E C O N O M I C S B U L L E T I N

Endogenous lifetime and economic growth revisited

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Abstract

Chakraborty [Journal of Economic Theory, 2004] introduces endogenous mortality in a two period overlapping generations model by postulating that the probability of surviving from the first period to the second depends on tax−funded public health. His central result on the existence of multiple steady states (including development traps) summarized in Proposition 1 is incorrect. This paper presents the correct proposition and its proof, and in the process, uncovers several new, interesting results. Contrary to Chakraborty's analysis, high mortality yet high capital nations may not be able to escape the poverty trap. Interestingly, TFP growth can help economies escape the vicious cycle of poverty.

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1 Introduction

A question that continues to intrigue macroeconomists and policymakers alike is this: Why is Africa so different from the rest of the world?[1] Indeed, in a recent speech, Alan Greenspan[2] touches on this question: "While, from a global perspective, wealth and the overall quality of life have risen, that success has not been evenly distributed across regions or countries. The economies of East Asia are often-repeated success stories. Some, including China, Malaysia, South Korea, and Thailand, stand out not only as growing very strongly, but also as having seen the greatest declines in poverty rates. ....But, sadly, the story in Africa has been quite different. Levels of per capita income in that continent have actually fallen. The poverty rate, which in 1970 matched the rate in Asia at the time, is estimated to have doubled to 40 percent by 1998." In fact, as Haber, North, and Weingast (2003) point out, two-thirds of African countries have either stagnated or shrunk in real per capita terms since the onset of independence in the early 1960s.

Easterly and Levine [1997] were among the first to pose this question; to them, the high ethnic fractionalization in the "dark continent" largely explains Africa's woes. In a fairly influential recent paper "Endogenous Lifetime and Economic Growth", Chakraborty [2004] indirectly revisits this question by exploring a new connection between pervasive ill-health and economic growth. As he points out, the probability that a average 15-year old would die before reaching age 60, was three times as high in sub-Saharan Africa as in the richer OECD economies. He goes on to suggest that when life expectancy is low, agents would place little emphasis on the future, and hence, would invest little in productive long-term assets, thereby getting stuck in a low level of real activity. In turn, poor health is largely explained by low public health spending which in turn is a direct outcome of the low level of real activity. In short, Africa is caught in a development trap induced by poor health.

More specifically, Chakraborty introduces endogenous mortality in an otherwise standard overlapping generations model with production of the classic Diamond [1965] variety. In particular, the probability with which young agents survive on to the second period depends on public health expenditures which are in turn funded by income taxes on labor income. The main result in Chakraborty [2004] contained in his Proposition 1 states that when the output elasticity of capital is high a development trap appears and countries differing in health and/or physical capital may not converge to similar living standards. It is important to note that such poverty traps do not arise in the standard Diamond model without the aid of several strong assumptions on preferences and technology that Chakraborty does not make.[3]

In this paper, we point out a crucial error in his statement of Proposition 1. We go on to correct the omission and in the process we uncover several new, interesting results. Contrary to his analysis, it turns out that high mortality nations even if they have high levels of capital may not be able to escape the poverty trap. This implies that the vicious cycle of poverty is far more persistent than what his analysis suggests. In addition, we show that the level of technological

[1] In the parlance of modern growth theory, this is often summarized as the puzzle of the persistent negative "Africa dummy" in cross-country growth studies.

[2] Remarks by Federal Reserve Chairman Alan Greenspan at Banco de Mexico's Second International Conference "Macroeconomic Stability, Financial Markets, and Economic Development," Mexico City, Mexico, November 12, 2002.

[3] See the discussion in Azariadis (2004).

development plays a crucial role in determining the persistence of the development trap. First, when technological development increases, a new long run equilibrium with higher real activity and reduced mortality appears. Second, this increase in technical efficiency drastically reduces the level of capital required to escape the development trap. Our results therefore suggest that high mortality and low capital nations can escape the low activity trap by raising their TFP.

2 The Model

We use the exact model outlined in Chakraborty [2004]. Here, young agents are born each period and inelastically supply one unit of labor, earning a wage w. The probability of a young agent surviving to the next period is given by the non-decreasing concave function $\phi_t \equiv \phi(h_t)$, where h_t denotes her health capital. We assume that $\phi(0) = 0$, $\lim_{h\to\infty} \phi(h) = \beta \leq 1$ and $\lim_{h\to 0} \phi(h) = \gamma < \infty$. Public health expenditure in period t is financed through a proportional tax $\tau_t \in (0,1)$ such that $h_t = \tau_t w_t$. At the end of each period, the young agents deposit their savings in a mutual fund, which earns a gross return on its investments of R_{t+1}, thus guaranteeing a gross return of $\hat{R}_{t+1} = R_{t+1}/\phi_t$ for the surviving old. The young agents born in period $t+1$ are not affected by the health capital of the previous generation.

A person born in period t maximizes her expected lifetime utility

$$U_t = \ln c_t^t + \phi_t \ln c_{t+1}^t,$$

subject to the budget constraints

$$c_t^t \leq (1-\tau_t) w_t - z_t, \; c_{t+1}^t \leq \hat{R}_{t+1} z_t.$$

Optimal savings takes the form $z_t = (1-\tau_t)\sigma_t w_t$, where $\sigma_t \equiv \frac{\phi_t}{1+\phi_t}$.

Final goods are produced using a constant returns to scale Cobb-Douglas technology $F(K,L) = AK^\alpha L^{1-\alpha}$, where $A > 0$ and $\alpha \in (0,1)$. Perfect competition ensures that $w_t = (1-\alpha) A k_t^\alpha$ and $R_t = 1 + \alpha A k_t^{\alpha-1} - \delta$, where k is the capital-labor ratio and δ is the depreciation rate of physical capital.

In the model set up above, using $z_t = k_{t+1}$, it follows that the general equilibrium law of motion for the capital-labor ratio is given by

$$k_{t+1} = (1-\tau)(1-\alpha)\sigma(k_t) A k_t^\alpha, \tag{1}$$

given $k_0 > 0$ and $h = \tau w(k) = \tau A(1-\alpha) k^\alpha$. We are now ready to re-state Proposition 1 of Chakraborty [2004].

3 Results

Before we proceed to make corrections, we restate his Proposition 1. (i) below.

Chakraborty's Proposition 1. (i). *The dynamic system described by (1)* [his equation (10)] *possesses two steady states* $\{0, \bar{k}\}$ *when* $\alpha < 1/2$, *only the positive one being asymptotically stable. When* $\alpha > 1/2$, *three steady states exist* $\{0, \bar{k}_1, \bar{k}_2\}$ *with* $\bar{k}_2 > \bar{k}_1$; *the two extreme steady states are asymptotically stable, the intermediate one is not.*

By means of a simple counterexample, it is easy to demonstrate that the *second* part of this result is incorrectly stated. Suppose $A = 5$, $\alpha = 0.55$, $\beta = 0.5$, $\tau = 0.2$, and $\phi(h) = \beta h/(1+h)$, a functional form for $\phi(h)$ that satisfies all of Chakraborty's assumptions on ϕ (as stated by him in his footnote 4). Then, it is easy to check that there exists a unique steady state, $k = 0$.

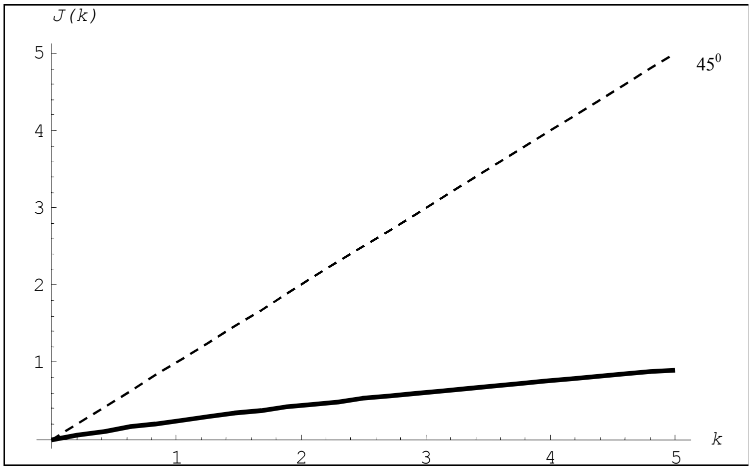

Figure 1: $J(k)$ versus k

As Figure 1 illustrates, the error appears because Chakraborty ignores the possibility that (1) may forever lie below the 45^o line thereby producing only the trivial steady state, $k = 0$. To see this, define $J(k)$ to be the right hand side of (1). Then, as proven in Lemma A.1 (Appendix A), $J(0) = 0$, $J'(k) \geq 0 \; \forall k \geq 0$, $\lim_{k\to\infty} J(k)/k < 1$ and $\lim_{k\to 0} J'(k) = 0$ if $\alpha > 1/2$. In other words, the J locus starts at 0, is non-decreasing, and eventually falls below the 45^o line; additionally, the 0 steady state is locally stable. From this Chakraborty erroneously concludes that the J locus "intersects the 45^o line from below at least once before falling below it." Indeed as we demonstrate below, $\alpha > 1/2$ is simply necessary but not sufficient for three steady states to exist.

Since the first part of the proposition (the case of $\alpha < 1/2$) is correct, henceforth we will focus our discussion only on the second part (the case of $\alpha > 1/2$).

The correct statement should read:

Proposition 1. (i). Suppose $\alpha > 1/2$ *and suppose* $k^* = \arg\max(J(k) - k)$ *satisfies* $J(k^*) > k^*$. Then at least three steady states exist. Naming the three smallest $\{0, \bar{k}_1, \bar{k}_2\}$ with $\bar{k}_2 > \bar{k}_1$; the two extreme steady states are asymptotically stable, the intermediate one is not.[4] If $J(k^*) = k^*$, there are exactly two steady states $\{0, \bar{k}_1\}$, where $\bar{k}_1$ is neither a repellor nor an attractor.

It turns out that even with the specific assumptions that Chakraborty makes, it is not possible to write down necessary and sufficient parametric conditions under which $J(k^*) > k^*$. However we can provide a sufficient condition under which there exists a $\hat{k}$ such that $J(\hat{k}) > \hat{k}$, implying the presence of at least three steady states. This sufficient condition is provided in the corollary below, and the proof is in the appendix.

[4]Note that Chakraborty [2004] ignores the possibility that there might be more than 3 steady states. Without specifying the ϕ function precisely, it is not possible to determine the maximum number of steady states.

Corollary to Proposition 1. (i). Suppose $\alpha > 1/2$ and

$$A > \frac{1}{(1-\alpha)} \left((1-\tau) \frac{\phi(\tau)}{1+\phi(\tau)} \right)^{-\alpha}. \tag{2}$$

Then at least three steady states exist. Naming the three smallest $\{0, \bar{k}_1, \bar{k}_2\}$ with $\bar{k}_2 > \bar{k}_1$; the two extreme steady states are asymptotically stable, the intermediate one is not.

This corollary provides a condition which is easy to check for a given ϕ function and set of parameters. For example, if $\phi = \beta \frac{h}{1+h}$ and for the parameter values provided above, namely $\alpha = 0.55$, $\beta = 0.5$ and $\tau = 0.2$, condition (2) becomes $A > 10.3$. Setting $A = 15$, this example provides three steady states 0, $\bar{k}_1 = 2.456 \times 10^{-6}$, and $\bar{k}_2 = 1.90$. Note that condition (2) always restricts A non-trivially, since $\frac{1}{(1-\alpha)} \left((1-\tau) \frac{\phi(\tau)}{1+\phi(\tau)} \right)^{-\alpha}$ is always strictly greater than 0.

While it is only possible to provide a sufficient condition under which there are at least three steady states in the general case, it is possible to provide a complete characterization of conditions under which (1) would admit at least three steady states for the special functional form for $\phi(.)$ mentioned in Chakraborty [2004] footnote 4. This characterization is provided in the corollary below. The proof of the corollary is relegated to the appendix.

Corollary to Proposition 1. (ii). Suppose $\phi(h) = \beta h/(1+h)$. Also suppose $\alpha > 1/2$ and

$$(A(1-\alpha))^{\frac{1}{2\alpha-1}} \left(\frac{(1-\alpha)}{\alpha} (1-\tau) \beta\tau\right)^{\frac{\alpha}{2\alpha-1}} \left(\frac{2\alpha-1}{1-\alpha}\right) > \tau (1+\beta) \tag{3}$$

holds. Then at least three steady states exist. Naming the three smallest $\{0, \bar{k}_1, \bar{k}_2\}$ with $\bar{k}_2 > \bar{k}_1$; the two extreme steady states are asymptotically stable, the intermediate one is not.

Notice that while (3) reveals that $\alpha > 1/2$ is necessary for the result to hold, since the left hand side of (3) is negative if $\alpha < 1/2$, clearly $\alpha > 1/2$ is not sufficient; for example, a sufficiently high value for A is needed.

In light of the amended proposition, some changes to the discussion on page 124 in Chakraborty [2004] are in order. While it is true that a poverty and ill-health trap exists for $\alpha > 1/2$, it is no longer given that it is possible to escape this trap if a country starts out with a high enough capital stock. In fact, as equation (3) reveals, unless there is a sufficiently high level of technological development (high enough A), the zero capital poverty trap is the *only* steady state, and it is stable.

Chakraborty limits his discussion of the implications of increasing A to the case where $\alpha < 1/2$. Our analysis reveals significant new benefits of doing so even when $\alpha > 1/2$ obtains. For general ϕ, it is easy to verify that $\partial J(k)/\partial A > 0$. The implication, as depicted in Figure 2, is strong. Consider two countries identical in all respects except that country $\mathcal{M}$ has a higher A than country $\mathcal{N}$. In such a setting, it is possible that country $\mathcal{N}$ is forever caught in the poverty trap (the only equilibrium) while country $\mathcal{M}$ seizes the potential to approach a high long run level of real activity (an equilibrium unavailable to country $\mathcal{N}$). This situation corresponds to countries $\mathcal{N}$ and $\mathcal{M}$ having $A = A_0$ and $A = A_1$ respectively in Figure 2. Perhaps more interestingly, when there are three steady states, namely $\{0, \bar{k}_1, \bar{k}_2\}$, increasing A decreases the intermediate steady state, $\bar{k}_1$, and this lowers the initial capital stock required to escape the twin traps of poverty and ill-health! This last benefit is in addition to the increase in $\bar{k}_2$, a point discussed by

Chakraborty. These last two effects are illustrated in Figure 2 as a country increases A from A_1 to A_2.

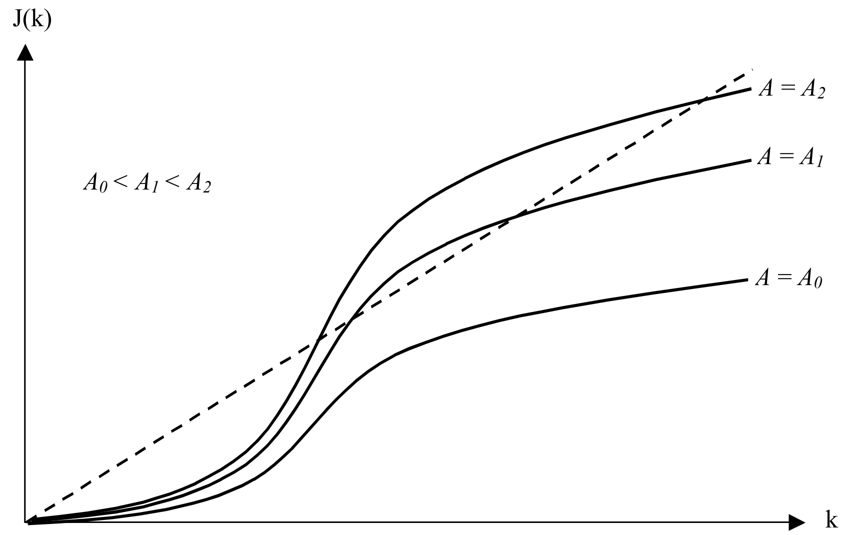

Figure 2: $J(k)$ versus k for $\alpha > \frac{1}{2}$ and different values of A.

A Appendix

A.1 Proof of Corollary to Proposition 1. (i).

Choose $\hat{k} = \left(\frac{1}{(1-\alpha)A}\right)^{\frac{1}{\alpha}}$ and note that $w(\hat{k}) = 1$. We wish to prove that $J(\hat{k}) > \hat{k}$. Using the definitions of J and $\hat{k}$, we obtain

$$J\left(\hat{k}\right) = (1-\tau)\sigma\left(\tau w\left(\hat{k}\right)\right)w(\hat{k}) = (1-\tau)\sigma\left(\tau\right) = (1-\tau)\frac{\phi\left(\tau\right)}{1+\phi\left(\tau\right)}.$$

For $J\left(\hat{k}\right) > \hat{k}$ we need

$$(1-\tau)\frac{\phi\left(\tau\right)}{1+\phi\left(\tau\right)} > \left(\frac{1}{\left(1-\alpha\right)A}\right)^{\frac{1}{\alpha}} \Leftrightarrow$$

$$A > \frac{1}{(1-\alpha)}\left((1-\tau)\frac{\phi\left(\tau\right)}{1+\phi\left(\tau\right)}\right)^{-\alpha},$$

which is exactly condition (2). ∎

A.2 Proof of Corollary to Proposition 1. (ii).

Proof: Rewrite $J(k)$ as $J(k) = (1-\tau)\sigma\left(k\right)w(k)$. Then steady states are fixed points to the equation $J(k) = k$. Note that 0 is a fixed point. It is easy to check that given $\phi(h) = \beta h/(1+h)$,

$$\sigma\left(k\right) = \frac{\beta\tau w(k)}{1+\tau w(k)\left(1+\beta\right)}.$$

Then $J(k) = k$ simplifies to

$$(1-\tau)\beta\tau w(k) = k\left[\frac{1}{w(k)} + \tau\left(1+\beta\right)\right]$$

and finally to $H(k) = c$ where

$$H(k) \equiv (1-\tau)\beta\tau A(1-\alpha)k^{\alpha-1} - \frac{1}{A(1-\alpha)}k^{-\alpha}$$

and $c \equiv \tau\left(1+\beta\right)$. Straightforward algebra establishes that $H'(\breve{k}) = 0$ where

$$\breve{k} = \left[\frac{\alpha}{A^2\beta(1-\alpha)^3(1-\tau)\tau}\right]^{\frac{1}{2\alpha-1}}.$$

Also, $\lim_{k\to 0} H(k) = -\infty$ and $\lim_{k\to\infty} H(k) = 0$. It remains to identify conditions under which $H(\breve{k})$ $> c$. Again straightforward but tedious algebra establishes that $H(\breve{k}) > c \Leftrightarrow$

$$(1-\tau)\beta\tau A(1-\alpha)\left[\frac{\alpha}{A^2\beta(1-\alpha)^3(1-\tau)\tau}\right]^{\frac{\alpha-1}{2\alpha-1}} - \frac{1}{A(1-\alpha)}\left[\frac{\alpha}{A^2\beta(1-\alpha)^3(1-\tau)\tau}\right]^{\frac{-\alpha}{2\alpha-1}} > \tau\left(1+\beta\right)$$

which simplifies to

$$A^{\frac{1}{2\alpha-1}}\left(\left(1-\tau\right)\beta\tau\right)^{\frac{\alpha}{2\alpha-1}}\left[\left(1-\alpha\right)^{\frac{2-\alpha}{2\alpha-1}}\alpha^{\frac{\alpha-1}{2\alpha-1}}-\left(1-\alpha\right)^{\frac{\alpha+1}{2\alpha-1}}\alpha^{\frac{-\alpha}{2\alpha-1}}\right]>\tau\left(1+\beta\right)$$

and finally to

$$\left(\frac{1-\alpha}{\alpha}\right)^{\frac{\alpha}{2\alpha-1}}\left(1-\alpha\right)^{\frac{1}{2\alpha-1}}A^{\frac{1}{2\alpha-1}}\left(\left(1-\tau\right)\beta\tau\right)^{\frac{\alpha}{2\alpha-1}}\left(\frac{2\alpha-1}{1-\alpha}\right)>\tau\left(1+\beta\right)\blacksquare$$

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