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A Note on the Profit Distribution among a Manufacturer and its Retailers

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Abstract

Examining two polar forms of restricted franchise contract, Nariu (2004) studied the pricing behavior of manufacturers and retailers and the market outcomes. This note provides a concise justification for his assumptions on contractual restraints. Introducing some fixed amount that a manufacturer must invest to build up its production facility, we show that a bargaining solution to distribute the total net profit among a manufacturer and its exclusive retailers assigns zero franchise fee payment to any retailers, if the investment is not large.

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1 Introduction

In distribution processes, there are independent intermediary agents who purchase manufacturers' products and sell them to consumers on behalf of manufacturers (vertical separation). In the literature, many authors have studied "vertical restraints" (e.g., exclusive sales territories) that reduce retailers' competition and examined the effect to the manufacturers' competition.[1] Nevertheless, how "contractual restraints" in franchise relationships affect the market outcomes has not been studied sufficiently yet.

Nariu (2004) examined restricted franchise contracts, assuming that retailers compete in quantity.[2] He showed that if two manufacturers operate and charge the full franchise fees, a manufacturer lowers its wholesale price when its rival manufacturer does the opposite, whereas it raises its wholesale price without franchise fees, when the rival raises its wholesale price.

It is, however, too strict to assume that no franchise fee can be charged. The aim of this note is to give a concise justification to his assumptions on contractual restraints from the viewpoint of bargaining theory. Introducing some fixed amount that a manufacturer must invest to build up its production facility, we show that a bargaining solution called *nucleolus* (Schmeidler (1969)) to distribute the total net profit among the manufacturer and its retailers assigns zero franchise fee payment to any retailers, if the investment cost is not so large.

To show this, our first step is to define the worth of a coalition of players that forms in the negotiation, i.e., the characteristic function derived from the original model of vertical separation. This note applies the one proposed by Tauman and Watanabe (2005). A manufacturer can transact with all or some of its retailers and so can the rival manufacturer. We define the worth of a coalition with a manufacturer as the total profit of the manufacturer and all its active retailers when the rival group uses the most offensive strategy (in terms of the number of its active retailers) against it. Any coalition without the manufacturer obtains nothing because of exclusive dealing clause, and the manufacturer obtains nothing without retailers.

The nucleolus always exists and specifies a single vector of payoffs in any bargaining problem (coalitional game) with sidepayments. Geometrically, it is almost in the center of core if the core is non-empty, and also it is almost in the center of shadow core if the core is empty. Hence, it is a reasonable solution when players have the equal bargaining power.

[1] See chapter 4 in Tirole (1988).

[2] This practice is quite often observed. That may be due to the franchise contracts in which retailers can gain more as they sell more outputs produced by manufacturers.

The outline of the paper is as follows. In section 2, a vertical separation model is first introduced, and then the characteristic function is derived from it. The definition of the nucleolus is provided there. Our proposition is given with its proof in section 3.

2 The Model

2.1 a duopoly case

Consider an industry of manufacturers, retailers and consumers. There are two manufacturers, each producing an identical good that is infinitely divisible. Their marginal cost of production is a constant $c(>0)$. For each manufacturer $i = 1, 2$, let $N(i) = \{1(i), \ldots, n(i)\}$, where $1 \leq n(i) < \infty$, be the set of retailers, each $j(i) \in N(i)$ purchasing manufacturer i's products and then selling them to consumers in the market. The franchise contract signed by manufacturer i and its retailer $j(i)$ entails the exclusive dealing clause, and so the profit of retailer $j(i)$ is zero without the franchise contract. The market is cleared at the price $p = \max(0, a - Q)$, where $a(>c)$ is a constant, $Q = \sum_{i=1,2} \sum_{j(i) \in N(i)} q_{j(i)}$ and $q_{j(i)}(\geq 0)$ is the quantity that retailer $j(i)$ purchases from manufacturer i and sells in the market.

There are two stages. (i) Anticipating the demand $q_{j(i)}$ by all its retailer $j(i) \in N(i)$, each manufacturer i sets its franchise fee $f_i(\geq 0)$ and wholesale price $w_i(\geq 0)$ independently and simultaneously, so as to maximize its profit

$$\pi_i = \sum_{j(i) \in N(i)} \{(w_i - c)q_{j(i)} + f_i\}. \tag{1}$$

Any franchise contract (f_i, w_i) is agreed upon insofar as $j(i)$ obtains non-negative profit, and it is verifiable to the court. (ii) Knowing (f_i, w_i) for both i, each retailer $j(i) \in N(i)$ for both i independently and simultaneously demands $q_{j(i)}$ units of manufacturer i's product and sells them to consumers at the market clearing price p, so as to maximize its profit

$$\pi_{j(i)} = (p - w_i)q_{j(i)} - f_i. \tag{2}$$

This is the original model in Nariu (2004).

2.2 the characteristic function

Let us introduce into the above model some fixed amount $F(>0)$ that each manufacturer $i = 1, 2$ must invest to build up its production facility. Further, each manufacturer i can activate the transaction with its own $t(i)$ retailers, $0 \leq t(i) \leq n(i)$. Let $\{i\} \cup N(i) = N^i$, $i = 1, 2$. We confine attention to N^1, without loss of generality. Any non-empty subset S of N^1 is called a coalition.

When manufacturers transact with $t(1) + t(2) (\neq 0)$ retailers, every active retailer $j(i)$ obtains the Cournot profit

$$\pi_{j(i)}(t(1), t(2)) = (\frac{a - w_i}{t(1) + t(2) + 1})^2 - f_i.$$

Hence, if $1 \in S$, the total profit of $S \subseteq N^1$ is

$$\Pi_S(t(1), t(2)) = \pi_i + \sum_{j(i) \in S} \pi_{j(i)}(t(1), t(2)) = t(1)(\frac{a - c}{t(1) + t(2) + 1})^2 - F,$$

where $0 \leq t(1) \leq |S \setminus \{1\}|$, by (1), (2) and F. Since the profit of inactive retailer $j(i)$ is zero due to the exclusive dealing clause, if $1 \notin S$,

$$\Pi_S(t(1), t(2)) = 0, \text{ regardless of } t(1) \text{ and } t(2).$$

In the spirit of von Neumann and Morgenstern, the worth of $S \subset N^1$ is the largest total Cournot profit of S given the most offensive strategy of the rival group N^2. We define a characteristic function $v : 2^{N^1} \to \mathbb{R}$ by

$$v(S) = \max_{0 \leq t(1) \leq |S \setminus \{1\}|} \min_{0 \leq t(2) \leq n(2)} \Pi_S(t(1), t(2)). \tag{3}$$

Assume $v(\emptyset) = 0$. The interpretation of v is briefly stated in calculating its values in the next section. Let (N^1, v) be the bargaining game to distribute the total profit of a manufacturer and retailers who belong to N^1.

2.3 a bargaining solution

Let $x = (x_1, (x_{j(1)})_{j(1) \in N^1 \setminus \{1\}})$ be a real-valued vector of payoffs that is specified by a bargaining solution to (N^1, v). The set of imputations is $I(v) = \{x \in \mathbb{R}^{n+1} \,|\, \sum_{k \in N^1} x_k = v(N^1), \text{ and } x_k \geq v(\{k\}) = 0 \;\; \forall k \in N^1\}$.

For any $x \in I(v)$, define a vector $e(x) = (e_1(x), \ldots, e_{2^{n+1}}(x)) \in \mathbb{R}^{2^{n+1}}$ of excesses of coalitions to x, where $e_1(x) \geq e_2(x) \geq, \cdots, \geq e_{2^{n+1}}(x)$ and $e_k(x) = v(S) - \sum_{k \in S} x_k$, $S \subset N^1$. The excess $e_k(x)$ represents the k-th largest complaint to an imputation x proposed in the bargaining.

Definition: The imputation x^* is the nucleolus $\text{Nu}(v)$ of the bargaining game (N^1, v) to distribute the total profit of a manufacturer and its retailers if and only if $e(x^*) \leq e(x)$ $\forall x \in I(v)$ in the lexicographic ordering on $\mathbb{R}^{2^{n+1}}$.

As is well known, $|\text{Nu}(v)| = 1$ for any game (N^1, v).

3 The Result and its Proof

Clearly, N^2 activates $t(2) = n(2)$ retailers to minimize $\Pi_S(t(1), t(2))$, whatever $t(1)$ is chosen by N^1. If $n(2) + 1 \leq |S \setminus \{1\}|$, then $\Pi_S(t(1), n(2)) = t(1)[(a-c)/(n(2)+t(1)+1)]^2 - F$ is maximized at $t(1) = n(2)+1$, since it is increasing in $t(1)$ until $n(2)+1$. If $n(2)+1 > |S \setminus \{1\}| = t$, it is maximized at $t(1) = t$. By the same argument, it turns out that N^2 "maximizes" its own total profit by minimizing the worth of N^1. Hence, this characteristic function is reasonable to analyze the situation described in Nariu (2004).

Only for simplicity, assume the following.

Assumption 1 *$n(1) = n(2) = n$ and $0 < F < n[(a-c)/(2n+1)]^2$.*

By the above argument and Assumption 1, the characteristic function v defined as (3) is simplified as follows: for any $S \subseteq N^1$,

$$v(S) = \begin{cases} 0 & 1 \notin S \text{ or } S = \{1\} \\ \max[0, t(\frac{a-c}{n+t+1})^2 - F] & 1 \in S,\ t = |S \setminus \{1\}|, \end{cases} \tag{4}$$

Let $t^* = \arg\max_{0 \leq t \leq n-1} t[(\dfrac{a-c}{n+t+1})^2 - A]$, where $A = \dfrac{1}{n+1}(n(\dfrac{a-c}{2n+1})^2 - F)$, and let

$$B(t) = t[\frac{a-c}{n+t+1}]^2, \text{ and so } A = \frac{1}{n+1}(B(n) - F).$$

A is the equal division of the total profit of N^1. Denote the Cournot price by $p^* = a - 2nq^*_{j(1)} (> c)$, where $q^*_{j(1)} = (a-c)/(2n+1)$ is the Cournot output of each firm.

Proposition 1 *Under Assumption 1 and 2,* Nu(v) *of (N^1, v) suggests the following: if $t^*([(a-c)/(n+t^*+1)]^2 - A) \leq F$ or $B(n) - (1/2)(n+1)[B(n) - B(n-1)] \leq F$, then*

$$w_1 = \frac{1}{n+1}(p^* + nc) \quad \textit{and} \quad f_1 = \frac{1}{n+1}F.$$

if $t^*([(a-c)/(n+t^*+1)]^2 - A) > F$ and $B(n) - (1/2)(n+1)[B(n) - B(n-1)] > F$, then

$$w_1 = p^* - \frac{1}{2(p^* - c)}[B(n) - B(n-1)] \quad \textit{and} \quad f_1 = 0.$$

Proof : Let $x(T) = \sum_{k \in T} x_k$, $T \subseteq N^1$. The proof consists of three lemmas.

Lemma 1 *Suppose that $t^*([(a-c)/(n+t^*+1)]^2 - A) \leq F$. Then,* Nu$(v) = x^*$, *where*

$$x^*_k = A = \frac{n}{n+1}(\frac{a-c}{2n+1})^2 - \frac{1}{n+1}F, \quad \forall k \in N^1. \tag{5}$$

Proof : If $1 \notin S$ or $S = \{1\}$, then $v(S) - x^*(S) \leq v(\{k\}) - x_k^* = -A$, $k \in S$, since $v(S) = v(\{k\}) = 0$. If $1 \in S \neq N^1$, then

$$\begin{aligned} v(S) - x^*(S) &= \max[0, t(\tfrac{a-c}{n+t+1})^2 - F] - (t+1)A \\ &\leq \max[-A, t(\tfrac{a-c}{n+t+1})^2 - F - (t+1)A] \\ &\leq -A \qquad \text{(by the supposition).} \end{aligned}$$

Hence, $e_1(x^*) = \max_{S \subset N^1} = [v(S) - \sum_{k \in S} x_k^*] = -A$. For any x such that $x \neq x^*$, there is $k \in N^1$ with $x_k < x_k^*$ and so $v(\{k\}) - x_k > v(\{k\}) - x_k^* = -A$. Therefore, $\mathrm{Nu}(v) = x^*$. ■

By (5), $x_k^* = (n/(n+1))(p^* - c)q_{j(i)}^* - (1/(n+1))F = (p^* - w_i)q_{j(i)}^* - f_i$. Hence, $w_i = (1/(n+1))(p^* + nc)$ and $f_i = (1/(n+1))F$.

Lemma 2 *Suppose that* $t^*([(a-c)/(n+t^*+1)]^2 - A) > F$ *and that* $B(n) - (1/2)(n+1)[B(n) - B(n-1)] > F$. *Then,* $\mathrm{Nu}(v) = x^*$, *where*

$$\begin{aligned} x_1^* &= n(\frac{a-c}{2n+1})^2 - nx_{j(1)}^* - F \\ x_{j(1)}^* &= (1/2)[n(\frac{a-c}{2n+1})^2 - (n-1)(\frac{a-c}{2n})^2], \quad \forall j(1) \in N(1). \end{aligned} \tag{6}$$

Proof : If $t^*([(a-c)/(n+t^*+1)]^2 - A) > F$, then $(n-1)[(a-c)/2n]^2 \geq F$. If $B(n) - (1/2)(n+1)[B(n) - B(n-1)] > F$, then $x_1^* > A > x_{j(1)}^*$. Hence, if $1 \in S \neq N^1$, then

$$\begin{aligned} v(S) - x^*(S) &= \max[0, t(\tfrac{a-c}{n+t+1})^2 - F] - (n(\tfrac{a-c}{2n+1})^2 - F - nx_{j(i)}^* + tx_{j(1)}^*) \\ &\leq \max[0, t(\tfrac{a-c}{n+t+1})^2 - tx_{j(1)}^* - F] - n(\tfrac{a-c}{2n+1})^2 + F + nx_{j(1)}^* \\ &\leq \max_{0 \leq t \leq n-1}\ t[(\tfrac{a-c}{n+t+1})^2 - x_{j(1)}^*] - n(\tfrac{a-c}{2n+1})^2 + nx_{j(1)}^* \\ &= (n-1)[(\tfrac{a-c}{2n})^2 - x_{j(1)}^*] - n(\tfrac{a-c}{2n+1})^2 + nx_{j(1)}^* \leq -x_{j(1)}^* \end{aligned}$$

If $1 \notin S$ or $S = \{1\}$, then $v(S) - x^*(S) \leq v(S) - x_{j(1)}^* = -x_{j(1)}^*$. Hence, $e_1(x^*) = \max_{S \subset N^1} = [v(S) - \sum_{k \in S} x_k^*] = -x_{j(1)}^*$. For any x such that $x \neq x^*$, there is $k \in N^1$ with $x_k < x_k^*$. If $k \neq 1$, then $v(\{k\}) - x_k > v(\{k\}) - x_k^* = -x_{j(1)}^*$. If $k = 1$, then there is a $j \neq 1$ with $x_j > x_{j(1)}^*$. Thus,

$$\begin{aligned} v(N^1 \setminus \{j\}) - x(N^1 \setminus \{j\}) &= v(N^1 \setminus \{j\}) - x(N^1) + x_j \\ &> v(N^1 \setminus \{j\}) - x^*(N^1) + x_{j(1)}^* \\ &= (n-1)(\tfrac{a-c}{2n})^2 - F - n(\tfrac{a-c}{2n+1})^2 + F + x_{j(1)}^* = -x_{j(1)}^*. \end{aligned}$$

Hence, for any x such that $x \neq x^*$, there is an S with $v(S) - x(S) > -x_{j(1)}^*$. Therefore, $\mathrm{Nu}(v) = x^*$. ■

By (6), $x_{j(1)}^* = (1/2)[n - B(n-1)/((a-c)/(2n+1))^2](p^* - c)q_{j(1)}^* = (p^* - w_1)q_{j(1)}^* - f_i$. So, $w_1 = p^* - (1/(2p^* - 2c))[B(n) - B(n-1)]$ and $f_1 = 0$.

Lemma 3 *Suppose that* $t^*[(a-c)/(n+t^*+1)]^2 - A) > F$ *but that* $B(n) - (1/2)(n+1)[B(n) - B(n-1)] \leq F$*. Then,* $\mathrm{Nu}(v) = x^*$ *if and only if*

$$x_k^* = A = \frac{n}{n+1}(\frac{a-c}{2n+1})^2 - \frac{1}{n+1}F, \quad \forall k \in N^1. \tag{7}$$

Proof : (only if) If $1 \notin S$ or $S = \{1\}$, then $v(S) - x^*(S) \leq v(\{k\}) - x_k^* = -x_k^*$, $^\forall k \in S$. Hence, $e_1(x^*) = \max_{S \subset N^1} = [v(S) - \sum_{k \in S} x_k^*] \geq \max(-x_1^*, -x_{j(1)}^*)$. For any $x \neq x^*$ with $x_1 < x_1^*$, there is a retailer $j \in N(1)$ such that $x_{j(1)}^* < x_j$. Since $\mathrm{Nu}(v) = x^*$,

$$\begin{aligned} v(N^1 \setminus \{j\}) - x(N^1 \setminus \{j\}) &= v(N^1 \setminus \{j\}) - x(N^1) + x_j \\ &> v(N^1 \setminus \{j\}) - x^*(N^1) + x_{j(1)}^* = B(n-1) - B(n) + x_{j(1)}^* \\ &\geq \max(-x_1^*, -x_{j(1)}^*). \end{aligned} \tag{8}$$

If $x_1^* < A < x_{j(1)}^*$, then $x_1^* + x_{j(1)}^* = x_1^* + ((n+1)A - x_1^*)/n < 2A$, and so $B(n) - (1/2)(n+1)[B(n) - B(n-1)] > F$, a contradiction. If $x_1^* > A > x_{j(1)}^*$, again $B(n) - (1/2)(n+1)[B(n) - B(n-1)] > F$ by (8). Hence, $\mathrm{Nu}(v) = x^*$ is the equal division of $B(n) - F$. (if) straightforward by the above argument. ■

Lemmas 1, 2 and 3 complete the proof. *Q.E.D.*

Proposition 1 can be easily extended to the case of $m(> 2)$. This paper used the maxmin approach, but we could show that the minmax approach coincides with it (Tauman and Watanabe (2005)). The characteristic function described in (4) is the same as the one used in Watanabe and Muto (2005).

References

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