

E C O N O M I C S   B U L L E T I N

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## No-shirking Conditions in Frictional Labor Markets

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### *Abstract*

A matching model, combined with a shirking model of efficiency wages, is examined. It depends on sources of unemployment variation whether the no-shirking condition (NSC) tends to be binding as the unemployment rate is lower. When only productivity varies, the NSC tends to be binding as the unemployment rate is higher, as in Rocheteau (2001). However, when only matching efficiency varies, the NSC tends to be binding as the unemployment rate is lower.

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# 1 Introduction

Wage rigidity appears important to explain aggregate behavior of frictional labor markets. However, our understanding about consequences of intra-firm work incentive problems to wage rigidity and labor market behavior appears still limited<sup>1</sup>. I examine how work incentive problems affect wage rigidity and variation in vacancy creation in a search and matching model (Pissarides 1985, 2000) in which employees' effort choice problem is incorporated by way of Shapiro and Stiglitz (1984) type efficiency wage arrangements. I examine economies with different kinds of disturbances to the economy. Variation in unemployment and wages can be generated by changes in either productivity, matching efficiency, vacancy costs, separation probabilities and so on. Of course, in the business cycle frequencies, variation in aggregate productivity appears most important. But, it is also important to consider economies with different sources of unemployment variation in order to better understand mechanisms of how incentives affect wage rigidity and, thereby, employment variation. This is how my analysis differs from Rocheteau (2001), who also provides a systematic analysis of a matching model with efficiency wages.

Suppose that workers face binary decisions of effort levels, say, work and shirk. In equilibrium in which workers choose to work every period, workers' gains from employment must be large enough compared to the cost of effort. I call this the no-shirking condition (NSC). I find that it depends on sources of unemployment variation whether the NSC tends to be binding as the unemployment is lower. When only productivity varies, the NSC tends to be binding as the unemployment is higher, as in Rocheteau. However, when only matching efficiency varies, the NSC tends to be binding as the unemployment rate is lower. This finding may help to better understanding the effects of no-shirking conditions on aggregate labor market behavior. As far as the author can tell, our understanding of matching models with no-shirking conditions is limited<sup>2</sup>.

This article is structured as follows. The next section builds a matching search model, combined with workers' shirking problems. Section 3 examines the relationship between the likelihood of no-shirking conditions being binding and the unemployment rate. Section 4 concludes. Omitted proofs are in the appendix.

## 2 The Model

### 2.1 Environment

Time is discrete:  $t \in \{1, 2, 3, \dots\}$ . The beginning of *period*  $t$  is called *date*  $t - 1$ , and its end *date*  $t$ . Only steady states are considered.

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<sup>1</sup>On the one hand, some authors argue that work incentive problems dampens employment fluctuations, since during recessions wages do not have to be high (Kimball 1994; Strand 1992). On the other, it is also known that rigid wages are generated in a matching economy with employees' effort choice problems (Costain and Jansen 2006; Gomme 1999).

<sup>2</sup>Gomme (1999), for instance, shows that the RBC model with no-shirking conditions generates strongly procyclical real wages. It is yet to be seen whether matching models with no-shirking conditions may deliver the similar result. Costain and Jansen (2006) examine the similar model.

**Agents and Preferences.** This economy is populated by a continuum of risk-neutral workers and firms, both of whom discount future payoffs at the common discount factor  $\beta \in [0, 1)$ . The preference of each agent is given by  $\mathbb{E} \sum_{t=1}^{\infty} \beta^t x_t$ , where  $x_t$  is consumption minus the cost of exerting effort for a worker, and it is a profit net of the cost of being vacant for a firm.

**Production Technology.** A pair of a worker and firm during employment in tenure  $t \in \{1, 2, 3, \dots\}$  produces  $e_t \cdot y$ , in which  $e_t \in \{0, 1\}$  is the worker's effort level,  $y \in \mathbb{R}_{++}$  is match quality, which is assumed to be the same across employment periods. The worker incurs a cost  $c \cdot e_t$  when choosing effort  $e_t \in \{0, 1\}$ , where  $c > 0$  is assumed. Each match separates with probability  $s \in [0, 1]$  each period for exogenous reasons.

**Search Technology.** Let  $\theta \equiv v/u$  be the vacancy-unemployment ratio, where  $v$  is a measure of vacant firms and  $u$  unemployed job seekers. Each unemployed worker contacts a vacant firm with probability  $\mu p(\theta)$  per period, where  $p'(\theta) > 0$ . Each vacant firm contacts a job-seeker with probability  $\mu q(\theta)$  per period, where  $q'(\theta) < 0$ . Functions  $p(\theta)$  and  $q(\theta)$  are related in such a way that  $p(\theta) = \theta q(\theta)$ . The parameter  $\mu > 0$  captures the degree of meeting efficiency in a frictional labor market. I assume that  $\lim_{\theta \rightarrow 0} p(\theta) = 0$ ,  $\lim_{\theta \rightarrow \infty} p(\theta) = 1/\mu$ ,  $\lim_{\theta \rightarrow 0} q(\theta) = 1/\mu$ , and  $\lim_{\theta \rightarrow \infty} q(\theta) = 0$ .<sup>3</sup>

**Asset Equations.** Now, I write down a bunch of asset equations. The value of unemployment  $U$  satisfies the following equations:

$$U = z + \beta \mu p(\theta) V + \beta [1 - \mu p(\theta)] U \quad (1)$$

where  $z$  is the flow utility of each unemployed job seeker,  $V$  the value of employment for each employee. The value of employment for each employee, exerting high effort,  $V$  must satisfy the following asset equation:

$$V = w - c + (1 - s)\beta V + s\beta U \quad (2)$$

where  $w$  is the flow wage level. The value of each filled job  $J$  must satisfy the following equation:

$$J = y - w + (1 - s)\beta J \quad (3)$$

**No-shirking Conditions.** It is assumed for simplicity that monitoring employees' effort is perfect in the sense that the employee's effort level is observed with no noise. However, it is assumed that effort is unverifiable. As a consequence, any agreements contingent on effort levels are not enforceable. When the worker chooses high effort ( $e=1$ ), the employment continues unless the match receives exogenous separation shocks. When the worker chooses low effort ( $e=0$ ), the employment is terminated. The employee chooses high effort ( $e=1$ ) if  $-c + w + \beta(1 - s)V + \beta sU \geq w + \beta U$ , which is rewritten as:

$$\beta(1 - s)(V - U) \geq c \quad (4)$$

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<sup>3</sup>One example satisfying these assumptions is  $q(\theta) = [1 - \exp(-\frac{1}{\theta})]/\mu$ .

We call this inequality the no-shirking condition, or NSC in what follows.

**Wage Determination.** When the worker and firm choose to match, they choose the wage level by bargaining, so that the no-shirking condition is satisfied. A flow wage is determined so that it maximizes the Nash product  $(V - U)^\gamma J^{1-\gamma}$  subject to the non-shirking inequality:

$$w = \arg \max (V - U)^\gamma J^{1-\gamma} \quad \text{s.t. (NSC)} \quad (5)$$

If the NSC is not binding, the solution is as the same as the standard Nash bargaining solution, which is given by  $(1 - \gamma)(V - U) = \gamma J$ . However, if the NSC is binding, the solution will be  $V - U = \frac{c}{\beta(1-s)}$ ,  $J = S - \frac{c}{\beta(1-s)}$ , where the match surplus  $S$  is defined by  $S \equiv V + J - U$ . By using equations (2) and (3), one can show that  $S = \frac{y-c-(1-\beta)U}{1-\beta(1-s)}$ .

**Free Entry of Vacancies.** Each vacant firm incurs recruiting cost  $k > 0$  per period in finding a worker. Entry into the labor market is free. This implies the following equation:

$$k = \beta\mu q(\theta)J \quad (6)$$

Now, we can define a steady state equilibrium of a search model with workers' shirking problems.

**Definition 1.** *A steady state equilibrium of a search matching model with workers' shirking problems is defined as a list  $\{U, V, J, w, \theta, u\}$  such that: (i)  $U, V,$  and  $J$  satisfy equations (1), (2) and (3); (ii)  $\theta$  satisfies equation (6); (iii)  $w$  satisfies the problem (5); and (iv)  $u$  satisfies the steady state accounting:  $(1 - u)s = u\mu p(\theta)$ .*

The definition given above is very standard except the part (iii), where wages must satisfy the NSC. If the NSC were not binding in equilibrium, the model would behave like the standard search matching models of Diamond, Mortensen and Pissarides. It is interesting to see when the NSC is likely to be binding. A steady state equilibrium of a search model with workers' shirking problems defined above is characterized by a pair  $(\theta, U)$  satisfying the following two equations:

$$U = z + \beta\mu p(\theta) \max\left[\gamma S, \frac{c}{\beta(1-s)}\right] + \beta U \quad (7)$$

$$\frac{k}{\beta\mu q(\theta)} = \min\left[(1 - \gamma)S, S - \frac{c}{\beta(1-s)}\right] \quad (8)$$

**Proposition 1.** *Consider a matching model with workers' shirking problems described above. Let*

$$\bar{\theta} \equiv \frac{y - c - z - \frac{c[1-\beta(1-s)]}{\beta(1-s)\gamma}}{k \frac{\gamma}{1-\gamma}}$$

$$\Psi(\theta) \equiv \frac{y - c - z - \frac{\gamma}{1-\gamma} k \theta}{1 - \beta(1-s)} - \frac{k}{\beta(1-\gamma)\mu q(\theta)}$$

Assume

$$\Psi(0) = \frac{y - c - z}{1 - \beta(1-s)} - \frac{k}{\beta(1-\gamma)} > 0$$

a. If

$$\Psi(\bar{\theta}) = \frac{c}{\beta(1-s)\gamma} - \frac{k}{\beta(1-\gamma)\mu q(\bar{\theta})} < 0$$

Then, a steady state equilibrium of the model, in which the NSC is not binding, exists and it is unique.

b. If

$$\Psi(\bar{\theta}) = \frac{c}{\beta(1-s)\gamma} - \frac{k}{\beta(1-\gamma)\mu q(\bar{\theta})} \geq 0$$

Then, a steady state equilibrium of the model, in which the NSC is binding, exists and it is unique.

One can find parameters satisfying the two boundary assumptions made in each part in Proposition 1. Pick the values of all parameters, except  $\mu$  and the shape of  $q(\theta)$  so that the first inequality  $\Psi(0) > 0$  is satisfied. Then, one can find the values of  $\mu$  and the shape of  $q(\theta)$  so that the second inequality ( $\Psi(\bar{\theta}) < 0$  in part a;  $\Psi(\bar{\theta}) \geq 0$  in part b) is satisfied.

### 3 No-shirking Conditions and Labor Markets

In this section, the relationship between the likelihood that no-shirking conditions bind and the unemployment rate is examined. Addressing this question serves the purpose of better understanding the relation between work incentive problems and labor market conditions.

**Proposition 2 (Variation in Worker Productivity).** *Consider a class of matching models defined above with two different levels of productivity  $y_G > y_B$ . There are no equilibria in which the NSC is binding in the state of lower unemployment rates and it is not in the state of higher unemployment rates.*

The key to understanding Proposition 2 is that the gains from employment for workers have to be sufficiently large for workers to be motivated to exert high effort. Hence, if employees' gains from employment were very large, the NSC would not be binding. Otherwise, the NSC would be binding.

Another key behind Proposition 2 is that when only productivity varies, the state of lower unemployment rates corresponds to the state of large match surplus, and vice versa. Recall that when the NSC is not binding, workers' gains from employment is a fixed proportion of the match surplus. Suppose that the NSC is binding in the state of lower unemployment rates. The reason why the NSC is binding is that the size of the match surplus is very small. Then, the match surplus is smaller in the state of higher unemployment rates than in the state of lower unemployment rates. The NSC would be binding, too, in this state of higher unemployment. Therefore, one can say that when the NSC is binding in the lower-unemployment-rate state, it is binding, too, in the higher-unemployment-rate state. Similarly, one can say that when the NSC is not binding in the higher-unemployment-rate state, it is not binding in the lower-unemployment-rate state.

Proposition 2 is silent about whether it is possible that the NSC is not binding in the state of lower unemployment rates and it is in the state of higher unemployment rates. The next proposition below shows this.

**Proposition 3 (Variation in Worker Productivity).** *Consider a class of matching models defined above with two different levels of productivity  $y_G > y_B$ . For  $j \in \{B, G\}$ , let*

$$\begin{aligned}\bar{\theta}_j &\equiv \frac{y_j - c - z - \frac{c[1-\beta(1-s)]}{\beta(1-s)\gamma}}{k \frac{\gamma}{1-\gamma}} \\ \Psi_j(\theta) &\equiv \frac{y_j - c - z - \frac{\gamma}{1-\gamma}k\theta}{1 - \beta(1-s)} - \frac{k}{\beta(1-\gamma)\mu q(\theta)}\end{aligned}$$

*Assume that:*

$$\begin{aligned}\Psi_j(0) &= \frac{y_j - c - z}{1 - \beta(1-s)} - \frac{k}{\beta(1-\gamma)} > 0 \text{ for both } j \\ \Psi_G(\bar{\theta}_G) &= \frac{c}{\beta(1-s)\gamma} - \frac{k}{\beta(1-\gamma)\mu q(\bar{\theta}_G)} < 0 \\ \Psi_B(\bar{\theta}_B) &= \frac{c}{\beta(1-s)\gamma} - \frac{k}{\beta(1-\gamma)\mu q(\bar{\theta}_B)} \geq 0\end{aligned}$$

*Then, there are equilibria in which the NSC is binding in state B and it is not in state G.*

Proof. The result follows from Proposition 1. ||

By following Rocheteau (2001), call the wages when the NSC is binding *efficiency wages* in what follows. Call the wages when the NSC is not binding *freely negotiated wages*. Proposition 2 and 3 suggest that efficiency wages are relevant when the unemployment is above a certain threshold, when

only workers' productivity varies. However, this equilibrium feature is not robust to differences in disturbances to the economy, as the next two propositions below show.

**Proposition 4 (Variation in Matching Efficiency).** *Consider a class of matching models defined above with two different levels of matching efficiency  $\mu_G > \mu_B$ . There are no equilibria in which the NSC is binding in the state of higher unemployment rates and it is not in the state of lower unemployment rates.*

The intuition behind Proposition 4 is similar to the one behind Proposition 2. It is important to notice that when only matching efficiency varies, the match surplus is larger in the state of higher unemployment rates than the other state. This is opposite to the case in which only workers' productivity varies. Given this observation, Proposition 4 can be understood in the similar way as Proposition 2.

**Proposition 5 (Variation in Matching Efficiency).** *Consider a class of matching models defined above with two different levels of matching efficiency  $\mu_G > \mu_B$ . For  $j \in \{B, G\}$ , let*

$$\begin{aligned}\bar{\theta} &\equiv \frac{y - c - z - \frac{c[1-\beta(1-s)]}{\beta(1-s)\gamma}}{k \frac{\gamma}{1-\gamma}} \\ \Psi_j(\theta) &\equiv \frac{y - c - z - \frac{\gamma}{1-\gamma} k \theta}{1 - \beta(1-s)} - \frac{k}{\beta(1-\gamma)\mu_j q(\theta)}\end{aligned}$$

Assume that:

$$\begin{aligned}\Psi(0) &= \frac{y - c - z}{1 - \beta(1-s)} - \frac{k}{\beta(1-\gamma)} > 0 \\ \Psi_B(\bar{\theta}) &= \frac{c}{\beta(1-s)\gamma} - \frac{k}{\beta(1-\gamma)\mu_B q(\bar{\theta})} < 0 \\ \Psi_G(\bar{\theta}) &= \frac{c}{\beta(1-s)\gamma} - \frac{k}{\beta(1-\gamma)\mu_G q(\bar{\theta})} \geq 0\end{aligned}$$

Then, there are equilibria in which the NSC is binding in state G and it is not in state B.

Proof. The result follows from Proposition 1. ||

Propositions 4 and 5 suggest that, in the economy with variations in only matching efficiency, efficiency wages are relevant if the unemployment rate is *below* a certain threshold. This is in contrast to the economy with variations in only worker productivity, in which efficiency wages are relevant if the unemployment rate is *above* a certain threshold.

Propositions 2, 3, 4 and 5 illustrate that it depends on sources of unemployment variation whether the NSC tends to be binding as the unemployment rate is lower. The punch-line is this: the size of match

surplus in the state of the lower unemployment rate is not necessarily larger than the one in the state of the higher unemployment rate. This enriches the insight of Rocheteau (2001). This point may be helpful to understand why in some literature work incentives problems dampen employment fluctuations (Strand 1992 e.g.), while in others work incentives amplify employment fluctuations (Costain and Jansen 2006 e.g.).

## 4 Conclusion

A matching model, combined with a shirking model of efficiency wages, was examined. When only productivity varies, the NSC tends to be binding as the unemployment is higher, as in Rocheteau (2001). However, when only matching efficiency varies, the NSC tends to be binding as the unemployment rate is lower. This finding may have important implications for practical labor market policies, such as working time regulation, which was examined in the similar framework by Rocheteau (2002). Moreover, my finding in this paper may help to address the issue raised by Hall (2005) and Shimer (2005)<sup>4</sup>. My conjecture tells that if procyclical productivity variation is more dominant than matching efficiency variation, then adding no-shirking conditions may help the DMP models to deliver larger fluctuations of vacancies and unemployment.

## 5 Appendix

### Proof of Proposition 1.

(part a) Consider, first, the case in which the NSC is not binding. Then, the equilibrium is given by  $(\theta, U)$  such that  $(1 - \beta)U = z + \beta\mu p(\theta)\gamma S$  and  $k = \beta\mu q(\theta)(1 - \gamma)S$ , where  $S = \frac{y-c-(1-\beta)U}{1-\beta(1-s)}$ . By eliminating  $U$ , one obtains  $\Psi(\theta) \equiv \frac{y-c-z-\frac{\gamma}{1-\gamma}k\theta}{1-\beta(1-s)} - \frac{k}{\beta(1-\gamma)\mu q(\theta)} = 0$ . By the two boundary assumptions stated in the proposition,  $\Psi(0) = \frac{y-c-z}{1-\beta(1-s)} - \frac{k}{\beta(1-\gamma)} > 0$ . Moreover,  $\Psi(\bar{\theta}) < 0$ . Since  $\Psi(\theta)$  is continuous in  $\theta$ , the intermediate value theorem implies that there exists  $\theta_e \in (0, \bar{\theta})$  such that  $\Psi(\theta_e) = 0$ . Since  $\Psi(\theta)$  is monotonically decreasing, such  $\theta_e$  is unique. It is easily checked that the NSC is not binding for  $\theta < \bar{\theta}$ .

(part b) Consider, next, the case in which the NSC is binding in equilibrium. Then, the equilibrium is given by  $(\theta, U)$  such that  $(1 - \beta)U = z + \beta\mu p(\theta)\frac{c}{\beta(1-s)}$  and  $k = \beta\mu q(\theta)[S - \frac{c}{\beta(1-s)}]$ . By eliminating  $U$ , one obtains  $\Xi(\theta) \equiv -\frac{k}{\beta\mu q(\theta)} + \frac{c}{\beta(1-s)} + \frac{y-c-z-\frac{\beta c \mu p(\theta)}{\beta(1-s)}}{1-\beta(1-s)} = 0$ . By the boundary assumptions stated in the proposition,  $\Xi(0) = -\frac{k}{\beta} + \frac{c}{\beta(1-s)} + \frac{y-c-z}{1-\beta(1-s)} > 0$ . (Note that this is implied by the assumed inequality  $\Psi(0) > 0$ .) Moreover,  $\Xi(\infty) < 0$ . Since  $\Xi(\theta)$  is continuous in  $\theta$ , the intermediate value theorem implies that there exists  $\theta_e \in (0, \infty)$  such that  $\Xi(\theta_e) = 0$ . Since  $\Xi(\theta)$  is monotonically decreasing, such  $\theta_e$  is unique. By construction, the NSC is binding for such  $\theta_e$ , since  $\Psi(\bar{\theta}) \geq 0$ . ||

### Proof of Proposition 2.

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<sup>4</sup>I thank Guillaume Rocheteau for suggesting this to me.



Consider a matching economy with two different levels of productivity:  $y_G > y_B$ . Conjecture that the unemployment rate is lower in state G than in state B, which will turn out to be the case below. Suppose, by way of contradiction, that there are equilibria in which the NSC is binding in state G and it is not in state B, i.e.,  $\beta(1-s)\gamma S_G \leq c$  and  $\beta(1-s)\gamma S_B > c$ . The two inequalities imply that  $S_G < S_B$ . Then, free entry equation in state B implies that  $\frac{k}{\beta\mu q(\theta_B)} = (1-\gamma)S_B$ . Free entry in state G implies that  $k = \beta\mu q(\theta_G)[S_G - \frac{c}{\beta(1-s)}]$ . Note that  $S_j$  is decreasing in  $\theta_j$  for each  $j$ . Therefore,  $\theta_j$  satisfying each free entry equation in state  $j$  exists and it is unique. We have that  $S_G - \frac{c}{\beta(1-s)} \leq (1-\gamma)S_G < (1-\gamma)S_B$ . Then, free entry equations imply that  $\theta_B > \theta_G$ . Recall that  $S_j \equiv \frac{y_j - c - \frac{\gamma}{1-\gamma}k\theta_j}{1-\beta(1-s)}$ . Since  $y_G > y_B$ , and  $\theta_B > \theta_G$ , we have that  $S_G > S_B$ . Contradiction. Therefore, there are no such equilibria in which the NSC is binding in state G and it is not in state B. Therefore, there are three kinds of possibilities: (i) the NSC is binding in neither state; (ii) the NSC is binding in both states; and (iii) the NSC is binding in state B and it is not in state G. One can easily show that  $u_G < u_B$  for every case. Consider case (i) first. Free entry in state  $j \in \{B, G\}$  implies that  $\frac{k}{\beta\mu q(\theta_j)} = (1-\gamma)S_j$ .  $y_G > y_B$  implies  $\theta_G > \theta_B$ . Steady state accounting implies that  $u_G < u_B$ . Consider, next, the case (ii). Free entry in state  $j$  implies that  $\frac{k}{\beta\mu q(\theta_j)} = S_j - \frac{c}{\beta(1-s)}$ . This, with  $y_G > y_B$ , implies  $\theta_G > \theta_B$ . This implies  $u_G < u_B$ . Finally, consider the case (iii), in which  $\beta(1-s)\gamma S_B \leq c$  and  $\beta(1-s)\gamma S_G > c$ . These two inequalities imply that  $S_G > S_B$ . Free entry in state G is  $\frac{k}{\beta\mu q(\theta_G)} = (1-\gamma)S_G$ . Free entry in state B is  $\frac{k}{\beta\mu q(\theta_B)} = S_B - \frac{c}{\beta(1-s)} \leq (1-\gamma)S_B$ . Then, we have that  $S_B - \frac{c}{\beta(1-s)} \leq (1-\gamma)S_B < (1-\gamma)S_G$ . Free entry equations imply that  $\theta_G > \theta_B$ , which implies  $u_G < u_B$ . The result follows. ||

#### Proof of Proposition 4.

Consider a matching economy with two different levels of matching efficiency:  $\mu_G > \mu_B$ . Conjecture that the unemployment rate is lower in state G than in state B, which will turn out to be the case below. Suppose, by way of contradiction, that there are equilibria in which the NSC is binding in state B and it is not in state G. Then, we have that  $\beta(1-s)\gamma S_G > c$  and  $\beta(1-s)\gamma S_B \leq c$ . The two inequalities imply  $S_G > S_B$ . Then, free entry equation in state G implies that  $\frac{k}{\beta\mu_G q(\theta_G)} = (1-\gamma)S_G$ . Free entry in state B implies that  $k = \beta\mu_B q(\theta_B)[S_B - \frac{c}{\beta(1-s)}]$ . Note that  $S_j$  is decreasing in  $\theta_j$  for each  $j$ . Therefore,  $\theta_j$  satisfying each free entry equation in state  $j$  exists and it is unique. We have that  $S_B - \frac{c}{\beta(1-s)} \leq (1-\gamma)S_B < (1-\gamma)S_G$ . Then, free entry equations imply that  $\theta_G > \theta_B$ . Recall that  $S_j \equiv \frac{y - c - \frac{\gamma}{1-\gamma}k\theta_j}{1-\beta(1-s)}$ . Since  $\theta_G > \theta_B$ , we have that  $S_G < S_B$ . Contradiction. Therefore, there are no equilibria in which the NSC is binding in state B and it is not in state G. What remains to show is that  $u_G < u_B$  for whatever the case. There are three kinds of possibilities: (i) the NSC is binding in neither state; (ii) the NSC is binding in both states; and (iii) the NSC is binding in state G and it is not in state B. One can easily show that  $u_G < u_B$  for every case. Consider case (i) first. Free entry in state  $j \in \{B, G\}$  implies that  $\frac{k}{\beta\mu_j q(\theta_j)} = (1-\gamma)S_j$ .  $\mu_G > \mu_B$  implies  $\theta_G > \theta_B$ . Steady state accounting implies that  $u_G < u_B$ . Consider, next, the case (ii). Free entry in state  $j$  implies that  $\frac{k}{\beta\mu_j q(\theta_j)} = S_j - \frac{c}{\beta(1-s)}$ . This, with  $\mu_G > \mu_B$ , implies  $\theta_G > \theta_B$ . This implies  $u_G < u_B$ . Finally, consider the case (iii), in which  $\beta(1-s)\gamma S_B > c$  and  $\beta(1-s)\gamma S_G \leq c$ . The two inequalities imply  $S_G < S_B$ . The definition of  $S_j$  implies  $\theta_G > \theta_B$ . The steady state accounting with  $\mu_G > \mu_B$  implies that  $u_G < u_B$ . The result follows.

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