesaran and Chudik (2010) are also supportive of the conjecture made concerning the asymptotic properties of the proposed methodology, as consistent and asymptotically Normal estimation of the dynamic effects of the dominant unit (common dynamic factor), as well as those of the neighborood units, is shown for an augmented least square regression including a finite number of lagged values for the dominat unit, despite the theoretical relationship involves an infinite order distribute lag relationship.

3 Monte Carlo results

Consider the following data generation process (DGP) for the series x_t

[8] Yet, details cannot be found in the published version of his paper.

$$\begin{aligned} x_t &= \mu_t + f_t + C(x_{t-1} - \mu_{t-1} - f_{t-1}) + v_t \\ v_t &\sim iid(0, \Sigma_v) \qquad \Sigma_v = diag\left(\sigma^2\right). \end{aligned} \tag{10}$$

Then, for the fractionally integrated and integrated cases it is assumed

$$\begin{aligned} (1-\phi L)(1-L)^d f_t &= \eta_t = \sqrt{h_t}' \psi_t \qquad \psi_t \sim iid(0,1) \\ [1-\alpha L - \beta L](1-L)^b \left(\eta_t^2 - \sigma_\eta^2\right) &= [1-\beta L]\left(\eta_t^2 - h_t\right), \end{aligned} \tag{11}$$

while for the non integrated case

$$\begin{aligned} (1-\phi L) f_t &= \eta_t = \sqrt{h_t}' \psi_t \qquad \psi_t \sim iid(0,1) \\ [1-\alpha L - \beta L]\left(\eta_t^2 - \sigma_\eta^2\right) &= [1-\beta L]\left(\eta_t^2 - h_t\right) \end{aligned} \tag{12}$$

Different values for the autoregressive idiosyncratic parameter ρ, common across the n cross-sectional units ($C = diag\,(\rho)$), have been considered, i.e. $\rho = \{0, 0.2, 0.4, 0.6, 0.8\}$, as well as for the fractionally differencing parameter $d = \{0, 0.2, 0.4, 0.6, 0.8, 1\}$ and the common factor autoregressive parameter ϕ, setting $\phi = \{0.2, 0.4, 0.6, 0.8\}$ for the non integrated case and $\phi = \{0, d/2\}$ for the fractionally integrated and integrated cases; $\phi > \rho$ is always assumed in the experiments. For the conditional variance equation it is assumed $\alpha = 0.05$ and $\beta = 0.90$ for the short memory case, and $\alpha = 0.05$, $\beta = 0.30$ and $b = 0.45$ for the long memory case. The inverse signal to noise ratio is given by σ^2/σ_η^2, taking values $\sigma^2/\sigma_\eta^2 = \{4, 2, 1, 0.5, 0.25\}$.

Moreover, in addition to structural stability case, i.e. $\mu_t = \mu = 0$, two break process structures have been considered for the component μ_t, i.e.

i) the single step change in the intercept at the midpoint of the sample case, i.e.

$$\mu_t = \begin{cases} 0 & t = 1, ..., T/2 \\ 4 & t = T/2 + 1, ..., T \end{cases};$$

ii) the two step changes equally spaced throughout the sample case, with the intercept increasing at one third of the way through the sample and then decreasing at a point two thirds of the length of the sample, i.e.

$$\mu_t = \begin{cases} 0 & t = 1, ..., T/3 \\ 4 & t = T/3 + 1, ..., 2T/3 \\ 2 & t = 2T/3 + 1, ..., T \end{cases}.$$

The sample size investigated is $T = 100, 500$, and the number of cross-sectional units is $N = 30$. The number of replications has been set to 2,000 for each case.

The ability of the proposed approach in estimating the unknown common factor and break process, and the autoregressive parameters in the mean equation, has then been assessed. As the factor loadings and common factors are not separately identified when estimated using PCA, the evaluation neglects the estimation of the factor loading parameters, which are set to unity in the present exercise. For the common factors the Theil inequality coefficient (IC) and the correlation coefficient (ρ) have been employed in the evaluation:

$$\begin{aligned} IC &= RMSFE/(\sqrt{\frac{1}{T}\sum_{t=1}^{T} x_t^{*2}} + \sqrt{\frac{1}{T}\sum_{t=1}^{T} \hat{x}_t^2}), \\ RMSFE &= \sqrt{\frac{1}{T}\sum_{t=1}^{T} (x_t^* - \hat{x}_t)^2} \\ Corr &= Cov(x_t^*, \hat{x}_t)/\sqrt{Var(x_t^*)Var(\hat{x}_t)}, \end{aligned}$$

where x_t^* is the true unobserved component and $\hat{x}_t$ is its estimated counterpart. The above statistics have been computed for each Monte Carlo replication and then averaged.

In particular, the following experiments have been carried out:

1) observed common stochastic factor and break process (or no break process) case: the ability of the model in estimating the autoregressive parameters ϕ and ρ is then assessed, and the cross-sectional dimension is set to the minimum possible value, i.e. $N = 2$;

2) unobserved common stochastic factors and known break points and fractional differencing parameter d case: the ability of the model in estimating the autoregressive parameters ϕ and ρ, as in 1), is assessed, as well as in estimating the unobserved common stochastic and deterministic features; for this case $N = 30$; the performance of the approach has also been assessed using different cross-sectional dimensions, i.e. $N = 5, 10, 15, 50$;

3) the above experiments have been carried out assuming a constant conditional variance in a first case and a time-varying conditional variance, as detailed in the DGP, in the second case.

3.1 Results

The results for the non integration case are reported in Tables 1-9, while Tables 10-18 refer to the fractionally integrated and integrated cases (simply referred as the *integrated* case, independent of the type of integration, thereafter). In all cases results refer to the estimated parameters for the first equation in the model. Since the results are virtually unaffected by the presence of conditional heteroskedasticity, for reasons of space, in the Tables only the heteroskedastic case is considered. Moreover, only the results for the $\phi = d/2$ case are reported for the integrated case, as similar results have been obtained for the $\phi = 0$ case.[9]

3.1.1 The observed common factor and break process case

Tables 1 and 10 refer to the observed common stochastic factor case, assuming a constant unconditional mean (i.e. no structural breaks in the mean), or, equivalently, an observed common break process in mean.

As shown in the Tables, for both the non integrated and integrated case, there is evidence of a small downward bias in the autoregressive parameters ϕ and ρ, increasing with the persistence of the common factor, i.e. ϕ (non integrated case) or d (integrated case), yet decreasing as the sample size T increases. For ρ the bias also decreases as the serial correlation spread, $\phi - \rho$ (non integrated case) or $d - \rho$ (integrated case), increases.

The average bias is however negligible already for a sample size of 100 observations: -0.02 (ρ) and -0.03 (ϕ) for the non integrated case, and -0.02 (ρ) and -0.04 (ϕ) for the integrated case (-0.003 (ρ) and -0.006 (ϕ), and -0.004 (ρ and ϕ), for the non integrated and integrated case, respectively, and $T = 500$).

Hence, in the observed common features case, the approach delivers unbiased estimates of all the parameters, independently of the sample size, number of cross-sectional units ($N = 2$ in the experiment), degree of serial correlation of the common (ϕ) and idiosyncratic (ρ) components, integration order (d), and (inverse) signal to noise ratio s/n^{-1}. As can be expected, precision increases with the sample size.

3.1.2 The unobserved common factor, with observed common break process, case

Tables 2 and 3 refer to the unobserved common factor case, under the assumption of observed common break process, or, equivalently, no structural breaks in the mean, for the non integrated case; results for the corresponding integrated case are reported in Tables 11 and 12.

As is shown in Tables 2 and 11, for both cases, there is again evidence of a downward bias in ρ and ϕ, which is always negligible for the ρ parameter (-0.02 and -0.03, on average, for the non integrated and integrated case,

[9] A full set of results is available upon request from the author.

respectively, and $T = 100$; -0.01 and -0.006, respectively, and $T = 500$), yet not always negligible for the ϕ parameter.

In particular, the bias in ϕ is increasing with the degree of persistence of the common factor d, the (inverse) signal to noise ratio s/n^{-1}, and the serial correlation spread, $\phi - \rho$ or $d - \rho$, yet decreasing with the sample size T.

For the selected cross-sectional sample dimension $N = 30$, it can be noted that, for the non integrated case, there are only few cases ($\phi - \rho = 0.4, 0.6, 0.8$) when a 10%, or larger, bias is found, occurring when the series are particularly noisy ($s/n^{-1} = 4$); for the stationary long memory case a 10%, or smaller, bias is found for $s/n^{-1} \geq 2$, while for the non stationary long memory case it is required $s/n^{-1} \geq 1$ and a (relatively) large sample ($T = 500$). Increasing the cross-sectional dimension N leads to better findings (see the next section).

On the other hand, for the ρ parameter the bias is decreasing as the serial correlation spread, $\phi - \rho$ or $d - \rho$, or the sample size T, increase, being insensitive to the (inverse) signal to noise ratio. In all cases the bias for ρ is smaller than for ϕ.

Concerning the estimation of the unobserved common stochastic factor, from Tables 3 and 12 it can be concluded that a cross-sectional dimension of 30 units would lead to a satisfactory outcome, as the Theil statistic is always below 0.2 (0.14 (0.10), on average, for $T = 100$ ($T = 500$) for the non integrated case; 0.06 (0.03), on average, for $T = 100$ ($T = 500$) for the integrated case). Moreover, the correlation coefficient between the actual and estimated common factors is always very high, 0.98 and 0.99, on average, respectively, for both sample sizes.

Results for smaller and larger cross-sectional samples PCA is a $\sqrt{N}$ consistent estimator of the unobserved common factors, for $N, T \to \infty$; as T and N increase, the performance of the feasible case should therefore approach the one found for the unfeasible (observed factor) case.

In Tables 4-5 and 13-14 a summary of results for different cross-sectional dimensions, i.e. $N = 5, 10, 15, 50$, for the unobserved common factor and no structural breaks or, equivalently, observed break process case, is reported. In the Tables only the bias for the ϕ parameter, as well as the correlation coefficient between the actual and estimated common factor, is reported, for reasons of space; statistics for the ρ parameter are not reported as well, as the latter is always unbiasedly estimated, independently of the cross-sectional dimension.[10]

[10]Detailed results for all cases are available upon request from the author.

As is shown in the Tables, the performance of the estimator crucially depends on T, N, and s/n^{-1}.

For the non integrated case, when the (inverse) signal to noise ratio is low, i.e. $s/n^{-1} \leq 0.5$, the downward bias is already mitigated by using a cross-sectional sample size as small as $N = 5$, for the case of $T = 100$ observations; as N increases similar results are obtained for higher s/n^{-1}, i.e. $N = 10, 15$ and $s/n^{-1} \leq 1$, or $N = 50$ and $s/n^{-1} \leq 4$. For a larger sample size, i.e. $T = 500$, similar conclusions hold, albeit, for the $N = 5$ case, the (inverse) signal to noise ratio can be higher, i.e. $s/n^{-1} \leq 1$; similarly for the $N = 10, 15$ case with $s/n^{-1} \leq 2$ (Table 4).

For the integrated case conditions are slightly more restrictive; in particular, for the stationary long memory case, when the (inverse) signal to noise ratio is low, i.e. $s/n^{-1} \leq 0.5$, the downward bias is already mitigated by setting $N = 5$ and $T = 100$; similar results are obtained for higher s/n^{-1} and N, i.e. $N = 10, 15$ and $s/n^{-1} \leq 1, 2$, or $N = 50$ and $s/n^{-1} \leq 4$. Similar conclusions can be drawn for $T = 500$, albeit, holding N constant, accurate estimation is obtained also for higher s/n^{-1}. Also Similarly for the non stationary long memory or the integrated cases; yet, holding T constant, either larger N, or lower s/n^{-1}, would be required to attain accurate estimation (Table 13).

The above findings are corroborated by the correlation coefficients between the actual and estimated common factors (Tables 5 and 14), which do show that satisfactory estimation of the unobserved common factor (a correlation coefficient higher than 0.9) can indeed be obtained also in the case of a small cross-sectional sample, provided tha the (inverse) signal to noise ratio is not too high, and/or the cross-sectional dimension is not too low ($s/n^{-1} \leq 1$ and $N = 5$; $s/n^{-1} \leq 2$ and $N = 10$; $s/n^{-1} \leq 4$ and $N = 15$).

3.1.3 The unobserved common factor, with known break points, case

Tables 6-9 and 15-18 refer to the unobserved common factor case, under the assumption of known break points, requiring therefore the estimation of the common break process as well.

As is shown in Tables 6-7 and 15-16, when the common break process needs to be estimated, the bias in the ρ parameter increases slightly, but for the small sample case only (the average bias is -0.04, independent of the break process design and integration properties; $T = 100$), as for the large sample case the performance is virtually unchanged (the average bias is -0.01 in all the cases; $T = 500$). As for the no break process, or observed break process, case, the bias is decreasing as the serial correlation spread, $\phi-$ ρ or

$d-\ \rho$, or the sample size T, increase, being unaffected by the (inverse) signal to noise ratio, and always very small.

Concerning the estimation of the ϕ parameter, similar to the case of no break process, or observed common break process, the bias is increasing with the degree of persistence of the common factor d, the (inverse) signal to noise ratio s/n^{-1}, and the serial correlation spread, $\phi-\ \rho$ or $d-\ \rho$, yet decreasing with the sample size T. Also, the downward bias is always more sizable for ϕ than for ρ.

Moreover, also the complexity of the break process may affect the estimation of the ϕ parameter, worsening as the number of break points increases, particularly when the sample is small ($T = 100$); yet, for the no integration case (Tables 6-7), already for $T = 500$ the performance is very satisfactory, independently of the complexity of the break process and the (inverse) signal to noise ratio s/n^{-1}; on the other hand, for $T = 100$ the performance is still satisfactory when the series are not too noisy ($s/n^{-1} \leq 1$), yielding at most a 10% bias.

Figures reported in Tables 8-9 confirm the above conclusions, pointing to a satisfactory estimation of the unobserved common factor and break process: the Theil statistic is always below 0.2 for the $T = 500$ case (0.11 and 0.13, on average, for the single break point and two-break points case, respectively) and below 0.3 for the $T = 100$ case (0.17 and 0.20, on average), while the actual and estimated common factors are strongly correlated: on average, the correlation coefficient is 0.96 (0.98) for the single break point case, and 0.93 (0.97) for the two-break point case, with $T = 100$ ($T = 500$). The almost perfect recovery of the common break process is not too surprising, as the series are generated as stationary stochastic fluctuations about a deterministic step function and the break points are known.

Concerning the integrated case, some differences can be noted relatively to the non integrated case; in fact, as shown in Table 18, albeit the recovery of the common break process is always very satisfactory across the various designs, independent of the sample size, performance worsens, not only as the complexity of the break process increases, but also as d increases: the average correlation coefficient between the estimated and actual break process falls from 1 when $d = 0.2$ (single break point case) to 0.93 when $d = 1$ (two-break point case).

Moreover, concerning the estimation of the common stochastic factor (Table 17), while for the covariance stationary case ($d < 0.5$), the results are very close to what obtained for the non integrated case, i.e. a Theil statistic always below 0.2 for the $T = 500$ case (0.12 and 0.14, on average, for the single break point and two-break points case, respectively) and below 0.3 for the $T = 100$ case (0.21 and 0.24, on average, respectively), and a very high correlation coefficient (0.94 and 0.91, on average, $T = 100$; 0.97 and 0.96, on average, $T = 500$), for the non stationary case performance is worse, featuring average Theil statistics of 0.32 (0.32) and 0.42 (0.44), respectively, for the single and two-break point case and $T = 100$ ($T = 500$); similarly the average correlation coefficient is 0.79 (0.78) and 0.68 (0.66), respectively. Consistent with the above results is also the finding of inaccurate estimation of the common factor autoregressive parameter ϕ, for the $d = 0.8$ and $d = 1$ case, reported in Tables 15-16.

Overall, the above findings are not surprising, as the difficulty in discriminating between long memory and structural breaks is a well known issue in the literature (see, for instance, Diebold and Inoue, 2001); the findings, however, provide some additional interesting insights, as the discrimination between the two sources of persistence, does appear to be feasible in the case of stationary long memory, becoming more difficult as the degree of stochastic persistence and the complexity of the break process increase. The counterintuitive finding of performance worsening as the (temporal) sample size increases, i.e. smaller bias and Theil statistics and larger correlation coefficients for $T = 100$ than $T = 500$, is also not surprising, as the latter is determined by the non stationary wandering of the stochastic process about the step function constituting the break process, and the larger is the sample, the wider the wandering can be expected.

4 Empirical application: the US financial crisis

The data investigated refer to the whole term structure for the US dollar LIBOR-OIS spreads, i.e. the one-week and two-week maturities and the one-month through the one-year maturities, for a total of 14 time series. The data frequency is daily and the sample runs from 20 June 2005 through 7 April 2009, for a total of 992 working days. The data source is REUTERS.

OIS-LIBOR spreads contain interesting information concerning counterparty risk, liquidity risk and investors' sentiments; the proposed methodology is then employed in order to gauge insights on their statistical features and on how the crisis has affected their statistical properties.

4.1 Persistence analysis

Structural break analysis has been performed by means of the Dolado et al. (2004) structural break test (DGM test), modified to account for a general and unknown structural break process (Morana, 2009) and the Bai and Perron (BP, 1998) test; fractional integration analysis has also been performed, by means of the Moulines and Soulier (1999) broad band log periodogram (BBLP) estimator.

As shown in Table 19, OIS spreads are strongly persistent, featuring an average fractional differencing parameter, across maturities, of about 0.93.

Significant break points, for the level of the 1-week OIS spread, can also be detected, i.e. August 9 2007 (observation 558), September 16 2008 (observation 844), and November 19 2008 (observation 893).[11] Moreover, August 9 2007 and September 16 2008 are also break points for the variance of the 1-week OIS spread. Similar findings hold for all the other maturities, with close (not always coincident) location of the break points, suggesting a delayed adjustment in the spreads, particularly for the longest maturities. For all the maturities the shifts in the mean level were rapid, yet only gradual. A smooth transition mechanism across regimes, rather than an abrupt step function process, would therefore seem to be appropriate for describing the break process in the OIS spreads: a dummy model, with break points set according to the BP test, featuring cubic spline transition across regimes, has therefore been employed. The latter specification is validated by the DGM test.

Persistence in the OIS spreads is however not only due to the break process, as strong evidence of long memory in the break-free series can also be found. The average figure across maturities is 0.49, with shorter maturities featuring stronger persistence than longer ones.

4.2 Copersistence analysis

The evidence of breaks and long memory in mean and variance for the OIS spreads has been taken into account in the estimation of the FI-HF-VAR model, also allowing for a change in persistence in the mean of the process.

As shown in Table 20, a single common break process accounts for almost 100% of total variance for the break process components across maturities (Figure 1, top plot); the latter factor accounts for over 90% of total variance for each spread with maturity beyond two months; for maturities within one month the percentage of explained variance is in the range 65% to 78%; the common break process might therefore bear the interpretation of *level* factor.

[11] The first two estimated break points are uncontroversial: in fact, August 9 2007 is the day the French bank BNP Paribas revealed its inability to value structured products for three of its investment funds, triggering the first panic wave in the US money market; September 16 2008 is the day after of Lehman Brothers bankruptcy, triggering the second panic wave. We remind to Brunnermaier (2009) for institutional details on the crisis.

The crisis did induce major changes in the level of the spreads, i.e. an average nine fold increase after August 9 2007, followed by another two fold increase after September 16 2008; then a 30% average contraction took place between November 19 2008 and April 7 2009 (the last day of the investigated sample).

Moreover, two common long memory factors account for almost 100% of total variance (Figure 1 middle plots); the first factor affects all the maturities, with impact weakest at the very short end of the term structure (below 55% within the 1-month maturity) and strongest for medium- long term maturities (over 80%), a kind of *curvature* effect; the second long memory factor is strictly related to the shortest end of the term structure (about 45% on average within the 1-month maturity), a kind of *slope* effect. The crisis has determined a significant increase in the persistence of the two common long memory factors, i.e. from 0.38 to 0.70 and from 0.51 to 0.73, for the first and second factor, respectively.

The two common long memory factors also feature long memory and structural breaks in their conditional variance: the estimated fractional differencing parameters are about 0.17 and 0.63 for the first and second factor, respectively.[12] As is shown in Figure 1 (bottom plots), the change in the level and range of variation of the conditional standard deviation process, after August 2007, was remarkable, i.e. a three fold and two fold increase, on average, respectively.

4.3 Impulse responses and forecast error variance decomposition

As shown in Table 24, as a consequence of the crisis, fluctuations at the very short end of the term structure (up to the 3-month maturity) have become more idiosyncratic in the short-term; in fact, while the common long memory factor shocks jointly accounted, on average, for about 96% (95%) of fluctuation at the 1-day (20-day) horizon for the pre-crisis period, for the crisis period the average figure falls to 68% (95%); also, the forecast variance decomposition supports the interpretation of the stochastic factors as curvature and slope factors, the former accounting for a larger proportion of variance for intermediate maturities (86% to 96%; 3- to 8-month maturity), the latter accounting for the bulk of fluctuations at the very short end of the OIS spread term structure (69% to 80%; up to the 1-month maturity).

Finally, as shown in Figure 2, major differences can be noted between the

[12] Adaptive FIGARCH(1,d,1) models, with cubic spline dummy intercept component for the conditional variance equation have been estimated for both (final) common long memory factors, using factor residuals computed from median estimated parameters.

pre-crisis and crisis periods, both in terms of magnitude and persistence of common factor shocks, as well as of response profiles. Differences can also be noted, within each period, across maturities, as displayed by the comparison between the 1-week and 1-year maturities.

Concerning *curvature* shocks (top four plots), both the persistence and magnitude of the impact increase, in general, with the maturity of the OIS spreads, also being larger for the crisis than the pre-crisis period; concerning the *slope* shocks (bottom four plots), a similar impact, in absolute terms, can be found across maturities for the pre-crisis and crisis period; in both cases, for the crisis period, shock dissipation occurs well beyond twenty days, taking longer than for the pre-crisis period; finally, concerning the effects of idiosyncratic shocks (not reported), a similar monotonic decay can be noted in the impulse response functions computed for the crisis and pre-crisis period, yet, with a much larger contemporaneous impact of shocks during the crisis; moreover, stronger persistence can be detected for shorter than longer maturities, with full dissipation occurring within eight and two days, respectively.

5 Conclusions

In the paper the fractionally integrated heteroskedastic factor vector autoregressive (FI-HF-VAR) model is introduced. Relatively to other FVAR models, the proposed approach singles out for its minimal pretesting requirements for implementation, performing well independently of the integration properties of the data and of the sources of persistence, i.e. deterministic or stochastic, and therefore accounting for common features of different kinds, i.e. common integrated (of the fractional or integer type) or non integrated stochastic factors, also featuring conditional heteroskedasticity, and common deterministic break processes. As data are modelled in deviations from the common features, accurate (and asymptotically normal and efficient) estimation can be achieved within the two-step iterated approach of Stock and Watson (2005), featuring therefore simplicity of implementation also in the case of large systems of dynamic equations. The two-step estimation procedure can also be implemented following the Granger and Jeon (2004) thick modelling strategy, providing median estimates of the parameters of interest and robust standard errors. An empirical application to US LIBOR-OIS spreads, over the period June 2005-April 2009, is also proposed, showing how the proposed methodology allows for accurate investigation of the properties of the data, from persistence and copersistence analysis to impulse responses and forecast error variance decomposition, for both common and idiosyncratic shocks. Monte Carlo results strongly support the proposed methodology.

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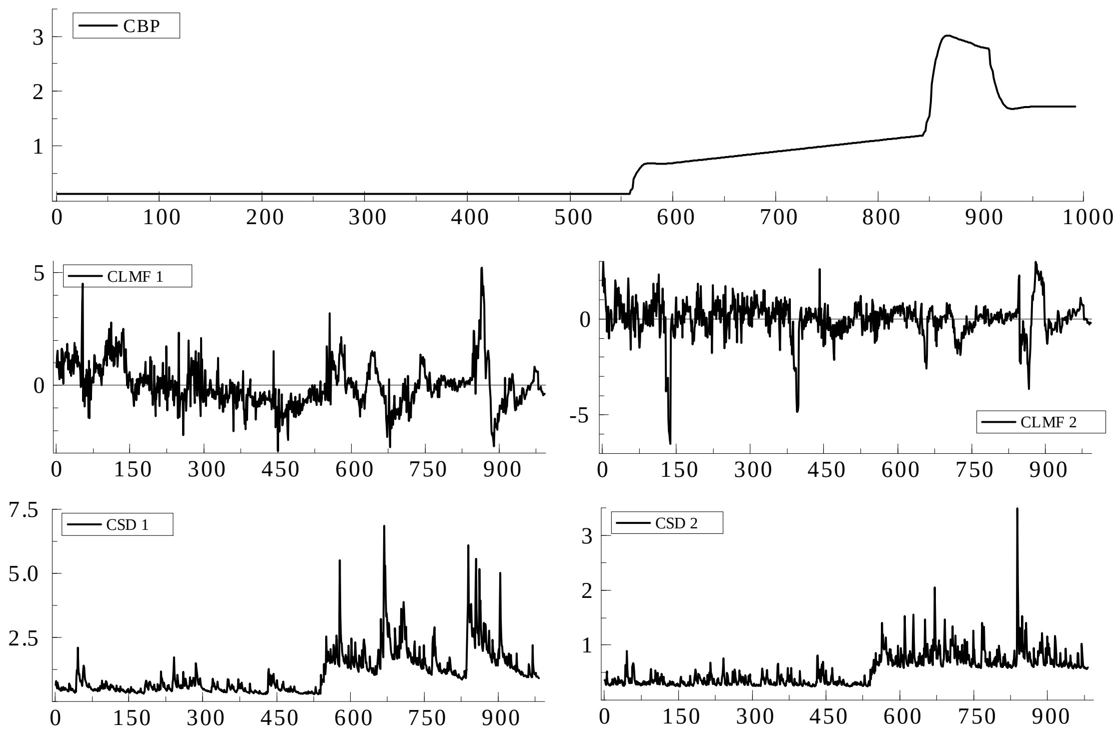

Figure 1: US LIBOR-OIS spreads; normalized common break process (CBP), common long memory factors (CLMF 1 and CLMF 2), and conditional standard deviations of the common long memory factors (CSD 1 and CSD 2).

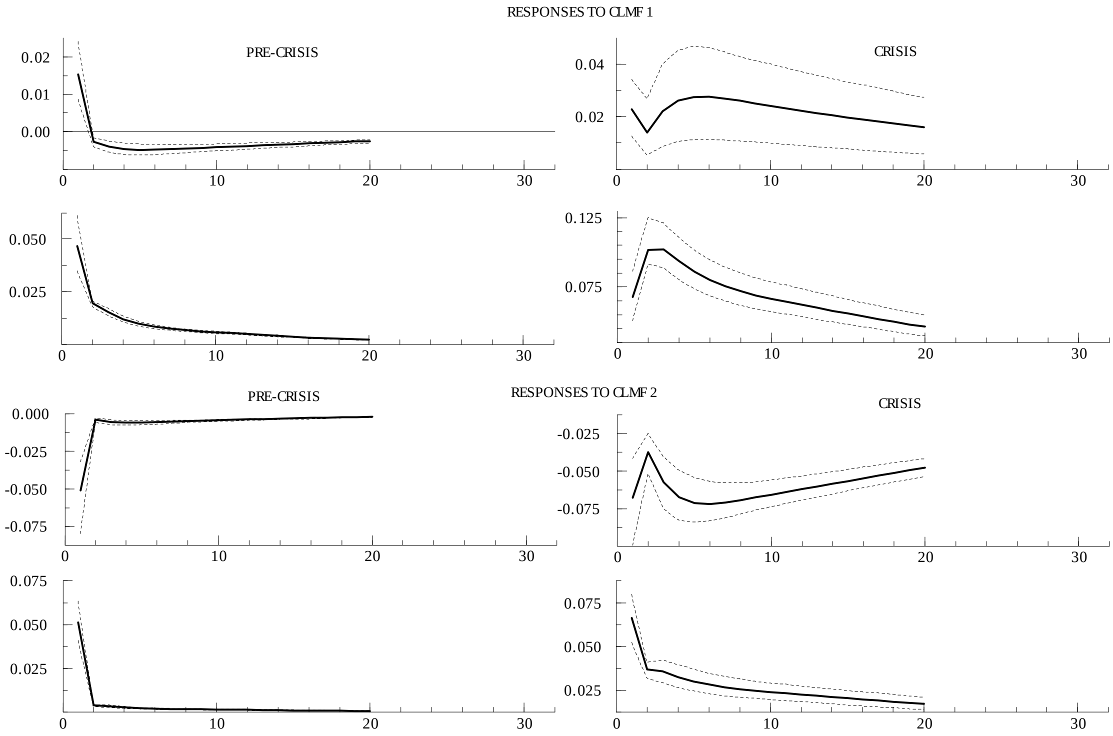

Figure 2: Impulse responses, with 90% confidence interval, to a unitary curvature factor (CLMF1) shock and slope factor (CLMF2) shock, for the pre-crisis (left hand side plots) and crisis (right hand side plots) periods, for the 1-week (1w) and 1-year (1y) maturities.

Table 1: Observed common stochastic factor and break process, heteroskedastic case, N=2: bias and RMSE of parameters.

common factor autoregressive parameter ϕ										
bias										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
any	-0.017	-0.022	-0.022	-0.026	-0.026	-0.026	-0.035	-0.035	-0.035	-0.035
root mean square error										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
any	0.104	0.100	0.100	0.092	0.092	0.092	0.085	0.085	0.085	0.085
autoregressive parameter ρ										
bias										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.008	-0.012	-0.020	-0.013	-0.016	-0.022	-0.010	-0.022	-0.025	-0.028
2	-0.011	-0.003	-0.018	-0.009	-0.022	-0.020	-0.014	-0.016	-0.022	-0.029
1	-0.010	-0.012	-0.012	-0.014	-0.019	-0.021	-0.010	-0.014	-0.022	-0.025
0.5	-0.009	-0.010	-0.017	-0.011	-0.019	-0.022	-0.010	-0.014	-0.024	-0.032
0.25	-0.010	-0.012	-0.017	-0.012	-0.014	-0.022	-0.013	-0.019	-0.024	-0.029
root mean square error										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.101	0.103	0.103	0.103	0.100	0.100	0.101	0.105	0.100	0.093
2	0.103	0.100	0.102	0.104	0.102	0.097	0.102	0.101	0.099	0.095
1	0.102	0.099	0.101	0.100	0.101	0.098	0.098	0.099	0.098	0.091
0.5	0.099	0.102	0.103	0.101	0.100	0.098	0.103	0.102	0.101	0.096
0.25	0.104	0.102	0.102	0.099	0.101	0.097	0.104	0.104	0.100	0.094
common factor autoregressive parameter ϕ										
bias										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
any	-0.003	-0.004	-0.004	-0.007	-0.007	-0.007	-0.006	-0.006	-0.006	-0.006
root mean square error										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
any	0.045	0.044	0.044	0.040	0.040	0.040	0.030	0.030	0.030	0.030
autoregressive parameter ρ										
bias										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.001	-0.002	-0.002	-0.003	-0.004	-0.004	0.000	-0.004	-0.004	-0.007
2	-0.003	-0.002	-0.002	-0.002	-0.004	-0.005	-0.003	-0.003	-0.003	-0.006
1	-0.004	-0.003	-0.003	-0.002	-0.004	-0.005	0.000	-0.004	-0.002	-0.005
0.5	-0.003	-0.003	-0.002	-0.002	-0.004	-0.004	-0.002	-0.003	-0.004	-0.005
0.25	-0.002	-0.003	-0.004	-0.003	-0.002	-0.003	-0.003	-0.003	-0.003	-0.006
root mean square error										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.046	0.046	0.043	0.045	0.044	0.042	0.044	0.045	0.041	0.038
2	0.045	0.044	0.043	0.045	0.044	0.041	0.045	0.044	0.042	0.037
1	0.044	0.045	0.044	0.045	0.045	0.042	0.045	0.045	0.042	0.037
0.5	0.044	0.045	0.045	0.045	0.043	0.044	0.046	0.043	0.042	0.037
0.25	0.046	0.046	0.044	0.045	0.045	0.040	0.044	0.043	0.043	0.037

The Table reports Monte Carlo bias and RMSE statistics, concerning the estimation of the common factor (***ϕ***) and idiosyncratic autoregressive parameter (***ρ***). Results are reported for various values of the common factor autoregressive parameter ***ϕ*** (0.2, 0.4, 0.6, 0.8), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6), assuming ***$\phi > \rho$***, and various values of the (inverse) signal to noise ratio ***$(s/n)^{-1}$*** (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 2, and the number of replications for each case is 2,000. The experiment refers to the case of observed autoregressive factor and common break process, or, equivalently, constant unconditional mean.

Table 2: Unobserved common stochastic factor, no structural break, heteroskedastic case, N=30: bias and RMSE of parameters.

N=30										
autoregressive common factor parameter ϕ										
bias										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.037	-0.063	-0.035	-0.078	-0.060	-0.037	-0.075	-0.064	-0.054	-0.038
2	-0.028	-0.044	-0.030	-0.055	-0.047	-0.038	-0.055	-0.050	-0.047	-0.039
1	-0.021	-0.033	-0.027	-0.044	-0.040	-0.036	-0.045	-0.042	-0.041	-0.039
0.5	-0.019	-0.027	-0.025	-0.037	-0.035	-0.034	-0.039	-0.039	-0.037	-0.037
0.25	-0.017	-0.025	-0.023	-0.033	-0.032	-0.032	-0.036	-0.036	-0.036	-0.036
root mean square error										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.116	0.133	0.106	0.144	0.123	0.099	0.132	0.118	0.104	0.083
2	0.110	0.115	0.105	0.119	0.110	0.102	0.107	0.101	0.097	0.088
1	0.108	0.107	0.103	0.108	0.103	0.100	0.096	0.093	0.091	0.088
0.5	0.107	0.103	0.102	0.102	0.100	0.099	0.090	0.089	0.088	0.087
0.25	0.107	0.102	0.101	0.099	0.098	0.098	0.087	0.086	0.086	0.086
autoregressive parameter ρ										
bias										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.010	-0.011	-0.021	-0.017	-0.024	-0.034	-0.018	-0.026	-0.033	-0.040
2	-0.012	-0.014	-0.020	-0.014	-0.022	-0.034	-0.019	-0.025	-0.036	-0.046
1	-0.010	-0.016	-0.025	-0.016	-0.026	-0.034	-0.021	-0.028	-0.040	-0.042
0.5	-0.011	-0.013	-0.022	-0.017	-0.024	-0.035	-0.016	-0.026	-0.036	-0.044
0.25	-0.012	-0.019	-0.022	-0.018	-0.023	-0.031	-0.018	-0.021	-0.035	-0.047
root mean square error										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.098	0.100	0.105	0.101	0.105	0.107	0.103	0.106	0.103	0.102
2	0.103	0.100	0.103	0.098	0.107	0.106	0.103	0.105	0.108	0.109
1	0.101	0.105	0.107	0.104	0.107	0.107	0.105	0.104	0.107	0.104
0.5	0.102	0.101	0.103	0.104	0.104	0.107	0.102	0.104	0.107	0.105
0.25	0.100	0.102	0.104	0.102	0.104	0.106	0.100	0.104	0.106	0.110
autoregressive common factor parameter ϕ										
bias										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.027	-0.045	-0.024	-0.054	-0.038	-0.023	-0.044	-0.036	-0.028	-0.020
2	-0.016	-0.026	-0.015	-0.031	-0.023	-0.015	-0.026	-0.022	-0.018	-0.014
1	-0.010	-0.016	-0.010	-0.019	-0.015	-0.011	-0.017	-0.014	-0.013	-0.011
0.5	-0.006	-0.010	-0.007	-0.012	-0.011	-0.009	-0.012	-0.011	-0.010	-0.009
0.25	-0.005	-0.007	-0.006	-0.009	-0.009	-0.008	-0.009	-0.009	-0.008	-0.008
root mean square error										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.059	0.078	0.056	0.086	0.066	0.050	0.071	0.060	0.051	0.041
2	0.050	0.058	0.049	0.058	0.050	0.044	0.049	0.044	0.040	0.036
1	0.047	0.049	0.046	0.046	0.043	0.040	0.039	0.037	0.035	0.034
0.5	0.047	0.046	0.045	0.042	0.040	0.039	0.034	0.034	0.033	0.033
0.25	0.046	0.045	0.044	0.040	0.039	0.039	0.033	0.032	0.032	0.032
autoregressive parameter ρ										
bias										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.002	-0.002	-0.005	-0.005	-0.004	-0.007	-0.002	-0.004	-0.007	-0.011
2	0.000	-0.002	-0.005	-0.002	-0.005	-0.007	-0.001	-0.005	-0.006	-0.011
1	-0.002	-0.004	-0.004	-0.004	-0.006	-0.008	-0.005	-0.005	-0.008	-0.009
0.5	-0.003	-0.002	-0.002	-0.003	-0.006	-0.009	-0.004	-0.006	-0.007	-0.009
0.25	-0.002	-0.002	-0.003	-0.006	-0.006	-0.005	-0.005	-0.004	-0.007	-0.010
root mean square error										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.045	0.046	0.045	0.046	0.044	0.042	0.045	0.046	0.042	0.039
2	0.045	0.044	0.044	0.045	0.044	0.041	0.045	0.044	0.042	0.040
1	0.045	0.045	0.043	0.045	0.045	0.043	0.044	0.043	0.043	0.038
0.5	0.045	0.045	0.045	0.044	0.045	0.044	0.045	0.045	0.042	0.039
0.25	0.045	0.045	0.044	0.045	0.045	0.041	0.044	0.046	0.043	0.040

The Table reports Monte Carlo bias and RMSE statistics, concerning the estimation of the common factor (ϕ) and idiosyncratic (ρ) autoregressive parameters. Results are reported for various values of the common factor autoregressive parameter ϕ (0.2, 0.4, 0.6, 0.8), various values of the idiosyncratic autoregressive parameter ρ (0, 0.2, 0.4, 0.6), assuming $\phi > \rho$, and various values of the (inverse) signal to noise ratio $(s/n)^{-1}$ (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 30, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved autoregressive factor and no breaks.

Table 3: Unobserved common stochastic factor, no structural break, heteroskedastic case, N=30: Monte Carlo Theil and correlation statistics.

N=30										
autoregressive common factor										
Theil index										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.187	0.183	0.185	0.174	0.176	0.185	0.175	0.177	0.182	0.195
2	0.140	0.140	0.142	0.139	0.141	0.146	0.153	0.154	0.158	0.165
1	0.108	0.111	0.112	0.116	0.117	0.121	0.140	0.141	0.143	0.147
0.5	0.086	0.092	0.093	0.102	0.102	0.105	0.132	0.132	0.133	0.136
0.25	0.072	0.081	0.081	0.093	0.093	0.095	0.127	0.127	0.128	0.130
correlation coefficient										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.938	0.944	0.943	0.956	0.955	0.949	0.973	0.972	0.968	0.959
2	0.968	0.972	0.971	0.978	0.977	0.974	0.986	0.986	0.984	0.979
1	0.984	0.986	0.985	0.989	0.988	0.987	0.993	0.993	0.992	0.990
0.5	0.992	0.993	0.993	0.994	0.994	0.993	0.997	0.996	0.996	0.995
0.25	0.996	0.996	0.996	0.997	0.997	0.997	0.998	0.998	0.998	0.997
Theil index										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.176	0.167	0.170	0.150	0.152	0.162	0.126	0.128	0.135	0.150
2	0.127	0.121	0.124	0.111	0.113	0.119	0.099	0.100	0.105	0.115
1	0.093	0.089	0.091	0.084	0.085	0.090	0.081	0.082	0.085	0.092
0.5	0.068	0.067	0.068	0.066	0.067	0.070	0.070	0.071	0.073	0.077
0.25	0.052	0.052	0.053	0.054	0.054	0.056	0.063	0.064	0.065	0.067
correlation coefficient										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.941	0.948	0.946	0.959	0.958	0.952	0.976	0.975	0.972	0.964
2	0.969	0.973	0.972	0.979	0.978	0.975	0.988	0.987	0.986	0.981
1	0.984	0.986	0.986	0.989	0.989	0.988	0.994	0.994	0.993	0.991
0.5	0.992	0.993	0.993	0.995	0.994	0.994	0.997	0.997	0.996	0.995
0.25	0.996	0.997	0.996	0.997	0.997	0.997	0.999	0.998	0.998	0.998

The Table reports Monte Carlo Theil index and correlation coefficient statistics, concerning the estimation of the unobserved common factor component. Results are reported for various values of the common factor autoregressive parameter ϕ (0.2, 0.4, 0.6, 0.8), various values of the idiosyncratic autoregressive parameter ρ (0, 0.2, 0.4, 0.6), assuming $\phi > \rho$, and various values of the (inverse) signal to noise ratio $(s/n)^{-1}$ (4, 2, 1, 05, 0.25). The sample size T is 100 and 500 observations, the number of cross-sectional units N is 30, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved autoregressive factor and no breaks.

Table 4: Unobserved common stochastic factor, no structural break, heteroskedastic case, N = 5, 10, 15, 50: bias of parameter ϕ.

autoregressive common factor parameter ϕ										
					bias N = 5					
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.101	-0.185	-0.101	-0.238	-0.170	-0.099	-0.230	-0.182	-0.142	-0.094
2	-0.073	-0.125	-0.075	-0.154	-0.115	-0.076	-0.146	-0.122	-0.100	-0.075
1	-0.049	-0.081	-0.055	-0.098	-0.078	-0.058	-0.096	-0.083	-0.073	-0.058
0.5	-0.035	-0.055	-0.041	-0.067	-0.056	-0.044	-0.067	-0.060	-0.054	-0.050
0.25	-0.029	-0.041	-0.033	-0.049	-0.043	-0.037	-0.052	-0.049	-0.046	-0.043
					bias N = 10					
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.072	-0.126	-0.072	-0.155	-0.113	-0.069	-0.150	-0.122	-0.098	-0.066
2	-0.046	-0.083	-0.053	-0.099	-0.079	-0.057	-0.098	-0.085	-0.071	-0.057
1	-0.032	-0.055	-0.042	-0.067	-0.056	-0.045	-0.070	-0.063	-0.057	-0.051
0.5	-0.025	-0.041	-0.034	-0.050	-0.045	-0.039	-0.054	-0.051	-0.048	-0.045
0.25	-0.018	-0.033	-0.029	-0.041	-0.038	-0.036	-0.046	-0.045	-0.043	-0.042
					bias N = 15					
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.058	-0.098	-0.056	-0.117	-0.086	-0.050	-0.118	-0.099	-0.081	-0.056
2	-0.037	-0.063	-0.042	-0.077	-0.060	-0.044	-0.082	-0.071	-0.064	-0.052
1	-0.030	-0.046	-0.037	-0.051	-0.045	-0.038	-0.061	-0.057	-0.053	-0.048
0.5	-0.023	-0.035	-0.030	-0.040	-0.036	-0.033	-0.051	-0.048	-0.047	-0.045
0.25	-0.019	-0.030	-0.028	-0.033	-0.032	-0.030	-0.045	-0.044	-0.044	-0.043
					bias N = 50					
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.027	-0.046	-0.027	-0.058	-0.045	-0.028	-0.062	-0.054	-0.047	-0.035
2	-0.023	-0.034	-0.026	-0.043	-0.037	-0.031	-0.049	-0.047	-0.043	-0.039
1	-0.019	-0.027	-0.024	-0.035	-0.033	-0.031	-0.043	-0.042	-0.042	-0.040
0.5	-0.017	-0.024	-0.023	-0.032	-0.031	-0.030	-0.040	-0.039	-0.039	-0.039
0.25	-0.016	-0.023	-0.022	-0.029	-0.029	-0.029	-0.038	-0.038	-0.038	-0.038
					bias N = 5					
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.091	-0.165	-0.087	-0.208	-0.144	-0.080	-0.190	-0.149	-0.112	-0.070
2	-0.058	-0.104	-0.057	-0.127	-0.089	-0.052	-0.110	-0.088	-0.067	-0.045
1	-0.036	-0.062	-0.034	-0.074	-0.052	-0.031	-0.063	-0.050	-0.040	-0.028
0.5	-0.021	-0.035	-0.021	-0.042	-0.031	-0.019	-0.036	-0.030	-0.024	-0.018
0.25	-0.014	-0.020	-0.012	-0.024	-0.018	-0.013	-0.022	-0.019	-0.016	-0.013
					bias N =10					
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.058	-0.104	-0.055	-0.129	-0.090	-0.052	-0.110	-0.087	-0.066	-0.043
2	-0.034	-0.061	-0.034	-0.075	-0.053	-0.032	-0.062	-0.050	-0.040	-0.028
1	-0.019	-0.035	-0.020	-0.042	-0.031	-0.020	-0.036	-0.030	-0.024	-0.018
0.5	-0.011	-0.020	-0.013	-0.025	-0.019	-0.014	-0.022	-0.019	-0.016	-0.013
0.25	-0.007	-0.012	-0.008	-0.015	-0.013	-0.010	-0.015	-0.013	-0.012	-0.010
					bias N = 15					
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.045	-0.078	-0.041	-0.094	-0.066	-0.038	-0.079	-0.063	-0.049	-0.032
2	-0.027	-0.046	-0.026	-0.054	-0.039	-0.024	-0.045	-0.037	-0.029	-0.022
1	-0.016	-0.026	-0.016	-0.031	-0.023	-0.016	-0.027	-0.023	-0.019	-0.015
0.5	-0.010	-0.015	-0.011	-0.018	-0.015	-0.011	-0.017	-0.015	-0.013	-0.011
0.25	-0.007	-0.010	-0.007	-0.013	-0.011	-0.009	-0.012	-0.011	-0.010	-0.009
					bias N = 50					
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.019	-0.028	-0.015	-0.036	-0.026	-0.016	-0.031	-0.026	-0.021	-0.015
2	-0.012	-0.016	-0.010	-0.021	-0.016	-0.011	-0.019	-0.017	-0.015	-0.012
1	-0.009	-0.010	-0.007	-0.014	-0.011	-0.009	-0.014	-0.012	-0.011	-0.010
0.5	-0.007	-0.006	-0.005	-0.010	-0.009	-0.008	-0.011	-0.010	-0.010	-0.009
0.25	-0.006	-0.005	-0.004	-0.008	-0.007	-0.007	-0.009	-0.009	-0.009	-0.008

The Table reports Monte Carlo bias statistics, concerning the estimation of the common factor (ϕ) autoregressive parameter. Results are reported for various values of the common factor autoregressive parameter $\boldsymbol{\phi}$ (0.2, 0.4, 0.6, 0.8), various values of the idiosyncratic autoregressive parameter $\boldsymbol{\rho}$ (0, 0.2, 0.4, 0.6), assuming $\boldsymbol{\phi > \rho}$, and various values of the (inverse) signal to noise ratio $\boldsymbol{(s/n)^{-1}}$ (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 5, 10, 15, 50, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved autoregressive factor and no breaks.

Table 5: Unobserved common stochastic factor, no structural break, heteroskedastic case, , N = 5, 10, 15, 50, bias of parameter ϕ.

autoregressive common factor										
correlation coefficient N = 5										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.742	0.764	0.756	0.801	0.795	0.774	0.863	0.861	0.845	0.809
2	0.846	0.859	0.856	0.885	0.882	0.870	0.925	0.923	0.913	0.892
1	0.914	0.922	0.920	0.937	0.936	0.928	0.960	0.959	0.953	0.942
0.5	0.954	0.959	0.957	0.967	0.966	0.962	0.979	0.979	0.976	0.970
0.25	0.976	0.979	0.978	0.983	0.983	0.980	0.990	0.989	0.988	0.985
correlation coefficient N = 10										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.843	0.857	0.853	0.884	0.881	0.867	0.924	0.921	0.913	0.889
2	0.913	0.921	0.919	0.937	0.935	0.928	0.959	0.958	0.953	0.941
1	0.954	0.959	0.957	0.967	0.966	0.962	0.979	0.978	0.976	0.969
0.5	0.976	0.979	0.978	0.983	0.983	0.980	0.990	0.989	0.988	0.984
0.25	0.988	0.989	0.989	0.992	0.991	0.990	0.995	0.995	0.994	0.992
correlation coefficient N = 15										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.886	0.898	0.894	0.918	0.916	0.906	0.947	0.945	0.939	0.922
2	0.939	0.946	0.944	0.957	0.956	0.950	0.972	0.972	0.968	0.959
1	0.968	0.972	0.971	0.978	0.977	0.974	0.986	0.985	0.984	0.979
0.5	0.984	0.986	0.985	0.989	0.988	0.987	0.993	0.993	0.992	0.989
0.25	0.992	0.993	0.993	0.994	0.994	0.993	0.996	0.996	0.996	0.995
correlation coefficient N = 50										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.962	0.966	0.965	0.973	0.972	0.969	0.983	0.983	0.981	0.975
2	0.981	0.983	0.982	0.986	0.986	0.984	0.992	0.991	0.990	0.988
1	0.990	0.991	0.991	0.993	0.993	0.992	0.996	0.996	0.995	0.994
0.5	0.995	0.996	0.996	0.997	0.996	0.996	0.998	0.998	0.998	0.997
0.25	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.998
correlation coefficient N = 5										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.750	0.772	0.765	0.811	0.806	0.786	0.877	0.873	0.859	0.827
2	0.849	0.864	0.860	0.891	0.888	0.875	0.933	0.930	0.922	0.901
1	0.916	0.925	0.922	0.941	0.939	0.931	0.965	0.963	0.958	0.947
0.5	0.955	0.960	0.959	0.969	0.968	0.964	0.982	0.981	0.979	0.972
0.25	0.977	0.979	0.979	0.984	0.984	0.981	0.991	0.990	0.989	0.986
correlation coefficient N =10										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.849	0.864	0.860	0.891	0.887	0.874	0.933	0.930	0.922	0.901
2	0.916	0.925	0.922	0.941	0.939	0.931	0.965	0.963	0.959	0.947
1	0.955	0.960	0.959	0.969	0.968	0.964	0.982	0.981	0.979	0.972
0.5	0.977	0.979	0.979	0.984	0.984	0.981	0.991	0.990	0.989	0.986
0.25	0.988	0.990	0.989	0.992	0.992	0.991	0.995	0.995	0.995	0.993
correlation coefficient N =1 5										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.891	0.903	0.899	0.923	0.920	0.911	0.954	0.952	0.946	0.931
2	0.941	0.948	0.946	0.959	0.958	0.952	0.976	0.975	0.972	0.964
1	0.969	0.973	0.972	0.979	0.978	0.975	0.988	0.987	0.986	0.981
0.5	0.984	0.986	0.986	0.989	0.989	0.987	0.994	0.994	0.993	0.991
0.25	0.992	0.993	0.993	0.995	0.994	0.994	0.997	0.997	0.996	0.995
correlation coefficient N = 50										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.963	0.968	0.966	0.975	0.974	0.970	0.985	0.985	0.983	0.978
2	0.981	0.983	0.983	0.987	0.987	0.985	0.993	0.992	0.991	0.989
1	0.991	0.992	0.991	0.994	0.993	0.992	0.996	0.996	0.996	0.994
0.5	0.995	0.996	0.996	0.997	0.997	0.996	0.998	0.998	0.998	0.997
0.25	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999

The Table reports Monte Carlo *correlation coefficients* between actual and estimated common factors. Results are reported for various values of the common factor autoregressive parameter ***ϕ*** (0.2, 0.4, 0.6, 0.8), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6), assuming ***ϕ > ρ***, and various values of the (inverse) signal to noise ratio ***$(s/n)^{-1}$*** (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 5, 10, 15, 50, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved autoregressive factor and no breaks.

Table 6: Unobserved common stochastic factor, single break point, heteroskedastic case, N=30: bias and RMSE of parameters.

N=30										
autoregressive common factor parameter ϕ										
bias										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.050	-0.078	-0.051	-0.099	-0.078	-0.057	-0.101	-0.091	-0.079	-0.062
2	-0.038	-0.058	-0.045	-0.076	-0.066	-0.057	-0.080	-0.076	-0.071	-0.064
1	-0.031	-0.047	-0.042	-0.062	-0.060	-0.055	-0.070	-0.068	-0.066	-0.064
0.5	-0.028	-0.042	-0.039	-0.056	-0.054	-0.053	-0.064	-0.064	-0.063	-0.062
0.25	-0.027	-0.039	-0.038	-0.053	-0.052	-0.052	-0.062	-0.061	-0.061	-0.061
root mean square error										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.125	0.151	0.123	0.170	0.143	0.119	0.166	0.151	0.135	0.111
2	0.118	0.130	0.118	0.142	0.130	0.120	0.138	0.132	0.125	0.116
1	0.113	0.120	0.117	0.127	0.124	0.120	0.125	0.122	0.119	0.117
0.5	0.111	0.116	0.115	0.121	0.119	0.118	0.118	0.117	0.116	0.115
0.25	0.111	0.115	0.114	0.117	0.117	0.116	0.114	0.115	0.114	0.114
autoregressive parameter ρ										
bias										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.021	-0.023	-0.036	-0.027	-0.035	-0.048	-0.031	-0.040	-0.050	-0.064
2	-0.021	-0.029	-0.033	-0.026	-0.038	-0.049	-0.034	-0.041	-0.051	-0.062
1	-0.023	-0.023	-0.038	-0.028	-0.031	-0.048	-0.022	-0.035	-0.051	-0.065
0.5	-0.025	-0.025	-0.034	-0.028	-0.038	-0.049	-0.034	-0.035	-0.049	-0.063
0.25	-0.020	-0.026	-0.035	-0.027	-0.033	-0.051	-0.027	-0.035	-0.048	-0.063
root mean square error										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.104	0.108	0.112	0.106	0.110	0.117	0.110	0.115	0.121	0.126
2	0.106	0.108	0.109	0.106	0.113	0.117	0.110	0.114	0.120	0.126
1	0.103	0.106	0.112	0.107	0.107	0.118	0.106	0.113	0.120	0.129
0.5	0.105	0.108	0.111	0.106	0.111	0.118	0.111	0.111	0.116	0.126
0.25	0.106	0.105	0.112	0.108	0.111	0.121	0.108	0.111	0.118	0.126
autoregressive common factor parameter ϕ										
bias										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.028	-0.048	-0.027	-0.057	-0.042	-0.026	-0.049	-0.040	-0.033	-0.024
2	-0.018	-0.029	-0.019	-0.034	-0.027	-0.019	-0.030	-0.026	-0.022	-0.018
1	-0.012	-0.018	-0.013	-0.022	-0.019	-0.015	-0.021	-0.018	-0.017	-0.015
0.5	-0.009	-0.013	-0.011	-0.016	-0.014	-0.012	-0.016	-0.015	-0.014	-0.013
0.25	-0.007	-0.010	-0.009	-0.013	-0.012	-0.011	-0.013	-0.013	-0.013	-0.012
root mean square error										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.060	0.081	0.059	0.091	0.072	0.054	0.076	0.065	0.055	0.044
2	0.053	0.060	0.052	0.063	0.055	0.047	0.053	0.048	0.043	0.038
1	0.049	0.051	0.048	0.051	0.047	0.044	0.042	0.039	0.038	0.036
0.5	0.048	0.048	0.046	0.045	0.044	0.043	0.037	0.036	0.035	0.034
0.25	0.047	0.046	0.046	0.043	0.042	0.042	0.034	0.034	0.034	0.033
autoregressive parameter ρ										
bias										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.002	-0.005	-0.006	-0.004	-0.007	-0.010	-0.005	-0.010	-0.010	-0.013
2	-0.003	-0.003	-0.009	-0.005	-0.008	-0.009	-0.006	-0.010	-0.008	-0.013
1	-0.004	-0.005	-0.005	-0.006	-0.008	-0.007	-0.007	-0.008	-0.010	-0.011
0.5	-0.006	-0.005	-0.007	-0.005	-0.008	-0.011	-0.007	-0.007	-0.009	-0.011
0.25	-0.004	-0.006	-0.005	-0.006	-0.007	-0.009	-0.006	-0.006	-0.009	-0.012
root mean square error										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
(s/n)$^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.044	0.046	0.045	0.045	0.045	0.044	0.044	0.046	0.044	0.041
2	0.045	0.044	0.045	0.045	0.045	0.043	0.045	0.046	0.043	0.042
1	0.045	0.045	0.045	0.046	0.045	0.042	0.046	0.046	0.043	0.039
0.5	0.046	0.043	0.046	0.045	0.046	0.044	0.046	0.044	0.043	0.041
0.25	0.044	0.045	0.044	0.045	0.044	0.043	0.045	0.045	0.044	0.040

The Table reports Monte Carlo bias and RMSE statistics, concerning the estimation of the common factor (***ϕ***) and idiosyncratic autoregressive parameter (***ρ***). Results are reported for various values of the common factor autoregressive parameter ***ϕ*** (0.2, 0.4, 0.6, 0.8), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6), assuming ***ϕ > ρ***, and various values of the (inverse) signal to noise ratio ***(s/n)$^{-1}$*** (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved autoregressive factor and known single break point.

Table 7: Unobserved common stochastic factor, two break points, heteroskedastic case, N=30: bias and RMSE of parameters.

N=30										
autoregressive common factor parameter ϕ										
bias										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***
4	-0.065	-0.096	-0.068	-0.115	-0.094	-0.070	-0.128	-0.116	-0.102	-0.082
2	-0.054	-0.076	-0.064	-0.092	-0.081	-0.072	-0.106	-0.100	-0.095	-0.088
1	-0.048	-0.066	-0.061	-0.078	-0.075	-0.071	-0.094	-0.092	-0.090	-0.087
0.5	-0.046	-0.061	-0.059	-0.072	-0.070	-0.069	-0.089	-0.088	-0.087	-0.087
0.25	-0.044	-0.058	-0.057	-0.069	-0.068	-0.068	-0.086	-0.085	-0.085	-0.085
root mean square error										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***
4	0.138	0.168	0.137	0.191	0.162	0.133	0.201	0.184	0.163	0.136
2	0.127	0.146	0.132	0.161	0.148	0.136	0.171	0.162	0.155	0.145
1	0.122	0.135	0.130	0.145	0.141	0.136	0.154	0.152	0.149	0.144
0.5	0.121	0.131	0.128	0.138	0.136	0.134	0.147	0.146	0.145	0.145
0.25	0.119	0.127	0.127	0.134	0.133	0.133	0.143	0.143	0.142	0.142
autoregressive parameter ρ										
bias										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***
4	-0.020	-0.024	-0.032	-0.026	-0.035	-0.049	-0.029	-0.038	-0.050	-0.062
2	-0.021	-0.025	-0.035	-0.027	-0.036	-0.047	-0.027	-0.037	-0.051	-0.062
1	-0.020	-0.022	-0.038	-0.024	-0.037	-0.051	-0.028	-0.039	-0.048	-0.064
0.5	-0.021	-0.020	-0.033	-0.027	-0.039	-0.048	-0.030	-0.037	-0.053	-0.059
0.25	-0.020	-0.022	-0.035	-0.026	-0.037	-0.048	-0.025	-0.042	-0.052	-0.061
root mean square error										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***	***ρ = 0***
4	0.105	0.105	0.107	0.107	0.110	0.116	0.107	0.113	0.119	0.124
2	0.104	0.105	0.109	0.107	0.113	0.116	0.105	0.114	0.119	0.124
1	0.104	0.103	0.114	0.107	0.115	0.118	0.110	0.114	0.118	0.127
0.5	0.103	0.105	0.111	0.108	0.115	0.117	0.108	0.113	0.121	0.122
0.25	0.103	0.105	0.111	0.107	0.111	0.117	0.105	0.114	0.122	0.122
autoregressive common factor parameter ϕ										
bias										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.032	-0.052	-0.030	-0.064	-0.048	-0.032	-0.054	-0.045	-0.038	-0.028
2	-0.022	-0.032	-0.022	-0.041	-0.033	-0.025	-0.036	-0.031	-0.027	-0.023
1	-0.016	-0.022	-0.017	-0.029	-0.025	-0.021	-0.026	-0.024	-0.022	-0.020
0.5	-0.014	-0.017	-0.014	-0.023	-0.021	-0.019	-0.021	-0.020	-0.019	-0.018
0.25	-0.012	-0.014	-0.013	-0.019	-0.019	-0.018	-0.019	-0.018	-0.018	-0.017
root mean square error										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.065	0.086	0.061	0.100	0.079	0.060	0.084	0.072	0.063	0.050
2	0.057	0.063	0.053	0.071	0.061	0.052	0.060	0.054	0.050	0.044
1	0.052	0.053	0.050	0.057	0.053	0.049	0.048	0.046	0.044	0.041
0.5	0.050	0.049	0.047	0.051	0.049	0.047	0.043	0.041	0.041	0.039
0.25	0.050	0.047	0.047	0.048	0.047	0.046	0.040	0.039	0.039	0.038
autoregressive parameter ρ										
bias										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	-0.006	-0.004	-0.007	-0.005	-0.007	-0.009	-0.005	-0.009	-0.010	-0.012
2	-0.003	-0.005	-0.006	-0.004	-0.008	-0.009	-0.005	-0.008	-0.008	-0.012
1	-0.004	-0.006	-0.008	-0.006	-0.007	-0.009	-0.009	-0.009	-0.011	-0.011
0.5	-0.006	-0.003	-0.007	-0.005	-0.007	-0.011	-0.006	-0.007	-0.011	-0.013
0.25	-0.005	-0.004	-0.007	-0.006	-0.008	-0.011	-0.006	-0.008	-0.010	-0.011
root mean square error										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.046	0.046	0.043	0.044	0.046	0.044	0.046	0.044	0.044	0.040
2	0.044	0.045	0.045	0.046	0.045	0.043	0.046	0.046	0.042	0.042
1	0.045	0.045	0.045	0.047	0.045	0.043	0.047	0.044	0.044	0.039
0.5	0.046	0.044	0.046	0.045	0.045	0.044	0.046	0.045	0.044	0.041
0.25	0.046	0.045	0.045	0.046	0.045	0.044	0.045	0.046	0.043	0.040

The Table reports Monte Carlo bias and RMSE statistics, concerning the estimation of the common factor (***ϕ***) and idiosyncratic autoregressive parameter (***ρ***). Results are reported for various values of the common factor autoregressive parameter ***ϕ*** (0.2, 0.4, 0.6, 0.8), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6), assuming ***$\phi > \rho$***, and various values of the (inverse) signal to noise ratio ***$(s/n)^{-1}$*** (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved autoregressive factor and known break points.

Table 8: Unobserved common stochastic factor, known break points, heteroskedastic case, N=30: Monte Carlo Theil and correlation statistics.

N=30										
common autoregressive factor										
1-break point case										
Theil index										
T = 100	$\phi = 0.2$	$\phi = 0.4$		$\phi = 0.6$			$\phi = 0.8$			
$(s/n)^{-1}$	$\rho = 0$	$\rho = 0$	$\rho = 0.2$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$
4	0.195	0.195	0.197	0.199	0.201	0.208	0.224	0.225	0.229	0.239
2	0.151	0.156	0.157	0.169	0.170	0.174	0.208	0.208	0.211	0.216
1	0.122	0.131	0.132	0.151	0.151	0.154	0.198	0.199	0.200	0.203
0.5	0.103	0.115	0.116	0.140	0.140	0.142	0.193	0.193	0.194	0.195
0.25	0.092	0.106	0.107	0.134	0.134	0.135	0.190	0.190	0.190	0.191
correlation coefficient										
T = 100	$\phi = 0.2$	$\phi = 0.4$		$\phi = 0.6$			$\phi = 0.8$			
$(s/n)^{-1}$	$\rho = 0$	$\rho = 0$	$\rho = 0.2$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$
4	0.932	0.934	0.933	0.936	0.935	0.929	0.932	0.931	0.928	0.919
2	0.961	0.961	0.960	0.957	0.957	0.954	0.945	0.945	0.943	0.939
1	0.977	0.975	0.974	0.968	0.968	0.967	0.952	0.952	0.951	0.949
0.5	0.984	0.982	0.982	0.974	0.974	0.973	0.956	0.956	0.956	0.954
0.25	0.988	0.985	0.985	0.977	0.977	0.977	0.958	0.958	0.958	0.957
Theil index										
T = 500	$\phi = 0.2$	$\phi = 0.4$		$\phi = 0.6$			$\phi = 0.8$			
$(s/n)^{-1}$	$\rho = 0$	$\rho = 0$	$\rho = 0.2$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$
4	0.178	0.170	0.173	0.157	0.160	0.168	0.141	0.142	0.148	0.162
2	0.130	0.126	0.128	0.120	0.122	0.128	0.117	0.118	0.122	0.131
1	0.097	0.096	0.097	0.095	0.097	0.100	0.102	0.103	0.105	0.111
0.5	0.074	0.075	0.076	0.080	0.080	0.083	0.093	0.094	0.095	0.098
0.25	0.058	0.062	0.062	0.070	0.070	0.072	0.088	0.089	0.089	0.091
correlation coefficient										
T = 500	$\phi = 0.2$	$\phi = 0.4$		$\phi = 0.6$			$\phi = 0.8$			
$(s/n)^{-1}$	$\rho = 0$	$\rho = 0$	$\rho = 0.2$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$
4	0.940	0.945	0.944	0.955	0.953	0.948	0.968	0.967	0.963	0.956
2	0.968	0.971	0.969	0.975	0.974	0.971	0.980	0.979	0.977	0.973
1	0.983	0.984	0.983	0.985	0.985	0.983	0.986	0.985	0.984	0.982
0.5	0.991	0.991	0.990	0.991	0.990	0.990	0.989	0.989	0.988	0.987
0.25	0.994	0.994	0.994	0.993	0.993	0.993	0.990	0.990	0.990	0.989
2-break point case										
Theil index										
T = 100	$\phi = 0.2$	$\phi = 0.4$		$\phi = 0.6$			$\phi = 0.8$			
$(s/n)^{-1}$	$\rho = 0$	$\rho = 0$	$\rho = 0.2$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$
4	0.204	0.209	0.211	0.221	0.223	0.229	0.267	0.268	0.270	0.278
2	0.162	0.173	0.174	0.195	0.196	0.199	0.254	0.254	0.256	0.260
1	0.136	0.151	0.152	0.180	0.180	0.182	0.246	0.246	0.247	0.249
0.5	0.119	0.138	0.139	0.171	0.171	0.172	0.242	0.242	0.243	0.244
0.25	0.110	0.131	0.131	0.166	0.166	0.167	0.240	0.240	0.241	0.241
correlation coefficient										
T =100	$\phi = 0.2$	$\phi = 0.4$		$\phi = 0.6$			$\phi = 0.8$			
$(s/n)^{-1}$	$\rho = 0$	$\rho = 0$	$\rho = 0.2$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$
4	0.923	0.922	0.920	0.916	0.915	0.909	0.890	0.889	0.886	0.877
2	0.953	0.949	0.948	0.938	0.937	0.935	0.904	0.904	0.902	0.898
1	0.968	0.963	0.962	0.949	0.948	0.947	0.911	0.911	0.910	0.908
0.5	0.976	0.970	0.970	0.954	0.954	0.954	0.915	0.915	0.914	0.913
0.25	0.980	0.974	0.973	0.957	0.957	0.957	0.917	0.917	0.917	0.916
Theil index										
T = 500	$\phi = 0.2$	$\phi = 0.4$		$\phi = 0.6$			$\phi = 0.8$			
$(s/n)^{-1}$	$\rho = 0$	$\rho = 0$	$\rho = 0.2$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$
4	0.185	0.178	0.180	0.166	0.168	0.176	0.157	0.158	0.164	0.176
2	0.139	0.136	0.138	0.132	0.133	0.139	0.136	0.137	0.140	0.148
1	0.109	0.109	0.110	0.110	0.111	0.114	0.124	0.124	0.126	0.131
0.5	0.089	0.092	0.093	0.097	0.097	0.099	0.117	0.117	0.118	0.121
0.25	0.077	0.082	0.082	0.089	0.090	0.091	0.113	0.114	0.114	0.115
correlation coefficient										
T = 500	$\phi = 0.2$	$\phi = 0.4$		$\phi = 0.6$			$\phi = 0.8$			
$(s/n)^{-1}$	$\rho = 0$	$\rho = 0$	$\rho = 0.2$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0$	$\rho = 0.2$	$\rho = 0.4$	$\rho = 0.6$
4	0.935	0.940	0.939	0.949	0.948	0.942	0.958	0.957	0.954	0.945
2	0.963	0.965	0.964	0.969	0.968	0.965	0.970	0.969	0.967	0.963
1	0.978	0.979	0.978	0.979	0.979	0.977	0.976	0.975	0.975	0.972
0.5	0.986	0.985	0.985	0.984	0.984	0.983	0.979	0.979	0.978	0.977
0.25	0.989	0.989	0.989	0.987	0.987	0.986	0.980	0.980	0.980	0.979

The Table reports Monte Carlo Theil index and correlation coefficient statistics, concerning the estimation of the unobserved common factor component. Results are reported for various values of the common factor autoregressive parameter ϕ (0.2, 0.4, 0.6, 0.8), various values of the idiosyncratic autoregressive parameter ρ (0, 0.2, 0.4, 0.6), assuming $\phi > \rho$, and various values of the (inverse) signal to noise ratio $(s/n)^{-1}$ (4, 2, 1, 05, 0.25). The sample size T is 100 and 500 observations, the number of cross-sectional units N is 30, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved autoregressive factor and known break points.

Table 9: Unobserved common stochastic factor, known break points, heteroskedastic case, N=30: Monte Carlo Theil and correlation statistics.

N=30										
common break process										
1-break point case										
Theil index										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.029	0.037	0.037	0.055	0.055	0.056	0.105	0.105	0.106	0.107
2	0.028	0.036	0.037	0.055	0.055	0.056	0.105	0.105	0.105	0.106
1	0.028	0.036	0.036	0.055	0.055	0.055	0.105	0.105	0.105	0.105
0.5	0.028	0.036	0.036	0.055	0.055	0.055	0.105	0.105	0.105	0.105
0.25	0.028	0.036	0.036	0.055	0.055	0.055	0.105	0.105	0.105	0.105
correlation coefficient										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Theil index										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.013	0.017	0.017	0.025	0.025	0.026	0.048	0.048	0.049	0.049
2	0.013	0.017	0.017	0.025	0.025	0.026	0.048	0.048	0.048	0.048
1	0.013	0.016	0.017	0.025	0.025	0.025	0.048	0.048	0.048	0.048
0.5	0.013	0.017	0.017	0.025	0.025	0.025	0.048	0.048	0.048	0.048
0.25	0.013	0.016	0.017	0.025	0.025	0.025	0.048	0.048	0.048	0.048
correlation coefficient										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2-break point case										
Theil index										
T = 100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.040	0.052	0.052	0.076	0.077	0.078	0.141	0.141	0.142	0.144
2	0.039	0.051	0.052	0.076	0.076	0.076	0.141	0.141	0.141	0.142
1	0.039	0.051	0.051	0.075	0.075	0.076	0.141	0.141	0.141	0.141
0.5	0.038	0.051	0.051	0.075	0.075	0.075	0.141	0.141	0.141	0.141
0.25	0.038	0.051	0.051	0.075	0.075	0.075	0.141	0.141	0.141	0.141
correlation coefficient										
T =100	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.997	0.995	0.995	0.988	0.988	0.987	0.957	0.958	0.957	0.955
2	0.997	0.995	0.995	0.988	0.988	0.988	0.958	0.957	0.958	0.957
1	0.997	0.995	0.995	0.988	0.988	0.988	0.958	0.958	0.957	0.958
0.5	0.997	0.995	0.995	0.988	0.988	0.988	0.958	0.957	0.958	0.958
0.25	0.997	0.995	0.995	0.988	0.988	0.988	0.958	0.958	0.958	0.958
Theil index										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.025	0.030	0.030	0.039	0.039	0.040	0.070	0.070	0.070	0.071
2	0.025	0.030	0.030	0.039	0.039	0.040	0.070	0.070	0.070	0.070
1	0.025	0.029	0.030	0.039	0.039	0.039	0.070	0.070	0.070	0.070
0.5	0.025	0.029	0.029	0.039	0.039	0.039	0.070	0.070	0.070	0.070
0.25	0.025	0.029	0.029	0.039	0.039	0.039	0.070	0.070	0.070	0.070
correlation coefficient										
T = 500	***ϕ = 0.2***	***ϕ = 0.4***		***ϕ = 0.6***			***ϕ = 0.8***			
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***
4	0.998	0.997	0.997	0.996	0.996	0.996	0.990	0.990	0.990	0.990
2	0.998	0.998	0.997	0.996	0.996	0.996	0.990	0.990	0.990	0.990
1	0.998	0.998	0.998	0.996	0.996	0.996	0.990	0.990	0.990	0.990
0.5	0.998	0.998	0.998	0.996	0.996	0.996	0.990	0.990	0.990	0.990
0.25	0.998	0.998	0.998	0.996	0.996	0.996	0.990	0.990	0.990	0.990

The Table reports Monte Carlo Theil index and correlation coefficient statistics, concerning the estimation of the unobserved common break process component. Results are reported for various values of the common factor autoregressive parameter $\boldsymbol{\phi}$ (0.2, 0.4, 0.6, 0.8), various values of the idiosyncratic autoregressive parameter $\boldsymbol{\rho}$ (0, 0.2, 0.4, 0.6), assuming $\boldsymbol{\phi > \rho}$, and various values of the (inverse) signal to noise ratio $\boldsymbol{(s/n)^{-1}}$ (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 30, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved autoregressive factor and known break points.

Table 10: Observed common stochastic factor and break processes, heteroskedastic case, N=2: bias and RMSE of parameters.

autoregressive common factor parameter φ															
bias															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
any	-0.012	-0.013	-0.013	-0.024	-0.024	-0.024	-0.064	-0.064	-0.064	-0.064	-0.040	-0.040	-0.040	-0.040	-0.040
root mean square error															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
any	0.138	0.135	0.135	0.130	0.130	0.130	0.145	0.145	0.145	0.145	0.129	0.129	0.129	0.129	0.129
autoregressive parameter ρ															
bias															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	-0.013	-0.012	-0.019	-0.007	-0.018	-0.021	-0.011	-0.018	-0.025	-0.029	-0.010	-0.013	-0.023	-0.028	-0.036
2	-0.013	-0.010	-0.016	-0.011	-0.015	-0.024	-0.009	-0.017	-0.021	-0.030	-0.010	-0.016	-0.026	-0.029	-0.036
1	-0.009	-0.009	-0.017	-0.009	-0.016	-0.023	-0.012	-0.010	-0.022	-0.027	-0.009	-0.015	-0.021	-0.032	-0.035
0.5	-0.008	-0.013	-0.014	-0.009	-0.017	-0.024	-0.015	-0.019	-0.023	-0.030	-0.009	-0.018	-0.022	-0.030	-0.038
0.25	-0.011	-0.009	-0.016	-0.010	-0.019	-0.023	-0.010	-0.018	-0.020	-0.031	-0.009	-0.015	-0.025	-0.029	-0.039
root mean square error															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.102	0.104	0.102	0.101	0.102	0.098	0.101	0.099	0.100	0.094	0.100	0.098	0.097	0.095	0.086
2	0.100	0.099	0.103	0.102	0.100	0.100	0.102	0.101	0.100	0.095	0.102	0.101	0.101	0.095	0.088
1	0.103	0.103	0.102	0.102	0.103	0.098	0.102	0.097	0.099	0.092	0.099	0.101	0.097	0.096	0.082
0.5	0.102	0.100	0.100	0.103	0.100	0.099	0.101	0.103	0.100	0.096	0.102	0.102	0.096	0.095	0.087
0.25	0.100	0.102	0.101	0.102	0.103	0.099	0.102	0.102	0.096	0.096	0.100	0.099	0.101	0.095	0.088
autoregressive common factor parameter φ															
bias															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
any	0.000	0.008	0.008	0.029	0.029	0.029	-0.032	-0.032	-0.032	-0.032	-0.008	-0.008	-0.008	-0.008	-0.008
root mean square error															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
any	0.064	0.063	0.063	0.077	0.077	0.077	0.073	0.073	0.073	0.073	0.055	0.055	0.055	0.055	0.055
autoregressive parameter ρ															
bias															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	-0.004	-0.002	-0.004	-0.001	-0.002	-0.003	-0.001	-0.004	-0.004	-0.005	-0.003	-0.003	-0.006	-0.005	-0.007
2	-0.003	-0.003	-0.003	-0.001	-0.002	-0.005	-0.003	-0.003	-0.005	-0.005	-0.003	-0.004	-0.005	-0.006	-0.006
1	-0.001	-0.001	-0.001	-0.004	-0.004	-0.005	-0.002	-0.003	-0.006	-0.004	-0.002	-0.005	-0.005	-0.003	-0.008
0.5	-0.002	-0.003	-0.002	-0.001	-0.003	-0.005	-0.002	-0.004	-0.005	-0.005	-0.003	-0.003	-0.004	-0.006	-0.006
0.25	0.000	-0.002	-0.004	-0.002	-0.002	-0.005	-0.002	-0.003	-0.005	-0.005	0.000	-0.003	-0.003	-0.006	-0.008
root mean square error															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.045	0.046	0.044	0.044	0.043	0.042	0.045	0.045	0.042	0.037	0.045	0.045	0.041	0.036	0.029
2	0.044	0.046	0.046	0.043	0.043	0.042	0.045	0.044	0.043	0.038	0.045	0.044	0.042	0.037	0.028
1	0.045	0.045	0.044	0.045	0.043	0.041	0.045	0.044	0.042	0.037	0.045	0.045	0.042	0.036	0.029
0.5	0.045	0.046	0.043	0.046	0.044	0.041	0.045	0.043	0.042	0.036	0.045	0.043	0.043	0.038	0.030
0.25	0.046	0.046	0.043	0.045	0.045	0.042	0.045	0.045	0.043	0.036	0.046	0.045	0.042	0.037	0.030

The Table reports Monte Carlo bias and RMSE statistics, concerning the estimation of the idiosyncratic (***ρ***) and common (***φ***) autoregressive parameters. Results are reported for various values of the common factor fractional differencing parameter ***d*** (0.2, 0.4, 0.6, 0.8, 1), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6, 0.8), assuming ***d > ρ*** and ***φ = d/2***, and various values of the (inverse) signal to noise ratio $(s/n)^{-1}$ (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 2, and the number of replications for each case is 2,000. The experiment refers to the case of observed long memory factor and common break process, or, equivalently, constant unconditional mean.

Table 11: Unobserved common stochastic factor, no structural break, heteroskedastic case, N=30: bias and RMSE of parameters.

autoregressive common factor parameter ϕ

bias

T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	-0.042	-0.066	-0.038	-0.102	-0.075	-0.054	-0.172	-0.146	-0.125	-0.108	-0.195	-0.157	-0.130	-0.109	-0.094
2	-0.025	-0.036	-0.022	-0.061	-0.049	-0.038	-0.123	-0.109	-0.100	-0.091	-0.121	-0.100	-0.087	-0.073	-0.066
1	-0.017	-0.020	-0.013	-0.038	-0.033	-0.028	-0.097	-0.090	-0.085	-0.080	-0.081	-0.069	-0.061	-0.055	-0.051
0.5	-0.013	-0.011	-0.008	-0.027	-0.024	-0.022	-0.083	-0.079	-0.076	-0.075	-0.058	-0.053	-0.048	-0.044	-0.043
0.25	-0.011	-0.006	-0.005	-0.022	-0.020	-0.019	-0.076	-0.074	-0.073	-0.072	-0.046	-0.043	-0.041	-0.039	-0.038

root mean square error

T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.141	0.162	0.138	0.190	0.161	0.143	0.268	0.235	0.208	0.189	0.305	0.254	0.221	0.194	0.178
2	0.137	0.141	0.134	0.153	0.141	0.135	0.208	0.192	0.182	0.172	0.210	0.188	0.173	0.159	0.152
1	0.137	0.136	0.134	0.137	0.133	0.131	0.178	0.170	0.165	0.161	0.167	0.156	0.147	0.142	0.138
0.5	0.137	0.135	0.135	0.131	0.130	0.130	0.164	0.159	0.156	0.155	0.146	0.141	0.137	0.134	0.133
0.25	0.138	0.135	0.135	0.129	0.128	0.128	0.156	0.154	0.153	0.152	0.136	0.134	0.132	0.131	0.131

autoregressive parameter ρ

bias

T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	-0.014	-0.018	-0.027	-0.018	-0.025	-0.038	-0.022	-0.027	-0.039	-0.048	-0.023	-0.028	-0.037	-0.045	-0.057
2	-0.016	-0.013	-0.026	-0.022	-0.028	-0.038	-0.021	-0.029	-0.037	-0.047	-0.019	-0.028	-0.037	-0.048	-0.056
1	-0.012	-0.018	-0.026	-0.018	-0.026	-0.036	-0.019	-0.030	-0.037	-0.050	-0.023	-0.025	-0.034	-0.046	-0.056
0.5	-0.014	-0.016	-0.026	-0.019	-0.026	-0.037	-0.022	-0.029	-0.041	-0.043	-0.024	-0.029	-0.040	-0.050	-0.055
0.25	-0.012	-0.013	-0.024	-0.024	-0.028	-0.035	-0.024	-0.027	-0.038	-0.049	-0.019	-0.028	-0.037	-0.044	-0.056

root mean square error

T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.102	0.101	0.105	0.102	0.106	0.109	0.103	0.105	0.111	0.110	0.105	0.105	0.110	0.105	0.110
2	0.102	0.100	0.104	0.105	0.108	0.109	0.105	0.106	0.109	0.111	0.106	0.105	0.108	0.110	0.109
1	0.101	0.105	0.107	0.104	0.103	0.107	0.103	0.108	0.111	0.112	0.103	0.104	0.104	0.106	0.108
0.5	0.102	0.102	0.104	0.103	0.107	0.110	0.104	0.108	0.110	0.105	0.106	0.107	0.111	0.111	0.107
0.25	0.101	0.103	0.107	0.105	0.107	0.106	0.106	0.106	0.109	0.110	0.106	0.108	0.109	0.104	0.107

autoregressive common factor parameter ϕ

bias

T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	-0.032	-0.061	-0.036	-0.063	-0.037	-0.018	-0.141	-0.114	-0.092	-0.077	-0.173	-0.135	-0.107	-0.084	-0.067
2	-0.016	-0.031	-0.017	-0.020	-0.006	0.005	-0.092	-0.076	-0.065	-0.056	-0.097	-0.076	-0.060	-0.048	-0.039
1	-0.008	-0.014	-0.007	0.004	0.011	0.017	-0.063	-0.055	-0.049	-0.045	-0.055	-0.044	-0.035	-0.029	-0.024
0.5	-0.003	-0.006	-0.002	0.016	0.020	0.023	-0.049	-0.045	-0.042	-0.039	-0.032	-0.026	-0.022	-0.018	-0.016
0.25	-0.001	-0.001	0.001	0.023	0.025	0.026	-0.041	-0.039	-0.037	-0.036	-0.020	-0.017	-0.015	-0.013	-0.012

root mean square error

T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.074	0.105	0.078	0.112	0.085	0.071	0.206	0.170	0.141	0.121	0.251	0.199	0.161	0.131	0.110
2	0.065	0.075	0.065	0.074	0.068	0.067	0.140	0.120	0.107	0.096	0.149	0.121	0.101	0.087	0.077
1	0.063	0.065	0.063	0.068	0.069	0.071	0.105	0.096	0.089	0.085	0.095	0.082	0.073	0.067	0.063
0.5	0.062	0.062	0.062	0.072	0.073	0.075	0.089	0.084	0.081	0.079	0.070	0.065	0.062	0.059	0.057
0.25	0.063	0.062	0.062	0.075	0.076	0.077	0.081	0.078	0.077	0.076	0.060	0.058	0.057	0.056	0.055

autoregressive parameter ρ

bias

T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	-0.003	-0.004	-0.004	-0.003	-0.006	-0.008	-0.004	-0.006	-0.007	-0.009	-0.004	-0.005	-0.006	-0.009	-0.011
2	-0.001	-0.002	-0.006	-0.004	-0.006	-0.007	-0.003	-0.007	-0.008	-0.009	-0.001	-0.005	-0.007	-0.009	-0.009
1	-0.004	-0.003	-0.006	-0.004	-0.005	-0.005	-0.004	-0.005	-0.009	-0.011	-0.005	-0.006	-0.007	-0.009	-0.011
0.5	-0.002	-0.004	-0.004	-0.001	-0.005	-0.008	-0.005	-0.006	-0.007	-0.008	-0.006	-0.006	-0.007	-0.009	-0.010
0.25	-0.002	-0.005	-0.007	-0.005	-0.006	-0.008	-0.004	-0.007	-0.006	-0.011	-0.002	-0.006	-0.008	-0.010	-0.010

root mean square error

T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.046	0.044	0.043	0.046	0.046	0.044	0.045	0.045	0.042	0.038	0.045	0.044	0.042	0.038	0.032
2	0.045	0.045	0.045	0.044	0.045	0.042	0.045	0.046	0.043	0.039	0.045	0.044	0.043	0.039	0.030
1	0.046	0.045	0.044	0.045	0.044	0.042	0.045	0.046	0.043	0.040	0.046	0.045	0.042	0.039	0.032
0.5	0.046	0.046	0.044	0.045	0.045	0.042	0.046	0.045	0.041	0.039	0.045	0.045	0.042	0.038	0.032
0.25	0.045	0.046	0.046	0.044	0.045	0.042	0.046	0.046	0.044	0.039	0.045	0.046	0.042	0.040	0.031

The Table reports Monte Carlo bias and RMSE statistics, concerning the estimation of the idiosyncratic (***ρ***) and common (***φ***) autoregressive parameters. Results are reported for various values of the common factor fractional differencing parameter ***d*** (0.2, 0.4, 0.6, 0.8, 1), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6, 0.8), assuming ***d > ρ*** and ***φ = d/2***, and various values of the (inverse) signal to noise ratio ***$(s/n)^{-1}$*** (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 30, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved long memory factor and no breaks, and known fractional differencing parameter.

Table 12: Unobserved common stochastic factor, no structural break, heteroskedastic case, N=30: Theil and correlation statistics.

common long memory factor															
Theil index															
T = 100	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.194	0.202	0.203	0.090	0.092	0.097	0.054	0.055	0.059	0.067	0.029	0.030	0.032	0.036	0.046
2	0.151	0.175	0.176	0.064	0.065	0.069	0.039	0.039	0.042	0.047	0.021	0.021	0.022	0.025	0.033
1	0.123	0.158	0.158	0.045	0.046	0.049	0.027	0.028	0.029	0.033	0.015	0.015	0.016	0.018	0.023
0.5	0.105	0.147	0.148	0.032	0.033	0.035	0.019	0.020	0.021	0.024	0.010	0.010	0.011	0.013	0.016
0.25	0.094	0.141	0.142	0.023	0.023	0.025	0.014	0.014	0.015	0.017	0.007	0.007	0.008	0.009	0.011
correlation coefficient															
T = 100	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.941	0.960	0.959	0.983	0.982	0.980	0.994	0.993	0.992	0.990	0.998	0.998	0.998	0.997	0.995
2	0.970	0.980	0.979	0.991	0.991	0.990	0.997	0.997	0.996	0.995	0.999	0.999	0.999	0.999	0.998
1	0.984	0.990	0.989	0.996	0.996	0.995	0.998	0.998	0.998	0.998	1.000	1.000	0.999	0.999	0.999
0.5	0.992	0.995	0.995	0.998	0.998	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	0.999
0.25	0.996	0.997	0.997	0.999	0.999	0.999	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000
Theil index															
T = 500	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.172	0.149	0.152	0.049	0.050	0.053	0.026	0.027	0.029	0.033	0.013	0.013	0.014	0.016	0.021
2	0.127	0.117	0.118	0.035	0.035	0.038	0.019	0.019	0.020	0.023	0.009	0.009	0.010	0.011	0.015
1	0.095	0.095	0.096	0.025	0.025	0.027	0.013	0.013	0.014	0.016	0.006	0.006	0.007	0.008	0.010
0.5	0.073	0.082	0.082	0.017	0.018	0.019	0.009	0.010	0.010	0.012	0.004	0.005	0.005	0.006	0.007
0.25	0.058	0.073	0.073	0.012	0.013	0.013	0.007	0.007	0.007	0.008	0.003	0.003	0.003	0.004	0.005
correlation coefficient															
T = 500	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.945	0.966	0.964	0.995	0.995	0.994	0.998	0.998	0.998	0.998	1.000	1.000	1.000	0.999	0.999
2	0.972	0.982	0.982	0.997	0.997	0.997	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
1	0.986	0.991	0.991	0.999	0.999	0.998	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000
0.5	0.993	0.996	0.995	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	0.996	0.998	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

The Table reports Monte Carlo Theil index and correlation coefficient statistics, concerning the estimation of the unobserved common long memory factor component. Results are reported for various values of the common factor fractional differencing parameter ***d*** (0.2, 0.4, 0.6, 0.8, 1), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6, 0.8), assuming ***d > ρ ρ*** and ***ϕ = d/2***, and various values of the (inverse) signal to noise ratio ***$(s/n)^{-1}$*** (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 30, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved long memory factor and no breaks, and known fractional differencing parameter.

Table 13: Unobserved common stochastic factor, no structural break, heteroskedastic case, N = 5, 10, 15, 50: bias of parameter ϕ.

autoregressive common factor parameter ϕ															
bias N = 5															
T = 100	***$d = 0.2\ \phi = 0.1$***	***$d = 0.4\ \phi = 0.2$***		***$d = 0.6\ \phi = 0.3$***			***$d = 0.8\ \phi = 0.4$***				***$d = 1.0\ \phi = 0.5$***				
$(s/n)^{-1}$	***$\rho = 0$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0.8$***
4	-0.127	-0.235	-0.151	-0.325	-0.244	-0.177	-0.429	-0.354	-0.284	-0.229	-0.554	-0.462	-0.382	-0.321	-0.264
2	-0.084	-0.157	-0.103	-0.219	-0.165	-0.119	-0.305	-0.248	-0.204	-0.163	-0.392	-0.317	-0.260	-0.212	-0.174
1	-0.054	-0.098	-0.064	-0.139	-0.105	-0.078	-0.206	-0.171	-0.144	-0.121	-0.251	-0.205	-0.167	-0.136	-0.114
0.5	-0.034	-0.055	-0.041	-0.085	-0.065	-0.050	-0.140	-0.120	-0.105	-0.093	-0.157	-0.128	-0.105	-0.089	-0.076
0.25	-0.021	-0.033	-0.025	-0.053	-0.043	-0.035	-0.104	-0.092	-0.084	-0.078	-0.099	-0.083	-0.071	-0.062	-0.055
bias N = 10															
T = 100	***$d = 0.2\ \phi = 0.1$***	***$d = 0.4\ \phi = 0.2$***		***$d = 0.6\ \phi = 0.3$***			***$d = 0.8\ \phi = 0.4$***				***$d = 1.0\ \phi = 0.5$***				
$(s/n)^{-1}$	***$\rho = 0$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0.8$***
4	-0.088	-0.156	-0.099	-0.218	-0.161	-0.115	-0.304	-0.246	-0.203	-0.166	-0.392	-0.318	-0.264	-0.215	-0.179
2	-0.055	-0.096	-0.062	-0.134	-0.103	-0.074	-0.207	-0.170	-0.145	-0.120	-0.255	-0.207	-0.169	-0.138	-0.117
1	-0.032	-0.056	-0.038	-0.079	-0.063	-0.048	-0.140	-0.121	-0.106	-0.094	-0.161	-0.129	-0.108	-0.091	-0.080
0.5	-0.020	-0.033	-0.024	-0.050	-0.039	-0.033	-0.104	-0.092	-0.084	-0.078	-0.100	-0.084	-0.072	-0.063	-0.057
0.25	-0.015	-0.020	-0.015	-0.033	-0.027	-0.025	-0.083	-0.077	-0.073	-0.069	-0.070	-0.061	-0.054	-0.050	-0.046
bias N = 15															
T = 100	***$d = 0.2\ \phi = 0.1$***	***$d = 0.4\ \phi = 0.2$***		***$d = 0.6\ \phi = 0.3$***			***$d = 0.8\ \phi = 0.4$***				***$d = 1.0\ \phi = 0.5$***				
$(s/n)^{-1}$	***$\rho = 0$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0$***	***$P = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0.8$***
4	-0.065	-0.117	-0.071	-0.166	-0.124	-0.089	-0.245	-0.201	-0.165	-0.138	-0.307	-0.248	-0.200	-0.168	-0.138
2	-0.041	-0.067	-0.043	-0.101	-0.077	-0.059	-0.167	-0.140	-0.121	-0.106	-0.192	-0.155	-0.128	-0.108	-0.091
1	-0.024	-0.038	-0.025	-0.062	-0.047	-0.038	-0.119	-0.103	-0.094	-0.085	-0.117	-0.098	-0.083	-0.072	-0.064
0.5	-0.016	-0.021	-0.015	-0.041	-0.033	-0.028	-0.092	-0.085	-0.079	-0.075	-0.079	-0.067	-0.059	-0.053	-0.048
0.25	-0.011	-0.012	-0.009	-0.028	-0.024	-0.022	-0.078	-0.074	-0.071	-0.069	-0.056	-0.051	-0.046	-0.043	-0.042
bias N = 50															
T = 100	***$d = 0.2\ \phi = 0.1$***	***$d = 0.4\ \phi = 0.2$***		***$d = 0.6\ \phi = 0.3$***			***$d = 0.8\ \phi = 0.4$***				***$d = 1.0\ \phi = 0.5$***				
$(s/n)^{-1}$	***$\rho = 0$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0.8$***
4	-0.028	-0.045	-0.025	-0.073	-0.056	-0.043	-0.132	-0.115	-0.104	-0.094	-0.132	-0.106	-0.090	-0.075	-0.067
2	-0.018	-0.026	-0.017	-0.046	-0.038	-0.033	-0.101	-0.093	-0.087	-0.082	-0.082	-0.068	-0.060	-0.052	-0.048
1	-0.013	-0.015	-0.012	-0.034	-0.029	-0.027	-0.085	-0.081	-0.077	-0.076	-0.055	-0.049	-0.044	-0.041	-0.039
0.5	-0.010	-0.011	-0.009	-0.025	-0.024	-0.022	-0.076	-0.075	-0.073	-0.072	-0.042	-0.039	-0.037	-0.035	-0.034
0.25	-0.009	-0.007	-0.007	-0.022	-0.021	-0.020	-0.072	-0.072	-0.070	-0.070	-0.035	-0.033	-0.032	-0.031	-0.031
bias N = 5															
T = 500	***$d = 0.2\ \phi = 0.1$***	***$d = 0.4\ \phi = 0.2$***		***$d = 0.6\ \phi = 0.3$***			***$d = 0.8\ \phi = 0.4$***				***$d = 1.0\ \phi = 0.5$***				
$(s/n)^{-1}$	***$\rho = 0$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0.8$***
4	-0.121	-0.227	-0.148	-0.305	-0.223	-0.152	-0.420	-0.338	-0.267	-0.208	-0.546	-0.452	-0.373	-0.303	-0.243
2	-0.080	-0.148	-0.094	-0.191	-0.134	-0.086	-0.288	-0.229	-0.181	-0.140	-0.379	-0.304	-0.245	-0.196	-0.156
1	-0.045	-0.085	-0.054	-0.101	-0.066	-0.037	-0.184	-0.147	-0.118	-0.093	-0.236	-0.187	-0.148	-0.117	-0.093
0.5	-0.025	-0.045	-0.029	-0.043	-0.023	-0.007	-0.116	-0.094	-0.078	-0.065	-0.137	-0.107	-0.084	-0.067	-0.053
0.25	-0.013	-0.023	-0.013	-0.009	0.002	0.010	-0.076	-0.065	-0.056	-0.049	-0.077	-0.060	-0.048	-0.038	-0.032
bias N = 10															
T = 500	***$d = 0.2\ \phi = 0.1$***	***$d = 0.4\ \phi = 0.2$***		***$d = 0.6\ \phi = 0.3$***			***$d = 0.8\ \phi = 0.4$***				***$d = 1.0\ \phi = 0.5$***				
$(s/n)^{-1}$	***$\rho = 0$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0.8$***
4	-0.080	-0.149	-0.095	-0.189	-0.133	-0.085	-0.288	-0.230	-0.182	-0.143	-0.377	-0.303	-0.244	-0.195	-0.156
2	-0.047	-0.087	-0.053	-0.102	-0.064	-0.037	-0.186	-0.149	-0.120	-0.097	-0.234	-0.185	-0.145	-0.115	-0.091
1	-0.027	-0.047	-0.028	-0.042	-0.022	-0.007	-0.119	-0.097	-0.080	-0.068	-0.136	-0.106	-0.083	-0.066	-0.052
0.5	-0.014	-0.023	-0.013	-0.009	0.002	0.011	-0.078	-0.067	-0.058	-0.052	-0.076	-0.058	-0.047	-0.037	-0.030
0.25	-0.008	-0.010	-0.006	0.010	0.015	0.020	-0.057	-0.051	-0.047	-0.043	-0.042	-0.033	-0.027	-0.022	-0.018
bias N =1 5															
T = 500	***$d = 0.2\ \phi = 0.1$***	***$d = 0.4\ \phi = 0.2$***		***$d = 0.6\ \phi = 0.3$***			***$d = 0.8\ \phi = 0.4$***				***$d = 1.0\ \phi = 0.5$***				
$(s/n)^{-1}$	***$\rho = 0$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0.8$***
4	-0.058	-0.110	-0.068	-0.134	-0.090	-0.054	-0.223	-0.177	-0.141	-0.112	-0.290	-0.230	-0.183	-0.146	-0.114
2	-0.032	-0.061	-0.036	-0.062	-0.037	-0.017	-0.141	-0.114	-0.092	-0.076	-0.172	-0.135	-0.106	-0.084	-0.066
1	-0.018	-0.031	-0.018	-0.020	-0.006	0.005	-0.090	-0.075	-0.064	-0.055	-0.098	-0.075	-0.060	-0.048	-0.038
0.5	-0.009	-0.014	-0.007	0.004	0.012	0.017	-0.062	-0.054	-0.049	-0.044	-0.054	-0.043	-0.034	-0.028	-0.023
0.25	-0.004	-0.005	-0.002	0.017	0.021	0.024	-0.048	-0.044	-0.041	-0.038	-0.032	-0.025	-0.021	-0.018	-0.015
bias N = 50															
T = 500	***$d = 0.2\ \phi = 0.1$***	***$d = 0.4\ \phi = 0.2$***		***$d = 0.6\ \phi = 0.3$***			***$d = 0.8\ \phi = 0.4$***				***$d = 1.0\ \phi = 0.5$***				
$(s/n)^{-1}$	***$\rho = 0$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0$***	***$\rho = 0.2$***	***$\rho = 0.4$***	***$\rho = 0.6$***	***$\rho = 0.8$***
4	-0.021	-0.034	-0.019	-0.034	-0.018	-0.005	-0.103	-0.084	-0.071	-0.060	-0.112	-0.087	-0.069	-0.054	-0.043
2	-0.011	-0.014	-0.006	-0.006	0.003	0.009	-0.069	-0.060	-0.052	-0.047	-0.062	-0.048	-0.038	-0.031	-0.025
1	-0.006	-0.004	0.000	0.009	0.013	0.016	-0.051	-0.046	-0.043	-0.040	-0.035	-0.028	-0.022	-0.019	-0.016
0.5	-0.003	0.001	0.004	0.016	0.019	0.020	-0.042	-0.040	-0.037	-0.036	-0.021	-0.017	-0.014	-0.012	-0.011
0.25	-0.002	0.004	0.005	0.020	0.021	0.022	-0.037	-0.036	-0.035	-0.034	-0.013	-0.012	-0.010	-0.009	-0.008

The Table reports Monte Carlo bias statistics, concerning the estimation of the common factors autoregressive parameter (ϕ). Results are reported for various values of the fractional differencing parameter ***d*** (0.2, 0.4, 0.6, 0.8, 1), various values of the idiosyncratic autoregressive parameter ρ (0, 0.2, 0.4, 0.6, 0.8), assuming $d > \rho$ and $\phi = d/2$, and various values of the (inverse) signal to noise ratio $(s/n)^{-1}$ (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 5, 10, 15, 50, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved long memory factor, no breaks, and known fractional differencing parameter.

Table 14: Unobserved common stochastic factor, no structural break, heteroskedastic case: Monte Carlo correlation coefficient statistics.

common long memory factor															
correlation coefficient N = 5															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.751	0.815	0.811	0.912	0.910	0.900	0.963	0.962	0.958	0.948	0.988	0.988	0.986	0.983	0.972
2	0.851	0.894	0.890	0.953	0.951	0.946	0.981	0.980	0.978	0.972	0.994	0.994	0.993	0.991	0.986
1	0.917	0.942	0.940	0.975	0.975	0.971	0.990	0.990	0.989	0.986	0.997	0.997	0.997	0.996	0.993
0.5	0.955	0.970	0.969	0.987	0.987	0.986	0.995	0.995	0.994	0.993	0.999	0.998	0.998	0.998	0.996
0.25	0.977	0.984	0.984	0.994	0.993	0.993	0.998	0.998	0.997	0.996	0.999	0.999	0.999	0.999	0.998
correlation coefficient N = 10															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.850	0.892	0.889	0.952	0.951	0.945	0.981	0.980	0.978	0.972	0.994	0.994	0.993	0.992	0.986
2	0.917	0.942	0.940	0.975	0.974	0.971	0.990	0.990	0.989	0.986	0.997	0.997	0.997	0.996	0.993
1	0.956	0.970	0.968	0.987	0.987	0.985	0.995	0.995	0.994	0.993	0.999	0.999	0.998	0.998	0.997
0.5	0.977	0.984	0.984	0.994	0.993	0.993	0.998	0.998	0.997	0.996	0.999	0.999	0.999	0.999	0.998
0.25	0.988	0.992	0.992	0.997	0.997	0.996	0.999	0.999	0.999	0.998	1.000	1.000	1.000	1.000	0.999
correlation coefficient N = 15															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.891	0.924	0.921	0.967	0.966	0.961	0.987	0.987	0.985	0.981	0.996	0.996	0.996	0.994	0.991
2	0.942	0.960	0.959	0.983	0.982	0.980	0.993	0.993	0.992	0.990	0.998	0.998	0.998	0.997	0.995
1	0.970	0.979	0.979	0.991	0.991	0.990	0.997	0.997	0.996	0.995	0.999	0.999	0.999	0.999	0.998
0.5	0.985	0.990	0.989	0.996	0.996	0.995	0.998	0.998	0.998	0.998	1.000	1.000	0.999	0.999	0.999
0.25	0.992	0.995	0.995	0.998	0.998	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	0.999
correlation coefficient N = 50															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.963	0.975	0.974	0.990	0.989	0.988	0.996	0.996	0.995	0.994	0.999	0.999	0.999	0.998	0.997
2	0.981	0.988	0.987	0.995	0.995	0.994	0.998	0.998	0.998	0.997	0.999	0.999	0.999	0.999	0.999
1	0.991	0.994	0.994	0.997	0.997	0.997	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	0.999
0.5	0.995	0.997	0.997	0.999	0.999	0.999	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000
0.25	0.998	0.998	0.998	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
correlation coefficient N = 5															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.765	0.836	0.832	0.969	0.968	0.964	0.991	0.991	0.990	0.987	0.998	0.998	0.997	0.997	0.994
2	0.860	0.907	0.904	0.984	0.984	0.981	0.996	0.995	0.995	0.993	0.999	0.999	0.999	0.998	0.997
1	0.922	0.950	0.948	0.992	0.992	0.991	0.998	0.998	0.997	0.997	1.000	0.999	0.999	0.999	0.999
0.5	0.959	0.974	0.973	0.996	0.996	0.995	0.999	0.999	0.999	0.998	1.000	1.000	1.000	1.000	0.999
0.25	0.979	0.987	0.986	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.9999	0.9999	0.9998	0.9998	0.9996
correlation coefficient N = 10															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.859	0.907	0.903	0.985	0.984	0.982	0.995	0.995	0.995	0.993	0.999	0.999	0.999	0.998	0.997
2	0.921	0.950	0.948	0.992	0.992	0.991	0.998	0.998	0.997	0.997	1.000	0.999	0.999	0.999	0.999
1	0.958	0.974	0.973	0.996	0.996	0.995	0.999	0.999	0.999	0.998	1.000	1.000	1.000	1.000	0.999
0.5	0.979	0.987	0.986	0.998	0.998	0.998	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
0.25	0.989	0.993	0.993	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
correlation coefficient N =1 5															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.899	0.935	0.932	0.990	0.989	0.988	0.997	0.997	0.996	0.995	0.999	0.999	0.999	0.999	0.998
2	0.946	0.966	0.964	0.995	0.995	0.994	0.998	0.998	0.998	0.998	1.000	1.000	1.000	0.999	0.999
1	0.972	0.982	0.982	0.997	0.997	0.997	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
0.5	0.986	0.991	0.991	0.999	0.999	0.999	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000
0.25	0.993	0.996	0.995	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
correlation coefficient N = 50															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.966	0.979	0.978	0.997	0.997	0.996	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	0.999
2	0.983	0.989	0.989	0.998	0.998	0.998	1.000	1.000	0.999	0.999	1.000	1.000	1.000	1.000	1.000
1	0.991	0.995	0.995	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	0.996	0.997	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	0.998	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

The Table reports Monte Carlo correlation coefficients between actual and estimated common factors. Results are reported for various values of the fractional differencing parameter **d** (0.2, 0.4, 0.6, 0.8, 1), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6, 0.8), assuming ***d > ρ*** and ***φ = d/2***, and various values of the (inverse) signal to noise ratio $(s/n)^{-1}$ (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 5, 10, 15, 50, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved long memory factor, no breaks, and known fractional differencing parameter.

Table 15: Unobserved common stochastic factor, single break point, heteroskedastic case, N=30: bias and RMSE of parameters.

autoregressive common factor parameter ϕ															
bias															
T = 100	*$d = 0.2$ $\phi = 0.1$*	*$d = 0.4$ $\phi = 0.2$*		*$d = 0.6$ $\phi = 0.3$*			*$d = 0.8$ $\phi = 0.4$*				*$d = 1.0$ $\phi = 0.5$*				
$(s/n)^{-1}$	*$\rho = 0$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0.8$*
4	-0.049	-0.083	-0.054	-0.122	-0.098	-0.074	-0.190	-0.168	-0.149	-0.131	-0.304	-0.281	-0.264	-0.251	-0.243
2	-0.035	-0.054	-0.040	-0.082	-0.071	-0.061	-0.148	-0.134	-0.124	-0.118	-0.260	-0.248	-0.238	-0.232	-0.227
1	-0.028	-0.036	-0.031	-0.063	-0.056	-0.051	-0.123	-0.117	-0.112	-0.110	-0.236	-0.229	-0.225	-0.222	-0.219
0.5	-0.023	-0.029	-0.026	-0.051	-0.048	-0.045	-0.111	-0.107	-0.105	-0.103	-0.223	-0.220	-0.218	-0.216	-0.215
0.25	-0.021	-0.025	-0.023	-0.045	-0.044	-0.043	-0.104	-0.102	-0.101	-0.100	-0.216	-0.215	-0.214	-0.213	-0.212
root mean square error															
T = 100	*$d = 0.2$ $\phi = 0.1$*	*$d = 0.4$ $\phi = 0.2$*		*$d = 0.6$ $\phi = 0.3$*			*$d = 0.8$ $\phi = 0.4$*				*$d = 1.0$ $\phi = 0.5$*				
$(s/n)^{-1}$	*$\rho = 0$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0.8$*
4	0.149	0.176	0.150	0.213	0.185	0.158	0.293	0.263	0.239	0.217	0.451	0.422	0.400	0.386	0.376
2	0.143	0.155	0.144	0.170	0.158	0.151	0.240	0.221	0.209	0.202	0.397	0.382	0.371	0.364	0.359
1	0.143	0.144	0.143	0.153	0.148	0.143	0.209	0.203	0.196	0.193	0.369	0.361	0.356	0.353	0.350
0.5	0.142	0.142	0.141	0.144	0.141	0.139	0.195	0.191	0.188	0.186	0.355	0.351	0.349	0.347	0.346
0.25	0.141	0.141	0.140	0.140	0.139	0.139	0.188	0.186	0.185	0.183	0.348	0.346	0.345	0.343	0.343
autoregressive parameter ρ															
bias															
T = 100	*$d = 0.2$ $\phi = 0.1$*	*$d = 0.4$ $\phi = 0.2$*		*$d = 0.6$ $\phi = 0.3$*			*$d = 0.8$ $\phi = 0.4$*				*$d = 1.0$ $\phi = 0.5$*				
$(s/n)^{-1}$	*$\rho = 0$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0.8$*
4	-0.023	-0.026	-0.039	-0.026	-0.037	-0.053	-0.026	-0.042	-0.055	-0.065	-0.029	-0.041	-0.051	-0.067	-0.082
2	-0.023	-0.029	-0.036	-0.027	-0.040	-0.050	-0.029	-0.039	-0.051	-0.067	-0.033	-0.041	-0.051	-0.064	-0.078
1	-0.022	-0.030	-0.038	-0.032	-0.041	-0.048	-0.033	-0.040	-0.052	-0.066	-0.028	-0.044	-0.053	-0.066	-0.077
0.5	-0.027	-0.024	-0.040	-0.032	-0.040	-0.054	-0.029	-0.042	-0.048	-0.063	-0.028	-0.040	-0.050	-0.065	-0.078
0.25	-0.022	-0.029	-0.039	-0.033	-0.042	-0.049	-0.031	-0.042	-0.053	-0.065	-0.030	-0.037	-0.054	-0.065	-0.081
root mean square error															
T = 100	*$d = 0.2$ $\phi = 0.1$*	*$d = 0.4$ $\phi = 0.2$*		*$d = 0.6$ $\phi = 0.3$*			*$d = 0.8$ $\phi = 0.4$*				*$d = 1.0$ $\phi = 0.5$*				
$(s/n)^{-1}$	*$\rho = 0$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0.8$*
4	0.106	0.108	0.113	0.107	0.112	0.121	0.108	0.116	0.124	0.129	0.108	0.118	0.119	0.131	0.139
2	0.103	0.108	0.112	0.107	0.115	0.118	0.108	0.113	0.121	0.131	0.111	0.113	0.120	0.127	0.134
1	0.103	0.109	0.113	0.110	0.116	0.116	0.108	0.112	0.120	0.129	0.106	0.116	0.123	0.129	0.133
0.5	0.104	0.106	0.115	0.112	0.116	0.124	0.108	0.116	0.120	0.126	0.108	0.115	0.120	0.127	0.134
0.25	0.104	0.107	0.113	0.112	0.115	0.122	0.108	0.117	0.123	0.129	0.109	0.112	0.123	0.129	0.139
autoregressive common factor parameter ϕ															
bias															
T = 500	*$d = 0.2$ $\phi = 0.1$*	*$d = 0.4$ $\phi = 0.2$*		*$d = 0.6$ $\phi = 0.3$*			*$d = 0.8$ $\phi = 0.4$*				*$d = 1.0$ $\phi = 0.5$*				
$(s/n)^{-1}$	*$\rho = 0$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0.8$*
4	-0.034	-0.060	-0.037	-0.071	-0.045	-0.025	-0.150	-0.125	-0.106	-0.090	-0.277	-0.252	-0.234	-0.219	-0.209
2	-0.019	-0.030	-0.017	-0.028	-0.015	-0.004	-0.105	-0.090	-0.080	-0.072	-0.229	-0.217	-0.207	-0.199	-0.193
1	-0.010	-0.014	-0.007	-0.006	0.002	0.008	-0.079	-0.072	-0.066	-0.062	-0.204	-0.197	-0.192	-0.188	-0.185
0.5	-0.006	-0.005	-0.002	0.007	0.011	0.014	-0.066	-0.062	-0.059	-0.057	-0.191	-0.186	-0.184	-0.182	-0.181
0.25	-0.004	0.000	0.001	0.014	0.016	0.017	-0.058	-0.056	-0.055	-0.054	-0.184	-0.181	-0.180	-0.179	-0.178
root mean square error															
T = 500	*$d = 0.2$ $\phi = 0.1$*	*$d = 0.4$ $\phi = 0.2$*		*$d = 0.6$ $\phi = 0.3$*			*$d = 0.8$ $\phi = 0.4$*				*$d = 1.0$ $\phi = 0.5$*				
$(s/n)^{-1}$	*$\rho = 0$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0.8$*
4	0.078	0.104	0.080	0.119	0.090	0.071	0.219	0.184	0.159	0.139	0.404	0.371	0.349	0.332	0.319
2	0.067	0.075	0.066	0.076	0.067	0.063	0.158	0.139	0.125	0.116	0.344	0.329	0.317	0.309	0.302
1	0.065	0.066	0.064	0.065	0.063	0.064	0.125	0.116	0.109	0.104	0.315	0.307	0.301	0.297	0.294
0.5	0.064	0.064	0.063	0.065	0.066	0.066	0.109	0.104	0.101	0.098	0.301	0.296	0.293	0.291	0.290
0.25	0.064	0.064	0.064	0.067	0.067	0.068	0.101	0.098	0.097	0.096	0.293	0.291	0.289	0.288	0.287
autoregressive parameter ρ															
bias															
T = 500	*$d = 0.2$ $\phi = 0.1$*	*$d = 0.4$ $\phi = 0.2$*		*$d = 0.6$ $\phi = 0.3$*			*$d = 0.8$ $\phi = 0.4$*				*$d = 1.0$ $\phi = 0.5$*				
$(s/n)^{-1}$	*$\rho = 0$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0.8$*
4	-0.004	-0.006	-0.007	-0.006	-0.007	-0.011	-0.008	-0.009	-0.011	-0.012	-0.007	-0.007	-0.009	-0.013	-0.016
2	-0.004	-0.007	-0.009	-0.007	-0.008	-0.009	-0.006	-0.009	-0.009	-0.012	-0.004	-0.007	-0.011	-0.012	-0.015
1	-0.004	-0.006	-0.007	-0.006	-0.008	-0.010	-0.006	-0.008	-0.010	-0.011	-0.006	-0.009	-0.010	-0.013	-0.016
0.5	-0.006	-0.005	-0.009	-0.007	-0.007	-0.010	-0.005	-0.008	-0.011	-0.012	-0.007	-0.010	-0.009	-0.013	-0.014
0.25	-0.004	-0.006	-0.010	-0.006	-0.008	-0.009	-0.005	-0.008	-0.011	-0.012	-0.005	-0.009	-0.011	-0.012	-0.015
root mean square error															
T = 500	*$d = 0.2$ $\phi = 0.1$*	*$d = 0.4$ $\phi = 0.2$*		*$d = 0.6$ $\phi = 0.3$*			*$d = 0.8$ $\phi = 0.4$*				*$d = 1.0$ $\phi = 0.5$*				
$(s/n)^{-1}$	*$\rho = 0$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0$*	*$\rho = 0.2$*	*$\rho = 0.4$*	*$\rho = 0.6$*	*$\rho = 0.8$*
4	0.045	0.045	0.045	0.045	0.043	0.044	0.047	0.046	0.045	0.041	0.046	0.045	0.043	0.041	0.036
2	0.046	0.045	0.046	0.046	0.046	0.043	0.046	0.045	0.043	0.040	0.046	0.045	0.044	0.040	0.036
1	0.045	0.046	0.045	0.046	0.044	0.043	0.045	0.046	0.045	0.040	0.047	0.047	0.043	0.041	0.037
0.5	0.046	0.044	0.047	0.047	0.046	0.045	0.045	0.045	0.043	0.041	0.046	0.047	0.043	0.042	0.034
0.25	0.045	0.044	0.046	0.045	0.046	0.043	0.045	0.046	0.044	0.042	0.045	0.046	0.043	0.040	0.035

The Table reports Monte Carlo bias and RMSE statistics, concerning the estimation of the idiosyncratic (***ρ***) and common (***ϕ***) autoregressive parameters. Results are reported for various values of the common factor fractional differencing parameter ***d*** (0.2, 0.4, 0.6, 0.8, 1), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6, 0.8), assuming ***d > ρ*** and ***ϕ = d/2***, and various values of the (inverse) signal to noise ratio ***$(s/n)^{-1}$*** (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 30, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved long memory factor, and known fractional differencing parameter and single break point.

Table 16: Unobserved common stochastic factor, two break points, heteroskedastic case, N=30: bias and RMSE of parameters.

autoregressive common factor parameter ϕ															
bias															
T = 100	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	-0.059	-0.095	-0.062	-0.141	-0.114	-0.093	-0.214	-0.191	-0.174	-0.159	-0.355	-0.333	-0.322	-0.311	-0.303
2	-0.045	-0.064	-0.051	-0.102	-0.090	-0.079	-0.174	-0.162	-0.153	-0.144	-0.320	-0.311	-0.302	-0.297	-0.294
1	-0.036	-0.048	-0.042	-0.082	-0.077	-0.071	-0.152	-0.145	-0.140	-0.137	-0.300	-0.296	-0.291	-0.287	-0.288
0.5	-0.032	-0.041	-0.038	-0.071	-0.069	-0.066	-0.139	-0.136	-0.134	-0.132	-0.290	-0.288	-0.287	-0.285	-0.284
0.25	-0.029	-0.036	-0.034	-0.066	-0.064	-0.063	-0.133	-0.131	-0.130	-0.130	-0.285	-0.284	-0.283	-0.282	-0.282
root mean square error															
T = 100	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.152	0.186	0.153	0.236	0.202	0.179	0.323	0.295	0.271	0.251	0.518	0.489	0.474	0.460	0.450
2	0.146	0.158	0.148	0.192	0.178	0.166	0.273	0.257	0.246	0.235	0.473	0.461	0.449	0.444	0.440
1	0.143	0.148	0.144	0.171	0.165	0.160	0.245	0.236	0.231	0.226	0.447	0.442	0.437	0.432	0.432
0.5	0.142	0.145	0.143	0.160	0.158	0.156	0.229	0.225	0.223	0.221	0.435	0.432	0.431	0.428	0.427
0.25	0.142	0.142	0.141	0.156	0.154	0.153	0.223	0.220	0.219	0.218	0.429	0.427	0.426	0.425	0.425
autoregressive parameter ρ															
bias															
T = 100	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	-0.021	-0.020	-0.038	-0.028	-0.039	-0.047	-0.031	-0.039	-0.050	-0.064	-0.026	-0.044	-0.052	-0.066	-0.074
2	-0.020	-0.033	-0.037	-0.028	-0.039	-0.049	-0.029	-0.042	-0.054	-0.066	-0.030	-0.038	-0.048	-0.060	-0.076
1	-0.024	-0.029	-0.033	-0.028	-0.038	-0.048	-0.028	-0.040	-0.051	-0.065	-0.026	-0.040	-0.054	-0.065	-0.078
0.5	-0.024	-0.024	-0.039	-0.031	-0.038	-0.050	-0.030	-0.039	-0.051	-0.062	-0.032	-0.040	-0.052	-0.062	-0.076
0.25	-0.025	-0.028	-0.038	-0.026	-0.037	-0.048	-0.031	-0.043	-0.049	-0.061	-0.032	-0.041	-0.052	-0.064	-0.078
root mean square error															
T = 100	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.105	0.103	0.112	0.107	0.116	0.116	0.111	0.112	0.119	0.128	0.107	0.117	0.119	0.130	0.131
2	0.106	0.111	0.113	0.107	0.113	0.117	0.111	0.117	0.125	0.128	0.107	0.114	0.116	0.122	0.131
1	0.107	0.106	0.113	0.108	0.113	0.118	0.106	0.115	0.120	0.129	0.107	0.116	0.123	0.128	0.134
0.5	0.107	0.105	0.116	0.109	0.112	0.119	0.106	0.114	0.120	0.124	0.111	0.113	0.121	0.124	0.133
0.25	0.105	0.106	0.111	0.108	0.112	0.116	0.109	0.116	0.118	0.122	0.111	0.117	0.119	0.126	0.135
autoregressive common factor parameter ϕ															
bias															
T = 500	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	-0.037	-0.063	-0.040	-0.148	-0.128	-0.110	-0.349	-0.333	-0.320	-0.312	-0.665	-0.657	-0.651	-0.647	-0.644
2	-0.023	-0.034	-0.021	-0.116	-0.104	-0.095	-0.324	-0.316	-0.310	-0.305	-0.654	-0.650	-0.647	-0.645	-0.644
1	-0.014	-0.017	-0.011	-0.098	-0.092	-0.088	-0.311	-0.307	-0.304	-0.301	-0.648	-0.647	-0.645	-0.644	-0.643
0.5	-0.010	-0.009	-0.006	-0.088	-0.086	-0.083	-0.304	-0.302	-0.300	-0.299	-0.646	-0.645	-0.644	-0.644	-0.643
0.25	-0.008	-0.005	-0.003	-0.084	-0.082	-0.081	-0.301	-0.300	-0.299	-0.298	-0.644	-0.644	-0.643	-0.643	-0.643
root mean square error															
T = 500	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.080	0.109	0.083	0.221	0.194	0.172	0.507	0.487	0.470	0.459	0.950	0.940	0.933	0.928	0.924
2	0.071	0.079	0.069	0.181	0.165	0.154	0.475	0.465	0.458	0.451	0.936	0.932	0.928	0.926	0.924
1	0.067	0.068	0.065	0.158	0.151	0.146	0.459	0.454	0.450	0.447	0.930	0.928	0.926	0.924	0.923
0.5	0.066	0.065	0.064	0.147	0.144	0.141	0.451	0.448	0.446	0.445	0.926	0.925	0.924	0.924	0.923
0.25	0.065	0.064	0.064	0.142	0.140	0.139	0.447	0.446	0.445	0.444	0.925	0.924	0.924	0.923	0.923
autoregressive parameter ρ															
bias															
T = 500	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	-0.004	-0.003	-0.008	-0.006	-0.007	-0.010	-0.006	-0.008	-0.010	-0.012	-0.005	-0.009	-0.011	-0.011	-0.015
2	-0.005	-0.007	-0.009	-0.005	-0.009	-0.010	-0.005	-0.008	-0.010	-0.012	-0.007	-0.007	-0.009	-0.012	-0.015
1	-0.004	-0.003	-0.008	-0.006	-0.007	-0.010	-0.003	-0.008	-0.011	-0.011	-0.006	-0.007	-0.011	-0.013	-0.015
0.5	-0.004	-0.007	-0.007	-0.005	-0.008	-0.009	-0.004	-0.007	-0.009	-0.014	-0.007	-0.007	-0.011	-0.013	-0.016
0.25	-0.006	-0.003	-0.007	-0.007	-0.008	-0.010	-0.007	-0.009	-0.010	-0.011	-0.005	-0.009	-0.010	-0.011	-0.016
root mean square error															
T = 500	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.046	0.044	0.045	0.047	0.046	0.045	0.045	0.045	0.044	0.040	0.044	0.045	0.044	0.040	0.035
2	0.046	0.046	0.046	0.046	0.046	0.044	0.045	0.046	0.044	0.039	0.046	0.044	0.044	0.041	0.036
1	0.045	0.044	0.045	0.047	0.045	0.043	0.044	0.045	0.044	0.040	0.045	0.045	0.044	0.041	0.036
0.5	0.047	0.045	0.045	0.044	0.047	0.044	0.046	0.044	0.044	0.041	0.044	0.046	0.044	0.041	0.037
0.25	0.046	0.045	0.046	0.046	0.046	0.043	0.046	0.045	0.043	0.040	0.045	0.045	0.044	0.039	0.037

The Table reports Monte Carlo bias and RMSE statistics, concerning the estimation of the idiosyncratic (***ρ***) and common (***ϕ***) autoregressive parameters. Results are reported for various values of the common factor fractional differencing parameter ***d*** (0.2, 0.4, 0.6, 0.8, 1), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6, 0.8), assuming ***d > ρ*** and ***ϕ = d/2***, and various values of the (inverse) signal to noise ratio ***$(s/n)^{-1}$*** (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 30, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved long memory factor, and known fractional differencing parameter and break points.

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Table 17: Unobserved common stochastic factor, known break points, heteroskedastic case, N=30: Monte Carlo Theil and correlation statistics.

N=30															
common long memory factor															
1-break point case															
Theil index															
T = 100	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.213	0.256	0.258	0.294	0.295	0.297	0.318	0.318	0.319	0.322	0.344	0.344	0.345	0.345	0.348
2	0.176	0.236	0.237	0.283	0.283	0.285	0.313	0.313	0.314	0.316	0.343	0.343	0.343	0.343	0.345
1	0.152	0.224	0.224	0.276	0.277	0.278	0.311	0.311	0.311	0.312	0.342	0.342	0.342	0.342	0.343
0.5	0.138	0.217	0.217	0.273	0.273	0.273	0.309	0.309	0.309	0.310	0.341	0.341	0.342	0.342	0.342
0.25	0.129	0.213	0.213	0.271	0.271	0.271	0.308	0.308	0.308	0.309	0.341	0.341	0.341	0.341	0.342
correlation coefficient															
T = 100	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.924	0.910	0.909	0.816	0.816	0.813	0.788	0.788	0.787	0.784	0.758	0.758	0.758	0.757	0.755
2	0.953	0.930	0.929	0.826	0.826	0.824	0.792	0.791	0.791	0.790	0.759	0.759	0.759	0.759	0.758
1	0.968	0.941	0.941	0.831	0.831	0.830	0.793	0.793	0.793	0.793	0.760	0.760	0.760	0.760	0.759
0.5	0.975	0.946	0.946	0.833	0.833	0.833	0.794	0.794	0.794	0.794	0.760	0.760	0.760	0.760	0.760
0.25	0.979	0.949	0.949	0.834	0.834	0.834	0.795	0.795	0.795	0.795	0.760	0.760	0.760	0.760	0.760
Theil index															
T = 500	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.178	0.167	0.168	0.310	0.311	0.312	0.326	0.326	0.327	0.327	0.326	0.326	0.326	0.326	0.327
2	0.134	0.138	0.139	0.306	0.306	0.307	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.325	0.326
1	0.104	0.120	0.121	0.304	0.304	0.304	0.324	0.324	0.324	0.324	0.325	0.325	0.325	0.325	0.325
0.5	0.084	0.109	0.110	0.303	0.303	0.303	0.324	0.324	0.324	0.324	0.325	0.325	0.325	0.325	0.325
0.25	0.071	0.103	0.104	0.302	0.302	0.302	0.323	0.323	0.323	0.323	0.325	0.325	0.325	0.325	0.325
correlation coefficient															
T = 500	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.942	0.954	0.953	0.794	0.794	0.793	0.778	0.778	0.778	0.777	0.776	0.776	0.776	0.775	0.775
2	0.968	0.971	0.970	0.798	0.797	0.797	0.779	0.779	0.779	0.779	0.776	0.776	0.776	0.776	0.776
1	0.982	0.980	0.979	0.799	0.799	0.799	0.780	0.780	0.780	0.779	0.776	0.776	0.776	0.776	0.776
0.5	0.989	0.984	0.984	0.800	0.800	0.800	0.780	0.780	0.780	0.780	0.776	0.776	0.776	0.776	0.776
0.25	0.993	0.986	0.986	0.800	0.800	0.800	0.780	0.780	0.780	0.780	0.776	0.776	0.776	0.776	0.776
2-break point case															
Theil index															
T = 100	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.231	0.296	0.296	0.375	0.375	0.377	0.423	0.424	0.424	0.425	0.464	0.464	0.464	0.464	0.466
2	0.197	0.278	0.279	0.368	0.368	0.369	0.421	0.421	0.421	0.422	0.463	0.463	0.463	0.464	0.464
1	0.176	0.268	0.269	0.364	0.364	0.364	0.420	0.420	0.420	0.420	0.463	0.463	0.463	0.463	0.464
0.5	0.164	0.263	0.263	0.362	0.362	0.362	0.419	0.419	0.419	0.419	0.463	0.463	0.463	0.463	0.463
0.25	0.157	0.260	0.261	0.360	0.360	0.361	0.418	0.418	0.419	0.419	0.463	0.463	0.463	0.463	0.463
correlation coefficient															
T = 100	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.907	0.870	0.870	0.731	0.730	0.728	0.677	0.677	0.676	0.674	0.632	0.632	0.631	0.631	0.629
2	0.937	0.891	0.891	0.741	0.741	0.740	0.681	0.681	0.681	0.680	0.633	0.633	0.633	0.632	0.631
1	0.952	0.902	0.902	0.747	0.746	0.746	0.684	0.684	0.683	0.683	0.634	0.634	0.634	0.633	0.633
0.5	0.960	0.908	0.908	0.749	0.749	0.749	0.685	0.685	0.685	0.684	0.634	0.634	0.634	0.634	0.633
0.25	0.964	0.911	0.910	0.751	0.751	0.750	0.685	0.685	0.685	0.685	0.634	0.634	0.634	0.634	0.634
Theil index															
T = 500	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.186	0.185	0.186	0.423	0.423	0.424	0.445	0.445	0.445	0.446	0.443	0.443	0.443	0.443	0.443
2	0.146	0.160	0.161	0.421	0.421	0.421	0.445	0.445	0.445	0.445	0.443	0.443	0.443	0.443	0.443
1	0.119	0.145	0.146	0.420	0.420	0.420	0.444	0.444	0.444	0.444	0.443	0.443	0.443	0.443	0.443
0.5	0.102	0.137	0.137	0.419	0.419	0.419	0.444	0.444	0.444	0.444	0.443	0.443	0.443	0.443	0.443
0.25	0.093	0.132	0.132	0.419	0.419	0.419	0.444	0.444	0.444	0.444	0.443	0.443	0.443	0.443	0.443
correlation coefficient															
T = 500	***d = 0.2 ϕ = 0.1***	***d = 0.4 ϕ = 0.2***		***d = 0.6 ϕ = 0.3***			***d = 0.8 ϕ = 0.4***				***d = 1.0 ϕ = 0.5***				
$(s/n)^{-1}$	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.935	0.941	0.940	0.675	0.674	0.674	0.651	0.651	0.651	0.650	0.652	0.652	0.652	0.652	0.651
2	0.961	0.958	0.957	0.678	0.678	0.678	0.652	0.652	0.652	0.651	0.653	0.653	0.652	0.652	0.652
1	0.975	0.966	0.966	0.680	0.680	0.680	0.652	0.652	0.652	0.652	0.653	0.653	0.653	0.653	0.652
0.5	0.982	0.971	0.971	0.681	0.681	0.681	0.653	0.653	0.653	0.653	0.653	0.653	0.653	0.653	0.653
0.25	0.985	0.973	0.973	0.681	0.681	0.681	0.653	0.653	0.653	0.653	0.653	0.653	0.653	0.653	0.653

The Table reports Monte Carlo Theil index and correlation coefficient statistics, concerning the estimation of the unobserved common long memory factor component. Results are reported for various values of the fractional differencing parameter ***d*** (0.2, 0.4, 0.6, 0.8, 1), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6, 0.8), assuming ***d > ρ*** and ***ϕ = d/2***, and various values of the (inverse) signal to noise ratio ***$(s/n)^{-1}$*** (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 30, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved long memory factor and known fractional differencing parameter and break points.

Table 18: Unobserved common stochastic factor, known break points, heteroskedastic case, N=30: Monte Carlo Theil and correlation statistics.

N=30															
common break process															
1-break point case															
Theil index															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.045	0.097	0.097	0.083	0.083	0.083	0.092	0.092	0.092	0.093	0.100	0.100	0.100	0.100	0.100
2	0.044	0.097	0.097	0.083	0.083	0.083	0.092	0.092	0.092	0.092	0.100	0.100	0.100	0.100	0.100
1	0.044	0.097	0.097	0.083	0.083	0.083	0.092	0.092	0.092	0.092	0.100	0.100	0.100	0.100	0.100
0.5	0.044	0.097	0.097	0.083	0.083	0.083	0.092	0.092	0.092	0.092	0.100	0.100	0.100	0.100	0.100
0.25	0.044	0.097	0.097	0.083	0.083	0.083	0.092	0.092	0.092	0.092	0.100	0.100	0.100	0.100	0.100
correlation coefficient															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Theil index															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.021	0.047	0.047	0.091	0.091	0.091	0.096	0.097	0.096	0.097	0.097	0.097	0.097	0.097	0.097
2	0.020	0.047	0.047	0.091	0.091	0.091	0.096	0.096	0.097	0.096	0.097	0.097	0.097	0.097	0.097
1	0.020	0.047	0.047	0.091	0.091	0.091	0.096	0.096	0.096	0.096	0.097	0.097	0.097	0.097	0.097
0.5	0.020	0.047	0.047	0.091	0.091	0.091	0.096	0.096	0.096	0.096	0.097	0.097	0.097	0.097	0.097
0.25	0.020	0.047	0.047	0.091	0.091	0.091	0.096	0.096	0.096	0.096	0.097	0.097	0.097	0.097	0.097
correlation coefficient															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2-break point case															
Theil index															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.061	0.129	0.129	0.116	0.116	0.116	0.129	0.129	0.129	0.129	0.139	0.139	0.138	0.139	0.139
2	0.060	0.128	0.128	0.116	0.116	0.116	0.129	0.129	0.129	0.129	0.138	0.138	0.138	0.138	0.139
1	0.060	0.128	0.128	0.116	0.116	0.116	0.129	0.129	0.129	0.129	0.138	0.138	0.138	0.138	0.139
0.5	0.060	0.128	0.128	0.116	0.116	0.116	0.129	0.129	0.129	0.129	0.138	0.138	0.139	0.138	0.139
0.25	0.060	0.128	0.128	0.116	0.116	0.116	0.129	0.129	0.129	0.129	0.138	0.138	0.138	0.139	0.139
correlation coefficient															
T = 100	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.992	0.967	0.968	0.950	0.950	0.950	0.940	0.940	0.940	0.940	0.931	0.931	0.931	0.931	0.930
2	0.993	0.967	0.968	0.950	0.950	0.950	0.940	0.940	0.940	0.940	0.931	0.931	0.931	0.931	0.931
1	0.993	0.968	0.968	0.950	0.950	0.950	0.940	0.940	0.940	0.940	0.931	0.931	0.931	0.931	0.931
0.5	0.993	0.968	0.968	0.950	0.950	0.950	0.940	0.940	0.940	0.940	0.931	0.931	0.931	0.931	0.931
0.25	0.993	0.968	0.968	0.950	0.950	0.950	0.940	0.940	0.940	0.940	0.931	0.931	0.931	0.931	0.931
Theil index															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.034	0.067	0.068	0.130	0.130	0.130	0.136	0.136	0.136	0.136	0.135	0.135	0.135	0.135	0.135
2	0.034	0.067	0.067	0.130	0.130	0.130	0.136	0.136	0.136	0.136	0.135	0.135	0.135	0.135	0.135
1	0.034	0.067	0.067	0.130	0.130	0.130	0.136	0.136	0.136	0.136	0.135	0.135	0.135	0.135	0.135
0.5	0.034	0.067	0.067	0.130	0.130	0.130	0.136	0.136	0.136	0.136	0.135	0.135	0.135	0.135	0.135
0.25	0.034	0.067	0.067	0.130	0.130	0.130	0.136	0.136	0.136	0.136	0.135	0.135	0.135	0.135	0.135
correlation coefficient															
T = 500	***d = 0.2 φ = 0.1***	***d = 0.4 φ = 0.2***		***d = 0.6 φ = 0.3***			***d = 0.8 φ = 0.4***				***d = 1.0 φ = 0.5***				
(s/n)^-1	***ρ = 0***	***ρ = 0***	***ρ = 0.2***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0***	***ρ = 0.2***	***ρ = 0.4***	***ρ = 0.6***	***ρ = 0.8***
4	0.997	0.990	0.990	0.939	0.939	0.939	0.935	0.935	0.935	0.935	0.935	0.935	0.935	0.935	0.935
2	0.997	0.990	0.990	0.939	0.939	0.939	0.935	0.935	0.935	0.935	0.935	0.935	0.935	0.935	0.935
1	0.997	0.990	0.990	0.939	0.939	0.939	0.935	0.935	0.935	0.935	0.9353	0.9353	0.935	0.935	0.935
0.5	0.997	0.990	0.990	0.939	0.939	0.939	0.935	0.935	0.935	0.935	0.9353	0.9353	0.935	0.935	0.935
0.25	0.997	0.990	0.990	0.939	0.939	0.939	0.935	0.935	0.935	0.935	0.9353	0.9353	0.935	0.935	0.935

The Table reports Monte Carlo Theil index and correlation coefficient statistics, concerning the estimation of the unobserved common long memory factor component. Results are reported for various values of the fractional differencing parameter ***d*** (0.2, 0.4, 0.6, 0.8, 1), various values of the idiosyncratic autoregressive parameter ***ρ*** (0, 0.2, 0.4, 0.6, 0.8), assuming ***d > ρ*** and ***φ = d/2***, and various values of the (inverse) signal to noise ratio ***(s/n)^-1*** (4, 2, 1, 05, 0.25). The sample size ***T*** is 100 and 500 observations, the number of cross-sectional units ***N*** is 30, and the number of replications for each case is 2,000. The experiment refers to the case of unobserved long memory factor and known fractional differencing parameter and break points.