E C O N O M I C S B U L L E T I N

The relationship between tax evasion and tax revenue in Chang, Lai and Chang (1999)

Martin Besfamille
Universidad Torcuato Di Tella

Abstract

Chang, Lai and Chang (1999) use a micro-founded short-term macroeconomic model, with an imperfectly competitive market, to analyze, among other issues, the relationship between tax evasion and tax revenue. They show that this relationship depends upon the market structure. In particular, when the market becomes perfectly competitive, this relationship can be non monotonic. Although CLC give an intuition of this result, based on the interaction of two opposite effects, they do not make explicit the form of this relationship. The goal of this note is precisely to show that, within the Chang, Lai and Chang (1999) model, one can completely characterize the shape of the relationship between tax evasion and tax revenue under perfect competition. Under some parametric conditions, the tax revenue decreases with tax evasion; otherwise, their relationship takes the form of a `Laffer curve'.

Citation: Besfamille, Martin, (2008) "The relationship between tax evasion and tax revenue in Chang, Lai and Chang (1999)." *Economics Bulletin,* Vol. 5, No. 14 pp. 1-6
Submitted: February 1, 2008. **Accepted:** April 2, 2008.
URL: http://economicsbulletin.vanderbilt.edu/2008/volume5/EB-08E60012A.pdf

1. Introduction

The macroeconomic consequences of tax evasion (in particular, the tax revenue loss) is an important issue[1] that has received relatively little attention in the theoretical literature (see Caballé and Panadés 2000). In a simple Keynesian model, Peacock and Shaw (1982) were the first to show that, provided the marginal net propensity to spend is less than unity, tax evasion decreases the tax revenue. In a more general framework, Ricketts (1984) confirms the negative effect of tax evasion upon the tax revenue. But later, Lai and Chang (1988), Von Zameck (1989), Lai, Chang and Chang (1995) and Chang and Lai (1996) show that tax evasion may even lead to an increase in the tax revenue. The drawback with all these articles is their lack of microeconomic foundations.

In order to correct this, Chang, Lai and Chang (1999) [hereinafter CLC] use a micro-founded short-term macroeconomic model, with an imperfectly competitive market, to analyze, among other issues, the relationship between tax evasion and tax revenue. They show that this relationship depends upon the market structure. When the market tends to be monopolistic, an increase in tax evasion decreases the tax revenue, as in Peacock and Shaw (1982) and Ricketts (1984). But, when the market becomes perfectly competitive, this relationship can be non monotonic. Although CLC give an intuition of this result, based on the interaction of two opposite effects, they do not make explicit the form of this relationship.

The goal of this note is precisely to show that, within the CLC model, one can completely characterize the shape of the relationship between tax evasion and tax revenue under perfect competition. Under some parametric conditions, the tax revenue decreases with tax evasion; otherwise, their relationship takes the form of a 'Laffer curve'.

2. The CLC model

We briefly present the CLC model and we explain the notations, when needed. The model, build-up on Dixon (1987) and Mankiw (1988), consists of a representative consumer, a number of identical firms and the government.

The representative consumer is price taker, has initial money balances and time endowment V and, owns all firms. After receiving his revenues from labour and profits, the consumer has to pay an income tax, at a rate of $t \in (0, 1)$.[2] CLC denote by $h \in [0, 1]$ the (exogenous) fraction of the tax that is effectively remitted to the government. The consumer's demands for goods, real money and leisure are derived from the maximization of his Cobb-Douglas utility function, subject to his budget constraint.

Firms only use labor to produce output, with a constant returns to scale technology. They are price takers in the labor market. But, in the product market, firms compete *à la* Cournot. CLC use the firms' profit margin μ as an index of the degree of competition in the product market.

The government spends its revenues from income taxation T in two ways: real expenditures G and a lump-sum transfer to the consumer.

[1] In the USA, the Internal Revenue Service (IRS) estimates a tax gap for the tax year 2001 of U$D 345 billion, an amount that represents almost 15% of the total tax revenue (see IRS 2006).

[2] Although CLC do not mention boundary restrictions on t, they undertake the analysis assuming them implicitly. In fact, when $t = 0$ or $t = 1$, the model may have no solution.

From the expression of the individual demands, the government's budget constraint and the markets' equilibrium conditions, CLC obtain the values of the aggregate output Y and the price level.

3. The relationship between tax evasion and tax revenue

CLC study the relationship between tax evasion and tax revenue doing a comparative static analysis with respect to the compliance parameter h. First, they find the equilibrium amount of the tax revenue

$$T = thY = th\frac{\alpha(1-ht)(1-\mu)V + \beta G}{\alpha(1-ht)(1-\mu)+\beta} \tag{1}$$

where $\alpha > 0$ and $\beta > 0$ are parameters of the utility function.[3] Then CLC compute

$$\begin{aligned}\frac{\partial T}{\partial h} &= \underbrace{tY}_{\text{Compliance effect}} + \underbrace{th\frac{\partial Y}{\partial h}}_{\text{Income effect}} \\ &= \frac{t\left\{\alpha(1-\mu)V[\alpha(1-ht)^2(1-\mu)+\beta(1-2ht)] + \beta[\alpha(1-\mu)+\beta]G\right\}}{[\alpha(1-ht)(1-\mu)+\beta]^2}\end{aligned} \tag{2}$$

As CLC state, an increase in the compliance parameter h (i.e. a decrease in tax evasion) has two opposite effects upon the tax revenue. The 'compliance effect' is direct and positive. The 'income effect' is indirect and negative because, as Peacock and Shaw (1982) show and CLC confirm, aggregate output decreases with compliance.

The sign of (2) depends upon the degree of competition in the product market. In particular, when the market is perfectly competitive (i.e. when $\mu = 0$), (1) and (2) become

$$\begin{cases} T = th\dfrac{\alpha(1-ht)V+\beta G}{\alpha(1-ht)+\beta} \\ \dfrac{\partial T}{\partial h} = t\dfrac{\alpha V[\alpha(1-ht)^2+\beta(1-2ht)]+\beta(\alpha+\beta)G}{[\alpha(1-ht)+\beta]^2} \end{cases} \tag{3}$$

As CLC state, the derivative in (3) has an ambiguous sign because, under perfect competition, both abovementioned effects are present.[4] But CLC do not go beyond this statement. In the following proposition, we complete their analysis by characterizing the form of the relationship between tax evasion and tax revenue.

Proposition *In Chang, Lai and Chang (1999), when the product market is perfectly competitive, the relationship between tax evasion and tax revenue can adopt only two forms. If* $(\alpha+\beta)G < \alpha V$ *and* $t > \dfrac{1}{\alpha}\left[\alpha+\beta-\sqrt{\dfrac{(\alpha+\beta)(V-G)\beta}{V}}\right]$, *this relationship is non*

[3] As the utility function is Cobb-Douglas, these parameters represent, at the consumer's optimum, the fraction of his real income spent in goods and leisure, respectively.

[4] This is not the case when the market tends to be monopolistic (i.e. when $\mu \to 1$). Under this circumstance, the income effect is absent.

monotonic and takes the form of a 'Laffer curve'. Otherwise, this relationship is monotonic decreasing.

Proof. See the Appendix.

Under some parametric configurations on α, β, G, V and t, the compliance effect dominates, for any $h \in [0, 1]$. Thus, the tax revenue decreases with evasion, confirming again the Peacock and Shaw's (1982) result. But, under other parametric configurations, this negative relationship may not hold. For lower values of h, the compliance effect dominates while, for higher values, it is the output effect that prevails. So the relationship between tax evasion and tax revenue takes the form of a 'Laffer curve'. This result completes the analysis of CLC when the product market is perfectly competitive.

Appendix: Proof of the Proposition

First, it is straightforward to prove that $T(h)$ and $\dfrac{\partial T}{\partial h}(h)$ given by (3) are continuous functions of $h \in [0, 1]$. These functions verify

$$T(0) = 0 \quad , \quad T(1) = t\frac{\alpha(1-t)V + \beta G}{\alpha(1-t) + \beta} > 0$$

and

$$\left.\frac{\partial T}{\partial h}\right|_{h=0} = t\frac{\alpha V(\alpha V + \beta G)}{(\alpha + \beta)} > 0 \quad , \quad \left.\frac{\partial T}{\partial h}\right|_{h=1} = t\frac{\alpha V[\alpha(1-t)^2 + \beta(1-2t)] + \beta(\alpha+\beta)G}{[\alpha(1-t) + \beta]^2}$$

for all $t \in (0, 1)$. The sign of the last derivative is, a priori, ambiguous. But we can obtain it, as the following lemma shows.

Lemma *When* $(\alpha + \beta)G < \alpha V$

- *if* $0 < t \leq \tau = \dfrac{1}{\alpha}\left[\alpha + \beta - \sqrt{\dfrac{(\alpha+\beta)(V-G)\beta}{V}}\right]$, $\left.\dfrac{\partial T}{\partial h}\right|_{h=1} \geq 0$
- *if* $\tau < t < 1$, $\left.\dfrac{\partial T}{\partial h}\right|_{h=1} < 0$.

When $(\alpha + \beta)G \geq \alpha V$, $\left.\dfrac{\partial T}{\partial h}\right|_{h=1} > 0$.

Proof. Observe that the sign of $\left.\dfrac{\partial T}{\partial h}\right|_{h=1}$ is given by the sign of its numerator. In order to study it, let's define the following quadratic function of t

$$F(t) = \alpha t^2 - 2t(\alpha + \beta) + \alpha + \beta$$

This function verifies

$$\lim_{t \to 0} F(t) = \alpha + \beta > 0 \quad , \quad \lim_{t \to 1} F(t) = -\beta < 0$$

and

$$F' < 0 \text{ for all } t \in (0,1)$$

On the one hand, if $(\alpha+\beta)G \geq \alpha V$, $F(t) > -\beta\dfrac{(\alpha+\beta)G}{\alpha V}$ for all $t \in (0,1)$. Hence, $\left.\dfrac{\partial T}{\partial h}\right|_{h=1} > 0$. On the other hand, if $(\alpha+\beta)G < V$, there exists a number

$$\tau = \frac{1}{\alpha}\left[\alpha+\beta-\sqrt{\frac{(\alpha+\beta)(V-G)\beta}{V}}\right] < 1^5$$

such that:

$$\begin{cases} \text{if } 0 < t \leq \tau \quad F(t) \geq -\beta\dfrac{(\alpha+\beta)G}{\alpha V} \Leftrightarrow \left.\dfrac{\partial T}{\partial h}\right|_{h=1} \geq 0 \\ \\ \text{if } \tau < t < 1 \quad F(t) < -\beta\dfrac{(\alpha+\beta)G}{\alpha V} \Leftrightarrow \left.\dfrac{\partial T}{\partial h}\right|_{h=1} < 0 \ \square \end{cases}$$

Next, we study the shape of $T(h)$ under each of these possible configurations of parameters.

<u>Case 1</u> $(\alpha+\beta)G < \alpha V$ and $\tau < t < 1$

In this case, as $\left.\dfrac{\partial T}{\partial h}\right|_{h=1} < 0$, $T(h)$ is unambiguously non monotonic. As the function $\dfrac{\partial T}{\partial h}(h)$ is continuos on $[0,1]$ and

$$\left.\frac{\partial T}{\partial h}\right|_{h=1} < 0 < \left.\frac{\partial T}{\partial h}\right|_{h=0}$$

there exists at least a critical number $\widehat{h}$ such that $\left.\dfrac{\partial T}{\partial h}\right|_{h=\widehat{h}} = 0$ (Bolzano's Theorem). Computing the derivative of $\dfrac{\partial T}{\partial h}$ when $h = \widehat{h}$, we obtain

$$\left.\frac{\partial}{\partial h}\left(\frac{\partial T}{\partial h}\right)\right|_{h=\widehat{h}} = -t\frac{\alpha V[2\alpha(1-ht)t + 2\beta t]}{[\alpha(1-ht)+\beta]^2} < 0 \tag{4}$$

This implies that any critical value $T(\widehat{h})$ is a local maximum. Using (4) again, we can easily prove by contradiction that, in fact, $T(h)$ has a unique local maximum. Therefore, in this first case, the shape of $T(h)$ takes the form of a 'Laffer curve'.

<u>Case 2</u> Either $(\alpha+\beta)G \geq \alpha V$ or $(\alpha+\beta)G < \alpha V$ and $0 < t < \tau$

In these cases, as $\left.\dfrac{\partial T}{\partial h}\right|_{h=1}$ is (weakly or strictly) positive, $T(h)$ can be either monotonic increasing or non monotonic. As $\left.\dfrac{\partial T}{\partial h}\right|_{h=0} > 0$, $T(h)$ has to have a local minimum to be non monotonic. But, due to (4), this is not possible. So, in these cases, $T(h)$ is monotonic increasing, and thus the relationship between tax evasion and tax revenue is monotonic decreasing $\square$

[5] The inequality holds because $V > G$ and, by assumption, $\alpha + \beta < 1$.

References

[1] Caballé, J., and J. Panadés (2000) "Tax Evasion and Economic Growth" *Public Finance/Finances Publiques* **52**, 318-340.

[2] Chang, W., and C. Lai (1996) "The Implication of Efficiency Wages on Tax Evasion and Tax Collections" *Public Finance Quarterly* **24**, 163-172.

[3] Chang, W., Lai, C., and J. Chang (1999) "Imperfect Competition and Tax Evasion" *Public Finance Review* **27**, 371-381.

[4] Dixon, H. (1987) "A simple model of imperfect competition with Walrasian features" *Oxford Economic Papers* **39**, 134-160.

[5] IRS (2006) "IRS Updates Tax Gap Estimates," IRS-2006-28.

[6] Lai, C., and W. Chang (1988) "Tax Evasion and Tax Collections: an Aggregate Demand-Aggregate Supply Analysis" *Public Finances/Finances Publiques* **43**, 138-146.

[7] Lai, C., Chang, J., and W. Chang (1995) "Tax Evasion and Efficient Bargains" *Public Finance/Finances Publiques* **50**, 96-105.

[8] Mankiw, G. (1988) "Imperfect competition and the Keynesian cross" *Economic Letters* **26**, 7-13.

[9] Peacock, A., and G. Shaw (1982) "Tax Evasion and Tax Revenue Loss" *Public Finance/Finances Publiques* **37**, 269-278.

[10] Ricketts, M. (1984) "On the Simple Macroeconomics of Tax Evasion: an Elaboration of the Peacock-Shaw Approach" *Public Finances/Finances Publiques* **39**, 420-424.

[11] Von Zameck, W. (1989) "Tax Evasion and Tax Revenue Loss: Another Elaboration of the Peacock-Shaw Approach" *Public Finances/Finances Publiques* **44**, 308-315.