E C O N O M I C S   B U L L E T I N

On the finite−sample power of modified Dickey−Fuller tests: The role of the initial condition

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Abstract

The relationship between the initial condition of time series data and the power of the Dickey−Fuller (1979) test and a number of modified Dickey−Fuller tests is examined. The results obtained extend the asymptotic analysis of Muller and Elliott (2003) by both focussing upon finite−sample power and examining previously unconsidered modified tests. It is shown that deviation of the initial condition from the underlying deterministic component of a time series increases the finite−sample power of the original Dickey−Fuller test, but removes the potential gains in power resulting from the use of modified tests. Interestingly, some variation in the properties of modified tests is noted. In addition to allowing evaluation of previous Monte Carlo studies of the finite−sample power of unit root tests, the results presented allow practitioners to select, and interpret the results of, alternative unit root tests in light of the initial condition of the data examined.

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1 Introduction

Following the seminal research of Dickey and Fuller (1979), a large literature has emerged considering the testing of the unit root hypothesis. While some authors have explored issues such as robustness to structural change and serial correlation, arguably the most prominent feature of this literature has been the development of tests which increase the known low power of the Dickey-Fuller (DF) test in the presence of near-integrated time series. This has resulted in a range of modified DF tests, each of which has been found to exhibit substantially greater power than the original DF test. However, in recent research, Müller and Elliott (2003) have shown that for a series, say, $\{y_t\}_{t=0}^{T}$, the deviation of the initial observation (y_0) from the modelled deterministic component of the series is crucial in determining the asymptotic power of unit root tests. Denoting this deviation as ξ, it is shown that the asymptotic power of the original DF test is positively related to ξ due to the large weighting the test places upon extreme deviations of y_0 from the underlying deterministic component of the series. In contrast, the noted increased power of the weighted symmetric DF test of Park and Fuller (1995) (see Pantula *et al.*, 1994) is explained by the moderate weighting the test places upon ξ, while the asymptotic power of the GLS-based DF test of Elliot *et al.* (1996) is found to result from the imposition of $y_0 = 0$. The present paper adds to the asymptotic analysis of Müller and Elliott (2003) by considering the relationship between the initial condition of a time series process and the finite-sample power properties of these modified DF tests. In addition to examining the properties of the original DF test and the above modified tests using either weighted symmetric estimation or local-to-unity detrending via GLS, further modified DF tests unconsidered by Müller and Elliott (2003) are also analysed. The additional tests examined are the maximum DF test of Leybourne (1995) and the recursively mean-adjusted DF test of Shin and So (2001). In previous studies, increased finite-sample power has been noted for these easily applied tests. In the present analysis the relationship between this increased power and the initial condition is examined.

This paper proceeds as follows. In section [2] the Dickey-Fuller test and the modified Dickey-Fuller tests are presented. Section [3] contains the Monte Carlo experimental design employed and simulation results obtained. Section [4] provides some concluding remarks.

2 Alternative unit root tests

In this section the original DF test and the four modified DF tests to be considered are presented.

2.1 The Dickey-Fuller test

Given a series $\{y_t\}_{t=0}^{T}$, the familiar Dickey-Fuller (1979) τ_μ test examines the unit root hypothesis ($H_0 : \phi < 0$) via the t-ratio of $\widehat{\phi}$ in the following regression:[1]

$$\Delta y_t = \mu + \phi y_{t-1} + \varepsilon_t. \tag{1}$$

2.2 The GLS detrended Dickey-Fuller test

To increase the power of the τ_μ test, Elliott *et al.* (1996) propose local-to-unity detrending via GLS. The resulting test of the unit root hypothesis, denoted as τ_μ^{GLS}, is then given as the t-ratio of $\widehat{\phi}_0$ in the regression:

$$\Delta \widetilde{y}_t = \phi_0 \widetilde{y}_{t-1} + \varepsilon_t,$$

where $\widetilde{y}_t$ is the locally detrended version of y_t. The revised series $\widetilde{y}_t$ is derived as $y_t - \widehat{\delta}$, with $\widehat{\delta}$ obtained from the regression:

$$y_t^\alpha = \delta z_t^\alpha + \eta_t,$$

where:

$$y_t^\alpha = \{y_0, (1 - \overline{\alpha}L)\, y_1, (1 - \overline{\alpha}L)\, y_2, ..., (1 - \overline{\alpha}L)\, y_T\},$$

and

$$z_t^\alpha = \{z_0, (1 - \overline{\alpha}L)\, z_1, (1 - \overline{\alpha}L)\, z_2, ..., (1 - \overline{\alpha}L)\, z_T\}.$$

For the intercept only model employed here, $z_t = 1$. Elliott *et al.* (1996) find that $\overline{\alpha} = 1 - 7/T$ results in the asymptotic power of the τ_μ^{GLS} test lying close to the power envelope. This suggested value is employed here.

2.3 The weighted symmetric Dickey-Fuller test

The weighted symmetric DF test of Park and Fuller (1995) results from the application of a double length regression, with the weighted symmetric estimator of the autoregressive parameter, denoted as $\widehat{\rho}_{ws}$, given as the value minimising:

$$Q_{ws}(\rho) = \sum w_t (y_t - \rho y_{t-1})^2 + \sum (1 - w_t)(y_t - \rho y_{t+1})^2,$$

[1] In this paper unit root tests are considered in their 'with intercept' form, as the recursively mean-adjusted DF test is available in this form only.

where $w_t = (t-1)/T$. The unit root hypothesis is then tested using via the statistic τ_μ^{ws}:

$$\tau_\mu^{ws} = \sigma_{ws}^{-1} (\widehat{\rho}_{ws} - 1) \left(\sum_{t=2}^{T-1} y_t^2 + T^{-1} \sum_{t=1}^{T} y_t^2 \right)^{0.5},$$

where $\sigma_{ws}^2 = (T-2)^{-1} Q_{ws} (\widehat{\rho}_{ws})$.

2.4 The maximum Dickey-Fuller test

A further modification proposed to increase the power of the DF test is provided by the maximum DF test of Leybourne (1995) which requires the joint application of forward and reverse regressions. Given a series of interest $\{y_t\}_{t=0}^{T}$, the DF test of (1) is applied to both $\{y_t\}$ and $\{z_t\}$, where $z_t = y_{T-t}$ for $t = 0, ..., T$. The maximum DF test, denoted as τ_μ^{max}, is then simply the maximum (less negative) of the two test statistics obtained.

2.5 The recursive mean adjusted Dickey-Fuller test

The final modified DF test to be considered is the recursively mean-adjusted DF test of Shin and So (2001). As Shin and So (2001) note, the use of mean-adjusted observations $(y_t - \overline{y})$ in the following regression results in correlation between the regressor $(y_{t-1} - \overline{y})$ and the error (ϵ_t):

$$y_t - \overline{y} = \gamma (y_{t-1} - \overline{y}) + \epsilon_t.$$

The resulting bias of the ordinary least squares estimator $\widehat{\gamma}$ has been calculated by Tanaka (1984) and Shaman and Stine (1988) as:

$$\mathsf{E} (\widehat{\gamma} - \gamma) = -T^{-1} (1 + 3\rho) + o \left(T^{-1} \right).$$

Shin and So (2001) propose the use of recursively mean-adjusted observations to overcome this correlation, with the recursive mean $(\overline{y}_t)$ calculated as:

$$\overline{y}_t = t^{-1} \sum_{i=1}^{t} y_i.$$

The recursively mean-adjusted DF test, denoted as τ_μ^{rec}, is then given as the t-test of $\gamma_0 = 1$ in the following regression:

$$y_t - \overline{y}_{t-1} = \gamma_0 (y_{t-1} - \overline{y}_{t-1}) + \epsilon_t.$$

3 Monte Carlo experimentation

To examine the relationship between the initial condition and the power properties of τ_μ, τ_μ^{GLS}, τ_μ^{ws}, τ_μ^{max} and τ_μ^{rec} tests, the following data generation process (DGP) of (2) and (3) is used in the

Monte Carlo analysis:

$$y_t = \rho y_{t-1} + \eta_t, \qquad t = 1, ..., T, \tag{2}$$

$$y_0 = \xi. \tag{3}$$

Following Müller and Elliott (2003), the deviation ξ is given as a function of the unconditional variance of y_t. More precisely, $\xi = \lambda\sigma_y$, with values $\lambda = \{0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0\}$ considered for the simulation analysis. The innovation series $\{\eta_t\}$ in (3) is generated using pseudo *i.i.d.* $\mathsf{N}(0,1)$ random numbers from the RNDNS procedure in the GAUSS. All experiments are performed over 25,000 simulations with three sample size considered: $T = \{100, 250, 500\}$. To observe the power of the alternative tests, a range of near-integrated processes are generated using $\rho = 1 - \gamma T^{-1}$, where $\gamma = \{-10, -5, -2.5\}$. Empirical rejection frequencies for the τ_μ, τ_μ^{GLS}, τ_μ^{ws}, τ_μ^{max} and τ_μ^{rec} tests are calculated at the 5% level of significance.[2]

The empirical rejection frequencies of the alternative tests are reported in Tables One to Three. Considering the results for the smallest sample size ($T = 100$) contained in Table One, it is apparent that for each of the near-integrated series examined ($\gamma = -10, -5, -2.5$) the τ_μ^{GLS} test is noticeably more powerful than its rivals when $\lambda = 0$. In contrast, the original DF test (τ_μ) exhibits the lowest power of all tests for $\lambda = 0$. To illustrate this, consider the results for $\{\gamma, \lambda\} = \{-10, 0.0\}$ where the τ_μ^{GLS} test has a rejection frequency of 73.28% compared to 61.14% for the second most powerful test $\left(\tau_\mu^{ws}\right)$, while the τ_μ test has a rejection frequency of 31.14%. However, as λ, and hence the size of the deviation ξ, are increased, the power of the τ_μ^{GLS} test falls dramatically towards zero, while the power of the τ_μ test increases. The properties of the τ_μ^{ws}, τ_μ^{max} and τ_μ^{rec} are found to be very similar, both in terms of the power exhibited for $\lambda = 0$ and the subsequent decline in power as λ is increased. However, the behaviour of these tests is not as extreme as that of the τ_μ^{GLS} test, either in terms of the maximum power achieved nor the reduction in power as ξ is increased. Inspection of Tables Two and Three shows that a similar pattern of behaviour exists for larger sample sizes of $T = (250, 500)$. The results obtained therefore show that while modifications to the original DF test do result in substantial gains in power when the deviation of the initial condition from the deterministic component of a series is zero or very small, the finite-sample power of all of these tests is inversely related to the size of the deviation ξ. However, while the previously unconsidered τ_μ^{max} and τ_μ^{rec} tests display similar behaviour to the τ_μ^{ws} test, the point optimal τ_μ^{GLS} test exhibits more pronounced sensitivity to the deviation ξ. In contrast to the modified tests, the power of original DF test increases with the size of the deviation. As a result, for moderate or large values of ξ, the original DF test is more powerful than the proposed powerful modified tests.

[2] It should be noted that all of the tests considered are invariant to ξ under the null hypothesis of a unit root.

TABLES ONE TO THREE ABOUT HERE

4 Conclusion

In this paper the role of the initial condition in determining the power of unit root tests has been explored. The present research adds to the recent seminal research of Müller and Elliott (2003) in two ways. Firstly, the asymptotic analysis of Müller and Elliott (2003) has extended by focussing upon the empirical power of the original and modified Dickey-Fuller tests for the type of finite samples typically employed in empirical research. Secondly, additional modified Dickey-Fuller tests unconsidered by Müller and Elliott (2003) have been examined. The results of the present analysis have shown that while the power of the original Dickey-Fuller test increases for larger values of the deviation ξ, the increased power resulting from modified tests disappears. Indeed, for moderate or large values of ξ, the original test is the most powerful test available. However, while Dickey-Fuller tests modified via the use of weighted symmetric estimation, recursive mean adjustment and forward and reverse regression display similar behaviour, the GLS-based test of Elliott *et al.* (1996) is shown to be the modified Dickey-Fuller most sensitive to the initial condition. The results have obvious implications for the practitioner. In addition to allowing a more informed appraisal of previously published Monte Carlo analyses of the finite-sample power of modified tests (see, *inter alia*, Pantula *et al.* 1994, Leybourne 1995, Shin and So 2001), the results presented also allow an appropriate test to be selected, or the results of alternative tests to be interpreted, in light of knowledge of the properties of the initial condition of the series examined.

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Table One: The empirical powers of the alternative unit root tests ($T = 100$)*

γ	λ	τ_μ	τ_μ^{GLS}	τ_μ^{ws}	τ_μ^{max}	τ_μ^{rec}
−10	0.0	31.14	73.28	61.14	58.69	58.39
	0.5	32.02	64.61	58.95	57.53	57.12
	1.0	33.91	39.52	51.90	52.33	51.82
	1.5	36.22	13.14	39.63	40.39	42.42
	2.0	40.82	1.96	27.33	25.35	31.64
	2.5	45.94	0.13	15.57	11.51	19.93
	3.0	53.57	0.00	6.98	3.43	9.90
	3.5	61.11	0.00	2.47	0.77	3.67
	4.0	69.65	0.00	0.71	0.08	0.90
−5	0.0	11.54	30.95	26.03	24.88	25.04
	0.5	11.78	24.97	23.19	22.66	22.59
	1.0	12.38	13.70	17.66	17.67	18.08
	1.5	13.58	4.99	10.96	10.71	11.83
	2.0	15.41	0.98	5.24	4.86	6.15
	2.5	18.18	0.17	1.93	1.66	2.44
	3.0	21.78	0.02	0.66	0.56	0.90
	3.5	26.08	0.00	0.08	0.07	0.10
	4.0	32.24	0.00	0.02	0.01	0.02
−2.5	0.0	6.68	13.44	12.57	12.41	12.34
	0.5	7.18	12.24	12.00	11.80	11.80
	1.0	7.38	8.41	8.95	8.93	9.18
	1.5	7.52	4.26	5.50	5.69	5.77
	2.0	8.35	1.77	2.96	2.98	3.28
	2.5	9.02	0.56	1.20	1.24	1.33
	3.0	9.99	0.12	0.39	0.38	0.42
	3.5	11.76	0.02	0.10	0.15	0.13
	4.0	12.83	0.00	0.02	0.01	0.02

* The reported results represent empirical rejection frequencies of the unit root hypothesis, measured in percentage terms, for the alternative tests calculated using the DGP of (10)-(12) using a sample size of 100 observations and 25,000 simulations.

Table Two: The empirical powers of the alternative unit root tests ($T = 250$)*

γ	λ	τ_μ	τ_μ^{GLS}	τ_μ^{ws}	τ_μ^{max}	τ_μ^{rec}
−10	0.0	30.70	75.50	62.55	57.86	57.24
	0.5	30.82	62.32	59.53	56.48	55.32
	1.0	32.14	31.49	50.64	50.88	50.20
	1.5	35.46	7.43	37.32	40.07	42.23
	2.0	39.71	0.65	22.73	24.46	32.20
	2.5	45.18	0.04	10.27	10.59	20.57
	3.0	50.96	0.00	3.14	2.90	9.87
	3.5	58.60	0.00	0.56	0.44	3.81
	4.0	67.93	0.00	0.06	0.04	0.98
−5	0.0	11.75	31.88	26.96	25.33	24.66
	0.5	11.82	25.46	24.30	23.28	22.92
	1.0	12.11	13.02	17.47	17.81	18.03
	1.5	13.04	3.91	9.86	10.38	11.74
	2.0	15.34	0.81	4.50	5.13	6.74
	2.5	17.69	0.10	1.36	1.58	2.76
	3.0	20.90	0.00	0.26	0.43	0.73
	3.5	24.91	0.00	0.06	0.06	0.24
	4.0	30.49	0.00	0.00	0.01	0.02
−2.5	0.0	7.09	14.04	13.85	13.22	13.10
	0.5	7.36	12.48	12.34	11.84	12.00
	1.0	7.42	8.24	8.99	8.93	9.30
	1.5	7.50	4.05	5.56	5.74	5.88
	2.0	7.79	1.51	2.64	2.83	3.10
	2.5	8.71	0.36	0.95	1.10	1.35
	3.0	9.59	0.12	0.32	0.44	0.52
	3.5	11.35	0.02	0.10	0.15	0.17
	4.0	12.97	0.00	0.01	0.02	0.04

* The reported results represent empirical rejection frequencies of the unit root hypothesis, measured in percentage terms, for the alternative tests calculated using the DGP of (10)-(12) using a sample size of 250 observations and 25,000 simulations.

Table Three: The empirical powers of the alternative unit root tests ($T = 500$)*

γ	α	τ_μ	τ_μ^{GLS}	τ_μ^{ws}	τ_μ^{max}	τ_μ^{rec}
−10	0.0	30.81	78.51	65.15	58.16	56.68
	0.5	31.60	64.39	62.61	57.19	55.44
	1.0	33.12	31.09	53.68	51.86	50.60
	1.5	35.97	7.03	39.00	40.67	41.98
	2.0	39.96	0.56	22.94	24.83	31.69
	2.5	45.20	0.01	9.15	10.22	20.00
	3.0	51.73	0.00	2.39	2.76	10.17
	3.5	59.81	0.00	0.34	0.42	3.69
	4.0	68.12	0.00	0.03	0.06	0.90
−5	0.0	11.58	34.08	28.50	25.01	23.98
	0.5	11.70	27.12	25.66	22.99	22.32
	1.0	13.14	13.59	19.12	18.22	18.24
	1.5	13.80	4.18	10.14	10.52	11.66
	2.0	15.76	0.80	4.25	4.98	6.54
	2.5	17.50	0.08	1.11	1.53	2.56
	3.0	21.20	0.00	0.20	0.36	0.89
	3.5	25.91	0.00	0.04	0.09	0.24
	4.0	31.35	0.00	0.00	0.02	0.03
−2.5	0.0	7.18	15.27	14.16	12.74	12.48
	0.5	7.22	13.36	12.98	11.82	11.56
	1.0	7.54	8.76	9.55	8.93	8.94
	1.5	8.13	4.42	5.96	5.74	5.93
	2.0	8.31	1.60	2.73	2.82	3.07
	2.5	9.15	0.51	0.98	1.20	1.34
	3.0	9.84	0.07	0.32	0.44	0.54
	3.5	11.38	0.01	0.10	0.13	0.14
	4.0	13.28	0.00	0.02	0.01	0.04

* The reported results represent empirical rejection frequencies of the unit root hypothesis, measured in percentage terms, for the alternative tests calculated using the DGP of (10)-(12) using a sample size of 500 observations and 25,000 simulations.