JENA ECONOMIC RESEARCH PAPERS

# 2009 – 101

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www.jenecon.de

ISSN 1864-7057

The JENA ECONOMIC RESEARCH PAPERS is a joint publication of the Friedrich Schiller University and the Max Planck Institute of Economics, Jena, Germany. For editorial correspondence please contact markus.pasche@uni-jena.de.

Impressum:

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seit 1558

# Government Spending Composition in a Simple Model of Schumpeterian Growth

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November 2009

**Preliminary draft – Do not quote**

Abstract:
This paper investigates the relevance of government purchasing behavior for innovation-based economic growth. We construct a parsimonious Schumpeterian growth model in which demand from the public sphere can effectively alter the economy's rate of technological change. We incorporate results of various empirical studies arguing that public sector demand acts as incentive for private innovation activities. In contrast to the standard Schumpeterian growth framework, we account for industry heterogeneity in terms of innovation potential. This extension allows to bring government demand policy within the realm of the growth policy debate. By varying the composition of its purchases, the government can induce a reallocation of private resources to stimulate the rate of technological change. This comes along with temporarily faster economic growth. Moreover, our welfare analysis implies that it is always worth implementing a policy in which industries benefit from public purchases subject to their specific innovation size.

*JEL classification*: E62, H54, H57, O31, O32, O41

*Keywords*: public demand, endogenous technological change, Schumpeterian growth

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I would like to thank Uwe Cantner, Marco Guerzoni, René Söllner, Sebastian von Engelhardt, Vera Popova and Ljubica Nedelkoska as well as the participants of the EMAEE conference in Jena and the ESSID Summer School in Barcelona for precious comments on earlier drafts of this paper. I am highly indebted to Oliver Kirchkamp for project supervision. I also gratefully acknowledge Sascha Rexhäuser's excellent research assistance.
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## 1. Introduction

This paper studies the influence of government purchasing behavior on innovation activity and economic growth within the framework of a Schumpeterian growth model. Long-run growth results from quality-improving innovation and is, in particular, driven by the technological composition of government demand. As innovation has been acknowledged to be a key determinant of long-run growth, the link between public demand and innovation is also relevant for economic growth. This paper is related to the literature emphasizing that market demand is one important driver of the rate and direction of technological change, typically referred to as "demand pull." Standard quality ladder models (Grossman & Helpman, 1991a and 1991b; Aghion & Howitt, 1992) completely neglect demand-pull effects because these authors assume all industries are symmetric with respect to their innovation potential. Relaxing the symmetry assumption allows us to explore the effects of a change in the composition of public demand spending on the long-run equilibrium and on the economy's transitional dynamics.

Our main result is that, when government purchases are relatively in favor of industries with above-average innovation size, the rate of technological change is stimulated. This unfolds temporarily higher economic growth. In the long run, however, the growth rate is completely determined by exogenous parameters. Both results combined imply that public demand policy influences the level of the balanced-growth equilibrium, while leaving the balanced-growth rate unaffected. We further show that government purchasing behavior might lead to indeterminacy of equilibria, thus having potential to explain business cycles as well as drastically different growth experiences of countries whose initial conditions were very similar. Finally, as the utility level of individuals provides an appropriate criterion for assessing the overall benefits of government policies, we investigate whether economic welfare can be raised by a change in the technological content of public demand spending. We find that from a normative viewpoint, it is always worth implementing a demand policy in which industries benefit from public purchases subject to their specific quality jump.

The paper suggests that the composition of government demand expenditure is an important determinant of firms' innovation activities, thereby contributing to the literature on the role of the demand side for innovation. Research in this field highlights a formidable array of possible explanations for the rate and the direction of technological change being sensible to demand conditions, which can be aggregated to two main grounds. On the one hand, demand "steers" firms to address certain problems (Rosenberg, 1969). Sophisticated users who are well aware of their needs and able to communicate them to the producers enable interaction between firms and users, leading to a decrease of uncertainty in the innovation process (von Hippel, 1982, 1986; Lundvall 1988; Guerzoni, 2007). On the other hand, the size of the pay-off to successful investment in innovation activities determines their attractiveness for firms. In the words of the U.S. sociologist Seabury C. Gilfillan (1935, pp. 58f.): "*Increasing population and/or industry stimulate invention, because they increase the absolute need for a device, and the number of potential finders, while the cost of finding remain the same. There are more mouths to eat the innovation, so to speak, and more eyes to find it*." Schmookler (1962, 1966) uses patent data to show that inventive activity tended to lag behind the peaks and valleys of output of a commodity. From this observation it can be inferred that market demand forces influence shifts in the allocation of resources to inventive activity. Schmookler (1966, p. 206) concludes concisely: "*[...] invention is largely an economic activity which, like other economic activities, is pursued for gain.*"[1] More recently, Gilfillan's and Schmookler's findings have been further explored by Acemoglu & Linn (2004) in their study on the emergence of new drugs. The authors find that a one percent increase in potential market size for a drug category leads to a four to 7.5 percent increase in the number of new drugs in that category entering the U.S. market. Thereby only a handful of the 1,400 new drugs approved over the last forty years have targeted so-called "tropical" diseases like malaria or tuberculosis, although these diseases are responsible for the death of millions of people every year.

Following the widespread recognition of the role of demand in affecting both the rate and direction of innovation, a stream of literature has emerged that focuses on *public* demand. An early study in this context was Project "HINDSIGHT," conducted on behalf of the U.S. Department of Defense (Sherwin & Isenson, 1967; Rothwell & Zegveld, 1982). A review of the development of 710 military innovations led to the key finding that nearly 95 percent of the innovations were motivated by a recognized defense need. Ruttan (2006) and Mowery (2008) go as far as to suggest that most of the general purpose technologies developed in the U.S. in the 20th century either would not have emerged without the impetus from government demand, or only with a considerable delay. Fridlund (2000) as well as Berggren & Laestadius (2003) attribute the observed major impact of the public sector in Scandinavian countries on the development of Nordic telecommunication to so-called "development pairs" defined as a

[1] Schmookler's findings are sometimes interpreted as supporting the statement that the primary stimulus for *innovation* comes from demand on the marketplace rather than being a result of major breakthroughs in science (e.g. Gilpin, 1975; Acemoglu & Linn, 2004). However, as Schmookler's work deals with inventions, not with commercially successful innovations, this extension is illegitimate (see also Mowery & Rosenberg, 1979, pp. 138f.).

long-term relation between industry and customers from the public sphere.[2] Moreover, Scandinavian governments often set challenging novelty requirements and insisted on the development of technical advances while the respective private counterpart hesitated. Complementing these case study results, quantitative studies of the influence of public demand on innovation typically support the conjecture that the size of public markets can provide an enormous stimulus to innovation (Lichtenberg, 1988; Aschhoff & Sofka, 2008).

Several factors can be identified why government demand might be critical for innovation. First, the total magnitude of government purchasing is considerable. In the U.S., the average size of public procurement markets[3] amounted to 420 billion USD in the fiscal year 2006, which is equivalent to about 4.5 percent of U.S. GDP (Federal Procurement Data Center, 2006). The European Union experienced a particularly pronounced growth of procurement volume since 1995. EUROSTAT data indicates that EU-15 procurement expenditure as a percentage of GDP more than doubled in the period 1995-2006, increasing from 1.41 percent to 3.15 percent. Although this figure is already non-negligible, it can be expected that the magnitude of public demand expenditure is significantly higher in reality. EUROSTAT data reflects government procurement subject to the obligations established by EU directives, which is only a fraction of total public procurement markets (EU, 2004).[4] In general, estimates of the importance of public procurement for OECD economies vary depending on the methodology used for their calculation and on the definition of procurement employed (Audet, 2002). Second, regarding the role of government demand, one line of argumentation rests on the importance of the inter-industrial composition of public purchases. Government demand is likely to affect decision making within supplier firms, particularly with respect to investment in R&D since in a number of markets the public sector is the first user of innovations, patents, and products (Dalpé et al., 1992). In general, public demand frequently constitutes a large fraction of total demand in sectors of significant technological content, such as environmental protection, transportation, and medical equipment (Edquist & Hommen, 2000; Edler & Georghiou, 2007). In some industries, however, government purchases comprise a relatively small portion of overall demand (Marron, 2003). The above figures on the quantitative relevance of public procurement become even more impressive when it is taken into account that government purchases are often concentrated in few specific markets. Third, an-

[2] Development pairs are one form of an innovation network, which is usually regarded to be conducive to user-producer interaction and interactive learning (Lundvall, 1988; Powell & Grodal, 2004).

[3] The term "procurement" refers "*to the function of purchasing goods or services from an outside body*" (Arrowsmith 2005, p. 1). If the state is the awarding authority for a procurement contract, "public procurement" takes place. "Public procurement" and "public demand" are treated as being synonymous throughout this paper.

[4] The European Commission emanates from an overall volume of public procurement in Europe of around 363 billion euro or 16.2 percent of European GDP (EU, 2008).

other viewpoint comes into play that addresses the interrelation between the demand and supply side. In various cases, the government acted as a demanding customer that was both willing and able to interact with supplier firms. It did not restrict its activity to the passive role of providing market incentives but actively showed inventors a beneficial path to pursue in their research efforts.

In essence, various studies that argue empirically suggest a significant impact of public demand spending on private innovation activities. However, of the several factors that drive this result we focus on government procurement as being part of the economic conditions affecting the profitability of innovations. Specifically, we develop an innovation-driven Schumpeterian growth model that allows us to investigate how the inter-industrial composition of public demand influences the pace of technological change as well as of economic growth.

The remainder of the paper is organized as follows. Section 2 provides a brief review of the literature on public expenditure and (endogenous) growth. We highlight a number of shortcomings of existing models that motivated this paper. Section 3 introduces the basic model. Section 4 characterizes the balanced-growth equilibrium and the steady-state properties of the model. Section 5 examines the transitional dynamics and analyzes the dynamic response of the economy to a permanent change in the technological composition of government purchases. Section 6 explores the welfare implications of the model, and Section 7 offers some concluding comments.

## 2. A Brief Overview of Public Spending in the Literature on (Endogenous) Growth

The role of public spending for aggregate economic activity is a subject with a long history in the growth literature. Within the neoclassical growth model, one traditional debate centered around the question whether fiscal policy can be an effective tool in mitigating the size of business cycle fluctuations (Turnovsky, 1977; Baxter & King, 1993). The macroeconomic implications of war expenditure have also been analyzed extensively (Braun & McGrattan, 1993). These approaches were amended by models that study the link between government spending and economic growth, where public spending is incorporated as a separate argument in the production function to reflect its impact on the productive capacity of the economy (Arrow & Kurz, 1970; Barro, 1990). While Arrow & Kurz (1970) modeled long-run growth as being solely driven by exogenous factors, Barro's (1990) seminal contribution implied responsiveness of equilibrium growth to government purchasing behaviour.[5] Later extensions of these models of "productive public spending" explored the role of the composition of government spending, basically distinguishing between public consumption and public investment (Ahmed & Yoo, 1995; Chen, 2006) or considering specific functional categories of public expenditure such as infrastructure (Fisher & Turnovsky, 1998) or education (Monteiro & Turnovsky, 2008). The emphasis on supply-side effects that is prevalent in this stream of literature, however, turns out to be an irritating characteristic when demand-side effects of public spending are in the center of attention. Moreover, models in the neoclassical tradition do not usually focus explicitly on the sources of technological change.

New inspiring insights into the mechanisms of the growth process have been provided by innovation-driven growth theory, a stratum of literature originated by Romer (1990), Grossman & Helpman (1991a and 1991b), and Aghion & Howitt (1992). Within this framework, technological change requires an intentional investment of resources by profit-seeking firms. It is further argued that "technology" is non-rival in nature so that innovation/invention costs must be incurred only once, and any newly discovered idea can be used for the production of infinite units. Non-rivalry of knowledge translates any uniform increase in the size of the market into an unambiguously larger profit flow for each successful innovator (Young, 1998). Demand conditions thus affect the incentive for firms to devote resources to R&D and (potentially) bear the risk of failure. It is all the more astonishing that policy exercises in this class of models, while encompassing the analysis of R&D subsidies (Segerstrom, 1998; Li, 2003), of public incentives for human capital accumulation (Arnold, 1998 and 2002; Bucci, 2003) as well as of industrial policy (Dinopoulos & Segerstrom, 1999; Giordani & Zamparelli, 2008) have so far largely neglected the view of public expenditure as a demand tool.

To the best of our knowledge, there are only two studies that attempted to fill this gap: Censolo & Colombo (2008) and Cozzi & Impullitti (2008). The former extends the original Grossman & Helpman (1991b, chap. 3) model of expanding product variety. Censolo and Colombo add the government as a third sector that demands a homogeneous consumption good and a differentiated commodity in order to produce utility-enhancing public services. They thoroughly investigate the long-run effects of changes in government demand composition on rates of innovative change and of economic growth. The transitional adjustment path is also analyzed. Their model, however, is insufficient to capture some of the essential features of innovation-based growth, leading to three points of criticism. First, innovations do not solely extend the scope of available products (horizontal innovation) but also improve the

[5] The title of the present paper, a variation of Barro's (1990) "Government Spending in a Simple Model of Endogenous Growth", is meant to acknowledge the pioneering role of Barro's work.

quality of existing ones (vertical innovation). Scherer (1980) cites survey evidence for the importance of vertical compared to horizontal innovations. According to him, firms devote 59 percent of their research outlays to product improvement, 28 percent to developing new products and 13 percent to enhance process technology. Hence quality-improved products replace inferior ones that perform similar functions. This immediately leads to the second point. A model of vertical innovation has the property that new technologies make the previous ones obsolete. By quality improvements, firms using inferior technologies will be squeezed out of the market. By contrast, in Censolo & Colombo (2008) every producer of an intermediate good competes on equal footing with all existing products as society uses old goods along with new ones. An obsolescence of products that might occur when an innovation takes hold, i.e., the Schumpeterian "creative destruction" effect, is not incorporated in their model. The third and last point deals with the uncertainty of research efforts. Censolo & Colombo (2008) treat R&D like other production activities – primary inputs are automatically converted into output. However, a number of scholars have stressed the inherent uncertainties associated with R&D. The stochastic nature of the innovation process is incorporated in most of the present models of vertical innovation.[6]

The above-mentioned points of criticism are resolved in Cozzi & Impullitti (2008). The authors develop a Schumpeterian growth model in which growth originates from firms engaging in R&D in order to improve the quality of existing products in a continuum of monopolistic competitive industries. The innovation process is stochastic in nature. Heterogeneity is introduced into the size of the quality improvements that occur when an innovation is found in an industry. Cozzi and Impullitti then elucidate how the distribution of public demand expenditure over the variety of heterogeneous industries affects the rate of technological change and the endogenous supply of skills. Although the authors elegantly set out the main features of Schumpeterian growth in a heterogeneous industry setting, their model suffers from a severe limitation. Neither is it possible to explicitly solve the model for the long-run equilibrium, nor can the transitional dynamics be illustrated analytically. Moreover, Cozzi and Impullitti omit a discussion of welfare implications of the inter-industrial composition of public demand.

Our paper is primarily inspired by Cozzi & Impullitti (2008). We maintain the basic ingredient of Cozzi and Impullitti's model, namely that the economy is populated by a continuum of heterogeneous industries. This allows us to account for the observable fact that government demand is not uniformly distributed across industries. In one crucial aspect we

[6] See Dinopoulos & Şener (2007) for a recent overview.

deviate from Cozzi and Impulliti, namely by imposing a specific assumption on how industries differ in terms of their innovation capacity. In this we draw upon the recent contribution by Minniti et al. (2008) who model the size of innovation as being Pareto distributed. This extension allows a more rigorous analytical treatment of the model compared to Cozzi and Impullitti. We can explicitly solve for the balanced-growth path (hereafter BGP) of the economy and for the transitional dynamics that lead to the BGP. In addition, we are able to make a normative statement on the optimality of the BGP in the decentralized economy and show how the government can ensure social optimum by adjusting the allocation of its demand expenditure across industries.

## 3. The Model

### *3.1 Description of the Model Economy*

The economy in the model is closed and consists of two sectors – a final goods (or manufacturing) sector and a research sector where firms seek for innovations. To avoid unnecessary complications and highlight the basic forces at work, labor is the only input factor used in both sectors and is not further differentiated. Labor supply decisions are treated as being exogenous.

As is standard in the Schumpeterian growth literature, there is a continuum of industries in the unit interval indexed by $\omega \in [0,1]$ in the economy under consideration. Each industry produces exactly one consumption good (or product line). The outputs of the various industries substitute only imperfectly for each other. As expansion of variety is not the focus of our model, the set of commodities is fixed in the progress of time. Vertical innovations improve the quality of the respective consumption good. Let the discrete variable $j \in \{0,1,2,...\}$ denote the quality level. Each innovation in industry $\omega$ leads to a jump in quality of the product in question from $j$ to $j+1$. The quality increments, denoted by $\lambda$, happen independently of each other – an improvement in one industry does not induce an improvement in any other industry. This idea can be illustrated by the metaphor of a quality ladder.

Following the specification introduced by Grossman & Helpman (1991a and 1991b), in a given point in time a good $\omega$ possesses a quality level of $\lambda^j$ if $j$ quality jumps of size $\lambda$ have happened so far. At time $t=0$, the state-of-the-art quality product in each industry is $j=0$; that is, one firm in each industry knows how to produce a $j=0$ quality product, and no firm knows how to produce any higher quality product. Further, in $t=0$ the quality of each good equals unity, i.e., $\lambda^0 = 1$. Over time, state-of-the art quality follows a progression up a quality ladder. Each step up the ladder, however, requires intentional R&D efforts by firms.

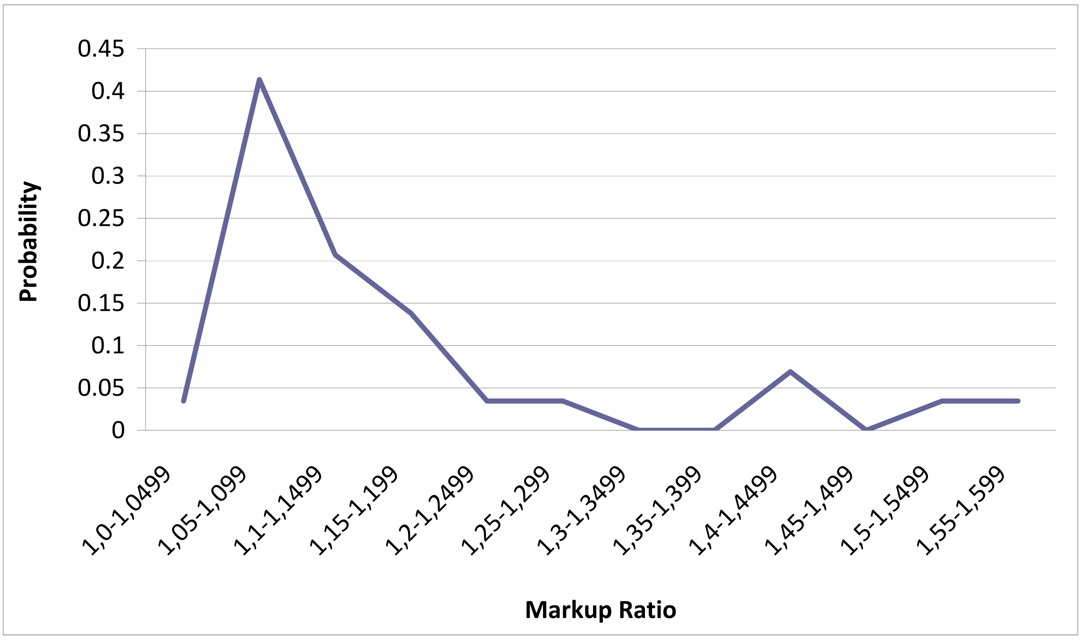
In previous Schumpeterian growth models (Grossman & Helpman, 1991a and 1991b; Aghion & Howitt, 1992; Segerstrom, 1998; Li, 2001 and 2003), different industries were usually treated as being structurally identical so that the economy could be regarded as if it consisted of only a single industry. Therefore, these approaches are only suitable when growth is analyzed on the macro level but cannot account for industry-specific effects of demand pull and technology push in the multitude of existing industries. The industrial organization literature presents overwhelming empirical evidence that the innovative behavior of firms varies across industries (Stadler, 1999). Geroski (1998) finds a considerable amount of heterogeneity on the firm level that does not disappear over time.

In order to overcome the symmetric treatment of industries, we assume the size of the quality jump after a successful innovation as being uncertain. In line with the recent work by Minniti et al. (2008), the realization of each R&D race is drawn independently from a Pareto distribution. Modeling uncertainty associated with the size of the quality jump to obey a Pareto distribution is supported by the patent literature. Scherer (1965) analyzes patent activities of the 500 largest firms in the U.S. and finds that the distribution of U.S. patent values (measured by profit returns) is highly skewed toward the low-value side, and heavy tailed to the high-value side. This evidence fits to the generic properties of a Pareto distribution quite well. Successive empirical work on patent values and citations often found the Pareto distribution as being accurate in describing the data. Harhoff et al. (2005), for instance, ask patent holders in Germany and in the U.S. to estimate the value of their inventions. The distribution of values yielded by this survey is strikingly close to the Pareto distribution for a wide range of patent values.[7]

The rationale to utilize the Pareto distribution for capturing heterogeneity on the industry level in a Schumpeterian growth model lies in the fact that this stream of models rests on the assumption that for each successful innovation a patent is granted. Moreover, the size of the quality jump associated with a successfully innovating firm affects its profitability for the innovator. For these reasons, empirical results indicating that patent values often follow a Pareto distribution are well suited to be applied to our model economy.

[7] Within a slightly different methodological framework, Kortum (1997) and Jones (2005) model the realization of new ideas (interpreted as productivity levels and production techniques, respectively) as being Pareto distributed.

Even more support for Pareto distributed innovation size can be derived from the empirical literature on markups of product prices over marginal cost. Schumpeterian growth models share the feature that quality jumps are understood as an indicator of monopoly power in an industry. More precisely, the quality jump is usually modeled as being equal to the markup of goods prices over marginal cost that a quality leader can charge. Oliveira Martins et al. (1996) estimate markups for 2-digit U.S. manufacturing industries for the period 1970-92. In Figure 1, we plot a stylized probability density function of their data.




Source: Own illustration, using data by Oliveira Martins et al. (1996, p. 30)

**Figure 1: Stylized probability density function of U.S. markup ratios**

It is apparent that the distribution of markups is right-skewed; the mass of the distribution is below the average economy-wide markup (equal to $1.17$). In fact, only one third of the industries turn out to have a markup above the mean.

### *3.2 Consumers*

Each household is modeled as a dynastic family whose size grows over time at an exogenous rate $n$ which also equals the rate of population growth. Each household member inelastically supplies labor services in exchange for wages. We normalize the total number of individuals at time $t = 0$ to unity, by appropriate choice of unit. Thus, the population of workers at time $t$ equals $L(t) = e^{nt}$. Intertemporal preferences of the representative household are given by:[8]

$$U = \int_0^{\infty} e^{nt} e^{-\rho t} \log u(t) dt, \quad (1)$$

where $\rho > 0$ denotes the rate of time preference, and $\log u(t)$ represents the flow of utility per household member at time $t$. Notice that the assumption $(\rho - n) > 0$ is needed to ensure convergence of the utility integral. Any individual's instantaneous utility is represented by:

$$\log u(t) = \int_0^1 \log \left[ \sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega,t) d(j,\omega,t) \right] d\omega. \quad (2)$$

Equation (2) describes Cobb-Douglas consumer preferences, where $d(j,\omega,t)$ is the consumption of quality $j$ in product line $\omega$ at time $t$. The utility derived by an individual from consumption is therefore determined by the quality-weighted amount of consumption, integrated (because we have a continuum of industries) over all industries $\omega \in [0,1]$. This formulation of instantaneous utility implies that a consumer enjoys one unit of good $\omega$ that was improved $j$ times as much as she would enjoy $\lambda(\omega,t)^j$ units of the good if it had never been improved, with $\lambda(\omega,t) > 1$.

The static utility function (2) contains the sum $\sum_{j=0}^{j^{\max}(\omega,t)} \lambda(\omega,t)^j d(j,\omega,t)$. It follows that, hypothetically, all existing quality levels ($j = 0,1,2..., j^{\max}$) of each product line could be consumed at a given point in time. However, we show later that in each product line only the good with the lowest quality-adjusted price will face demand.

The representative household maximizes lifetime utility (1) subject to the following intertemporal budget constraint:

$$B(0) + \int_0^{\infty} w(s) e^{-\int_0^t [r(\tau)-n]d\tau} ds - \int_0^{\infty} e^{-\int_0^t [r(\tau)-n]d\tau} T(s) ds = \int_0^{\infty} e^{-\int_0^t [r(\tau)-n]d\tau} c(s) ds, \quad (3)$$

where $B(0)$ is the ex ante endowment of asset holdings of the representative household, $w(t)$ is the wage rate earned by each individual, $T(t)$ is a per capita lump-sum tax and $c(t)$ is the flow of individual consumer expenditure. Consumer spending is given by:

$$c(t) = \int_0^1 \left[ \sum_{j=0}^{j^{\max}(\omega,t)} p(j,\omega,t) d(j,\omega,t) \right] d\omega,$$

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[8] Our infinite-horizon representative agent framework can be justified by referring to Barro (1974). However, Kirman (1992) points out some critical aggregation issues involved here.

where $p(j,\omega,t)$ is the price of product $\omega$ with quality $j$ at time $t$.

The household maximization problem is solved in three stages: first, the allocation of expenditure at any given point in time for each product across available quality levels; second, the allocation of expenditure on the different product lines $\omega$; and, third, the time path of expenditure such that intertemporal utility reaches a maximum.

It can be easily shown that an individual is indifferent between quality vintage $j$ and $j-1$ if $p(j,\omega)/p(j-1,\omega)=\lambda(\omega)$. If the quality leader in industry $\omega$ charges a price marginally below $\lambda(\omega)$, the next best quality faces no demand. The elasticity of substitution between goods of different quality vintages within the same industry is infinite. To break ties, we make the assumption that if a household member is indifferent between two quality vintages, she will buy the higher quality product.

From the formulation of the consumption index in (2) it follows that goods of different vintages in each industry are perceived as perfect substitutes, once the quality adjustment is made. As already noted, products of different industries enter utility symmetrically, and the elasticity of substitution between every pair of industries equals minus one. This yields the static demand functions:

$$d(j,\omega,t)=\begin{cases}\dfrac{c(t)}{p(j,\omega,t)} & j=j^{max}(\omega,t)\\ 0 & \text{otherwise}\end{cases} \tag{4}$$

The dynamic optimization problem, i.e., the allocation of lifetime expenditure over time, consists of maximizing discounted utility (1) subject to (2), (3), and (4). The solution of the optimal control problem obeys the Keynes-Ramsey rule:

$$\frac{\dot{c}(t)}{c(t)}=r(t)-\rho. \tag{5}$$

This intertemporal optimization condition implies that a constant consumption expenditure path is optimal when the market interest rate is equal to $\rho$. A rate above $\rho$ induces consumers to increase savings “today” and spend more “tomorrow,” resulting in a rise of consumption over time.

Since preferences are homothetic, aggregate demand at time $t$ in industry $\omega$, denoted by $D(j,\omega,t)$, is given by $D(j,\omega,t)=d(j,\omega,t)L(t)$.

### *3.3 Product Markets*

The constant returns to scale production function $Y = L_Y$ holds for *any* quality level in industry $\omega$. The firms within each industry compete over prices. Only a single firm possesses the technology to produce the highest quality product, while its product has a quality advantage of $\lambda$ over the next best quality in the industry. The optimal strategy for the quality leader is to set the *limit price* $p_L(\omega,t)$, preventing any other firm in the industry from offering its product without losses (Grossman & Helpman, 1991a and 1991b; Segerstrom, 1998).[9] The quality leader will set a quality-adjusted price below the unit costs of its nearest competitor, while that competitor will come up with a price equal to its own marginal cost. Hence the highest price the quality leader can set to capture the entire industry market is its lead over the next best quality follower, implying $p_L(\omega,t) = \lambda(\omega,t)w = \lambda(\omega,t)$.[10] There is no incentive for the quality leader to set a price above the limit price because if she did, she would lose all of its customers.

We now introduce government demand (i.e., government procurement spending) into the model. Per capita public demand spending in industry $\omega$ at time $t$ is denoted by $G(\omega,t) \geq 0$, for all $\omega \in [0,1]$ and $t \geq 0$. Because we wish to isolate wealth effects from the distortionary effects of taxation, we assume that the government uses lump-sum tax revenues to finance its procurement expenditure. We further assume that the government balances its budget at any time. To avoid unnecessary complications, we abstract from modeling any effects of public demand expenditure on individual utility or on marginal productivity of private input factors in manufacturing or research.

Since static consumer demand (4) is unit elastic and the quality leader charges a price of $\lambda(\omega,t)$ both for private consumers and the government, the quantity of a state-of-the-art quality product in each industry $\omega$ sold to private consumers equals $L(t)c(t)/\lambda(\omega,t)$, while public demand for products at the quality frontier in each industry $\omega$ is equal to $L(t)G(\omega,t)/\lambda(\omega,t)$. Given that marginal production cost are unity (recall that labor is the numeraire), the quality leader in each industry $\omega$ earns a profit flow of:

[9] Note that Li (2001, 2003) as well as Minniti et al. (2008) develop quality-ladder models in which the producer of the state-of-the-art quality can charge the unconstrained monopoly price. Whether or not she can do that and still leaves no positive profit to producers of previous vintages depends on the size of the quality jump and the degree of substitutability of different vintages. Aghion & Howitt (1998, chap. 2) label as "drastic innovation" the case when the monopoly price can be set and the producers of the next best qualities are still squeezed out of the market as well as the case when the limit price is set as "non-drastic innovation".

[10] Limit pricing obviously leads to tension between static and dynamic optimality (static optimality requires marginal cost pricing). But without positive profits innovation would cease, and so would growth.

$$\pi(\omega,t)=\left\{\left[\lambda(\omega,t)-1\right]\frac{c(t)L(t)}{\lambda(\omega,t)}+\left[\lambda(\omega,t)-1\right]\frac{L(t)G(\omega,t)}{\lambda(\omega,t)}\right\}. \tag{6}$$

In equation (6), $\left[\lambda(\omega,t)-1\right]$ is to be interpreted as the markup factor over marginal cost. Thus, the parameter $\lambda(\omega,t)$ describes the degree of monopoly power.

### *3.4 R&D Races*

Free entry into each R&D race prevails so that firms may target their research effort at any industry. Labor is the only input used in R&D and can be freely allocated between manufacturing and research. The frictionless nature of the labor market implies that workers earn the same wage in R&D as in manufacturing, $w=1$. Firms conduct R&D activities in industries in which they are not the current quality leader. This excludes the case in which a firm producing the current state-of-the-art quality in industry $\omega$ accumulates patents in that industry.[11] The aim of each firm's R&D efforts is a superior quality and to monopolize the market by achieving a patent (with infinite patent length). All firms have access to the same R&D technology. In industry $\omega$ at time *t*, a firm engaged in R&D that employs $l_i(\omega,t)$ units of labor faces a Poisson arrival rate of innovation, $I_i(\omega,t)$, equal to:

$$I_i(\omega,t)=\frac{Al_i(\omega,t)}{X(\omega,t)}, \tag{7}$$

where $A>0$ is a given technology parameter, and $X(\omega,t)>0$ is a function that captures the difficulty of conducting R&D, taken as given by each R&D firm. The "innovation production function" as specified in (7) takes into account the stochastic in the R&D process. For firm *i* in industry $\omega$, lagging behind the state-of-the-art quality at time *t*, $I_i(\omega,t)dt$ indicates the probability to win the R&D race and become the next quality leader within the time interval $[t,t+dt]$. In (7), the time interval $dt$ approaches zero. Hence $I_i(\omega,t)$ is to be interpreted as the instantaneous probability of firm *i* being successful in finding the next higher quality product per unit of time.

[11] The effect that monopolists may systematically have less incentive to innovate than potential rivals, eventually ceding technological leadership, was first described by Arrow (1962) and is a common feature in the literature of Industrial Organization (Fudenberg et al., 1983; Fudenberg & Tirole, 1985) as well as of R&D-driven endogenous growth models. The occurrence of this effect can be explained as follows. A two-step quality advantage of the monopolist comes along with smaller profits than the gain of a one-step quality improvement in another industry. Therefore, the monopolist will direct all R&D resources to other industries to become the market leader there. As it is the dominant strategy for quality leaders not to invest in further improving their technology, the monopoly will only remain as long as no better technology is found in the R&D sector.

We can conveniently aggregate across firms to obtain the industry-wide arrival rate of innovation by assuming that the probability of winning an R&D race is independent across firms, across industries, and over time. It follows that (7) holds for each firm at any time irrespective of the workforce employed intra- or inter-industrially. The industry-wide arrival rate of innovation reads:

$$I\left(\omega,t\right)=\frac{AL_I\left(\omega,t\right)}{X\left(\omega,t\right)} \tag{8}$$

where $L_I(\omega,t)=\sum_i l_{I,i}(\omega,t)$ denotes the industry-wide R&D labor employment, and $I(\omega,t)=\sum_i I_i(\omega,t)$ is the cumulated arrival rate of innovation of all firms in industry $\omega$ at time $t$.

The empirically uncomfortable "scale effect" property[12] of early R&D-driven endogenous growth models (Romer, 1990; Grossman & Helpman, 1991a and 1991b; Aghion & Howitt, 1992) is removed by assuming that R&D difficulty grows in each industry at a rate proportional to the arrival of innovation (Segerstrom, 1998):

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)}=\mu I(\omega,t), \tag{9}$$

where $\mu>0$ is exogenously given and $X\left(\omega,0\right)=X_0$ for all $\omega$. An ever increasing R&D difficulty, as formalized in equation (9), reflects the idea of rational behavior of R&D firms (Li, 2003). During each R&D race, firms may choose between an infinite array of research projects with varying degree of R&D difficulty, $X(\omega,t)$. While the most promising research projects are tried first, these may fail, making firms switch to less promising projects with a higher degree of R&D difficulty. With this in mind, innovating becomes more difficult over time. and technological opportunities vanish because of a series of research failures.

This idea of "fishing out" of innovations, which causes a fall in relative productivity of R&D inputs, is consistent with empirical observations. Because Schumpeterian growth models are characterized by the assumption that a steady part of innovations (in fact 100 percent) is patented, patent statistics can be a natural judge of these models. In the second half of the 20th century, patents granted in the U.S. to residents showed a certain degree of stability, fluctuating around 40,000-50,000 per year, while the number of researchers increased greatly. This implies a sharp decrease in patent-to-researcher ratio (Kortum, 1997). Segerstrom (2000,

[12] "Scale effect" here means a positive relationship between the long-run growth rate of the economy and the population size. Its underlying intuition has been nicely described by Jones (2004, p. 14): "*A larger population means more Mozarts and Newtons, and more Wright brothers, Sam Waltons, and William Shockleys.*" Empirical studies, however, typically reject such population size level effects (see especially Jones, 1995a).

Table 1) finds a comparable development of the patent-to-researcher ratio for a number of highly developed countries such as the U.S., France, Japan, Sweden, and Great Britain.[13]

Once a firm becomes successful in finding an innovation, the size of that innovation is drawn from a Pareto distribution with shape parameter $1/\kappa$ and a scale parameter equal to one.[14] The probability density function of a Pareto distribution with these properties reads:

$$g(\lambda) = \frac{1}{\kappa}\lambda^{-\frac{1+\kappa}{\kappa}}, \ \ \lambda \in [1,\infty), \tag{10}$$

where $\kappa \in (0,1)$ is a parameter that measures the degree of dispersion or heterogeneity of the Pareto distribution. The higher $\kappa$, the fatter the upper tail of the distribution of quality increments. The median of the Pareto distribution equals $2^{\kappa}$ and the mean is given by $1/(1-\kappa)$. Both median and mean increase in $\kappa$, while the mean is always larger than the median.

For analytical tractability, and to make the analysis of transitional dynamics less tedious, we assume that the initial distribution of $\lambda$ values is given by $g(\lambda)$ at $t = 0$. Then, as the R&D dynamics start off and successfully innovating firms draw new values of $\lambda$, the distribution of $\lambda$ values does not change over time. Notice further that $X(\omega,t) = X_0$ for all $\omega$ means $I(\omega,0) = I_0$ (constant) for all $\omega$. Hence a symmetric equilibrium path must exist along which $I(\omega,t) = I(t)$ and $X(\omega,t) = X(t)$ for all $\omega$. As Grossman & Helpman (1991a and 1991b), Segerstrom (1998), Li (2003) and Minniti et al. (2008), we focus on this symmetric equilibrium.

We are now in the position to derive the optimal amount of labor $l_i(\omega,t)$ that each firm *i* employs in R&D. Let $\upsilon^e(\omega,t)$ be the expected discounted reward for R&D successes in industry $\omega$ at time *t*. By hiring $l_i(\omega,t)$ units of labor in R&D for a time interval $dt$, firm *i* expects to realize $\upsilon^e(\omega,t)$ with probability $I_i(\omega,t)$. The optimization problem to be solved by firm *i* at each point in time can then be written as:

$$\max_{l_i} \upsilon^e(\omega,t)\frac{A l_i(\omega,t)}{X(t)} - l_i(\omega,t).$$

Profit maximization yields the first order condition for an interior solution:

$$\upsilon^e(\omega,t) = \frac{X(\omega,t)}{A}. \tag{11}$$

[13] Benjamin F. Jones (2005) sheds some light on the consequences of an ever rising R&D difficulty for the organization of innovation activity. The author analyzes U.S. patent data and presents evidence on inventors striving for narrower, more specialized expertise and showing a greater reliance on teamwork.
[14] We here adopt the specification used in Minniti et al. (2008).

The RHS of (11) is equivalent to the marginal effective cost of innovating. Equation (11) implies that the expected reward for a successful innovation, $\upsilon^e(\omega,t)$, has to increase when R&D difficulty grows in order to provide sufficient incentives for firms to participate in an R&D race. Only if (11) holds for all $\omega$, can a symmetric equilibrium exist, where the innovation rate $I(t)$ is positive, finite, and the same across all industries. In the next section, we determine the expected value of the uncertain profit stream of finding a product of superior quality, $\upsilon^e(\omega,t)$.

### *3.5 Stock Market and Specification of Public Demand*

Firms that participate in an R&D race issue securities on a perfect financial market.[15] R&D-performing firms are thus financed by consumers' savings channeled to them through the stock market. Thus, consumers are allowed to choose the R&D sectors where to employ their savings by buying securities. The claims pay nothing in the event that research efforts fail, but entitle the claimants to the income stream associated with quality (and industry) leadership if the efforts succeed.[16] In addition to the (risky) investment in R&D-performing firms, consumers can also buy a risk-free bond with the rate of return $r(t)$. The interest rate $r(t)$ adjusts to clear the capital market at each moment in time. The absence of profitable arbitrage opportunities makes the expected rate of return on securities issued by R&D firms equal to the risk-free rate of return $r(t)$. The no arbitrage condition for the stock market is then given by (Blanchard & Fischer, 1989, p. 215):

$$\frac{\pi^e(\omega,t)}{\upsilon^e(\omega,t)}dt+\frac{\dot{\upsilon}^e(\omega,t)}{\upsilon^e(\omega,t)}(1-I(t)dt)dt-I(t)dt=r(t)dt, \tag{12}$$

where $\pi^e(\omega,t)$ denotes the expected profits earned by a successful innovator.

The first term on the LHS of (12) describes the accrued dividend paid to the consumers during time interval $dt$. The second term shows possible capital gains of a firm's share. However, the value of the quality leader will only appreciate if the respective quality leader is able to maintain her position – this happens with a probability $1-I(t)dt$. The third term represents the capital loss shareholders will suffer in case a better quality is found during the time interval $dt$. Because a producer of the latest quality vintage who loses her leadership due

[15] In other words, all moral hazard and adverse selection problems which – as empirical observations imply – exist mainly for young firms when they attempt to raise capital funds for risky R&D investments, are completely neglected. The integration of imperfect capital markets is a primary aim of newer models of endogenous growth (e.g., King & Levine, 1993; Aghion & Howitt, 2005).

[16] As there is no physical capital in the economy, shares of R&D-performing firms are the only existing commercial paper.

to a new innovation is immediately squeezed out of the market – causing her stock value to shrink to zero instantly – shareholders lose everything in this case. The third term is thus equal to the probability of the arrival of an innovation per unit of time, $I(t)dt$. The RHS of (12) describes the alternative investment in a safe bond.

Dividing (12) by $dt$ and calculating the limit $dt \to 0$ yields:

$$\frac{\pi^e(\omega,t)}{\upsilon^e(\omega,t)} + \frac{\dot{\upsilon}^e(\omega,t)}{\upsilon^e(\omega,t)} = r(t) + I(t). \tag{13}$$

In the stock market equilibrium, the expected dividend rate plus the expected rate of capital gains is equal to the rate of return of the risk-free security plus a risk premium. An expression for $\dot{\upsilon}^e(\omega,t)/\upsilon^e(\omega,t)$ can be obtained by using (11), and the dividend rate becomes:

$$\frac{\pi^e(\omega,t)}{\upsilon^e(\omega,t)} = r(t) + I(t) - \frac{\dot{X}(t)}{X(t)}.$$

Before we can derive an expression for the expected profits of a firm winning an R&D race, $\pi^e(\omega,t)$, we have to be more concrete on how the government allocates its demand expenditure among the various industries in our model economy.

Once a firm wins an R&D race in industry $\omega$, the government observes the realized quality jump and then decides how much to purchase from the new quality leader. Specifically, we model public demand spending as a linear combination of two rules. On the one extreme, there is a perfectly symmetric rule in which each industry in the economy faces the same government demand. On the other extreme, there is a rule that allocates public spending in proportion to the quality jump that occurs in a particular industry; the higher the quality jump in industry $\omega$, the more the successful innovator in this industry benefits from public demand. Formalizing this idea yields the following public demand rule:

$$G(\omega,t) = (1-\gamma)\bar{G} + \gamma\left(\bar{G} + \varepsilon\right), \ 0 \le \gamma \le 1 \tag{14}$$

where $\bar{G} \equiv \int_0^1 G(\omega)d\omega$, $\varepsilon \equiv \begin{cases} -\varepsilon_1 & \text{for } \lambda(\omega,t) < \dfrac{1}{1-\kappa} \\ \varepsilon_2 & \text{for } \lambda(\omega,t) \ge \dfrac{1}{1-\kappa} \end{cases}$ as well as $0 < \varepsilon_1 < \bar{G}$ and $0 < \varepsilon_2 < \bar{G}$.

The demand policy rule (14) necessitates some further remarks. In (14), $\bar{G}$ denotes the average per capita public procurement, i.e., the value of public demand spending a quality leader in each industry $\omega$ would receive if the government spread its expenditure $G(\omega)$ evenly across all industries. However, treating all industries equally is not the only option for the government to spend its financial resources in our model. Any increase in the fiscal policy parameter $\gamma$ will lead to a public demand policy that more heavily promotes industries with above-average quality jumps.[17] This second part of (14) can be nicely interpreted if, just for illustrative purposes, we assume $\gamma = 1$. This would mean that if the quality increment coming along with an innovation in industry $\omega$ is smaller than the average economy-wide quality increment, public purchases in this industry are lower than they would have been if the government had distributed its expenditure symmetrically over all industries. On the other hand, if an innovator in industry $\omega$ drew a value of $\lambda$ above the average quality jump, she would benefit more from public spending than under the perfectly symmetric demand policy rule.

It is worth stressing that (14) imposes a "bang-bang solution" on public demand expenditure.[18] For each $\gamma > 0$, once an industry experiences a quality jump above (below) economy-wide average, the government abruptly spends more (less) in this industry, irrespective of how far beyond the average this industry finds itself after the quality jump. It is easy to show that the strictly positive values $\varepsilon_1$ and $\varepsilon_2$, which indicate how much government spending in "low-jump" respectively "high-jump" industries deviates from average spending, cannot be chosen independently.[19]

As stated above, the distribution of $\lambda$ values does not change over time. Thus, although there is uncertainty at the industry level concerning the size of the quality jump that occurs after an innovation arrives, there is always the same share of industries with quality increments above respectively below average at the macro level. Moreover, we make the simplifying assumption that average per capita public demand expenditure, $\bar{G}$, is fixed in the progress of time.

After solving for the expected profits of a firm winning an R&D race by taking into account (14)[20] we obtain an expression for $\upsilon^e(\omega,t)$:

$$\upsilon^e(\omega,t) = \frac{\frac{\kappa}{1+\kappa} L(t)\left(c(t) + \bar{G} + \gamma\Gamma\right)}{r(t) + I(t) - \frac{\dot{X}(t)}{X(t)}}, \tag{15}$$

where $\Gamma \equiv \varepsilon_2 \left\{ 1 \big/ \left[ 1 - (1-\kappa)^{1/\kappa} \right] - 1 \right\}$ is a strictly positive value. In (15), an innovator's profits are discounted using the risk-free rate of return $r(t)$ and the instantaneous probability that the

[17] This becomes even more obvious when one observes that (14) can be rewritten as $G(\omega,t) = \bar{G} + \gamma\varepsilon$.

[18] Bang-bang solution is a term used in optimal control theory. If the optimal control switches from one extreme to the other at certain times (i.e., is never strictly in between the bounds) then that control is referred to as a bang-bang solution.

[19] See App. A for a formal derivation of the parameter restriction.

[20] The mathematical details are relegated to App. B.

firm loses its leadership position, $I(t)$, adjusted by the increase in R&D difficulty over time, $\dot{X}(t)/X(t)$. Here the effect of "creative destruction" is revealed: the more research (is expected to) occur in an industry, the shorter, *ceteris paribus*, the expected duration of the monopoly profits and the smaller the incentive to innovate.

We now define a new endogenous variable that serves as a measure of relative (i.e., population-adjusted) R&D difficulty: $x(t) \equiv X(t)/L(t)$. We can then express (15) as:

$$\upsilon^e(\omega,t) = \frac{\frac{\kappa}{1+\kappa} L(t)\left(c(t)+\bar{G}+\gamma\Gamma\right)}{r(t)+I(t)-\frac{\dot{x}(t)}{x(t)}-n}. \tag{16}$$

By subtracting the rate of population growth, *n*, in the denominator of (16), we also take into account that aggregate consumer markets, and thus profits earned by a successful innovator, increase over time. Notice that (16) also holds outside the balanced-growth equilibrium derived below.

Combing equations (11) and (16) and recalling that $x(t) \equiv X(t)/L(t)$ gives us the following *R&D equilibrium condition*:

$$\frac{x(t)}{A} = \frac{\frac{\kappa}{1+\kappa}\left(c(t)+\bar{G}+\gamma\Gamma\right)}{r(t)+I(t)-\frac{\dot{x}(t)}{x(t)}-n}. \tag{17}$$

Profit maximization of R&D firms imposes that in the research equilibrium the marginal revenue product of an innovation [RHS of (17)] must equal its marginal cost [LHS of (17)] at each point in time.

### *3.6 Labor Market*

Labor demand in manufacturing equals aggregate demand from both private and public consumers (recall that the production function in manufacturing reads $Y = L_Y$ and that we assume market clearing). Total employment in manufacturing is then given by:

$$L_Y(t) = \int_0^1 \left\{ \frac{c(t)L(t)}{\lambda(\omega,t)} + \frac{G(\omega)L(t)}{\lambda(\omega,t)} \right\} d\omega$$

$$= \int_0^1 L(t)\left\{ c(t)\int_1^\infty \lambda^{-1} g(\lambda) d\lambda + \int_1^\infty G(\omega)\lambda^{-1} g(\lambda) d\lambda \right\} d\omega .$$

Using the Pareto density function given in (10) as well as the public demand rule as specified in (14) and (A.3), total employment necessary to satisfy private and public consumers' demand for the consumption good can be calculated as:

$$L_Y(t) = L(t)\frac{c(t) + \bar{G} - \gamma\kappa\Gamma}{1+\kappa}.$$

An equation for R&D labor can be derived from solving (8) for the R&D input of a firm in industry $\omega$, then aggregating over the continuum of industries $\omega \in [0,1]$, while taking into account that we assume symmetric behavior, where the industry-level innovation rate $I(\omega,t)$ is the same across industries at each point in time. We obtain:

$$L_I(t) = \frac{I(t)X(t)}{A}.$$

Labor-market clearing implies that $L(t) = L_Y(t) + L_I(t)$ is always fulfilled, which, when slightly rewritten, gives the *resource constraint* of the economy:

$$1 = \frac{c(t) + \bar{G} - \gamma\kappa\Gamma}{1+\kappa} + \frac{I(t)x(t)}{A}. \tag{18}$$

The labor market equilibrium in (18) holds for all *t* outside the BGP by assumption that factor markets clear instantaneously. Equation (18) completes the description of the model.

## 4. Balanced-Growth Equilibrium

We now solve for the BGP of the model, where all endogenous variables grow at a constant (although not necessarily at the same) rate and research intensity $I(t)$ is common across industries. According to (8), constant growth rate of R&D difficulty *X* constrains *I* to be constant over time. For that reason, $\dot{x}/x = \dot{c}/c = 0$ is implied by (18). Then, $r(t) = \rho$ prevails by (5), meaning that the market interest rate must be equal to the rate of time preference in the BGP. Equations (9), (17), and (18) represent a system of three equations in three unknowns $x$, $c$, and $I$. Solving this system of equations allows us to uniquely determine balanced-growth equilibrium values for all endogenous variables.

We first derive an expression for the equilibrium research intensity. Taking the logarithm of the RHS of (8) and differentiating with respect to time yields, using (9):

$$I^* = \frac{n}{\mu}. \tag{19}$$

According to equation (19), the balanced-growth value of the research intensity is completely pinned down by the population growth rate, *n*, and the parameter governing the R&D difficulty, $\mu$.

Having determined the equilibrium value of *I*, we are now in the position to solve for the balanced-growth values of $x$ and $c$. Given that $I^* = n/\mu$ and $r = \rho$ in the balanced-growth equilibrium, R&D equilibrium condition (17) can be written as:

$$\frac{x(t)}{A} = \frac{\frac{\kappa}{1+\kappa}\left(c(t) + \bar{G} + \gamma\Gamma\right)}{\rho + n\left(\frac{1}{\mu} - 1\right)}. \tag{20}$$

Equation (20) defines a negative linear relationship between per capita private consumption expenditure, *c*, and relative R&D difficulty, *x*. The resource constraint (18) becomes:

$$1 = \frac{c(t) + \bar{G} - \gamma\kappa\Gamma}{1+\kappa} + \frac{n}{\eta A}x(t), \tag{21}$$

defining a positive linear relationship between per capita private consumption expenditure, *c*, and relative R&D difficulty, *x*. Equation (20) is an upward sloping line in $(c, x)$ space, while (21) is a downward sloping linear function in $(c, x)$ space. Necessary and sufficient condition for both lines to have a unique and positive intersection is given by $\bar{G} < 1$. Solving the system of linear equations in (20) and (21) by applying Cramer's rule uniquely determines the balanced-growth equilibrium values of $x$ and $c$ as:

$$x^* = \frac{A\kappa\mu\left(1+\gamma\Gamma\right)}{n\left(1+\kappa-\mu\right)+\mu\rho}, \tag{22}$$

$$c^* = \frac{\mu\rho\left(1+\kappa+\gamma\kappa\Gamma-\bar{G}\right) - n\left[\bar{G}\left(1+\kappa-\mu\right)+\left(1+\kappa\right)\left(\mu-1\right)+\gamma\kappa\mu\Gamma\right]}{n\left(1+\kappa-\mu\right)+\mu\rho}. \tag{23}$$

Along a BGP, the fraction of the labor force devoted to R&D can be determined as follows. From (8), the R&D labor share reads $nx/(\eta A)$. Substituting into this expression using (22) yields:

$$\left(\frac{L_I}{L}\right)^* = \frac{\kappa n\left(1+\gamma\Gamma\right)}{n\left(1+\kappa-\mu\right)+\mu\rho}. \tag{24}$$

We are now in the position to analyze the long-run effects of a change in the parameters governing public demand expenditure. By differentiating (22) with respect to the appropriate parameter, it is readily established that relative R&D difficulty in balanced-growth equilibrium, $x^*$, is an increasing function of $\gamma$ unaffected by changes of $\bar{G}$. In the same vein, the equilibrium value of average per capita private consumption expenditure, $c^*$, increases in $\gamma$ but falls in $\overline{G}$. The latter simply reflects the fact that as the government increases its (average) demand spending, it takes away resources from the private sector, thereby reducing private consumption one-for-one.[21] The balanced-growth equilibrium share of R&D employment in (24) is an increasing function of $\gamma$ and does not depend on $\overline{G}$. Notice further that the balanced-growth values of $x$, $c$, and $L_I / L$ are all positively affected by an increase in $\kappa$. The larger the expected size of innovations, the higher the values of the endogenous variables along the BGP.

Finally, we calculate the balanced-growth rate of the economy. Because we refrain from capital accumulation the concept of growth in the model relates to growth in each individual's utility. This property is shared by all Schumpeterian growth models in which firms' R&D efforts are directed toward increasing the product quality, and per capita consumption does not change in equilibrium. However, even if the same amount of goods is consumed per person, individual utility in (2) augments if R&D turns out to be successful. To obtain an explicit expression for the utility growth rate, we substitute for consumer demand in (2) by using (4):

$$\log u(t) = \int_0^1 \log\left[\frac{c(t)}{\lambda(\omega,t)}\right] d\omega + \int_0^1 j^{\max}(\omega,t) \log\left[\lambda(\omega,t)\right] d\omega, \tag{25}$$

where $\int_0^1 j^{\max}(\omega,t)d\omega$ is a measure for the number of quality improvements aggregated over all industries $\omega \in [0,1]$. The index $j^{\max}$ increases when firms are successful in innovating and firms engage in innovative R&D in all industries throughout time in any steady-state equilibrium. In each industry $\omega$, the (Poisson distributed) probability of exactly $m$ improvements within a time interval of length $\tau$ can be calculated as:

$$f(m,\tau) = (I\tau)^m e^{-I\tau} / m!,$$

where $f(m,\tau)$ represents the measure of products that are improved exactly $m$ times in an interval of length $\tau$. Following Davidson & Segerstrom (1998, p. 562), $\int_0^1 j^{\max}(\omega,t)d\omega$ then equals $tI$. Taking this and (19) into account, differentiating (25) with respect to time yields the following balanced-growth rate of per capita utility:[22]

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[21] This result of complete crowding-out is a consequence of our assumption that goods purchased by the government neither affect households' utility nor firms' production processes.

[22] Notice that the first integral on the RHS of (25) is constant along the BGP. We further exploit the fact that quality jumps follow a Pareto distribution, so $\int_0^1 \log\left[\lambda(\omega,t)\right] d\omega = \kappa$ [using (10)].

$$\frac{\dot{u}(t)}{u(t)} \equiv g^{*} = \frac{n}{\mu}\kappa . \tag{26}$$

We summarize the balanced-growth properties of our model economy by establishing the following proposition:

**Proposition 1.** *Existence and uniqueness of BGP*

*If $\bar{G} < 1$, a unique balanced-growth equilibrium always exists, where per capita private consumer expenditure, c, relative R&D difficulty, x, innovation rate in each industry, I, share of R&D workers in employment, $L_I / L$, and the rate of per capita utility growth, g, are all constant and given by (19), (22), (23), (24), and (26), respectively.*

According to (26), the long-run rate of utility growth is proportional to the population growth rate. It is particularly noteworthy that (demand) policy changes have only temporary effects on growth. Such being the case, our model belongs to the class of so-called semi-endogenous growth models.[23] Before discussing further the result of policy ineffectiveness in altering long-term economic growth it is worth stressing three implications of (26).

First, utility growth in (26) proceeds smoothly even though every good is improved after an irregular interval of time has elapsed. The economy-wide growth rate does not change in equilibrium because a large number of goods exists so that the proportion of goods improved in a given time interval remains constant in steady state. Naturally, the smooth pace of progress on the macro level just masks the potential turmoil on the meso level.

Second, utility growth increases in *n*. An obvious criticism against this result is that, in fact, countries with a quickly growing population do not usually exhibit the highest economic growth rates. One possible reply to this criticism is that the parameter *n* need not be tied to a specific country. Rather, the parameter could be interpreted as a measure of world population growth since the quality leader in each industry is not restricted to operating on domestic markets (Segerstrom 1998, p. 1299). However, as we have not incorporated foreign trade in our model, we do not want to give undue emphasis to this aspect.

Third, the steady-state growth rate in (26) is an increasing function of $\kappa$, the parameter determining the expected size of innovations. This is a new result in the literature on innovation-driven growth models. According to equation (26), an increase in the average size of innovations stimulates growth. This is due to the fact that the equilibrium R&D intensity $I^{*}$

[23] This stratum of literature originated from the seminal paper by Jones (1995b). See Jones (1999) for an illuminating essay on the differences between semi-endogenous and fully-endogenous growth models.

does not depend on $\kappa$, making the probability of arrival of innovations independent of their expected size.

The model retains the standard neoclassical prediction that the balanced-growth rate of the economy is beyond the influence of policy. All of the values on the RHS of (26) are usually regarded as being outside of the range of government actions. This property of the model is due to our specification of R&D technology. This specification incorporates the idea of diminishing returns to knowledge to eliminate the scale effect of long-run economic growth. In consequence, sustained growth in R&D input is necessary just to maintain a given rate of technological change. However, the stark prediction that follows from that, namely that long-run economic growth is independent of what fraction of society's resources is assigned to R&D, should not be seen as a weakness of the model. Empirical support for the conjecture that long-run growth rates depend on macroeconomic policy is fragile at best. Stockey & Rebelo (1995) find that the U.S. GDP growth rate did not respond significantly after the share of income tax on GDP grew from two to 15 percent in the early 1940s. Jones (1995a, p. 521) formulates concisely: "*[...] nothing in the US experience during the last century appears to have had a permanent effect on growth*." Jones (1997) pointedly illustrates the accuracy of this statement with the following example: if one tried to predict U.S. per capita GDP in 1994 by extrapolating a constant-growth path using data from 1870 to 1929, the predicted value would only understate per capita GDP by 10.6 percent. The empirical fragility of a relationship between long-run growth and economic policies is precisely what is to be expected when referring to the model's results.

Policy ineffectiveness appears to be an inconvenient model feature *prima facie*. But a number of authors highlight the importance of studying transitional dynamics *vis-à-vis* balanced-growth dynamics (e.g., Temple, 2003). Simulation exercises of innovation-based growth models typically suggest slow convergence to the long-run equilibrium. On the basis of his model of expanding product variety, Jones (1995b) finds that it takes approximately 62 years to close half of the gap between the initial per capita income and its balanced-growth value. Steger (2003) calibrates the Schumpeterian growth model by Segerstrom (1998) with the help of U.S. data. He detects a particularly slow rate of convergence to the BGP: it takes almost 40 years to go half the distance to the steady state. Employing a calibrated version of our model indicates a half-life of between 28 and 29 years for a wide range of parameter values.[24] Given these results, it is important to investigate how public policy influences the growth rate along the transition path and thereby the level of the balanced-growth path. We

[24] More details on the simulation model are available upon request.

will show below that a change in $\gamma$, which is the parameter that governs the allocation of government demand spending, can have far-reaching consequences for the transitional adjustment path of the economy. However, before we are able to investigate the effects of changes in the composition of public demand on the economy's transitional dynamics, we have to determine stability properties of the BGP.

## 5. Transitional Dynamics

### *5.1 Local Stability Analysis*

If the value of the relative R&D difficulty, $x$, by pure chance, took an initial value of $x = x^*$, per capita private consumption, $c$, would immediately jump to its balanced-growth value $c = c^*$ due to the optimizing behavior of households and the equilibrium conditions (17) and (18). The model economy would reach its dynamic equilibrium *ab initio* and would stay there as long as no macroeconomic shock occurred. If $x \neq x^*$ initially, it is imperative to investigate the dynamic movement of the endogenous variables in the model.

In particular, we want to show that there exists (exactly one or infinitely many) equilibrium transition path(s) satisfying (5), (9), (17), and (18) for all $t$ that converge to the balanced-growth equilibrium. In other words, we want to determine whether or not the economy would be led back to the steady-state equilibrium if it deviated from it at some point in time. Studying the dynamical system consisting of the "equations of motion" for $x(t)$ and $c(t)$ gives the following proposition.

**Proposition 2.** *Stability of the balanced-growth equilibrium*
*Assuming that all industries $\omega \in [0,1]$ begin with the same relative R&D difficulty $x(0) = X_0$, the BGP $\left(c^*, x^*\right)$ of the dynamical system under study, given by (22) and (23), is either locally saddle-path stable or locally indeterminate, but never instable.*
*(i) Local saddle-path stability arises for $\gamma < \gamma_{crit}$,*

*where* $\gamma_{crit} = \dfrac{n\left[\bar{G}(1+\kappa-\mu)+(1+\kappa)(\mu-1)\right]-\mu\rho(1+\kappa-\mu)}{\kappa\mu(\rho-n)\Gamma}$. *If so, there exists a unique one-dimensional stable manifold in $(c, x)$ space with the property that trajectories beginning on this manifold converge to the steady state $\left(c^*, x^*\right)$, while all other trajectories diverge. There-*

*fore, the equilibrium-growth path (EGP) will be locally unique in the neighborhood of the BGP.*

*(ii) Local indeterminacy arises for* $\gamma > \gamma_{crit}$.

*This means that all the trajectories satisfying the laws of motion of the endogenous variables, (C.1) and (C.2), and begin in the neighborhood of the BGP* $\left(c^*, x^*\right)$*, converge back to the steady state. There is a continuum of equilibrium paths, and the stable manifold has dimension two.*

**Proof**. See Appendix C.

Schumpeterian growth models typically exhibit saddle-path stability (e.g., Segerstrom, 1998; Minniti et al., 2008).[25] The dynamical system approaches its stationary solution (reflecting the BGP), given that some boundary conditions (initial and/or final conditions) are satisfied. Nonetheless, multiple equilibria existing for a given set of fundamentals is an issue that has attracted considerable attention in the last two decades. The possibility of multiple equilibria has the potential for explaining business cycles as well as drastically different growth experiences of countries whose wealth and endowment levels were similar in the past (Benhabib & Farmer, 1999). Especially the latter argument is important from an empirical perspective. In the presence of indeterminacy, countries that are initially identically endowed may develop along widely different paths with respect to consumption, investment and allocation of productive factors. The latter is frequently observed in empirical studies, contrasting sharply with the robustness of theoretical predictions of Schumpeterian (and other) growth models. For this reason, we deem the possibility of indeterminacy that the model provides a particularly intriguing result.

The stability analysis above outlines the region in the parameter space of the demand policy parameter $\gamma$ for which path multiplicity is consistent with the unique equilibrium position of the economy. As noted by Benhabib & Farmer (1994, p. 19), the key to indeterminacy is the presence of some sort of market imperfection, e.g., "*[...] spillover effects of knowledge acquisition.*" More recently, Palivos et al. (2003) as well as Hu et al. (2008) developed a modified version of Barro's (1990) seminal model of productive public spending, emphasizing that government expenditure may play a role in indeterminacy. We find further support for this result even without introducing public purchases as a complementary input in private production.

[25] According to our knowledge, Cozzi (2007) is the only author that has so far accounted for the possibility of indeterminacy in Schumpeterian growth models.

### 5.2 *Phase Space Analysis*

Having analytically derived the local stability properties of the dynamical system (C.1) and (C.2), we want to facilitate the interpretation and understanding of our results by performing a qualitative phase diagram analysis. To illustrate the dynamics associated with the system diagrammatically, we first recall that the $(\dot{x}=0)$ demarcation curve is downward sloping in $(c,x)$ space and the $(\dot{c}=0)$ demarcation curve is upward sloping (downward sloping) for $1+\kappa>\mu$ $(1+\kappa<\mu)$ in $(c,x)$ space. The intersection of the two demarcation curves represents the intertemporal equilibrium of our model economy, $(c^*,x^*)$. At any other point, either $c$ or $x$ (or both) would change over time, in directions designated by the signs of the time derivatives $\dot{c}$ and $\dot{x}$ at that particular point.

We happen to have $\dot{x}>0$ to the left of the $(\dot{x}=0)$ demarcation curve and $\dot{x}<0$ to the right of that curve. These signs are based on the fact that we get from (C.1):

$$\frac{\delta\dot{x}}{\delta c}=-\frac{A\mu}{1+\kappa}<0.$$

As we move continually from west to east in the phase space, $\dot{x}$ steadily decreases corresponding to an increase in $c$ so that the sign of $\dot{x}$ must pass through three stages, in the order $+\|0\|-$.

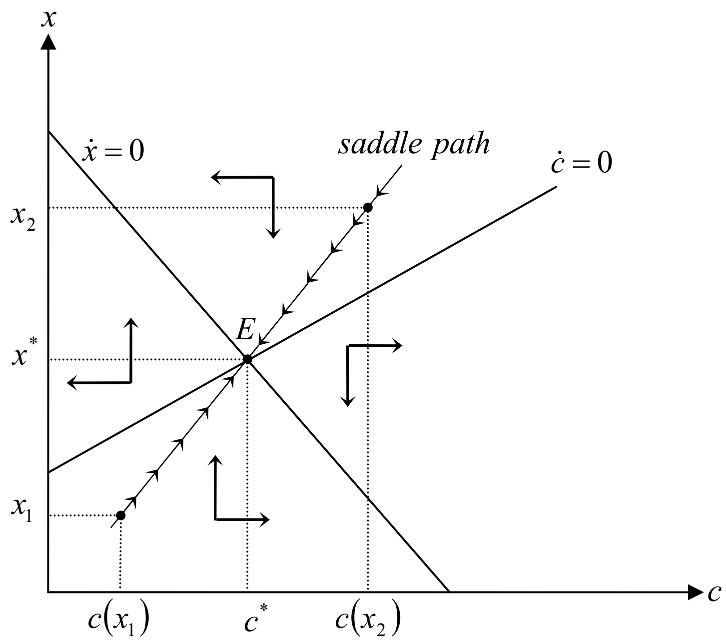
Analogously, we use (C.2) to find:

$$\frac{\delta\dot{c}}{\delta x}=-\frac{Ac\left[c(1+\kappa-\mu)+\bar{G}(1+\kappa-\mu)+(1+\kappa)(\mu-1)+\gamma\kappa\mu\Gamma\right]}{(1+\kappa)x^2}.$$

From the previous section we can infer that $\delta\dot{c}/\delta x<0$ $\left(\delta\dot{c}/\delta x>0\right)$ for $\gamma<\gamma_{crit}$ $(\gamma>\gamma_{crit})$. A negative (positive) derivative $\delta\dot{c}/\delta x$ implies that as we move continually from south to north in the phase space, $\dot{c}$ steadily decreases (increases) corresponding to an increase in $x$ so that the sign of $\dot{c}$ must pass through three stages, in the order $+\|0\|-$ $(-\|0\|+)$.

The complete phase diagram for the case of saddle-path stability is displayed in Figure 2.[26]

[26] Figure 2 is drawn to depict the case where the $(\dot{c}=0)$ demarcation curve is upward sloping and saddle-path stability prevails. In the case of a downward sloping $(\dot{c}=0)$ demarcation curve, Figure 2 must be redrawn with the $(\dot{x}=0)$ isocline being steeper than the $(\dot{c}=0)$ isocline. The reason for the latter is easy to comprehend when we note that both demarcation curves are never congruent. This would be a contradiction to our result of a unique intersection of both curves. However, the $(\dot{c}=0)$ isocline being more steeply downward sloping than the $(\dot{x}=0)$ isocline would require that there exist a value of $\mu$ for which congruency of both curves is al-




Source: Own illustration

**Figure 2: Saddle-path stability of the balanced-growth equilibrium**

The directional arrows in the phase diagram above serve to map out the dynamic movement of the system from any conceivable initial point. There exists a unique trajectory that leads toward the BGP, denoted by $E$. Along these stable branches, per capita private consumption and relative R&D difficulty strive directly and consistently toward their long-run values $c^*$ and $x^*$, given by (22) and (23). For any initial value of the state variable $x$, the control variable $c$ immediately jumps to a value such that the economy is located on the saddle path. This is due to the assumption of rational, forward-looking agents in the economy. If the R&D difficulty initially took a value below $x^*$, e.g., $x_1$, per capita private consumption would instantly jump to $c(x_1)$, lying on the saddle path. Subsequently, both variables increase until $(c^*, x^*)$ is reached. As long as no exogenous shock occurs, $c$ and $x$ remain at their long-run values. Analogously, both variables exhibit a downward movement over time if the economy starts at $[x_2, c(x_2)]$.

Notice that the R&D labor share, $L_I(t)/L(t)$, is nonconstant along the saddle path. From the resource constraint (18), it follows that:

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lowed. This can be excluded. Notice further that the slope of the $(\dot{c}=0)$ isocline being positive or negative does not affect the stability properties of the BGP.

$$\frac{L_I(t)}{L(t)} = 1 - \frac{c(t) + \overline{G} - \gamma\kappa\Gamma}{1+\kappa}$$

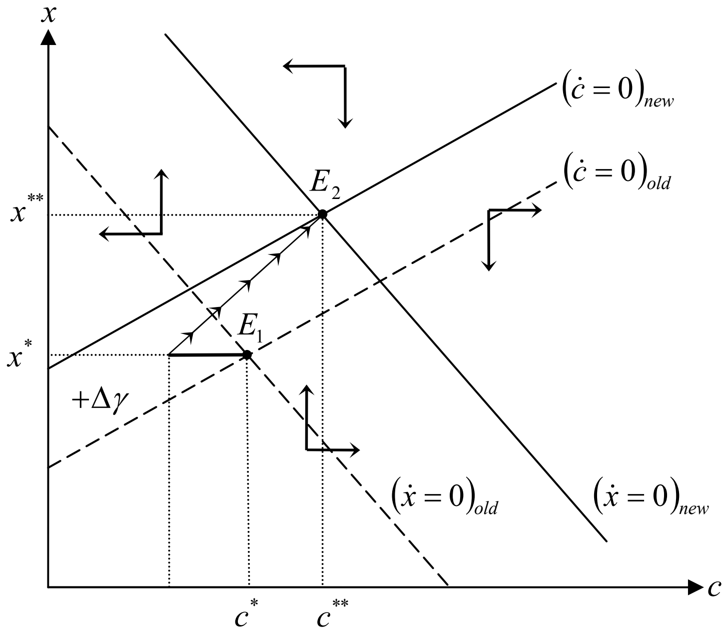
For starting values $\left[x_1, c\left(x_1\right)\right]$ left of the intertemporal equilibrium, $E$, $L_I(t)/L(t)$ decreases because $c(t)$ increases. According to the Keynes-Ramsey rule (5), an increase in $c(t)$ can be attributed to the interest rate being higher than the rate of time preference. A relatively high interest rate, however, implies that expected R&D revenues are heavily discounted [see (16)], which lowers the incentive to invest in respectively employ R&D labor. The same reasoning can be used to explain the upward movement of $L_I(t)/L(t)$ for starting values $\left[x_2, c\left(x_2\right)\right]$ to the right of $E$.

### *5.3 The Dynamic Effects of an Increase in $\gamma$*

The main result of the paper is derived in this section. We here study how a reshuffling of public demand spending in favor of industries with an above-average quality jump, that is, an increase in $\gamma$, affects the steady-state properties of our model as well as the transition toward the new balanced-growth equilibrium. We restrict our attention to the case of saddle-path stability of the BGP.

Assume that, initially, the economy rests in the balanced-growth equilibrium $\left(c^*, x^*\right)$, denoted by $E_1$. Assume further that a permanent and unanticipated increase occurs in $\gamma$. A look at (C.1) and (C.2) reveals that an increase in $\gamma$ causes the $\left(\dot{x}=0\right)$ isocline to shift rightward and the $\left(\dot{c}=0\right)$ isocline to shift upward. This is illustrated in Figure 3.[27]

[27] We again focus on the case of an upward sloping $\left(\dot{c}=0\right)$ demarcation curve. As before, all qualitative results remain the same for the $\left(\dot{c}=0\right)$ demarcation curve being downward sloping.




Source: Own illustration

**Figure 3: Dynamic effects of an increase in $\gamma$**

To identify the short-run effects of a change in $\gamma$, observe first that according to (24) the equilibrium R&D labor depends on $\gamma$ in a positive manner. Interpreted economically, an increase in $\gamma$ raises aggregate expected profits from winning an R&D race instantly [see (B.4)]. Firms respond by investing more heavily in R&D. Thus, a rise in $\gamma$ stimulates R&D, leading immediately to a decrease in per capita private consumption, $c$, because the additional labor force needed in R&D has to be withdrawn from the manufacturing sector.

In the medium run, relative R&D difficulty, $x$, experiences an upward movement caused by the temporarily rising innovation intensity, $I$ (due to the additional workforce employed in R&D). By the same token, per capita private consumption, $c$, increases over time. This can be seen from the equation for the interest rate, which, using (17), reads:

$$r(t)=\frac{A\left[\overline{G}\kappa+c(t)\left(\kappa-\mu\right)+\mu\left(1+\kappa-\overline{G}\right)+\left(1+\mu\right)\gamma\kappa\Gamma\right]}{\left(1+\kappa\right)x(t)}-I(t). \tag{27}$$

The increase in $\gamma$ leads to a temporary upward shift of $r(t)$ above its steady-state level $r^{*}=\rho$. When we argue with the Keynes-Ramsey rule in (5), this implies $\dot{c}(t)/c(t)>0$. The intuition behind the increase in the interest rate relates to the stock market where households can channel their savings. The stock market valuation of a new quality leader increases due to the higher expected monopoly profits earned by a successful innovator. Efficiency on financial markets requires that the expected rate of return from holding a stock of a quality leader be equal to the riskless market interest rate that can be obtained through complete diversification. However, as $x$ appears in the denominator in (27), the increase in $x$ will eventually bring $r$ back to its balanced-growth level. An increase in $x$ implies a fall in profitability of R&D projects. Firms that want to engage in R&D are thus less willing to pay high rates of return for the households' savings.

In the long run, $c$ and $x$ reach their new balanced-growth values $c = c^{**}$ and $x = x^{**}$. In the new dynamic equilibrium, denoted by $E_2$ in Figure 3, both $c$ and $x$ have increased compared to the situation before the policy change occurred. It is noteworthy that the upward movement of per capita private consumption in the new steady state is to be attributed to the assumption that the distribution of quality increments in the economy is Pareto. Due to its right-skewness, the median of the Pareto distribution is always smaller than its mean. Hence the mass of the distribution is on the low-value side. With reference to the model economy, this means that there are more industries with quality jumps below rather than above the mean. It is straightforward to conclude that the higher the value of $\gamma$, the lower is the absolute size of public demand. The tax base needed to finance government demand shrinks accordingly, leaving private households more resources to be spent for consumption.

Observe that for $x(t)$ to rise over time in the transition to the new steady state, (9) implies that the innovation rate in each industry, $I(t)$, temporarily exceeds its balanced-growth value, $I^* \left(= n/\mu\right)$. Thus, a permanent redistribution of public spending that privileges industries with a quality jump higher than the economy-wide average generates a temporarily faster rate of technological change. The reason why an increase in $\gamma$ contributes to a temporary acceleration of technological change may be labeled *R&D incentive* effect. A change in the technological composition of public procurement expenditure, privileging more promising industries (with respect to the quality increments), causes aggregate expected monopoly profits to increase. This happens because higher quality jumps imply higher markups over marginal cost and, thus, higher reward for successful innovation activities [see (B.4) and (16)]. Innovations are stimulated because firms direct relatively more resources to R&D, which induces a higher demand for R&D labor. The consequence of this increase in the relative size of the R&D sector is an acceleration of technological change, and, according to (26), a temporarily faster rate of economic growth. As discussed in Section 4, accelerated growth eventually comes to a halt because innovating becomes progressively more difficult over time.

Proposition 3 recapitulates the dynamic effects of an increase in $\gamma$.

**Proposition 3.** *Dynamic effects of an increase in* $\gamma$

*A permanent increase in* $\gamma$*, the parameter that governs the allocation of public procurement spending,*

*(i) permanently increases per capita private consumer expenditure (c),*

*(ii) permanently increases relative R&D difficulty (x),*

*(iii) permanently increases share of R&D workers in employment (*$L_I / L$*),*

*(iv) temporarily increases the rate of technological change (I),*

*(v) temporarily increases the rate of utility growth (g), and*

*(vi) has no effect on long-run utility growth.*

Whether or not our theoretical results are plausible is difficult to evaluate because empirical research on the impact of public demand spending on innovation and growth has been poorly represented by the literature. To the best of our knowledge, econometric studies that deal with the effects of a change in the technological composition of government procurement on economic variables have yet to be conducted. However, empirical work exploring the potential link between public demand spending in general and technological change provides some preliminary support for our theoretical findings. Guellec and Van Pottelsberghe (2000) use R&D data of 17 OECD countries covering the period 1981-1996 to shed light on the question whether public R&D expenditure is complementary or substitutable to business R&D. The authors conclude that direct government funding of R&D (including R&D procurement) crowds in privately financed R&D. Lichtenberg (1988) analyzes 169 U.S. manufacturing firms in the period 1979-1984. He tests the hypothesis, and estimates the quantity, of private investment in "signalling" R&D in order to attract public procurement contracts. Lichtenberg finds that the size of government demand can have a large positive impact on private R&D expenditure. Saal (2001) investigates whether U.S. military procurement expenditure represented an important determinant of growth in total factor productivity in the period 1973-1986. The author shows that few manufacturing industries that were directly defense dependent had substantially greater productivity growth than other manufacturing industries, although in the aggregate this positive effect of procurement-driven technological change vanishes. A recent study by Aschhoff & Sofka (2008) uses firm survey data for Germany. Their results indicate that, if public procurement contracts are regarded as important for a firm's innovation activities, these contracts have a positive and significant effect on innovation success (measured as sales with new-to-the-market products).

It is clear that neither the level of the BGP nor the magnitude of the growth rate are ultimate goals. From a theoretical perspective, the focus should rather be on welfare as the key variable of public policy. The next section is thus devoted to the derivation of the social optimum where maximum utility is realized.

## 6. Social Optimum

In this section, we explore the model's properties when a benevolent social planner exists who is able to allocate all resources in the economy such that welfare of the representative family is maximized. We compare the decentralized solution found in Section 4 to the optimal solution without public demand. We then solve for the value of the policy parameter, $\gamma$, that generates the optimal solution in our original model framework.

The objective function of the social planner is given by (1), which is optimized subject to the resource constraint of the economy. The social planner's control variable is research intensity, $I$, her state variable is relative R&D difficulty, $x$. To economize space, the optimal control problem is set up and solved in Appendix D. We show that there is a unique balanced-growth solution in which the optimal R&D intensity coincides with that obtained in the decentralized case:

$$I^{opt} = \frac{n}{\mu}. \tag{28}$$

The optimal share of R&D workers in employment is equal to:

$$\left(\frac{L_I}{L}\right)^{opt} = \frac{\kappa n}{\kappa n + \mu\rho}. \tag{29}$$

Comparing (29) to the R&D labor share in the decentralized economy in (24), it is readily established that overinvestment (underinvestment) in R&D compared to the socially optimal R&D level occurs if $n\gamma\kappa\Gamma + n\mu + \mu\gamma\rho\Gamma > n$ $\left(n\gamma\kappa\Gamma + n\mu + \mu\gamma\rho\Gamma < n\right)$. *Ceteris paribus*, the economy is characterized by an insufficiently low amount of resources directed toward R&D, the lower the values of $\kappa$, $\rho$, and $\mu$.

Raising, or lowering, private R&D expenditure closer to the social optimum is possible by the appropriate choice of the public demand policy parameter, $\gamma$. Hence the government can, in principle, increase welfare by adjusting the technological composition of its purchases. It is straightforward to show that the balanced-growth equilibrium in the decentralized economy is optimal for:

$$\gamma^{opt} = \frac{n(1-\mu)}{n\kappa\Gamma + \mu\rho\Gamma}, \tag{30}$$

where the value of $\gamma^{opt}$ lies within the boundaries of zero and one if the condition $n\kappa\Gamma + n\mu + \mu\rho\Gamma \geq n$ holds.[28] A particularly important fact to be noted is that the value of $\gamma^{opt}$ is strictly positive. In other words, it is always useful to choose an asymmetric spending rule that relatively favors industries with above-average quality jumps to restore efficiency.

These welfare results can be understood by looking more closely at the various external effects associated with R&D at work in the model (see also Aghion & Howitt, 1992 and 1998; Segerstrom, 1998).

First, there is a positive externality called *consumer surplus* effect. A successful innovation leads to a higher quality of the respective good, whereas consumers still have to pay the limit price, $\lambda$, for it. This externality continues infinitely into the future, since an innovator who adds the $(j+1)^{\text{th}}$ improvement builds upon the knowledge accumulated over the last $j$ improvements, i.e., all later innovations build upon the already higher utility level resulting from the most recent innovation. Innovators do not consider this positive impact on consumers' utility when deciding about the optimal R&D level [see (11)] so that, *ceteris paribus*, R&D efforts are too low under *laissez-faire*. The social planner takes into account that the benefits of the next innovation will last forever.

Second, each successful innovation generates a *business stealing* effect. A firm undertaking R&D investment does not internalize the loss to the previous monopolist caused by an innovation. This negative externality also affects the profits of all other industry leaders because the income loss suffered by shareholders of the displaced firm means less demand and thus less profit for the remaining monopolists. According to the business stealing effect, too much is, *ceteris paribus*, invested in R&D compared to the social optimum. The social planner takes into account that an innovation destroys the social return from the previous innovation.

Finally, the *intertemporal R&D spillover* effect associated with research failures represents a second reason why firms may overinvest in R&D activities from a social perspective. Due to our specification of the research technology in (8) and (9), R&D difficulty, $X$, is an increasing function of the innovation rate, $I$. The higher the R&D investment in industry $\omega$,

[28] Only if the parameter constellation satisfied $n\kappa\Gamma + n\mu + \mu\rho\Gamma = n$, is the value of $\gamma^{opt}$ that maximizes welfare equal to one.

the more difficult it becomes to realize the next innovation in that particular industry. The parameter $\mu$ determines how fast this increase in R&D difficulty takes place. The negative externality is further aggravated because more researchers are needed to overcome the decreasing research productivity, leaving, *ceteris paribus*, fewer workers in the manufacturing sector. Because the goods market always clears, this goes hand in hand with lower consumer demand in equilibrium and with a decrease in profits for the quality leader in each industry $\omega \in [0,1]$. While the increased R&D costs incurred by other firms are ignored in a firm's R&D profit maximization calculus, the social planner takes them into account.

Identifying these externalities enables us to shed light on the intuition behind the result that, *ceteris paribus*, the amount of resources channeled into R&D is too low from a normative viewpoint when $\kappa$ and $\mu$ are small. In our model, when $\kappa$ (the parameter that governs the expected size of innovations) is large, new products represent rather "radical" technological breakthroughs as quality improvements over existing products are immense. In that case, innovative firms are able to charge a relatively high markup of price over marginal cost. Under these circumstances, the negative business stealing effect associated with R&D investment dominates. In the pursuit of other firms' profits, an excessive fraction of the economy's resources is devoted to R&D from a social viewpoint. On the other hand, when $\mu$ takes a small value, innovation only becomes slightly more difficult over time [see (9)] so the BGP of the economy is characterized by innovations occurring frequently. Successfully innovating firms earn monopoly profit flows for a relatively short period of time before they are driven out of business by further innovation. A small $\mu$ prevailing thus results in a situation where the positive consumer surplus effect dominates. Innovating firms benefit only briefly from their discoveries, but the benefits to consumers last forever. Under these circumstances, the fraction of the economy's resources directed to R&D is too small.

Given the normative results of the model, we now refer to the relation between the optimal policy intervention, on the one hand, and the average size of innovation or the speed of increases in R&D difficulty, on the other hand. First, holding all other parameters fixed, (30) implies that the optimal technological composition of public demand, $\gamma^{opt}$, depends negatively on $\mu$; the smaller $\mu$, the higher is the welfare-maximizing share of asymmetry in public demand spending. As any increase in $\gamma$ stimulates private R&D, this finding is in line with the interpretation of the welfare result above. Second, in (30) $\gamma^{opt}$ is a globally decreasing function of $\kappa$ for parameter values of $\mu < 1$. For $\mu > 1$, $\gamma^{opt}$ increases in $\kappa$. There is no unambiguous relationship between the demand policy parameter $\gamma^{opt}$ and the size of innovation.[29] Provided the average size of innovation becomes larger, industries with "radical" technological breakthroughs should be treated (less) favorably with respect to government demand compared to industries with rather "incremental" innovations if the R&D difficulty in (9) grows sufficiently fast (slow).

## 7. Conclusions

In this paper, we have developed a generalized version of a Schumpeterian growth model that incorporates a typical trait of real economies, namely the presence of industries characterized by different innovation size. This asymmetry causes the distribution of monopoly profits from successful innovation to be highly skewed toward the low-value side, with a long tail to the high-value side. We use the model to analyze the dynamic effects of a change in the technological content of government demand spending.

Our paper provides some arguments that bring the inter-industrial composition of public purchases within the realm of the debate on innovation and growth policy. We find that a change in the composition of public demand expenditure that relatively favors industries with above-average quality jumps temporarily fosters technological change and economic growth due to an *R&D incentive* effect. A government that channels its demand toward industries with a relatively high innovation capacity increases aggregate expected profits. The higher reward for successful innovation activities stimulates technological change because firms allocate relatively more resources to R&D, thereby inducing a higher demand for R&D labor. The model shows that the considerable increase in the wages of skilled workers relative to unskilled workers that occurred in U.S. during the 1980s, while at about the same time government procurement expenditure shifted out of low-tech industries and into high-tech industries, as highlighted recently by Cozzi & Impullitti (2008), was no coincidence.

We further show that the presence of a public procurement policy that differentiates between industries can generate indeterminacy of the balanced-growth equilibrium; i.e., there might exist multiple convergence paths that depend on initial conditions, eventually leading to the same long-run outcome. This result suggests that government purchasing behavior can be one of potentially many factors that explain why different countries with similar initial conditions diverge (temporarily) in their subsequent growth performance. In addition, we find that

[29] Previous literature on Schumpeterian growth comes to comparable conclusions, albeit when analyzing policy intervention by means of R&D subsidies. Grossman & Helpman (1991) find that private R&D should be subsidized when the size of quality improvements is either very small or very large. Li's (2003) model basically implies the same U-shaped relation between the optimal subsidy and the size of quality increments.

public procurement privileging industries with above-average innovation size unambiguously raises welfare above the *laissez-faire* equilibrium.

However, drawing policy recommendations from our model should be approached with some caution. In this respect, two caveats of the model should be noted. First, due to our assumption of full employment of labor in every instant of time, we neglect search unemployment that may come along with the reallocation of labor force from one sector to another (Aghion & Howitt, 1994). In a more realistic model setup, the welfare gain resulting from a public demand policy relatively in favor of industries with above-average innovation size should be offset against the social costs entailed by such asymmetric policy intervention. Second, in the real world it may be tolerably difficult for public authorities to identify and also to pick "winning industries" (Giordani & Zamparelli, 2008). On the one hand, the assumption that the government has the ability to recognize winners is not highly unrealistic provided that the distribution of quality increments is indeed time invariant and equivalent to the distribution of industrial markups. Hall (1988), Roeger (1995), and Oliveira Martins et al. (1996) present empirical estimates of industrial markups for U.S. manufacturing industries, providing the government with an indication as to the industries where public demand expenditure should be directed from the model's viewpoint. On the other hand, willingness to pick winners may be threatened by the presence of lobbies capable of influencing policy makers' decisions in their favor and by purchasing conservatism of public authorities. To substantiate the latter, a recent survey of the U.K. environmental sector shows that 66 percent of the interviewed companies regarded the procurement process as harmful for their innovativeness as tender specifications locked suppliers into traditional technologies (DTI, 2006). In general, public sector employees seem to exhibit a substantial degree of risk aversion (Buurman et al., 2009).

Bearing in mind these qualifications of the model, we believe our research demonstrates a substantial degree of policy relevance. Recently, some major initiatives have been launched on both the European and the national levels, encouraging public authorities to focus their demand on high-tech and innovative solutions (see, e.g., Edler & Georghiou, 2007). We see a potential for extending the current analysis to questions concerning the deliberate utilization of public procurement as a tool for innovation policy. One important issue in this respect is to contrast the impact of public procurement on the economy with more "traditional" innovation policy instruments, especially R&D subsidies.

A further aspect not addressed by the paper is the potential importance of government purchasing behavior in understanding private production processes. We here refer to a large stream of literature in which publicly provided goods and services are treated as a complementary input for private production (Arrow & Kurz, 1970; Barro, 1990). Empirical work on this topic was stimulated by Aschauer (1989) who stresses that physical infrastructure has the most explanatory power for private-sector productivity in the U.S. in the period 1949-1985. We believe that accounting for this so-called "productive public spending" in the model provides a fruitful path for future research.

Finally, we intend to analyze the empirical plausibility of the model's predictions. Using disaggregated U.S. and European procurement data at the industry level, we will subject the model to a detailed quantitative assessment. If it survives such an assessment, the model could be used to further explore questions about the interactions between public demand, technological change and growth.

## References

**Acemoglu, D. (2008)** Introduction to Modern Economic Growth. Princeton University Press.

**Acemoglu, D. and Linn, J. (2004)** Market Size in Innovation: Theory and Evidence from the Pharmaceutical Industry. *The Quarterly Journal of Economics*, **119**, 1049-1090.

**Aghion, P. and Howitt, P. (1992)** A Model of Growth Through Creative Destruction. *Econometrica,* **60(2),** 323-351.

**Aghion, P. and Howitt, P. (1994)** Growth and Unemployment. *Review of Economic Studies*, **61**, 477-494.

**Aghion, P. and Howitt P. (1998)** Endogenous Growth Theory, MIT Press.

**Aghion, P. and Howitt, P. (2005)** Growth with Quality-Improving Innovations: An Integrated Framework. In: **P. Agion and S.N. Durlauf** *Handbook of Economic Growth*, Amsterdam et al.: North Holland, 67-110.

**Ahmed, S. and Yoo, B.S. (1995)** Fiscal Trends in Real Economic Aggregates. *Journal of Money, Credit and Banking*, **27**, 985-1001.

**Arnold, L.G. (1998)** Growth, Welfare, and Trade in an Integrated Model of Human-Capital Accumulation and Research. *Journal of Macroeconomics*, **20**, 81-105.

**Arnold, L.G. (2002)** On the Effectiveness of Growth-Enhancing Policies in a Model of Growth Without Scale Effects. *German Economic Review*, **3**, 339-346.

**Arrow, K.J. (1962)** Economic welfare and the allocation of resources for inventions. In: **R.R. Nelson (Ed.)** *The Rate and Direction of Inventive Activity*. Princeton University Press, 609-625.

**Arrow, K.J. and Kurz, M. (1970)** Public Investment, the Rate of Return, and Optimal Fiscal Policy. Baltimore, MD: The Johns Hopkins University Press.

**Arrowsmith, S. (2005)** The Law of Public and Utilities Procurement. London: Sweet & Maxwell.

**Aschauer, D.A. (1989)** Is Public Expenditure Productive? *Journal of Monetary Economics*, **23**, 177–200.

**Aschhoff, B. and Sofka, W. (2008)** Innovation on Demand – Can Public Procurement Drive Market Success of Innovations? ZEW Discussion Paper No. 08-052. Zentrum für Europäische Wirtschaftsforschung, Mannheim.
http://madoc.bib.uni-mannheim.de/madoc/volltexte/2008/2084/pdf/dp08052.pdf, on 14th April 2009.

**Audet, D. (2002)** Government procurement: A synthesis Report. *OECD Journal of Budgeting*, **2**, 149-194.

**Barro, R.J. (1974)** Are Government Bonds Net Wealth? *Journal of Political Economy*, **82**, 1095-1117.

**Barro, R.J. (1990)** Government Spending in a Simple Model of Endogenous Growth. *Journal of Political Policy,* **98,** 103-125.

**Barro, R.J. and Sala-i-Martin, X. (1995)** Economic Growth. McGraw-Hill.

**Baxter, M. and King, R.G. (1993)** Fiscal policy in general equilibrium. *American Economic Review*, **83**, 315-334.

**Benhabib, J. and Farmer, R. (1994)** Indeterminacy and increasing returns. *Journal of Economic Theory*, **63**, 19-41.

**Benhabib, J. and Farmer, R. (1999)** Indeterminacy and sunspots in macroeconomics. In: Handbook of Macroeconomics, vol. 1. Elsevier Science B.V., pp. 387–448.

**Benhabib, J. and Perli, R. (1994)** Uniqueness and Indeterminacy: On the Dynamics of Endogenous Growth. *Journal of Economic Theory*, **63**, 113-142.

**Berggren, C. and Laestadius, S. (2003)** Co-Development and Composite Clusters – The Secular Strength of Nordic Telecommunication. *Industrial and Corporate Change*, **12**, 91-114.

**Blanchard, O.J. and Fischer, S. (1989)** Lectures on Macroeconomics. Cambridge: MIT Press.

**Bond, E., Wang, P., and Yip, C.K. (1996)** A General Two-Sector Model of Endogenous Growth with Human and Physical Capital. Balanced Growth and Transitional Dynamics. *Journal of Economic Theory*, **68**, 149-173.

**Braun, R. and McGrattan, E.R. (1993)** The macroeconomics of war and peace. NBER Macroeconomics Annual 1993.

**Bucci, A. (2003)** R&D, Imperfect Competition and Growth with Human Capital Accumulation. *Scottish Journal of Political Economy*, **50**, 417-439.

**Buurman, M., Dur, R. and Van den Bossche, S. (2009)** Public Sector Employees: Risk Averse and Altruistic? Tinbergen Institute Discussion Paper No. TI 2009-067/1.

**Chen, B. (2006)** Economic growth with an optimal public spending composition. *Oxford Economic Papers*, **58**, 123-136.

**Chiang, A.C. and Wainwright, K. (2005)** Fundamental Methods of Mathematical Economics. Irwin Professional Pub.

**Censolo, R. and Colombo, C. (2008)** Public consumption composition in a growing economy. *Journal of Macroeconomics*, **30**, 1479-1495.

**Cozzi, G. (2007)** Self-fulfilling prophecies in the quality ladders economy. *Journal of Development Economics*, **84**, 445-464.

**Cozzi, G. and Impullitti, G. (2008)** Government Spending Composition, Technical Change and Wage Inequality. *Journal of the European Economic Association*, Forthcoming.

**Dalpé, R., DeBresson, C. and Xiaoping, H. (1992)** The Public Sector as First User of Innovations. *Research Policy,* **21,** 251-263.

**Davidson, C. and Segerstrom, P.S. (1998)** R&D Subsidies and Economic Growth. The RAND Journal of Economics, **29**, 548-577.

**Dinopoulos, E. and Segerstrom, P.S (1999)** The dynamic effects of contingent tariffs. *Journal of International Economics*, **47**, 191-222.

**Dinopoulos, E. and Şener, F. (2007)** New Directions in Schumpeterian Growth Theory. In: **H. Hanusch and A. Pyka (Ed.)** *Edgar Companion to Neo-Schumpeterian Economics*. Cheltenham: Edward Elgar.

**Dinopoulos, E. and Thompson, P. (1999)** Scale Effects in Schumpeterian Models of Economic Growth. *Journal of Evolutionary Economics*, **9**, 157-185.

**DTI (Department of Trade and Industry) (2006)** Environmental Innovation: Bridging the Gap Between Environmental Necessity and Economic Opportunity. First Report of the Environmental Innovations Advisory Group.
http://www.berr.gov.uk/files/file34987.pdf, on 14th April 2009.

**Edler, J. and Georghiou, L. (2007)** Public Procurement and Innovation – Resurrecting the Demand Side. *Research Policy,* **36,** 949-963.

**Edquist, C. and Hommen, L. (2000)** Public Technology Procurement and Innovation Theory. In: **C. Edquist, L. Hommen and L. Tsipouri (Ed.)** *Public Technology Procurement and Innovation*. Kluwer Academic Publishers, 5-70.

**EU (2004)** A report on the functioning of public procurement markets in the EU: benefits from the application of EU directives and challenges for the future.
http://ec.europa.eu/internal_market/publicprocurement/docs/public-proc-market-final-report_en.pdf, on 14th April 2009.

**EU (2008)** The LIME assessment framework (LAF): a methodological tool to compare, in the context of the Lisbon Strategy, the performance of EU Member States in terms of GDP and in terms of twenty policy areas affecting growth. European Economy, Occasional Papers No. 41. Brussels.
http://ec.europa.eu/economy_finance/publications/publication13275_en.pdf, on 9th September 2009.

**Federal Procurement Data Center (2006)** Federal Procurement Report FY 2006. Arlington.

**Fisher, W.H. and Turnovsky, S.J. (1998)** Public Investment, Congestion, and Private Capital Accumulation. *The Economic Journal*, **108**, 399-413.

**Fridlund, M. (2000)** Switching Relations and Trajectories: The Development Procurement of the Swedish AXE Switching Technology. In: **C. Edquist, L. Hommen and L. Tsipouri (Ed.)** *Public Technology Procurement and Innovation*. Kluwer Academic Publishers, 143-

165.

**Fudenberg, D. and Tirole, J. (1985)** Preemption and Rent Equalization in the Adoption of New Technology. *American Economic Review,* **52,** 383-401.

**Fudenberg, D., Gilbert, R.J., Stiglitz, J.E. and Tirole, J. (1983)** Preemption. Leapfrogging, and Competition in Patent Races. *European Economic Review,* **22,** 3-31.

**Gantmacher, F.R. (1960)** The Theory of Matrices. New York: AMS Chelsea Publishing.

**Geroski, P.A. (1998)** An applied econometrician's view of large company performance. *Review of Industrial Organization*, **13**, 271-293.

**Gilfillan, S. C. (1935)** The Sociology of Invention. Chicago: Follett Publishing Company.

**Gilpin, R. (1975)** Technology, Economic Growth and International Competitiveness. Study prepared for the Subcommittee on Economic Growth of the Congressional Joint Economic Committee. Washington, D.C.: U.S. Government Printing Office.

**Giordani, P.E. and Zamparelli, L. (2008)** The Importance of Industrial Policy in Quality-Ladder Growth Models. *The B.E. Journal of Macroeconomics*, **8**, Article 1.

**Grossman, G.M. and Helpman, E. (1991a)** Quality Ladders in the Theory of Growth. *Review of Economic Studies,* **58,** 43-61.

**Grossman, G.M. and Helpman, E. (1991b)** Innovation and Growth in the Global Economy. Cambridge Mass., London: MIT Press.

**Guellec, D. and Van Pottelsberghe, B. (2000)** The Impact of Public R&D Expenditure on Business R&D. OECD Science, Technology and Industry Working Papers, 2000/4, OECD publishing.

**Guerzoni, M. (2007)** Size and sophistication: the two faces of demand. CESPRI Working Papers, No.197.
ftp://ftp.unibocconi.it/pub/RePEc/cri/papers/WP197Guerzoni.pdf, 15th April 2009.

**Hall, R.E. (1988)** The Relation between Price and Marginal Cost in U.S. Industry. *Journal of Political Economy*, **96**, 921-947.

**Harhoff, D., Scherer, F. and Vopel, K. (2005)** Exploring the Tail of Patented Invention Value Distributions. In: **D. Harhoff, F. M. Scherer, K. Vopel (Eds.)**, *Patents: Economics, Policy and Measurement*. Cheltenham, U.K. and Northampton, Mass.: Elgar, 251-281.

**Hu, Y., Ohdoi, R. and Shimomure, K. (2008)** Indeterminacy in a two-sector endogenous growth model with productive government spending. *Journal of Macroeconomics*, **30**, 1104-1123.

**Jones, B.F. (2005)** The Burden of Knowledge and the 'Death of the Renaissance Man': Is Innovation Getting Harder? NBER Working Paper No. 11360.

**Jones, C.I. (1995a)** Time Series Tests of Endogenous Growth Models. *Quarterly Journal of Economics*, **110**, 495-525.

**Jones, C.I. (1995b**) R&D-Based Models of Economic Growth. *Journal of Political Economy*, **103**, 759-784.

**Jones, C.I. (1997**) The Upcoming Slowdown in U.S. Economic Growth. NBER Working Paper No. 6284.

**Jones, C.I. (1999)** Growth: With or Without Scale Effects? *American Economic Review*, **89**, 139-144.

**Jones, C.I. (2004)** Growth and Ideas. NBER Working Paper No. 10767.

**Jones, C.I. (2005)** The Shape of Production Functions and the Direction of Technological Change. *Quarterly Journal of Economics*, **120**, 517-549.

**King, R.G. and Levine, R. (1993)** Finance, Entrepreneurship and growth: theory and evidence. *Journal of Monetary Economics*, **32**, 513-542.

**Kirman, A.P. (1992)** Whom or what does the representative individual represent? *Journal of Economic Perspectives*, **6**, 117-136.

**Kortum, S. (1997)** Research, Patenting, and Technological Change. *Econometrica*, **65**, 1389-1419.

**Li, C.W. (2001)** On the Policy Implications of Endogenous Technological Progress. Economic Journal, Vol. 111, pp. 164-179.

**Li, C.W. (2003)** Endogenous Growth without Scale Effects: Comment. American Economic Review, Vol. 93, pp. 1009-1017.

**Lichtenberg, F.R. (1988)** The Private R&D Investment Response to Federal Design and Technical Competitions. *American Economic Review*, **78**, 550-559.

**Lorenz, H.W. (1993)** Nonlinear Dynamical Economics and Chaotic Motion. Lecture Notes in Economics and Mathematical Systems. Berlin: Springer. Second Edition.

**Lundvall, B.-A. (1988)** Innovation as an interactive process: from user-producer interaction to the national system of innovation. In: **G. Dosi, C. Freeman, R. Nelson, G. Silverberg, L. Soete (Ed.)**, *Technical Change and Economic Theory*. Pinter, London, 349-369.

**Marron, D. (2003)** Greener Public Purchasing as an Environmental Policy Instrument. In: **OECD (Ed.)**, The Environmental Performance of Public Procurement: Issues of Policy Coherence. OECD, Paris, 21-49.

**Minniti, A., Parello, C.A. and Segerstrom, P.S. (2008)** A Schumpeterian Growth Model with Heterogenous Firms. MPRA Working Paper. http://mpra.ub.uni-muenchen.de/13674/, 14th April 2009.

**Monteiro, G. and Turnovsky, S.J. (2008)** The Composition of Productive Government Expenditure: Consequences for Economic Growth and Welfare. *Indian Growth and Development Review*, **1**, 57-83.

**Mowery, D.C. (2008)** National security and national innovation systems. *Journal of Technology Transfer*.

**Mowery, D.C. and Rosenberg, N. (1979)** The Influence of Market Demand upon Innovation: a Critical Review of some Recent Empirical Studies. *Research Policy*, **8**, 102-153.

**Oliveira Martins, J., S. Scarpetta and D. Pilat (1996)** Mark-Up Ratios in Manufacturing Industries: Estimates for 14 OECD Countries. OECD Economics Department Working Papers, No. 162, OECD Publishing.

**Palivos, T., Yip, C.K. and Zhang, J. (2003)** Transitional Dynamics and Indeterminacy of Equilibria in an Endogenous Growth Model with a Public Input. *Review of Development Economics*, **7**, 86-98.

**Powell, W.W. and Grodal, S. (2004)** Networks of innovators. In**: J. Fagerberg., D.C. Mowery and R.R Nelson (Eds.)** *The Oxford handbook of innovation*. Oxford/New York: Oxford University Press.

**Roeger, W. (1995)** Can Imperfect Competition Explain the Difference between Primal and Dual Productivity Measures? Estimates for US Manufacturing. *Journal of Political Economy*, **103**, 316-330.

**Romer, P. M. (1990)** Endogenous Technological Change. *Journal of Political Economy*, **98**, 71-102.

**Rosenberg, N. (1969)** Direction of technological change – inducement mechanisms and focusing devices. *Economic Development and Cultural Change*, **18**, 1-24.

**Rothwell, R. and Zegveld, W. (1982)** Industrial Innovation and Public Policy. Preparing for the 1980s and the 1990s. London: Pinter.

**Ruttan, V.W. (2006)** Is War Necessary for Economic Growth? Military Procurement and Technology Development. New York: Oxford University Press.

**Saal, D. (2001)** The Impact of Procurement-Driven Technological Change on U.S. Manufacturing Productivity Growth. *Defense and Peace Economics*, **12**, 537-568.

**Scherer F. M. (1965)** Firm Size, Market Structure, Opportunity, and the Output of Patented Inventions. *American Economic Review*, **55**, 1097-1123.

**Scherer, F.M. (1980)** Industrial Market Structure and Economic Performance. 2nd ed., Chicago: Rand McNally.

**Schmookler, J. (1962)** Economic Sources of Inventive Activity. *Journal of Economic History*, **XXII(1)**, 1-20.

**Schmookler, J. (1966)** Invention and Economic Growth. Cambridge, Mass: Harvard University Press.

**Segerstrom, P.S. (1998)** Endogenous Growth without Scale Effects. *American Economic Review*, **88**, 1290-1310.

**Segerstrom, P.S. (2000)** The Long-Run Growth Effects of R&D Subsidies. *Journal of Economic Growth*, **5**, 277-305.

**Sherwin, C.W. and Isenson, R.S. (1967)** Project HINDSIGHT. *Science*, **156**, 1571-1577.

**Simon, C.P. and Blume, L. (1994)** Mathematics for Economists. New York: W.W. Norton & Company.

**Stadler, M. (1999)** Demand Pull and Technology Push Effects in the Quality Ladder Model. http://tobias-lib.ub.uni-tuebingen.de/volltexte/2005/2102/pdf/172.pdf, 14th April 2009.

**Steger T. (2003)** The Segerstrom Model: Stability, Speed of Convergence and Policy Implications. *Economics Bulletin*, **15**, 1-8.

**Stokey, N. and Rebelo, S. (1995)** Growth effects of flat-rate taxes. *Journal of Political Economy*, **103**, 519–550.

**Temple, J. (2003)** The Long-Run implications of Growth Theories. *Journal of Economic Surveys*, **17(3)**, 497-510.

**Turnovsky, S.J. (1977)** Macroeconomic analysis and stabilization policy. Cambridge: Cambridge University Press.

**Von Hippel, E. (1982)** Appropriability of innovation benefit as a predictor of the source of innovation, *Research Policy*, **11**, 95-115.

**Von Hippel, E. (1986)** Lead Users: A Source of Novel Product Concepts. *Management Science,* **32,** 791-805.

## Appendix A: Determining the unique ratio between ε1 and ε2

In this Appendix, we derive the relation between $\varepsilon_1$ and $\varepsilon_2$ for the public demand rule to be feasible. By definition, $\int_0^1 G(\omega)d\omega \equiv \bar{G}$. Substituting the public demand rule for $G(\omega)$ yields:

$$\int_0^1\int_1^\infty (1-\gamma)\bar{G}+\gamma\left(\bar{G}+\varepsilon\right)d\lambda d\omega$$

$$=\int_0^1\left\{(1-\gamma)\int_1^\infty \bar{G}g(\lambda)d\lambda+\gamma\left[\int_1^{\frac{1}{1-\kappa}}(\bar{G}-\varepsilon_1)g(\lambda)d\lambda+\int_{\frac{1}{1-\kappa}}^\infty(\bar{G}+\varepsilon_2)g(\lambda)d\lambda\right]\right\}d\omega, \qquad \text{(A.1)}$$

where $g(\lambda)$ is the Pareto density function with scale parameter equal to one and share parameter equal to $1/\kappa$. According to (10), we can express $g(\lambda)$ as $1/\kappa\,\lambda^{-(1+\kappa)/\kappa}$, which allows us to rewrite (A.1) as:

$$\int_0^1\left\{\frac{(1-\gamma)}{\kappa}\bar{G}\int_1^\infty \lambda^{-\frac{1+\kappa}{\kappa}}d\lambda+\frac{\gamma}{\kappa}\left[(\bar{G}-\varepsilon_1)\int_1^{\frac{1}{1-\kappa}}\lambda^{-\frac{1+\kappa}{\kappa}}d\lambda+(\bar{G}+\varepsilon_2)\int_{\frac{1}{1-\kappa}}^\infty\lambda^{-\frac{1+\kappa}{\kappa}}d\lambda\right]\right\}d\omega.$$

Computing the integral above gives:

$$\int_0^1 G(\omega)d\omega=(1-\gamma)\bar{G}+\gamma\left[\varepsilon_1(-1+(1-\kappa)^{\frac{1}{\kappa}}+\varepsilon_2(1-\kappa)^{\frac{1}{\kappa}}\right]. \qquad \text{(A.2)}$$

By definition, the term on the RHS of (A.2) is equal to $\bar{G}$. It is now straightforward to show that this relation determines a unique ratio between $\varepsilon_1$ and $\varepsilon_2$ equal to:

$$\frac{\varepsilon_1}{\varepsilon_2}=\frac{(1-\kappa)^{\frac{1}{\kappa}}}{1-(1-\kappa)^{\frac{1}{\kappa}}}. \qquad \text{(A.3)}$$

Because the RHS of (A.3) is strictly positive but smaller than one, it follows that $\varepsilon_1<\varepsilon_2$.

## Appendix B: Calculation of the expected profit stream earned by an industry leader

When we take into account (6), the expected value of the profit flow that accrues to the winner of a R&D race in industry $\omega$ at time $t$ can be written as (suppressing time arguments):

$$\pi^e(\omega,t)=\int_1^\infty\left[(\lambda-1)\frac{cL}{\lambda}g(\lambda)+(\lambda-1)\frac{G(\omega)L}{\lambda}g(\lambda)\right]d\lambda. \qquad \text{(B.1)}$$

The first term in the integral on the RHS of (B.1) represents the profits an industry leader gains from private demand, while the second term captures the profits resulting from government purchases. We can substitute for the Pareto density function, $g(\lambda)$, and for public demand spending, $G(\lambda)$, by using (10) and (14). Equation (B.1) becomes:

$$\pi^e(\omega,t) = \int_1^\infty \left\{ \frac{cL}{\kappa}(\lambda-1)\frac{1}{\lambda}\lambda^{-\frac{1+\kappa}{\kappa}} + \frac{L}{\kappa}(\lambda-1)\frac{1}{\lambda}\lambda^{-\frac{1+\kappa}{\kappa}}\left[(1-\gamma)\bar{G}+\gamma\left(\bar{G}+\varepsilon\right)\right]\right\}. \qquad \text{(B.2)}$$

The term $(\lambda-1)(1/\lambda)\lambda^{-(1+\kappa)/\kappa}$ can be simplified to $(\lambda-1)\lambda^{-2-1/\kappa}$. Having this in mind, we can compute integral (B.2) as being equal to:

$$\pi^e(\omega,t) = \frac{\kappa}{1+\kappa}cL + (1-\gamma)\frac{\kappa}{1+\kappa}\bar{G}L + \gamma\frac{\kappa}{1+\kappa}L\left\{(\varepsilon_1-\bar{G})\left[-1+2(1-\kappa)^{\frac{1}{\kappa}}\right]+2(\varepsilon_2+\bar{G})\left[(1-\kappa)^{\frac{1}{\kappa}}\right]\right\}$$

In Appendix A, we showed that there exists a specific relation between $\varepsilon_1$ and $\varepsilon_2$, given in (A.3). We now make use of this result to eliminate $\varepsilon_1$. Using (A.3), the integral above boils down to:

$$\pi^e(\omega,t) = \frac{\kappa}{1+\kappa}cL + \frac{\kappa}{1+\kappa}L\left[\bar{G}+\gamma\varepsilon_2\left(-1+\frac{1}{1-(1-\kappa)^{\frac{1}{\kappa}}}\right)\right]. \qquad \text{(B.3)}$$

Notice that $0 < 1-(1-\kappa)^{1/\kappa} < 1$ for all $\kappa \in (0,1)$, and thus $1\big/\left[1-(1-\kappa)^{1/\kappa}\right] > 1$, leaving the term in round brackets on the RHS of (B.3) positive. Rearranging (B.3) eventually allows us to write the expected profit stream as:

$$\pi^e(\omega,t) = \frac{\kappa}{1+\kappa}L\left(c+\bar{G}+\gamma\Gamma\right), \qquad \text{(B.4)}$$

where $\Gamma \equiv \varepsilon_2\left\{1\big/\left[1-(1-\kappa)^{1/\kappa}\right]-1\right\} > 0$ was defined for notational simplicity and is completely determined by parameter values.

## Appendix C: Local stability analysis

To determine stability properties of the balanced-growth equilibrium requires constructing a system of two differential equations describing the evolution of the endogenous variables out of steady state. Above we defined $x(t) \equiv X(t)/L(t)$. Taking the logarithm and differentiating with respect to time [recalling (9)] gives:

$$\frac{\dot{x}(t)}{x(t)} = \mu I(t) - n\,.$$

We use (18) to obtain an expression for the rate of innovation, $I(t)$. After substituting this rate in the equation above, we arrive at a differential equation for $\dot{x}(t)$ solely containing $x(t)$ and $c(t)$ as endogenous variables:

$$\dot{x}(t) = \frac{A\mu\left[1+\kappa+\gamma\kappa\Gamma-\bar{G}-c(t)\right]}{\left(1+\kappa\right)x(t)}x(t)-nx(t). \tag{C.1}$$

The Keynes-Ramsey rule in (5) yields a condition for the evolution of per capita private consumption. However, we have to substitute for interest rate $r(t)$ because it is non-constant over time. Solving (17) for $r(t)$ gives us an expression for the interest rate at each instant of time. Plugging the respective value of $r(t)$ into (5) yields:

$$\dot{c}(t) = \frac{A\left[\left(c(t)+\bar{G}\right)\left(1+\kappa-\mu\right)+\left(1+\kappa\right)\left(\mu-1\right)+\gamma\kappa\mu\Gamma\right]}{\left(1+\kappa\right)x(t)}c(t)-\rho c(t), \tag{C.2}$$

where we again used the resource constraint (18) to get an expression for $I(t)$.

Equations (C.1) and (C.2) represent a system of two nonlinear, autonomous differential equations. Notice that the $(\dot{x}=0)$ isocline is identical to the resource constraint (18) in $(c,x)$ space. Thus, it is downward sloping. The $(\dot{c}=0)$ isocline is upward sloping in $(c,x)$ space for $(1+\kappa)>\mu$ and downward sloping in $(c,x)$ space for $(1+\kappa)<\mu$. However, whether the slope of the $(\dot{c}=0)$ demarcation line is positive or negative has no impact on the stability properties of the economy's BGP.

The dynamical system (C.1) and (C.2) is nonlinear. A first order Taylor expansion around the steady state, (22) and (23), provides a linear approximation of the system (e.g., Acemoglu, 2008, chap. IX). It follows from the *Hartman-Grobman theorem* and the *Stable/Unstable Maniford theorem* (see Lorenz, 1993) that in a sufficiently small neighborhood of the balanced-growth equilibrium the linear approximation has the same general stability properties as the original nonlinear system. Moreover, the following matrix provides sufficient information to judge the system's stability (Chiang & Wainwright, 2005, chap. 19):

$$J = \begin{pmatrix} \dfrac{\delta\dot{c}}{\delta c} & \dfrac{\delta\dot{c}}{\delta x} \\ \dfrac{\delta\dot{x}}{\delta c} & \dfrac{\delta\dot{x}}{\delta x} \end{pmatrix},$$

where $J$ is the so-called Jacobian matrix, which contains all first partial derivatives of (C.1) and (C.2) with respect to the endogenous variables, $c$ and $x$. For notational convenience, we denote the Jacobian matrix evaluated at the balanced-growth equilibrium by $J_E$ and its elements by $a$, $b$, $c$, and $d$:

$$J_E = \begin{pmatrix} \frac{\delta \dot{c}}{\delta c} & \frac{\delta \dot{c}}{\delta x} \\ \frac{\delta \dot{x}}{\delta c} & \frac{\delta \dot{x}}{\delta x} \end{pmatrix}_{(c=c^*, x=x^*)} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{C.3}$$

where

$$a = \frac{[1+\kappa-\mu]\left[\mu\rho\left(1-\bar{G}+\kappa+\gamma\kappa\Gamma\right)-n\left(\bar{G}\left(1+\kappa-\mu\right)+\left(1+\kappa\right)\left(\mu-1\right)+\gamma\kappa\mu\Gamma\right)\right]}{\kappa\left(1+\kappa\right)\mu\left(1+\gamma\Gamma\right)},$$

$$b = \frac{\rho\left[\mu\rho\left(-1+\bar{G}-\kappa-\gamma\kappa\Gamma\right)+n\left(\bar{G}\left(1+\kappa-\mu\right)+\left(1+\kappa\right)\left(\mu-1\right)+\gamma\kappa\mu\Gamma\right)\right]}{A\kappa\mu\left(1+\gamma\Gamma\right)},$$

$$c = -\frac{A\mu}{1+\kappa},$$

$$d = -n.$$

The eigenvalues of $J_E$, represented by $\vartheta$, are the solutions of the characteristic equation:

$$\vartheta^2 - (a+d)\vartheta + (ad-bc) = 0. \tag{C.4}$$

These eigenvalues determine the stability properties of the system. If the Jacobian in (C.3) does not have an eigenvalue with negative real part, the BGP is unstable. If the Jacobian in (C.3) has exactly one eigenvalue with negative real part, the BGP is saddle-path stable. Letting the economy start from any initial value of $x$, $c$ jumps immediately to the corresponding saddle-path value, and over time both $x$ and $c$ converge to their steady-state values given by (22) and (23). In other words, in the case of saddle-path stability the equilibrium-growth path (hereafter EGP) of $x$ and $c$, on which the variables converge to their balanced-growth values, is unique. If the Jacobian in (C.3) has two eigenvalues with negative real parts, the BGP is fully stable. Irrespective of the initial value of $x$, both endogenous variables, $x$ and $c$, will eventually approach their steady-state values. Expressed differently, an infinite number of EGP exist (indeterminacy).

At this point we want to determine the sign of the real parts of the eigenvalues of $J_E$. To be able to do so, we make use of the following theorem:

**Theorem 1.** *The number of roots with negative real parts equal two minus the number of sign changes in the scheme*

$$1 \,||\, -Tr\left(J_E\right) \,||\, Det\left(J_E\right)$$

**Proof**. This is an application to our special case of the more general *Routh-Hurwitz theorem* (see Gantmacher, 1960).

Recalling that $Tr(J_E)$ is equal to the sum and $Det(J_E)$ is equal to the product of the eigenvalues of $J_E$ (see, e.g., Simon & Blume, 1994, Theorem 23.9), we apply Theorem 1 and establish that a necessary and sufficient condition for saddle-path stability is $Det(J_E)$ being negative. If $Det(J_E) > 0$, indeterminacy (instability) arises for $Tr(J_E) < 0$ $(Tr(J_E) > 0)$.

The determinant of the Jacobian evaluated at the BGP reads in its general form:

$$Det(J_E) = ad - bc\,.$$

After some simplifications, we arrive at:

$$Det(J_E) = \frac{\left[n(1+\kappa-\mu)+\mu\rho\right]\left\{\left[\mu\rho\left(\bar{G}-1-\kappa-\gamma\kappa\Gamma\right)\right]+n\left[\bar{G}(1+\kappa-\mu)+\mu+\kappa\left(\mu+\gamma\mu\Gamma-1\right)-1\right]\right\}}{\kappa(1+\kappa)\mu(1+\gamma\Gamma)}.$$

The sign of $Det(J_E)$ depends critically on the parameter $\gamma$. It is straightforward to show that $Det(J_E) < 0$ if $\gamma < \gamma_{crit}$ and $Det(J_E) > 0$ if $\gamma > \gamma_{crit}$, where:

$$\gamma_{crit} = \frac{n\left[\bar{G}(1+\kappa-\mu)+(1+\kappa)(\mu-1)\right]-\mu\rho(1+\kappa-\mu)}{\kappa\mu(\rho-n)\Gamma}. \tag{C.5}$$

We used above our necessary-and-sufficient condition for the existence of a balanced-growth equilibrium, namely $\bar{G} < 1$. Notice that in the following we impose that the parameter values always satisfy the restriction:

$$\mu\rho\left(1+\kappa+\kappa\Gamma-\bar{G}\right) \le n\left[\bar{G}(1+\kappa-\mu)+\mu+\kappa\left(\mu+\mu\Gamma-1\right)-1\right], \tag{C.6}$$

which ensures that $\gamma_{crit} \in [0,1]$. We conclude from the preceding discussion that the balanced-growth equilibrium, given by (22) and (23), is locally saddle-path stable for values of $\gamma$ below $\gamma_{crit}$.

In order to investigate the stability properties of our dynamical system for values of $\gamma$ above $\gamma_{crit}$, we consider now the trace of the Jacobian in (C.3). We use (C.5) to substitute for $\gamma$ in the equation for the trace. More precisely, we add an (infinitesimal small) number $\chi \in \Re^{++}$ to the RHS of (C.5), and plug it into the expression for the trace. By doing so we make sure that we are in the parameter range of a positive $Det(J_E)$. Some major cancellation occurs, and we get:

$$Tr(J_E)_{(\gamma=\gamma_{crit}+\chi)} = \frac{(1-\bar{G})(1+\kappa)n\left[n(1+\kappa-\mu)+\mu\rho\right]+\chi\kappa\mu\Gamma(n-\rho)\left[n(2+2\kappa-\mu)+\rho(\mu-\kappa-1)\right]}{(1+\kappa)\left\{(\bar{G}-1)\left[n(1+\kappa-\mu)+\mu\rho\right]+\chi\kappa\mu(\rho-n)\Gamma\right\}}.$$

Making use of (C.6), one can show after some tedious algebra that $Tr(J_E)$ is always negative, provided that both eigenvalues of $J_E$ are of the same sign, i.e., $Det(J_E) > 0$. It follows that both eigenvalues of $J_E$ must be negative, if $\gamma > \gamma_{crit}$, indicating the existence of multiple EGP that eventually approach the BGP (local indeterminacy).

## Appendix D: Social optimum

This Appendix considers the social optimum. First, we observe that production of each vintage of consumption goods in industry $\omega \in [0,1]$ requires the same technology. Hence it cannot be optimal to produce consumption goods at a lower quality level than state-of-the-art quality. When only goods at the current quality frontier are produced in each industry, subutility (2) becomes:

$$\log u(t) = \int_0^1 \log\left[\lambda^{j^{\max}(\omega,t)}(\omega,t) d\left(j^{\max},\omega,t\right)\right] d\omega, \tag{D.1}$$

where $j^{\max}(\omega,t)$ denotes the highest quality level in industry $\omega$ at time $t$. Equation (D.1) can be rearranged to:

$$\log u(t) = \int_0^1 \log\left[\lambda^{j^{\max}(\omega,t)}(\omega,t)\right] d\omega + \int_0^1 \log\left[d\left(j^{\max},\omega,t\right)\right] d\omega .$$

To see that the social planner will choose the same R&D intensity $X$ in each industry, observe that the quantity of each state-of-the-art product consumed by an individual equals:

$$\frac{c(t)}{1+\kappa} = 1 - \int_0^1 \frac{L_I(\omega,t)}{L(t)} d\omega . \tag{D.2}$$

For given resources devoted to R&D at time $t$, $\int_0^1 L_I(\omega,t)/L(t)\, d\omega$, the social planner wants to choose the distribution of R&D input across industries that maximizes:

$$\frac{d}{dt}\int_0^1 \log\left[\lambda^{j^{\max}(\omega,t)}(\omega,t)\right] d\omega$$

$$= \frac{d}{dt}\int_0^1 \log\left[\lambda(\omega,t)\right] j^{\max}(\omega,t)\, d\omega$$

$$= \frac{d}{dt}\int_0^1 j^{\max}(\omega,t)\left[\int_1^\infty \log(\lambda) g(\lambda)\, d\lambda\right] d\omega ,$$

where the probability density function $g(\lambda)$ is Pareto and equal to $1/\kappa\, \lambda^{-(1+\kappa)/\kappa}$. Solving the integral above, while noting that $\int_0^1 j^{\max}(\omega,t)\, d\omega = \int_0^1\int_1^t I(\omega,\tau)\, d\tau$, gives:

$$\frac{d}{dt}\int_0^1 \log\left[\lambda^{j^{\max}(\omega,t)}\left(\omega,t\right)\right]d\omega = \kappa\int_0^1 \frac{AL_I(\omega,t)}{X\left(\omega,t\right)}d\omega, \tag{D.3}$$

where we used (8) to substitute for $I\left(\omega,t\right)$.

Equation (D.3) implies that at each point in time, the social planner wants to carry out all R&D in industries with the lowest $X\left(\omega,t\right)$. Executing this policy throughout time will lead to $X\left(\omega,t\right)=X(t)$ and $I\left(\omega,t\right)=I(t)$ for all $t$ (at some point in time all industry-specific R&D difficulties are equalized).

We define now $I(t)\equiv\dot{\phi}(t)$ so that with $X$ and $I$ not varying across industries, the following expression for $X(t)$ can be obtained, using (9):

$$X(t)=X_0 e^{\mu\int_0^t I(\tau)d\tau}=X_0 e^{\mu\phi(t)}.$$

Solving (8) for $L_I(t)$ and aggregating over all industries $\omega\in[0,1]$, equation (D.2) can be written as:

$$\frac{c(t)}{1+\kappa}=1-\frac{I(t)X(t)}{AL(t)}$$

We plug this expression into the discounted utility function (1), while noting that $L(t)=e^{nt}$ $\left(L_0\equiv 1\right)$, to obtain:

$$U(t)=\int_0^\infty e^{-(\rho-n)t}\left[\phi(t)\kappa+\log\left(1-\frac{I(t)X_0 e^{\mu\phi(t)}}{Ae^{nt}}\right)\right]dt\,. \tag{D.4}$$

The social planner maximizes $U(t)$ given in (D.4) with respect to the control variable $I(t)$, subject to the state equation $\dot{\phi}(t)=I(t)$ and the control constraint $0\le I(t)\le A/x(t)$. The RHS of the latter inequality results from (8), at the point where $L_I=L$.

As shown in Segerstrom (1998), this maximization problem can be reformulated using $x(t)$ instead of $\phi(t)$ as state variable. By solving $x(t)=X_0 e^{\mu\phi(t)}/e^{nt}$ for $\phi(t)$ and plugging the resulting expression into (D.4), the dynamic optimization problem the social planner solves takes the form:[30]

$$\max_I\int_0^\infty e^{-(\rho-n)t}\left[\kappa\frac{\log x+nt-\log X_0}{\mu}+\log\left(1-\frac{Ix}{A}\right)\right]dt$$

s.t.

[30] Notice that the time argument of functions is dropped in the remainder of this Appendix for the sake of notational convenience.

$$\frac{\dot{x}(t)}{x(t)} = \mu I(t) - n \text{ (state equation),}$$

$$x(0) = X_0 > 0 \text{ (given), and}$$

$$0 \leq I(t) \leq \frac{A}{x(t)} \text{ (control constraint).}$$

Dropping the constant term $\int_0^\infty e^{-(\rho-n)t}\left(\kappa\frac{nt-\log X_0}{\mu}\right)dt$, the current-value Hamiltonian for the social planner's problem is:

$$H = \frac{\kappa}{\mu}\log x + \log\left(1-\frac{Ix}{A}\right) + \xi\left(\mu I - n\right)x, \tag{D.5}$$

with $\xi$ denoting the costate variable associated with the state variable $x$.

For the application of Pontryagin's maximum principle, the Hamiltonian has to be concave in its control variable $I$ (Chiang & Wainwright 2005, chap. 20), which is indeed the case for our maximization problem. By Pontryagin's maximum principle, we obtain as first-order condition:

$$\frac{\delta H}{\delta I} = x\left(\frac{1}{Ix-A} + \mu\xi\right) = 0, \tag{I}$$

and as costate equation:

$$\dot{\xi} = (\rho-n)\xi - \frac{\delta H}{\delta x} = -\frac{\kappa}{\mu x} + \rho\xi + I\left(\frac{1}{A-Ix} - \mu\xi\right). \tag{II}$$

The transversality condition reads:

$$\lim_{t\to\infty} e^{-(\rho-n)t}x(t)\xi(t) = 0. \tag{III}$$

We solve (I) for $I$ and substitute the resulting expression into (II), which yields an equation of motion for the state variable $\xi$. Again solving (I) for $I$ and plugging the resulting value into the state equation, we obtain a differential equation that governs the evolution of relative R&D difficulty, $x$.

The resulting two-dimensional system of nonlinear, autonomous differential equations can be written as:

$$\dot{\xi} = -\frac{\kappa}{\mu x} + \rho\xi, \tag{D.6}$$

$$\dot{x} = A\mu - nx - \frac{1}{\xi}. \tag{D.7}$$

In the balanced-growth equilibrium, (I) implies that $\dot{x} = \dot{\xi} = 0$. The $(\dot{x} = 0)$ isocline is upward sloping and the $(\dot{\xi} = 0)$ isocline is downward sloping in $(x, \xi)$ space. There is a unique intersection of both curves in the positive orthand of $(x, \xi)$ space, giving rise to the following balanced-growth solution of the social planner's problem:

$$\xi^{opt} = \frac{\kappa n + \mu\rho}{A\rho\mu^2},$$

$$x^{opt} = \frac{A\kappa\mu}{\kappa n + \mu\rho}.$$

Stability inferences of the BGP can be made by setting up the "maximized Hamiltonian", which is obtained when we substitute the value of $I$ derived from (I) into the Hamiltonian (D.5). According to Proposition 10 in Arrow & Kurz (1970) respectively Theorem 7.14 in Acemoglu (2008), if the maximized Hamiltonian is a strictly concave function of the state variable $x$ for any given $\xi$, jumping onto the saddle path at time $t = 0$ and staying on the saddle path until the BGP is reached represents an optimal path for the economy. It is easy to show that in our case the second-order derivative of the maximized Hamiltonian is strictly negative. Thus, we can establish that the unique BGP resulting from solving the social planner's optimal control problem is saddle-path stable.

The socially optimal research intensity is the same as in the decentralized case, which can be seen when setting the state equation equal to its balanced-growth value, $\dot{x} = 0$. This gives equation (30) in the main text. Further, the socially optimal R&D labor share (equation (31) in the main text) can be calculated by substituting $x^{opt}$ for $x$ in $L_I / L = nx / \mu A$.