E C O N O M I C S   B U L L E T I N

The poverty trap with high fertility rates

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Abstract

The aim of this paper is to clarify the causes of the poverty trap resulting from a negative correlation between income and fertility, in a manner that is consistent with the data across and within countries. This paper points out that a higher fertility rate is the cause of the poverty trap, because of its educational cost aspect.

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1. Introduction

It is known well that a high fertility rate is one of the most remarkable features of the developing countries (Birdsall 1988). And the fact is commonly recognized to be one of the serious problems which developing countries face. [1] This common recognition is drawn from the natural inference that a high fertility rate is not only the result of poverty, but the causes of the poverty. The recent paper, which has the recognition that a high fertility rate is the causes of the poverty, is Kremer and Chen (1999) [2] , in which the influence by endogenizing fertility rates shows up through the labor market.

In this paper, we deal with an overlapping generations model similar to Glomm and Ravikumar (1992), in which human capital investment through formal schooling is the engine of growth. However, to investigate a relationship between a fertility rate and educational opportunities created by parents (for example, a quality of schools), in our model the fertility rate is determined endogenously, whereas Glomm and Ravikumar (1992) assume an exogenous population size. In contrast with Kremer and Chen (1999), in this paper, the influence by endogenizing fertility rates appears in change of a quality of schools (not through the labor market), and this change of the quality of schools affects the effort of children. This process generates multiple steady states.

The remainder of the paper is organized as follows. Section 2 describes the behavior of individuals. The model is solved in Section 3. Section 4 establishes the evolution of human capital and contains the welfare analysis. Section 5 concludes the paper.

2. The Model

In this paper, we consider an overlapping generations model in which individuals live for two periods. In the first period, the individuals decide the amount of time for accumulating human capital, given the level of their parents' human capital and the quality of schools. In the second period, they obtain income and decide consumption volume, fertility rates, and the quality of schools of their children.

The preferences of an individual born at time t are represented by $l_t + c_{t+1} + \ln n_{t+1} + A \ln e_{t+1}$, where $A \in (0, 1)$, l_t is leisure at time t, c_{t+1} is consumption at time t+1, n_{t+1} is fertility rate at time t+1, and e_{t+1} is the quality of schools at time t+1. Due to this quasi-linear utility function, a higher income level leads people to have fewer children, as shown below. As mentioned in Kremer and Chen (1999, 2000), this function is a convenient vehicle by which to consider the influence of the negative correlations between the income level and fertility and between the quality of schools and fertility. If a log-linear utility function is assumed, the fertility rate becomes constant. However, this is not consistent with the actual data, that is, the negative correlation between income and fertility. [3]

[1] Because of the importance of this issue, there are many related studies (Barro and Becker 1989; Becker and Barro 1986, 1988; Becker *et al.* 1990; Dahan and Tsiddon 1998; Dessy 2000; Galor and Weil 2000; Iyigun 2000; and Kremer and Chen 1999).

[2] In Becker *et al.* (1990), the multiplicity derives from the assumption on the rates of return on investments in human capital. Dessy (2000) focuses on an economic value for children's time. In Iyigun (2000), a high fertility rate is only the result of poverty.

[3] There is empirical evidence for the view that substitution effects are important (Schultz 1981). In addition, Kremer and Chen (1999, 2000) study the relation between the fertility differentials and inequality, emphasizing the importance of the substitution effect based on empirical evidence.

All individuals are endowed with one divisible unit of time in each of the two periods. At time t, children allocate l_t units of their endowment toward leisure and devote the remaining $1 - l_t$ units toward accumulating human capital:

$$h_{t+1} = \theta(1 - l_t)^{\beta} e_t^{\gamma} h_t^{\delta}, \qquad \theta > 0, \tag{1}$$

where h_t is the stock of human capital of their parents. We assume that $\beta, \gamma, \delta \in (0, 1)$, so that all factors exhibit diminishing returns, and we assume $A > \beta$ to satisfy the second order condition for the utility maximization. At time t+1, an individual's income is the same as his or her human capital h_{t+1}, assuming a linear technology. Raising each child requires a time commitment of φ. An adult who chooses to have n_{t+1} children, will spend a portion φn_{t+1} of his or her time caring for children, and the remaining portion $1 - \varphi n_{t+1}$ working. An individual allocates his or her income $(1 - \varphi n_{t+1})h_{t+1}$ between consumption c_{t+1} and the educational cost of his or her children $e_{t+1}n_{t+1}$. Thus, the budget constraint becomes $c_{t+1} = h_{t+1} - (e_{t+1} + \varphi h_{t+1})n_{t+1}$.

3. Utility Maximization

We solve the utility maximization problem backwards.

3.1 Period 2

Each member of the t-th generation maximizes his or her utility with respect to c_{t+1}, n_{t+1}, and e_{t+1}, given h_{t+1}. The first order conditions are as follows:

If $c_{t+1} > 0$, $(h_{t+1} > 1)$

$$\begin{cases} e_{t+1} = \frac{A\varphi h_{t+1}}{1-A} \\ n_{t+1} = \frac{1-A}{\varphi h_{t+1}} \\ c_{t+1} = h_{t+1} - 1 \end{cases} \tag{2}$$

If $c_{t+1} = 0$ [4], $(h_{t+1} \leq 1)$

$$\begin{cases} e_{t+1} = \frac{A\varphi h_{t+1}}{1-A} \\ n_{t+1} = \frac{1-A}{\varphi} \\ c_{t+1} = 0 \end{cases} \tag{3}$$

3.2 Period 1

In the first period, a young individual chooses leisure to maximize the objective function. Here we define h_t as h^L to satisfy $\partial U_t/\partial l_t|_{l_t=1-A\beta} = 0$, and also as h^H to satisfy $\partial U_t/\partial l_t|_{l_t=0} = 0$.

<**case 1 :** $h_t \leq h^L$ > $\quad l_t = 1 - A\beta$.

<**case 2 :** $h^L \leq h_t \leq h^H$ > $\quad h_t^{\gamma+\delta} = \frac{1-l_t+\beta-A\beta}{\beta\theta(1-l_t)^{\beta}(\frac{A\varphi}{1-A})^{\gamma}}$.

<**case 3 :** $h_t \geq h^H$ > $\quad l_t = 0$.

[4] For the sake of simplicity, we suppose the minimum level of consumption is zero.

In case 1, because the level of parental human capital is too low, individuals do not have any incentive to accumulate human capital. Thus individuals choose the maximum leisure time available so long as the second period's consumption is non-negative. In case 2, individuals determine leisure as an interior solution. In case 3, because the marginal utility of leisure is always negative, individuals choose the minimum leisure time available, which is 0. Figure 1 describes the relationship between the parent's human capital and the level of effort (or the time of leisure) of their children.

4. Dynamics

Corresponding to the choice of the effort level in the preceding section, the evolution of human capital is determined:

$$<\textbf{case 1 :}\ h_t \le h^L > \qquad h_{t+1} = \theta(A\beta)^\beta(\frac{A\varphi}{1-A})^\gamma h_t^{\gamma+\delta} \ . \tag{4}$$

$$<\textbf{case 2 :}\ h^L \le h_t \le h^H > \qquad h_t^{\gamma+\delta} = \frac{h_{t+1}}{\theta(\beta h_{t+1} - \beta + A\beta)^\beta(\frac{A\varphi}{1-A})^\gamma} \ . \tag{5}$$

$$<\textbf{case 3 :}\ h_t \ge h^H > \qquad h_{t+1} = \theta(\frac{A\varphi}{1-A})^\gamma h_t^{\gamma+\delta} \ . \tag{6}$$

4.1 Steady states

Since h_t takes a constant value h in steady states, from (4), (5), and (6), h satisfies the following conditions:

$$\text{the maximum leisure steady state} \quad ; \quad h^{1-\gamma-\delta} = \theta(A\beta)^\beta(\frac{A\varphi}{1-A})^\gamma \ , \tag{7}$$

$$\text{the interior solution steady state} \quad ; \quad \beta^\beta h^{\gamma+\delta-1}(h-1+A)^\beta = \frac{1}{\theta(\frac{A\varphi}{1-A})^\gamma} \ , \tag{8}$$

$$\text{the minimum leisure steady state} \quad ; \quad h^{1-\gamma-\delta} = \theta(\frac{A\varphi}{1-A})^\gamma \ . \tag{9}$$

Four cases of the evolution of human capital exist (figure 2-5). As φ and θ increase, the graphs shift up. Figure 3 shows nontrivial multiple steady states. [5] This case consists of the maximum leisure and the interior solution steady states. The interior solution steady state E_2 shown in figure 3 describes a situation, where individuals enjoy leisure, too, at the same time as they are diligent in their studies. But this model connotes a possibility: Such situation may be accompanied by another situation, where no one has an incentive to make an effort. The cause of the possibility is a key distinguishing feature and is explained by characteristics of this model. The intuitional explanation is as follows: Because the total cost of education is given by the quality of schools e_{t+1} multiplied by the number of children n_{t+1}, a decrease in the income level h_t that is accompanied by an increase in the fertility rate (see (2)) raises the total cost of education, with the quality of schools fixed.

[5] See Appendix 1.

Thus the decrease in h_t provides individuals with an environment in which educational investment is more difficult. This change makes the educational investment decline more. In addition, from (1) the decrease in the quality of schools makes the level of children's effort decline because of its decline of the efficiency. Thus the small difference in the income level h_t causes the large difference in the next period's human capital h_{t+1}. This mechanism creates the multiple steady states and also a development trap.

4.2 Welfare

In figure 3, E_2 is a steady state that realizes a higher income level than E_1. However, E_1 is the maximum leisure steady state. Thus, when the economy has multiple steady states, this model requires welfare analysis.

Proposition 1 *Let us define the lifetime utility of an individual of the t-th generation born at the steady state E_1, as $\bar{U}_t^1$, and the lifetime utility of an individual of the t-th generation born at the steady state E_2, as $\bar{U}_t^2$. Then, we have the following relationship: $\bar{U}_t^2 > \bar{U}_t^1$*

Proof. See Appendix 2.

5. Conclusion

This paper clarifies the cause of the poverty trap that results from the negative correlation between income and fertility. To capture the population explosion that developing countries face, we incorporate the inverse relationship between fertility and income into this model. In low-income countries, high fertility rates make educational costs higher. Low income and higher educational costs reduce the income level children face in the next period and thus, reduce the opportunity cost of having children. This effect generates the multiple steady states.

Appendix 1. The existence of the multiple steady states

Since the graph of the LHS of (8) has one peak, if $\gamma + \delta + \beta < 1$, and (h^L, h^L_{+1}) and (h^H, h^H_{+1}) are below the 45-degree line, multiple steady states may exist (h^i_{+1} means the level of human capital in the next period, when the current period's level of human capital is h^i $(i = L, H)$). One of the multiple steady states satisfies (7) and the other satisfies (8). Thus, the existence of above multiple steady states requires two conditions to be met. One of these is the condition that leads (h^L, h^L_{+1}) and (h^H, h^H_{+1}) to be below the 45-degree line, and the other is the condition by which (8) has steady states.

First, we show the condition that leads (h^L, h^L_{+1}) and (h^H, h^H_{+1}) to be below the 45-degree line. If (h^L, h^L_{+1}) is below the 45-degree line, the parameters satisfy the following inequality:

$$(A\beta)^{\frac{\beta}{\gamma+\delta}} \Big(\theta(\frac{A\varphi}{1-A})^{\gamma}\Big)^{\frac{1}{\gamma+\delta}} < 1 \ . \tag{10}$$

If (h^H, h^H_{+1}) is below the 45-degree line, the parameters satisfy the following inequality:

$$(\frac{\beta}{1+\beta-A\beta})^{\frac{1-\gamma-\delta}{\gamma+\delta}} \Big(\theta(\frac{A\varphi}{1-A})^{\gamma}\Big)^{\frac{1}{\gamma+\delta}} < 1 \ . \tag{11}$$

From $A\beta < 1$, we have $A < 1/(1 + \beta - A\beta)$, so that if $\gamma + \delta + \beta < 1$ and Inequality (11) hold, Inequality (10) always holds.

Next, we show the condition by which (8) has steady states. When $\gamma + \delta + \beta < 1$ and Inequality (11) hold, if the maximum value of the LHS of (8) is greater than the RHS of (8), steady states exist that satisfy (8). Since the LHS of (8) takes a maximum value at $(1 - A)(\gamma + \delta - 1)/(\gamma + \delta + \beta - 1)$, the maximum value of the LHS of (8) is greater than the RHS of (8), when the following inequality holds:

$$\frac{\beta^{2\beta}(1-\gamma-\delta-\beta)^{1-\gamma-\delta-\beta}}{(1-A)^{1-\gamma-\delta-\beta}(1-\gamma-\delta)^{1-\gamma-\delta}} > \frac{1}{\theta(\frac{A\varphi}{1-A})^{\gamma}} \ .$$

A combination of Inequality (11) with the above inequality generates the following inequality:

$$\frac{\beta^{2\beta}(1-\gamma-\delta-\beta)^{1-\gamma-\delta-\beta}}{(1-A)^{1-\gamma-\delta-\beta}(1-\gamma-\delta)^{1-\gamma-\delta}} > \frac{1}{\theta(\frac{A\varphi}{1-A})^{\gamma}} > (\frac{\beta}{1+\beta-A\beta})^{1-\gamma-\delta} \ .$$

Thus, when the parameters satisfy this inequality, there are two stable steady states (Figure 3). Q.E.D.

Appendix 2. Proof of Proposition 1

$\bar{U}_t^1 (= U(l_t^1, c_{t+1}^1, e_{t+1}^1, n_{t+1}^1))$ and $\bar{U}_t^2 (= U(l_t^2, c_{t+1}^2, e_{t+1}^2, n_{t+1}^2))$ express the maximum lifetime utility at points E_1 and E_2, respectively.

Assume that an individual born at equilibrium E_2 chooses leisure time $\hat{l}_t$ that makes next period's income $\hat{h}_{t+1}^2$ and h_{t+1}^1 the same level. $\hat{l}_t$ is determined by the next equation.

$$\theta(1 - l_t^1)^\beta (e_t^1)^\gamma (h_t^1)^\delta = \theta(1 - \hat{l}_t)^\beta (e_t^2)^\gamma (h_t^2)^\delta$$

From the comparison of E_1 and E_2, $h_t^1 < h_t^2$ and $e_t^1 < e_t^2$ always holds. Therefore, it is clear that $l_t^1 < \hat{l}_t$ always stands. When the second period's income is the same level, the utility from the second period's choices is also the same level. Thus, the lifetime utility differential results from the differential of leisure time. Since $l_t^1 < \hat{l}_t$ always holds, the next relation holds.

$$\begin{aligned} \bar{U}_t^1 &= U(l_t^1, c_{t+1}^1, e_{t+1}^1, n_{t+1}^1) \\ &< U(\hat{l}_t, c_{t+1}^1, e_{t+1}^1, n_{t+1}^1) \\ &< U(l_t^2, c_{t+1}^2, e_{t+1}^2, n_{t+1}^2) \\ &= \bar{U}_t^2 \end{aligned}$$

Because the choices $(\hat{l}_t, c_{t+1}^1, e_{t+1}^1, n_{t+1}^1)$ at the steady state E_2 are different from the optimal choices that appear in the third formulation, the relation between the second and third formulation always holds. Q.E.D.

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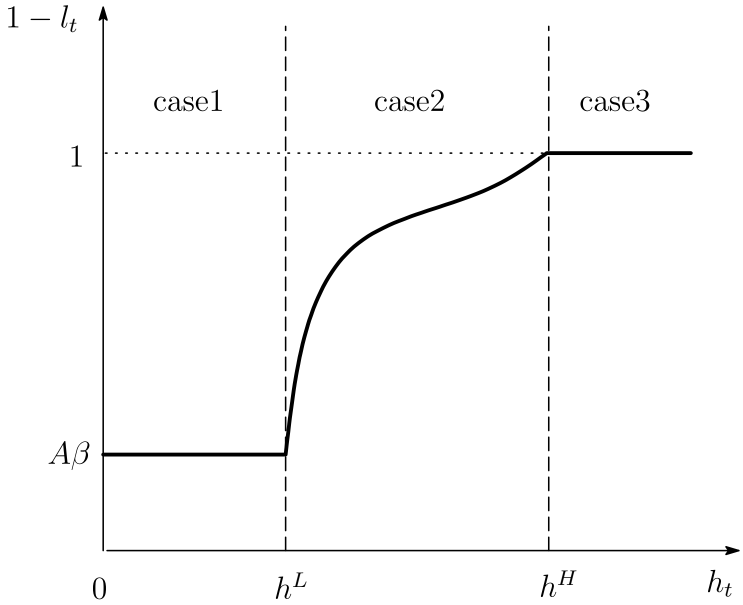

Figure 1: The relation between the parental human capital and the level of effort of their children

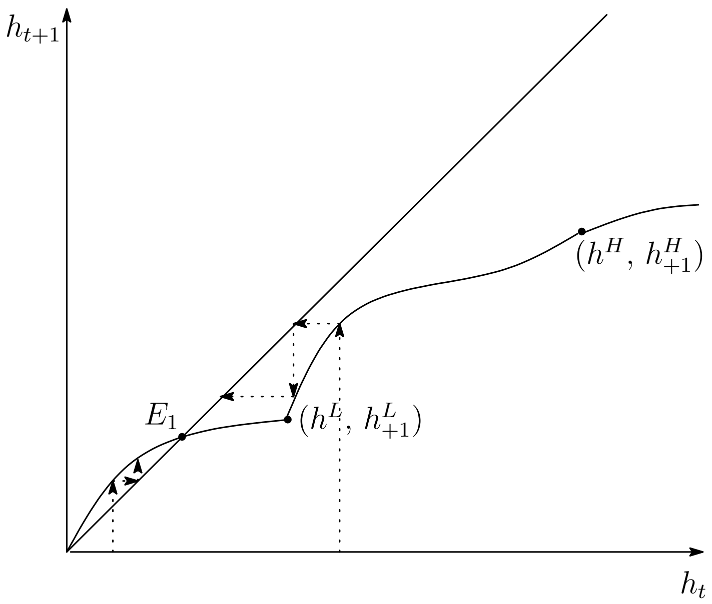

Figure 2: The evolution of human capital: The maximum leisure steady state E_1

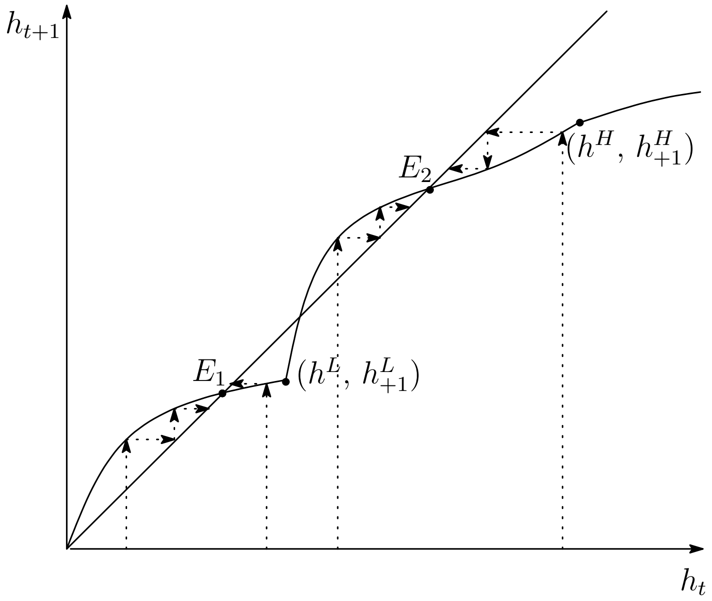

Figure 3: The evolution of human capital: The maximum leisure steady state E_1 and the interior solution steady state E_2

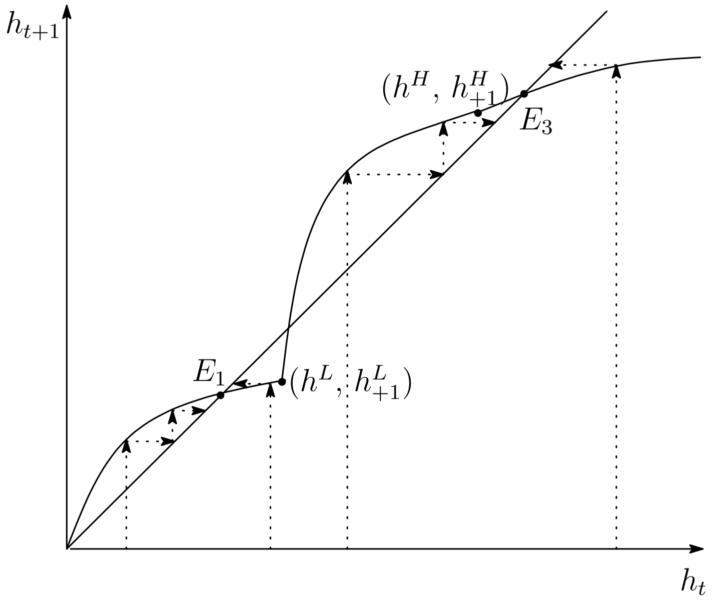

Figure 4: The evolution of human capital: The maximum leisure steady state E_1 and the minimum leisure steady state E_3

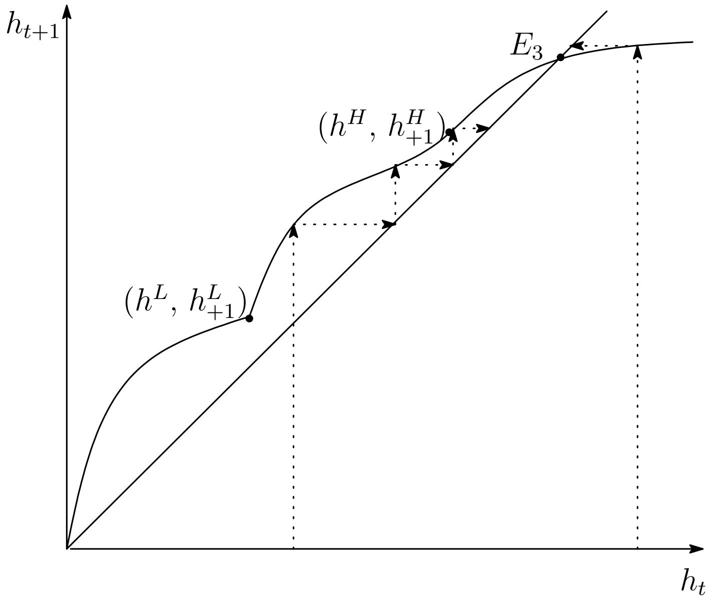

Figure 5: The evolution of human capital: The minimum leisure steady state E_3