E C O N O M I C S   B U L L E T I N

Information Aggregation Under Strategic Delay

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Abstract

In this paper, we show that consumers delay their buying to learn the unknown quality of a product. Agents receive imperfect but informative signals about the unknown quality. Then, each one simultaneously decides whether or not to buy the product in one of the two periods. Consumers with moderate tastes will strategically delay their buying to the second period even though they receive a good signal. They deduce the true quality by observing the mass of first period buyers. We avoid equilibrium non-existence problem by using agents with different private values.

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1 INTRODUCTION

The inter-temporal price discrimination literature (e.g. Stokey (1979), Tirole (1989)) shows that a monopolist that faces heterogenous consumers has to decrease the price at each period. The reason is that, after the high-taste consumers buy their unit demand and exit the market, the firm has no choice but to decrease the price to sell to the remaining lower-taste consumers. However, this makes some consumers delay their buying to the next period to take advantage of the lower prices. In this paper, by writing a model in which consumers value not only their private taste but also the quality of the product, we show that some consumers with moderate tastes delay their buying strategically to learn the unknown quality of the good.

Social learning literature shows that agents can learn the unknown state by observing the actions of others or the outcome of these actions. For example, Caplin and Leahy (1994) show that if each agent receives a stream of private information (correlated with the true state), then the rest can aggregate information and learn the unknown state by observing the mass of agents taking a specific action. Gunay (2008) shows that information aggregation is possible in an infinite state world even when there is externality. In this paper, our agents also aggregate information perfectly by observing the mass of first period buyers.

In our paper, we have a continuum of consumers who will decide whether or not to buy one unit of durable "uncertain quality" good in one of the two periods. A consumer's utility depends on her taste for quality and the good's unknown intrinsic quality. Consumers' taste for quality is uniformly distributed and they each observe a private binary signal about the quality. Agents with high enough taste will buy the good in the first period after receiving a good signal. In the second period, the rest will deduce the quality by observing the mass who bought the good during the first period. Since consumers know that uncertainty will be resolved, the ones who have moderate preferences will strategically delay buying the good to the second period.

In Caplin and Leahy (1994), (a continuum of) agents with extreme bad news know that if they change their actions, the state will be revealed. Therefore, all these agents may have incentive to delay their actions strategically. However, if they all delay their actions, then the state will not be revealed. Hence, a symmetric equilibrium (even a mixed one) may not exist. The reason for the possible equilibrium non-existence is that agents with extreme bad news are all identical. In our paper, by using heterogenous consumers, we do not run into the equilibrium non-existence problem. Some agents with high enough taste are better off by not delaying their action; hence, an equilibrium exists.

Our paper also falls into the strategic delay literature. In Hendricks and Kovenock (1989), Aoyagi (1998), and Frisell (2003), two agents/firms receive signals of different strength. Agents delay their actions hoping that the other agent received a more informative signal in the first two papers. In Frisell (2003), depending on externality, the firm with less information may move first or second. In our paper, agents with good news and high-enough tastes move first; that is, buy in the first period. Also, in our paper learning the state by observing a finite number of agents' actions is impossible unlike the aforementioned papers.

Gunay (2008b) analyzes the pricing decision of the firms when consumers have strategic delay incentives. However, in that paper, consumers do not have private information, and hence, they do not aggregate information (unlike this paper).

2 THE MODEL

A continuum of consumers are indexed by their private taste parameters θ_i which is uniformly distributed on $[0, 1]$. Each consumer decides whether to buy a good of unknown quality $\gamma \in [0, 1]$ in one of the two periods. The quality is drawn from a continuous distribution $F(\gamma)$ on [0,1]; nobody observes the true quality $\widetilde{\gamma}$ that will be fixed throughout the game.

A type θ_i consumer who owns the good will have an expected utility of $\theta_i E(\gamma)$ at each period where E denotes expectation. Note that each consumer's total valuation depends on her private taste and the common parameter (quality) unlike the inter-temporal price discrimination literature. We assume that our good is a durable good (no depreciation) so a consumer who buys the good in the first period will derive $2\theta_i E(\gamma)$ total expected utility.[1] If he does not buy the good, the normalized per period payoff to any type of consumer is $y \in (0, 1]$. There are no production costs. The price of the product is fixed.[2]

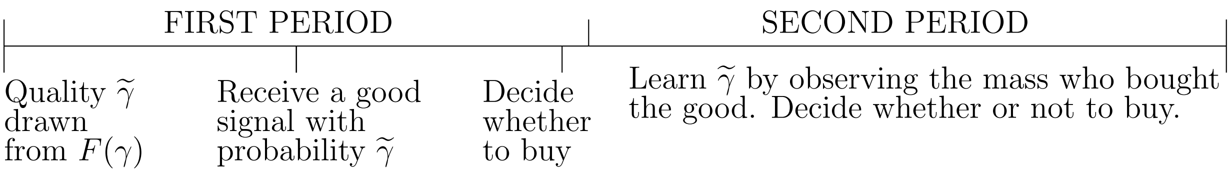

Figure 1: TIMING OF THE GAME

The timing of the game is as follows. At the beginning of period 1, each agent receives a (private) signal 1 with probability $\widetilde{\gamma}$ and a (private) signal 0 with probability $1 - \widetilde{\gamma}$.[3] Therefore, signals are imperfect but informative. After receiving the signal, each consumer updates her prior and then decide whether to buy the good at period 1. At period 2, after observing the mass who bought the good in period 1, agents (who have not already bought) decide whether or not to buy the good. They derive their payoffs and the game ends.

After getting a signal of 1 or 0, the consumers will have two different posteriors which we will denote as $F_g(\gamma)$ and $F_b(\gamma)$, respectively.

[1]We assume no discounting. If consumers discount the future, then waiting will be more costly; hence, this will decrease the strategic delay incentive (but not change the qualitative results).

[2]The intertemporal price discrimination literature shows that consumers expect that prices will decrease; hence, they delay their buying. In this paper, we want to show that even though there is no price effect, some consumers delay their buying. Hence, we assume that the prices are fixed. Also note that we implicitly assume price is fixed to zero. We could have had a slightly different modelling approach and could have assumed $y = 0$ but *price* is fixed to a positive value with the consumer's total surplus being equal to $2\theta_i E(\gamma) - price$. The results will not change with such a modelling approach.

[3]Signals are iid; however, we can make the signals dependent without changing any qualitative results.

2.1 Myopic Consumers

We will compare the myopic agents case with the strategic agents case. Our first proposition proves that the myopic agents can aggregate information by observing m, the mass of first period buyers. Note that they cannot delay buying strategically. We look for pure strategy Bayesian-Nash equilibrium in which consumers use threshold (cutoff) strategies.[4] Before proving our result, first we will state two assumptions.

Assumption 1: $\int_0^1 \gamma dF_b(\gamma) = E_b(\gamma) < y$
Assumption 2: $\int_0^1 \gamma dF_g(\gamma) = E_g(\gamma) > y$.

With Assumption 1, we ensure that even the agent who values the good most ex-ante, i.e., $\theta = 1$ agent, does not buy it after receiving a "zero" signal. In other words, anybody who receives a bad signal does not buy the good since $\theta \int_0^1 \gamma dF_b(\gamma) \leq \int_0^1 \gamma dF_b(\gamma) = E_b(\gamma) < y$, for all $\theta \in [0, 1]$. Bad signals are powerful enough to change the decisions.[5] Assumption 2 ensures that agent with $\theta = 1$ will buy the good after receiving a "one" signal. This, in turn, guarantees that a positive mass of agents will buy the good after receiving a "one" signal by continuity.

Proposition 1 *The agents who wait to decide in the second period will aggregate information by observing the first period buyers m. Specifically,*

$$\widetilde{\gamma} = \frac{m}{(1 - \frac{y}{E_g(\gamma)})} \tag{1}$$

Proof We will find the threshold type $\widehat{\theta_1^M}$ that will be just indifferent between buying and delaying. The superscript M indicates that agents are myopic, the subscript 1 indicates that the threshold level is for the first period. The threshold agent can be calculated from the equation below. The left hand side is the payoff from buying in the first period (after receiving a one signal and updating the prior).

$$\widehat{\theta_1^M} \int_0^1 \gamma dF_g(\gamma) = \widehat{\theta_1^M} E_g(\gamma) = y \tag{2}$$

Since the payoff is increasing in type θ, a mass of $(1 - \widehat{\theta_1^M})$ agents are potential buyers. However, by assumption 1, any agent who receives a "zero" signal will not buy the good. Only $\widetilde{\gamma}$ of them will receive a one signal; therefore, only a mass of $\widetilde{\gamma}(1 - \widehat{\theta_1^M})$ will buy it (by the law of large numbers). Then, anybody who observes the mass will learn the true quality $\widetilde{\gamma}$ since

$$m = \widetilde{\gamma}(1 - \widehat{\theta_1^M}) = \widetilde{\gamma}(1 - \frac{y}{E_g(\gamma)}) \Rightarrow \widetilde{\gamma} = \frac{m}{(1 - \frac{y}{E_g(\gamma)})}$$

[4]Because of assumption 1 and the fact that we use cutoff strategies, only the agents who are above a threshold (cutoff) type and **who received a signal 1** buy the good.

[5]Our results will hold by weakening Assumption 1 but this requires a little bit more work to prove the result. For instance, Example 1 below shows a case in which Assumption 1 does not hold.

Note that $(1 - \frac{y}{E_g(\gamma)}) \in (0,1)$ by Assumption 2, by $y \in (0,1]$, and by $F(.)$ being a continuous distribution with having a support on $[0,1]$.

Proposition 1 is a variant of Caplin and Leahy (1994) and Gunay (2008) since it proves that perfect information aggregation is possible in the absence of strategic delay.

If agents were not myopic, we had to deal with strategic delay. The next subsection deals with this issue.

2.2 Strategic Agents

We look for pure strategy Bayesian-Nash equilibrium in which consumers use threshold (cutoff) strategies. We will again find the threshold type $\widehat{\theta_1^S}$ that is indifferent between buying and delaying where the superscript S denotes strategic agents and subscript 1 denotes the period. For this threshold type, the expected payoff from buying (in the first period) is $2\widehat{\theta_1^S}E_g(\gamma)$, after receiving a one-signal.

The payoff from delaying is as follows. The delaying agent will get a payoff of y in the first period. In the second period, after learning the $\widetilde{\gamma}$ (proposition 2 will prove that agents will learn $\widetilde{\gamma}$), the $\widehat{\theta_1^S}$ agent will not buy the good if $\widetilde{\gamma} < \frac{y}{\widehat{\theta_1^S}}$, and hence, receives a payoff of y in the second period. She/he will buy the good if $\widetilde{\gamma} > \frac{y}{\widehat{\theta_1^S}}$, and receives a payoff of $\widetilde{\gamma}\widehat{\theta_1^S}$. Now we can write equation 3 which equates the "delaying" payoff to the "buying in the first period" payoff.[6]

$$y + \int_0^{\frac{y}{\widehat{\theta_1^S}}} y dF_g(\gamma) + \int_{\frac{y}{\widehat{\theta_1^S}}}^1 \gamma \widehat{\theta_1^S} dF_g(\gamma) = 2\widehat{\theta_1^S}E_g(\gamma) \tag{3}$$

We need the following assumptions that are modified from the myopic case. These assumptions ensure that zero signals are powerful enough to make even the $\theta = 1$ agent not to buy the good in the first period and to make sure that some agents will buy the good in the first period (after receiving a one signal).

Assumption 3

$$y + \int_0^y y dF_b(\gamma) + \int_y^1 \gamma dF_b(\gamma) > 2E_b(\gamma) \tag{4}$$

Assumption 4

$$y + \int_0^y y dF_g(\gamma) + \int_y^1 \gamma dF_g(\gamma) < 2E_g(\gamma) \tag{5}$$

Proposition 2 *(Information aggregation under strategic delay) Given assumption 3 and 4 hold, a mass of agents $\widetilde{\gamma}(1 - \widehat{\theta_1^S})$ will buy the good in the first period. The others will wait for the second period to decide. Everyone will learn the true $\widetilde{\gamma}$ in the beginning of the second period by observing the mass of buying agents.*

[6]The integrals are defined only for the region $[0,1]$. In all other regions, the value of integrals are zero.

Agents with high-enough tastes (and with good signal) buy the good without delaying. The rest delay their actions and aggregate the privately held information by observing the mass of buyers. Unlike Caplin and Leahy (1994), we do not have equilibrium non-existence problem since our agents are heterogenous.

Let us contrast our result with the intertemporal price discrimination literature, (e.g. Stokey (1979) and Tirole (1989)) which explains consumers' waiting by the "price decrease expectation" motives. In this note, we explain consumers' waiting by the strategic delay motives; i.e., consumers wait to learn more about the unknown quality. The difference between this paper and Gunay (2008b) is the fact that the latter one does not have any information aggregation feature since consumers do not receive signals.

Example 1 and Figure 2 show that agents with moderate tastes will delay their buying strategically although there is no price effect here.

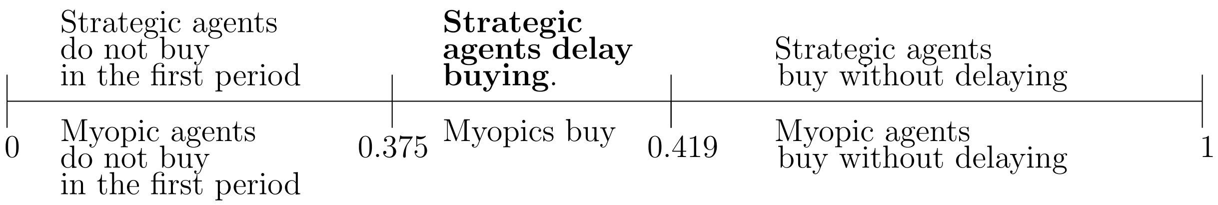

Figure 2: How do different types decide **after receiving a good signal** in the first period?

Example 1[7] Let $F(\gamma)$ be a beta distribution with parameters $\alpha = 1, \beta = 1$. Let $y = \frac{1}{4}$. For the myopic agents, the cutoff type that is indifferent between buying and delaying in the first period will be

$$E_g(\gamma)\widehat{\theta_1^M} = \frac{1}{4} \quad \Longrightarrow \quad \widehat{\theta_1^M} = (\frac{1}{4})(\frac{3}{2}) = \frac{3}{8} = 0.375 \tag{6}$$

since the agents who receive a good signal will have a beta posterior with parameters $\alpha = 2, \beta = 1$ with $E_g(\gamma) = \frac{2}{3}$.

The threshold type when agents act strategically can be found from solving the following equation:

$$\frac{2}{3}\widehat{\theta_1^S} + \frac{2}{3}\widehat{\theta_1^S} = \frac{1}{4} + \frac{1}{4}\int_0^{\frac{1}{4\widehat{\theta_1^S}}} 2p dp + \widehat{\theta_1^S}\int_{\frac{1}{4\widehat{\theta_1^S}}}^1 2p^2 dp \tag{7}$$

The unique answer to the solution of equation 7 is $\widehat{\theta_1^S} = 0.41941$. Then, agents who receive good signal in the range $[0.317, 0.419]$ will delay their buying strategically. By the law of large numbers, this mass is equal to $(0.41941 - 0.375)\widetilde{\gamma}$ (where $\widetilde{\gamma}$ will be learned by observing the total mass of buyers in the first period.)

[7]In this example, the assumption $E_b(\gamma) < y$ does not hold but this does not have any effect on the threshold levels.

3 CONCLUSION AND DISCUSSION

In a setup in which consumers decide whether and when to buy a good of unknown quality, we show that consumers with moderate tastes delay their buying strategically to the next period. Unlike the intertemporal price discrimination literature, we show that some agents wait strategically to learn the unknown quality. Our other result is that agents can still aggregate information even after some agents delay their buying.

We admit that fixing the price is a limitation of this paper; however, this is an intended choice to show that consumers delay their buying for reasons other than the price expectation motives. An extension of this paper should investigate how firms will choose their prices when consumers learn unknown quality by aggregating privately held information by observing the sales number in the previous period.

4 Appendix

Proof of Proposition 2 If we can show that there is a unique threshold type $\widehat{\theta_1^S}$ who is indifferent between buying and delaying, then everyone will learn the true $\widetilde{\gamma}$. This is because the mass m who buys will be equal to $\widetilde{\gamma}(1-\widehat{\theta_1^S})$, which implies $\widetilde{\gamma} = \frac{m}{(1-\widehat{\theta_1^S})}$ (by assumption 3 and by the law of large numbers).

To show uniqueness, we define a function $G(\theta)$.

$$G(\theta) = \underbrace{2\theta \int_0^1 \gamma dF_g(\gamma)}_{Buying\ payoff} - \underbrace{(y + y\int_0^{\frac{y}{\theta}} dF_g(\gamma) + \theta \int_{\frac{y}{\theta}}^1 \gamma dF_g(\gamma))}_{Delaying\ payoff} \tag{8}$$

If we can show that $G(0) < 0$, $G(1) > 0$ and the derivative of this function is positive, then we can conclude that $G(.)$ has a unique root which is $\widehat{\theta_1^S}$ by the setup of G function. It is straightforward to see from equation 8 that $G(0) = -2y < 0$. $G(1) > 0$ holds by assumption 4.

Next, we will show that the derivative of $G(.)$ is positive.

$$\frac{dG(\theta)}{d\theta} = 2\int_0^1 \gamma dF_g(\gamma) - \frac{d}{d\theta}[y\int_0^{\frac{y}{\theta}} dF_g(\gamma)] - \int_{\frac{y}{\theta}}^1 \gamma dF_g(\gamma) - \theta[\frac{d}{d\theta}\int_{\frac{y}{\theta}}^1 \gamma dF_g(\gamma)] \tag{9}$$

We can use Leibniz's rule to simplify the second and fourth term above:

$$\frac{dG(\theta)}{d\theta} = 2\int_0^1 \gamma dF_g(\gamma) - ydF_g(\frac{y}{\theta})(\frac{-y}{\theta^2}) - \int_{\frac{y}{\theta}}^1 \gamma dF_g(\gamma) + \theta(\frac{y}{\theta})dF_g(\frac{y}{\theta})(\frac{-y}{\theta^2}) \tag{10}$$

Since the second and fourth term cancels each other, we get:

$$\frac{dG(\theta)}{d\theta} = 2\int_0^1 \gamma dF_g(\gamma) - \int_{\frac{y}{\theta}}^1 \gamma dF_g(\gamma) > 0 \tag{11}$$

Hence, the function $G(.) = 0$ has a unique root which is $\widehat{\theta_1^S}$. Since agents can calculate $\widehat{\theta_1^S}$ and observe m, they can learn the true γ from the equation $\widetilde{\gamma} = \frac{m}{(1-\widehat{\theta_1^S})}$.

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