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Fixed Fee Licenses and Welfare Reducing Innovation: A Note

Shuji Takahashi
Gradudate School of Economics, Hitotsubashi University

Abstract

This note shows that the fixed fee oligopolistic license model developed by Kamien and Tauman (1986) yields the result that the private returns from innovation can be greater than the social returns when the number of firms in the industry is equal to or larger than 3. This result implies that an innovation does not always improve welfare, even when it is profitable for the innovator. We also show that the auction license model yields the same result as the fixed fee.

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1 Introduction

A widespread notion in studies on innovation and patent protection is that the social returns from innovation (the welfare gain) are greater than the private returns (the license income or the profit earned by the innovator). In other words, it is generally assumed that any profitable innovation is welfare-improving. For example, theoretical studies on patent protection, Green and Scotchmer (1995) and Denicolo (2000), are based on this notion.

A model under-pinning this notion that the social returns from innovation are greater than the private returns is the one developed by Arrow (1962), which has become famous and has been widely used in the literature.[1]

The reason behind this notion is familiar to us. The profitable innovation increases not only the producer's surplus, but also, due to lowering the price, increases the consumer's benefit which has not been appropriated by the innovator.

However, the reason why this notion is so widespread remains unclear.[2] Indeed, Arrow's results is simple, but, this simplicity depends on a factor — the perfect competition (or monopoly).

The purpose of this paper is to re-examine the proposition that the social returns from innovation are greater than the private returns using the fixed fee license model developed by Kamien and Tauman (1986). Their model assumes that the market structure before the innovation is the textbook case of an Cournot oligopoly with n-firms with identical in marginal costs.

We show the theoretically that for $n \geq 3$, the patent holder can earn a license income that is greater than the social returns from innovation. This result implies that when the development costs are taken into account, an innovation does not always improve overall welfare, even when it is profitable for the innovator.[3]

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[1] For a simple illustrations of Arrow's model, see Tirole (1990)[page. 390], and Dasgupta and Stiglitz (1980).

[2] In fact, Katz and Shapiro (1985), for example, have developed a model consisting of two firms with non-identical costs that shows that a minor innovation by a firm with extremely high marginal costs reduces overall welfare. This result is the reverse of this idea.

[3] It is a well-known fact that the amount of the license income is effected by many factors, although the welfare effects of each type of licensing have not been fully researched. There are many types of licensing that are more profitable than a fixed fee, as shown in Kamien (1992). Royalties can also be more profitable than a fixed fee license, as shown in Sen (2005).

The remainder of this paper is organized as follows. In section 2, we introduce Kamien and Tauman (1986)'s model. In section 3, we compare the private returns and the social returns, and show our results and the intuitive reason of our result. In section 4, we shows that a similar result is obtained in the auction model. The conclusion is in section 5.

2 Kamien and Tauman's Model and Equilibrium

The model used here is the fixed fee license model developed by Kamien and Tauman (1986). The model and equilibrium are as follows.

The Model Consider an industry with n firms and one patent holder. The inverse demand function is $p(Q) = a - Q$. The marginal costs of production with an existing technology are c.

There is a patentee with an innovation that lowers marginal costs from c to $c - \epsilon$. The patent holder can sell a license of this patent for the fixed fee α. The license income is $\pi_{PH} \equiv k\alpha$, where k denotes the number of licensees. Let us define $(a-c)/\epsilon$ as K (the larger K, the smaller ϵ for a given $a-c$).

The interaction between the patent holder and n firms is characterized by the following three-stage game. In the first stage, the patent holder chooses the value of α at which the license is offered to n firms. In the second stage, each firm chooses whether to buy or not to buy the license for a given value of α. Let S denote the set of firms which choose to buy in this stage, and let $k \equiv |S|$. The third stage consists of "Cournot quantity competition," where firm i's profit without fee payment is

$$\pi_i = \begin{cases} p(Q)q_i - (c-\epsilon)q_i & \text{for} \quad i \in S \\ p(Q)q_i - cq_i & \text{for} \quad i \notin S \end{cases}.$$

Equilibrium The game is solved by backward induction. In the third stage, the values of q_i and π_i are determined. The result is as follows:

$$q_i^*(k) = \begin{cases} \frac{\epsilon(K-k)}{n+1} + \epsilon & \text{for} \quad i \in S, \text{and } k \leq K \\ \frac{\epsilon(K-k)}{n+1} & \text{for} \quad i \notin S, \text{and } k \leq K \\ \frac{\epsilon(K+1)}{k+1} & \text{for} \quad i \in S, \text{and } k \geq K \\ 0 & \text{for} \quad i \notin S, \text{and } k \geq K \end{cases} \tag{1}$$

$$Q^*(k) = \begin{cases} \frac{\epsilon(k+nK)}{n+1} & \text{for} \quad k \leq K \\ k\frac{\epsilon(K+1)}{k+1} & \text{for} \quad k \geq K \end{cases} \tag{2}$$

In the second stage, the optimal value of k is determined.[4] The optimal values of k and α are given by the condition $\pi_i^*(k) - \alpha - \pi_j^*(k-1) = 0$ for $i \in S$, and $j \notin S$. From this condition, we obtain

$$\alpha(k) = \frac{n\epsilon^2}{(n+1)^2}\left[2K + (n+2) - 2k\right], \text{ for } k \leq K, \tag{3}$$

which yields the "private price" of the license.[5]

In the first stage, the patent holder maximizes $\pi_{PH}, = \alpha(k)k$ for k. The equilibrium values of k and π_{PH} are as follows:[6]

Let $A_0(K) \equiv \{n|2 \leq n \leq (2/3)(K+1)\}$, $A_1(K) \equiv \{n|(2/3)(K+1) \leq n \leq 2(K-1)\}$, $A_2(K) \equiv \{n|2(K-1) \leq n\}$, then

$$k^* = \begin{cases} n, & n \in A_0(K) \\ \frac{1}{2}\left(K + 1 + \frac{n}{2}\right), & n \in A_1(K) \\ K, & n \in A_2(K) \end{cases} \tag{4}$$

$$\pi_{PH}^*(k^*) = \begin{cases} \frac{2n^2\epsilon^2}{(n+1)^2}\left(K + 1 - \frac{n}{2}\right), & n \in A_0(K) \\ \frac{n\epsilon^2}{2(n+1)^2}\left(K + 1 + \frac{n}{2}\right)^2, & n \in A_1(K) \\ \frac{n(n+2)\epsilon^2}{(n+1)^2}K, & n \in A_2(K) \end{cases} . \tag{5}$$

3 The Private Versus the Social Returns from Innovation

The Social Returns Now, we derive the social returns from innovation, which are not considered by Kamien and Tauman (1986). Let W(k) denote the overall welfare when the number of licensees is k, and let SV(k) denote the welfare improvement from the license. That is, SV(k) $\equiv$ W(k) $-$ W(0), where W(k) $= \{(a + P(Q^*(k)))/2\}Q^*(k) - [cQ^*(k) - k\epsilon q_i^*(k)]$.

[4] Below, for simplicity of exposition, we omit the case $k \geq K$.

[5] This equation is the same as the original equation (A.5) in Kamien and Tauman (1986)[page. 486].

[6] Our equations (4) and (5) are *not* the same as the original equations (1) and (2) in Kamien and Tauman (1986)[page. 475-476]. The original equations are misprinted. For example, the original equation (1) is not in accord with the equation in their proof in the appendix, and their original equation (2) for the case $n \in A_0(K)$ does not yield their numerical example of $5.12\epsilon^2$. Of course, our equations (4) and (5) are the correct versions.

By substituting (1) and (2) into SV(k), we obtain

$$\mathrm{SV}(k) = \beta(k)k \quad (6)$$

$$\beta(k) \equiv \frac{\epsilon^2}{(n+1)^2}\left[(n+2)K - \left(n+\frac{3}{2}\right)k + (n+1)^2\right]. \quad (7)$$

Conveniently, function (6) does not contain a constant term and takes a similar form as $\pi^*_{PH} = \alpha(k)k$. We can regard $\beta(k)$ as the "social price" of the license. By substituting (4) into (6), we obtain

$$\mathrm{SV}(k^*) = \begin{cases} \frac{n\epsilon^2}{(n+1)^2} & \text{for} \quad n \in A_0(K) \\ \frac{\epsilon^2}{(n+1)^2}\frac{1}{2}\left(K+1+\frac{n}{2}\right) & \text{for} \quad n \in A_1(K) \\ \frac{K\epsilon^2}{(n+1)^2} & \text{for} \quad n \in A_2(K) \end{cases} \quad (8)$$

The Private Versus the Social Returns from Innovation Let us compare $\pi^*_{PH}(k^*)$ and SV(k^*).[7] Let D represent the fixed development costs of this technology. The innovation is profitable if $D \leq \pi^*_{PH}$. We obtain the following proposition.

Proposition 1 *(i) For $n \geq 3$, there always exists a value of K which yields $SV(k^*) < \pi^*_{PH}(k^*)$. (ii) Directly from (i), it follows that, for $n \geq 3$, there exists a case in which the innovation is profitable for the innovator but reduces overall welfare. That is, for $n \geq 3$, there exists a combination of values for K and D which satisfies both $D \leq \pi_{PH}(k^*)$ and $SV(k^*) - D < 0$ simultaneously.*

proof (i) The sign of $\mathrm{SV}(k^*) - \pi^*_{PH}(k^*)$ is as follows:

$$\mathrm{sign}\left[\mathrm{SV}(k^*) - \pi^*_{PH}(k^*)\right] = \begin{cases} \mathrm{sign}\left[B_0(n,K)\right] & \text{for} \quad n \in A_0(K) \\ \mathrm{sign}\left[B_1(n,K)\right] & \text{for} \quad n \in A_1(K) \\ + & \text{for} \quad n \in A_2(K) \end{cases} \quad (9)$$

$$B_0(n,K) \equiv -(n-2)K - \frac{3}{2}n + 1 + n^2 \quad (10)$$

$$B_1(n,K) \equiv \left(\frac{5}{2} - n\right)K + \left(\frac{n^2}{2} + \frac{1}{4}n + \frac{1}{2}\right). \quad (11)$$

[7] The direct calculation for $\mathrm{SV}(k^*) - \pi^*_{PH}(k^*)$ is tedious. We can avoid this by calculating $\beta(k^*) - \alpha(k^*)$ instead. The calculation of $\pi^*_{PH}(k^*) - \mathrm{SV}(k^*)$ or $\beta(k^*) - \alpha(k^*)$ both yields Proposition 1

The sign of $B_0(n, K)$ is negative for $K \geq (n-2)^{-1}(n^2 - (3/2)n + 1)$ and $n \geq 3$.

It is easy to find a value for K which satisfies $n \in A_0(K)$ and $K \geq (n-2)^{-1}(n^2 - (3/2)n + 1)$. The sign of $B_1(n, K)$ is also determined.

(ii) As shown in (i), we can find a value for n and K, which satisfies $\pi^*_{PH}(k^*) > \mathrm{SV}(k^*)$. It is obvious that $\pi^*_{PH}(k^*) - D > \mathrm{SV}(k^*) - D$. Let us consider a profitable innovation such as $\pi^*_{PH}(k^*) - D = 0$. From this, we obtain $0 > \mathrm{SV}(k^*) - D$. [QED]

Example Consider the case $n = 4$ and $K = 5$.[8] The equilibrium is $k^* = 4$, $\pi^*_{PH}(k^*) = 5.12\epsilon^2$, and $\mathrm{SV}(k^*) \approx 5.28\epsilon^2$. Thus, in this case, $\pi^*_{PH}(k^*) < \mathrm{SV}(k^*)$. However, the result is reversed when $n = 4$ and $K = 6$. In this case, the equilibrium is $k^* = 4$, $\pi^*_{PH}(k^*) = 6.4\epsilon^2$, and $\mathrm{SV}(k^*) \approx 6.24\epsilon^2$.

Intuitive Explanation We can show the the intuitive explanation of Proposition 1. The key factor is the profit of the firm using the *existing* technology, a similar to "profit stealing".[9]

The SV is the sum of the consumer's gain and the firm's gain, that is, $\mathrm{SV}(k) \equiv \{\mathrm{CS}(k) - \mathrm{CS}(0)\} + \{k\pi_i(k) + (n-k)\,\pi_j(k) - n\pi_j(0)\}$, for $i \in S$, and $j \notin S$. From this equation, we obtain

$$\mathrm{SV}(k) - \left\{k(\pi^*_i(k) - \pi^*_j(k))\right\} \equiv \{\mathrm{CS}(k) - \mathrm{CS}(0)\} + \left\{n(\pi^*_j(k) - \pi^*_j(0))\right\}. \quad (12)$$

Recall that the value of π^{PH*} is determined by $k(\pi^*_i(k) - \pi^*_j(k-1))$ which take a close value to the second term of LHS of (12). Therefore, the value of LHS of (12) can take a close value to $\mathrm{SV}(k^*) - \pi^{PH*}(k^*)$.

The first term of RHS is the consumer's gain which is positive or zero. But, the second term of RHS, which is the firm's gain using the existing technology, is negative, since $\pi^*_j(k)$ is decreasing in k. Thus, if the later term dominates the former, the sign of $\mathrm{SV}(k^*) - \pi^{PH*}(k^*)$ can be negative, and Kamien and Tauman (1986)'s model really yields such a domination.

4 Auction Licenses

[8] This is the case considered by Kamien and Tauman (1986)[page. 478].

[9] As for the profit stealing and patent system, see Varian et al. (2004)(p.65-66)

Finally, we refer to the auction model developed by Katz and Shapiro (1986). The mathematical characteristics of the result is simpler in the auction model than in the fixed fee model.

In the auction model, the amount of the license income is $k(\pi_i^*(k) - \pi_j^*(k)) = \pi^{PH}(k)$. By substituting this equation into (12), we obtain, for $k \leq K$

$$\mathrm{SV}(k) - \pi^{PH}(k) = \{\mathrm{CS}(k) - \mathrm{CS}(0)\} + \left\{n(\pi_j^*(k) - \pi_j^*(0))\right\}. \qquad (13)$$

The equilibrium number of licensee is as follows.[10]

$$k^* = \begin{cases} n & \text{if} \ \ K \geq \frac{3n-1}{2} \\ \frac{n+1+2K}{4} & \text{if} \ \ \frac{3n-1}{2} \geq K \geq \frac{n+1}{2} \\ K & \text{if} \ \ \frac{n+1}{2} \geq K \end{cases} \qquad (14)$$

By evaluating the sign of the RHS of (13) in k^*, we obtain the following proposition.[11]

Proposition 2 *(i) For $n > 2$,*

$$SV(k^*) - \pi_{PH}^*(k^*) \gtreqless 0 \Leftrightarrow K \lesseqgtr g(n),$$

where

$$g(n) \equiv \frac{n+1}{2}\frac{2n+1}{2n-1}.$$

(ii) For $2 < n < \infty$, $g(n) \in (\frac{n+1}{2}, \frac{3n-1}{2})$.

Proposition 2(i) implies that the private returns exceeds the social one when the cost reduction of the innovation is small ($K < g(n)$), and vis versa. Proposition 2(ii) implies the threshold value $g(n)$ locates in the interval $(\frac{n+1}{2}, \frac{3n-1}{2})$ which is exactly equal to the interior of the interval of K of the second case of (14).

5 Conclusion and Remark

[10]For the equation (14), see Kamien (1992)
[11]The proof is in Appendix.

This paper reconsidered the idea — first formally presented by Arrow (1962) and uncritically accept in many studies — that any profitable innovation leads to an improvement in welfare. The analysis was based on the fixed fee oligopolistic model developed by Kamien and Tauman (1986). We find that for $n \geq 3$, the patent holder can earn a license income that is greater than the social returns from the innovation.

Note also that our result is obtained from a very simple model. No complicated model is required to analyze a kind of the "excessive" private returns or "insufficient" social returns — a topic on which little research has been done.

Our result may also throw a new light on the effects of spillovers. Aoki and Tauman (2001), for example, have shown that spillovers from licensing do not always reduce the incentive to license, but always reduce the license income. We may expect spillovers to correct for excessive private returns and raise the overall welfare of society.

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Appendix

A Calculations for $\mathbf{W}(k)$

As shown in the previous part, the definition of $\mathrm{W}(k)$ is

$$\mathrm{W}(k) = \{(a + P(Q^*(k)))/2\}Q^*(k) - cQ^*(k) + k\epsilon q_i^*(k), \text{for } k \leq K. \quad (15)$$

Among the terms of (15), q_i^* is given by (1), and Q^* is given by $kq_i^* + (n-k)q_j^*$ for $i \in S$, and $j \notin S$. From (1), we obtain

$$Q^* = \frac{k\epsilon + n\epsilon K}{n+1}, \text{ for } k \leq K. \quad (16)$$

By substituting (1) and (16) into (15), we obtain

$$\begin{aligned} \mathrm{W}(k) &= \frac{1}{2}\left\{\frac{\epsilon^2}{2}\frac{(2(n+1)K - k - nK)(k + nK)}{(n+1)^2} + \frac{k\epsilon^2(K-k)}{(n+1)} + k\epsilon^2\right\} \\ &= \frac{\epsilon^2}{2(n+1)^2}\left\{2Kk + (n+2)nK^2 - k^2\right\} + \frac{k\epsilon^2(K-k)}{(n+1)} + k\epsilon^2 \quad (17) \end{aligned}$$

The social value ($\mathrm{SV}(k)$) is

$$\begin{aligned} \mathrm{SV}(k) &\equiv \mathrm{W}(k) - \mathrm{W}(0) \\ &= \frac{\epsilon^2}{2(n+1)^2}\left(2Kk + n(n+2)K^2 - k^2\right) + \frac{k\epsilon^2(K-k)}{(n+1)} + k\epsilon^2 - \frac{n(n+2)\epsilon^2K^2}{2(n+1)^2} \\ &= \frac{\epsilon^2}{2(n+1)^2}\left(2Kk - k^2\right) + \frac{k\epsilon^2(K-k)}{n+1} + k\epsilon^2 \\ &= \frac{k\epsilon^2}{(n+1)^2}\left[(n+2)K - \left(n + \frac{3}{2}\right)k + (n+1)^2\right] \quad (18) \end{aligned}$$

The equation (18) is the equation (6) in the main text.

B Proof of Proposition 2

(i) At first, let determine the value of two terms of the RHS of (13) for $k \leq K$. The value of the first term is

$$\mathrm{CS}(k) - \mathrm{CS}(0) = \frac{\epsilon^2 k}{2(n+1)^2}(k + 2nK) \quad (19)$$

which is obtained from $\text{CS}(k) \equiv 1(1/2)Q^*(k)$ and (2). The value of the second term is

$$n(\pi_j^*(k) - \pi_j^*(0)) = -n\frac{\epsilon^2 k}{(n+1)^2}(2K - k) \tag{20}$$

which is obtained from (9b) shown in Kamien (1992).

Next, we determine the sign of $\text{SV}(k) - \pi^{PH}(k)$ in the equilibrium value of k. The equation (19) and (20) implies

$$\text{Sign}\left(\text{SV}(k) - \pi^{PH}(k)\right) \gtreqless 0 \Leftrightarrow \text{Sign}\left(k - \frac{2n}{2n+1}K\right) \gtreqless 0 \tag{21}$$

The equilibrium value of k is shown in (14). By substituting (14) into the RHS of (21), we obtain the following result:

$$\text{Sign}\left(\text{SV}(k) - \pi^{PH}(k)\right) = \begin{cases} - & \text{for} \quad K \geq \frac{3n-1}{2} \\ -orzero & \text{for} \quad K \geq g(n) \\ + & \text{for} \quad K < g(n) \\ + & \text{for} \quad \frac{n+1}{2} \geq K \end{cases} \quad \text{for } n > 2. \tag{22}$$

The formula (22) is that of Proposition 2.

Some calculation yields $g(n) < (3n-1)/2$ for $n \geq 3$, $g(n) > (n+1)/2$ for $n > 2$. Therefore, we obtain $(n+1)/2 < g(n) < (3n-1)/2$ for $n > 2$.
(ii) We obtain $g(n) \in (\frac{n+1}{2}, \frac{3n-1}{2})$, directly from the following calculation:

$$\lim_{n\to\infty}\left(\frac{n+1}{2} - g(n)\right) = 0, \quad \text{and} \lim_{n\to 2}\left(\frac{3n-1}{2} - g(n)\right) = 0.$$

[QED]

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