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On the Core of an Economy with Multilateral and Multidimensional Environmental Externalities

Charles Figuières
University of Bristol

Magali Verdonck
Université Catholique de Louvain, CORE

Abstract

For simple economic models of transfrontier pollution, Chander and Tulkens (1995) and (1997) have offered a formula for transfers to sustain international cooperation on a voluntary basis and which deter coalitionnal free−riding under some reasonable behaviours of countries not in the coalition. Their scheme rests on the assumption that pollution is a scalar. Relaxing this assumption, interesting interactions among pollutants arise that call for a new formula. In this paper we extend Chander and Tulkens formula for this more realistic multidimensional context, and thereby enhance the pratical and theoretical relevance of their seminal analysis.

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1 Introduction

For simple fundamental economic models of transfrontier pollution Chander and Tulkens (1995) and (1997) (CT hereafter) have offered a scheme of transfers to implement Pareto efficient decisions. The transfers they have proposed have the two following properties, which are particularly desirable when dealing with the cooperation of interacting sovereign countries: *i)* the international optimum is a voluntary one, *ii)* it is robust against the temptation of free-riding by some of the countries (or groups of countries). This scheme has also proved useful in related questions, for instance in the study of stock-pollution (Germain et al 1998 and 2002), and of negotiation process (Chander and Tulkens 1992, Chander 1995).

CT's scheme applies when the production of a private good induces the emission of a single pollutant, or equivalently to situations where, for some reasons, the single pollution problem under consideration can be treated in isolation of other pollution problems. Otherwise interesting problems of interaction between pollutants arise that call for an integrated analysis.[1] Clearly an integrated analysis is also needed for the definition of coalition-proof transfers. Our paper offers an extension of CT's transfers formula to a multidimensional framework and thus substantially enhance its practical and theoretical relevance.

The paper is organised as follows. Section 2 presents a multi-pollutant variant of CT's' model. Section 3 analyzes coalitional behaviours that could potentially object against an optimum; we then build on this equilibrium concept to offer a formula that Nash-implements an efficient decisions profile which is robust against the temptation to deviate by any coalition. Section 4 concludes.

2 An economy with many pollutants

The model presented in this section is a multi-pollutant version of CT, from which we shall borrow the basic notation.

Consider a set N of n countries and two kinds of commodities: i) a ho-

[1] Interactions between pollutants is one of the salient aspects of the so-called Integrated Assessment models that analyze the impacts of a warmer climate and possible mitigation policies. See for instance Nordhaus (1993), Nordhaus and Yang (1996), Nordhaus and Boyer (1999).

mogenous private good, whose quantities are denoted $x \geq 0$ when consumed and $y \geq 0$ when produced, ii) pollutant discharged.

We depart from CT's model in that we introduce any number of pollutants, say $m \geq 1$, instead of only one; think for instance about water pollution, air pollution, etc...

In the notations to follow superscripts refer to countries, whereas subscripts refer to commodities, and bold characters are used for vectors or matrices. The quantities of pollutant h discharged in country i are denoted by $p_h^i \geq 0$, and $\mathbf{p}_h \in R_+^n$ is the n-dimensional vector made of the non negative components p_h^i. For notational convenience it will also prove useful to introduce $\mathbf{p}^i \in R_+^m$, the m-dimensional vector of pollutant discharges in country i, and $\mathbf{p}^{-i} \in R_+^{(n-1)} \times R_+^m$ the $(n-1) \times m$ matrix of pollutant discharges produced by the remaining countries.

The addition of discharges h by all the countries result in a total ambient pollutant quantity of type h, denoted by $z_h \leq 0$, where:

$$z_h = -\sum_{j=1}^{n} p_h^j, \qquad h = 1, ..., m \,. \tag{1}$$

We call these ambient pollutants "public bads" : they have the characteristics of public goods, but their effect on all the countries is negative.

In country i the representative agent is endowed with a linear utility function defined over his consumption of the private homogenous good x^i and the public bads quantities $(z_1, ..., z_m)$. Formally this utility function reads as:

$$V^i(x^i, z_1, ..., z_m) = x^i + \sum_{h=1}^{m} \pi_h^i \, z_h \ , \ \ \pi_h^i \geq 0 \ , \ \forall i \ , \ \forall h \,. \tag{2}$$

Note that utilities are country specific. The essence of the problem comes from the heterogeneity of national preferences.

The production of the private good denoted y^i generates as a by-product some unavoidable pollution; in other words the emissions p_h^i can be seen as inputs for production in country i, with technologies specified as:

$$y^i = \sum_{h=1}^{m} \log p_h^i \,. \tag{3}$$

A feasible state of the economy is a vector $(x^1, ..., x^n, \mathbf{p}_1, ..., \mathbf{p}_m, z_1, ..., z_m)$, that satisfies (1) and the resource constraint:

$$\sum_{i=1}^{n} x^i = \sum_{i=1}^{n}\sum_{h=1}^{m} \log p_h^i \ . \tag{4}$$

In this context with international public bads it is possible to single out a unique Pareto efficient decisions vector; it is such that the social marginal disutility of each pollutant is equal to the marginal utility this pollutant produces by increasing the private good consumption. Therefore this vector satisfies the $n \times m$ familiar conditions:

$$\pi_h^N = \frac{1}{p_h^i} \ , \qquad i = 1, ..., n \text{ and } h = 1, ..., m \ , \tag{5}$$

where $\pi_h^N = \sum_{j=1}^{n} \pi_h^j$.

Thanks to the efficiency conditions (5), we can compute the corresponding levels of public bads:

$$\widehat{z}_h = -\frac{n}{\pi_h^N} \qquad h = 1, ..., m \ .$$

From (5) it follows that there is a unique production vector $\widehat{\mathbf{y}} = (\widehat{y}^1, ..., \widehat{y}^n)$; but there is an infinite number of combinations for the private good consumptions $\widehat{\mathbf{x}} = (\widehat{x}^1, ..., \widehat{x}^n)$ consistent with (4) and therefore an infinite number of Pareto efficient states, denoted $(\widehat{x}^1, ..., \widehat{x}^n, \widehat{\mathbf{p}}_\mathbf{1}, ..., \widehat{\mathbf{p}}_\mathbf{m}, \widehat{z}_1, ..., \widehat{z}_m)$, which we shall refer to as *feasible international optima*.

For further use the notation $\widehat{U}^i = V^i(\widehat{x}^i, \widehat{z}_1, ..., \widehat{z}_m)$ stands for state i's utility evaluated at an international optimum.

3 Coalition-proof transfers to sustain an international optimum

The cooperative game with transferable payoffs corresponding to our economy is defined by the pair $[N, w]$ of its players set N (the set of countries) and the characteristic function $w(S)$ that associates with every subset S of the n players a number called the worth of S. Following CT we assume that when a coalition forms, the players outside the coalition behave non cooperatively in their own best individual interest, taking the other players' decisions as given. This leads us to consider a Nash equilibrium with respect to a coalition S where the coalition is viewed as a single player.[2]

Each country faces the following budget constraint for the private good consumption: $x^i = y^i$, $i \in N$. Using this budget constraint, (1), (2), and (3), player i's payoff function is:

$$U^i(\mathbf{p}^i, \mathbf{p}^{-i}) = \sum_{h=1}^{m} \log p_h^i + \sum_{h=1}^{m} \pi_h^i \left(-\sum_{j=1}^{n} p_h^j \right) .$$

Formally, a partial agreement Nash equilibrium with respect to a coalition consists of a decision profile $(\tilde{\mathbf{p}}_1, ..., \tilde{\mathbf{p}}_m)$ which simultaneously solves the following problems:

$i)$ inside the coalition: $\underset{\{\mathbf{p}^i\}_{i \in S}}{Max} \sum_{i \in S} U^i(\mathbf{p}^i, \mathbf{p}^{-i})$, given the profile of decisions outside the coalition

$ii)$ outside the coalition: $\underset{\mathbf{p}^i}{Max}\, U^i(\mathbf{p}^i, \mathbf{p}^{-i})$, taking as given the other players' decisions.

The above equilibrium concept underlies the γ-characteristic function $w^\gamma(S)$, which is defined as $w^\gamma(S) = \sum_{i \in S} U^i(\tilde{\mathbf{p}}^{\mathbf{i}}, \tilde{\mathbf{p}}^{-\mathbf{i}})$.

The first order conditions for the maximization problem of the members of the coalition are, for each pollutant h:

$$\sum_{j \in S} \pi_h^j = \pi_h^S = \frac{1}{p_h^i} , \qquad i \in S .$$

The first order conditions for the maximization problems outside the coalition, for each h, are:

$$\pi_h^i = \frac{1}{p_h^i} , \qquad i \in N \setminus S .$$

[2]See CT for a detailed exposition of this concept and a discussion of the other possible assumptions made in the literature as for the behaviours of the non-members of S.

The outcome associated with this concept corresponds to a state of the economy called a *partial agreement equilibrium with respect to a coalition* denoted $(\widetilde{x}^1, ..., \widetilde{x}^n, \widetilde{\mathbf{p}}_{\mathbf{1}}, ..., \widetilde{\mathbf{p}}_{\mathbf{m}}, \widetilde{z}_1, ..., \widetilde{z}_m)$. We denote $\widetilde{U}^i = V^i(\tilde{x}^i, \tilde{z}_1, ..., \tilde{z}_m)$ the utility of state i associated to the partial agreement equilibrium with respect to a coalition.

As a particular case, when the coalition under consideration is made of only one country the above concept boils down to a Nash equilibrium. This Nash equilibrium corresponds to a so-called *disagreement equilibrium* for the economy denoted $(\bar{x}^1, ..., \bar{x}^n, \bar{\mathbf{p}}_1, ..., \bar{\mathbf{p}}_m, \bar{z}_1, ..., \bar{z}_m)$. The notation $\overline{U}^i = V^i(\bar{x}^i, \bar{z}_1, ..., \bar{z}_m)$ is used for state i's utility computed at the disagreement equilibrium.

It is easy to see that for any coalition S with more than two countries, $\sum \widehat{x}^i \leq \sum \widetilde{x}^i < \sum \bar{x}^i$, and $\widehat{z}_h \geq \widetilde{z}_h > \bar{z}_h$ for each pollutant h, meaning that the international optimum is characterized by a resource allocation favoring less pollution and less private good.

Having described the countries' behaviours, individually or in coalition, we now turn to the definition of transfers that would lead to an feasible international optimum, with the condition of this outcome being coalition-proof.

A strategy profile of all the countries is said to belong to the γ-core of the game $[N, w^\gamma(S)]$ if for any coalition $S \subseteq N$ the payoffs it yields to the members of S is not lower than $w^\gamma(S)$, i.e. the payoff that S can achieve by itself. In their models with only one public bad, CT have proposed a specific formula for transfers leading to an international optimum in the γ-core, thus with the attractive properties that: *i*) it is individually rational, *ii*) it is also robust to free-riding by coalitions in the sense captured by the *partial agreement equilibrium with respect to a coalition.* In an extended context with several pollutants what would be the adequate form for such transfers? We propose the following formula:

$$\theta^i\left(\mathbf{p}^i, \mathbf{p}^{-i}\right) = -\left[U^i\left(\mathbf{p}^i, \mathbf{p}^{-i}\right) - \overline{U}^i\right] + K^i\left[\sum_i U^i\left(\mathbf{p}^i, \widehat{\mathbf{p}}^{-\mathbf{i}}\right) - \overline{U}\right] \quad (6)$$

$$\text{where } K^i = \frac{\min\left\{\frac{\pi_1^i}{\pi_1^N}, ..., \frac{\pi_m^j}{\pi_m^N}\right\}}{\sum\limits_{j=1}^{n} \min\left\{\frac{\pi_1^j}{\pi_1^N}, ..., \frac{\pi_m^j}{\pi_m^N}\right\}} \in [0,1]\ ,\ \sum_{j=1}^{n} K^i = 1 \text{ and } \overline{U} = \sum_{j=1}^{n} \overline{U}^j.$$

When confronted to the transfer (6) each country's problem now reads as:

$$\max_{\mathbf{p}^i} U^i\left(\mathbf{p}^i, \mathbf{p}^{-i}\right) + \theta^i\left(\mathbf{p}^i, \mathbf{p}^{-i}\right) = \overline{U}^i + K^i\left[\sum_i U^i\left(\mathbf{p}^i, \widehat{\mathbf{p}}^{-\mathbf{i}}\right) - \overline{U}\right] ,$$

and obviously it is a dominant strategy for each country to implement the efficient decisions. When efficient choices are adopted, under our scheme the transfer (6) to each country consists of two parts: a payment that covers its payoff variation incurred by the shift from a Nash equilibrium to the optimum, and a share of the total utility surplus extracted by moving to the international optimum. The following remarks are in order:

- those transfers are balanced since $\sum_{j=1}^{n} \theta^i\left(\widehat{\mathbf{p}}^{\mathbf{i}}, \widehat{\mathbf{p}}^{-\mathbf{i}}\right) = 0$;
- state i's utility with transfers, denoted w^i, is:

$$w^i = \widehat{U}^i + \theta^i\left(\widehat{\mathbf{p}}^{\mathbf{i}}, \widehat{\mathbf{p}}^{-\mathbf{i}}\right) = \overline{U}^i + K^i(\widehat{U} - \overline{U}) > \overline{U}^i \;,\; \forall i \;, \tag{7}$$

  where $\widehat{U} = \sum_j \widehat{U}^j$, so they are individually rational;
- in CT the sharing rule K^i was defined by the relative intensity of country i's preference for the (unique) public bad component of the problem. The sharing rule we propose in this paper is defined using a combination of the previous idea with the *min* function. When there is only one public bad, our sharing rule boils down to the CT one. Otherwise, this new formula makes use, for each state, only of that public bad with the smallest relative incentive to pursue the international interest.

The central property is that such transfers prevent free-riding by coalitions in the following sense:

Theorem: *The imputation* $\mathbf{w} = (w^1, ..., w^n)$ *belongs to the* γ*-core of the game* $[N, w^\gamma]$.

Proof: see Appendix A.

This result has two virtues: *i)* it establishes the non-emptiness of the γ*-core, ii)* it allows one to compute a γ*-core* allocation. As in CT the robustness against free riding of the γ*-core* imputation can be restated as follows: given the sharing of $w^{\gamma}(N)$ proposed to all the players, if some coalition S contemplates the possibility to free ride in order to achieve an other arrangement on her own, the other players acting as rational singletons is sufficient to make this free riding less attractive than the proposed solution.

4 Concluding remarks

In a quite specific framework (log additive production functions, linear damage functions) where the production of a private good induces the emission of multiple pollutants, our extension of CT's formula features two important properties: *i*) there exists a transfer scheme such that a coalition-proof international optimum can be achieved and *ii*) the share of country i in the ecological surplus redistribution is solely related to that input with the smallest relative marginal (dis-)utility. This new formula substantially enhances the practical relevance of CT's seminal papers; also all the existing theoretical analyses where this formula has been used could be extended to many pollutants. Further research should test the robustness of this new formula using more general specifications for the production functions and utility functions. This task is currently taken up by the authors.

Appendix

A An imputation in the γ-core

Suppose that the imputation $\mathbf{w}$ does not belong to the core. Then, there would exist a coalition S and a partial agreement equilibrium with respect to S such that:

$$\sum_{i\in S}\widetilde{U}^{i} > \sum_{i\in S}w^{i} \tag{8}$$

Consider then the alternative imputation $\widehat{\mathbf{w}} = (\widehat{w}^{1}, ..., \widehat{w}^{n})$ with typical elements defined as:

$$\widehat{w}^{i} = \widehat{U}^{i} + \widehat{\theta}^{i}$$

where the new transfer $\widehat{\theta}^i$ is

$$\widehat{\theta}^i = -(\widehat{U}^i - \widetilde{U}^i) + K^i(\widehat{U} - \widetilde{U})$$

As we now show, the inequality (8) implies that:

1. $\sum_{i \in S} \widehat{w}^i \geq \sum_{i \in S} \widetilde{U}^i > \sum_{i \in S} w^i$
2. $\sum_{i \in N \setminus S} \widehat{w}^i \geq \sum_{i \in S} w^i$

but then the imputation $\widehat{\mathbf{w}}$ induces an aggregate welfare for all countries that is higher than $\mathbf{w}$, an impossibility that proves the theorem.

It is easy to show 1:

$$\begin{aligned} \sum_{i \in S} \widehat{w}^i &= \sum_{i \in S} (\widehat{U}^i + \hat{\theta}^i) \\ &= \sum_{i \in S} \widetilde{U}^i + \underbrace{\sum_{i \in S} K^i(\widehat{U} - \widetilde{U})}_{\geq 0} \geq \sum_{i \in S} \widetilde{U}^i \end{aligned}$$

To show 2, it is sufficient to show that $\widehat{\theta}^j \geq \theta^j, \forall j \in N \setminus S$:

$$-(\widehat{U}^i - \widetilde{U}^i) + K^i(\widehat{U} - \widetilde{U}) \geq -(\widehat{U}^i - \overline{U}^i) + K^i(\widehat{U} - \overline{U})$$

$$\widetilde{U}^i - \overline{U}^i \geq K^i(\widetilde{U} - \overline{U})$$

From the Nash first order conditions it follows that:

$$\pi_h^i = \frac{1}{\bar{p}_h^i}\ , \qquad i = 1, ..., n \text{ and } h = 1, ..., m\ . \tag{9}$$

or equivalently $\bar{p}_h^i = 1/\pi_h^i$. This is a dominant strategy equilibrium and the private good production of a typical country evaluated at such an equilibrium is $\bar{y}^i = \sum_{h=1}^{m} \log \bar{p}_h^i$. As for the ambient pollutants we have:

$$\bar{z}_h = -\sum_{j=1}^{n} \frac{1}{\pi_h^j} \qquad h = 1, ..., m\ .$$

Observe that: $\bar{z}_h < \hat{z}_h$ for each pollutant h and $\sum \bar{x}^i > \sum \hat{x}^i$, meaning that the international optimum is characterized by a resource allocation favoring less pollution and less private good.

Given (2) and the first order conditions for the players outside the coalition at the disagreement equilibrium and at the partial agreement equilibrium, one can write:

$$\sum_{h=1}^{m} \pi_h^i (\tilde{z}_h - \bar{z}_h) \geq K^i \sum_{j=1}^{n} \left[(\tilde{x}^j - \bar{x}^j) + \sum_{h=1}^{m} \pi_h^j (\tilde{z}_h - \bar{z}_h) \right]$$

$$\sum_{h=1}^{m} (\pi_h^i - K^i \pi_h^N) \underbrace{(\tilde{z}_h - \bar{z}_h)}_{\geq 0} \geq K^i \underbrace{\sum_{j=1}^{n} (\tilde{x}^j - \bar{x}^j)}_{\leq 0} \tag{10}$$

Using the definition of K^i we know that:

$$\pi_h^i - K^i \pi_h^N \geq 0 \ , \quad \forall i, \forall h \ .$$

The inequality (10) is therefore always verified and the theorem is proved.

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