## An incomplete contracts approach to financial contracting: a comment

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## Abstract

We show that the principal result of Aghion and Bolton (1992) related to the optimality properties of contingent control allocations under incomplete contracting environment holds only if an additional condition is satisfied.

I thank Juha-Pekka Niinimäki and Tuomas Takalo for perceptive comments.

Submitted: April 22, 2002. Accepted: May 28, 2002.

**Citation:** Vauhkonen, Jukka, (2002) "An incomplete contracts approach to financial contracting: a comment." *Economics Bulletin*, Vol. 7, No. 1 pp. 1–3

URL: http://www.economicsbulletin.com/2002/volume7/EB-02G30001A.pdf

In their seminal paper, Aghion and Bolton (1992) (hereafter AB) examine a long-term financial contracting problem between a wealth-constrained entrepreneur and a wealthy investor in an incomplete contracting environment. At date 0, the entrepreneur, who has no funds, seeks funding from the investor to cover the set-up cost K of an investment project. At date 2, the project yields monetary returns and unobservable and unverifiable private benefits for the entrepreneur. The sizes of the monetary returns and private benefits depend on the date 1 realization of the state of the world and the action taken at date 3/2. As the agents have potentially conflicting interests, they may prefer different actions. The potential conflict over the choice of action cannot always be solved by ex-ante contracts since, by assumption, contracts can be contingent only on publicly verifiable signals but not on the true state of the world or the action. Then, the right to decide what action to choose (control) becomes important. In this framework, AB show that different control allocations are efficient for different entry returns and private benefits.

In Proposition 5(iii) of their paper, AB argue that for some sufficiently high values of the set-up cost K, a contingent control allocation where control is allocated to the investor when the signal realization s = 0 and to the entrepreneur when s = 1 dominates unilateral control allocations (entrepreneur control and investor control). The purpose of this comment is to show that the contingent control allocation stated in their Proposition 5 dominates *investor control* only if the values of the monetary returns satisfy an additional condition. Furthermore, we show that this additional condition is not satisfied in their numerical example.

The model and notation are described in section II of AB. For further usage, we write out AB's expression for the investor's expected return under contingent control (AB's equation 6) and their Proposition 4. The investor's expected return  $p_c$  under contingent control allocation ( $a_0 = 0$ ;  $a_1 = 1$ ) with a transfer schedule t(s,r) = 0 for all s and r is:

$$\boldsymbol{p}_{c} = q \Big[ \boldsymbol{b}^{g} y_{g}^{g} + (1 - \boldsymbol{b}^{g}) y_{b}^{g} \Big] + (1 - q) \Big[ \boldsymbol{b}^{b} y_{g}^{b} + (1 - \boldsymbol{b}^{b}) y_{b}^{b} \Big].$$
(1)

Their Proposition 4 reads as follows:

**Proposition 1.** When monetary benefits are not comonotonic with total returns, a necessary and sufficient condition for the first-best action plan to be feasible under investor control is  $\mathbf{p}_4 \equiv (qy_b^g + (1-q)y_b^b)y_g^g / y_b^g \ge K$ .

It is important to note that one assumption underlying the definition of  $p_4$  is that the entrepreneur's share of the final monetary returns, determined in the initial contract, is set as a *constant*  $\bar{t} = 1 - y_g^g / y_b^g$ .

AB state that contingent control dominates investor control when the investor's expected returns under contingent control and investor control are determined by equation (1) and Proposition 1, respectively. More specifically, AB claim that if the set-up cost *K* belongs to the non-empty interval  $(\mathbf{p}_4, qy_g^g + (1-q)y_b^b)$ , a contingent control allocation  $(\mathbf{a}_0 = 0, \mathbf{a}_1 = 1)$  strictly dominates investor control when  $(\mathbf{b}^g, \mathbf{b}^b) \rightarrow (1, 0)$ .

Their reasoning goes as follows (see their p. 485). By Proposition 4, the first-best action plan is not implemented under investor control when  $K > p_4$ , since investor control implements the inefficient action  $a_b$  in state  $q_g$ . As a result, the aggregate payoffs under investor control are bounded away from the first-best aggregate payoffs. On the other hand, the aggregate payoffs under contingent control converge to the first-best aggregate payoffs as  $(\mathbf{b}^g, \mathbf{b}^b) \rightarrow (1,0)$ . This argumentation leads them to conclude that contingent control allocation strictly dominates investor control under the conditions stated in their Proposition 5.

However, their reasoning is not valid. To see this, consider an alternative investor control contract with a *signal-contingent* transfer schedule  $(t_0 = 0, t_1 = \bar{t})$ , that is, a contract giving the investor the right to choose the action and stipulating that the entrepreneur's share of the monetary returns  $t_0 = 0$  when the signal realization s = 0, and  $t_1 = \bar{t} = 1 - y_g^g / y_b^g$  when the signal realization s = 1. Correspondingly, the investor's share of the monetary returns is 1 when s = 0 and  $y_g^g / y_b^g$  when s = 1. Given this contract, the investor's expected (post-renegotiation) return under investor control is

$$\boldsymbol{p}_{4}' = q \Big[ \boldsymbol{b}^{g} y_{g}^{g} + (1 - \boldsymbol{b}^{g}) y_{b}^{g} \Big] + (1 - q) \Big[ \boldsymbol{b}^{b} y_{b}^{b} (y_{g}^{g} / y_{b}^{g}) + (1 - \boldsymbol{b}^{b}) y_{b}^{b} \Big].$$
(2)

Now, AB's reasoning is not valid, since  $p'_4$  also converges to the first-best when  $(b^g, b^b) \rightarrow (1, 0)$ . Therefore, to show that contingent control dominates investor control we must show that  $p_c - p'_4 > 0$  for some values of K. The difference  $p_c - p'_4$  is positive only if the monetary returns satisfy the following condition:

$$\Phi = y_g^b - y_b^b \left( y_g^g / y_b^g \right) > 0.$$
(3)

Thus, AB's Proposition 5(iii) is correct if and only if the condition  $\Phi > 0$  is satisfied. This condition is not necessarily satisfied in AB. For example, in their numerical example  $\Phi = -25$ , which implies that there are no values of K in their numerical example such that contingent control strictly dominates investor control.

## REFERENCE

Aghion, P., and P. Bolton (1992) "An incomplete contracts approach to financial contracting" *Review of Economic Studies* **59**, 473-494.