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Traditional public schools versus charter schools: a comparison of technical efficiency

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Abstract

This paper addresses the now famous question of "Does Money Matter?" in public education. While the general consensus is that additional expenditures may improve educational outcomes, this is by no means a guarantee. Indeed, some studies indicate that a school's resources are not an important determinant of student performance. As Adkins and Moomaw (2003) suggest, the true relationship between resources and performance may become more apparent in a better specified model accounting for technical inefficiency. Along these lines, we attempt to measure the technical efficiency gains of charter schools over traditional public schools using a stochastic frontier production model.

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1. Introduction

The usual argument in favor charter schools centers on the idea that charter schools bring about increased competition for students, encouraging the adoption of more effective teaching methods at lower costs. Teske, et. al. (2000) suggest that the presence of charter schools spurs the managers of traditional public schools to increase efficiency and adopt more innovative methods of teaching.[1] But, while traditional public schools have improved in response to charter schools, the majority of the evidence suggests that charter school performance has outpaced that of public schools.[2] King (2007) argues that for-profit charter schools face a greater incentive to expand enrollment than do not-for-profit charters and provides evidence that student achievement is indeed higher at for-profit charter schools.

The literature mentioned above addresses only the first of two claimed benefits of charter schools: improved teaching methods leading to improved student achievement. However, it is unclear whether this improved performance is the result of increased resources or the more efficient use of resources. Theory would suggest that the organizational structure of a school should have a significant effect on the efficiency of a school, with for-profit charters being more efficient than both not-for-profit charters and traditional public schools. In order to test this hypothesis, we need some measure of efficiency. A modern approach to measuring and modeling inefficiency in the production literature is to use stochastic frontier analysis. This approach can be extended to education.

In Stochastic Frontier Analysis efficiency, or more precisely inefficiency, is measured as the distance between some stochastic frontier and the actual production or cost point. That inefficiency is usually assumed to be function of a set of exogenous variables. In the case of education, we are therefore adding an inefficiency term to the typical education production function, and that inefficiency term is a function of school related variables. This approach gives us a measure of inefficiency and we can determine if school organization influences inefficiency.

While the use of stochastic frontier models in education is not new, their use in the comparison of traditional public schools and charter schools is. Ruggiero and Vitaliano (1999) use a stochastic cost function approach to measure efficiency in New York Schools. They find that urban schools tend to be more cost efficient. Adkins and Moomaw (2003) examine Oklahoma public school districts and find that spending does affect technical efficiency.

2. Stochastic Production Frontier Analysis

Consider a determinist production frontier model

$$y_i = f(x_i, \beta) \tag{1}$$

[1] See also Holmes, et. al (2003) and Hoxby (2003) for further evidence of traditional public school improvements in response to the presence of charter schools.
[2] Hoxby (2004) finds evidence that charter school students across the nation are 3.2 and 5.2 percent more likely to be proficient in math and reading, respectively, and Hoxby and Rockoff (2004) find that in Chicago achievement scores are roughly six percentiles higher for students who enroll in charter schools by grade five.

where y_i is the output for producer i, x_i is the vector of inputs for producer i, β is a vector of parameters, and f(*) is technology transforming inputs into the outputs. In the case of education, y_i is typically test scores or graduation rates, and x_i typically contains expenditures per student, student characteristics, and teacher characteristics.

By incorporating a random component, we can allow output to differ randomly between producers. This stochastic production frontier can be specified as

$$y_i = f(x_i, \beta)e^{v_i} \tag{2}$$

where v_i is an independent and identically distributed random variable typically assumed to be normally distributed.

We can further augment the model, by allowing producers to produce at a point below the production frontier. Let

$$y_i = f(x_i, \beta)TE_i e^{v_i} \tag{3}$$

where $0 < TE_i \leq 1$ represents a producers technical efficiency. If $TE_i = 1$ then the firm is producing on their frontier and are considered technically efficient. If $TE_i < 1$, the firm is producing below its frontier and has some degree of technical inefficiency. Letting $TE_i = e^{-u_i}$ and taking the natural log of both sides,

$$ln(y_i) = ln(f(x_i, \beta)) + v_i - u_i \tag{4}$$

Equation 4 represents a typical stochastic production frontier model.[3] In earlier stochastic frontier analysis, u_i was assumed to follow some one-sided distribution such as a truncated normal or exponential distribution. In more recent analysis, the model is augmented to incorporate exogenous influences into the measure of technical efficiency (Battese and Coelli 1995.) Let

$$u_i = g(z_i, \gamma) + e_i \tag{5}$$

where z is a vector of exogenous parameters affecting efficiency, γ is vector of parameters, and e_i is an i.i.d. random variable that follows a truncated normal distribution. If g(.) is a linear function, we could write $u_i \sim N^+(\gamma' z_i, \sigma_u^2)$.

If we let $v_i \sim N(0, \sigma_v^2)$ and $u_i \sim N^+(\gamma' z_i, \sigma_u^2)$, the parameter vector $(\beta, \gamma, \sigma_v^2, \sigma_u^2)$ can be estimated via maximum likelihood. Additionally, e^{u_i} is a measure of the technical inefficiency for producer i.

[3] See Meeusen and van den Broeck 1977 and Aigner, Lovell, and Schmidt 1977 for early work on stochastic frontier analysis.

3. Empirical Model

In order to create an educational stochastic production frontier model that incorporates technical effects, we need to do the following: 1) determine what variable will represent educational output, 2) determine which variables are direct inputs, 3) determine which variables affect efficiency, and 4) specify the functional forms for f(x, β) and g(x, γ)

3.1. Output

In this model, educational output will be measured as a school's average achievement on a standardized test score. Standardized test scores are by far the most prolific measure of educational output used in the literature. The main reason for this is because of the availability of the data and their ease of use. In addition, test scores provide a quantitative look at the performance of a school and since they are standardized, they allow researchers to compare schools in different locations.

Hanushek (1986), though indicating that test scores are not an ideal measure of educational output, gives three reasons for their use. First he notes that test scores have value in and of themselves. Parents, educators, and government officials all look toward test scores as a measure of how well schools are doing. Furthermore, test scores are important as a selection mechanism for further education. Therefore, test scores may change real outputs such as wages. Finally, Hanushek states that test scores may be a good measure for elementary education where cognitive skills are of particular importance.

3.2. Input and Efficiency Variables

Determining which variables affect efficiency and which shift the frontier is difficult. Typically, those variables which affect efficiency are considered to be exogenous to the particular production decisions. The choice of which variables affect the frontier and which variables affect efficiency is often a judgment call (Kumbhakar and Lovell 2000). For this study, we make the choice in two different ways. We make a judgment call based on past empirical research (most notably Adkins and Moomaw) and we also use a statistical criteria based on log-likelihoods.

3.3. Intuitive Model

As with output, characterizing educational inputs is not an easy task. First of all, there has to be a distinction between quality and quantity of an educational resource. For example, teacher characteristics can vary widely even though they may have equal pay and the same number of students. However, quality of resources is more difficult to quantify and less data is available describing those characteristics. Therefore, in many studies only the quantities of educational resources are presented as educational inputs.

All inputs to a student's educational achievements do not come from school; rather, there are other non-school inputs that could have consequences on educational output. Parental involvement, the living environment, and peer groups can contribute significantly to student achievement. These influences further confound the data problem since data on these personal characteristics are not readily available. For this reason, researchers include general family characteristics such as parental income and education in an attempt to control for these factors when estimating an education production function. While we admit it is an imperfect measure, we include in our model a percent minority variable to capture the effects of non-school attributes.

The primary focus of our study is to examine the use and effectiveness of school inputs across traditional public schools and charter schools. Thus, we include in our analysis school specific inputs including teaching expenditure per pupil (teach), administration expenditure per pupil (admin), and supplies spending per pupil (supply). We attempt to capture the quality effects of these expenditures in our measure of technical efficiency.

Of particular interest in this analysis is the affect of school organization on school efficiency. Therefore, we include a dummy variable to indicate whether the school is a for-profit charter school (profit) and a dummy variable to indicate whether the school is a non-for-profit charter school (nonprofit).

We model teacher quality as an exogenous variable that only affects technical efficiency. We use average teacher salaries (salary), average years of experience (exp), and percentage of teachers with advanced degrees (Hdegree) as variables measure teacher quality. As mentioned above, we attempt to control for non-school inputs with a percent minority variable.[4] The input variables and the efficiency variables do not have to be mutually exclusive.[5] Therefore, we also include the charter dummy variables as efficiency variables.

We will assume that the production technology has a Cobb-Douglas form. This is similar to much of the educational production function literature. As with many stochastic frontier models, we will assume that g is a linear function of z. Following equations (3) and (4) we specify the following educational stochastic production frontier model, which serves as our intuitive empirical model (model 1):

$$\begin{aligned} ln(\text{test scores}) &= \beta_0 + \beta_1 ln(\text{teach}) + \beta_2 ln(\text{admin}) + \beta_3 ln(\text{supplies}) \\ &\quad + \beta_4 ln(\text{service}) + \beta_5 \text{profit} + \beta_6 \text{nonprofit} + v_i - u_i \end{aligned} \tag{6}$$

where

$$u_i = \gamma_0 + \gamma_1 \text{exp} + \gamma_2 \text{profit} + \gamma_3 \text{nonprofit} + \gamma_4 \text{salary} + \gamma_5 \text{minority} + \gamma_6 \text{hdegree} \tag{7}$$

with $v_i \sim N(0, \sigma_v^2)$, and $u_i \sim N^+(\gamma' z_i, \sigma_u^2)$.

3.4. Information criterion based model

Given the ambiguities involved in selecting a model based on a judgment call, we also select an alternative model using log likelihoods.

We begin with the entire set of potential variables and try all the combinations of input and efficiency variables (given our variables, this equates to 256 possible combinations). We then select the model that gives the highest log likelihood. There are no restrictions on which variables should appear as input variables and which variables should appear as efficiency variables. However, we do allow the charter dummy variables to appear both as efficiency and as input variables.

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[4] A superior measure for controlling for student quality would be a percentage of students participating in free and reduced school lunch program. Unfortunately, this data is not available for many charter schools, preventing its inclusion in our empirical model.

[5] See Huang and Liu 1993 for an example of a stochastic frontier model where technical efficiency is not neutral with respect to its effect on input usage.

4. Data and Estimation

We use data for Arizona public school districts and Arizona charter schools obtained from the Arizona Department of Education for year 2001. Table I presents descriptive statistics for all the variables included in our analysis.

Test scores are averages of the math, reading, and language from the 3rd grade Stanford Achievement test. Expenditure variables are defined as follows: *admin* is the total administration spending divided by total enrollment, *supply* is the total supply expense divided by total enrollment, *instruct* is total classroom expense divided by total enrollment, and *support* is the total support and other expenses divided by total enrollment. *Hdegree* is the percentage of teachers with a master's degree or better. *exper* is our experience variable and it is the percentage of teachers that have seven or more years of teaching experience, and *minority* is the average percentage of minority students who took the Stanford Achievement Test. Finally, *TeachSal* is the total classroom expense divided by the number of teachers in a school.

On average, district schools tend to spend considerably more per pupil on administration, instruction, and support. Spending per pupil on supplies, however, is similar across school types. Average tests scores are also similar for the three different types of schools. District schools had a higher percentage of instructors with higher degrees and with more experience. Finally, district schools serviced a higher percentage of minority students.

The intuitive model and the automatically selected model are estimated via maximum likelihood using Ox (Doornik 2005.) For the logged variables and variables in percentage form, the parameter estimates, β, are interpreted as elasticities. The parameter estimates, γ, are interpreted as the percentage decrease in technical efficiency when there is a one unit change in the efficiency variable. For example, a coefficient estimate of one would be interpreted as follows: a one unit change in the variable causes a one percent reduction in median technical efficiency. The marginal effect, evaluated on the frontier, of a charter school on a percentage change in test scores can be computed as the difference between the production and inefficiency coefficients associated with the two dummy variables (β_5-γ_2 for for-profit charter schools and β_6-γ_3 for not-for-profit charter schools in equation 7.)

Table II reports the results for our judgment model (model 1). Administration spending tends to shift the frontier downward, while instructional spending tends to shift the frontier outward. A one percent increase in administration spending decreases average test scores by 0.09 %. A one percent increase in classroom spending increases average test scores by 0.084%. Spending on supplies and support were insignificant in this model.

In the efficiency equation, a higher percentage of advanced degrees and higher salaries had no statistically significant effect on efficiency. A more experienced teaching staff greatly reduced inefficiency, while an increase in the minority percentage tends to increase inefficiency, but by a small amount.

Table I. Descriptive Statistics

	Public n=178		Not for Profit Charter n=47		For Profit Charter n=12	
Variable	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev
score	46.21	15.33	45.02	21.48	49.75	23.23
admin	$1,009.20	$569.75	$510.72	$473.32	$655.78	$23.23
supply	$231.18	$156.40	$203.56	$179.16	$108.51	$23.23
instruct	$4,121.55	$1,862.83	$2,111.68	$670.02	$2,126.74	$727.09
support	$2,331.75	$1,255.49	$1,321.30	$850.90	$1,528.51	$23.23
Hdegree	0.36	0.16	0.23	0.17	0.16	0.13
exper	0.56	0.16	0.39	0.21	0.32	0.18
Minority	0.49	0.32	0.36	0.34	0.20	0.17
TeachSal	$64,284.96	$22,745.93	$35,938.68	$16,837.77	$43,373.46	$18,612.34

The charter school variables appeared to have a mixed effect. First of all, they appeared to have higher frontiers compared to their district counter parts. However they also appeared to be more inefficient. The total effects are -0.405306 and -0.331598 for for-profit and not-for-profit charter schools respectively.

This makes sense for many new markets. Managers are venturing into new waters and, as such, are trying many different methods. Some of these risks pay off, but many do not. Until enough experimentation has been conducted by managers in the industry they are likely to lag behind public schools in terms of efficiency merely because public schools have found the methods that work "best" for them. Charter schools are still searching for the most efficient method. The high turnover in the industry is indicative of this: many schools are failing financially because their methods were not efficient. Others are expanding and consolidating. It will be interesting to repeat this study in another 10 years or so to see how far charters have come.

Table II. Maximum likelihood parameter estimates for Model 1

dependent variable: ln(score)				
	Coefficient	Std.Error	t-value	t-prob
Constant	4.25634	0.3121	13.6	0.00
lnsupply	0.009661	0.02405	0.402	0.688
lnsupport	-0.02306	0.04776	-0.483	0.63
lnadmin	-0.08978	0.04364	-2.06	0.041
lninstruct	0.084491	0.03596	2.35	0.02
PCH	0.266812	0.1566	1.7	0.09
NPCH	0.207168	0.07953	2.6	0.01
Constant	-0.13232	0.2695	-0.491	0.624
Hdegree	-0.18993	0.29	-0.655	0.513
TeachSal	-0.00296	0.02667	-0.111	0.912
PCH	0.672118	0.294	2.29	0.023
NPCH	0.538766	0.161	3.35	0.001
exper	-0.73885	0.2698	-2.74	0.007
Minority	0.01368	0.002057	6.65	0.00
sigma2S	0.183717	0.03068	5.99	0.00
Gamma	0.936928	0.04011	23.4	0.00
log-likelihood	-47.3344			
observations	236		parameters	16
AIC.T	126.6688		AIC	0.536732

Table III reports the results for the automatically selected model (model 2). It is interesting to note that the automatically selected model has the expenditure per student variables listed as efficiency variables. This should not be too surprising for public schools. Total available funding for public and some charter schools are determined from outside sources, such revenues from local property taxes. Additionally, schools have some mandated support services which would further constrain a schools ability to allocate moneys to instruction and administration. Therefore, it isn't entirely unreasonable to argue that the spending variables are exogenous to the production decision.

This model fits the data better than model 1. It has a log-likelihood ratio of -42.14 compared to the -47.33 for model 1. The major difference between the models is the location of the expenditure variables and the degree and salary variables. The results, however, tend to be very similar in terms of the overall effects on test scores.

Salaries are still insignificant, but now a one percent increase in higher degrees results in a 0.2 percent increase in average test scores. Percentage of minority students still increases inefficiency, while experience tends to decrease inefficiency. Administration spending tends to increase inefficiency and classroom spending tends to decrease inefficiency. The charter dummies have the same effects as in model 1. That is, they increase the frontier but increase inefficiency. There overall effects are also similar.

Table III. Maximum likelihood parameter estimates for Model 2

dependent variable: ln(score)

	Coefficient	Std.Error	t-value	t-prob
Constant	4.13268	0.1062	38.9	0.00
Hdegree	0.2033	0.102	1.99	0.048
TeachSal	-0.00347	0.01451	-0.239	0.811
PCH	0.336595	0.1597	2.11	0.036
NPCH	0.244193	0.07549	3.23	0.001
Constant	-0.06677	0.8806	-0.0758	0.94
exper	-0.64767	0.2603	-2.49	0.014
Minority	0.013905	0.001993	6.98	0.00
lnsupply	-0.05495	0.04911	-1.12	0.264
lnsupport	0.004888	0.08643	0.0566	0.955
lnadmin	0.286377	0.09971	2.87	0.004
lninstruct	-0.23747	0.1168	-2.03	0.043
PCH	0.704171	0.3063	2.3	0.022
NPCH	0.54472	0.1662	3.28	0.001
sigma2S	0.181858	0.03041	5.98	0.00
Gamma	0.926688	0.03816	24.3	0.00
log-likelihood	-42.14			
observations	236		parameters	16
AIC.T	116.28		AIC	0.492712

5. Conclusions and Directions for Future Work

This paper explores differences in technical efficiency between traditional public schools and both profit and non-profit charter schools using a stochastic frontier model. We find that administration spending tends to have a negative effect on test scores, while classroom spending tends to have a positive effect on test scores. Spending on support services or supplies had little effect on test scores. More experienced teachers tended to increase efficiency, while minority percentage tended to decrease efficiency.

Charter schools, both for profit and not for profit, appeared to have higher frontiers, but lower levels of efficiency when compared to traditional schools. The overall effect of charter schools on test scores is mixed.

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