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Omitted Asymmetric Persistence and Conditional Heteroskedasticity

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Abstract

We show that asymmetric persistence induces ARCH effects, but the LM-ARCH test has power against it. On the other hand, the test for asymmetric dynamics proposed by Koenker and Xiao (2004) has correct size under the presence of ARCH errors. These results suggest that the LM-ARCH and the Koenker-Xiao tests may be used in applied research as complementary tools.

1 Introduction

It is widely acknowledged that many economic and financial time series display asymmetric persistence. In effect, Beaudry and Koop (1993) showed that positive shocks to US GDP are more persistent than negative shocks, indicating asymmetric business cycle dynamics. More recently, Nam et al. (2005) identified asymmetry in return dynamics for daily returns on the S&P 500 and used that to develop optimal technical trading strategies. However, much applied research is still conducted assuming implicitly the existence of symmetric dynamics, which may lead to model misspecification if dynamic asymmetry is indeed present. In this note, we show that a type of conditional heteroskedasticity arises when asymmetric persistence is ignored by the practitioner. Our Monte Carlo experiments suggest, however, that the LM-ARCH test has power against this asymmetric-persistence-induced-ARCH effect. We also investigate the presence of asymmetric persistence in financial time series and propose some new research that exploits this feature. This paper is organized as follows: Section 2 describes the theoretical model and shows how omitted asymmetric dynamics

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lead to conditional heteroskedasticity. In section 3, we explain how to test the null hypothesis of symmetric dynamics. Section 4 presents the Monte Carlo results and an empirical example. Section 5 concludes.

2 The Econometric Model

Koenker and Xiao (2004) introduced the so-called quantile autoregressive model (QAR model) which we briefly describe below.

Let $\{U_t\}$ be a sequence of i.i.d. standard uniform random variables, and consider the p^{th} order autoregressive process,

$$y_t = \theta_0(U_t) + \theta_1(U_t)y_{t-1} + ... + \theta_p(U_t)y_{t-p}, \quad (1)$$

where the $\theta_j^{\prime}s$ are unknown functions $[0,1] \rightarrow R$ that we want to estimate. We refer to this model as the $QAR(p)$ model.[1] The $QAR(p)$ model (1) can be reformulated in more conventional random coefficient notation as

$$y_t = \mu + \alpha_{1,t}y_{t-1} + ... + \alpha_{p,t}y_{t-p} + u_t, \quad (2)$$

where $\mu = E[\theta_0(U_t)]$, $u_t = \theta_0(U_t) - \mu$, and $\alpha_{j,t} = \theta_j(U_t)$, for $j = 1, ..., p$. Thus, $\{u_t\}$ is an i.i.d. sequence of random variables with distribution function $F(\cdot) = \theta_0^{-1}(\cdot + \mu)$, and the $\alpha_{j,t}$ coefficients are functions of this u_t innovation random variable. Now assume that $E(\alpha_{j,t}) = \alpha_j$ and re-write (2) as

$$y_t = \mu + (\alpha_{1,t} - \alpha_1 + \alpha_1)y_{t-1} + ... + (\alpha_{p,t} - \alpha_p + \alpha_p)y_{t-p} + u_t, \quad (3)$$

or

$$y_t = \mu + \alpha_1 y_{t-1} + ... + \alpha_p y_{t-p} + v_t, \quad (4)$$

with

$$v_t = (\alpha_{1,t} - \alpha_1)y_{t-1} + ... + (\alpha_{p,t} - \alpha_p)y_{t-p} + u_t. \quad (5)$$

Equations (4) and (5) show what happens when asymmetric persistence is not accounted for: a new form of conditional heteroskedasticity arises since $E[v_t^2|\mathcal{F}_{t-1}] \neq 0$, where $\mathcal{F}_t$ is the $\sigma - field$ generated by $\{y_s, s \leq t\}$.

3 Testing for Asymmetric Dynamics

Provided the right hand side of (1) is monotone increasing in U_t, it follows that the r^{th} quantile function of y_t can be written as

$$Q_y(\tau|y_{t-1}, ..., y_{t-p}) = \theta_0(\tau) + \theta_1(\tau)y_{t-1} + ... + \theta_p(\tau)y_{t-p}, \quad (6)$$

or somewhat more compactly as

$$Q_y(\tau|\mathcal{F}_{t-1}) = x_t^T\theta(\tau), \quad (7)$$

[1] More regularity conditions underlying model (1) are found in Koenker and Xiao (2004).

where $x_t = (1, y_{t-1}, ..., y_{t-p})^T$.

There will be symmetric persistence when the parameters $\theta_j(\tau)$, $j = 1, ..., p$, are constant over τ (i.e. $\theta_j(\tau) = \alpha_j$). This hypothesis can be represented as $H_0 : R\theta(\tau) = r$ by taking $R = \left[0_{px1} \vdots I_p\right]$ and $r = [\alpha_1, ..., \alpha_p]^T$, with unknown parameters $\alpha_1, ..., \alpha_p$. Koenker and Xiao (2004) propose to test H_0 using the following quantile process.

$$\widehat{V}_n(\tau) = \sqrt{n}\left[R\widehat{\Omega}_1^{-1}\widehat{\Omega}_0\widehat{\Omega}_1^{-1}R^T\right]^{-1/2}(R\widehat{\theta}(\tau) - \widehat{r}), \tag{8}$$

where $\widehat{\Omega}_1$ is the consistent estimator of $\Omega_1 = \lim n^{-1}\sum_{t=1}^{n} f_{t-1}\left[F_{t-1}^{-1}(\tau)\right] x_t x_t^T$ with F_{t-1} being the conditional distribution function with derivative $f_{t-1}(\cdot)$. Hence, $f_{t-1}\left[F_{t-1}^{-1}(\tau)\right]$ is the conditional quantile density function. $\widehat{\Omega}_0$ is the consistent estimator of $\Omega_0 = E(x_t x_t^T) = \lim n^{-1}\sum_{t=1}^{n} x_t x_t^T$. Finally, $\widehat{r} = [\widehat{\alpha}_1, ..., \widehat{\alpha}_p]^T$ where $\widehat{\alpha}_j$ is the least squares estimator of α_j, $j = 1, ..., p$, and $\widehat{\theta}(\tau)$ are the autoregression quantiles obtained solving the linear programing problem as in Koenker and Basset (1978).

Estimation of Ω_0 is straightforward: $\widehat{\Omega}_0 = n^{-1}\sum_{t=1}^{n} x_t x_t^T$. For the estimation of Ω_1, see Koenker and Machado (1999). Under H_0, Koenker and Xiao (2004) show that

$$\widehat{V}_n(\tau) \Rightarrow B_q(\tau) + O_p(1), \tag{9}$$

where " $\Rightarrow$ " signifies weak convergence and $B_q(\tau)$ represents a q-dimensional standard Brownian bridge. Thus, the necessity of estimating r introduces a drift component in addition to the simple Brownian bridge process. Hence, we can either accept the absence of the asymptotically-distribution-free nature of the test and use a resampling strategy to determine critical values or we can, following Koenker and Xiao (2002), apply the Khmaladze transformation to $\widehat{V}_n(\tau)$ in order to restore the asymptotically-distribution-free nature of inference.[2] We adopt the latter approach.

4 Monte Carlo Simulation

We consider the following DGP

$$y_t = \alpha_{1,t} y_{t-1} + u_t, \tag{10}$$

with the following specifications

(i) $u_t \sim i.i.d.\ N(0, 1)$ and $\alpha_{1,t} = 0.0$;

(ii) $u_t \sim i.i.d.\ N(0, 1)$ and $\alpha_{1,t} = -0.3$ if $u_t < 0$, and $\alpha_{1,t} = 0.3$ if $u_t \geq 0$;

(iii) $u_t = \sqrt{h_t}\epsilon_t$, $\epsilon_t \sim i.i.d.\ N(0, 1)$ with $h_t = 1 + 0.5\epsilon_{t-1}^2$ and $\alpha_{1,t} = 0.0$.

Thus, specification (i) assumes that there are no asymmetric dynamics and the process $\{y_t\}$ is simply a sequence of i.i.d. random variables with a Gaussian distribution. Asymmetric dynamics are present in the specification (ii), which

[2] For a complete discussion on Khmaladze transformation, see Koenker and Xiao (2002).

is just a special case of the equation (2) with $\mu = 0$, $p = 1$ and $E(\alpha_{1,t}) = 0$. Finally, specification (iii) corresponds to an ARCH(1) process.

We are interested in the size and power of the tests for asymmetric persistence and ARCH effects under the DGP and specifications above described. In order to implement the LM-ARCH test, we use least squares to estimate the auxiliary regression $y_t = \beta y_{t-1} + v_t$, and then we compute the test statistic nR^2 where, as usual, R^2 is the uncentered determination coefficient of the regression $\widehat{v}_t^2 = b_0 + b_1\widehat{v}_{t-1}^2 + ... + b_k\widehat{v}_{t-k}^2 + \eta_t$, where k is determined according the Schwarz criterion and, under the null hypothesis, $b_1 = ... = b_k = 0$.

In order to test for asymmetric dynamics, we implement the test described in Section 3. We consider $p = 1$, $p = 2$ and $p = 3$. The results are very similar for these lag choices and, therefore, we decide to report only the results for $p = 1$. Still, in order to implement such a test, one needs to estimate nuisance parameters. Estimation of nuisance parameters requires the choice of a bandwidth h_n. Following Koenker and Xiao (2002), we used $0.6 * h_n^B$, where h_n^B is the bandwidth choice proposed by Bofinger (1975).

We consider 10,000 replications and sample sizes n equal to 500, 1,000 and 4,000 observations.[3] Table 1 displays the Monte Carlo results. We first recall that the quantile regression is robust against fat-tail innovation distributions, which means that the test for asymmetric persistence should have good size under our third specification of the DGP.[4] On the other hand, we showed in Section 2 that omitted asymmetric persistence leads to ARCH effects. Therefore, we expect that the LM-ARCH test has power not only against the baseline form of ARCH, specification (iii), but also against the asymmetric-persistence-induced-ARCH effects.

Table 1. Size of 5% Tests

Specifications	**n = 500**	**n = 1000**	**n = 4000**
H_0 : **Symmetric Persistence**			
(i)	0.0352	0.0347	0.0359
(ii)	0.3379	0.7146	0.9997
(iii)	0.0336	0.0393	0.0452
H_0 : **No ARCH Effect**			
(i)	0.0439	0.0437	0.0490
(ii)	0.4323	0.7361	0.9998
(iii)	0.9955	1.0000	1.0000

Results in Table 1 confirm what we expected: the test for symmetric persistence is robust against the presence of the ARCH effect in the sense that it has empirical size close to nominal size under specification (iii). It also has empirical size close to nominal size under specification (i) and good power under specification (ii). On the other hand, the test for ARCH effects has the correct

[3] The R codes are available at www.fgv.br/aluno/bneri.

[4] It is well known that even if ϵ_t has a Gaussian distribution, the unconditional distribution of u_t in (10) is non-Gaussian with heavier tails than a Gaussian distribution (see Bollerslev, 1986, p. 313).

size for specification (i) and power under specifications (ii) and (iii). Therefore, the LM-ARCH test has the attractive feature of detecting not only the baseline form of ARCH, but also the ARCH effects induced by asymmetric persistence. As a simple empirical example, we investigate whether the daily return on S&P 500 index displays asymmetric persistence. The null hypothesis of symmetric dynamics is rejected at 5% (critical value is 2.140), as shown in Table 2, when 1 and 2 lags are used. This confirms previous findings of Nam et al. (2005) that reported the presence of asymmetric dynamic in return series.

Table 2. Test Statistics

Number of Observations	**1 Lag**	**2 Lags**
6000 (from 03-27-1981 to 12-31-2004)	2.248512	2.805515
5500 (from 03-20-1983 to 12-31-2004)	2.340302	2.643725
5000 (from 03-11-1985 to 12-31-2004)	2.471118	2.266379

5 Conclusion

The presence of dynamic asymmetry in time series leads to a special type of conditional heteroskedasticity that is not the baseline (textbook) case. We show, however, that this new form of ARCH can be detected by the LM-ARCH test. Additionally, the Koenker-Xiao test is specific to asymmetric persistence in the sense that it correctly rejects symmetric persistence when asymmetric persistence is present, but it does not reject the symmetric persistence when the (symmetric persistence) baseline form of ARCH is present. These results suggest that the LM-ARCH and the Koenker-Xiao tests may be used in applied research as complementary tools: if the null hypothesis of the LM-ARCH test is rejected, then one may apply the Koenker-Xiao test to determine whether such a rejection was caused by asymmetric-persistence-induced ARCH effects. If there is evidence of dynamic asymmetry, then the practitioner should account for it by using the model described in section 2.

We report evidence of asymmetric dynamics in the returns on S&P 500. New research that exploits the presence of asymmetric dynamics in economic and financial time series would be very fruitful. In particular, asymmetric persistence in return series might be used to improve measures of risk (such as Value-at-Risk) and develop optimal technical trading strategies.

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